Master of science-level in Mechanical Engineering
Academic Year 2019-2020, Second Semester

Automatic Control (05LSLQD, 05LSLNE)

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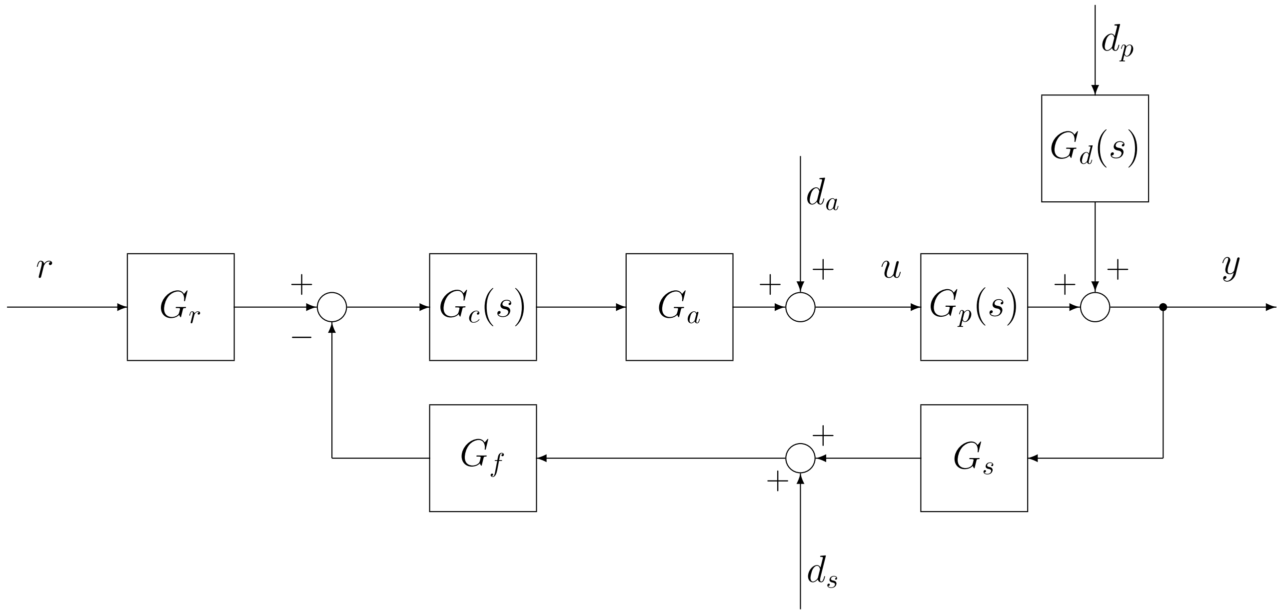
Homework n. 4

Main learning objectives

Upon successful completion of this homework, students will be able to analyze and translate steady-state requirements . More precisely, students will

1. Be able to design the feedback controller G_f from the steady-state gain of the feedback control system.
2. Be able to evaluate the number of poles of the cascade controller $G_c(s)$ at the origin and the minimum of the absolute value of the the cascade controller steady-state gain (K_c) from requirements on the steady-state output error in the presence of polynomial references .
3. Be able to evaluate the number of poles of the cascade controller $G_c(s)$ at the origin and the minimum of the absolute value of the the cascade controller steady-state gain (K_c) from requirements on the steady-state output error in the presence of polynomial disturbances .
4. Be able to derive the control system type from steady-state requirements on the output error in the presence of either polynomial references or polynomial disturbances .
5. Be able to derive constraints either on the sensitivity function S or on the complementary sensitivity function T from steady-state requirements on the output error in the presence of sinusoidal disturbances .

Consider the feedback control system below.



For problem $P1$ to problem $P8$, students are asked to analyze and translate steady-state requirements (specifications).

Problem 1 — Given

$$G_p(s) = \frac{25}{s^3 + 3.3s^2 + 2s}$$

$G_s = 1$
$G_a = 0.095$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ \mid D_{a0} \mid \leq 5.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad \mid a_p \mid \leq 2 \cdot 10^{-2},\ \ \omega_p \leq 0.02$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad \mid a_s \mid \leq 10^{-1},\ \ \omega_s \geq 40$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 1$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $\mid e_r^\infty \mid \leq 1.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $\mid e_{d_a}^\infty \mid \leq 1.5 \cdot 10^{-2}$
(S4) Steady-state output error in the presence of d_p: $\mid e_{d_p}^\infty \mid \leq 5 \cdot 10^{-4}$.
(S5) Steady-state output error in the presence of d_s: $\mid e_{d_s}^\infty \mid \leq 5 \cdot 10^{-4}$.
(S6) Rise time: $t_r \leq 3$ s
(S7) Settling time: $t_{s,\,5\%} \leq 12$ s
(S8) Step response overshoot: $\hat{s} \leq 10\%$

