Formatting instructions: Write clearly and neatly in an organized manner. For each question display your answer clearly—perhaps by putting it in a box so the marker can easily find it.

1. Find an equation in normal form $ax + by + cz = d$ for the plane P that passes through the point $(3, 0, -1)$ and contains the line $(x, y, z) = (1-2t, t, 2)$.

$\boxed{-3x-6y-2z=-7}$

2. Find the shortest distance between the lines $(-1, 1, 4) + t[1, 1, -1]$ and $(5, 3, -3) + s[-2, 0, 1]$. Do not use calculus.

$\boxed{=\sqrt{6}}$

3. Let P_1 be the plane through the origin containing the vectors $[1, 2, -1]$ and $[0, 1, 1]$.
Let P_2 be the plane through the point $(1, 1, 1)$ parallel to the vectors $[-1, 2, 2]$ and $[3, 4, -2]$.
(a) Find an equation for P_1 in normal form $ax + by + cz = 0$. $\boxed{3x-y+z=0}$
(b) The planes P_1 and P_2 intersect in a line. Find a parametric equation for this line.

$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} + t\begin{bmatrix} -6 \\ -18 \\ 0 \end{bmatrix}}$ $\boxed{x=-6t}$ $\boxed{y=3-18t}$ $\boxed{z=3}$

4. (a) Find the least squares approximation to the solution of the system of equations:
$2x + y = 1$
$x - y = -1$
$x + 2y = 1$

$\boxed{x=0}$ $\boxed{y=\frac{2}{3}}$

(b) Let P be the plane passing through the origin and the points $U = (2, 1, 1)$ and $V = (1, -1, 2)$. Use your answer to (a) to find the foot of the perpendicular dropped from the point $W = (1, -1, 1)$ to the plane P.

$\boxed{\therefore \left(\frac{2}{3}, \frac{-2}{3}, \frac{4}{3}\right) \text{ is foot of } \perp \text{ drop from } W}$

5. Let A be the point $(0, 1, 0)$ and B be the point $(0, 0, 2)$. Let S be the sphere

$$(x-2)^2 + y^2 + z^2 = 4$$

(a) Provide a good diagram of the (x, y, z) coordinate system showing the points A and B and the sphere S.
* on Solution page

(b) Consider the family of all planes that pass through both the points A and B. This forms a one-parameter family of planes in the sense that one additional piece of information will specify the plane. One of these planes is of course the *y-z* coordinate plane which has equation $x = 0$. The remaining planes in the family do not pass through the origin and they can be described by the general equation

$$ax + 2y + z = 2.$$

Here the single parameter is a. Do check that for every a, this plane passes through A and B. Your job is to find the value of a for which the plane is tangent to the sphere S. Do not use calculus.

$\boxed{a=-2}$ $\boxed{\therefore -2x+2y+z=2}$

1. $P = (3,0,-1)$ $\quad (x,y,z) = (1-2t, t, 2)$

$= (1,0,2) + t(-2,1,0)$

line passes through $Q = (1,0,2)$ and $\bar{v} = (-2,1,0)$

$\bar{PQ} = (1,0,2) - (3,0,-1) = (-2,0,3)$

$$\bar{PQ} \times \bar{v} = \begin{vmatrix} a & b & c \\ -2 & 0 & 3 \\ -2 & 1 & 0 \end{vmatrix} = a\begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} - b\begin{vmatrix} -2 & 3 \\ -2 & 0 \end{vmatrix} + c\begin{vmatrix} -2 & 0 \\ -2 & 1 \end{vmatrix}$$

$$= a(-3) - b(6) + c(-2)$$

$$-3x - 6y - 2z = 0$$

$$-3(x-3) - 6(y-0) - 2(z+1) = 0$$

$$-3x + 9 - 6y - 2z - 2 = 0$$

$$\boxed{-3x - 6y - 2z = -7}$$

check:

$-3(3) - \cancel{6(0)} - 2(-1) = -7$

$-9 - 0 + 2 = -7$

$-7 = -7$

check:

