PERTURBATIVE BOOTSTRAP

of the Wilson line defect CFT

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based on the work with J. Barrat, G. Peveri, Y. Xu

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Rethinking
Quantum Field Theory

PAPERS FOR THE CLUB

Perturbative bootstrap of the Wilson-line defect CFT: multipoint correlators

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Perturbative bootstrap of the Wilson-line defect CFT: bulk-defect-defect correlators

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MOTIVATIONS

WHY DEFECTS

"[...] all the available tools in Quantum Field Theory have been applied to this subject [defects]: perturbation theory, lattice, conformal bootstrap, holographic methods, supersymmetric localization, and more"
from the webpage of the workshop *Defects, from condensed matter to quantum gravity*, Pollica 2024

$\mathcal{N} = 4$ SUSY set-up

- AdS/CFT correspondence
  [Giombi, Roiban, Tseytlin, '17][Erikson, Semenoff, Zarembo, '00]
- Supersymmetric localization
  [Giombi, Komatsu, '18][Drukker, '20]
- Superconformal Ward identity
  [Liendo, Meneghelli, '16][Liendo, Meneghelli, Mitev, '18][Barrat, Liendo, Peveri, Plefka, '21][Bliard, '24]
- Superconformal bootstrap
  [Ferrero, Meneghelli, '23][Ferrero, Meneghelli, '23][Ferrero, Meneghelli, '23]
- Bootstrability
  [Cavaglià, Gromov, Julius, Preti, '22][Cavaglià, Gromov, Julius, Preti, '22][Cavaglià, Gromov, Julius, Preti, '22][Cavaglià, Gromov, Julius, Preti, '22]

non-SUSY set-up

- Conformal bootstrap
  [Billò, Goncalves,Lauria,Meineri, '17][Poland, Rychkov, Vichi'18][Iliesiu, Kologlu, Mahajan, Perlmutter, Simmons-Duffin, '18]
- Analytic bootstrap for $O(N)$ model
  [Bianchi, Bonomi, de Sabbata, '22][Bianchi, Bonomi, de Sabbata, Gimenez-Grau, '23]
- Locality
  [Levine, Paulos, '23][Levine, Paulos, '24]
- ϵ-expansion
  [Cuomo, Komargodski, Mezei, '21][Barrat, Liendo, Van Vliet '23][Giombi, Helfenberger, Khanchandani, '23]
- Applications to gravity
  [Bachas, Chen, '24]

WHY $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM: the maximally extended supersymmetric theory in four space-time dimensions with a multiplet representation of spin $\ell \leq 1$.

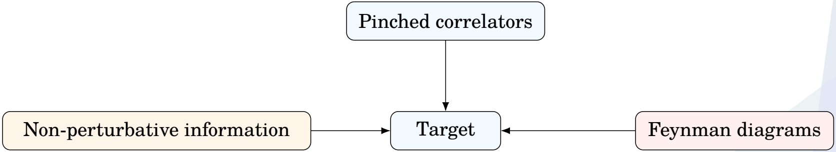
Although the ultimate goal may be calculating non-SUSY amplitudes, SUSY theories provide an excellent testing ground. Looking at super-Yang-Mills offers a lot of insight into how one can deal with the problems in QCD.

Some selected topics of interest

- Leading transcendental weight contribution identical in QCD and $\mathcal{N} = 4$ SYM
- Presence of exact and all order results (e.g. Γ_{cusp} [Beisert, Eden, Staudacher, '07], 4-gluon amplitude [Anastasiou, Bern, Dixon, Kosower '03])
- Duality between amplitudes and Wilson loops [Alday, Maldacena, '07]
- Position space description of event shapes [Henn, Sokatchev, Yan, Zhiboedov, '19]
- Analytic bootstrap [Bern, Dixon, Dunbar, Kosower, '94]

THE PERTURBATIVE BOOTSTRAP

The perturbative bootstrap framework puts together Feynman diagram computation for weak coupling expansion of correlators and non-perturbative information given by the symmetries of the set-up.



SET-UP AND WORKFLOW

LINE DEFECTS

Defects: extended operators preserving a specific subgroup of the original symmetry.

p-dimensional defects preserve a p-dimensional conformal symmetry too.

$$SO(d+1,1) \longrightarrow SO(p+1,1) \otimes SO(q)$$

A defect extends the set of conformal data to compute

BULK CFT

- $\langle \mathcal{O}_i \mathcal{O}_j \rangle \rightarrow$ anomalous dimension γ_i
- $\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle \rightarrow$ 3-point coefficient λ_{ijk}

DEFECT CFT

- $\langle \hat{\mathcal{O}}_i \rangle \rightarrow$ anomalous dimension $\hat{\gamma}_i$, 1-point coefficient a_i
- $\langle \hat{\mathcal{O}}_i \mathcal{O}_j \rangle \rightarrow$ 2-point coefficient b_{ij}
- $\langle \hat{\mathcal{O}}_i \hat{\mathcal{O}}_j \hat{\mathcal{O}}_k \rangle \rightarrow$ 3-point coefficient $\hat{\lambda}_{ijk}$

