01OAIQD – Dynamic Design of Machines

Academic year 2020-2021

Discrete linear systems

Emission date: October 16, 2020		**GROUP:**

EXERCISE 1

The undamped system presented in Figure 1 is characterized by two torsion bars (negligible inertia) and two flywheels. The second one is excited by a harmonic excitation $M(t) = M_0 \sin(\omega t)$. It is requested to study the dynamic behavior of the system computing:

a) the natural frequencies and the corresponding mode shapes;
b) the potential and kinetic energy associated to the different subsystems for each mode.
c) the maximum shear stresses in the torsion bars using the configuration space and the state space approach.

Data: G = 7.7·10^{10} N/m^2, l_I = 0.5 m, l_{II} = 1.0 m, d_I = 12.8 mm, d_{II} = 14.1 mm, J_{p1} = 6 kgm^2, J_{p2} = 4 kgm^2, M_0 = 100 Nm, ω = 8 rad/s.

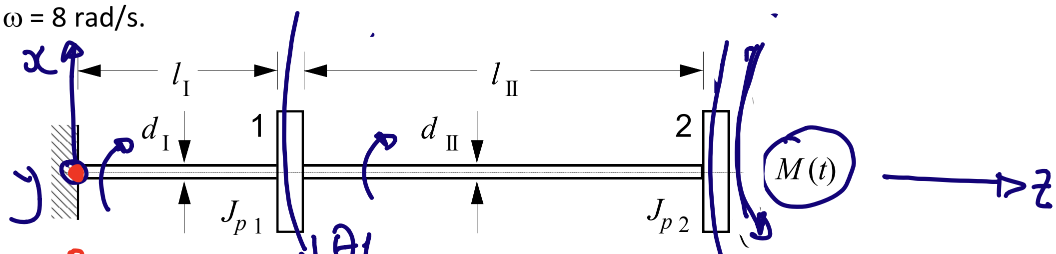


Figure 1

ω_{n1} = 5.6 rad/s, ω_{n2} = 12.7 rad/s, $\{\theta\}_1$ = $\{0.58\ 1\}^T$, $\{\theta\}_2$ = $\{1\ -0.87\}^T$, τ_I = 390 MPa, τ_{II} = 16 MPa.

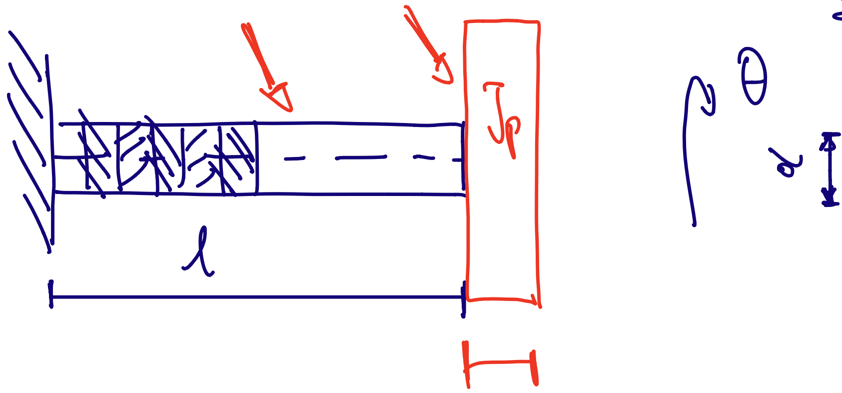
EXERCISE 2

The undamped system represented in Figure 2 is characterized by two bars having a diameter d (with negligible inertia) and two masses. A harmonic excitation $F(t) = F_0 \sin(\omega t)$ is applied on the mass 2.

It is requested to:

- compute the amplitude of the oscillations by adopting the modal analysis,
- compute the maximum stress in bar 1 and bar 2.

