Master of science-level in Mechanical Engineering
Academic Year 2019-2020, Second Semester

Automatic Control (05LSLQD, 05LSLNE)

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Homework n. 1

Main learning objectives

Upon successful completion of this homework, students will

1. Be able to compute the state and output response of LTI systems.
2. Be able to evaluate and analyze the natural modes of LTI systems.
3. Be able to study the internal stability of LTI systems.
4. Be able to study the BIBO stability of LTI systems.

Problem 1

Find the zero state output response of a system with transfer function $H_1(s)$ below, in the presence of a unitary step input. Is the system BIBO stable?

$$H_1(s) = \frac{s+1}{0.5s^2 + 3.5s + 5}$$

Problem 2

Find the zero state output response of a system with transfer function $H_2(s)$ below, in the presence of the input $u(t) = e^{-2t}$. Is the system BIBO stable?

$$H_2(s) = \frac{1}{s^3 + 3s^2 + 2s}$$

Problem 3

Find the zero state output response of a system with transfer function $H_3(s)$ below, in the presence of the impulse input. Is the system BIBO stable?

$$H_3(s) = \frac{200(s+5)}{s^3 + 21s^2 + 220s + 200}$$

Problem 4

Given the following dynamical system:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= 8\,x_1(t) + 2\,x_2(t) + u(t) \\
y(t) &= 3\,x_1(t) + x_2(t)
\end{aligned}$$

find the output response in the presence of a step input with amplitude 7, ($u(t) = 7\varepsilon(t)$), and zero initial state. Is the system internally stable? Is the system BIBO stable?

Problem 5

Given the following linear system with 2 inputs (u_1, u_2) and two outputs (y_1, y_2):

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -10 & -6.5 \end{bmatrix} x(t) + \begin{bmatrix} 0 & -1 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} 1.6 & 1 \\ -1.6 & -1 \end{bmatrix} x(t)$$

Find the transfer function $G(s)$ from input u_2 to output y_2. Is the system internally stable? Is the system BIBO stable?

Problem 6

Given the continuous time LTI system:

$$\dot{x}(t) = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 5 \\ 8 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -1 & 3 \end{bmatrix} x(t) + 8u(t)$$

find the state and the output response, in the presence of a step input with amplitude 2, ($u(t) = 2\varepsilon(t)$), given the the initial state $x(0) = \begin{bmatrix} 2 & 2 \end{bmatrix}^T$. Is the system internally stable? Is the system BIBO stable?

Problem 7

Given the continuous time LTI system:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} x(t)$$

find the state and the output response, in the presence of a zero input, given the the initial state $x(0) = \begin{bmatrix} 2 & 5 \end{bmatrix}^T$. Is the system internally stable? Is the system BIBO stable?

Problem 8

Given the continuous time LTI system:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 10 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

find the output response, in the presence of a zero input $u_1(t)$, while the input $u_2(t)$ is an impulse with amplitude 2, and zero initial state. Is the system internally stable? Is the system BIBO stable?

Problem 9

Given a continuous time LTI system with:

$$A = \begin{bmatrix} -3 & -1 & 0.2 & 3 \\ 0 & -0.2 & 0.3 & 1 \\ 0 & 0 & 0.5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 10

Given a continuous time LTI system with:

$$A = \begin{bmatrix} 0.3 & 0.2 & 1 & 2 \\ -0.2 & 0.3 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.8 & -2.4 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 11

Given a continuous time LTI system with:

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -0.4 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 12

Given a continuous time LTI system with:

$$A = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -16 & -8 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 13

Given a system with the transfer function:

$$H_{13}(s) = \frac{1 - 2s}{(s^2 + 12s + 20)(s + 4)}$$

Is the system BIBO stable?.

Problem 14

Given a continuous time LTI system with:

$$A = \begin{bmatrix} -0.3 & 0 & 1 & 3 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 15

Given a continuous time LTI system with:

$$A = \begin{bmatrix} 0 & -0.01 & 0 & 0 \\ 1 & -0.1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 16

Given a continuous time LTI system with:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0.4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Find the system natural modes. Is the system internally stable?

