

**Automatic Control (05LSLQD, 05LSLNE)**

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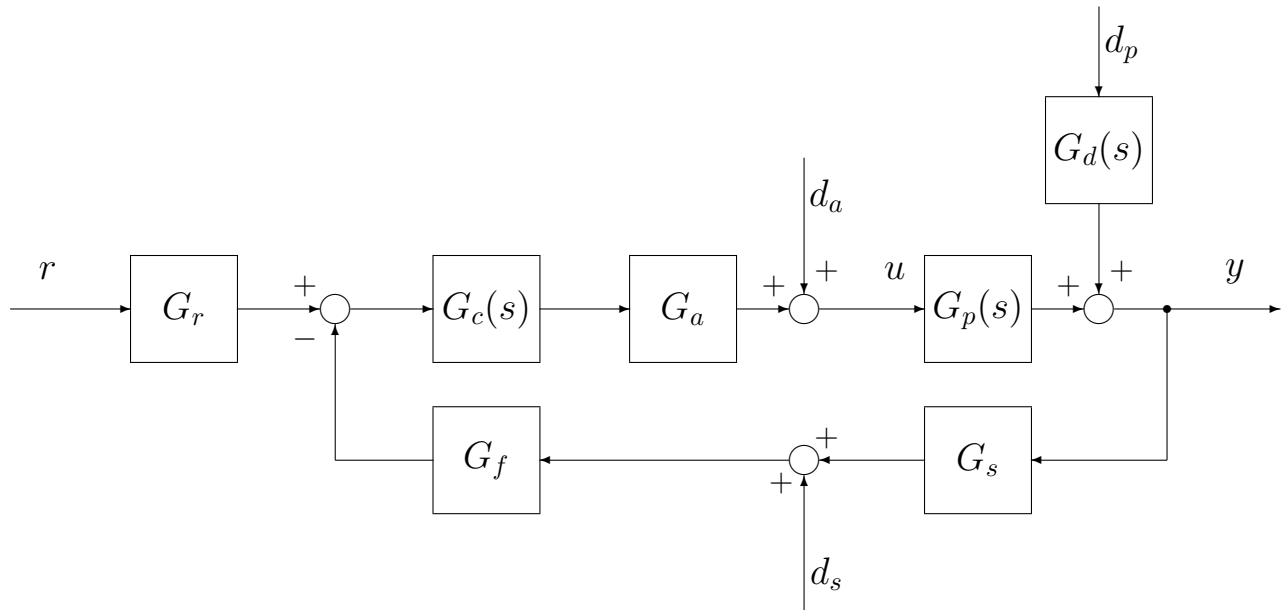
**Homework n. 5**

**Main learning objectives**

Upon successful completion of this homework, students will be able to **analyze and translate transient requirements** . More precisely, students will

1. Be able to valuate the **damping factor of the prototype second order system** from the required overshoot.
2. Be able to valuate the **sensitivity peak  $S_{po}$  and the complementary sensitivity peak  $T_{po}$  of the prototype second order system** from the required overshoot.
3. Be able to draw on the phase-magnitude plane, **the magnitude constant curves corresponding to  $S_{po}$  and  $T_{po}$**  respectively.
4. Be able to **draw the Nichols plot** of the frequency response of a given transfer function, together with the **magnitude constant curves** corresponding to  $S_{po}$  and  $T_{po}$  respectively.
5. Be able to evaluate **a lower bound of  $\omega_c$  (cross-over frequency)** from the required rise time and the settling time.

Consider the feedback control system below.



For problem  $P1$  to problem  $P8$ , students are asked to analyze and translate transient requirements (specifications).

**Problem 1**

— Given

$$G_p(s) = \frac{25}{s^3 + 3.3s^2 + 2s}$$

$$G_s = 1$$

$$G_a = 0.095$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}; \quad |D_{a0}| \leq 5.5 \cdot 10^{-3};$$

$$d_p(t) = a_p \sin(\omega_p t), \quad |a_p| \leq 2 \cdot 10^{-2}, \quad \omega_p \leq 0.02 \text{ rad s}^{-1}.$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 10^{-1}, \quad \omega_s \geq 40 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 1$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| \leq 1.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| \leq 1.5 \cdot 10^{-2}$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| \leq 5 \cdot 10^{-4}$ .(S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| \leq 5 \cdot 10^{-4}$ .(S6) Rise time:  $t_r \leq 3 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} \leq 12 \text{ s}$ (S8) Step response overshoot:  $\hat{s} \leq 10\%$ **Problem 2**

