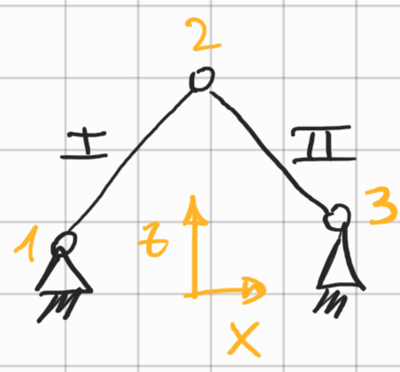
Ex 3 - Tutorial 3

[NOTE: in 2 we have an internal constraint



global

⊛ Structure 3A

→ only hinges $\Longrightarrow$ axial loads only are transmitted

$\Downarrow$

$\boxed{\text{truss elements}}$

local

Stiffness matrix of truss element (local)

$$\underbrace{\frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{[k_T]} \begin{Bmatrix} u_{x_1} \\ u_{z_1} \\ u_{x_2} \\ u_{z_2} \end{Bmatrix} = \begin{Bmatrix} f_{x_1} \\ 0 \\ f_{x_2} \\ 0 \end{Bmatrix}$$

The local matrix is rotated into the global reference system

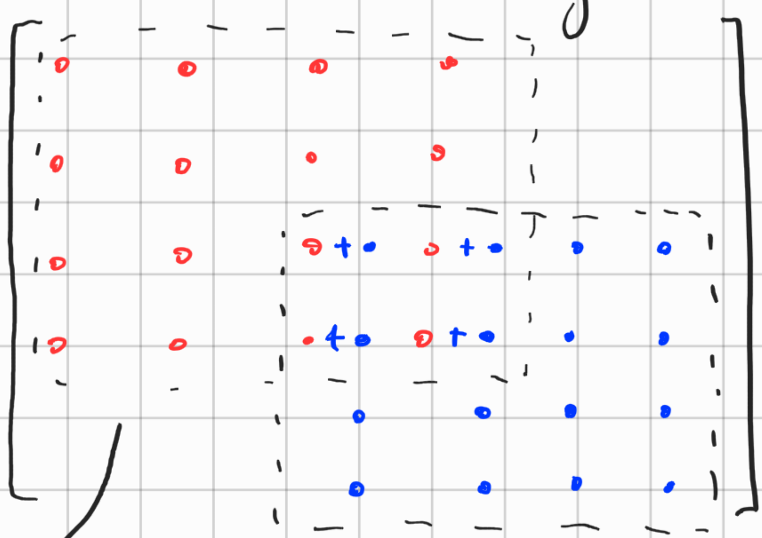
element I, Global

$$
\left.
\begin{aligned}
\left[K_T^{IG}\right] &= \left[R(\vartheta_I)\right]^T \left[K_T^{I}\right] \left[R(\vartheta_I)\right] \\
\left[K_T^{IIG}\right] &= \left[R(\vartheta_{II})\right]^T \left[K_T^{II}\right] \left[R(\vartheta_{II})\right]
\end{aligned}
\right] \circledast
$$

where $R(\vartheta)$ is the rotation matrix and

$$\vartheta_I = \alpha \quad , \quad \vartheta_{II} = \pi -)$$

Now we can assemble the global matrix

$[K_G] =$ 

$[K_T^{IG}]$ • $[K_T^{IIG}]$ •

The external load acts on the DOF u_{z2}

global

$$[K_G]\begin{Bmatrix} u_{x1} \\ u_{z1} \\ u_{x2} \\ u_{z2} \\ u_{x3} \\ u_{z3} \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ F_{z1} \\ 0 \\ -P \\ F_{x3} \\ F_{z3} \end{Bmatrix}$$