Problem 17

Given the following dynamical system:

$$\dot{x}(t) = \begin{bmatrix} -1 & -0.25 & -0.25 \\ -0.5 & -0.75 & -0.25 \\ -0.5 & 0.25 & -1.25 \end{bmatrix} x(t) + \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 0 & 1.5 & -0.5 \end{bmatrix} x(t)$$

find the zero state and the zero input state and output response, given the the initial state $x(0) = \begin{bmatrix} 0.01 & 0.1 & 0.2 \end{bmatrix}^T$; the input $u(t)$ is a step with amplitude 2. Is the system internally stable? Is the system BIBO stable?

Results for some of the given problems

Problem 1

$$Y_1(s) = \frac{-0.5333}{s+5} + \frac{0.3333}{s+2} + \frac{0.2}{s} \tag{1}$$

$$y_1(t) = -0.5333e^{-5t} + 0.3333e^{-2t} + 0.2 \tag{2}$$

The system is BIBO stable.

Problem 2

$$Y_2(s) = \frac{0.75}{s+2} + \frac{0.5}{(s+2)^2} - \frac{1}{s+1} + \frac{0.25}{s} \tag{3}$$

$$y_2(t) = (0.75 + 0.5t)\, e^{-2t} - e^{-t} + 0.25 \tag{4}$$

The system is not BIBO stable.

Problem 3

$$Y_3(s) = -4.4198\frac{s+10}{(s+10)^2+10^2} + 16.022\frac{10}{(s+10)^2+10^2} + \frac{4.4199}{s+1} \tag{5}$$

$$y_3(t) = -4.4198e^{-10t}\cos(10t) + 16.022e^{-10t}\sin(10t) + 4.4199e^{-t} \tag{6}$$

The system is BIBO stable.

Problem 4

$y_4(t) = [2.0417 \cdot e^{4t} + 0.5833 \cdot e^{-2t} - 2.625]\, \varepsilon(t)$.

Problem 5

$G_5(s) = \dfrac{1.6(s+0.25)}{(s+2.5)(s+4)}$.

Problem 6

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 3.0\overline{6}e^{5t} - 0.\overline{6}e^{-t} - 0.4 \\ 6.1\overline{3}e^{5t} + 0.\overline{6}e^{-t} - 4.8 \end{bmatrix} \varepsilon(t)$$

$$y(t) = Cx(t) + Du(t) = \begin{bmatrix} -1 & 3 \end{bmatrix} \begin{bmatrix} 3.0\overline{6}e^{5t} - 0.\overline{6}e^{-t} - 0.4 \\ 6.1\overline{3}e^{5t} + 0.\overline{6}e^{-t} - 4.8 \end{bmatrix} \varepsilon(t) + 8 \cdot 2\varepsilon(t) =$$
$$= \left(15.\overline{3}e^{5t} + 2.\overline{6}e^{-t} + 2\right) \varepsilon(t)$$

Problem 7

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1.75e^{3t} + 0.25e^{-t} \\ 5.25e^{3t} - 0.25e^{-t} \end{bmatrix} \varepsilon(t)$$

$$y(t) = Cx(t) = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 1.75e^{3t} + 0.25e^{-t} \\ 5.25e^{3t} - 0.25e^{-t} \end{bmatrix} \varepsilon(t) = \left(0.5e^{-t} - 3.5e^{3t}\right) \varepsilon(t)$$

Problem 8

$$y(t) = \left[2.3094e^{-0.5t} \cos(0.866t - 1.5708)\right] \varepsilon(t)$$

Problem 9

The eigenvalues of matrix A are:

$$\begin{aligned} &\lambda_1 = -3 \rightarrow \mathbb{R}e(\lambda_1) < 0 \\ &\lambda_2 = -0.2 \rightarrow \mathbb{R}e(\lambda_2) < 0 \\ &\lambda_3 = 0.5 \rightarrow \mathbb{R}e(\lambda_3) > 0 \\ &\lambda_4 = 0 \rightarrow \mathbb{R}e(\lambda_4) = 0 \end{aligned}$$

with corresponding natural modes:

$$\begin{aligned} &\lambda_1 \rightarrow e^{-3t} \\ &\lambda_2 \rightarrow e^{-0.2t} \\ &\lambda_3 \rightarrow e^{0.5t} \\ &\lambda_4 \rightarrow e^{0t} = 1 \end{aligned}$$

Such modes are:

