01OAIQD – Dynamic Design of Machines

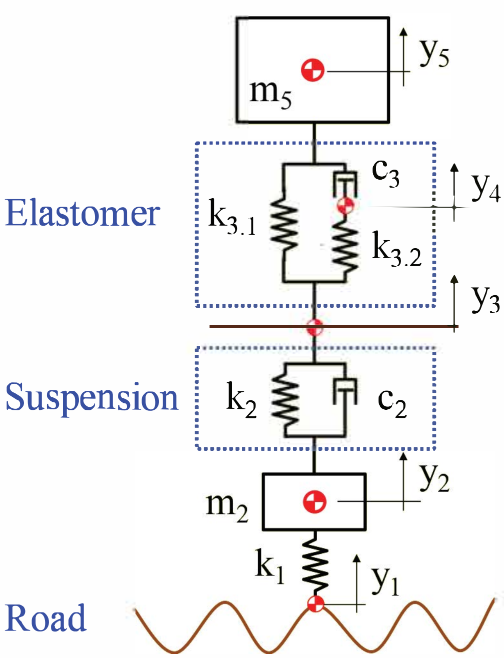
Academic year 2019-2020

Discrete linear systems

Emission date: November 22, 2019		**GROUP:**

EXERCISE 1) QUARTER CAR MODEL

Figure 1 shows the so-called quarter car model, one of the simplest models used to study the dynamic behavior of motor vehicle suspensions. The upper mass m_5 simulates the part of the mass of the car body (the sprung mass) that can be considered supported by a given wheel, while the lower mass m_2 simulates the wheel and all the parts that can be considered as rigidly connected with the unsprung mass. The two masses are connected by a spring-damper system simulating the suspension (k_2,c_2) and the silent block (elastometer, $k_{3.1}$, $k_{3.2}$,c_3). The unsprung mass is connected to the ground with a second spring simulating the radial stiffness of the tire. The point at which the tire contacts the ground is assumed to move in a vertical direction with a given law y_1(t), and it simulates the motion on uneven ground.



sprung mass	m_5 = 400 kg;
unsprung mass	m_2 = 30 kg;
spring stiffness	k_2 = 24 kN/m;
spring damping	c_2 = 1200 Ns/m;
elastomer stiffness	$k_{3.1}$ = 150 kN/m;
elastomer stiffness	$k_{3.2}$ = 90 kN/m;
elastomer damping	c_3 = 800 Ns/m;
tire stiffness	k_1 = 190 kN/m;

Figure 1

For the given quarter car model

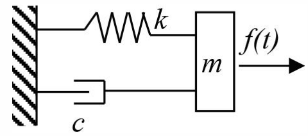
- determine the equations of dynamic equilibrium using the Lagrangian approach,
- arrange the equations in matrix form,
- find the natural frequencies and the mode shapes of the system,

- Consider as input a harmonic excitation with amplitude y_1 of 3 mm @ 5 Hz. Compute the power dissipated in the elastomeric member.

EXERCISE 2

Determine the unit impulse response of the mass-damper-spring system in figure. Plot the response and

- evaluate the first peaks of the response;
- calculate the damping ratio of the system

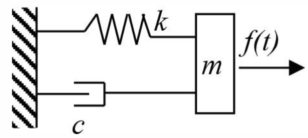


k = 100 N/m, m=2 kg, c = 2 N/m s.

EXERCISE 3

Calculate the unit step response *s*(*t*) of the mass-damper-spring system of exercise 1 integrating the impulse response. Plot *s*(*t*) versus t.

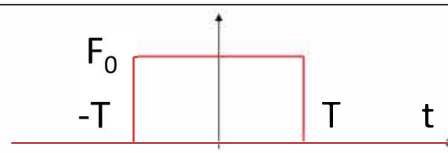
- Determine the response for the undamped system (c=0) and,
- for the damped system as given in the picture.
- Which is the response at infinite?



EXERCISE 4

Use the concept of unit step function and calculate the response of system in exercise 1 to the rectangular pulse shown in figure. Plot the first 8 peaks around –T and T.