WILSON LINE DEFECT

Conformal line defects preserve some of the original conformal symmetry

$$SO(5,1) \to SO(2,1) \otimes SO(3)$$

Starting from $N = 4$ SYM, we can also preserve half of the supercharges by defining the Wilson line defect

$$\mathcal{W}_\ell = \frac{1}{N} \mathrm{tr}\, \mathcal{P} \exp \int_{-\infty}^{+\infty} d\tau \left(i\dot{x}^\mu A_\mu + |\dot{x}| \theta^i \phi_i(\tau) \right)$$

where $\theta^2 = 1$ to preserve half of the supercharges.
The expectation value of the Wilson line is protected.

$$\langle \mathcal{W}_\ell \rangle = 1$$

The resulting supersymmetry break is

$$\boxed{PSU(2,2|4) \to OSP(4^*|4) \supset \text{1d CFT}}$$

SYMMETRY BREAKING [Liendo, Meneghelli '16]

$$PSU(2,2|4) \to OSP(4^*|4)$$

$N=4$ SYM

- 32 supercharges
- $SO(5,1)$ conformal symmetry
- $SO(6)$ R-simmetry

Wilson line defect

- 16 supercharges
- $SO(2,1)$ conformal symmetry
- $SO(5)$ R-simmetry

$$\left\{\phi^1, ..., \phi^6\right\} \longrightarrow \left\{\phi^1, ..., \phi^5\right\} + \phi^6$$

SCALAR INSERTIONS [Giombi, Roiban, Tseytlin '17]

The operators we consider are scalar insertions on the line

$$\hat{\mathcal{O}}_{\hat{\Delta}}(u,\tau) = \frac{1}{\sqrt{n_{\hat{\Delta}}}} \mathcal{W}_\ell \left[(u \cdot \phi(\tau))^{\hat{\Delta}} \right]$$

where $n_{\hat{\Delta}}$ is a normalization factor and u is the $SO(5)$ vector describing the insertion of the protected scalar fields at the point τ on the line.

We can define multipoint correlation functions for defect operators using the $4d$ theory

$$\left\langle \hat{\mathcal{O}}_{\hat{\Delta}_1} ... \hat{\mathcal{O}}_{\hat{\Delta}_n} \right\rangle_{1d} := \frac{1}{N} \left\langle \operatorname{tr} \mathcal{P}\, \hat{\mathcal{O}}_{\hat{\Delta}_1} ... \hat{\mathcal{O}}_{\hat{\Delta}_n} \exp \int_{-\infty}^{+\infty} d\tau \left(i\dot{x}^\mu A_\mu + |\dot{x}| \phi_6(\tau) \right) \right\rangle_{4d}$$

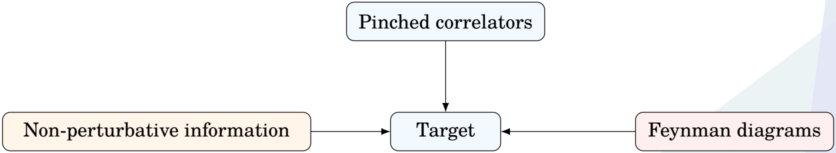
and bulk-defect-defect form factors in a similar fashon

$$\langle \mathcal{O}_{\Delta_1} \hat{\mathcal{O}}_{\hat{\Delta}_2} \hat{\mathcal{O}}_{\hat{\Delta}_3} \rangle = \frac{1}{N} \langle \mathcal{O}_{\Delta_1} \operatorname{tr} \mathcal{P} \left[\hat{\mathcal{O}}_{\hat{\Delta}_2} \hat{\mathcal{O}}_{\hat{\Delta}_3} \exp \int_{-\infty}^{+\infty} d\tau \left(i\dot{x}^\mu A_\mu + |\dot{x}| \phi_6(\tau) \right) \right] \rangle_{4d} \,.$$

PERTURBATIVE BOOTSTRAP: WORKFLOW

The computation of correlation functions consists in the following steps:

1. Use of super-conformal symmetry (Ward identities) to constrain the correlator
2. Design of a suitable ansatz (using symbols) for the solution function
3. Input of perturbative information (minimal set of Feynman diagrams) to constrain the ansatz
4. Input of lower-point results (pinching) to fix the last degrees of freedom



MULTIPOINT CORRELATORS

DEFINITION AND CONVENTIONS

2-point

$$\boxed{\langle \Delta_1 \Delta_2 \rangle = \delta_{\hat{\Delta}_1 \hat{\Delta}_2} (12)^{\hat{\Delta}_1}}\,,$$

$$(ij) = \frac{u_i \cdot u_j}{\tau_{ij}^2}\,.$$

3-point

$$\boxed{\langle \Delta_1 \Delta_2 \Delta_3 \rangle = \lambda_{\Delta_1 \Delta_2 \Delta_3} (12)^{2\Delta_{123}} (23)^{2\Delta_{231}} (31)^{2\Delta_{312}}}\,,$$

