01OAIQD – Dynamic Design of Machines

Academic year 2019-2020

EXERCISES:

EXERCISE 1

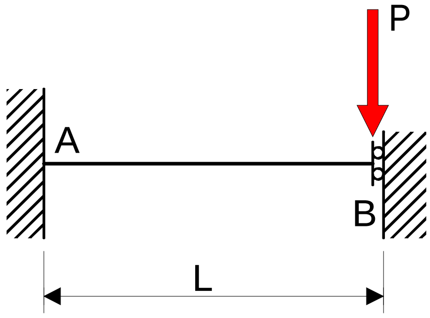


Figura 1

The beam sketched in figure 1, is constrained so that at the end A all the degrees of freedom are locked while at the end B it is allowed only the translation in the vertical direction. The stricture is loaded with a force P, lumped in the point B.
It is requested to

- assemble the stiffness matrix;
- compute the reaction forces.

Data

- Section: rectangular with base b = 10 mm and height h = 20 mm
- Material: iron steel, E = 210.000 MPa
- Length: L = 1 m
- Load: P = 100 kN

EXERCISE 2

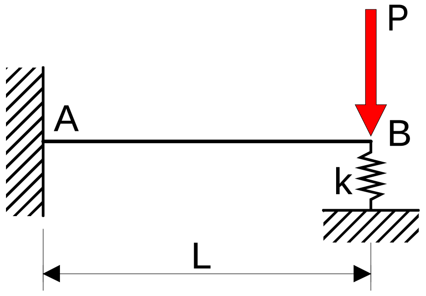


Figura 2

The beam sketched in figure 2, is constrained so that at the end A all the degrees of freedom are locked. The structure is loaded with a lumped vertical load applied in B.
It is requested to define the stiffness matrix and compute the reaction forces in A and the defection in B.

Data

- Section: rectangular with base b = 10 mm and height h = 20 mm
- Material: iron steel, E = 210.000 MPa
- Length: L = 1 m
- Load: P = 10 kN
- Stiffness of the elastic member: k = 10000 N/m

EXERCISE 3

Le structure in figures 5a, 5b, 5c similar from the geometrical point of view, have the same loads but different constraints. It is requested to solve the structures.

Data

Beam AC: base b = 15 mm; height h = 30 mm
Beam BC: base b = 10 mm; height h = 20 mm
Geometry: $\alpha = \beta = 45°$
Material: Iron Steel, E = 210.000 MPa
Length: L = 1 m
External load: P = 10000 N

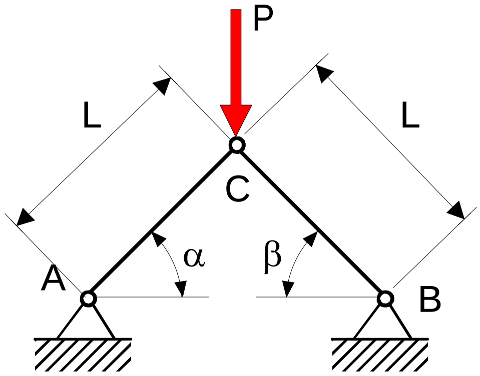


Figure 3a

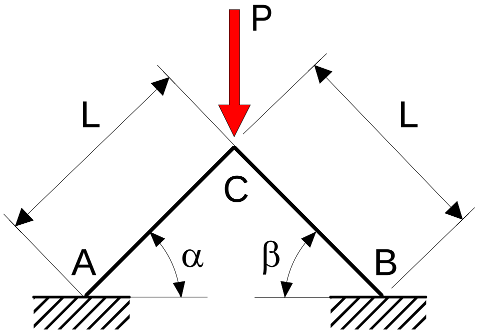


Figure 3c

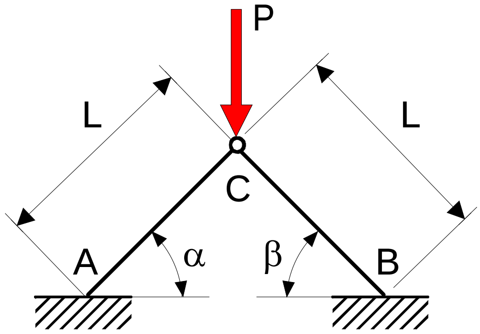


Figure 3b

EXERCISE 4

Compute the reaction forces of the structure in figure 6.