Problem 2 — Given

$$G_p(s) = \frac{40}{s^2 + 3s + 4.5}$$

$G_s = 1$
$G_a = -0.09$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ \mid D_{a0} \mid \leq 8.5 \cdot 10^{-3};$
$d_p(t) = D_{p0}t;\ \mid D_{p0} \mid \leq 3 \cdot 10^{-3};$
$d_s(t) = a_s \sin(\omega_s t), \quad \mid a_s \mid \leq 10^{-2},\ \ \omega_s \geq 50$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 1$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $\mid e_r^\infty \mid \leq 3.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $\mid e_{d_a}^\infty \mid \leq 1.75 \cdot 10^{-2}$
(S4) Steady-state output error in the presence of d_p: $\mid e_{d_p}^\infty \mid \leq 1 \cdot 10^{-3}$
(S5) Steady-state output error in the presence of d_s: $\mid e_{d_s}^\infty \mid \leq 2 \cdot 10^{-4}$.
(S6) Rise time: $t_r \leq 2.5$ s
(S7) Settling time: $t_{s,\,5\%} \leq 10$ s
(S8) Step response overshoot: $\hat{s} \leq 8\%$

Problem 3 — Given

$$G_p(s) = \frac{100}{s^2 + 5.5s + 4.5}$$

$G_s = 1$
$G_a = 0.014$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ | D_{a0} | \leq 1.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad | a_p | \leq 16 \cdot 10^{-2},\ \ \omega_p \leq 0.03$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad | a_s | \leq 2 \cdot 10^{-1},\ \ \omega_s \geq 60$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 1$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $| e_r^\infty | \leq 1.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $| e_{d_a}^\infty | \leq 4.5 \cdot 10^{-3}$
(S4) Steady-state output error in the presence of d_p: $| e_{d_p}^\infty | \leq 2 \cdot 10^{-3}$.
(S5) Steady-state output error in the presence of d_s: $| e_{d_s}^\infty | \leq 8 \cdot 10^{-4}$.
(S6) Rise time: $t_r \leq 2$ s
(S7) Settling time: $t_{s,\,5\%} \leq 8$ s
(S8) Step response overshoot: $\hat{s} \leq 12\%$

Problem 4 — Given

$$G_p(s) = \frac{-30}{s^3 + 3s^2 + 2s}$$

$G_s = 1$
$G_a = 0.006$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ | D_{a0} | \leq 2.5 \cdot 10^{-3};$
$d_p(t) = D_{p0} t;\ | D_{p0} | \leq 8.5 \cdot 10^{-3};$
$d_s(t) = a_s \sin(\omega_s t), \quad | a_s | \leq 5 \cdot 10^{-2},\ \ \omega_s \geq 40$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 1$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $| e_r^\infty | \leq 2.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $| e_{d_a}^\infty | \leq 1 \cdot 10^{-2}$
(S4) Steady-state output error in the presence of d_p: $| e_{d_p}^\infty | \leq 1.5 \cdot 10^{-3}$
(S5) Steady-state output error in the presence of d_s: $| e_{d_s}^\infty | \leq 5 \cdot 10^{-4}$.
(S6) Rise time: $t_r \leq 3.5$ s
(S7) Settling time: $t_{s,\,5\%} \leq 14$ s
(S8) Step response overshoot: $\hat{s} \leq 15\%$

Problem 5 — Given

$$G_p(s) = \frac{25}{s^3 + 3.3s^2 + 2s}$$

$G_s = 2$
$G_a = 0.38$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0}t;\ \mid D_{a0} \mid \leq 5.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad \mid a_p \mid \leq 2 \cdot 10^{-2},\ \ \omega_p \leq 0.02$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad \mid a_s \mid \leq 10^{-1},\ \ \omega_s \geq 40$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 4$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $\mid e_r^\infty \mid < 1.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $\mid e_{d_a}^\infty \mid < 5.8$
(S4) Steady-state output error in the presence of d_p: $\mid e_{d_p}^\infty \mid < 3.6 \cdot 10^{-4}$.
(S5) Steady-state output error in the presence of d_s: $\mid e_{d_s}^\infty \mid < 1.25 \cdot 10^{-4}$.
(S6) Rise time: $t_r < 2.5$ s
(S7) Settling time: $t_{s,\,5\%} < 5$ s
(S8) Step response overshoot: $\hat{s} < 12\%$