boundary reactions

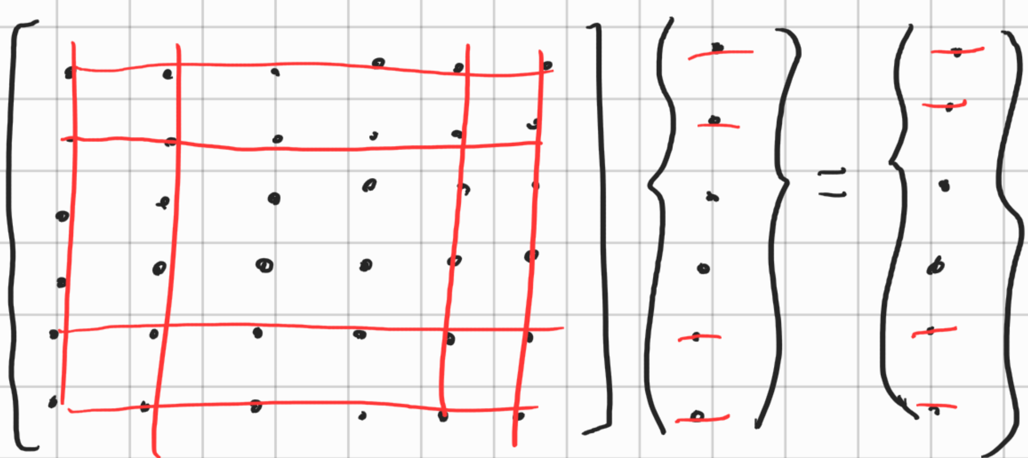
Let's apply the boundary conditions

→ the hinges act on nodes 1 and 3

$$\Rightarrow \quad u_{x1} = u_{z1} = u_{x3} = u_{z3} = 0$$

we can cancel rows and columns

1 2 4 and 5



We have a 2×2 system

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{Bmatrix} U_{x2} \\ U_{z2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -P \end{Bmatrix}$$

we solve the system to obtain U_{x_2} and U_{z2}

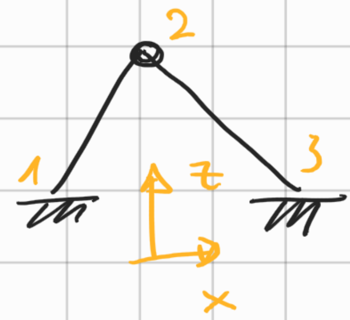
Then, we can compute the reactions of the constraints by solving this product

$$[K_G] \begin{Bmatrix} 0 \\ 0 \\ U_{x2} \\ U_{z2} \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} f_{x_1} \\ f_{z_1} \\ 0 \\ -P \\ f_{x3} \\ f_{z3} \end{Bmatrix}$$

$\Rightarrow$ we get $f_{x_1}, f_{z_1}, f_{x3}, f_{z3}$

**

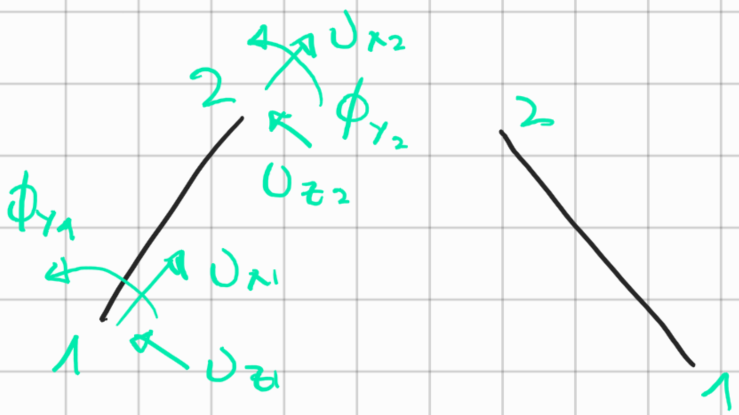
⊛ Structure 3B



global

Clamp at nodes 1 and 3

$\Longrightarrow$ beam elements



local

Stiffness matrix of beam element (local)

↳ both axial and bending

$$[K_b]\begin{Bmatrix} u_{x1} \\ u_{z1} \\ \phi_{y1} \\ u_{x2} \\ u_{z2} \\ \phi_{y2} \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ F_{z1} \\ M_{y1} \\ F_{x2} \\ F_{z1} \\ M_{y2} \end{Bmatrix}$$

Now we apply rotations to the local matrices (as done in ⊛)

$\Rightarrow$ we get matrices of the elements in the global reference frame

$[K_b^{IG}]$ and $[K_b^{IIG}]$

Now we can assemble the global matrix. Note that in node 2 we have a hinge. The hinge does not constrain the rotation

$\Rightarrow$ hence, the rotations ϕ_y of beam I and beam II at node 2 are not identical

