CISC-102

PRACTICE PROBLESM WITH SOLUTIONS – PROOFS

Problems

(1) Find the argument form for the following argument and determine whether it is valid. Can we conclude that the conclusion is true if the premises are true?
If Socrates is human, then Socrates is mortal.
Socrates is human.

$\therefore$ Socrates is mortal.

(2) What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

Let m be an integer

(a) Prove directly that (m even) $\rightarrow$ (m^3 even)

(b) Prove by contrapositive that (m^3 even) $\rightarrow$ (m even).

(3) Prove that the following three statements are equivalent, for an integer n:

(a) n is even

(b) $n + 4$ is even

(c) $(n + 4)^2 - 1$ is odd.

(4) Prove the proposition $P(0)$, where $P(n)$ is the proposition "If n is a positive integer, then $n^2 > n$." What kind of proof did you use?

(5) Prove using the notion of without loss of generality that $5x + 5y$ is an odd integer when x and y are integers of opposite parity.

Solutions

(1) This is modus tollens. The

first statement is $p \to q$, where p is "Socrates is human" and q is "Socrates is mortal." The second statement is q. The third is p. Modus tollens is valid. We can therefore conclude that the conclusion of the argument (third statement) is true, given that the hypotheses (the

first two statements) are true.

(2) First we use universal instantiation to conclude from "For all x, if x is a man, then x is not an island" the special case of interest, "If Manhattan is a man, then Manhattan is not an island." Then we form the contrapositive (using also double negative): "If Manhattan is an island, then Manhattan is not a man." Finally, we use modus ponens to conclude that Manhattan is not a man. Alternatively, we could apply modus tollens.

(3) (a) Let m be an even integer. Then there exists an integer k such that $m = 2k$. So $m^3 = (2k)^3 = 2(4k^3)$. Since k is an integer, so is $4k^3$, hence m^3 is even.

(b) The contrapositive is (m odd) $\to$ (m^3 odd). Let $m = 2k + 1$ be an odd integer. Then $m^3 = (2k+1)^3 = 8k^3+12k^2+6k+1 = 2(4k^3+6k^2+6k)+1$ Since k is an integer, so is $4k^3 + 6k^2 + 6k$. Therefore m^3 is odd.

(4) We will prove $(a) \to (b) \to (c) \to (a)$:
$(a) \to (b)$: If n is even then $n = 2k$ for an integer k. Then $n + 4 = 2k + 4 = 2(k + 2)$. So $n + 4$ is even.
$(b) \to (c)$: If $n + 4$ is even then so is $(n + 4)^2$ (square of an even number is even). So $(n + 4)^2 - 1$ is odd.
$(c) \to (a)$: If $(n+4)^2 - 1$ is odd, then $(n+4)^2$ is even, and hence so is $n+4$. So $n + 4 = 2l$ for some integer l. But then $n = 2(l - 2)$ and n is even.

(5) The proposition is vacuously true because 0 is not a positive integer. Vacuous proof.

(6) Because x and y are of opposite parities, we can assume, without loss of generality, that x is even and y is odd. This tells us that $x = 2m$for some integer m and $y = 2n+1$ for some integer n. Then $5x+5y = 5(2m)+5(2n+1) = 10m + 10n + 1 = 10(m + n) + 1 = 2.5(m + n) + 1$, which satisfies the definition of being an odd number.