01OAIQD – Dynamic Design of Machines

Academic year 2019-2020

THEORY EXERCISES ON FE DYNAMICS AND ROTORDYNAMICS: choose the correct answer. Briefly explain in few lines, eventually resorting to graphs, schemes or formulas, the reason of your choice and exclusion of the other possible answers.

1. If the shear deformation is considered in the study of the vibration motion of a beam,
   [A] all the natural frequencies of the model increases,
   [B] all the natural frequencies of the model decreses,
   [C] a negligible effect on the natural frequencies if the beam is not slender enough.

2. In absence of damping, the bending natural frequencies of a rotor:
   [A] Depend on the spin speed only in presence of gyroscopic effect
   [B] Depend on the spin speed only in absence of gyroscopic effect
   [C] Always depend on the spin speed

3. If the spin speed of an undamped rotor is equal to its critical speed
   [A] The amplitude of the orbit grows linearly with time
   [B] The amplitude of the orbit grows exponentially with time
   [C] The amplitude of the orbit grows squarely with time

4. If the spin speed of a rotor falls in a field of instability
   [A] The amplitude of the orbit grows linearly with time
   [B] The amplitude of the orbit grows exponentially with time
   [C] The amplitude of the orbit grows squarely with time

5. To increase the threshold of instability of a rotor it is possible
   [A] To reduce unbalance
   [B] To increase the rotating damping and decrease non rotating damping
   [C] To increase non rotating damping and decrease rotating damping

6. In general, the vibrating motion of a Jeffcott rotor is constituted
   [A] Of the sum of two forward and one backward whirl
   [B] Of the sum of two forward whirls

[C] Of the sum of one forward and one backward whirl

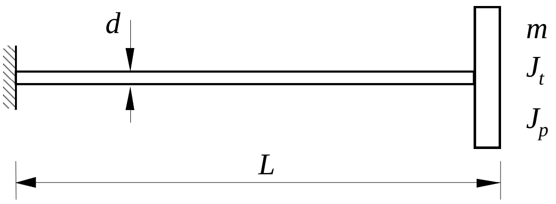
7. The self-centering can be verified
   [A] In subcritical region
   [B] In supercritical region
   [C] In sub- or supercritical fields, depending on the value of rotating damping.

8. In case of a rotor where $J_p < J_t$
   [A] There are two critical speeds
   [B] There is only one critical speed
   [C] There are no critical speeds

9. In the four degrees of freedom model for the free vibration of an undamped rotor
   [A] There are four values of ω (two positive and two negative) equal to pairs in modulus for each value of Ω
   [B] There are four values of ω (two positive and two negative) different in modulus for each value of Ω
   [C] There are four values of ω (two positive and two negative) equal to pairs in modulus for $\Omega = 0$ and different in modulus for $\Omega > 0$

EXERCISES:

1) Compute the first bending natural frequency of the structure shown in the figure, which is composed by a cantilever shaft (whose mass can be neglected) having a flywheel of mass m and inertia J_t attached to its end. Compare the results obtained studying the full system with those obtained using the Guyan reduction.



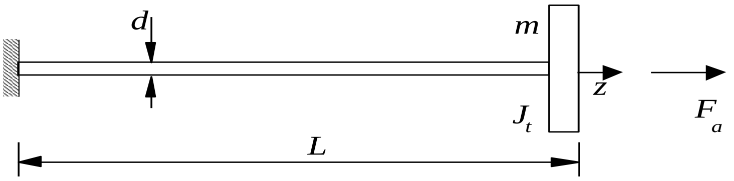
Data:
$E = 2\cdot 1011$ N/m^2, $\nu = 0.3$, L = 1 m,
d = 0.03 m, m_I = 10 kg, m_{II} = 100 kg
J_t = 1 kgm^2,

The stiffness matrix of the beam element (due to the slenderness of the shaft we refer to the Euler formulation) is:

$$[K] = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

ω_{1I}=43.87 rad/s, ω'_{1I}=44.13 rad/s (Guyan), ω_{1II}=15.273 rad/s, ω'_{1II}=15.274 rad/s (Guyan)

2) The rotor shown in the figure is composed by a flywheel of mass m and moments of inertia J_t and J_p attached to the end of a shaft with negligible mass.



Assuming to orientate the structure vertically and to consider the effect of gravity, calculate the critical speed in the following three cases:

a) Z axis coincident with respect to the gravitational field (mass below the constraint)
b) Z axis non coinciding with gravitational field (mass above the constraint)
c) Absence of gravity (orientation is indifferent)

Data:
$E = 2{\cdot}10^{11}$ N/m^2, L = 0.3 m, d = 2.5 mm, m = 0.5 kg, J_t =0.0 1 kgm^2
J_p =0.0 2 kgm^2

Stiffness matrix for the beam element (considering the slenderness we refer to the Euler formulation) and geometrical matrix.

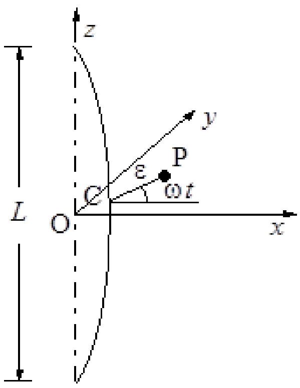
$$[K]=\frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \qquad [K_g]=\frac{F_a}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix}$$

Solution:
Ω_a=13.9 rad/s , Ω_b=9.1 rad/s, Ω_c=11.8 rad/s.