Data

- Section: rectangular with base b = 10 mm and height h = 20 mm
- Material: iron steel, E = 210.000 MPa
- Length L= 1 m

External load q = 10000 N/m

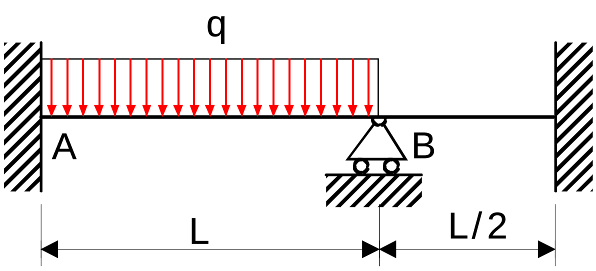


Figure 4

EXERCISE 5

Compute the reaction forces and the vertical displacement in point C.

Data

- beam AC: area A_1 = 3000 mm^2,
- moment of inertia = 3.5 10^7 mm^4 ,
- beam BC: area A_2 = 615 mm^2
- Geometry: α = 45°
- Material: iron steel, E = 210.000 MPa
- Length: L = 1 m
- External load: q = 570 N/mm

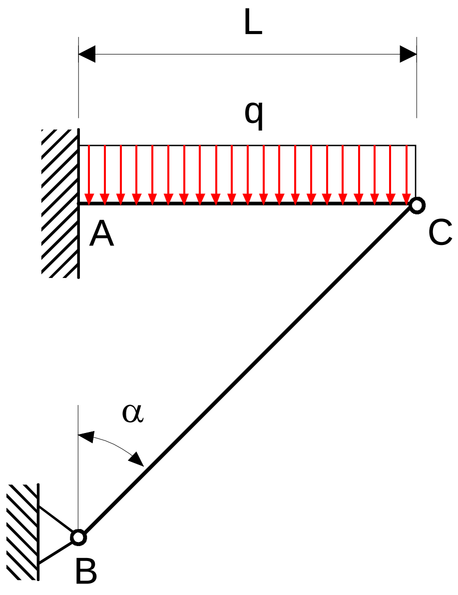


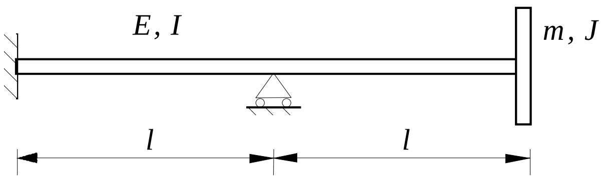
Figure 5

EXERCISE 6

Compute the natural frequencies and the corresponding mode shapes of the systems presented in the Figure. It is characterized by a beam with negligible inertia and a rigid body with mass m and moment of inertia J.

Data: $E = 2.0 \cdot 10^5$ N/mm^2, $l = 0.1$ m, $I = 4.91 \cdot 10^{-10}$ m^4, m = 5 kg, J = 0.05 kgm^2.