Solution EX.1.



$$\boxed{\mathcal{L} = T - U}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}_i}\right) - \frac{\partial \mathcal{L}}{\partial x_i} + \frac{\partial \bar{F}}{\partial \dot{x}_i} = \frac{\partial(\delta L)}{\partial \delta x_i}$$

$\bar{F} = 0$ because we do not have damping

T

U

δL

- $T = \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2$

• $\bar{U} = \frac{1}{2} k_I \left(\theta_1 - \underbrace{\cancel{\theta_0}}_{=0}\right)^2 + \frac{1}{2} k_{II} \left(\theta_2 - \theta_1\right)^2$

$$k_I = \frac{G I_{p1}}{l_I} \qquad I_{p1} = \frac{\pi d_I^4}{32} = 2.6 \cdot 10^{-9} \, m^4$$

$$k_{II} = \frac{G I_{p2}}{l_{II}} \qquad I_{p2} = \frac{\pi d_{II}^4}{32} = 3.9 \cdot 10^{-9} \, m^4$$

$$k_I = 400.4 \, \frac{N \cdot m}{rad}$$

$$k_{II} = 300.3 \, \frac{N \cdot m}{rad}$$

T not depending on θ

V // // // $\dot{\theta}$

so

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_i}\right) + \frac{\partial U}{\partial \theta_i} + \frac{\partial \bar{F}}{\partial \dot{\theta}_i} = \frac{\partial \delta L}{\partial \delta \theta_i}$$

$\theta_i = \theta_1$ or $\theta_i = \theta_2$

$$T = \frac{1}{2} J_{p1} \dot{\theta}_1^2 + \frac{1}{2} J_{p2} \dot{\theta}_2^2$$

$$U = \frac{1}{2} k_I (\theta_1 - \underbrace{\cancel{\theta_0}}_{=0})^2 + \frac{1}{2} k_{II} (\theta_2 - \theta_1)^2$$

θ_1

$$\frac{\partial T}{\partial \dot{\theta}_1} = J_{p1} \cdot \dot{\theta}_1 \; ; \; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = J_{p1} \ddot{\theta}_1$$

$$\frac{\partial U}{\partial \theta_1} = k_I \theta_1 + k_{II} (\theta_2 - \theta_1)(-1)$$

$$\boxed{\delta L = M_t \cdot \delta \theta_2} \; ; \; \frac{\partial \delta L}{\partial \delta \theta_1} = 0$$

1st equation :

$$J_{p1}\ddot{\theta}_1 + k_{II}(\theta_2-\theta_1)(-1) = 0$$

$$J_{p1}\ddot{\theta}_1 + k_I\theta_1 + k_{II}\theta_1 - k_{II}\theta_2 = 0$$

$$\boxed{J_{p1}\ddot{\theta}_1 + (k_I + k_{II})\theta_1 - k_{II}\theta_2 = 0}$$

θ_2] $\dfrac{\partial T}{\partial \dot{\theta}_2} = J_{p2}\dot{\theta}_2$; $\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial \dot{\theta}_2}\right) = J_{p2}\ddot{\theta}_2$

$$\frac{\partial U}{\partial \theta_2} = k_{II}(\theta_2-\theta_1)$$

$$\frac{\partial \delta L}{\partial \delta\theta_2} = \frac{\partial(M_t \cdot \delta\theta_2)}{\partial \delta\theta_2} = M_t$$

$$\boxed{J_{p2}\ddot{\theta}_2 - k_{II}\theta_1 + k_{II}\theta_2 = M_t}$$

$$\begin{bmatrix} J_{p1} & 0 \\ 0 & J_{p2} \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_I + k_{II} & -k_{II} \\ -k_{II} & k_{II} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_t \end{Bmatrix}$$

$$[M] \qquad \{\ddot{\theta}\} \qquad\qquad [k] \qquad \{\theta\} = \{F\}$$

$$[M]\{\ddot{\theta}\} + [k]\{\theta\} = \{0\}$$

$$\{\theta\} = \{\theta_0\} e^{i\omega t} \quad ; \quad \{\dot{\theta}\} = + i\omega \{\theta_0\} e^{i\omega t}$$

$$\ddot{\theta} = -\omega^2 \{\theta_0\} e^{i\omega t}$$

$\{\theta_0\}$ and ω are the values that we have to compute

$$\left(-\omega^2 \begin{bmatrix} J_{p1} & 0 \\ 0 & J_{p2} \end{bmatrix} + \begin{bmatrix} k_I + k_{II} & -k_{II} \\ -k_{II} & k_{II} \end{bmatrix} \right) \begin{Bmatrix} \theta_{01} \\ \theta_{02} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\det\left(-\omega^2\begin{bmatrix} J_{p1} & 0 \\ 0 & J_{p2}\end{bmatrix} + \begin{bmatrix} k_I + k_{II} & -k_{II} \\ -k_{II} & k_{II}\end{bmatrix}\right) = 0$$