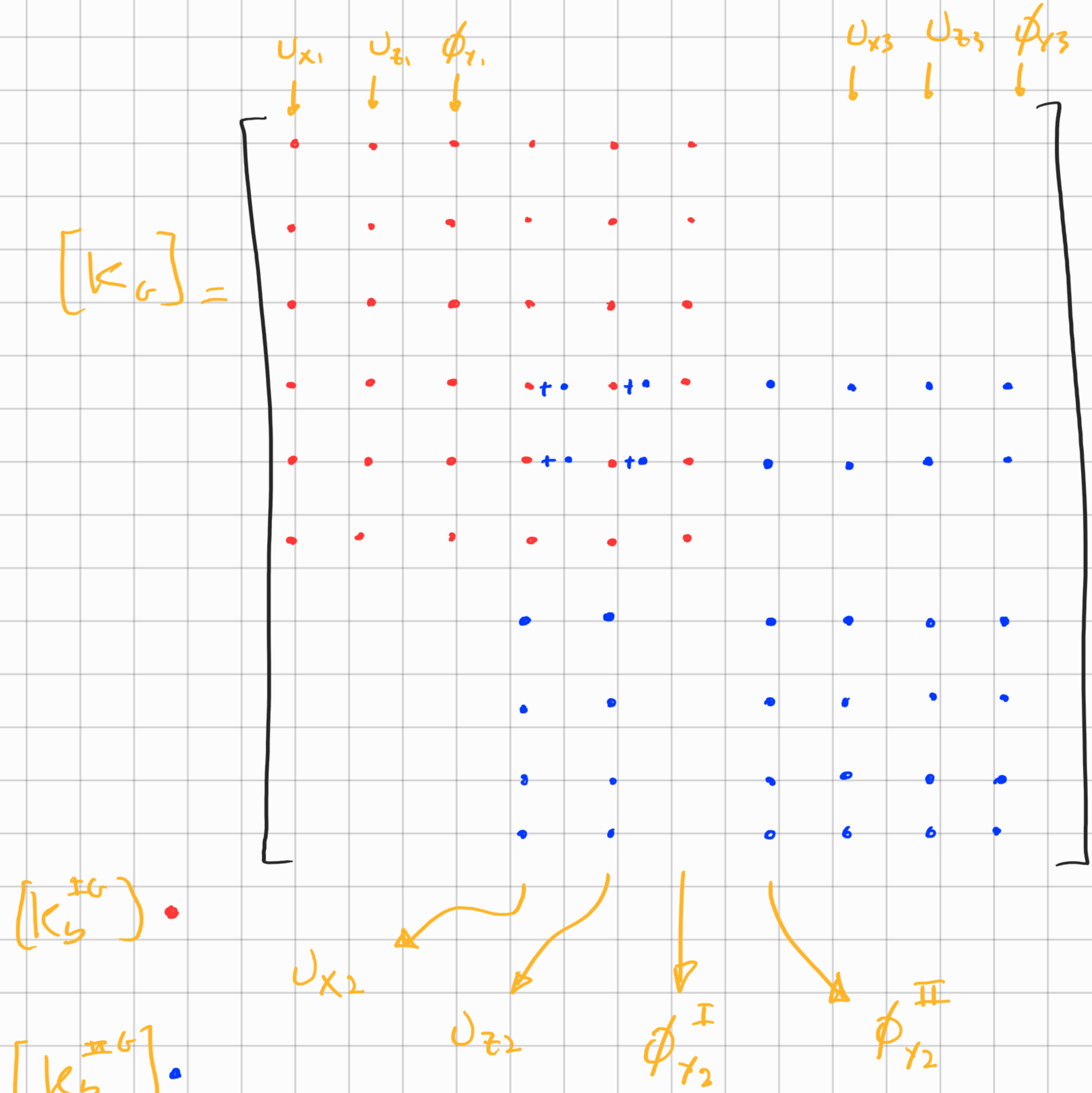
$\Downarrow$

we have to define two separated DOFs

ϕ_{y3}^{I} rotation DOF of beam I at node 2

ϕ_{y3}^{II} rotation DOF of beam II at node 2

Let's assemble the matrices



- Both the elements gives a stiffness contribution to the translating DOFs of node 2 (in the matrix there is the sum of red and blue dots)

The external loads act on node 2 (on DoF U_{Z2}).