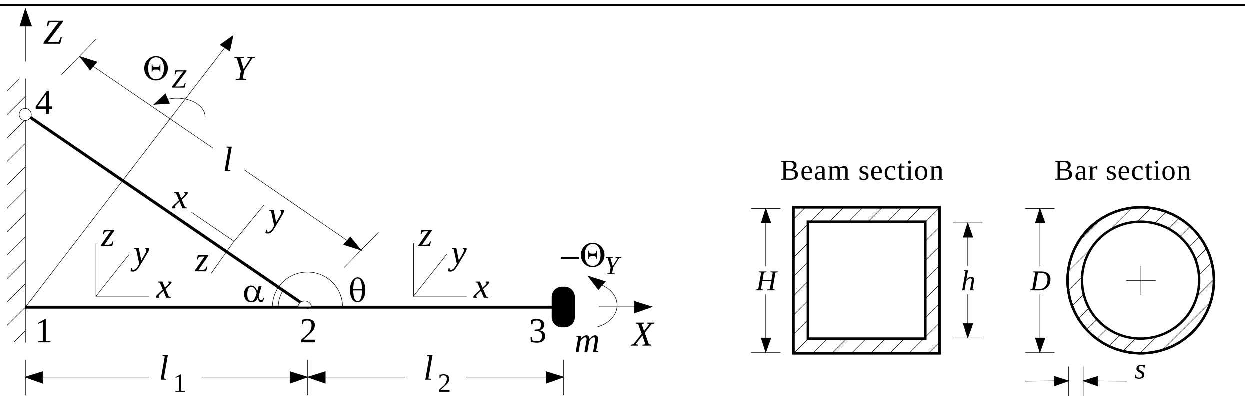
$$[K] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \qquad [K] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$



ω_1 = 106 rad/s, ω_2 = 452 rad/s.

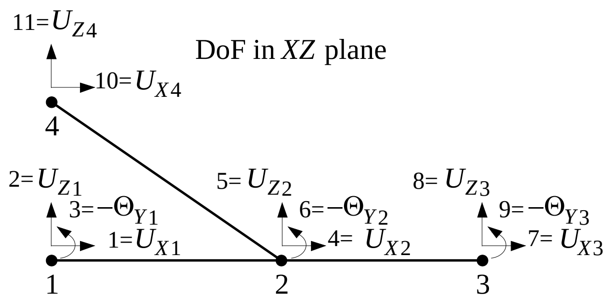
EXERCISE 7

Consider the dynamic system sketched in the figure. It consists of an arm (1-3) and a and a stiffening bar (2-4). Both are constrained on the left side. The arm (1-3) supports the mass *m* ant the right end. The material is aluminium alloy 2024 T6 (ASM 4112 standard).



Working scheme

Two beam elements (1–2 and 2–3) and one bar element (2–4), axial and flexural vibrations coupled.



System's data

Parameter	Symbol	SI units	Value
Gravity acceleration	g	m/s^2	9.81
Instrument mass	m	kg	15
Young's elastic modulus	E	N/m^2	$7.2 \cdot 10^{10}$
Ultimate strength	R_m	N/m^2	$328 \cdot 10^6$
Elastic strength	$R_{p0.2}$	N/m^2	$216 \cdot 10^6$
Fatigue limit	σ_{D-1}	N/m^2	$115 \cdot 10^6$
Beam length (element 1–2)	l_1	m	0.300
Beam length (element 2–3)	l_2	m	0.123
Beams external height	H	m	0.100
Beams internal height	h	m	0.094
Bar length (element 2–4)	$l = l_1/\cos\alpha$	m	$l = l_1/\cos(20°)$
Bar orientation	θ	degree	160
Bar external diameter	D	m	0.035
Bar radial thickness	s	m	0.001

Excitation

- motion of the constraints (nodes 1 and 4) with harmonic oscillation (simultaneous along *X*,*Y* and *Z* axes);
- acceleration of the constraints (nodes 1 and 4) along *Y* axis, due to a shock.

Harmonic oscillation simultaneous along *X* and *Z*	
Frequency λ_e (Hz)	Intensity
8.5	ampl. 10 mm
35	acc. 3g
50	acc. 1g

Table 1: typology of excitations

It is required to:

- Perform the dynamic analysis of the free behavior of the system;
- Perform the dynamic analysis of the forced behavior of the system (harmonic excitations as explained in table 1). It is suggested to plot the maximum displacement at the end of the arm.
- Make the resistance verification of the structure with respect of the MIL-STD 810C standard as summarized in table 2. It is suggested to plot the maximum stress in the frequency range.