$$\left|\begin{array}{c|c} -\omega^2 J_{p1} + k_I + k_{II} & -k_{II} \\ \hline -k_{II} & k_{II} - J_{p2}\,\omega^2 \end{array}\right| = 0$$

$$\left(-\omega^2 J_{p1} + k_I + k_{II}\right)\left(k_{II} - J_{p2}\,\omega^2\right) - k_{II}^2 = 0$$

$$\left(J_{p1}\cdot J_{p2}\right)\omega^4 - \left(J_{p1}\cdot k_{II} + J_{p2} k_I + J_{p2}\, k_{II}\right)\omega^2 +$$

$$+ k_I \cdot k_{II} = 0$$

$$24\,\omega^4 - 46.02\cdot\omega^2 + 1.2024\cdot 10^5 = 0$$

$$a x^4 + b x^2 + c = 0$$

$$* \quad \omega_{1/2}^2 = \begin{cases} 31.18 \left(\frac{rad}{s}\right)^2 \Rightarrow \omega_{m1} = 5.6 \frac{rad}{s} \\ 160.68 \left(\frac{rad}{s}\right)^2 \Rightarrow \omega_{m2} = 12.7 \frac{rad}{s} \end{cases}$$

$$\left[\begin{array}{c|c} -\omega^2 J_{p1} + k_I + k_{II} & -k_{II} \\ \hline -k_{II} & -\omega^2 J_{p2} + k_{II} \end{array}\right] \begin{Bmatrix} \Theta_{01} \\ \Theta_{02} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Theta_{02} = 1$$

$$\boxed{\left(-\omega^2 J_{p1} + k_I + k_{II}\right) \Theta_{01} - k_{II} = 0}$$

$$\boxed{-k_{II}\Theta_{01} - \omega^2 J_{p2} + k_{II} = 0}$$

$$\Theta_{01} = \frac{-\omega^2 J_{p2} + k_{II}}{k_{II}} = 1 - \omega^2 \frac{J_{p2}}{k_{II}}$$

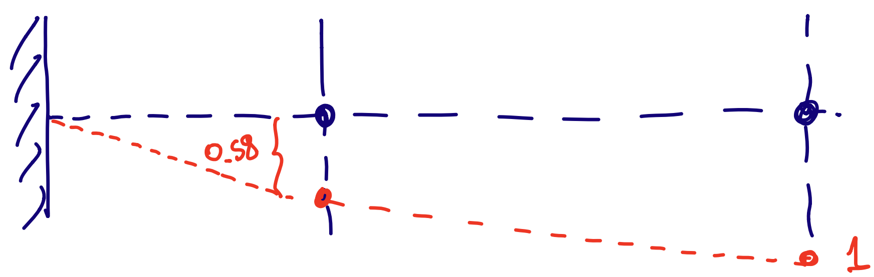
$$\Theta_{01} = 1 - 1.332 \cdot 10^{-2} \omega^2$$

$$\omega = \omega_{m1} = 5.6 \ \frac{rad}{s} \Rightarrow \theta_{01} = 0.58$$

$$\omega = \omega_{m2} = 12.7 \ \frac{rad}{s} \Rightarrow \theta_{01} = -0.88$$

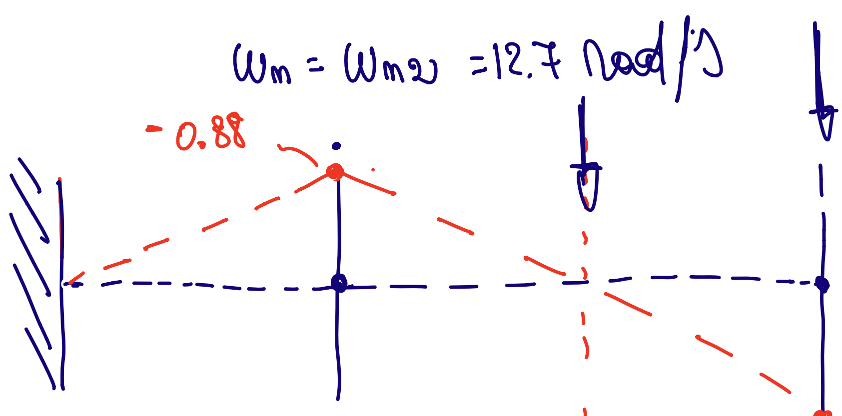
$$\omega_{m1} \Rightarrow \begin{Bmatrix} 0.58 \\ 1 \end{Bmatrix} ; \quad \omega_{m2} \Rightarrow \begin{Bmatrix} -0.88 \\ 1 \end{Bmatrix}$$

