01OAIQD – Dynamic Design of Machines

Academic year 2019-2020

Discrete linear systems

Emission date: October 18, 2019		GROUP:

EXERCISE 1

An undamped system (presented in Figure 1) is made with two torsion bars (negligible inertia) and two flywheels. One of the flywheels (number 2) is loaded with a harmonic excitation $M(\mathrm{t}) = M_0 \sin(\omega t)$. Analyze the dynamic behavior of the system by computing:

a) natural frequencies and corresponding mode shapes;
b) potential and kinetic energy associated to the different subsystems for each mode.
c) maximum shear stresses in the torsion bars using the configuration space and the state space approach.

Data: $G = 7.7 \cdot 10^{10}$ N/m^2, $l_\mathrm{I} = 0.5$ m, $l_\mathrm{II} = 1.0$ m, $d_\mathrm{I} = 12.8$ mm, $d_\mathrm{II} = 14.1$ mm, $J_{p1} = 6$ kgm^2, $J_{p2} = 4$ kgm^2, $M_0 = 100$ Nm, $\omega = 8$ rad/s.

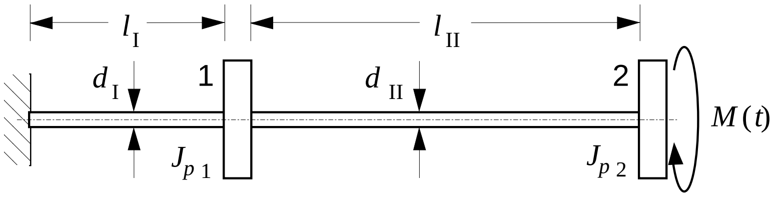

Figure 1

$\omega_1 = 5.6$ rad/s, $\omega_2 = 12.7$ rad/s, $\{\theta\}_1 = \{0.58\ 1\}^\mathrm{T}$, $\{\theta\}_2 = \{1\ -0.87\}^\mathrm{T}$, $\tau_\mathrm{I} = 390$ MPa, $\tau_\mathrm{II} = 16$ MPa.

EXERCISE 2

Consider a 1 dof system having a stiffness $k = 10\ kN/m$ and a mass $m = 4\ kg$. A logarithmic decrement $\delta = 0.3$ was measured on it. Additionally the system is subject to a harmonic excitation $F(t) = F_0 \sin(\omega t)$ where $F_0 = 20$ N.

Compute the:

amplitude, phase angle, real part, imaginary part relative to an excitation frequency

1) $\omega = 0.9\omega_n$

2) $\omega = 2.5\omega_n$

assuming a viscous (a) and hysteretic (b) damping model.

a) $\omega = 0.9\ \omega_n$: $x_0 = 9.59 \cdot 10^{-3}$ m, $\Phi = -24.3°$, $\Re = 8.74 \cdot 10^{-3}$ m, $\Im = -3.95 \cdot 10^{-3}$ m;

$\omega = 2.5\omega_n : x_0 = 3.81 \cdot 10^{-4}$ m, $\Phi = -177.4°$, $\Re = -3.80 \cdot 10^{-4}$ m, $\Im = -1.73 \cdot 10^{-5}$ m;

b) $\omega = 0.9\omega_n : x_0 = 9.41 \cdot 10^{-3}$ m, $\Phi = -26.7°$, $\Re = 8.40 \cdot 10^{-3}$ m, $\Im = -4.22 \cdot 10^{-3}$ m;

$\omega = 2.5\omega_n : x_0 = 3.81 \cdot 10^{-4}$ m, $\Phi = -179.0°$, $\Re = -3.81 \cdot 10^{-4}$ m, $\Im = -6.93 \cdot 10^{-6}$ m.

EXERCISE 3

The undamped system represented in Figure 2 is characterized by two bars having a diameter d (with negligible inertia) and two masses. A harmonic excitation $F(t) = F_0 \sin(\omega t)$ is applied on the mass 2.

It is requested to:

- compute the amplitude of the oscillations by adopting the modal analysis,
- compute the maximum stress in bar 1 and bar 2.

