CISC 102: Discrete Mathematics for Computing I

Fall 2020

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Introduction

Instruction Team

- Instructor:
  - Hazem Abbas hazem.abbas@queensu.ca

- TAs:
  - There are 13+ TAs.
  - Names and emails will be posted on onQ.

Textbooks

- **[LL]** Seymour Lipschutz and Marc Lipson. *Schaum's Outline of Discrete Mathematics*. McGraw Hill, 2010.

- **[KR]** Kenneth Rosen. *Discrete Math and Its Applications*. McGraw Hill, 2019.

- [LPV] L. Lovász, J. Pelikán, K. Vesztergombi. *Discrete Mathematics Elementary and Beyond*. Springer, 2003.

Assessment

Quizzes	(20%)	9 online (one/topic)
Assignments	(40%)	9 online (one/topic)
Exams (2)	(40%)	Two online (20% each) to be held in week 6 and week 12
Bonus	4%	Based on the quality of Dicussion Forums postings

A minimum of 50% must be obtained on the **two exams** (together) to pass the course.

Course Communication

- Course material will be on OnQ
- Video/audio Lectures will be uploaded
- Discussion Forums will be used to post questions of general nature. There is a General Forum and aForum for each Topic of the course.
- You still can email the instructor/TAs for specific issues
- A weekly live session to answer questions & solve problems

Motivation

- Discrete math is used in cryptography allowing us the convenience of online shopping
- Learning discrete mathematics is the direct pre-requisite to mastering algorithm design and analysis skills
- You should view this course as a *language* course. You will be learning the language of mathematics and computing!
- Math can be fun.
- Math is beautiful!

slides borrowed from Dr. David Rapport' 2019 course

Motivation

- The basis of all of digital information processing is: *Discrete manipulations of discrete structures represented in memory*
- It's the basic language and conceptual foundation for all of computer science
- Discrete math concepts are also widely used throughout math, science, engineering, economics, biology, etc., …
- A generally useful tool for rational thought!

Based on slides Dr. M. P. Frank and Dr. J.L. Gross
Provided by McGraw-Hill

Uses for Discrete Math in Computer Science

- Advanced algorithms & data structures
- Programming language compilers & interpreters
- Computer networks
- Operating systems
- Computer architecture
- Database management systems
- Cryptography
- Error correction codes
- Graphics & animation algorithms, game engines, etc.…
- i.e., the whole field!

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Motivation

- The equation

$$e^{j\pi} + 1 = 0$$

  consists of the most important concepts in mathematics:
- numbers
  - 0,1 (integers)
  - π, e (irrational real numbers)
  - i (a complex number)
- operations

  $+$ $\times$ and exponentiation (exp. function)
- and the relation

  $=$

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Motivation

- The expression

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

is known as the *binomial identity*.

- Does the binomial identity seem like a mess you would like to avoid?

- By the end of the term you will be able to read this and other similar "complicated" looking mathematical expressions.

slides borrowed from Dr. David Rapport' 2019 course

Motivation

- Math is a human *invention* just like music, painting, sculpture, poetry, hockey, basketball, soccer, fishing …
- And how do you become *proficient* at music, painting, hockey … ?

  Practice, practice, practice.

- 10,000 "rule" holds that **10,000 hours** of "deliberate practice" are needed to become world-class in any field. (Working 40 hours per week for a 4 years university degree gets you about half way there)
- The assignment that you do for this course can be viewed as "deliberate practice".

slides borrowed from Dr. David Rapport' 2019 course

The Perfect Introductory Problem: Counting handshakes

- Alice is having a birthday party at her house, and has invited six friends: Bob, Carl, Diane, Eve, Frank, and George.

- They all shake hands with each other.

- Q: How many handshakes?

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The Perfect Introductory Problem: Counting handshakes

- *George* says, “ I know the answer and I can prove it to you. There are 7 of us, so I shake hands with 6 other people. That’s also true for everyone else. So the total number of handshakes is $6 \times 7 = 42$.”
- *Frank* says, “ I have another way of working this out. Suppose there’s only two of us, just George and I. That’s 1 handshake.
- *George*: “No that’s two handshakes! I shake your hand and you shake my hand. That makes 2 handshakes.
- Who’s right?