$e^{-3t} \rightarrow$ exponentially convergent

$e^{-0.2t} \rightarrow$ exponentially convergent

$e^{0.5t} \rightarrow$ exponentially divergent

$e^{0t} = 1 \rightarrow$ bounded mode

Problem 10
Matrix A is block triangular:

$$A = \begin{bmatrix} 0.3 & 0.2 & 1 & 2 \\ -0.2 & 0.3 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -0.8 & -2.4 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ \mathbf{0} & A_{22} \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} 0.3 & 0.2 \\ -0.2 & 0.3 \end{bmatrix}, A_{22} = \begin{bmatrix} 0 & 1 \\ -0.8 & -2.4 \end{bmatrix}$$

Eigenvalues of A are the same as those of blocks A_{11} and A_{22}.

Eigenvalues of A_{11}: $\lambda_{1,2} = 0.3 \pm 0.2j \rightarrow \mathbb{R}e(\lambda_{1,2}) > 0$

Eigenvalues of A_{22}: $\lambda_3 = -0.4 \rightarrow \mathbb{R}e(\lambda_3) < 0$ e $\lambda_4 = -2 \rightarrow \mathbb{R}e(\lambda_4) < 0$.

The corresponding modes are:

$$\begin{aligned} &\lambda_{1,2} \rightarrow e^{0.3t}\cos(0.2t), e^{0.3t}\sin(0.2t) \\ &\lambda_2 \rightarrow e^{-0.4t} \\ &\lambda_3 \rightarrow e^{-2t} \end{aligned}$$

$e^{0.3t}\cos(0.2t), e^{0.3t}\sin(0.2t) \rightarrow$ exponentially divergent modes

$e^{-0.4t} \rightarrow$ exponentially convergent mode

$e^{-2t} \rightarrow$ exponentially convergent mode

Problem 11

$$\begin{aligned} &\lambda_1 = -1 \rightarrow \mathbb{R}e(\lambda_1) < 0 \\ &\lambda_2 = -0.4 \rightarrow \mathbb{R}e(\lambda_2) < 0 \\ &\lambda_3 = -5 \rightarrow \mathbb{R}e(\lambda_3) < 0 \end{aligned}$$

The natural modes are:

$$\begin{aligned} &\lambda_1 \rightarrow e^{-t} \\ &\lambda_2 \rightarrow e^{-0.4t} \\ &\lambda_3 \rightarrow e^{-5t} \end{aligned}$$

$e^{-t} \rightarrow$ exponentially convergent mode

$e^{-0.4t} \rightarrow$ exponentially convergent mode

$e^{-5t} \rightarrow$ exponentially convergent mode

Problem 12

Matrix A is block diagonal:

$$A = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & -0.1 & 0 & 0 \\ 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} A_{11} & & \mathbf{0} \\ & A_{22} & \\ \mathbf{0} & & A_{33} \end{bmatrix},$$

$$A_{11} = [0.2], A_{22} = [-0.1], A_{33} = \begin{bmatrix} -4 & 1 \\ 0 & -4 \end{bmatrix}$$

Eigenvalues of A are the same as those of blocks A_{11}, A_{22} and A_{33}.

Eigenvalue of A_{11}: $\lambda_1 = 0.2 \rightarrow \mathbb{R}e(\lambda_1) > 0$
Eigenvalues of A_{22}: $\lambda_2 = -0.1 \rightarrow \mathbb{R}e(\lambda_2) < 0$.
Eigenvalue of A_{33}: $\lambda_3 = \lambda_4 = -4 \rightarrow \mathbb{R}e(\lambda_{3,4}) < 0$.

The natural modes are:

$\lambda_1 \rightarrow e^{0.2t}$
$\lambda_2 \rightarrow e^{-0.1t}$
$\lambda_{3,4} \rightarrow e^{-4t}, t \cdot e^{-4t}$

$e^{0.2t}$ $\rightarrow$ exponentially divergent mode

$e^{-0.1t}$ $\rightarrow$ exponentially convergent mode

$e^{-4t}, t \cdot e^{-4t}$ $\rightarrow$ exponentially convergent modes

Problem 13
The poles of the transfer function are:

$p_1 = -2 \rightarrow \mathbb{R}e(\lambda_1) < 0$
$p_2 = -10 \rightarrow \mathbb{R}e(\lambda_2) < 0$
$p_3 = -4 \rightarrow \mathbb{R}e(\lambda_3) < 0$

The system is BIBO stable

Problema 14
The system has two exponentially convergent modes and one oscillating (bounded) mode.