$$\omega = \omega_{m1} = 5.6 \ rad/s$$



$\theta_{01} = 0.58$ $\theta_{02} = 1$

$$\omega_m = \omega_{m2} = 12.7 \ rad/s$$



$\theta_{01} = -0.88$ $\theta_{02} = 1$

MODAL ANALYSIS

$$\text{mode } 1 \Leftrightarrow \omega_{m1} = 5.6 \frac{\text{rad}}{s} \Rightarrow \begin{Bmatrix} \theta_{01} \\ \theta_{02} \end{Bmatrix} = \begin{Bmatrix} 0.58 \\ 1 \end{Bmatrix}$$

$$U_1 = \frac{1}{2} \theta_{01}^2 k_I + \frac{1}{2} k_{II} (\theta_{02} - \theta_{01})^2$$

$$U_{11} = \frac{\frac{1}{2} \theta_{01}^2 k_I}{U_1} = 0.72$$

$$U_{12} \quad \frac{\frac{1}{2} k_{II} (\theta_{02} - \theta_{01})^2}{U_1} = 0.28$$

$$T_1 = \frac{1}{2} J_{p1} (\omega_{m1} \cdot \theta_{01})^2 + \frac{1}{2} (\omega_{m1} \theta_{02})^2$$

$$T_{11} = \frac{\frac{1}{2} J_{p1} (\omega_{m1} \theta_{01})^2}{T_1} = 0.3354$$

$$T_{12} = \frac{\frac{1}{2} J_{p2} \left(\omega_{n1} \Theta_{02}\right)^2}{T_1} = 0.6646$$

$$\text{mode } 2 \Rightarrow \omega_{n2} = 12.7 \frac{\text{rad}}{s} \;;\; \begin{Bmatrix} \Theta_{01} \\ \Theta_{02} \end{Bmatrix} = \begin{Bmatrix} -0.88 \\ 1 \end{Bmatrix}.$$

$$V_{21} = 0.274$$

$$V_{22} = 0.726$$

FORCED RESPONSE

$$\begin{bmatrix} J_{p1} & 0 \\ 0 & J_{p2} \end{bmatrix} \begin{Bmatrix} \ddot{\Theta}_1 \\ \ddot{\Theta}_2 \end{Bmatrix} + \begin{bmatrix} k_I + k_{II} & -k_{II} \\ -k_{II} & k_{II} \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ M_0 \sin \omega t \end{Bmatrix}$$

$$\begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} \Theta_{01} \\ \Theta_{02} \end{Bmatrix} \sin \omega t$$

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} = \begin{Bmatrix} \theta_{01} \\ \theta_{02} \end{Bmatrix} \cos \omega t \, (\omega)$$

$$\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} = -\begin{Bmatrix} \theta_{01} \\ \theta_{02} \end{Bmatrix} \sin \omega t \, (\omega^2)$$

$$\left(-\omega^2 \begin{bmatrix} J_{p1} & 0 \\ 0 & J_{p2} \end{bmatrix} + \begin{bmatrix} k_I + k_{II} & -k_{II} \\ -k_{II} & k_{II} \end{bmatrix} \right) \begin{Bmatrix} \theta_{01} \\ \theta_{02} \end{Bmatrix} \cancel{\sin \omega t} =$$

$$= \begin{Bmatrix} 0 \\ M_0 \end{Bmatrix} \cancel{\sin \omega t}$$

$$\begin{bmatrix} 316.7 & -300.3 \\ -300.3 & 44.7 \end{bmatrix} \begin{Bmatrix} \theta_{01} \\ \theta_{02} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 100 \end{Bmatrix}$$

$$\theta_{01} = -0.394 \text{ rad}$$

$$\theta_{02} = -0.416 \text{ rad}$$

$$\tau_I = \frac{M_t}{I_{p1}} \frac{d_I}{2} = \frac{k_I(\theta_1-\theta_0)}{I_{p1}} \frac{d_I}{2} = 390 \text{ MPa}$$

$$\tau_{II} = \frac{M_t}{I_{p2}} \frac{d_I}{2} = \frac{k_{II}(\theta_2-\theta_1)}{I_{p2}} \frac{d_{II}}{2} \approx 16 \text{ MPa}$$