— Given

$$G_p(s) = \frac{40}{s^2 + 3s + 4.5}$$

$$G_s = 1$$

$$G_a = -0.09$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}; \quad |D_{a0}| \leq 8.5 \cdot 10^{-3};$$

$$d_p(t) = D_{p0}t; \quad |D_{p0}| \leq 3 \cdot 10^{-3};$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 10^{-2}, \quad \omega_s \geq 50 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 1$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| \leq 3.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| \leq 1.75 \cdot 10^{-2}$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| \leq 1 \cdot 10^{-3}$ (S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| \leq 2 \cdot 10^{-4}$ .(S6) Rise time:  $t_r \leq 2.5 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} \leq 10 \text{ s}$ (S8) Step response overshoot:  $\hat{s} \leq 8\%$

**Problem 3**

— Given

$$G_p(s) = \frac{100}{s^2 + 5.5s + 4.5}$$

$$G_s = 1$$

$$G_a = 0.014$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}; \quad |D_{a0}| \leq 1.5 \cdot 10^{-3};$$

$$d_p(t) = a_p \sin(\omega_p t), \quad |a_p| \leq 16 \cdot 10^{-2}, \quad \omega_p \leq 0.03 \text{ rad s}^{-1}.$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 2 \cdot 10^{-1}, \quad \omega_s \geq 60 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 1$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| \leq 1.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| \leq 4.5 \cdot 10^{-3}$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| \leq 2 \cdot 10^{-3}$ .(S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| \leq 8 \cdot 10^{-4}$ .(S6) Rise time:  $t_r \leq 2 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} \leq 8 \text{ s}$ (S8) Step response overshoot:  $\hat{s} \leq 12\%$ **Problem 4**

— Given

$$G_p(s) = \frac{-30}{s^3 + 3s^2 + 2s}$$

$$G_s = 1$$

$$G_a = 0.006$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}; \quad |D_{a0}| \leq 2.5 \cdot 10^{-3};$$

$$d_p(t) = D_{p0}t; \quad |D_{p0}| \leq 8.5 \cdot 10^{-3};$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 5 \cdot 10^{-2}, \quad \omega_s \geq 40 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 1$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| \leq 2.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| \leq 1 \cdot 10^{-2}$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| \leq 1.5 \cdot 10^{-3}$ (S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| \leq 5 \cdot 10^{-4}$ .(S6) Rise time:  $t_r \leq 3.5 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} \leq 14 \text{ s}$ (S8) Step response overshoot:  $\hat{s} \leq 15\%$

**Problem 5**

— Given

$$G_p(s) = \frac{25}{s^3 + 3.3s^2 + 2s}$$

$$G_s = 2$$

$$G_a = 0.38$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}t; \quad |D_{a0}| \leq 5.5 \cdot 10^{-3};$$

$$d_p(t) = a_p \sin(\omega_p t), \quad |a_p| \leq 2 \cdot 10^{-2}, \quad \omega_p \leq 0.02 \text{ rad s}^{-1}.$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 10^{-1}, \quad \omega_s \geq 40 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 4$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| < 1.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| < 5.8$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| < 3.6 \cdot 10^{-4}$ .(S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| < 1.25 \cdot 10^{-4}$ .(S6) Rise time:  $t_r < 2.5 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} < 5 \text{ s}$ (S8) Step response overshoot:  $\hat{s} < 12\%$ **Problem 6**

— Given

$$G_p(s) = \frac{40}{s^3 + 3s^2 + 4.5s}$$

$$G_s = 3$$

$$G_a = -0.27$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}; \quad |D_{a0}| \leq 8.5 \cdot 10^{-3};$$

$$d_p(t) = D_{p0}t^2; \quad |D_{p0}| \leq 3 \cdot 10^{-3};$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 10^{-2}, \quad \omega_s \geq 50 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 3$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| < 3.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| < 1.75 \cdot 10^{-2}$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| < 0.375$ (S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| < 3.3 \cdot 10^{-5}$ .(S6) Rise time:  $t_r < 2.35 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} < 8 \text{ s}$ (S8) Step response overshoot:  $\hat{s} \leq 9\%$

