

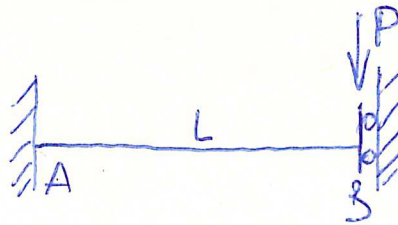
TUTORIAL 3

1.1

Ex. 1



a)



b)

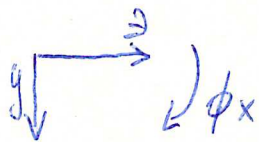
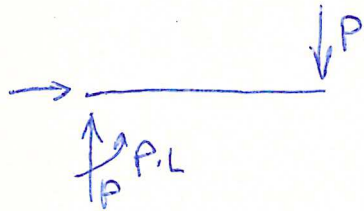
Beam section: rectangular, $b = 10 \text{ mm}$ $h = 20 \text{ mm}$

$$E = 212000 \text{ MPa}$$

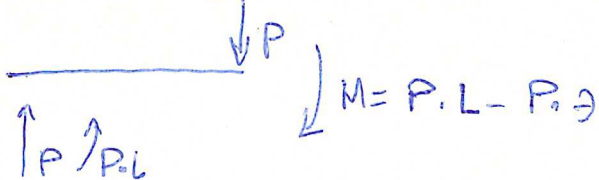
$$L = 1 \text{ m}$$

$$P = 100 \text{ kN}$$

a)



$$\phi_x = \frac{dy}{dx}$$



Linear elasticity

$$EI \frac{d^2 y}{dz^2} = P \cdot L - P \cdot z$$

$$EI \frac{dy}{dz} = P \cdot L \cdot z - P \cdot \frac{z^2}{2} + C_1$$

1st integration

$$EI y = P \cdot L \cdot \frac{z^2}{2} - \frac{P \cdot z^3}{6} + C_1 z + C_2 \quad \text{2nd integration}$$

Behaviour at the boundaries

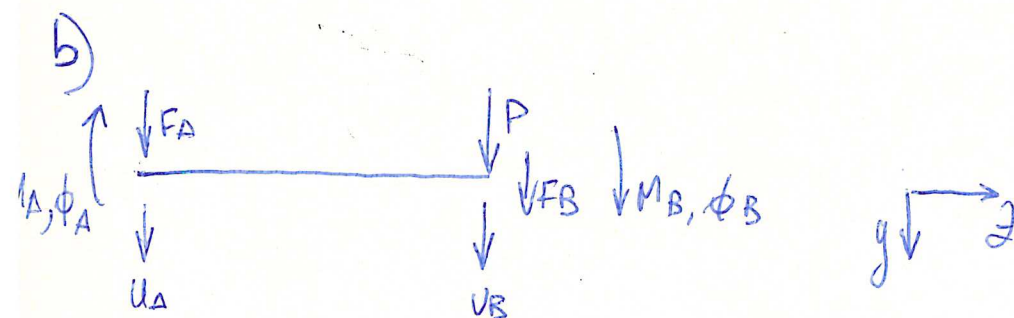
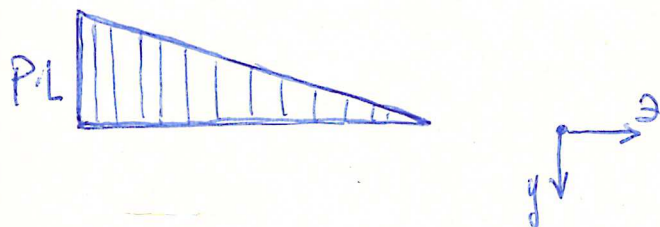
$$y|_{z=0} = 0 \rightarrow C_2 = 0$$

$$\phi_x|_{z=0} = 0 \rightarrow C_1 = 0$$

$$\Rightarrow y = \frac{P}{E \cdot I} \left(L \frac{z^2}{2} - \frac{z^3}{6} \right)$$

$$\phi_x = \frac{dy}{dz} = \frac{P}{EI} \left(Lz - \frac{z^2}{2} \right)$$

$$\Rightarrow v_B = y|_L = \frac{P}{EI} \left(\frac{L^3}{2} - \frac{L^3}{6} \right) = \frac{P \cdot L^3}{3EI}$$



The selected element is a beam with the only flexural behaviour.

$$\begin{Bmatrix} F_A \\ M_A \\ F_B \\ M_B \end{Bmatrix} = \begin{bmatrix} \downarrow & \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \downarrow & \downarrow & \downarrow & \downarrow \end{bmatrix} \begin{Bmatrix} v_A \\ \phi_A \\ v_B \\ \phi_B \end{Bmatrix}$$