$-3(1-2t) - 6(t) - 2(2) = -7$

$-3 + \cancel{6t} - \cancel{6t} - 4 = -7$

$-7 = -7$

2. $\bar{P}(t) = (-1,1,4) + t[1,1,-1]$ $\quad \bar{Q}(s) = (5,3,-3) + s[-2,0,1]$

$P(t) = (-1+t,\ 1+t,\ 4-t)$ $\quad Q(s) = (5-2s,\ 3,\ -3+s)$

① $\overline{PQ} = P - Q = \begin{bmatrix} -1+t-(5-2s) \\ 1+t-3 \\ 4-t-(-3+s) \end{bmatrix} = \begin{bmatrix} t+2s-6 \\ t-2 \\ -t-s+7 \end{bmatrix}$

$\overline{PQ} \cdot \overline{P(t)} = 0$ & $\overline{PQ} \cdot Q(s) = 0$

② $\overline{PQ} \cdot \overline{P(t)} = 0$ $\begin{bmatrix} t+2s-6 \\ t-2 \\ -t-s+7 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = (t+2s-6) + (t-2) - (-t-s+7)$

$= t + 2s - 6 + t - 2 + t + s - 7$

$= 3t + 3s - 15$

$\underline{3t + 3s = 15}$ (1)

$\overline{PQ} \cdot \overline{Q(t)} = 0$ $\begin{bmatrix} t+2s-6 \\ t-2 \\ -t-s+7 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} = -2(t+2s-6) + 0(t-2) + (-t-s+7)$

$= -2t - 4s + 12 - t - s + 7$

$= -3t - 5s + 19$

$\underline{3t + 5s = 19}$ (2)

③ $3t + 5s = 19$

$(-)\ 3t + 3s = 15$

$2s = 4$

$\boxed{s = 2}$

$3t + 3(2) = 15$

$3t = 9$

$\boxed{t = 3}$

④ $\overline{PQ} = \begin{bmatrix} (3)+2(2)-6 \\ 3-2 \\ -(3)-(2)+7 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}}$

$\|\overline{PQ}\| = \sqrt{1^2 + 1^2 + 2^2}$ $\boxed{= \sqrt{6}}$ $\quad \boxed{\therefore \|\overline{PQ}\| = \sqrt{6}}$

3. P_1: $\bar{u} = [1,2,-1]$ $\bar{v} = [0,1,1]$ through origin $(0,0,0)$

P_2: $\bar{w} = [-1,2,2]$ $\bar{Q} = [3,4,-2]$ through $(1,1,1)$

a) ① $\bar{u} \cdot \bar{v} = \begin{bmatrix} a & b & c \\ 1 & 2 & -1 \\ 0 & 1 & 1 \end{bmatrix} = a\begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} - b\begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} + c\begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix}$

$= a(3) - b(1) + c(1)$

$\boxed{\therefore 3x - y + z = 0}$

b) $P2$: equation $\bar{w} \cdot \bar{Q} = \begin{bmatrix} a & b & c \\ -1 & 2 & 2 \\ 3 & 4 & -2 \end{bmatrix} = a\begin{vmatrix} 2 & 2 \\ 4 & -2 \end{vmatrix} - b\begin{vmatrix} -1 & 2 \\ 3 & -2 \end{vmatrix} + c\begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}$

②

$= a(-4-8) - b(2-6) + c(-4-6)$

$= -12a + 4b - 10c$

$-12x + 4y - 10z = 0$

$-12(x-1) + 4(y-1) - 10(z-1) = 0$

$-12x + 12 + 4y - 4 - 10z + 10 = 0$

$\boxed{P_2: -12x + 4y - 10z = -18}$ & $\boxed{P_1: 3x - y + z = 0}$

③ $x = 0$ $4y - 10z = -18$
$-y + z = 0$ $\rightarrow z = y$

$4y - 10y = -18$
$-6y = -18$
$\boxed{y = 3}$ $\boxed{z = 3}$

$(0,3,3)$

$P_1 \cdot P_2 = \begin{bmatrix} a & b & c \\ -12 & 4 & -10 \\ 3 & -1 & 1 \end{bmatrix} = a\begin{vmatrix} 4 & -10 \\ -1 & 1 \end{vmatrix} - b\begin{vmatrix} -12 & -10 \\ 3 & 1 \end{vmatrix} + c\begin{vmatrix} -12 & 4 \\ 3 & -1 \end{vmatrix} = a(4-10) - b(-12+30) + c(12-12)$