F_0 = 50 N, T = 5 s

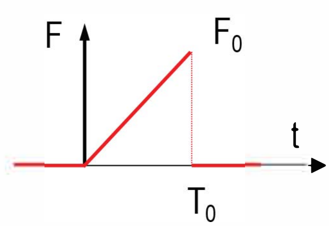


EXERCISE 5

Derive an expression for the response of the system in exercise 1 to a ramp force *F*(t) = $F_0 \cdot t$ in terms of the convolution integral. Consider both undamped and damped systems.

generic decaying oscillatory $\frac{Bs+C}{(s+a)^2+\omega_d^2}$ L^-1 $\rightarrow$ $e^{-at}\left(B\cos(\omega_d t)+\frac{C-aB}{\omega_d}\sin(\omega_d t)\right)$

F_0 = 10 N, T_0 = 5ms



EXERCISE 6

A single degree of freedom dynamic system (k = 2000 N/m, m = 0.3 kg, neglect the damping) is shocked by an acceleration having peak 4000 m/s² and duration 0.5 ms. Determine the equation of motion by using both the impulse response method and the Duhamel integral (assume a linear acceleration profile). Plot the displacements versus time and compare the results (at least in term of maximum amplitude).
Repeat the calculation when the same shock lasts for 20 ms. Explain the difference with the previous case.

MULTIPLE CHOICE PROBLEMS: select the right answer. Explain in few sentences (eventually with the support of graphs, schems and formulas) the reason of the correct answer.

1) The hysteretic damping model is applicable to
   [A] linear models subject to any type of excitation
   [B] only linear models subject to harmonic excitation
   [C] linear and non-linear models subject to harmonic or multi harmonic excitation

2) The Duhamel integral can be applied only
   [A] For linear systems,
   [B] with shock inputs

[C] if the system is undamped.

3) At resonance
[A] elastic forces balance exactly inertia forces;
[B] damping forces balance exactly inertia forces;
[C] elastic forces balance exactly damping forces.

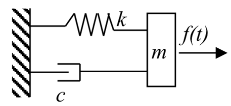
4) The dynamic compliance of a system with viscous damping
[A] is expressed by a complex number;
[B] tends to zero when the forcing function tends to zero;
[C] is always expressed by a real number.

5) The response of an undamped linear system at its resonant frequency
[A] is infinitely large;
[B] grows linearly in time to infinity;
[C] grows exponentially in time to infinity.

EXERCISE 2

Determine the unit impulse response of the mass-damper-spring system in figure.
Plot the response and

- evaluate the first peaks of the response;
- calculate the damping ratio of the system



k = 100 N/m, m=2 kg, c = 2 N/m s.

$$x(t) = e^{-\sigma t}\left\{ x(0)\cos(\omega_p \cdot t) + \frac{1}{\omega_p}\left(\dot{x}(0) + \sigma x(0)\right) \cdot \sin(\omega_p t)\right\}$$

$$\sigma = \xi \omega_n$$

$$\omega_p = \omega_n \sqrt{1-\xi^2}$$

$$\boxed{\xi = \frac{c}{c_{cr}} = \frac{c}{2\sqrt{k \cdot m}}} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

$$m(\underbrace{\dot{x}_{in}}_{0} - \dot{x}_{fin}) = \int_{-\varepsilon}^{+\varepsilon} f_0\, \delta(t)\, dt = f_0$$

$$m\dot{x}_{fin} = f_0 \Rightarrow \dot{x}_{fin} = \frac{f_0}{m}$$

$$\dot{x}_{fin} = \dot{x}(0) = \frac{\cancel{I_0}^{=1}}{m} = \frac{1}{m}$$

$$x(t) = e^{-\sigma t}\left\{ \underbrace{x(0)}_{=0}\cos(\omega_p \cdot t) + \frac{1}{\omega_p}\left(\dot{x}(0) + \sigma \underbrace{x(0)}_{=0}\right) \cdot \sin(\omega_p t)\right\}$$