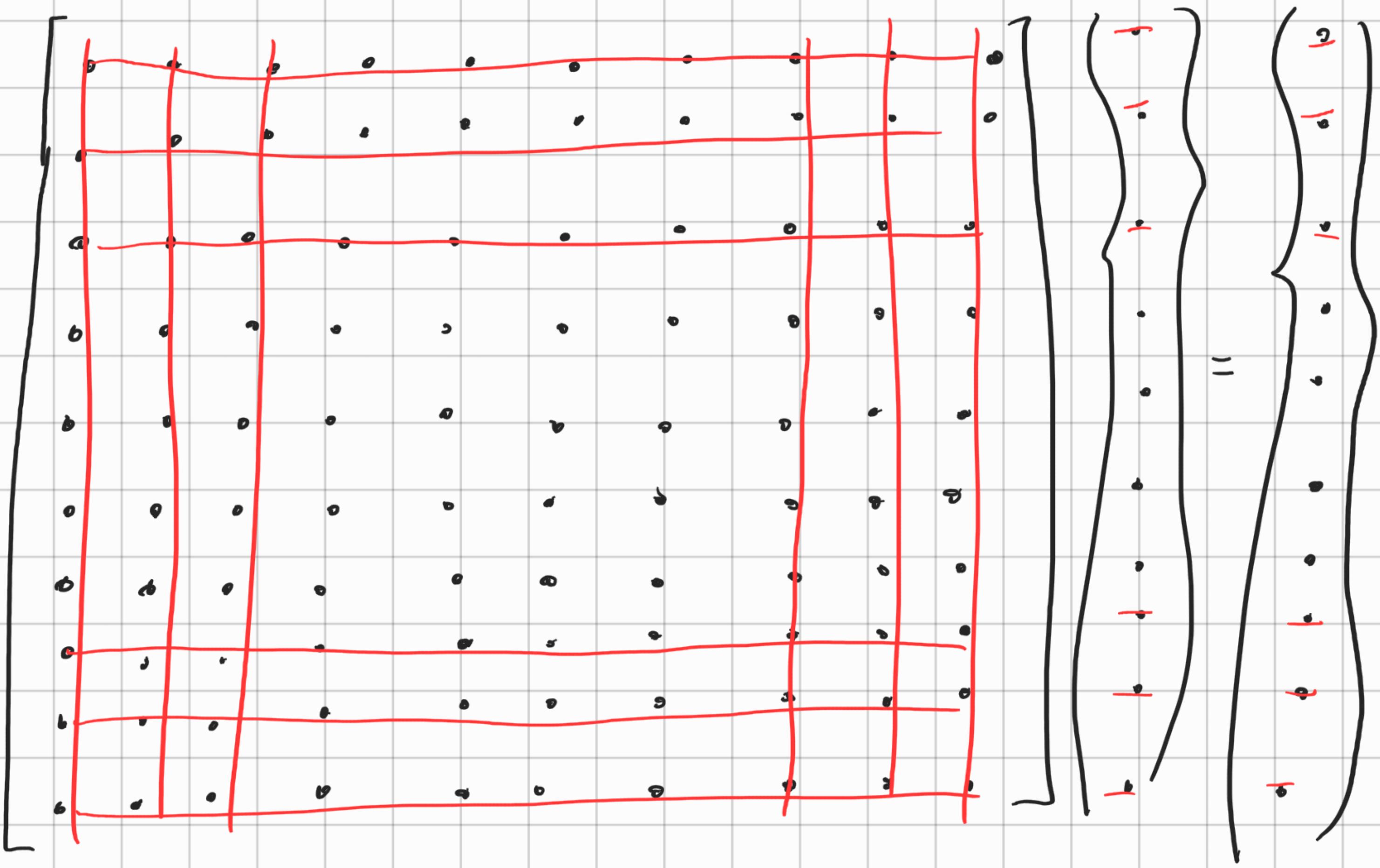
Hence, the static equilibrium is

$$[K_G]\begin{Bmatrix} U_{X1} \\ U_{Z1} \\ \phi_{Y1} \\ U_{X2} \\ U_{Z2} \\ \phi_{Y2}^{I} \\ \phi_{Y2}^{II} \\ U_{X3} \\ U_{Z3} \\ \phi_{Y3} \end{Bmatrix} = \begin{Bmatrix} F_{X1} \\ F_{Z1} \\ M_{Y1} \\ 0 \\ -P \\ 0 \\ 0 \\ F_{X3} \\ F_{Z3} \\ M_{Y3} \end{Bmatrix}$$

Now we can apply the boundary conditions

$$U_{X1} = U_{Z1} = \phi_{Y1} = U_{X3} = U_{Z3} = \phi_{Y3}$$

$\Rightarrow$ we can remove the corresponding rows and columns



As result, we have to solve a 4x4 system.