Problem 6 — Given

$$G_p(s) = \frac{40}{s^3 + 3s^2 + 4.5s}$$

$G_s = 3$
$G_a = -0.27$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ \mid D_{a0} \mid \leq 8.5 \cdot 10^{-3};$
$d_p(t) = D_{p0}t^2;\ \mid D_{p0} \mid \leq 3 \cdot 10^{-3};$
$d_s(t) = a_s \sin(\omega_s t), \quad \mid a_s \mid \leq 10^{-2},\ \ \omega_s \geq 50$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 3$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $\mid e_r^\infty \mid < 3.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $\mid e_{d_a}^\infty \mid < 1.75 \cdot 10^{-2}$
(S4) Steady-state output error in the presence of d_p: $\mid e_{d_p}^\infty \mid < 0.375$
(S5) Steady-state output error in the presence of d_s: $\mid e_{d_s}^\infty \mid < 3.3 \cdot 10^{-5}$.
(S6) Rise time: $t_r < 2.35$ s
(S7) Settling time: $t_{s,\,5\%} < 8$ s
(S8) Step response overshoot: $\hat{s} \leq 9\%$

Problem 7 — Given

$$G_p(s) = \frac{100}{s^3 + 5.5s^2 + 4.5s}$$

$G_s = 0.5$
$G_a = 0.112$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0}t;\ | D_{a0} | \leq 1.5 \cdot 10^{-3};$
$d_p(t) = a_p \sin(\omega_p t), \quad | a_p | \leq 16 \cdot 10^{-2},\ \ \omega_p \leq 0.03$ rad s^{-1}.
$d_s(t) = a_s \sin(\omega_s t), \quad | a_s | \leq 2 \cdot 10^{-1},\ \ \omega_s \geq 60$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 8$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $| e_r^\infty | < 1.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $| e_{d_a}^\infty | < 2.14$
(S4) Steady-state output error in the presence of d_p: $| e_{d_p}^\infty | < 5.1 \cdot 10^{-3}$.
(S5) Steady-state output error in the presence of d_s: $| e_{d_s}^\infty | < 1.6 \cdot 10^{-3}$.
(S6) Rise time: $t_r < 1.8$ s
(S7) Settling time: $t_{s,\,5\%} < 6$ s
(S8) Step response overshoot: $\hat{s} < 13\%$

Problem 8 — Given

$$G_p(s) = \frac{-30}{s^3 + 3s^2 + 2s}$$

$G_s = 10$
$G_a = 0.06$
$G_r = 1$
$G_d(s) = 1;$
$d_a(t) = D_{a0};\ | D_{a0} | \leq 2.5 \cdot 10^{-3};$
$d_p(t) = D_{p0}t^2;\ | D_{p0} | \leq 8.5 \cdot 10^{-3};$
$d_s(t) = a_s \sin(\omega_s t), \quad | a_s | \leq 5 \cdot 10^{-2},\ \ \omega_s \geq 40$ rad s^{-1}.

Specifications
(S1) Steady-state gain of the feedback control system: $K_d = 10$
(S2) Steady-state output error when the reference is a ramp $(R_0 = 1)$: $| e_r^\infty | < 2.5 \cdot 10^{-1}$
(S3) Steady-state output error in the presence of d_a: $| e_{d_a}^\infty | < 1 \cdot 10^{-2}$
(S4) Steady-state output error in the presence of d_p: $| e_{d_p}^\infty | < 0.94$
(S5) Steady-state output error in the presence of d_s: $| e_{d_s}^\infty | < 1.6 \cdot 10^{-5}$.
(S6) Rise time: $t_r < 2.5$ s
(S7) Settling time: $t_{s,\,5\%} < 13$ s
(S8) Step response overshoot: $\hat{s} < 14\%$

Some results of given problems

Problem P1

(S2) $| e_r^\infty | \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0, \quad | K_c | \geq 5.614$
(S3) $| e_{d_a}^\infty | \leq 1.5 \cdot 10^{-2} \Rightarrow \nu \geq 0, \quad | K_c | \geq 3.8596$
(S4) $| e_{d_p}^\infty | \leq 5 \cdot 10^{-4} \Rightarrow M_S^{LF} \approx -32$ dB, $\omega_c \geq 0.25$ rad/s.
(S5) $| e_{d_s}^\infty | \leq 5 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -46$ dB, $\omega_c \leq 1.4$ rad/s.

Problem P2

(S2) $| e_r^\infty | \leq 3.50 \cdot 10^{-1} \Rightarrow \nu \geq 1, \quad | K_c | \geq 3.5714$
(S3) $| e_{d_a}^\infty | \leq 1.75 \cdot 10^{-2} \Rightarrow \nu \geq 0$. Due to (S2), $\nu \geq 1 \Rightarrow | e_{d_a}^\infty | = 0$ and no constraints on $| K_c |$.
(S4) $| e_{d_p}^\infty | \leq 1.00 \cdot 10^{-3} \Rightarrow \nu \geq 1, \quad | K_c | \geq 3.75$
(S5) $| e_{d_s}^\infty | \leq 2 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -34$ dB, $\omega_c \leq 3.5$ rad/s.