**Problem 7**

— Given

$$G_p(s) = \frac{100}{s^3 + 5.5s^2 + 4.5s}$$

$$G_s = 0.5$$

$$G_a = 0.112$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}t; \quad |D_{a0}| \leq 1.5 \cdot 10^{-3};$$

$$d_p(t) = a_p \sin(\omega_p t), \quad |a_p| \leq 16 \cdot 10^{-2}, \quad \omega_p \leq 0.03 \text{ rad s}^{-1}.$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 2 \cdot 10^{-1}, \quad \omega_s \geq 60 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 8$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| < 1.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| < 2.14$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| < 5.1 \cdot 10^{-3}$ .(S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| < 1.6 \cdot 10^{-3}$ .(S6) Rise time:  $t_r < 1.8 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} < 6 \text{ s}$ (S8) Step response overshoot:  $\hat{s} < 13\%$ **Problem 8**

— Given

$$G_p(s) = \frac{-30}{s^3 + 3s^2 + 2s}$$

$$G_s = 10$$

$$G_a = 0.06$$

$$G_r = 1$$

$$G_d(s) = 1;$$

$$d_a(t) = D_{a0}; \quad |D_{a0}| \leq 2.5 \cdot 10^{-3};$$

$$d_p(t) = D_{p0}t^2; \quad |D_{p0}| \leq 8.5 \cdot 10^{-3};$$

$$d_s(t) = a_s \sin(\omega_s t), \quad |a_s| \leq 5 \cdot 10^{-2}, \quad \omega_s \geq 40 \text{ rad s}^{-1}.$$

Specifications(S1) Steady-state gain of the feedback control system:  $K_d = 10$ (S2) Steady-state output error when the reference is a ramp ( $R_0 = 1$ ):  $|e_r^\infty| < 2.5 \cdot 10^{-1}$ (S3) Steady-state output error in the presence of  $d_a$ :  $|e_{d_a}^\infty| < 1 \cdot 10^{-2}$ (S4) Steady-state output error in the presence of  $d_p$ :  $|e_{d_p}^\infty| < 0.94$ (S5) Steady-state output error in the presence of  $d_s$ :  $|e_{d_s}^\infty| < 1.6 \cdot 10^{-5}$ .(S6) Rise time:  $t_r < 2.5 \text{ s}$ (S7) Settling time:  $t_{s, 5\%} < 13 \text{ s}$ (S8) Step response overshoot:  $\hat{s} < 14\%$

**Problem 9**

Draw the Nichols plot of the frequency response of the transfer functions listed below, together with the magnitude constant curves corresponding to  $S_{po}$  and  $T_{po}$ . To this end, students may want to consider values of  $S_{po}$  and  $T_{po}$  derived from overshoot requirements randomly generated in the range from 5% to 15%.