Problema 15
The system has 3 exponentially convergent modes and one exponentially divergent mode.

Problema 16
The system has 3 exponentially divergent mode.

Problem 17

$$
\begin{aligned}
X_{\ell 1}(s) &= \frac{0.01s^2 - 0.055s - 0.0525}{s^3 + 3s^2 + 2.75s + 0.75} = \frac{0.105}{s+1.5} - \frac{0.05}{s+1} - \frac{0.045}{s+0.5} \\
X_{\ell 2}(s) &= \frac{0.1s^2 + 0.17s + 0.0825}{s^3 + 3s^2 + 2.75s + 0.75} = \frac{0.105}{s+1.5} - \frac{0.05}{s+1} + \frac{0.045}{s+0.5} \\
X_{\ell 3}(s) &= \frac{0.2s^2 + 0.37s + 0.1575}{s^3 + 3s^2 + 2.75s + 0.75} = \frac{0.105}{s+1.5} + \frac{0.05}{s+1} + \frac{0.045}{s+0.5}
\end{aligned} \tag{7}
$$

$$
\begin{aligned}
x_{\ell 1}(t) &= 0.105e^{-1.5t} - 0.05e^{-t} - 0.045e^{-0.5t} \\
x_{\ell 2}(t) &= 0.105e^{-1.5t} - 0.05e^{-t} + 0.045e^{-0.5t} \\
x_{\ell 3}(t) &= 0.105e^{-1.5t} + 0.05e^{-t} + 0.045e^{-0.5t}
\end{aligned} \tag{8}
$$

$$
Y_\ell(s) = \frac{0.05s^2 + 0.07s + 0.045}{s^3 + 3s^2 + 2.75s + 0.75} = \frac{0.105}{s+1.5} - \frac{0.1}{s+1} + \frac{0.045}{s+0.5} \tag{9}
$$

$$
y_\ell(t) = 0.105e^{-1.5t} - 0.1e^{-t} + 0.045e^{-0.5t} \tag{10}
$$

$$
\begin{aligned}
X_{f1}(s) &= \frac{-4s^2 - 9s - 5.5}{s^4 + 3s^3 + 2.75s^2 + 0.75s} = \frac{1.3333}{s+1.5} - \frac{2}{s+1} + \frac{8}{s+0.5} - \frac{7.3333}{s} \\
X_{f2}(s) &= \frac{4s^2 + 11s + 6.5}{s^4 + 3s^3 + 2.75s^2 + 0.75s} = \frac{1.3333}{s+1.5} - \frac{2}{s+1} - \frac{8}{s+0.5} + \frac{8.6667}{s} \\
X_{f3}(s) &= \frac{3s + 3.5}{s^4 + 3s^3 + 2.75s^2 + 0.75s} = \frac{1.3333}{s+1.5} + \frac{2}{s+1} - \frac{8}{s+0.5} + \frac{4.6667}{s}
\end{aligned} \tag{11}
$$

$$
\begin{aligned}
x_{f1}(t) &= 1.3333e^{-1.5t} - 2e^{-t} + 8e^{-0.5t} - 7.3333 \\
x_{f2}(t) &= 1.3333e^{-1.5t} - 2e^{-t} - 8e^{-0.5t} + 8.6667 \\
x_{f3}(t) &= 1.3333e^{-1.5t} + 2e^{-t} - 8e^{-0.5t} + 4.6667
\end{aligned} \tag{12}
$$

$$
Y_f(s) = \frac{6s^2 + 15s + 8}{s^4 + 3s^3 + 2.75s^2 + 0.75s} = \frac{1.3333}{s+1.5} - \frac{4}{s+1} - \frac{8}{s+0.5} + \frac{10.6667}{s} \tag{13}
$$

$$
y_f(t) = 1.3333e^{-1.5t} - 4e^{-t} - 8e^{-0.5t} + 10.6667 \tag{14}
$$

The system is asymptotically stable. The system is BIBO stable.

