CISC 102: Discrete Mathematics for Computing I

Fall 2020

Hazem Abbas
hazem.abbas@queensu.ca

Topic 0-2: Proofs

Motivation (1)

- "Mathematical proofs, like diamonds, are hard and clear, and will be touched with nothing but strict reasoning." - *John Locke*
- Mathematical proofs are, in a sense, the only true knowledge we have
- They provide us with a guarantee as well as an explanation (and hopefully some insight)

Motivation (2)

- Mathematical proofs are necessary in CS
  - You must always (try to) prove that your algorithm
    - <u>terminates</u>
    - is <u>sound</u>, <u>complete</u>, <u>optimal</u>
    - finds <u>optimal solution</u>
  - You may also want to show that it is more efficient than another method
  - Proving certain properties of data structures may lead to new, more efficient or simpler algorithms
  - Arguments may entail assumptions. You may want to prove that the assumptions are valid

Proofs

Theorem: <*something*> is true

Proof:

<*valid assumption #1*>

<*valid assumption #2*>

. . . .

<*valid assumption #n*>

therefore <*something*> is true

Theorem: if <*something*> is true, then <*something_else*> is true

Proof:

assume <*something*> is true

<*valid assumption #1*>

<*valid assumption #2*>

. . . .

<*valid assumption #n*>

therefore <*something_else*> is true

Direct Proof

Theorem: Let x, y, z be integer numbers. If $x > y$ and $y > z$ then $x > z$.

Proof:

Assume $x > y$ and $y > z$

$x > y \rightarrow x = y + a$, for some positive integer a

$y > z \rightarrow y = z + b$, for some positive integer b

$\rightarrow x = z + b + a = z + c$, $c = a + b$ *must be a* positive integer

$x > z$ *QED "quod erat demonstrandum"*,

meaning "what was to be shown

// or ■

Next Time

Sets