$$H(s) = \frac{1}{s(s+2)(s+4)} \quad (1)$$

$$H(s) = \frac{-0.1(1-2s)}{s(s+0.2)(1+s)} \quad (2)$$

$$H(s) = \frac{1}{s^2(s+3)} \quad (3)$$

$$H(s) = \frac{2(1+0.5s)}{(1+s)(1-s)^2} \quad (4)$$

$$H(s) = \frac{s^2+1}{(s-2)(s+2)(s+4)} \quad (5)$$

$$H(s) = \frac{0.125(1+s^2)}{s(1+0.25s)(1+0.5s)} \quad (6)$$

$$H(s) = \frac{s-2}{(s+2)(s^2+1)} \quad (7)$$

$$H(s) = \frac{0.25}{s(1-0.5s)^2} \quad (8)$$

$$H(s) = \frac{1}{(s^2+1)(s+2)} \quad (9)$$

$$H(s) = \frac{-(1-s)}{s(1+s^2)} \quad (10)$$

$$H(s) = \frac{s+1}{(s+2)(s^2+4s+5)} \quad (11)$$

$$H(s) = \frac{(s+3)(s^2+2s+2)}{s(s-1)(s+2)(s+4)} \quad (12)$$

$$H(s) = \frac{1}{2s^3} \quad (13)$$

$$H(s) = \frac{s^3+4s^2+7s+6}{s^4+5s^3+10s^2+11s+3} \quad (14)$$

$$H(s) = \frac{(s^2-1)}{s^3+s^2+s-3} \quad (15)$$

## Some results of given problems

### Problem P1

- (S2)  $|e_r^\infty| \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0, \quad |K_c| \geq 5.614$
- (S3)  $|e_{da}^\infty| \leq 1.5 \cdot 10^{-2} \Rightarrow \nu \geq 0, \quad |K_c| \geq 3.8596$
- (S4)  $|e_{dp}^\infty| \leq 5 \cdot 10^{-4} \Rightarrow M_S^{LF} \approx -32 \text{ dB}, \omega_c \geq 0.25 \text{ rad/s.}$
- (S5)  $|e_{ds}^\infty| \leq 5 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -46 \text{ dB}, \omega_c \leq 1.4 \text{ rad/s.}$
- (S6)  $t_r \leq 3 \Rightarrow \omega_c \geq 0.66 \text{ rad/s}$
- (S7)  $t_s \leq 12 \Rightarrow \omega_c \geq 0.31 \text{ rad/s}$
- (S8)  $\hat{s} \leq 10\% \Rightarrow \zeta \geq 0.59, \quad T_{po} \leq 1.049 = 0.41 \text{ dB}, \quad S_{po} \leq 1.361 = 2.7 \text{ dB}$

### Problem P2

- (S2)  $|e_r^\infty| \leq 3.50 \cdot 10^{-1} \Rightarrow \nu \geq 1, \quad |K_c| \geq 3.5714$
- (S3)  $|e_{da}^\infty| \leq 1.75 \cdot 10^{-2} \Rightarrow \nu \geq 0. \text{ Due to (S2), } \nu \geq 1 \Rightarrow |e_{da}^\infty| = 0 \text{ and no constraints on } |K_c|.$
- (S4)  $|e_{dp}^\infty| \leq 1.00 \cdot 10^{-3} \Rightarrow \nu \geq 1, \quad |K_c| \geq 3.75$
- (S5)  $|e_{ds}^\infty| \leq 2 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -34 \text{ dB}, \omega_c \leq 3.5 \text{ rad/s.}$
- (S6)  $t_r \leq 2.5 \Rightarrow \omega_c \geq 0.81 \text{ rad/s}$
- (S7)  $t_s \leq 10 \Rightarrow \omega_c \geq 0.33 \text{ rad/s}$
- (S8)  $\hat{s} \leq 8\% \Rightarrow \zeta \geq 0.63, \quad T_{po} \leq 1.024 = 0.21 \text{ dB}, \quad S_{po} \leq 1.33 = 2.5 \text{ dB}$

### Problem P3

- (S2)  $|e_r^\infty| \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 1, \quad |K_c| \geq 21.429$
- (S3)  $|e_{da}^\infty| \leq 4.5 \cdot 10^{-3} \Rightarrow \nu \geq 0. \text{ Due to (S2), } \nu \geq 1 \Rightarrow |e_{da}^\infty| = 0 \text{ and no constraints on } |K_c|.$
- (S4)  $|e_{dp}^\infty| \leq 2 \cdot 10^{-3} \Rightarrow M_S^{LF} \approx -38 \text{ dB}, \omega_c \geq 0.54 \text{ rad/s.}$
- (S5)  $|e_{ds}^\infty| \leq 8 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -48 \text{ dB}, \omega_c \leq 1.9 \text{ rad/s.}$
- (S6)  $t_r \leq 2 \Rightarrow \omega_c \geq 0.972 \text{ rad/s}$
- (S7)  $t_s \leq 8 \Rightarrow \omega_c \geq 0.498 \text{ rad/s}$
- (S8)  $\hat{s} \leq 12\% \Rightarrow \zeta \geq 0.56, \quad T_{po} \leq 1.078 = 0.65 \text{ dB}, \quad S_{po} \leq 1.39 = 2.9 \text{ dB}$