STATE SPACE

$$[M]\{\ddot{\theta}\} + [K]\{\theta\} = \begin{Bmatrix} 0 \\ M_t \end{Bmatrix} \text{ input}$$

$$\left.\begin{matrix} \tau_I \\ \tau_{II} \end{matrix}\right\} \text{OUTPUT}$$

$$\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} = -\begin{bmatrix} M \end{bmatrix}^{-1} \begin{bmatrix} K \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} + \begin{bmatrix} M \end{bmatrix}^{-1} \begin{Bmatrix} 0 \\ M(t) \end{Bmatrix}$$

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix}$$

$$\begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} = \left[\begin{array}{c|c} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & -\begin{bmatrix} M \end{bmatrix}^{-1} \begin{bmatrix} K \end{bmatrix} \\ \hline \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}\right] \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \theta_1 \\ \theta_2 \end{Bmatrix} + \begin{bmatrix} \begin{bmatrix} M \end{bmatrix}^{-1} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{bmatrix} \begin{Bmatrix} 0 \\ M(t) \end{Bmatrix}$$

$$\underset{(4\times 1)}{\{\dot{z}\}} = \underset{(4\times 4)}{\begin{bmatrix} A \end{bmatrix}} \underset{(4\times 1)}{\{z\}} + \underbrace{\underset{(4\times 2)}{\begin{bmatrix} B \end{bmatrix}} \underset{(2\times 1)}{\{u\}}}_{(4\times 1)}$$

$$\tau_I = \frac{M_t}{I_{pI}} \frac{d_I}{2} = \frac{k_I(\theta_1 - \cancel{\theta_0})}{I_{pI}} \frac{d_I}{2}$$

$$\tau_I = \frac{k_I \cdot d_I}{2 I_{pI}} \cdot \theta_1$$

$$\tau_{II} = \frac{M_t}{I_{pII}} \frac{d_{II}}{2} = \frac{k_{II}(\theta_2 - \theta_1)}{I_{pII}} \frac{d_{II}}{2} =$$

$$= -\frac{k_{II}\, d_{II}}{2 I_{pII}} \theta_1 + \frac{k_{II}\, d_{II}}{2 I_{pII}} \theta_2$$

$$\{y\} = [C]\{z\} + [D]\{u\}$$

$$\begin{Bmatrix} \tau_I \\ \tau_{II} \end{Bmatrix} = \underset{[C]}{\begin{bmatrix} 0 & 0 & \frac{k_I \cdot d_I}{2 I_{pI}} & 0 \\ & & -\frac{k_{II}\, d_{II}}{2 I_{pII}} & \frac{k_{II}\, d_{II}}{2 I_{pII}} \end{bmatrix}} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \theta_1 \\ \theta_2 \end{Bmatrix} +$$

$$+ \underset{[D]}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \begin{Bmatrix} 0 \\ M_t \end{Bmatrix}$$

- determine how the bar diameter should be modified to decrease the stress. A design constraint imposes that $d_m \leq 1.25d$, where d_m is the modified diameter.

Data: l_1 = 0.4 m, l_2 = 0.2 m, d = 0.02 m, $E = 2.06 \cdot 10^{11}$ Pa, m_1 = 50 kg, m_2 = 20 kg, ω = 1700 rad/s, F_0 = 2000 N.

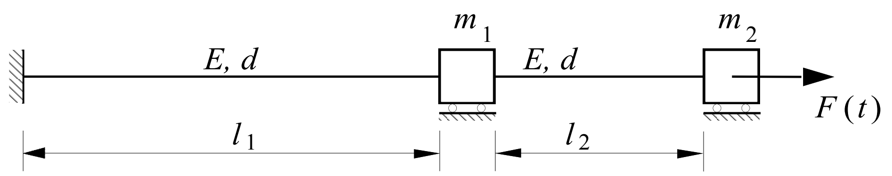


Figure 2

$x_{0,1} = -4.59 \cdot 10^{-5}$ m, $x_{0,2} = -4.89 \cdot 10^{-5}$ m.

EXERCISE 3

Consider the belt drive system represented in Figure 3. The main data are reported in Table 1.