$= -6a - 18b + (0)c$

$\boxed{\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 3 \end{bmatrix} + t\begin{bmatrix} -6 \\ -18 \\ 0 \end{bmatrix}}$

$\boxed{\begin{aligned} x &= -6t \\ y &= 3 - 18t \\ z &= 3 \end{aligned}}$

4. $2x + y = 1$
$x - y = -1$
$x + 2y = 1$

$$\begin{array}{cc} \bar{u} & \bar{v} \end{array} \quad \bar{\beta} \quad \bar{w}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

a) $$\begin{bmatrix} \bar{u}\cdot\bar{u} & \bar{u}\cdot\bar{v} \\ \bar{v}\cdot\bar{u} & \bar{v}\cdot\bar{v} \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \bar{u}\cdot\bar{w} \\ \bar{v}\cdot\bar{w} \end{bmatrix}$$

$\bar{u}\cdot\bar{u} = 6$ $\quad \bar{v}\cdot\bar{v} = 6$ $\quad \bar{v}\cdot\bar{w} = 4$

$\bar{u}\cdot\bar{v} = 3$ $\quad \bar{u}\cdot\bar{w} = 2$

$$\begin{bmatrix} 6 & 3 \\ 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ 3 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{27} \begin{bmatrix} 6 & -3 \\ -3 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{27} \begin{bmatrix} 0 \\ 18 \end{bmatrix} \qquad \boxed{x = 0} \qquad \boxed{y = \frac{18}{27} = \frac{2}{3}}$$

b) plane through $v = (2,1,1)$ & $v = (1,-1,2)$

$$\begin{array}{cc} \bar{u} & \bar{v} \end{array} \quad \bar{\beta} \quad \bar{\varepsilon} \quad \bar{w}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 0 \\ 2 \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 1 & 2 \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix} \Rightarrow \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

$$u \cdot v = \begin{bmatrix} a & b & c \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{bmatrix} = a\begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} - b\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + c\begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = a(3) - b(3) + c(-3)$$

$0 = 3x - 3y - 3z$

$x = 1 + \frac{1}{3}t$ $\quad 0 = 3(1 + \frac{1}{3}t) - 3(-1 - \frac{1}{3}t) - 3(1 - \frac{1}{3}t)$

$y = -1 - \frac{1}{3}t$

$z = 1 - \frac{1}{3}t$ $\quad 0 = 3 + t + 3 + t - 3 + t$

$t = -1$

Sub in $t = -1$

$z = 1 - \frac{1}{3}(-1)$

$\boxed{z = \frac{4}{3}}$

$\boxed{\therefore \text{ coor of } \perp \text{ dropped from } w \text{ is } \left(\frac{2}{3}, \frac{-2}{3}, \frac{4}{3}\right)}$