### Problem P4

- (S2)  $|e_r^\infty| \leq 2.5 \cdot 10^{-1} \Rightarrow \nu \geq 0, \quad |K_c| \geq 44.4$
- (S3)  $|e_{da}^\infty| \leq 1.0 \cdot 10^{-2} \Rightarrow \nu \geq 0, \quad |K_c| \geq 41.6$
- (S4)  $|e_{dp}^\infty| \leq 1.5 \cdot 10^{-3} \Rightarrow \nu \geq 0, \quad |K_c| \geq 62.963$
- (S5)  $|e_{ds}^\infty| \leq 5 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -40 \text{ dB}, \omega_c \leq 2 \text{ rad/s.}$
- (S6)  $t_r \leq 3.5 \Rightarrow \omega_c \geq 0.55 \text{ rad/s}$
- (S7)  $t_s \leq 14 \Rightarrow \omega_c \geq 0.32 \text{ rad/s}$
- (S8)  $\hat{s} \leq 15\% \Rightarrow \zeta \geq 0.52, \quad T_{po} \leq 1.13 = 1.1 \text{ dB}, \quad S_{po} \leq 1.45 = 3.2 \text{ dB}$



**Problem P5**

- (S2)  $|e_r^\infty| \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$ . Due to (S3),  $\nu \geq 1 \Rightarrow |e_r^\infty| = 0$  and no constraints on  $|K_c|$ .  
 (S3)  $|e_{da}^\infty| \leq 5.8 \Rightarrow \nu \geq 1, |K_c| \geq 0.01$   
 (S4)  $|e_{dp}^\infty| \leq 3.6 \cdot 10^{-4} \Rightarrow M_S^{LF} \approx -35 \text{ dB}, \omega_c \geq 0.30 \text{ rad/s}$ .  
 (S5)  $|e_{ds}^\infty| \leq 1.25 \cdot 10^{-4} \Rightarrow M_T^{HF} \approx -52 \text{ dB}, \omega_c \leq 1 \text{ rad/s}$ .  
 (S6)  $t_r \leq 2.5 \Rightarrow \omega_c \geq 0.78 \text{ rad/s}$   
 (S7)  $t_s \leq 5 \Rightarrow \omega_c \geq 0.80 \text{ rad/s}$   
 (S8)  $\hat{s} \leq 12\% \Rightarrow \zeta \geq 0.56, T_{po} \leq 1.078 = 0.65 \text{ dB}, S_{po} \leq 1.39 = 2.9 \text{ dB}$

**Problem P6**

- (S2)  $|e_r^\infty| \leq 3.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$ . Due to (S4),  $\nu \geq 1 \Rightarrow |e_r^\infty| = 0$  and no constraints on  $|K_c|$ .  
 (S3)  $|e_{da}^\infty| \leq 1.75 \cdot 10^{-2} \Rightarrow \nu \geq 0$ . Due to (S4),  $\nu \geq 1 \Rightarrow |e_{da}^\infty| = 0$  and no constraints on  $|K_c|$ .  
 (S4)  $|e_{dp}^\infty| \leq 0.375 \Rightarrow \nu \geq 1, |K_c| \geq 0.01$   
 (S5)  $|e_{ds}^\infty| \leq 3.3 \cdot 10^{-5} \Rightarrow M_T^{HF} \approx -40 \text{ dB}, \omega_c \leq 2.49 \text{ rad/s}$ .  
 (S6)  $t_r \leq 2.35 \Rightarrow \omega_c \geq 0.85 \text{ rad/s}$   
 (S7)  $t_s \leq 8 \Rightarrow \omega_c \geq 0.44 \text{ rad/s}$   
 (S8)  $\hat{s} \leq 9\% \Rightarrow \zeta \geq 0.61, T_{po} \leq 1.036 = 0.30 \text{ dB}, S_{po} \leq 1.35 = 2.58 \text{ dB}$

**Problem P7**

- (S2)  $|e_r^\infty| \leq 1.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$ . Due to (S3),  $\nu \geq 1 \Rightarrow |e_r^\infty| = 0$  and no constraints on  $|K_c|$ .  
 (S3)  $|e_{da}^\infty| \leq 2.14 \Rightarrow \nu \geq 1, |K_c| \geq 0.0501$   
 (S4)  $|e_{dp}^\infty| \leq 5.1 \cdot 10^{-3} \Rightarrow M_S^{LF} \approx -30 \text{ dB}, \omega_c \geq 0.34 \text{ rad/s}$ .  
 (S5)  $|e_{ds}^\infty| \leq 1.6 \cdot 10^{-3} \Rightarrow M_T^{HF} \approx -48 \text{ dB}, \omega_c \leq 1.90 \text{ rad/s}$ .  
 (S6)  $t_r \leq 1.8 \Rightarrow \omega_c \geq 1.07 \text{ rad/s}$   
 (S7)  $t_s \leq 6 \Rightarrow \omega_c \geq 0.80 \text{ rad/s}$   
 (S8)  $\hat{s} \leq 13\% \Rightarrow \zeta \geq 0.54, T_{po} \leq 1.095 = 0.79 \text{ dB}, S_{po} \leq 1.41 = 2.99 \text{ dB}$