Problem P3

(S2) $| e_r^\infty | \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 1, \quad | K_c | \geq 21.429$
(S3) $| e_{d_a}^\infty | \leq 4.5 \cdot 10^{-3} \Rightarrow \nu \geq 0$. Due to (S2), $\nu \geq 1 \Rightarrow | e_{d_a}^\infty | = 0$ and no constraints on $| K_c |$.
(S4) $| e_{d_p}^\infty | \leq 2 \cdot 10^{-3} \Rightarrow M_S^{LF} \approx -38$ dB, $\omega_c \geq 0.54$ rad/s.
(S5) $| e_{d_s}^\infty | \leq 8 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -48$ dB, $\omega_c \leq 1.9$ rad/s.

Problem P4

(S2) $| e_r^\infty | \leq 2.5 \cdot 10^{-1} \Rightarrow \nu \geq 0, \quad | K_c | \geq 44.4$
(S3) $| e_{d_a}^\infty | \leq 1.0 \cdot 10^{-2} \Rightarrow \nu \geq 0, \quad | K_c | \geq 41.6$
(S4) $| e_{d_p}^\infty | \leq 1.5 \cdot 10^{-3} \Rightarrow \nu \geq 0, \quad | K_c | \geq 62.963$
(S5) $| e_{d_s}^\infty | \leq 5 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -40$ dB, $\omega_c \leq 2$ rad/s.

Problem P5

(S2) $| e_r^\infty | \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$. Due to (S3), $\nu \geq 1 \Rightarrow | e_r^\infty | = 0$ and no constraints on $| K_c |$.
(S3) $| e_{d_a}^\infty | \leq 5.8 \quad \Rightarrow \nu \geq 1, \quad | K_c | \geq 0.01$
(S4) $| e_{d_p}^\infty | \leq 3.6 \cdot 10^{-4} \Rightarrow M_S^{LF} \approx -35$ dB, $\omega_c \geq 0.30$ rad/s.
(S5) $| e_{d_s}^\infty | \leq 1.25 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -52$ dB, $\omega_c \leq 1$ rad/s.

Problem P6

(S2) $| e_r^\infty | \leq 3.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$. Due to (S4), $\nu \geq 1 \Rightarrow | e_r^\infty | = 0$ and no constraints on $| K_c |$.
(S3) $| e_{d_a}^\infty | \leq 1.75 \cdot 10^{-2} \Rightarrow \nu \geq 0$. Due to (S4), $\nu \geq 1 \Rightarrow | e_{d_a}^\infty | = 0$ and no constraints on $| K_c |$.
(S4) $| e_{d_p}^\infty | \leq 0.375 \Rightarrow \nu \geq 1, \quad | K_c | \geq 0.01$
(S5) $| e_{d_s}^\infty | \leq 3.3 \cdot 10^{-5} \Rightarrow M_T^{HF} \approx -40$ dB, $\omega_c \leq 2.49$ rad/s.

Problem P7

(S2) $| e_r^\infty | \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$. Due to (S3), $\nu \geq 1 \Rightarrow | e_r^\infty | = 0$ and no constraints on $| K_c |$.
(S3) $| e_{d_a}^\infty | \leq 2.14 \quad \Rightarrow \nu \geq 1, \quad | K_c | \geq 0.0501$
(S4) $| e_{d_p}^\infty | \leq 5.1 \cdot 10^{-3} \Rightarrow M_S^{LF} \approx -30$ dB, $\omega_c \geq 0.34$ rad/s.
(S5) $| e_{d_s}^\infty | \leq 1.6 \cdot 10^{-3} \Rightarrow M_T^{HF} \approx -48$ dB, $\omega_c \leq 1.90$ rad/s.

Problem P8

(S2) $| e_r^\infty | \leq 2.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$. Due to (S4), $\nu \geq 1 \Rightarrow | e_r^\infty | = 0$ and no constraints on $| K_c |$.
(S3) $| e_{d_a}^\infty | \leq 1 \cdot 10^{-2} \Rightarrow \nu \geq 0$. Due to (S4), $\nu \geq 1 \Rightarrow | e_{d_a}^\infty | = 0$ and no constraints on $| K_c |$.
(S4) $| e_{d_p}^\infty | \leq 0.94 \Rightarrow \nu \geq 1, \quad | K_c | \geq 0.1005$
(S5) $| e_{d_s}^\infty | \leq 1.6 \cdot 10^{-5} \Rightarrow M_T^{HF} \approx -50$ dB, $\omega_c \leq 1.13$ rad/s.