The procedure is the same as **

Structure 3C

Here, the beam elements matrices are the same as the ones of structure 3B

$\hookrightarrow$ $[K_b^{I\,G}]$ and $[K_b^{II\,G}]$

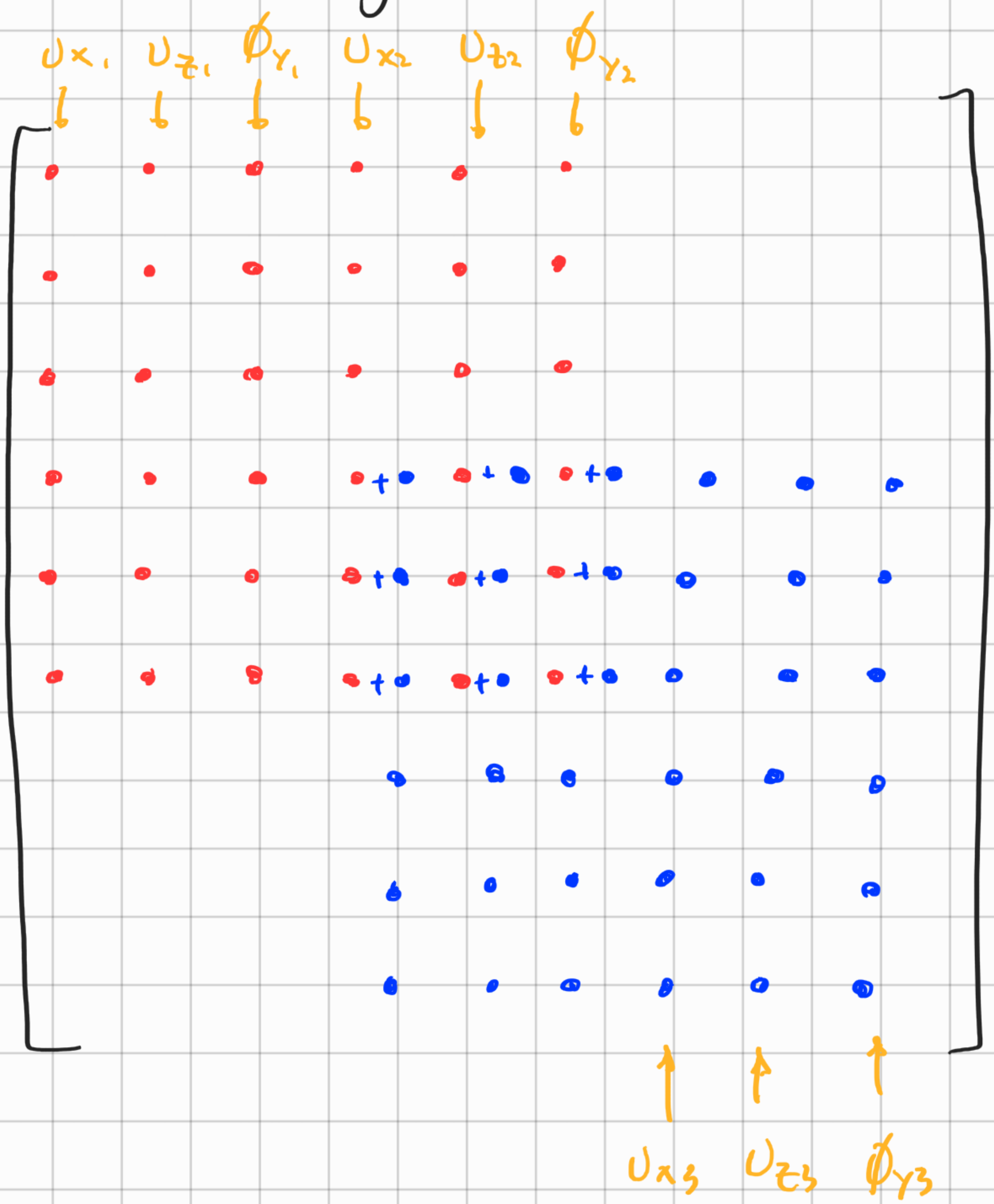
The present structure differs from the 3B one because it has a clamp at node 2

$\hookrightarrow$ the assembly is the only difference

- in structure 3C, the clamp makes rotations $\phi_{2Y}^{I} \equiv \phi_{2Y}^{II}$ $\leftarrow$ we call it ϕ_{2Y}

$\Longrightarrow$ hence, the global matrix will be

$[K_G] =$



$[K_b^{IG}]$

$[K_b^{IIG}]$

The static equilibrium is

$$[K_G]\begin{Bmatrix} U_{x1} \\ U_{z1} \\ \phi_{y1} \\ U_{x2} \\ U_{z2} \\ \phi_{y2} \\ U_{x3} \\ U_{z3} \\ \phi_{y3} \end{Bmatrix} = \begin{Bmatrix} F_{x1} \\ F_{z1} \\ M_{y1} \\ 0 \\ -P \\ 0 \\ F_{x3} \\ F_{z3} \\ M_{y3} \end{Bmatrix}$$

The boundary conditions and the solving procedure is the same as structure 3B.

