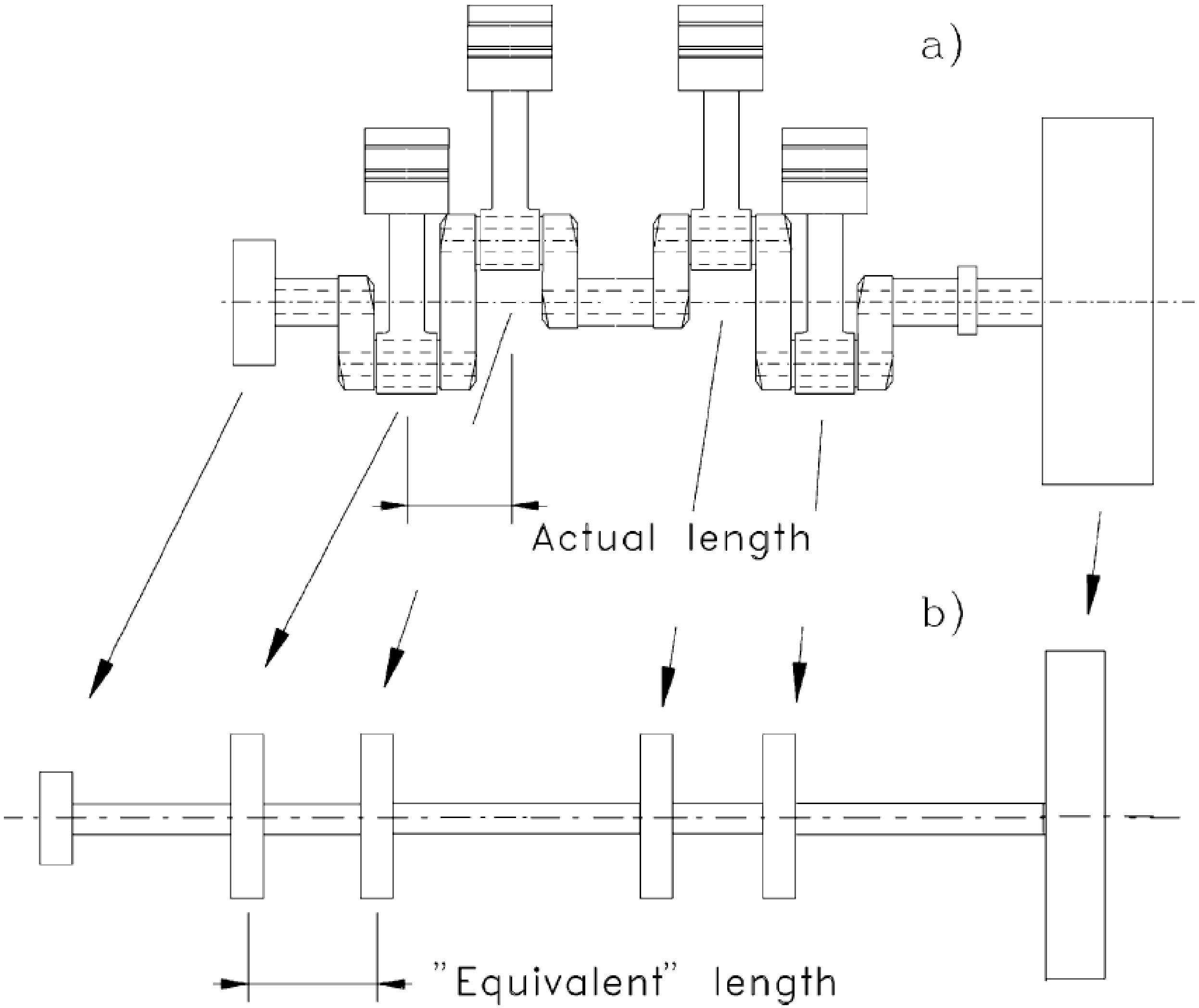
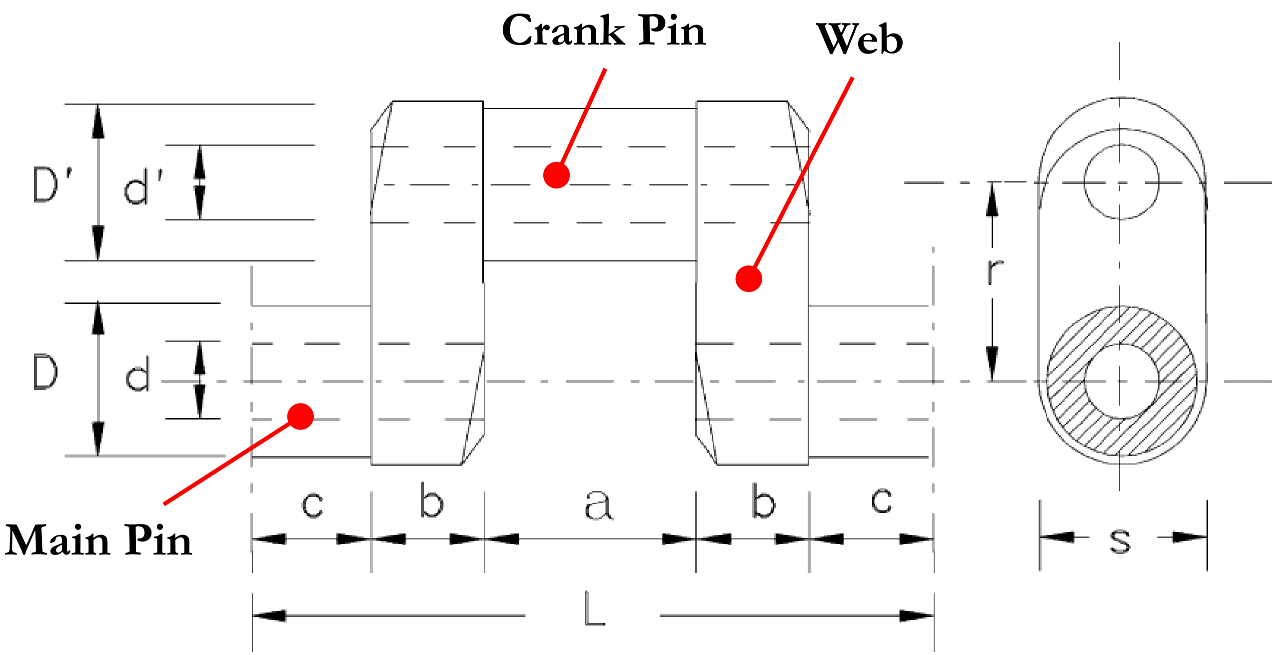