3) A rotor with mass m can be modeled as a Jeffcott rotor having a deformable shaft on stiff supports with distance L between the supports. The shaft has a full circular section area with stiffness $k = 48EI/L^3$ ($I = \pi d^4/64$). Given that the mass is fixed to the rotor with eccentricity ε and that at rated speed the radius of the orbit is equal to z_0=-1.2ε, calculate:

a) The critical speed Ω_{cr};
b) The diameter of the shaft d;
c) The maximum bending stress on the shaft and the reactions on the supports at rated speed Ω.

Data: $E = 2{\cdot}10^{11}$ N/m^2, L = 0.5 m, m = 15 kg, ε = 100 μm, Ω = 10000 rpm.



Solution:
Ω_{cr} = 427 rad/s, d = 29 mm, R = 165 N, σ = 17 MPa.

4) A rotor with mass m can be modeled as a Jeffcott rotor having a deformable shaft on stiff supports with distance L between the supports. The shaft has a full circular shaft with stiffness $k = 48EI/L^3$ $(I = \pi d^4/64)$.Given that the mass is fixed to the rotor with eccentricity ε, and that the rotating damping c_r is null, calculate:

a) The critical speed Ω_{cr}
b) The nonrotating damping c_n necessary to keep the radius of the orbit z_0 below 1.25ε;
c) The radius of the orbit z_0 when crossing the critical speed assuming the value of c_n calculated in the previous question.

Data: $E = 2\cdot10^{11}$ N/m^2, L = 0.5 m, d = 0.029 m, m = 15 kg, ε = 10 μm, Ω = 8000 turns/min.

Solution:

Ω_{cr} = 427 rad/s, $c_n = 3.61\cdot10^3$ Ns/m, z_0 = 17.5 μm.

5) A rotor with mass m can be modeled as a Jeffcott rotor having a deformable shaft with bending stiffness k. The bearings have a known viscous damping coefficient c_n; the relative total damping $\zeta = (c_n + c_r) / c_{cr}$ is evaluated with tests. Calculate the threshold of instability of the rotor.

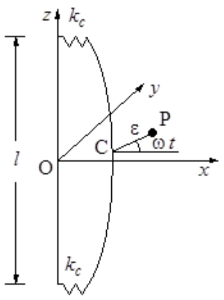
Data: k = 50 kN/m, m = 30 kg, ζ = 0.05, c_n = 100 Ns/m.

Solution:

Ω_{th} = 226 rad/s.

6) A Jeffcott rotor is constituted by a mass m, having eccentricity ε, fixed in the middle of a deformable shaft with length l and diameter d suspended on elastic supports having stiffness k_c. Assuming structural rotating damping, having loss factor η_r , and viscous nonrotating damping c_n , calculate:

d) The critical speed of the rotor
e) The viscous nonrotating damping c_n needed to guarantee stability
f) The amplitude and phase of the response to static unbalance at rated speed Ω.



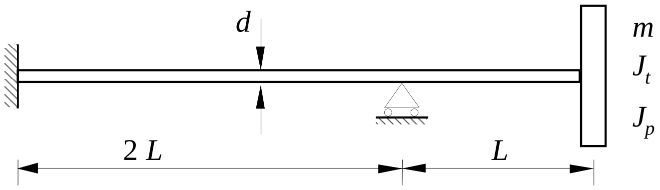
Data: m = 8 kg, l = 0.4 m, d = 0.015 m, η_r = 0.002, $E = 2\cdot10^{11}$ N/m^2, $k_c = 5\cdot10^5$ N/m, ε = 6 μm, Ω = 150 rad/s.

Solution:

Ω_{cr} = 184 rad/s, c_n = 4.05 Ns/m, $z_0 = 1.18\cdot10^{-5}$ m, $\phi \approx -0.4°$.

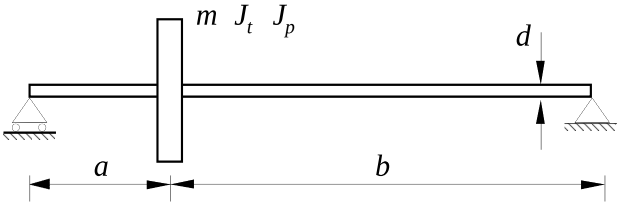
7) Obtain the bending critical speeds and the corresponding mode shapes for the system shown in the figure. The system is represented by two beam elements using the lumped parameter approach and neglecting the distribution of the mass along the beam elements.

Data: $E = 2\cdot10^{11}$ N/m^2, L = 0.5 m, d = 0.015 m, m = 3 kg, J_t = 0.04 kgm^2, J_p = 0.08 kgm^2.



Ω_{cr} = 41.5 rad/s.

8) A rotor having stiff supports, deformable shaft whose inertia can be neglected, has inertia properties m, J_t, J_p. The flywheel is attached with an eccentricity ε and angular error χ ($\alpha = 0$). Calculate the critical speed and the amplitudo of the response due to unbalance at spin speed equal to Ω.



Data: $E = 2 \cdot 10^{11}$ N/m^2, a = 0.08 m, b = 0.32 m, d = 0.022 m, m = 6 kg, J_t = 1.4 kgm^2, J_p = 2.6 kgm^2, ε = 20 μm, χ = 0.01 rad, Ω = 2000 rad/s.

Ω_{cr} = 1491 rad/s, $z_{0,\varepsilon} = -5.1 \cdot 10^{-5}$ m, $z_{0,\chi} = 1.0 \cdot 10^{-3}$ m, $\phi_{0,\varepsilon} = -1.0 \cdot 10^{-5}$ rad, $\phi_{0,\chi} = -1.0 \cdot 10^{-2}$ rad.