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The Perfect Introductory Problem: Counting handshakes

- The handshake problem is stated imprecisely and is lacking a clear *definition* of what one *handshake* is.
- We could say that the act of two people touching hands constitutes one handshake, but that too leaves open questions about what part of each hand touches.
- We convert the handshake problem into a math problem using proper mathematical notation.
- The basic building block will be the **set**, where a set is just a collection of distinct objects. (*The topic of "Sets" will be detailed later*)

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The Perfect Introductory Problem: Counting handshakes

- Let $S = \{a, b, c, d, e, f, g\}$ denote the set of party goers, and a handshake can be represented as a two-element subset of S (For example {a,b} denotes the handshake between Alice and Bob.)

- Q: How many two-element subsets are there of the set S?
  for n elements: $\frac{1}{2}n(n-1)$

- The handshake problem seems frivolous, but it is actually a representation of an important mathematical concept.

- For example if we wanted to know which handshake was the "best" we would have to compare $n(n-1)/2$ of them (if n = 35 mil, population of Canada, how many comparisons?!)

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Additional counting problems involving subsets

Suppose that S is a set consisting of n elements.

Q1. How many one element subsets are there of the set S? - (Easy)

Q2. How many zero element subsets are there of the set S? - (Easy)

Q3. How many n element subsets are there of the set S? - (Easy)

Q4. Suppose $n \geq 3$. How many *three* element subsets are there of the set S? - (Harder, to be solved later.)

Q5. Suppose $0 \leq k \leq n$ what is a formula for the number of k element subsets of the set S?

(More general and harder to be solved later.)

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Problems from Schaum's Notes (SN)

Which of the following sets are equal?

1.26 Which of the following sets are equal?

$A = \{x \mid x^2 - 4x + 3 = 0\}$,
$B = \{x \mid x^2 - 3x + 2 = 0\}$,
$C = \{x \mid x \in \mathbb{N}, x < 3\}$,
$D = \{x \mid x \in \mathbb{N}, x \text{ is odd}, x < 5\}$,
$E = \{1, 2\}$,
$F = \{1,2,1\}$,
$G = \{3, 1\}$,
$H = \{1,1,3\}$.

- To determine the elements of sets A and B, you need to be able to factor quadratic equations. This is a topic that you may or may not be familiar with.
- For this course, it is assumed that you are able to do this factoring or pick up this skill on your own.
- All examples that you will see in this course will have *integer* solutions.
- Here's a link to a web page with some good tips for factoring quadratic equations: https://www.mathsisfun.com/algebra/factoring-quadratics.html

slides borrowed from Dr. David Rapport' 2019 course

So, what's this class about?

- What are "discrete structures" anyway?
  - "***Discrete***" -Composed of distinct, separable parts. (Opposite of *continuous*)

    Discrete vs *continuous*, *digital* vs *analog*
  - "***Structures***"-Objects built up from simpler objects according to some definite pattern.
  - "***Discrete Mathematics***" -The study of discrete, mathematical (i.e., well-defined conceptual) objects and structures.

Discrete Objects/Concepts and Structures

- DM I
  - Propositions
  - Proofs
  - Sets
  - Functions
  - Relations
  - Integers
  - Summations
  - Sequences
  - Permutations
  - Combinations

- DM II
  - Orders of Growth
  - Algorithms
  - Graphs
  - Trees
  - Boolean Functions / Logic Circuits
  - Automata
  - Cryptography

Course Contents

1. Sets
2. Logic
3. Proofs
4. Functions and Relations
5. Number Theory
6. Induction
7. Sequences and Summations
8. Recursion
9. Counting

Learning Outcomes

- To complete this course students will demonstrate their ability to:
  1. understand standard Mathematics notation used in the field of Computing
  2. recognize the difference between a proof and a counter example
  3. formulate elementary proofs using mathematical induction
  4. recognize comparative magnitudes of functions
  5. read and understand some elementary logical proofs

Next Time

LaTeX