Table of Laplace Transforms

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
$\epsilon(t)$	$\frac{1}{s}$	(1)
$e^{at}f(t)$	$F(s-a)$	(2)
$\epsilon(t-a)$	$\frac{e^{-as}}{s}$	(3)
$f(t-a)\epsilon(t-a)$	$e^{-as}F(s)$	(4)
$\delta(t)$	1	(5)
$\delta(t-t_0)$	e^{-st_0}	(6)
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	(7)
$f'(t)$	$sF(s) - f(0)$	(8)
$f^n(t)$	$s^n F(s) - s^{(n-1)} f(0) -$	
	$\cdots - f^{(n-1)}(0)$	(9)
$\int_0^t f(x)g(t-x)dx$	$F(s)G(s)$	(10)
$t^n \ (n = 0, 1, 2, \dots)$	$\frac{n!}{s^{n+1}}$	(11)
$t^x \ (x \geq -1 \in \mathbb{R})$	$\frac{\Gamma(x+1)}{s^{x+1}}$	(12)
$\sin \omega t$	$\frac{k}{s^2 + \omega^2}$	(13)
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	(14)
e^{at}	$\frac{1}{s-a}$	(15)
$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$	(16)
$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$	(17)
$\frac{e^{at} - e^{bt}}{a-b}$	$\frac{1}{(s-a)(s-b)}$	(18)

$f(t)$	$\mathcal{L}[f(t)] = F(s)$	
$\frac{ae^{at} - be^{bt}}{a-b}$	$\frac{s}{(s-a)(s-b)}$	(19)
te^{at}	$\frac{1}{(s-a)^2}$	(20)
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$	(21)
$e^{\sigma t} \sin \omega t$	$\frac{\omega}{(s-\sigma)^2 + \omega^2}$	(22)
$e^{\sigma t} \cos \omega t$	$\frac{s-\sigma}{(s-\sigma)^2 + \omega^2}$	(23)
$e^{\sigma t} \sinh \omega t$	$\frac{\omega}{(s-\sigma)^2 - \omega^2}$	(24)
$e^{\sigma t} \cosh \omega t$	$\frac{s-\sigma}{(s-\sigma)^2 - \omega^2}$	(25)
$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$	(26)
$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$	(27)
$t \sinh \omega t$	$\frac{2\omega s}{(s^2 - \omega^2)^2}$	(28)
$t \cosh \omega t$	$\frac{s^2 + \omega^2}{(s^2 - \omega^2)^2}$	(29)
$\frac{\sin \omega t}{t}$	$\arctan \frac{\omega}{s}$	(30)
$\frac{1}{\sqrt{\pi t}} e^{-a^2/4t}$	$\frac{e^{-a\sqrt{s}}}{\sqrt{s}}$	(31)
$\frac{a}{2\sqrt{\pi t^3}} e^{-a^2/4t}$	$e^{-a\sqrt{s}}$	(32)
$\mathrm{erfc}\left(\frac{a}{2\sqrt{t}}\right)$	$\frac{e^{-a\sqrt{s}}}{s}$	(33)

Useful Matlab commands

Following is a list of commands which are useful for this homework. If you type `help control`, you get the complete list of commands included in the Control System Toolbox of Matlab. Use `help` in MATLAB for more information on how to use any of these commands.

- `help`: Matlab help documentation.
- `abs`: returns the absolute value of of a complex number.
- `angle`: returns the phase angles, in radians, of a complex number.
- `squeeze`: Remove singleton dimensions.
- `eig`: Compute the eigenvalues and eigenvectors of a matrix.
- `roots`: Find the roots of a polynomial.
- `tf`: Creation of transfer functions or conversion to transfer function. `s = tf('s')` specifies the transfer function $H(s) = s$ (Laplace variable).
- `zpk`: Create zero-pole-gain models or convert to zero-pole-gain format.
- `eps`: Floating-point relative accuracy.
- `minreal(sys)`: Minimal realization and pole-zero cancellation.
- `minreal(sys,tol)`: further specifies the tolerance `tol` used for pole-zero cancellation or state dynamics elimination. The default value is `tol=sqrt(eps)` and increasing this tolerance forces additional cancellations.
- `inv`: Matrix inverse. `inv(X)` is the inverse of the square matrix X.
- eye: Identity matrix. `eye(N)` is the N-by-N identity matrix.
- `tfdata`: Quick access to transfer function data. `[num,den] = tfdata(sys)` returns the numerator(s) and denominator(s) of the transfer function sys.
- `residue`: Partial-fraction expansion (residues).