CISC-102

PRACTICE PROBLESM WITH SOLUTIONS – LOGIC

(1) Let p, q, and r be the propositions
p : You have the flu, q : You miss the final examination, and r : You pass the course.
Express each of these propositions as an English sentence.

(a) $q \to \neg r$

(b) $p \vee q \vee r$

(c) $(p \to \neg r) \vee (q \to \neg r)$

(d) $(p \wedge q) \vee (\neg q \wedge r)$

Answer:

(a) If you miss the final exam, then you do not pass the course.

(b) You have the flu, or miss the final exam, or pass the course.

(c) It is either the case that if you have the flu then you do not pass the course or the case that if you miss the final exam then you do not pass the course (or both, it is understood).

(d) Either you have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

(2) Write each of these statements in the form "if p, then q" in English.

(a) I will remember to send you the address only if you send me an e-mail message.

(b) The Red Wings will win the Stanley Cup if their goalie plays well.

(c) It is necessary to have a valid password to log on to the server.

Answer:

(a) If I am to remember to send you the address, then you will have to send me an e-mail message. (This has been slightly reworded so that the tenses make more sense.)

(b) If their goaltender plays well, then the Red Wings will win the Stanley Cup.

(c) If you log on to the server, then you have a valid password.

(3) Are the two logical expressions $P \wedge \neg Q$ and $\neg(\neg P \vee Q)$ logically equivalent? Answer this question using a truth table (or tables)

Answer:

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \vee Q$	$\neg(\neg P \vee Q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	T	T	F	T	F

So the answer is Yes, they are logically equivalent.

(4) Is $\neg(P \rightarrow Q)$ logically equivalent to $P \wedge \neg Q$?

Answer:

P	Q	$\neg Q$	$P \rightarrow Q$	$\neg(P \rightarrow Q)$	$P \wedge \neg Q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

Yes, they are logically equivalent.

(5) Determine the truth value of each of these statements if the domain consists of all real numbers.

(a) $\exists(x^3 = -1)$

(b) $\exists(x^4 < x^2)$

(c) $\forall x((-x)^2 = x^2)$

(d) $\forall(2x > x)$

Answer:

(a) Since $(-1)^3 = 01$, this is true.

(b) Since $(\frac{1}{2})^4 < (\frac{1}{2})^2$ this is true.

(c) Since $(-x)^2 = ((-1)x)^2 = (-1)^2x^2 = x^2$, we know that$\forall x((-x)^2 = x^2)$ is true. d) Twice a positive number is larger than the number, but this inequality is not true for negative numbers or 0. Therefore $\forall(2x > x)$ is false.

(6) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

(a) Something is not in the correct place.

(b) All tools are in the correct place and are in excellent condition.

(c) Everything is in the correct place and in excellent condition.

(d) Nothing is in the correct place and is in excellent condition.

(e) One of your tools is not in the correct place, but it is in excellent condition.

Answer:
Let $R(x)$ be "x is in the correct place", let $E(x)$ be "x is in excellent condition," let $T(x)$ be "x is a [or your] tool," and let the domain of discourse be all things.

(a) There exists something not in the correct place: $\exists x \neg R(x)$.

(b) If something is a tool, then it is in the correct place place and in excellent condition: $\forall x(T(x) \rightarrow (R(x) \wedge E(x)))$.

(c) $\forall x(R(x) \wedge E(x))$

(d) This is saying that everything fails to satisfy the condition: $\forall x \neg(R(x) \wedge E(x))$.

(e) There exists a tool with this property: $\exists x(T(x) \wedge \neg R(x) \wedge E(x))$.

(7) Suppose the domain of the propositional function $P(x, y)$consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

(a) $\exists x P(x, 3)$
(b) $\forall P(1, y)$
(c) $\exists y \neg P(2, y)$
(d) $\forall x \neg P(x, 2)$

Answer:

(a) $P(1, 3) \vee P(2, 3) \vee P(3, 3)$
(b) $P(1, 1) \wedge P(1, 2) \wedge P(1, 3)$
(c) $\neg P(2, 1) \vee \neg P(2, 2) \vee \neg P(2, 3)$
(d) $\neg P(1, 2) \wedge \neg P(2, 2) \wedge P(32)$

(8) Express the negation of the statement $\exists x(-5 < x < -1)$ in terms of quantifiers without using the negation symbol.
Answer:
$\forall x((x \leq -5) \vee (x \geq -1))$