a)
Actual length
b)
"Equivalent" length



Equivalent length according to Carter's model

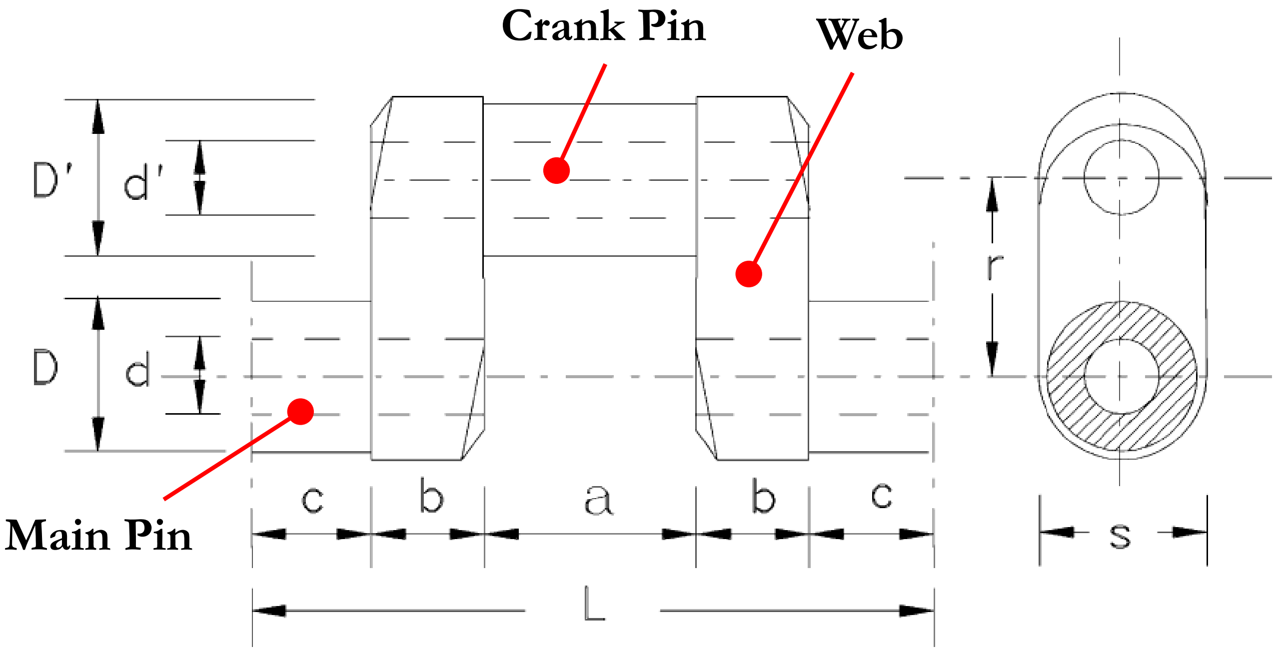
$$L_{eq} = 2c + 0.8b + \frac{3}{4}\frac{D^4 - d^4}{D'^4 - d'^4}a + \frac{3}{2}\frac{D^4 - d^4}{bs^3}r = 0.150\ m$$