$$\Delta_{ijk} = \frac{1}{2}(\Delta_i + \Delta_j - \Delta_k)\,.$$

4-point

$$\boxed{\langle \Delta_1 \Delta_2 \Delta_3 \Delta_4 \rangle = \mathcal{K}_{\Delta_1 \Delta_2 \Delta_3 \Delta_4} \mathcal{A}_{\Delta_1 \Delta_2 \Delta_3 \Delta_4}(x; r, s)}\,,$$

$$x := \frac{\tau_{12}\tau_{34}}{\tau_{13}\tau_{24}}\,, \qquad r := \frac{(u_1 \cdot u_2)(u_3 \cdot u_4)}{(u_1 \cdot u_3)(u_2 \cdot u_4)}\,, \qquad s := \frac{(u_1 \cdot u_4)(u_2 \cdot u_3)}{(u_1 \cdot u_3)(u_2 \cdot u_4)}\,.$$

5-point

$$\boxed{\langle 11112 \rangle = \mathcal{K}_{11112} \mathcal{A}_{11112}(\{x; r, s, t\})}\,,$$

$$\mathcal{A}_{11112} = \sum_{i=1}^{6} R_i F_i(x_1, x_2)\,, \qquad \{R_i\} = \left\{ 1, \frac{r_1}{x_1^2}, \frac{s_1}{\bar{x}_1^2}, \frac{r_2}{x_2^2}, \frac{s_2}{\bar{x}_2^2}, \frac{t_{12}}{x_{12}^2} \right\}\,,$$

6-point

$$\boxed{\langle 111111 \rangle = \mathcal{K}_{111111} \mathcal{A}_{111111}(\{x; r, s, t\})}\,.$$

WHAT IS KNOWN

WEAK COUPLING

- n-point recursion relation up to NLO [Barrat, Liendo, Peveri, Plefka '23]
- 4-point function at NNLO using integrability methods [Cavaglià, Gromov, Julius, Preti, '22]
- ϕ_6 scaling dimension up to 5 loop [Grabner, Gromov, Julius, '20]

STRONG COUPLING

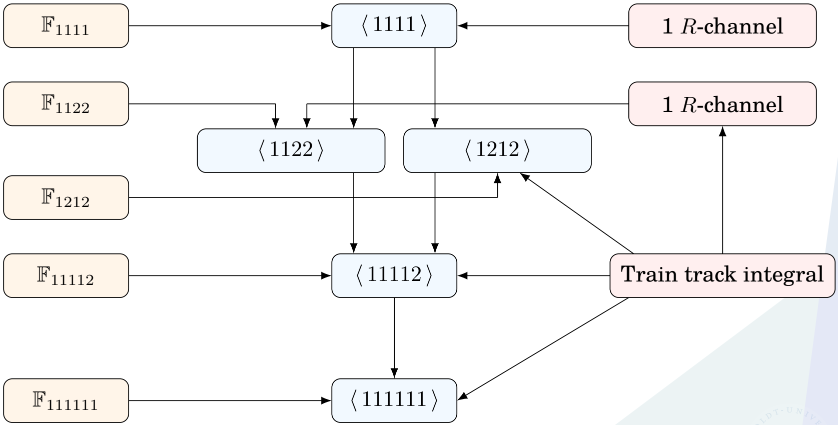
- 4-point function up to $\mathrm{N}^3\mathrm{LO}$ [Ferrero, Meneghelli '22]
- 5-point function up to NLO [Barrat, Bliard, Ferrero, Meneghelli, Peveri, *to appear*]
- ϕ_6 scaling dimension up to 4 loop [Ferrero, Meneghelli '22]

EXACT RESULTS

- Closed form of the topological sector $\mathbb{F}$ [Giombi, Komatsu, '18]
- Precise numerical results for 4-point function [Cavaglià, Gromov, Preti, '23]

PERTURBATIVE BOOTSTRAP: MULTIPOINT

The perturbative bootstrap table for multipoint correlators



NON-PERTURBATIVE CONSTRAINTS - 1

PINCHING

Higher-weight operators formed by bringing together fields from distinct points

$$\left\langle \hat{\mathcal{O}}_{\hat{\Delta}_1}(u_1,\tau_1)\ldots\hat{\mathcal{O}}_{\hat{\Delta}_{n-1}+\hat{\Delta}_n}(u_{n-1},\tau_{n-1})\right\rangle_{1d} =$$
$$\lim_{n\to n-1}\left\langle \hat{\mathcal{O}}_{\hat{\Delta}_1}(u_1,\tau_1)\ldots\hat{\mathcal{O}}_{\hat{\Delta}_{n-1}}(u_{n-1},\tau_{n-1})\hat{\mathcal{O}}_{\hat{\Delta}_n}(u_n,\tau_n)\right\rangle_{1d}$$

CROSSING

Relations arising after identifying the line endpoints at infinity, consisting in the exchange of external points

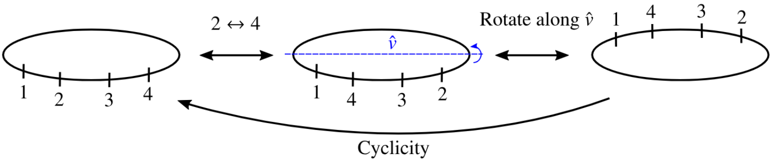


figure adapted from [Liendo, Meneghelli, '18]