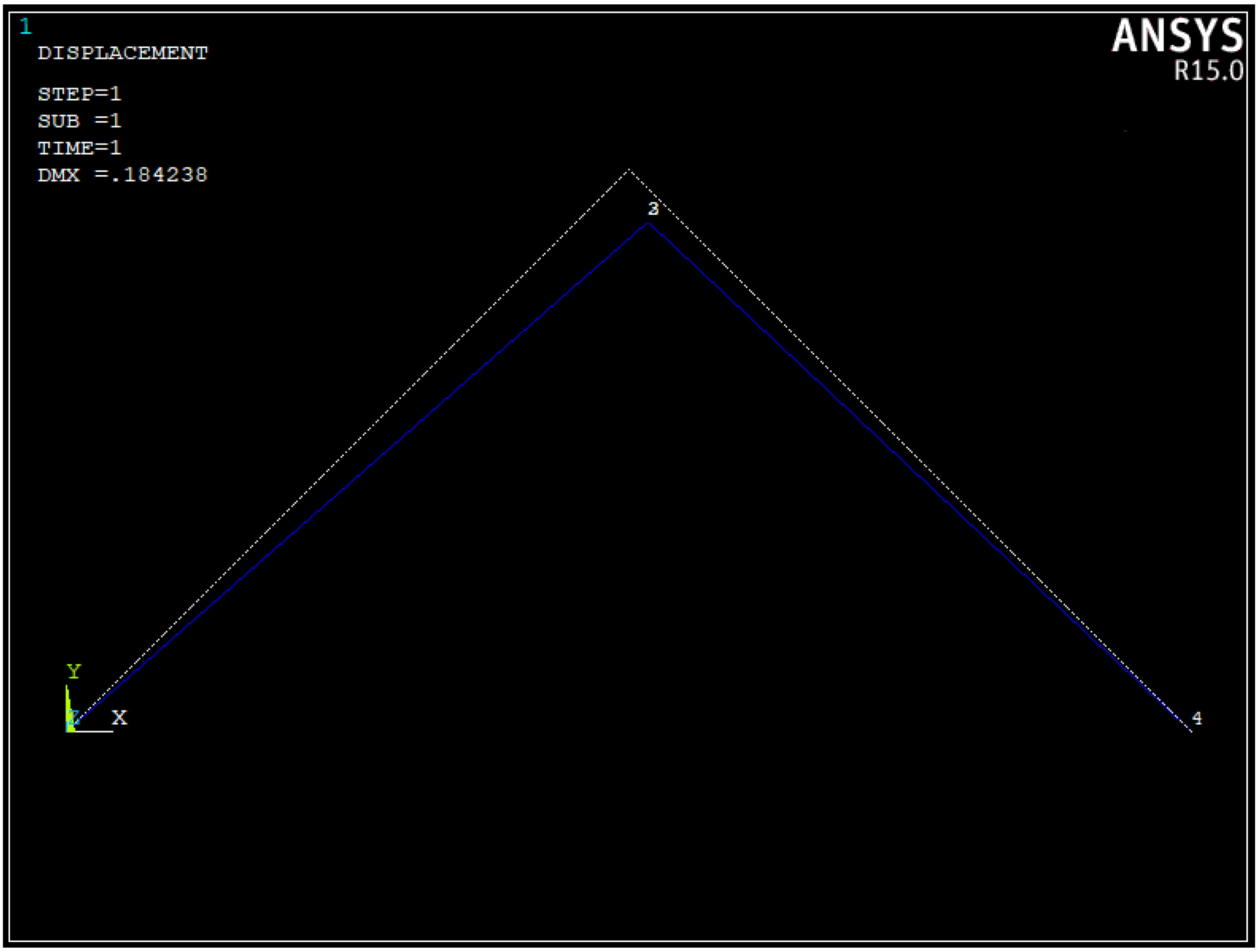
Validation: Matlab vs Ansys

The commercial FE software Ansys has been used to check the solution to Exercise 3 (Tutorial 3) that has been implemented in Matlab. I have used Ansys Mechanical APDL 15. In the Appendix you can find the Ansys code. I attach also the Matlab code.