Second moment of area of the Main Pin

$$I_{p,MP} = \pi\frac{D^4 - d^4}{32} = 7.854\ 10^{-8}\ m^4$$

Equivalent stiffness of the Crankshaft

$$\boxed{k_{eq,C} = \frac{GI_{p,MP}}{L_{eq}} = 4.040\ 10^4 Nm/rad}$$

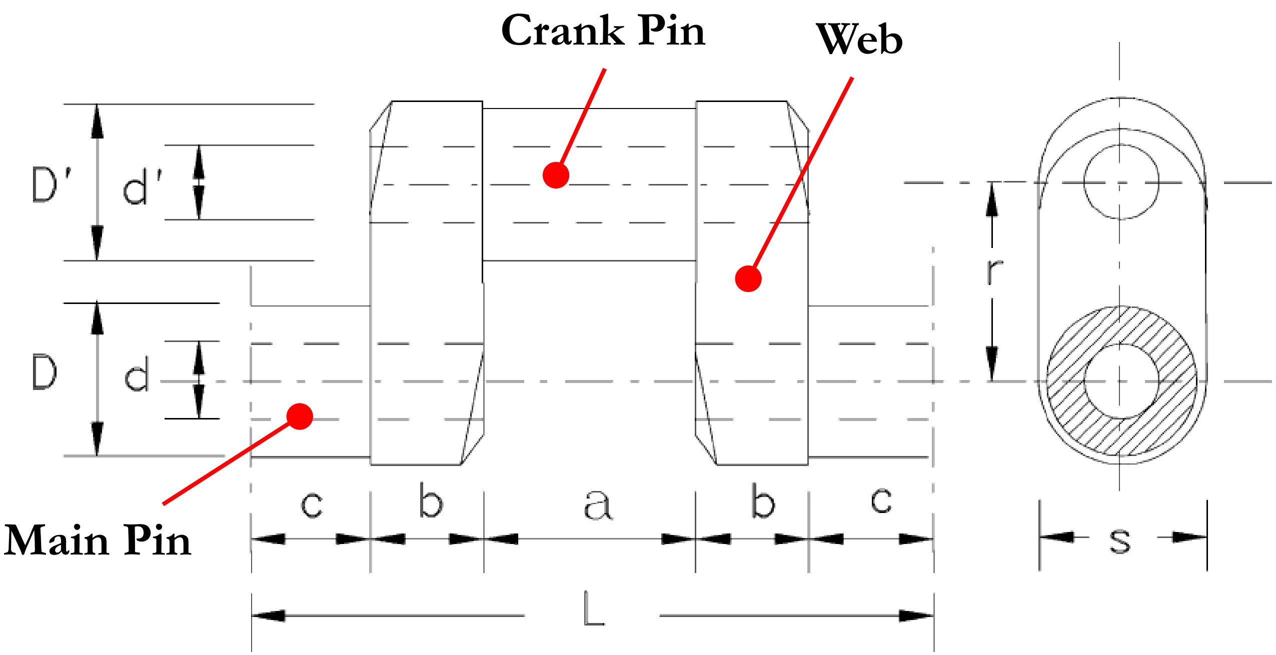


Inertia of the Main Pin

$$J_{MP} = \pi \frac{D^4 - d^4}{32} c\rho = 1.225\ 10^{-5}\ kgm^2$$

Inertia of the Crank Pin

$$J_{CP} = \pi \frac{D'^4 - d'^4}{32} a\rho + \pi \frac{D'^2 - d'^2}{4} a\rho r^2 = 3.157\ 10^{-4}\ kgm^2$$