Data: $l_1 = 0.4$ m, $l_2 = 0.2$ m, $d = 0.02$ m, $E = 2.06 \cdot 10^{11}$ Pa, $m_1 = 50$ kg, $m_2 = 20$ kg, $\omega = 1700$ rad/s, $F_0 = 2000$ N.

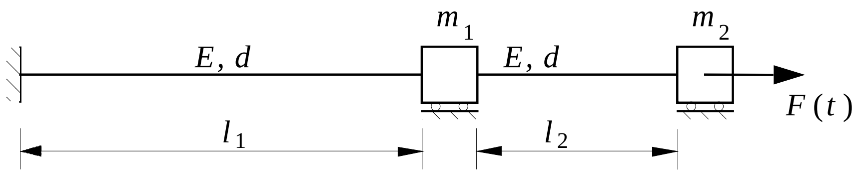

Figure 2

$x_{0,1} = -4.59 \cdot 10^{-5}$ m, $x_{0,2} = -4.89 \cdot 10^{-5}$ m.

EXERCISE 4

Consider the belt drive system represented in Figure 3. The main data are reported in Table 1.

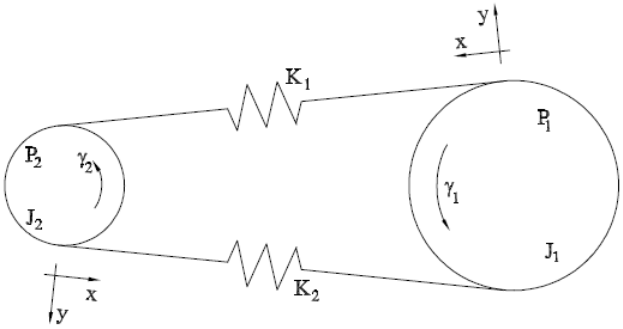

Figure 3.

$J_1 = 0.2$	Kgm^2
$J_2 = 0.15$	Kgm^2
$r_1 = 0.07$	m
$r_2 = 0.04$	m
$l = 0.5$	m

Table 1.

The Young modulus E of the belt in the axial direction is equal to $2.02 \cdot 10^9$ N/m^2 while the transversal section area A is equal to $4.26 \cdot 10^{-5}$ m^2.

It is requested to compute the two natural frequencies of the systems.

$\omega_1 = 0$ Hz, $\omega_2 = 17.5$ Hz,

EXERCISE 5

For the two degree of freedom system depicted in Figure 4

- determine the equations of dynamic equilibrium using the Langangian approach,
- arrange the equations in matrix form,
- find the natural frequencies and the mode shapes of the system,
- compute the modal matrices (modal mass and modal stiffness),
- mass-normalize the mode shapes,
- Check the mass-orthogonality of the eigenvectors.

Make the calculation using the following two sets of data

$k_1 = k_2 = 400000$ N/m $k_3 = 800000$ N/m; $m_1 = m_2 = 2$ kg

$k_1 = 400000$ N/m $k_2 = 200000$ N/m $k_3 = 800000$ N/m; $m_1 = 2$ kg $m_2 = 2.5$ kg

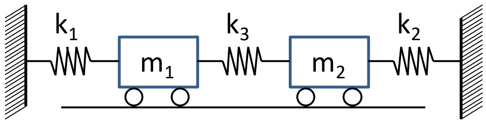

Figure 4

EXERCISE 6

A dynamic system with three flywheels is depicted in Figure 5. Neglecting the inertia of the shafts

- write the dynamic equations of equilibrium and
- determine natural frequencies and mode shapes of the system (torsional motion only)
-

d_1=30 mm	d_2=20 mm
L_1=100 mm	L_2=80 mm
m_1=1 kg	m_2=0.5 kg

D_1=120 mm D_2=60 mm D_3=100 mm

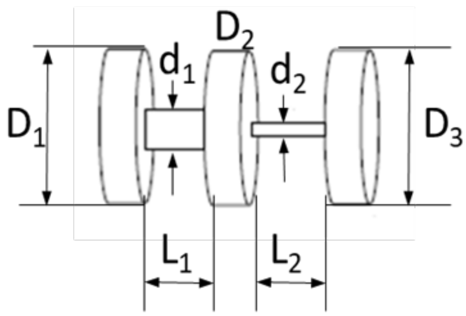

Figure 5

EXERCISE 7

For the two degree of freedom depicted in the Figure 6

- determine the equations of dynamic equilibrium (Lagrange approach).
- Find natural frequencies and modal shapes of the system.
- Use the eigenvectors to compute the modal matrices.