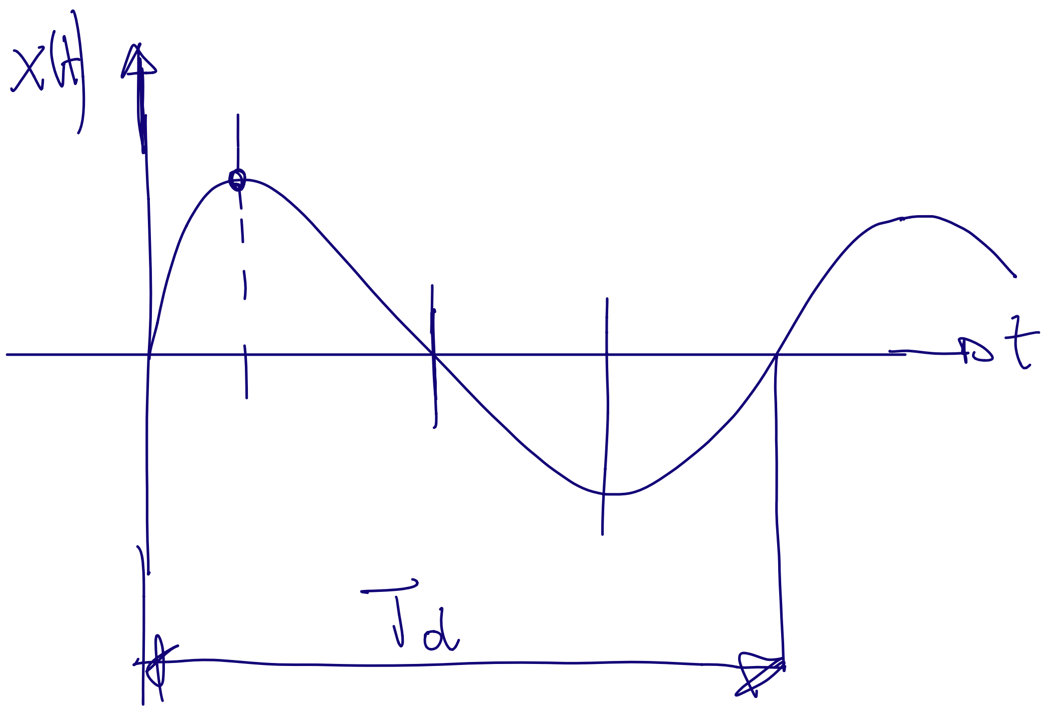
$$x(0) = 0$$

$$\dot{x}(0) = \frac{1}{m}$$

$$x(t) = e^{-\sigma t} \cdot \frac{1}{\omega_p}\dot{x}(0) \cdot \sin(\omega_p \cdot t)$$

$$\boxed{x(t) = e^{-\xi \omega_n t} \cdot \frac{1}{\omega_n\sqrt{1-\xi^2}} \frac{1}{m} \sin\left(\omega_n\sqrt{1-\xi^2} \cdot t\right)}$$

$$T_d = \frac{1}{\omega_p} = \frac{1}{\omega_n\sqrt{1-\xi^2}}$$

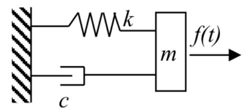


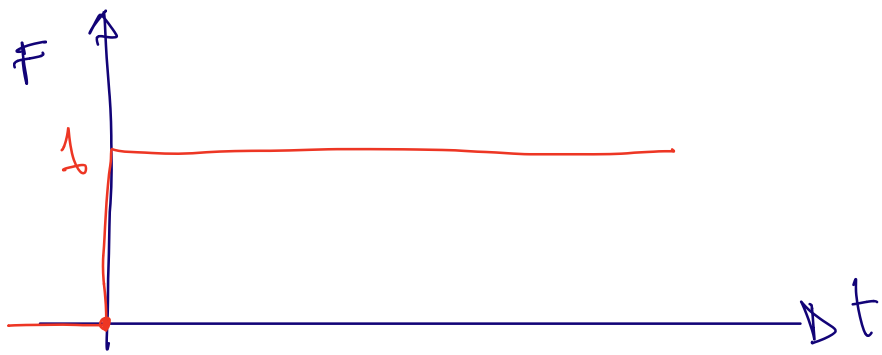
$$t = \frac{1}{4} T_d$$

EXERCISE 3

Calculate the unit step response $s(t)$ of the mass-damper-spring system of exercise 1 integrating the impulse response. Plot $s(t)$ versus t.