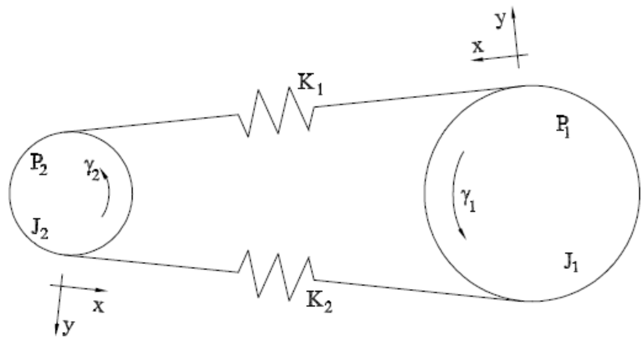


Figure 3.

$J_1 = 0.2$	Kgm^2
$J_2 = 0.15$	Kgm^2
$r_1 = 0.07$	m
$r_2 = 0.04$	m
$l = 0.5$	m

Table 1.

The Young modulus E of the belt in the axial direction is equal to $2.02 \cdot 10^9$ N/m^2 while the transversal section area A is equal to $4.26 \cdot 10^{-5}$ m^2.

It is requested to compute the two natural frequencies of the systems.

$\omega_1 = 0$ Hz, $\omega_2 = 17.5$ Hz,

EXERCISE 4

For the two degree of freedom system depicted in Figure 4

- determine the equations of dynamic equilibrium using the Langangian approach,
- arrange the equations in matrix form,
- find the natural frequencies and the mode shapes of the system,
- compute the modal matrices (modal mass and modal stiffness),
- mass-normalize the mode shapes,
- Check the mass-orthogonality of the eigenvectors.

Make the calculation using the following two sets of data

$k_1 = k_2 = 400000$ N/m	$k_3 = 800000$ N/m;	$m_1 = m_2 = 2$ kg	
$k_1 = 400000$ N/m	$k_2 = 200000$ N/m	$k_3 = 800000$ N/m; $m_1 = 2$ kg	$m_2 = 2.5$ kg

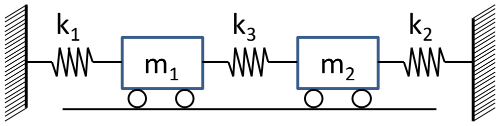


Figure 4

EXERCISE 5

A dynamic system with three flywheels is depicted in Figure 5. Neglecting the inertia of the shafts

- write the dynamic equations of equilibrium
- determine natural frequencies and mode shapes of the system (torsional motion only)

d_1=30 mm d_2=20 mm
L_1=100 mm L_2=80 mm
m_1=1 kg m_2=0.5 kg
m_3=2 kg
$G=7.7*10^{10}$ N/m^2

D_1=120 mm D_2=60 mm D_3=100 mm

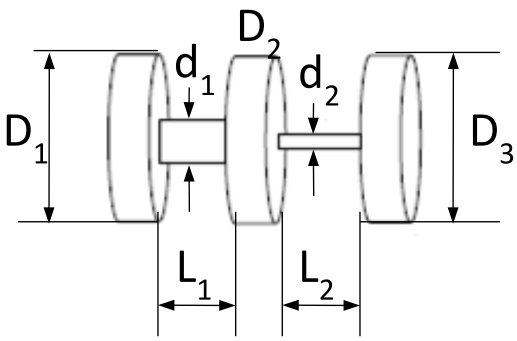


Figure 5

EXERCISE 6

For the two degree of freedom depicted in the Figure 6

- determine the equations of dynamic equilibrium (Lagrange approach).
- Find natural frequencies and modal shapes of the system.
- Use the eigenvectors to compute the modal matrices.

Data

$k_1 = k_2 = 20000$ N/m

m=950 kg IC=1400 kg m^2

L_1=1.0 m L_2=1.5 m

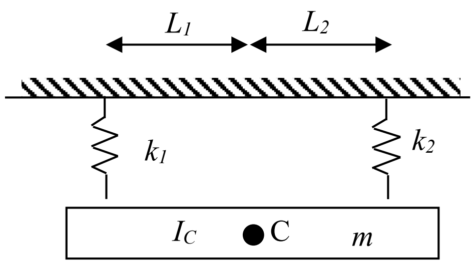


Figure 6.