NON-PERTURBATIVE CONSTRAINTS - 2

SUPERCONFORMAL WARD IDENTITIES

Set of constraints encoding the effect of superconformal symmetry on correlators
[Bliard, '24][Barrat, Meneghelli, Müller, '24]

$$\sum_{i\neq j}^{n_R} \beta_i \left(\frac{1}{2}\partial_{x_i} + \alpha_i \partial_{r_i} - \bar{\alpha}_i \partial_{s_i} + \alpha_{ij}\partial_{t_{ij}} \right) \mathcal{A}_{\Delta_1 \dots \Delta_n} \Big|_{r_i \to \alpha_i x_i, s_i \to \bar{\alpha}_i \bar{x}_i, t_{ij} \to \alpha_{ij} x_{ij}} = 0\,,$$

The easiest form of the solution to the WI proceeds in (5+1) steps:

1. Introduce a new basis for the correlator:

$$\mathcal{A}_n = \sum_{j=1}^{n_R} \tilde{R}_j G_j(x_1, \dots, x_{n-3})\,.$$

2. Impose the topological limit to be $G_1(x_1, \dots, x_{n-3})$
3. The maximum number of derivatives of G_i do not appear after applying the WI

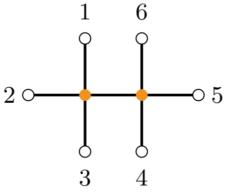
$$\tilde{R}_j \Big|_{r_i \to \alpha_i x_i, s_i \to \bar{\alpha}_i \bar{x}_i, t_{ij} \to \alpha_{ij} x_{ij}} = 0\,.$$

4. The remaining R_i are chosen such to satisfy the WI
5. The functions corresponding to the elements in point 3 are directly related to the simplest channels at weak coupling
6. The functions G_i corresponding to the elements in point 4 can be chosen to satisfy simple crossing relations

ONE INTEGRAL TO RULE THEM ALL

A central part of the perturbative bootstrap is to demonstrate that higher-point correlators at NNLO can be computed by imposing symmetry constraints, provided we know **one** integral.

$B_{123,456} =$



$$B_{123,456} = \frac{b_{123,456}(x_1, x_2, x_3)}{8192\pi^{10}\tau_{15}^2\tau_{24}^2\tau_{36}^2},$$

ONE INTEGRAL TO RULE THEM ALL [Rodrigues, '24]

$$
\begin{aligned}
b_{123,456} = \frac{x_{13}^2}{x_1 x_2 \bar{x}_3 x_{12}} \Big(& -G(1,x_1)G(1,x_2)G(1,x_3) + G(1,x_2)G(x_3,x_1)G(1,x_3) \\
& - G(1,x_1)G(x_3,x_2)G(1,x_3) + G(x_3,x_1)G(x_3,x_2)G(1,x_3) \\
& - 2G(1,0,x_1)G(1,x_3) + 2G(1,x_2,x_1)G(1,x_3) + 2G(x_3,0,x_1)G(1,x_3) \\
& - 2G(x_3,x_2,x_1)G(1,x_3) + G(x_3,x_1)G(0,1,x_2) - G(1,x_1)G(0,x_3,x_2) \\
& + G(x_3,x_1)G(1,0,x_2) + G(0,x_3)(-G(1,x_2)G(x_3,x_1) + G(1,x_1)(G(1,x_2) \\
& + G(x_3,x_2)) + 2G(1,0,x_1) - 2G(1,x_2,x_1)) - G(x_3,x_2)G(1,x_2,x_1) \\
& + G(x_3,x_2)G(1,x_3,x_1) + G(1,x_1)G(1,x_3,x_2) - G(x_3,x_1)G(1,x_3,x_2) \\
& - G(1,x_1)G(x_3,0,x_2) - G(1,x_2)G(x_3,1,x_1) + G(0,x_2)(-2G(0,x_3)G(1,x_1) \\
& + 2G(1,x_3)G(1,x_1) - 2G(1,x_3)G(x_3,x_1) - G(1,x_3,x_1) + G(x_3,1,x_1)) \\
& + G(1,x_1)G(x_3,1,x_2) - G(x_3,x_1)G(x_3,1,x_2) + G(1,x_2)G(x_3,x_2,x_1) \\
& + G(1,0,x_3,x_1) - G(1,x_2,x_3,x_1) + G(1,x_3,0,x_1) - G(1,x_3,x_2,x_1) \\
& - G(x_3,0,1,x_1) - G(x_3,1,0,x_1) + G(x_3,1,x_2,x_1) + G(x_3,x_2,1,x_1) \Big) \,.
\end{aligned}
$$

INTERLUDE: GONCHAROV POLYLOGARITHMS

Iterated integrals of the kind

$$G(a_1,\dots,a_n,x)=\int_0^x \frac{dt}{t-a_1}G(a_2,\dots,a_n,t)\,,$$

with $G(x)=1$. An example is

$$G(0,\dots,0,a,x)=-\mathrm{Li}_n\left(\frac{x}{a}\right).$$

The symbol is an effective way of describing Goncharov polylogarithm and a crucial part of our ansatz building.

