CISC 102 (Fall 20)
Homework #2: Logic (Solutions) #2: Logic (25 Points)

Student Name/ID:. . . . .

Solutions are due before 11:59 PM on **Thursday 2 September 2020**.

1. (4 pts) Let p, q, and r be the propositions
   p: You have the flu, q: You miss the final examination, and r: You pass the course.
   Express each of these propositions as an English sentence.

   (a) $q \to \neg r$
   Solution:
   If you miss the final exam, then you do not pass the course.

   (b) $p \vee q \vee r$
   Solution:
   You have the flu, or miss the final exam, or pass the course.

   (c) $(p \to \neg r) \vee (q \to \neg r)$
   Solution:
   It is either the case that if you have the flu then you do not pass the course or the case that if you miss the final exam then you do not pass the course (or both, it is understood).

   (d) $(p \wedge q) \vee (\neg q \wedge r)$
   Solution:
   Either you have the flu and miss the final exam, or you do not miss the final exam and do pass the course.

2. (2 pts) Write each of these statements in the form "if p, then q" in English.

   (a) I will remember to send you the address only if you send me an e-mail message.
   Solution:
   If I am to remember to send you the address, then you will have to send me an e-mail message. (This has been slightly reworded so that the tenses make more sense.)

   (b) The Red Wings will win the Stanley Cup if their goalie plays well.
   Solution:
   If their goaltender plays well, then the Red Wings will win the Stanley Cup.

3. (3 pts) State the converse, contrapositive, and inverse of each of these conditional statements.

(a) If it snows tonight, then I will stay at home.
Solution:
Converse: If I stay home, then it will snow tonight. Contrapositive: If I do not stay at home, then it will not snow tonight. Inverse: If it does not snow tonight, then I will not stay home.

(b) I go to the beach whenever it is a sunny summer day.
Solution:
Converse: Whenever I go to the beach, it is a sunny summer day. Contrapositive: Whenever I do not go to the beach, it is not a sunny summer day. Inverse: Whenever it is not a sunny day, I do not go to the beach.

(c) When I stay up late, it is necessary that I sleep until noon.
Solution:
Converse: If I sleep until noon, then I stayed up late. Contrapositive: If I do not sleep until noon, then I did not stay up late. Inverse: If I don't stay up late, then I don't sleep until noon.

4. (2 pts) Are the two logical expressions $P \wedge \neg Q$ and $\neg(\neg P \vee Q)$ logically equivalent? Answer this question using a truth table (or tables)
Solution:

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \vee Q$	$\neg(\neg P \vee Q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	T	T	F	T	F

So the answer is Yes, they are logically equivalent.

5. (2 pts) is $\neg(P \to Q)$ logically equivalent to $P \wedge \neg Q$?
Solution:

P	Q	$\neg Q$	$P \to Q$	$\neg(P \to Q)$	$P \wedge \neg Q$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

Yes, they are logically equivalent.

6. (2 pts)

(a) Prove that $A \subseteq B \to A \cap \bar{B} = \varnothing$

Solution:

(a) Proof: Suppose $A \subseteq B$

Now suppose $A \cap \bar{B} \neq \varnothing$

$\Rightarrow \exists x$ such that $x \in A$ and $x \in \bar{B}$ which is a

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow A \nsubseteq B$

contradiction. Thus $A \cap \bar{B} = \varnothing$

Therefore $A \subseteq B \to A \cap \bar{B} = \varnothing$

(b) Prove that $A \cap \bar{B} = \varnothing \to A \subseteq B$

Solution:

Proof: Consider the contrapositive: $A \nsubseteq B \to A \cap \bar{B} \neq \varnothing$

Suppose $A \nsubseteq B$

$\Rightarrow \exists x$ such that $x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in \bar{B}$

$\Rightarrow x \in A \cap \bar{B}$

$\Rightarrow A \cap \bar{B} \neq \varnothing$

Therefore $A \nsubseteq B \to A \cap \bar{B} \neq \varnothing$

Therefore $A \cap \bar{B} = \varnothing \to A \subseteq B$

7. (2 pts) Show that $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$ are logically equivalent.

Solution:

We can solve this either by logical equivalence or by truth table:

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\begin{aligned}
(p \to q) \vee (p \to r) &\equiv (\neg p \vee q) \vee (\neg p \vee r) && \text{Definition of } \to \\
&\equiv (\neg p \vee \neg p) \vee (q \vee r) && \text{Commutative and Associative} \\
&\equiv \neg p \vee (q \vee r) && \text{Idempotent} \\
&\equiv p \to (q \vee r) && \text{Definition of } \to
\end{aligned}
$$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$q \vee r$	$p \to (q \vee r)$
F	F	F	T	T	T	F	T
F	F	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T
T	T	F	T	F	T	T	T
T	T	T	T	T	T	T	T

8. (3 pts) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

(a) There is a person in your class who cannot swim.

Solution:

Let $S(x)$ be "x can swim." Then we have $\exists x \neg S(x)$ the first way, or $\exists x(C(x) \wedge \neg S(x))$ the second way.

(b) All students in your class can solve quadratic equations.
Solution:
Let $Q(x)$ be "x can solve quadratic equations." Then we have $\forall x Q(x)$ the first way, or $\forall x(C(x) \rightarrow Q(x))$ the second way.

(c) Some student in your class does not want to be rich.
Solution:
Let $R(x)$ be "x wants to be rich." Then we have $\exists x \neg R(x)$ the first way, or $\exists x(C(x) \wedge \neg R(x))$ the second way.

9. (3 pts) Express the negations of these propositions using quantifiers, and in English.

(a) Every student in this class likes mathematics.
Solution:
In English, the negation is "Some student in this class does not like mathematics." With the obvious propositional function, this is $\exists x \neg L(x)$.

(b) There is a student in this class who has never seen a computer.
Solution:
In English, the negation is "Every student in this class has seen a computer." With the obvious propositional function, this is $\forall x S(x)$.

(c) There is a student in this class who has taken every mathematics course offered at this school.
Solution:
In English, the negation is "For every student in this class, there is a mathematics course that this student has not taken." With the obvious propositional function, this is $\forall x \exists c \neg T(x, c)$.

10. (2 pts)

(a) Suppose that $Q(x)$ is the statement $x + 1 = 2x$. What are the truth values of $\forall x Q(x)$ and $\exists x Q(x)$?
Solution: Since $x + 1 = 2x$ is true if and only if $x = 1$, we see that $Q(x)$ is true if and only if $x = 1$. It follows that $\forall x Q(x)$ is false and $\exists x Q(x)$ is true.

(b) Let $P(m, n)$ be "n is greater than or equal to m" where the domain (universe of discourse) is the set of nonnegative integers. What are the truth values of $\exists n \forall m P(m, n)$ and $\forall m \exists n P(m, n)$?
Solution: For every positive integer n there is an integer m such that $n < m$ (take $m = n + 1$ for instance). Hence $\exists n \forall m P(m, n)$ is false. For every integer m there is an integer n such that $n \geq m$ (take $n = m + 1$ for instance). Hence $\forall m \exists n P(m, n)$ is true.