RTG 2575:
Rethinking
Quantum Field Theory

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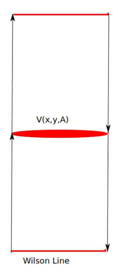
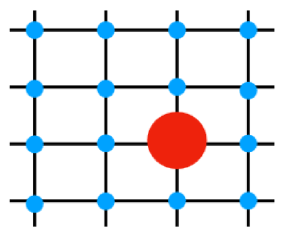
Perturbative bootstrap of the Wilson-line defect CFT

Motivation and Summary

- The study of conformal defects is relevant in the study of different phenomena at both high and low energy, such as



confinement in gauge theories



magnetic impurities in condensed matter systems

- The introduction of defects allows for a controlled symmetry breaking.
- Line defects preserve 1-dimensional conformal symmetry.

$$SO(d+1,1) \to SO(p+1,1) \otimes SO(d-p)$$

We focus on two type of correlators at weak coupling

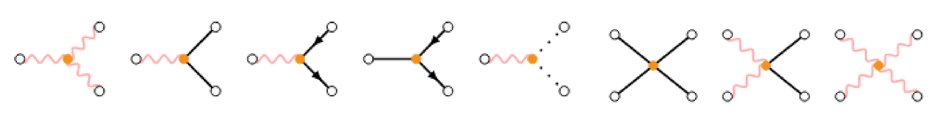
- multipoint line correlators, of high interest in the bootstrap program [1-8]
- bulk-defect-defect correlators, a simple set-up rarely studied and strongly constrained [9]

Are these correlators fully solved by symmetry?

Set up: the Wilson-line defect

We work in $\mathcal{N}=4$ SYM

$$\text{Field content}: 1A_\mu, 4\psi, 6\phi_i$$



Introducing defect operators extends the set of conformal data we can compute. Starting from $\mathcal{N}=4$ SYM we can preserve half of the supercharges by defining the Wilson-line defect [1]

$$\mathcal{W}_\ell = \frac{1}{N}\text{tr}\mathcal{P}\exp\int_{-\infty}^{+\infty} d\tau\,\left(i\dot{x}^\mu A_\mu + |\dot{x}|\theta^i\phi_i(\tau)\right),\quad \theta^2=1$$

with new Feynman vertices

The residual supersymmetry in this frameword allows for the calculation of correlators both at weak and strong coupling with different techniques: examples are dispersion relations two-point bulk correlators [6] and bootstrability for multi-point defect correlators [7-8].

Defining the correlators

Built of single-trace half-BPS bulk operators and line protected operators

$$\mathcal{O}_\Delta(u,x) := \text{tr}\,(u\cdot\phi(x))^\Delta \quad \hat{\mathcal{O}}_{\hat{\Delta}}(\hat{u},\tau) := \mathcal{W}_\ell[(\hat{u}\cdot\phi(\tau))^{\hat{\Delta}}]$$

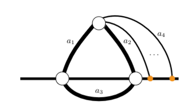
Constructed by inserting the line fields in $\mathcal{W}_\ell[\ldots]$ inside the trace of the Wilson line [1].

- Multipoint defect correlators

$$\langle\,\hat{\mathcal{O}}_{\hat{\Delta}_1}\ldots\hat{\mathcal{O}}_{\hat{\Delta}_n}\,\rangle := \frac{1}{N}\langle\,\text{tr}\mathcal{P}\,[\hat{\mathcal{O}}_{\hat{\Delta}_1}\ldots\hat{\mathcal{O}}_{\hat{\Delta}_n}\exp\int d\tau\,\left(i\dot{x}_\mu A_\mu + |\dot{x}|\phi^6\right)]\,\rangle_{4d}\,,$$

- Bulk-defect-defect correlators

$$\langle\,\mathcal{O}_{\Delta_1}\hat{\mathcal{O}}_{\hat{\Delta}_2}\hat{\mathcal{O}}_{\hat{\Delta}_3}\,\rangle = \frac{1}{N}\langle\,\mathcal{O}_{\Delta_1}\text{tr}\mathcal{P}\left[\hat{\mathcal{O}}_{\hat{\Delta}_2}\hat{\mathcal{O}}_{\hat{\Delta}_3}\exp\int_{-\infty}^{\infty} d\tau\,(iA_0(\tau)+\phi^6(\tau)\right]\,\rangle_{4d}$$