- Determine the response for the undamped system (c=0) and,
- for the damped system as given in the picture.
- Which is the response at infinite?





$$x(t) = e^{-\sigma t}\left[x_{01}\cos(\omega_p \cdot t) + x_{02}\sin(\omega_p \cdot t)\right] + \frac{f_0}{k}$$

$$x(0) = 0 \; ; \quad \dot{x}(0) = 0$$

$$x_{01} = -\frac{f_0}{k} \qquad x_{02} = \frac{f_0}{k}\,\frac{\xi}{\sqrt{1-\xi^2}}$$

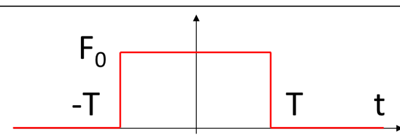
$$x(t) = \frac{f_0}{k}\, g(t) \quad \text{where}$$

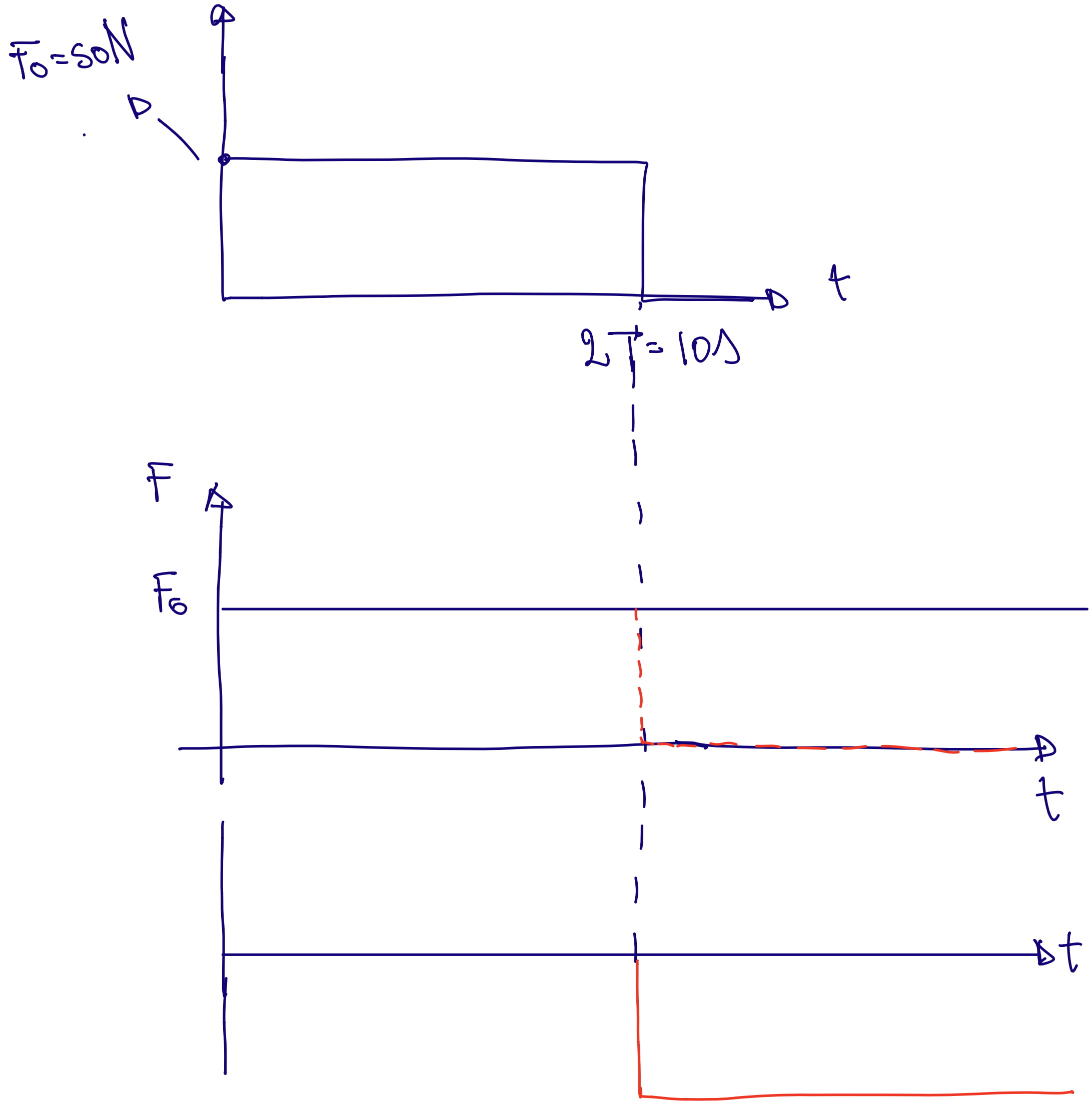
$$g(t) = 1 - e^{-\xi \omega_n t}\left[\cos\left(\omega_n \sqrt{1-\xi^2}\cdot t\right) + \frac{\xi}{\sqrt{1-\xi^2}} \sin\left(\omega_n \sqrt{1-\xi^2}\cdot t\right)\right]$$