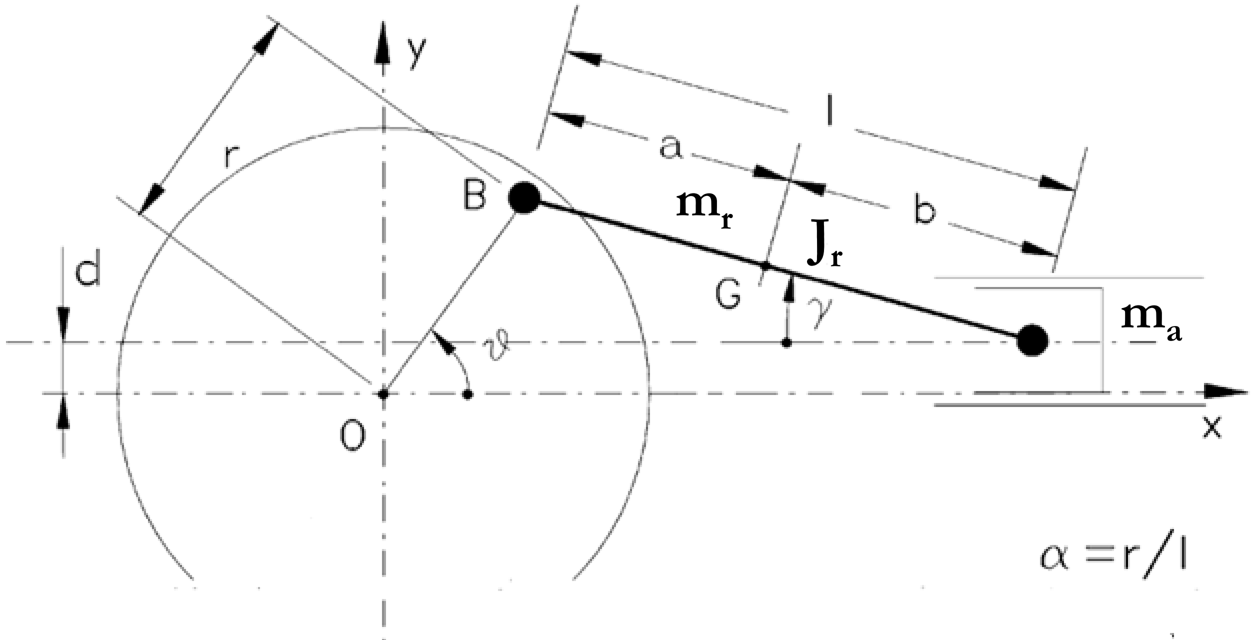
Inertia of the Web

$$J_W = \frac{sr^3}{12} b\rho + \, m_W \left(\frac{r}{2}\right)^2 = \frac{1}{3} sbr^3 \rho = 1.860\ 10^{-4}\ kgm^2$$

Inertia of the Crank

$$\boxed{J_C = 2J_{MP} + 2J_W + J_{CP} = 7.122\ 10^{-4}\ kgm^2}$$

Equivalent Rod



Equivalent Rod

Same mass as the real rod

$$m_r = m_{r1} + m_{r2}$$

Same center of mass position

$$x_G\, m_r = x_1\, m_{r1} + x_2 m_{r2}$$

$$a\, m_r = 0\, m_{r1} + L\, m_{r2}$$

$$\boxed{m_{r2} = \frac{a}{L} m_r = 0.402\ kg}$$

$$\boxed{m_{r1} = \frac{b}{L} m_r = 1.011\ kg}$$

Same moment of inertia

$$J_r = J_0 + m_{r1} a^2 + m_{r2} b^2$$

$$\boxed{J_0 = J_r + m_r ab = -3.876\ 10^{-3}\ kgm^2}$$

Equivalent average moment of inertia

$$J_{eq} = J_C + m_{r1}r^2 + (m_{r2}+m_a)r^2 f_1(\vartheta) + J_0 f_2(\vartheta)$$

$$f_1(\vartheta) \approx a_0 = \frac{8 + 2\alpha + \alpha^4}{16} = 0.514$$

First order approximation, constant terms only

$$f_2(\vartheta) \approx c_0 = \alpha^2 \frac{4 + \alpha^2}{8} = 0.054 \qquad \alpha = \frac{r}{L} = 0.325$$

$$\boxed{J_{eq} = J_C + m_{r1}r^2 + (m_{r2}+m_a)r^2 a_0 + J_0 c_0 = 4.463\ 10^{-3}\ kgm^2}$$