**Problem P8**

- (S2)  $|e_r^\infty| \leq 2.5 \cdot 10^{-1} \Rightarrow \nu \geq 0$ . Due to (S4),  $\nu \geq 1 \Rightarrow |e_r^\infty| = 0$  and no constraints on  $|K_c|$ .  
 (S3)  $|e_{da}^\infty| \leq 1 \cdot 10^{-2} \Rightarrow \nu \geq 0$ . Due to (S4),  $\nu \geq 1 \Rightarrow |e_{da}^\infty| = 0$  and no constraints on  $|K_c|$ .  
 (S4)  $|e_{dp}^\infty| \leq 0.94 \Rightarrow \nu \geq 1, |K_c| \geq 0.1005$   
 (S5)  $|e_{ds}^\infty| \leq 1.6 \cdot 10^{-5} \Rightarrow M_T^{HF} \approx -50 \text{ dB}, \omega_c \leq 1.13 \text{ rad/s}$ .  
 (S6)  $t_r \leq 2.5 \Rightarrow \omega_c \geq 0.77 \text{ rad/s}$   
 (S7)  $t_s \leq 13 \Rightarrow \omega_c \geq 0.33 \text{ rad/s}$   
 (S8)  $\hat{s} \leq 14\% \Rightarrow \zeta \geq 0.53, T_{po} \leq 1.111 = 0.92 \text{ dB}, S_{po} \leq 1.43 = 3.09 \text{ dB}$

## Useful Matlab commands

Following is a list of commands which are useful for this homework. If you type `help control`, you get the complete list of commands included in the Control System Toolbox of Matlab. Use `help` in MATLAB for more information on how to use any of these commands.

- `help`: Matlab help documentation.
- `figure`: Create a new figure or redefine the current figure, see also `subplot`, `axis`.
- `hold`: Hold the current graph, see also `figure`.
- `axis`: Set the scale of the current plot, see also `plot`, `figure`.
- `plot`: Draw a plot, see also `figure`, `axis`, `subplot`.
- `xlabel/ylabel`: Add a label to the horizontal/vertical axis of the current plot, see also `title`, `text`, `gtext`.
- `title`: Add a title to the current plot.
- `text`: Add a piece of text to the current plot, see also `title`, `xlabel`, `ylabel`, `gtext`.
- `subplot`: Divide the plot window up into pieces, see also `plot`, `figure`.
- `abs`: returns the absolute value of of a complex number.
- `angle`: returns the phase angles, in radians, of a complex number.
- `squeeze`: Remove singleton dimensions.
- `bode`: Draw the Bode plot, see also `logspace`, `margin`, `nyquist1`.
- `nyquist`: Draw the Nyquist plot.
- `nyquist1`: Draw the Nyquist plot, see also `nyquist`. Note this command was written to replace the MATLAB standard command `nyquist` to get more accurate Nyquist plots.
- `grid`: Draw the grid lines on the current plot.
- `logspace`: Provides logarithmically spaced vector.
- `dcgain`: Computes the steady-state (D.C. or low frequency) gain of LTI models.
- `tf`: Creation of transfer functions or conversion to transfer function. `s = tf('s')` specifies the transfer function  $H(s) = s$  (Laplace variable).
- `zpk`: Create zero-pole-gain models or convert to zero-pole-gain format.
- `minreal`: Minimal realization and pole-zero cancellation.
- `tfdata`: Quick access to transfer function data. `[num,den] = tfdata(sys)` returns the numerator(s) and denominator(s) of the transfer function `sys`.
- **nichols** : Draws the Nichols plot of the frequency response of LTI models.
- **myngridst** : Draws the constant magnitude loci related to  $T_{po}$  (complementary sensitivity resonance peak) and  $S_{po}$  (sensitivity resonance peak) on the Nichols plane. This is not a native matlab command. This matlab function is provided by the teacher and should be copied in the working directory.