$$t \leq 0 \qquad x(t) = 0$$

$$t > 0 \qquad x(t) =$$

EXERCISE 4

Use the concept of unit step function and calculate the response of system in exercise 1 to the rectangular pulse shown in figure. Plot the first 8 peaks around –T and T. F_0 = 50 N, T = 5 s	



$t \leq 0$

$0 < t \leq 10\,s$

$t > 10\,s$

$t \leq 0 \qquad x(t) = 0$

$0 < t \leq 10\,s$

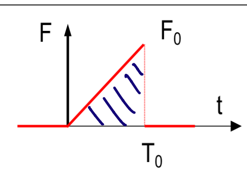
$$x(t) = \frac{F_0}{k}\, g_1(t)$$

$t \geq 10\,s$

$$\boxed{x(t) = \frac{F_0}{k}\, g_1(t) - \frac{F_0}{k}\, g_1(t-2T)}$$

EXERCISE 5

Derive an expression for the response of the system in exercise 1 to a ramp force $F(t) = F_0 \cdot t$ in terms of the convolution integral. Consider both undamped and damped systems.



F_0 = 10 N/s, T_0 = 5 ms

$$x(0) = 0$$

$$\dot{x}(0) = \frac{I_0}{m}$$

$$x(t) = e^{-\sigma t} \cdot \frac{1}{\omega_p} \dot{x}(0) \cdot \sin(\omega_p \cdot t)$$

$$I_0 = \frac{1}{2} F_0 T_0 = \frac{1}{2} 10 \frac{N}{\cancel{s}} 5 \cdot 10^{-3} \cancel{s} = 25 \, 10^{-3}$$

$$x(t) = e^{-\sigma t} \frac{1}{\omega_p} \frac{I_0}{m} \sin(\omega_p t)$$

DUHAMEL

$$x(t) = A_1(t) \sin(\omega_p \cdot t) - B_1(t) \cos(\omega_p \cdot t)$$

$$A_1(t) = \frac{1}{m \, \omega_p \cdot e^{-\xi \omega_m t}} \int_0^t \frac{F_0}{T_0} \cdot \tau \, e^{\xi \omega_m \tau} \cos(\omega_p \cdot \tau) d\tau$$

$$B_1(t) = \frac{1}{m\omega_p e^{-\xi\omega_n}} \int_0^t \frac{F_0}{T_0}\tau \, e^{\xi\omega_n\tau} \sin(\omega_p \tau)\, d\tau$$

$$F(\tau) = \frac{F_0}{T_0}\tau$$

when $t > T_0$

$$x(t) = A_1(T_0)\sin(\omega_p \cdot t) \; +$$

$$- B_1(T_0)\cos(\omega_p \cdot t)$$