Free response	Forced response	
	Harmonic oscillation simultaneous along *X* and *Z*	
λ_{n1}>max(λ_e)=50 Hz	max(U_{X3}, U_{Z3})≤3 mm $\sigma \le R_m/1.575$ $\sigma \le R_{p0.2}/1.155$ $\sigma \le \sigma_{D-1}$	

Table 2: Verification (MIL-STD 810C)

APPENDIX

Geometrical characteristics of the structure

The characteristics of the 2 beams are the same, since they differ only for their length:

$$A_T = H^2 - h^2 = 0.1^2 - 0.094^2 = 1.1640 \cdot 10^{-3} \text{ m}^2$$

$$I = \frac{H^4}{12} - \frac{h^4}{12} = \frac{1}{12}\left(0.1^4 - 0.094^4\right) = 1.8271 \cdot 10^{-6} \text{ m}^4$$

The characteristics of the bar are:

$$l = l_1 / \cos\alpha = 0.3/\cos(20) = 3.1925 \cdot 10^{-1} \text{ m}$$

$$A_A = \frac{\pi}{4}\left(D^2 - (D-2s)^2\right) = \frac{\pi}{4}\left(0.035^2 - 0.033^2\right) = 1.0681 \cdot 10^{-4} \text{ m}^2$$

Elements' stiffness matrix

Beams 1-2 and 2-3 in the *XZ* plane

$$[k_{XZ}]^{1-2}_{2-3} = E\begin{bmatrix} A_T/l_{1,2} & 0 & 0 & -A_T/l_{1,2} & 0 & 0 \\ 0 & 12\,I/l_{1,2}^3 & 6\,I/l_{1,2}^2 & 0 & -12\,I/l_{1,2}^3 & 6\,I/l_{1,2}^2 \\ 0 & 6\,I/l_{1,2}^2 & 4\,I/l_{1,2} & 0 & -6\,I/l_{1,2}^2 & 2\,I/l_{1,2} \\ -A_T/l_{1,2} & 0 & 0 & A_T/l_{1,2} & 0 & 0 \\ 0 & -12\,I/l_{1,2}^3 & -6\,I/l_{1,2}^2 & 0 & 12\,I/l_{1,2}^3 & -6\,I/l_{1,2}^2 \\ 0 & 6\,I/l_{1,2}^2 & 2\,I/l_{1,2} & 0 & -6\,I/l_{1,2}^2 & 4\,I/l_{1,2} \end{bmatrix}$$

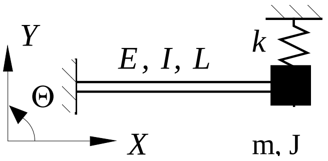
Bar 2-4 in the *XZ* plane

$$[k_{XZ}]_{2-4} = \frac{EA}{l}\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta & -\cos^2\theta & -\cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & -\cos\theta\sin\theta & -\sin^2\theta \\ -\cos^2\theta & -\cos\theta\sin\theta & \cos^2\theta & \cos\theta\sin\theta \\ -\cos\theta\sin\theta & -\sin^2\theta & \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$$

EXERCISE 8

Compute the first bending natural frequency of the system represented in the figure. The inertia contribution of the beam having a lenght L is negligible.

Data: ($E = 2{\cdot}10^5$ MPa, $L = 1000$ mm, $I = 7.5{\cdot}10^5$ mm^4, $k = 100$ N/mm, $m = 20\,[1+(U+2P)/30]$ kg, J=0.2 kg·m^2).



$$[K] = \begin{bmatrix} 12EI/L^3 & 6EI/L^2 & -12EI/L^3 & 6EI/L^2 \\ 6EI/L^2 & 4EI/L & -6EI/L^2 & 2EI/L \\ -12EI/L^3 & -6EI/L^2 & 12EI/L^3 & -6EI/L^2 \\ 6EI/L^2 & 2EI/L & -6EI/L^2 & 4EI/L \end{bmatrix}$$

ω_1 = ...21.39...... Hz