Stiffness matrix

$$K = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1 + k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix}$$

Mass matrix

$$M = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

$$det(-\omega^2[M] + [K]) = 0$$

Potential Energy

$$U=\frac{1}{2}k_5(\vartheta_6-\vartheta_5)^2+\frac{1}{2}k_3(\vartheta_4-\vartheta_3)^2+\frac{1}{2}k_2(\vartheta_3-\vartheta_2)^2+\frac{1}{2}k_1(\vartheta_2-\vartheta_1)^2$$

Kinetic Energy

$$T=\frac{1}{2}J_6\dot{\vartheta}_6{}^2+\frac{1}{2}J_5\dot{\vartheta}_5{}^2+\frac{1}{2}J_4\dot{\vartheta}_4{}^2+\frac{1}{2}J_3\dot{\vartheta}_3{}^2+\frac{1}{2}J_2\dot{\vartheta}_2{}^2+\frac{1}{2}J_1\dot{\vartheta}_1{}^2$$

$$\tau=\frac{\vartheta_5}{\vartheta_4}=\frac{\dot{\vartheta}_5}{\dot{\vartheta}_4}$$

$$U=\frac{1}{2}k_5(\vartheta_6-\tau\vartheta_4)^2+\frac{1}{2}k_3(\vartheta_4-\vartheta_3)^2+\frac{1}{2}k_2(\vartheta_3-\vartheta_2)^2+\frac{1}{2}k_1(\vartheta_2-\vartheta_1)^2$$

$$T=\frac{1}{2}J_6\dot{\vartheta}_6{}^2+\frac{1}{2}J_5\tau^2\dot{\vartheta}_4{}^2+\frac{1}{2}J_4\dot{\vartheta}_4{}^2+\frac{1}{2}J_3\dot{\vartheta}_3{}^2+\frac{1}{2}J_2\dot{\vartheta}_2{}^2+\frac{1}{2}J_1\dot{\vartheta}_1{}^2$$

$$\vartheta_6^* = \frac{\vartheta_6}{\tau}$$

$$U = \frac{1}{2}\underbrace{k_5\tau^2}_{k_5^*}(\vartheta_6^* - \vartheta_4)^2 + \frac{1}{2}k_3(\vartheta_4 - \vartheta_3)^2 + \frac{1}{2}k_2(\vartheta_3 - \vartheta_2)^2 + \frac{1}{2}k_1(\vartheta_2 - \vartheta_1)^2$$

$$T = \frac{1}{2}\underbrace{J_6\tau^2}_{J_6^*}\dot{\vartheta}_6^{*\,2} + \frac{1}{2}\underbrace{J_5\tau^2}_{J_5^*}\dot{\vartheta}_4^{\,2} + \frac{1}{2}J_4\dot{\vartheta}_4^{\,2} + \frac{1}{2}J_3\dot{\vartheta}_3^{\,2} + \frac{1}{2}J_2\dot{\vartheta}_2^{\,2} + \frac{1}{2}J_1\dot{\vartheta}_1^{\,2}$$

Stiffness matrix

$$\boldsymbol{K} = \begin{bmatrix} k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k_1 + k_2 & -k_2 & 0 & 0 \\ 0 & -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & 0 & -k_3 & k_3 + k_5^* & -k_5^* \\ 0 & 0 & 0 & -k_5^* & k_5^* \end{bmatrix}$$

Mass matrix

$$\boldsymbol{M} = \begin{bmatrix} J_1 & 0 & 0 & 0 & 0 \\ 0 & J_2 & 0 & 0 & 0 \\ 0 & 0 & J_3 & 0 & 0 \\ 0 & 0 & 0 & J_4 + J_5^* & 0 \\ 0 & 0 & 0 & 0 & J_6^* \end{bmatrix}$$

$$det(-\omega^2 \boldsymbol{M} + \boldsymbol{K}) = 0$$

With degrees of freedom

$$\begin{Bmatrix} \vartheta_1 \\ \vartheta_2 \\ \vartheta_3 \\ \vartheta_4 \\ \vartheta_6^* \end{Bmatrix}$$