↑
from constraints

Constraints:

$$u_A = 0$$

$$\phi_A = 0$$

$$\phi_B = 0$$

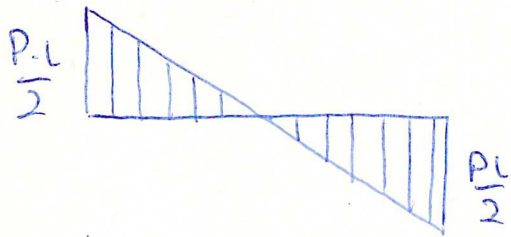
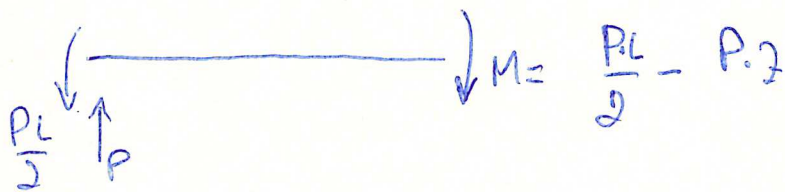
$$*) F_B = K_{33} v_B \Rightarrow v_B = \frac{P}{K_{33}} = \frac{P \cdot L^3}{12EI}$$

(1,2)

$$*) F_A = K_{13} v_B = -12 \frac{EI}{L^3} \frac{P \cdot L^3}{12EI} = -P$$

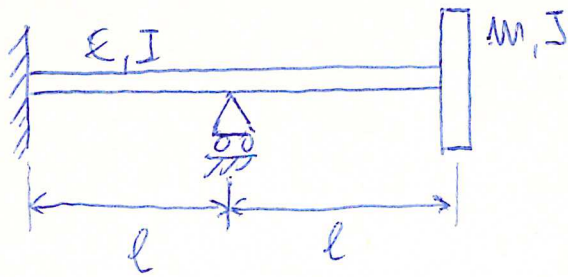
$$*) M_A = K_{23} v_B = -6 \frac{EI}{L^2} \frac{P \cdot L^3}{12EI} = -\frac{PL}{2}$$

$$*) M_B = K_{43} v_B = -\frac{PL}{2}$$



TUTORIAL 3

Ex. 6



$$E = 2.0 \cdot 10^5 \text{ N/mm}^2$$

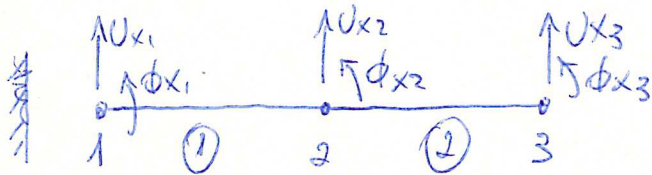
$$l = 2.1 \text{ m}$$

$$I = 4.91 \cdot 10^{10} \text{ mm}^4$$

$$m = 5 \text{ kg}$$

$$J = 0.05 \text{ kg m}^2$$

$$[k] = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$



Constraints

$$U_{x1} = 0$$

$$\phi_{x1} = 0$$

$$U_{x2} = 0$$

Map

| | U_{x1} | ϕ_{x1} | U_{x2} | ϕ_{x2} | U_{x3} | ϕ_{x3} |
|---|----------|-------------|----------|-------------|----------|-------------|
| ① | 1 | 2 | 3 | 4 | | |
| ② | | | 1 | 2 | 3 | 4 |
| ③ | x | x | x | 1 | 2 | 3 |

$$M_g, K_g = \begin{bmatrix} 1_{11} & 1_{12} & 1_{13} & & & \\ 1_{21} & 1_{22} & 1_{23} & & & \\ 1_{31} & 1_{32} & 1_{33}+2_{11} & 1_{34}+2_{12} & 2_{13} & 2_{14} \\ 1_{41} & 1_{42} & 1_{43}+2_{21} & 1_{44}+2_{22} & 2_{23} & 2_{24} \\ & & 2_{31} & 2_{32} & 2_{33} & 2_{34} \\ & & 2_{41} & 2_{42} & 2_{43} & 2_{44} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1_{44}+2_{22} & 2_{23} & 2_{24} \\ 2_{32} & 2_{33} & 2_{34} \\ 2_{42} & 2_{43} & 2_{44} \end{bmatrix}$$

$$M_G = M_G + \begin{bmatrix} \oplus & \oplus \\ & m \\ & J \end{bmatrix}$$

$$\omega_n^2 = \sqrt{\frac{kg}{Mg}} \Rightarrow \omega_n = \begin{matrix} 106.3 \text{ rad/s} \\ 452.5 \text{ rad/s} \end{matrix}$$