[ω_1=5.9 rad/s, ω_2=7.3 rad/s, $(Y/A)_1$= -1.55, $(Y/A)_2$= 0.95]

EXERCISE 7

The output to a harmonic excitation of a vibrating structure is described only by the first mode:

$\{q\}_1 = \{1/\sqrt{5} \quad 2/\sqrt{5}\}^T$. Take into account that the modal mass is 2 kg, the modal stiffness is 5000 N/m, the modal force is equal to 10+(2U+P)/3 N. The excitation frequency is equal to $0.8\omega_1$, and the damping is negligible. Compute the amplitude of oscillation of the first degree of freedom.

U= last digit of your registration n°

P= digit before the last of your registration n°

x_{01} = ... m

EXERCISE 8

A two degrees of freedom undamped system has the following modal shapes:

$\{q\}_1 = \{1 \quad 0.8165\}^T$, $\{q\}_2 = \{1 \quad -0.8165\}^T$.

The stiffness matrix, the mass matrix and the vector of the excitation forces are:

$$[K] = 10^3 \begin{bmatrix} 10 & -8 \\ -8 & 15 \end{bmatrix} \frac{\mathrm{N}}{\mathrm{m}}, \quad [M] = \begin{bmatrix} 4 & 0 \\ 0 & 6 \end{bmatrix} \mathrm{kg}, \quad \{F\} = 10^3 (1 + \frac{2U+P}{20}) \begin{Bmatrix} 5 \\ 8 \end{Bmatrix} \mathrm{N}$$

Compute the modal force corresponding to the first mode.

U= last digit of your registration n°

P= digit before the last of your registration n°

$\overline{F}_1$ = ... N

EXERCISE 9

Consider a vibrating system. The dynamic response is characterized using only the first mode $\{q\}_1 = \{1/\sqrt{7} \ 2/\sqrt{7}\}^T$. The modal mass is equal to 3 kg, the modal stiffness is equal to 3000 N/m, the modal force is equal to 10+(2*U*+*P*)/3 N. The excitation frequency is equal to $0.8 \cdot \omega_1$, the damping is negligible. Compute the amplitude of oscillation of the degree of freedom n° 1.

U= last digit of your registration n°
P= digit before the last of your registration n°

x_{01} = …………… m

EXERCISE 10

Consider the system reported in the picture.

The eigenvector matrix is

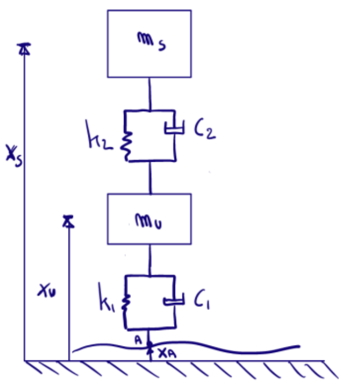
$$\emptyset = \begin{bmatrix} -1 & 0.01 \\ -0.1 & -1 \end{bmatrix};$$

$$x = [x_u,\ x_s]';$$

The vector η at $\omega=\omega_{n1}$ is equal to:

$$\eta = [5*10^{-3},\ 2*10^{-3}]';$$

Compute the displacement of the x_s at the first natural frequency.
x_s = ……………………… [m]



MULTIPLE CHOICE PROBLEMS: select the right answer. Explain in few sentences (eventually with the support of graphs, schems and formulas) the reason of the correct answer.

1) The Lagrange equations can be written
   [A] only for undamped systems,
   [B] only for damped and undamped systems subject to vibration
   [C] also for non conservative systems.

2) For a damped multi degree of freedom system it is not possible to obtain a complete modal decoupling. This sentence is
   [A] Always correct
   [B] Never correct
   [C] Not correct only in some cases.

3) An external excitation applied to a dynamic system is considered quasi static if
   [A] Its frequency is much lower with respect to the lowest natural frequency of the dynamic system.
   [B] Its frequency is equal or lower with respect to the lowest natural frequency of the dynamic system.
   [C] Its frequency is equal or higher with respect to the lowest natural frequency of the dynamic system.

4) A gyroscopic matrix present in the equations of motion of a mechanical system
   [A] Is symmetric
   [B] Is skew symmetric and may never cause instability.
   [C] Is skew symmetric and may cause instability.

5) The solution of the homogeneous equation describes
   [A] the behavior of the dynamic system that is free to move;
   [B] the behavior of the dynamic system after a step input;
   [C] the behavior of the dynamic system excited by a harmonic inputs.