The workflow

- Fix the correlator dependence from external variables
- Apply all non perturbative constraints to reduce the number of Feynman diagrams to compute
- Using the minimum amount of perturbative information, fix the entire correlator

Multi-point defect correlators

Non-perturbative constraints

- Conformal symmetry: the correlator is a function of $n-3$ cross-ratios; in higher dimension the growth is quadratic with n

$$\langle\,\hat{\mathcal{O}}_{\hat{\Delta}_1}\ldots\hat{\mathcal{O}}_{\hat{\Delta}_n}\,\rangle = \mathcal{K}\mathcal{F}(\{\chi,r,s,t\})$$

where r_i, s_j and t_{ij} are the R-symmetry cross ratios.

- Crossing symmetry [3] the correlator is symmetric under $\langle\,\hat{\mathcal{O}}_1(\tau_1)\ldots\hat{\mathcal{O}}_1(\tau_n)\,\rangle = \langle\,\hat{\mathcal{O}}_1(\tau_n)\ldots\hat{\mathcal{O}}_1(\tau_1)\,\rangle$
- Superconformal Ward identities [3-4][9][16-17]: the function $\mathcal{F}(\{\chi,r,s,t\})$ is annihilated by the differential operator

$$\sum_{i\neq j}^{n_R}\beta_i\left(\frac{1}{2}\partial_{x_i}+\alpha_i\partial_{r_i}-\bar{\alpha}_i\partial_{s_i}+\alpha_{ij}\partial_{t_{ij}}\right)\mathcal{A}_{\Delta_1\ldots\Delta_n}\big|_{r_i\to\alpha_i x_i, s_i\to\bar{\alpha}_i\bar{x}_i, t_{ij}\to\alpha_{ij}x_{ij}} = 0\,,$$

which are more general than the ones previously conjectured in [3-4].
For the 4-point function, the solution to the WI reduces three R-symmetry channels to one function

$$\mathcal{F}(\chi,r,s,t) = \mathbb{F}_4 + \frac{\partial}{\partial\chi}\left(\frac{(1-\chi)(r-\chi^2)+\chi(s-(1-\chi)^2)}{(1-\chi)-\chi}f(\chi)\right)$$

For the 5-point function, after applying the Ward identities and eliminating $G_{1,\ldots,4}$, the five-point function solution takes the schematic form

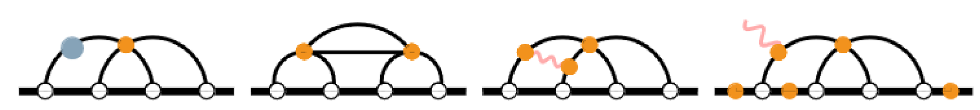
$$\mathcal{A}_{11112} = \frac{t_{12}}{x_{12}^2}\mathbb{F}_{11112} + \rho f_1 + \partial_{x_1}(\xi_1 f_1) + \partial_{x_2}(\xi_2 f_1) + \partial_{x_2}(\eta f_2)\,,$$

while for the 6-point punction the solution of the Ward identities reduces the 15 $R-$symmetry channels to 4 functions related by crossing and one that is zero at NNLO.

Perturbative constraints and results

- 4-point $\langle\,\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\hat{\mathcal{O}}_1\,\rangle$

We manage to reproduce the NNLO bootsrablity result in [7-8] by computing the easiest set of Feynamn diagrams to $O(\chi^0)$, $O(\chi\log\chi)$ and $O(\chi\log^2\chi)$