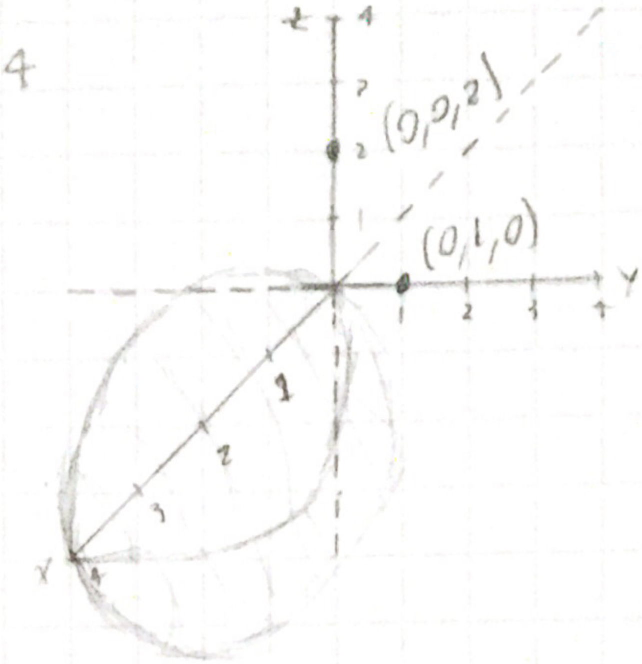
$x = -1 + \frac{1}{3}(-1)$

$\boxed{= \frac{2}{3}}$

$y = -1 - \frac{1}{3}(-1)$

$\boxed{y = \frac{-2}{3}}$

5. $(x-2)^2+y^2+z^2=4$



b) $ax+2y+z=2$

① $R=[a,2,1]$

$(x,y,z)=(2,0,0)+t[a,2,1]$

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\0\\0\end{bmatrix}+t\begin{bmatrix}a\\2\\1\end{bmatrix}=\boxed{\begin{bmatrix}at+2\\2t\\t\end{bmatrix}}$$

② $z^2=4-(x-2)^2-y^2$

$z=\sqrt{4-(x-2)^2-y^2}$

$=2-(x-2)-y$

$=-x-y+4$

*Let $z=t$ *

$ax+2y+z=2$

$\vec{n}\cdot\vec{AB}=0$

$$\begin{bmatrix}a\\2\\1\end{bmatrix}\cdot\begin{bmatrix}0\\-1\\2\end{bmatrix}=0$$

$\vec{AB}=B-A=[0,0,2]-[0,1,0]$

$=\underline{[0,-1,2]}$

$\vec{AT}=T-A=[x,y,z]-[0,1,0]$

$=\underline{[x,y-1,z]}$

$CA\cdot AT=0$ $\begin{bmatrix}a\\2\\1\end{bmatrix}\cdot\begin{bmatrix}x\\y-1\\z\end{bmatrix}\neq 0$

$\underline{\vec{CT}\cdot\vec{AB}=\vec{CT}\cdot\vec{AT}}$

③ $T-C=[x,y,z]-[2,0,0]=\underline{[x-2,y,z]}$

$$\vec{CT}\cdot\vec{AB}=\begin{bmatrix}x-2\\y\\z\end{bmatrix}\cdot\begin{bmatrix}0\\-1\\2\end{bmatrix}$$

$0=-y+2z$

$\underline{2z=y}$

$\vec{CT}\cdot\vec{AT}=0$

$$0=\begin{bmatrix}x-2\\y\\z\end{bmatrix}\cdot\begin{bmatrix}x\\y-1\\z\end{bmatrix}\Rightarrow \underline{0=x^2-2x+y^2-y+z}$$

$(x-2)^2+y^2+z^2=4$

① $2z-y=0$

② $x^2-2x+y^2-y+z=0$

③ $(x-2)^2+y^2+z=4$

$\boxed{x=\frac{2}{3}}$ $\boxed{y=\frac{4}{3}}$ $\boxed{z=\frac{2}{3}}$

$\vec{AT}=[x,y-1,z]$

$\boxed{\vec{AT}=[\frac{2}{3},\frac{1}{3},\frac{2}{3}]}$

$\vec{AT}\cdot\vec{n}=0$

$$\begin{bmatrix}2/3\\1/3\\2/3\end{bmatrix}\cdot\begin{bmatrix}a\\2\\1\end{bmatrix}=0$$

$\frac{2}{3}a+\frac{2}{3}+\frac{2}{3}=0$

$2a+4=0$

$\boxed{a=-2}$

$\boxed{\therefore -2x+2y+z=2}$