$$\mathcal{S}\left(G(a_1,\dots,a_n,x)\right)=x-a_n\otimes\ldots\otimes x-a_1$$

Symbols obey the useful properties as

$$A\otimes(x\cdot y)\otimes B=A\otimes x\otimes B+A\otimes y\otimes B\,.$$

$$\mathcal{S}\left(G(a_1,\dots,a_n,x)G(b_1,\dots,b_m,x)\right)=\mathcal{S}\left(G(a_1,\dots,a_n,x)\right)\sqcup\!\sqcup\,\mathcal{S}\left(G(b_1,\dots,b_m,x)\right)\,,$$

FOUR-POINT CORRELATORS

For four external operators, there are 3 R-symmetry channels in the correlator, therefore three G_i to solve for. A solution to the WI is for the correlator to have the form

$$\boxed{\mathcal{A} = \frac{1}{2}\left(\frac{r}{x^2} + \frac{s}{\bar{x}^2}\right)\mathbb{F} + \partial_x(\xi f(x))\,,}$$

where the auxiliary function ξ is defined as

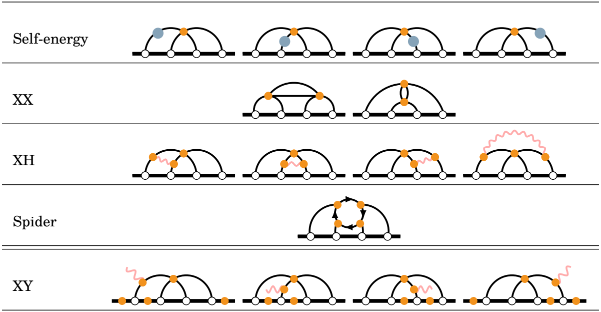
$$\xi = 1 - \frac{r}{x} - \frac{s}{\bar{x}}\,.$$

The algorithm presented for solving the WI in this configuration leads to

1. $G_1(x) = \mathbb{F}$
2. $\tilde{R}_2\Big|_{r\to\alpha x, s\to\bar{\alpha}\bar{x}} = 0\,.$
3. $f'(x) = G_2(x) \sim F_1(x)$ corresponding to the simplest perturbative channel

For each (protected) configuration of four scalar operators, the knowlegde of $f(x)$ is sufficient to determine the whole correlator.

$\langle\, 1111\, \rangle_{NNLO}$: DIAGRAMS AND SOLUTION



$$f^{(2)}(x) = \frac{1}{64\pi^4}\Bigg(\frac{\pi^4}{15} + 3\zeta_3 G(1,x) + \frac{\pi^2}{3}(G(1,0,x) - G(0,1,x) + G(1,1,x))$$
$$+ 2(G(1,1,0,1,x) - G(0,0,1,0,x) + G(0,0,1,1,x) - G(1,1,0,0,x)$$
$$+ G(0,1,0,0,x) - G(1,0,1,1,x)) + G(1,0,1,0,x) - G(0,1,0,1,x)$$
$$+ G(1,0,0,1,x) - G(0,1,1,0,x)\Bigg)\,.$$

$\langle\, 1212 \,\rangle_{NNLO}$: DIAGRAMS AND SOLUTION

Fully determined by a pinching of the B integral

$$= -\frac{1}{8} I_{24} K_{13,24}\,.$$

The solution to the Ward identities as

$$f^{(2)}(x) = \frac{1}{64\pi^4}\bigg(-\frac{7\pi^4}{180} + \frac{1}{2}(G(0,0,0,1,x) + G(0,0,1,0,x) + G(0,1,0,1,x) + G(0,1,1,0,x) - G(1,0,0,1,x) - G(1,0,1,0,x) - G(1,1,0,1,x) - G(1,1,1,0,x) + 2G(1,0,1,1,x) - 2G(0,1,0,0,x) + 2G(1,1,0,0,x) - 2G(0,0,1,1,x))\bigg)\,.$$

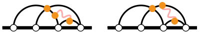
$\langle\, 1122 \,\rangle_{NNLO}$: DIAGRAMS AND SOLUTION

It is convenient to decompose the correlator into its connected and disconnected components

$$\mathcal{A}_{1122} = \mathcal{A}_{1122}^{(\text{disc})} + \mathcal{A}_{1122}^{(\text{conn})} \, .$$

though with necessary subtractions for overcounted terms:

$$\mathcal{A}_{1122}^{(\text{disc})}\Big|_{(13)(24)} = \frac{n_1^2}{n_2} \; \mathcal{A}_{1111}|_{(13)(24)} - (\text{overcounted}) \, .$$

XX	
XH	
XY (subtractions)	