Data

$k_1 = k_2 = 20000$ N/m

m=950 kg IC=1400 kg m^2

L_1=1.0 m L_2=1.5 m

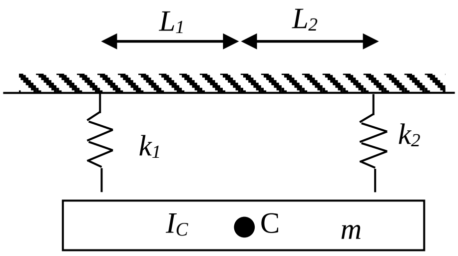

Figure 6.

[ω_1=5.9 rad/s, ω_2=7.3 rad/s, $(Y/A)_1$= -1.55, $(Y/A)_2$= 0.95]

EXERCISE 8

The output to a harmonic excitation of a vibrating structure is described only by the first mode:
$\{q\}_1 = \{1/\sqrt{5} \quad 2/\sqrt{5}\}^T$. Take into account that the modal mass is 2 kg, the modal stiffness is 5000 N/m, the modal force is 10+(2U+P)/3 N. The excitation frequency is equal to 0.8ω_1, and the damping is negligible. Compute the amplitude of oscillation of the first degree of freedom.

x_{01} = ... m

EXERCISE 9

A two dwgrees of freedom undamped system has the following modal shapes:

$\{q\}_1 = \{1 \quad 0.8165\}^T$, $\{q\}_2 = \{1 \quad -0.8165\}^T$.

The stiffness matrix, the mass matrix and the vector of the excitation forces are:

$$[K] = 10^3 \begin{bmatrix} 10 & -8 \\ -8 & 15 \end{bmatrix} \frac{\text{N}}{\text{m}}, \quad [M] = \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix} \text{kg}, \quad \{F\} = 10^3 (1 + \frac{2U+P}{20}) \begin{Bmatrix} 5 \\ 8 \end{Bmatrix} \text{N}$$

Compute the modal force corresponding to the first mode.

$\overline{F}_1$ = ... N

EXERCISE 10

Consider a vibrating system. The dynamic response is characterized using only the first mode $\{q\}_1 = \{1/\sqrt{7} \; 2/\sqrt{7}\}^T$. The modal mass is equal to 3 kg, the modal stiffness is equal to 3000 N/m, the modal force is equal to 10+(2U+P)/3 N. The excitation frequency is equal to 0.8·ω_1, the damping is negligible.

Compute the amplitude of oscillation of the degree of freedom n° 1.

x_{01} = ... m

MULTIPLE CHOICE PROBLEMS: select the right answer. Explain in few sentences (eventually with the support of graphs, schems and formulas) the reason of the correct answer.

1) The Lagrange equations can be written
   [A] only for undamped systems,
   [B] only for damped and undamped systems subject to vibration
   [C] also for non conservative systems.

2) For a damped multi degree of freedom system it is not possible to obtain a complete modal decoupling. This sentence is
   [A] Always correct
   [B] Never correct
   [C] Not correct only in some cases.

3) An external excitation applied to a dynamic system is considered quasi static if
   [A] Its frequency is much lower with respect to the lowest natural frequency of the dynamic system.
   [B] Its frequency is equal or lower with respect to the lowest natural frequency of the dynamic system.
   [C] Its frequency is equal or higher with respect to the lowest natural frequency of the dynamic system.

4) A gyroscopic matrix present in the equations of motion of a mechanical system
   [A] Is symmetric
   [B] Is skew symmetric and may never cause instability.
   [C] Is skew symmetric and may cause instability.

5) A mechanical system is characterized by a damping factor $\xi=0.6$. This means that its structure is characterized mainly by
   [A] light alloys,
   [B] steel alloys,
   [C] a dedicated damping device.