Structure 3A

MATLAB	ANSYS
U_{x2} = 0.0661 mm	U_{x2} = 0.0661 mm
U_{z2} = -0.1720 mm	U_{z2} = -0.1720 mm
F_{x1} = 5000 N	F_{x1} = 5000 N
F_{z1} = 5000 N	F_{y1} = 5000 N
F_{x3} = -5000 N	F_{x4} = -5000 N
F_{z3} = 5000 N	F_{y4} = 5000 N

Ansys undeformed (white) vs deformed (blue) shape:

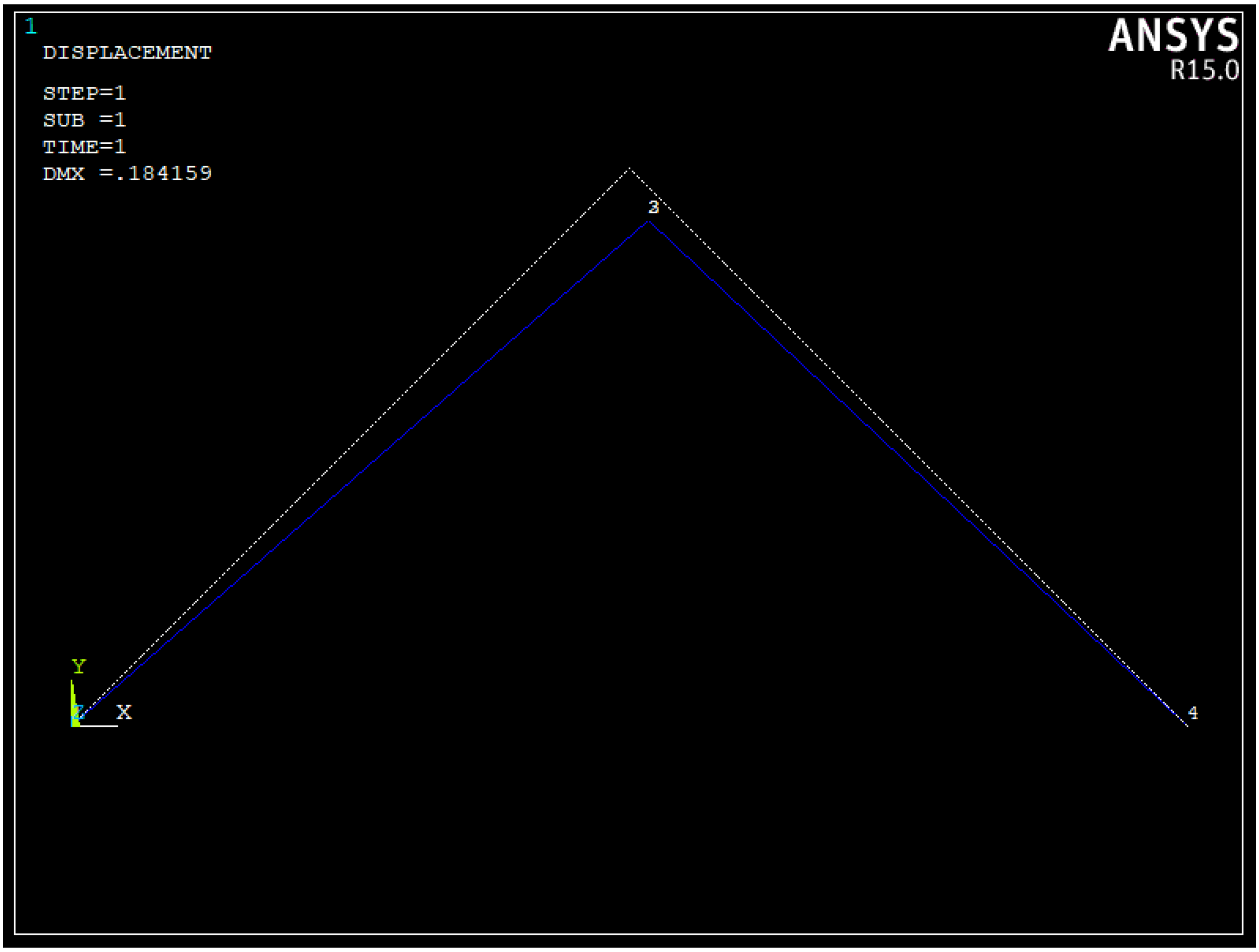


Structure 3B

MATLAB	ANSYS
U_{x2} = 0.0661 mm	$U_{x2} = U_{x3}$ = 0.0661 mm
U_{z2} = -0.1719 mm	$U_{y2} = U_{y3}$ = -0.1719 mm
Φ^{I}_{y2} = 0.2524 10^{-3} rad	Φ_{z2} = 0.2524 10^{-3} rad