The solution to the Ward identities is

$$\begin{aligned} f^{(2)}(x) = \frac{1}{64\pi^4} \bigg(& \frac{17\pi^4}{180} + \frac{3}{2}\zeta_3 G(1,x) + \frac{\pi^2}{6}(4G(1,0,x) - 2G(0,1,x) + G(1,1,x)) \\ & - 2(G(1,0,1,1,x) + G(1,1,0,0,x) + G(0,0,1,0,x) - G(0,0,1,1,x) \\ & - G(0,1,0,0,x)) - G(0,1,0,1,x) - G(0,1,1,0,x) + G(1,0,0,1,x) \\ & + G(1,0,1,0,x) + \frac{3}{2}G(1,1,0,1,x) + \frac{1}{2}G(1,1,1,0,x) \bigg) \, . \end{aligned}$$

FIVE-POINT CORRELATORS

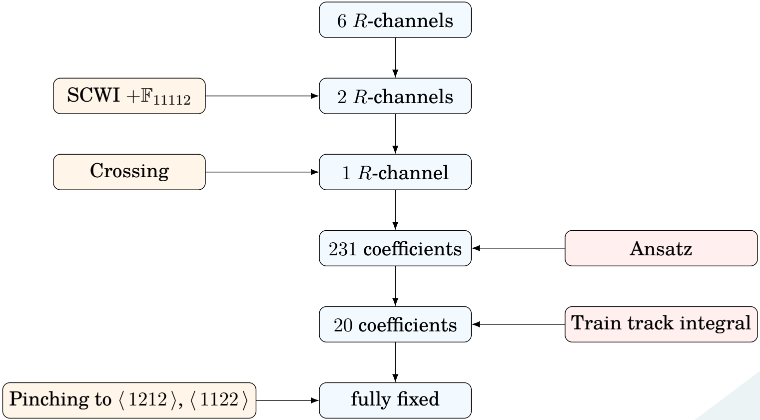
After applying the Ward identities and eliminating $G_{1,\dots,4}$, the five-point function solution takes the form

$$\boxed{\mathcal{A}_{11112} = \frac{t_{12}}{x_{12}^2}\mathbb{F}_{11112} + \rho f_1 + \partial_{x_1}(\xi_1 f_1) + \partial_{x_2}(\xi_2 f_1) + \partial_{x_2}(\eta f_2)\,,}$$

The auxiliary functions are given by

$$\begin{aligned}
\xi_1 &= \frac{\bar{x}_1}{\bar{x}_2}\left(1 - \frac{r_1}{x_1} - \frac{s_1}{\bar{x}_1}\right),\\
\xi_2 &= 1 - \frac{r_1}{x_1} - \frac{s_2}{\bar{x}_2} + \frac{t_{12}}{x_{12}}\,,\\
\eta &= \frac{x_2}{x_1}\left(\frac{r_1}{x_1} - \frac{r_2}{x_2} - \frac{t_{12}}{x_{12}}\right),\\
\rho &= -\frac{1}{\bar{x}_2}\left(1 - \frac{r_1}{x_1^2}\right).
\end{aligned}$$

$\langle 11112 \rangle_{NNLO}$: SOLUTION WORKFLOW



The knowledge of lower-point functions, non-perturbative information and the train track integral fully determines $\langle 11112 \rangle$ at NNLO.

$\langle\, 11112\, \rangle_{NNLO}$: SOME DETAILS

ANSATZ

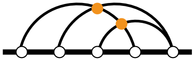
To construct the appropriate Goncharov polylogarithms, we write the Ansatz in symbols

$$f^{(2)}(x_1, x_2) = c_0 + \sum_{i,j} c_{ij} \xi_i \otimes \xi_j + \sum_{i,j,k,l} c_{ijkl} \xi_i \otimes \xi_j \otimes \xi_k \otimes \xi_j \,, \quad \xi_l = \{x_1, \bar{x}_1, x_2, \bar{x}_2, x_{12}\} \,.$$

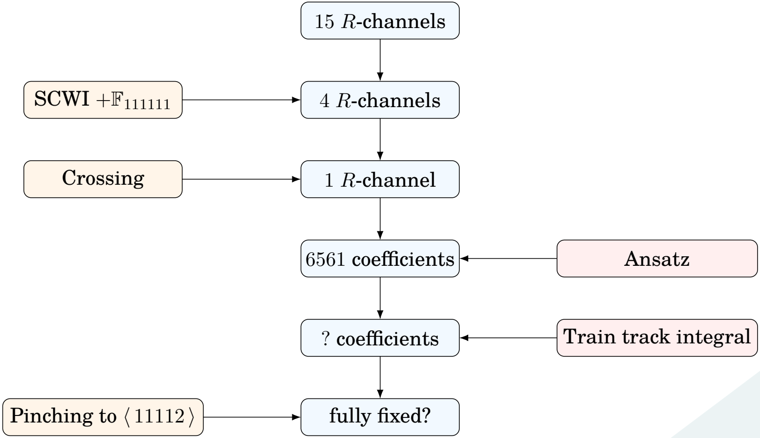
The ansatz is not quite yet in a usable form, as it is not finite for every choice of coefficients. Requiring finiteness fixes 6 out of the 21 free coefficients.