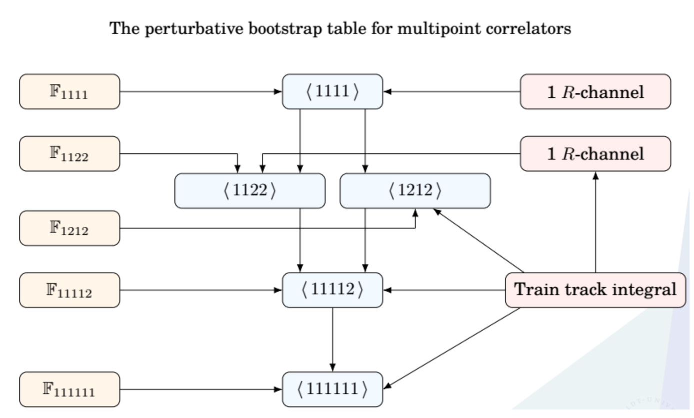


The result can be written as a sum of HPLs as [7-9]

$$\begin{aligned} f^{(2)}(\chi) = {} & 4H_{1,3}-4H_{2,2}+8H_{3,0}+8H_{3,1}-8H_{1,1,2}+4H_{1,2,0}+8_{H1,2,1}-8H_{2,0,0} \\ & -4H_{2,1,0}-8H_{1,1,0,0}+\frac{4}{3}\pi^2H_2-\frac{4}{3}\pi^2H_{1,0}+\frac{4}{3}\pi^2H_{1,1}-12\zeta_3H_1+\frac{8\pi^4}{15}\chi \end{aligned}$$

- Multipoint correlators at NNLO

The 5- and 6-point correlators at NNLO are uniquely determined by one 6-point polylogarithmic integral and lower point functions.



Bulk-defect-defect correlators

Non-perturbative constraints

- Superconformal symmetry fixes the correlator to depend only one space-time and one R-symmetry cross ratio

$$\langle\,\mathcal{O}_{\Delta_1}(u_1,x_1)\hat{\mathcal{O}}_{\hat{\Delta}_2}(\hat{u}_2,\tau_2)\hat{\mathcal{O}}_{\hat{\Delta}_3}(\hat{u}_3,\tau_3)\,\rangle \sim \sum_{j=0}^{n}\left(\frac{\zeta}{\chi}\right)^j F_j(\chi)\,.$$

- The Ward identity takes the form [10]

$$\left(\partial_\chi+\partial_\zeta\right)\mathcal{A}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}(\zeta;\chi)\bigg|_{\zeta=\chi} = 0\,.$$

whose solution is simply $\sum_{j=0}^{n}F_j(\chi) = \mathbb{F}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}$.

- Locality[18-19] prescribes the correlators to have branch cuts only for $\chi\to 0^-$, when the two operators cross.

Can we fix more elements via block expansions or intregrated constraints?

Perturbative constraints and results

- For any three operators, the number of non-zero channels grows with the perturbative order until it reaches the total

$$\langle\,\mathcal{O}_4(u_1,x_1)\hat{\mathcal{O}}_2(\hat{u}_2,\tau_2)\hat{\mathcal{O}}_2(\hat{u}_3,\tau_3)\,\rangle \sim F_0(\chi)\text{ starts at LO}$$
$$+\left(\frac{\zeta}{\chi}\right)F_1(\chi)\text{ starts at NLO} + \left(\frac{\zeta}{\chi}\right)^2 F_2(\chi)\text{ starts at NNLO}$$

- Up to NLO the solution to the WI fixes all correlators up to simple integrals expressed as trigonometric functions. [10]

$$\mathcal{A}_{211}(\zeta;\chi) = \mathbb{F}_{211} + \frac{\lambda^3}{512\pi^6 N}\frac{1}{\sqrt{n_2}\hat{n}_1}\left(\pi^2+4\arccos^2\left(\sqrt{\chi}\right)\right)\left(\frac{\zeta}{\chi}-1\right)$$

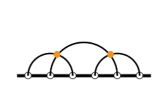
- The absence of $\log\chi$ in the series expansion can be explained by cancellations of coefficients in the superblock expansion and has been proven for all correlators using locality constraints

A redefinition of the cross-ratio is promising for finding NNLO results.

Outlook

We plan to test this method in different follow-ups, possibly including

- Supersymmetry is useful but not crucial, can we use the information we have from the 6-point diagram



to study a line defect in Fishnet, still integrable?

- $O(N)$ model and Yukawa theory in $d=4-\epsilon$ [5][13-15]
- defects of different dimension: surfaces, boundaries

References

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