$\Phi^{II}_{y2} = 0.1122\ 10^{-3}$ rad	$\Phi_{z3} = 0.1122\ 10^{-3}$ rad
$F_{x1} = 4997.2$ N	$F_{x1} = 4997.2$ N
$F_{z1} = 5002.3$ N	$F_{y1} = 5002.3$ N
$M_{y1} = 3577.9$ Nmm	$M_{z1} = 3577.9$ Nmm
$F_{x3} = -4997.2$ N	$F_{x4} = -4997.2$ N
$F_{z3} = 4997.7$ N	$F_{y4} = 4997.7$ N
$M_{y3} = 314.3$ Nmm	$M_{z4} = -314.3$ Nmm

Ansys undeformed (white) vs deformed (blue) shape:



Structure 3C

MATLAB	ANSYS
$U_{x2} = 0.0661$ mm	$U_{x2} = U_{x3} = 0.0661$ mm
$U_{z2} = -0.1719$ mm	$U_{y2} = U_{y3} = -0.1718$ mm
$\Phi_{y2} = 0.2293\ 10^{-3}$ rad	$\Phi_{z2} = \Phi_{z3} = 0.1922\ 10^{-3}$ rad
$F_{x1} = 4997.2$ N	$F_{x1} = 4993.6$ N
$F_{z1} = 5003.7$ N	$F_{y1} = 5002.3$ N
$M_{y1} = 3905.1$ Nmm	$M_{z1} = 4428.9$ Nmm
$F_{x3} = -4997.2$ N	$F_{x4} = -4993.6$ N
$F_{z3} = 4996.3$ N	$F_{y4} = 4997.7$ N
$M_{y3} = 13.3$ Nmm	$M_{z4} = -1166.4$ Nmm

Ansys undeformed (white) vs deformed (blue) shape:

Ansys code

Copy and paste the specified code sections according to the desired structure.

!CODE FOR ALL THE THREE STRUCTURES

finish

/clear

! [mm kg s]

/prep7

E=210000

```
b1 = 15
h1 = 30
b2 = 10
h2 = 20

L = 1000

AREA1=b1*h1
I1=(b1*h1*h1*h1)/12

AREA2=b2*h2
I2=(b2*h2*h2*h2)/12

et,1,beam3
et,2,link1

r,1,AREA1,I1,h1
r,2,AREA2,I2,h2
r,3,AREA1
r,4,AREA2

mp,ex,1,E


n,1
n,4,2*L*sqrt(2)/2
n,2,L*sqrt(2)/2,L*sqrt(2)/2
n,3,L*sqrt(2)/2,L*sqrt(2)/2

mat,1


!STRUCTURE 3A
```

```
type,2

real,3
e,1,2

real,4
e,4,3

!LOADS
f,2,fy,-10000

!CONSTRAINTS
d,1,ux
d,1,uy
d,4,ux
d,4,uy

cp,next,ux,2,3
cp,next,uy,2,3


!STRUCTURE 3B

type,1

real,1
e,1,2

real,2
e,4,3

!LOADS
```

```
f,2,fy,-10000

!CONSTRAINTS
d,1,all
d,4,all

cp,next,ux,2,3
cp,next,uy,2,3


!STRUCTURE 3C

type,1

real,1
e,1,2

real,2
e,4,3

!LOADS
f,2,fy,-10000

!CONSTRAINTS
d,1,all
d,4,all

cp,next,ux,2,3
cp,next,uy,2,3
cp,next,rotz,2,3

!CODE FOR ALL THE THREE STRUCTURES
/solu
```

```
solve

/post1

!DISPLAY NODE NUMBERING
/PNUM,ELEM,0

!DEFORMED+UNDEFORMED SHAPE
PLDISP,1

!LIST NODAL DISPLACEMENT
PRNSOL,U,COMP

!LIST NODAL ROTATIONS
PRNSOL,ROT,COMP

!LIST NODAL FORCES
PRRSOL,



!DISPLAY NODE NUMBERING
/PNUM,KP,0
/PNUM,LINE,0
/PNUM,AREA,0
/PNUM,VOLU,0
/PNUM,NODE,1
/PNUM,TABN,0
/PNUM,SVAL,0
/NUMBER,0
!*
/PNUM,ELEM,0
/REPLOT
!*
```