TRAIN TRACK INTEGRAL

On the input side, it is crucial to note that the channel F_1 is now determined by a single diagram:

 $= -\dfrac{1}{8} B_{125,345} \,,$

$\langle\, 111111 \,\rangle_{NNLO}$: WORK IN PROGRESS



The integrability conditions of the ansatz reduce the number of coefficients to 1380.

BULK-DEFECT-DEFECT CORRELATORS

DEFINITION AND CONVENTIONS

$$\boxed{\langle\, \mathcal{O}_{\Delta_1}(u_1, x_1)\hat{\mathcal{O}}_{\hat{\Delta}_2}(\hat{u}_2, \tau_2)\hat{\mathcal{O}}_{\hat{\Delta}_3}(\hat{u}_3, \tau_3)\,\rangle = \mathcal{K}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}\mathcal{A}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}(\zeta;\chi)\,,}$$

The prefactor $\mathcal{K}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}$ is defined as

$$\mathcal{K}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3} := \frac{(u_1\cdot\hat{u}_2)^{\hat{\Delta}_2}(u_1\cdot\hat{u}_3)^{\hat{\Delta}_3}(u_1\cdot\theta)^{\Delta_{123}}}{x_{12}^{2\hat{\Delta}_2}x_{13}^{2\hat{\Delta}_3}\vec{x}_1^{2\Delta_{123}}}\,,$$

with

$$x_{1j}^2 := \vec{x}_1^2 + \tau_{1j}^2\,, \qquad \Delta_{123} := \Delta_1 - \hat{\Delta}_2 - \hat{\Delta}_3\,.$$

The spacetime and R-symmetry cross-ratios are defined as

$$\chi := \frac{\vec{x}_1^2\tau_{23}^2}{(\vec{x}_1^2+\tau_{12}^2)(\vec{x}_1^2+\tau_{13}^2)}\,, \qquad \zeta := \frac{(u_1\cdot\theta)^2(\hat{u}_2\cdot\hat{u}_3)}{(u_1\cdot\hat{u}_2)(u_1\cdot\hat{u}_3)}\,.$$

The form of the function $\mathcal{A}$ is

$$\boxed{\mathcal{A}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}(\zeta;\chi) = \sum_{j=0}^{n}\left(\frac{\zeta}{\chi}\right)^j F_j(\chi)\,.}$$

NON-PERTURBATIVE CONSTRAINTS - 3

SUPERCONFORMAL SYMMETRY

Given the particularly reduced number of superconformal invariants, the correlator obey a superconformal Ward identity

$$\left((\partial_\chi + \partial_\xi)\mathcal{A}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}\right)|_{\xi\to\chi^2} = 0 \longrightarrow \sum_{j=0}^{n} F_j(\chi) = \mathbb{F}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}\,.$$

LOCALITY

The superconformal block expansion

$$\mathcal{A}_{\Delta_1\hat{\Delta}_2\hat{\Delta}_3}(\zeta;\chi) = \sum_{\hat{\Delta},\ell,k} b_{\Delta_1\hat{\Delta}}\lambda_{\hat{\Delta}_2\hat{\Delta}_3\hat{\Delta}}\mathcal{G}_{\hat{\Delta},\ell,k}(\chi,\zeta)$$

is made of blocks $\mathcal{G}_{\hat{\Delta},\ell,k}(\chi,\zeta)$ with non-physical singularities and branch cuts. Such singularities must cancel in correlation functions, resulting in a second expansion manifestly local. [Kabat, Lifschytz, '16][Levine, Paulos, '23]

PERTURBATIVE INFORMATION

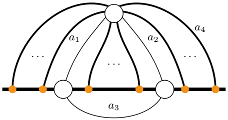
NUMBER OF R-SYMMETRY CHANNELS

The number of R-symmetry channels that are non-zero reduces to 2 at NLO at weak coupling, meaning that the correlation function becomes

$$\mathcal{A}_{\Delta_1 \hat{\Delta}_2 \hat{\Delta}_3}(\zeta;\chi) = F_0(\chi) + \left(\frac{\zeta}{\chi}\right) F_1(\chi)\,.$$

MASTER INTEGRALS

The diagrams appearing in $F_1(\chi)$ at NLO are of the kind



,

The integrals to solve are all of the kind

$$\int_{-\infty}^{a} d\tau_1 I_{1\tau_1} = \frac{\pi + 2\arctan\left(\frac{a}{|x_\perp|}\right)}{2|x_\perp|} \qquad \int_{a}^{+\infty} d\tau_1 I_{1\tau_1} = \frac{\pi - 2\arctan\left(\frac{a}{|x_\perp|}\right)}{2|x_\perp|}$$

RESULTS AND TRANSCENDENTALITY

The solution to the WI combined with the knowledge of the master integrals gives the NLO result of bulk-defect-defect correlation functions for operators of arbitrary dimensions. For $\langle\, \mathcal{O}_2 \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_1 \,\rangle$

$$F_1^{(1)}(\chi) = \frac{\lambda^3}{512\pi^6\, N} \frac{1}{\sqrt{n_2}\hat{n}_1} \left(\pi^2 + 4\, \mathbf{arccos}^2 \left(\sqrt{\chi} \right) \right),$$

The NLO expression of the $\langle\, \mathcal{O}_2 \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_1 \,\rangle$ does not present any power of $\log(\chi)$ in the series expansion for $\chi \to 0$, nor does any other form-factor.

How can we explain this absence?

LOWER TRANSCENDENTALITY

SUPERBLOCK EXPANSION

$$\mathcal{A}_{\Delta_1 \hat{\Delta}_2 \hat{\Delta}_3}(\zeta; \chi) = \sum_{\hat{\Delta}, \ell, k} b_{\Delta_1 \hat{\Delta}} \lambda_{\hat{\Delta}_2 \hat{\Delta}_3 \hat{\Delta}} \mathcal{G}_{\hat{\Delta}, \ell, k}(\chi, \zeta)$$

for $\langle\, \mathcal{O}_2 \hat{\mathcal{O}}_1 \hat{\mathcal{O}}_1 \,\rangle$ we predict a total coefficient of $\log(\chi)$ that is the sum of $b^{(0)}_{2\hat{\mathcal{O}}} \lambda^{(0)}_{11\hat{\mathcal{O}}} \gamma_{\hat{\mathcal{O}}}$. This contribution to the $\log(\chi)$ term of the expansion vanishes. We need an explanation that is valid beyond the single example.

LOCALITY

The $\log(\chi)$-proportional parts of the discontinuity of the conformal blocks

$$\mathcal{G}_{\hat{\Delta}, \Delta_{12}}(\chi) = \chi^{\hat{\Delta}/2} \, {}_2F_1 \left(\frac{1}{2} \left(\hat{\Delta} - \Delta_{12} \right), \frac{1}{2} \left(\hat{\Delta} + \Delta_{12} \right); \hat{\Delta} + \frac{1}{2}; \chi \right)$$

for $\chi \to 1$ generate a linearly independent set of functions. The locality condition imposes the vanishing of such terms, and therefore of the $\log(\chi)$ piece of the small χ expansion. We checked this statement for a correlator in the non-SUSY $O(N)$ model.

CONCLUSIONS AND OUTLOOK

MULTIPOINT - SUMMARY

- We use superconformal symmetry and pinching limits to reduce the amount of perturbative information needed
- We develop a systematic algorithm to find a suitable solution to the WI for this application
- We benchmark our method by computing $\langle\, 1111 \,\rangle$ at NNLO and $\langle\, 111111 \,\rangle$ at NLO
- We compute new 4-point correlators at NNLO: $\langle\, 1212 \,\rangle$, $\langle\, 1122 \,\rangle$
- Using non-perturbative information and one single integral, we determine $\langle\, 11112 \,\rangle$ at NNLO
- The NNLO result for $\langle\, 111111 \,\rangle$ is under way.

MULTIPOINT - PERSPECTIVES

BEYOND WEAK COUPLING

- Numerical analysis of the correlator $\langle\, 111111\, \rangle$, building upon numerical bootstrap [Antunes, Harris, Kaviraj, Schomerus, '23].
- Connection with the bootstrability techniques applied so far to four-point functions [Cavaglià, Gromov, Julius, Preti, '22][Cavaglià, Gromov, Julius, Preti, '22][Cavaglià, Gromov, Julius, Preti, '22][Cavaglià, Gromov, Julius, Preti, '22]
- A preliminary study of the superconformal blocks is necessary and under way [Barrat, Bliard, Ferrero, Meneghelli, Peveri, *to appear*

THE ROLE OF THE TRAIN-TRACK

- Study of the role of train-track integrals at further perturbative orders
- The collapse of elliptic complexity for 6-point train-track may hold for higher points
- Train-track integrals would be the ones appearing in a line-defect of the fishnet theory
- Fascinating integrable non-supersymmetric framework to study exact correlators

BULK-DEFECT-DEFECT - SUMMARY

- We use superconformal symmetry and pinching limits to reduce the amount of perturbative information needed
- We compute the master integrals necessary to describe all bulk-defect-defect correlators at NLO
- We use locality constraints to explain the loss of transcendentality of the results
- We observe the same loss of transcendentality in a non-SUSY set-up, the $O(N)$ model

BULK-DEFECT-DEFECT - PERSPECTIVES

FURTHER CONSTRAINTS FOR LOCALITY

- We can write an expansion of the correlators in local blocks, serving as a second block-expansion [Levine, Paulos, '23][Levine, Paulos, '24]
- This expansion can be used to determine relations among conformal data and can further constrain the correlator

THE SPACE OF FUNCTIONS

- The NLO analysis did not give access to the space of transcendental functions describing bulk-defect-defect correlators
- Locality constraints predict a loss of transcendentality for NNLO as well: this could be used to bootstrap NNLO correlators
- There are NNLO correlators with 3 channels, with one of them simpler than the others. Further constraints are needed to apply the perturbative bootstrap set-up

THANK YOU!

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$$\mathcal{W}_\ell = \frac{1}{N}\mathrm{tr}\,\mathcal{P}\,\exp\int_{-\infty}^{+\infty} d\tau|\dot{x}|\left(aX(\tau)+b\bar{X}(\tau)\right)$$