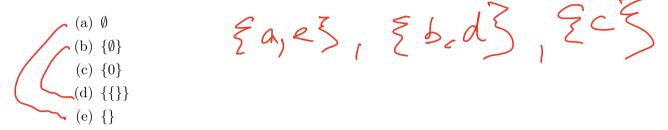
Student ID Page 2 of 9

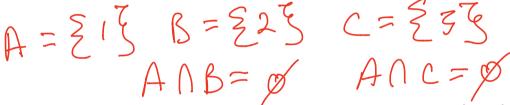
Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. There are 9 pages to this exam. Note that ( x ) denotes the question is worth x points.

1. (4) Group the following into sets that are equal.

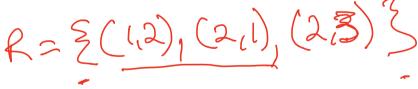


2. (4) Show that there are sets A, B, C such that  $A \cap B = A \cap C$  and  $B \neq C$ .

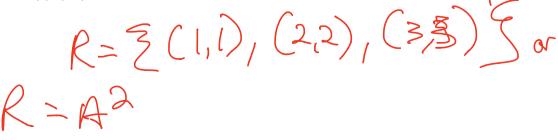


- 3. In each of the following give an example of a relation R on the set  $A = \{1, 2, 3\}$  such that:
  - (a) (2) R is both symmetric and antisymmetric.

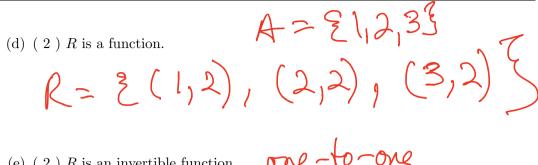
(b) ( 2 )  ${\cal R}$  is neither symmetric nor antisymmetric.

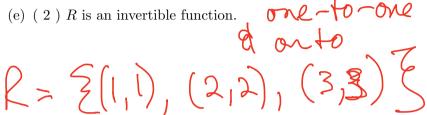


(c) (2) R is reflexive.



Student ID Page 3 of 9





4. Let R be the relation on the natural numbers defined by

$$R = \{(x, y) : x, y \in \mathbb{N}, x^2 + y^2 \le 8\}.$$

(a) (2) Write out the elements of R as a set of ordered pairs.



(b) (2) Is R an equivalence relation, and explain why or why not?

Petlexive, Symmetric, transitive

No Ris not an equivareable rise

(c) (2) Is R a partial order and explain why or why not?

Antisymmetric, transitive, reflexive

No Ris not a partial order

be cause Ris not reflexive.

Ris not antisymmetric.

Student ID Page 4 of 9

5. The Division Algorithm Theorem can be written as:

$$a = bq + r$$
 and  $0 \le r < |b|$ 

For each of the following statements state whether it is true or false, and if it is false correct it.



(b) (2) q and r are unique.



6. Find the quotient q and remainder r, as given by the Division Algorithm theorem for the following examples.

(a) (2) 
$$a = 23$$
,  $b = 8$   
 $23 = (2)(8) + 7$   
(b) (2)  $a = -23$ ,  $b = 8$   
 $-23 = (-3)(8) + 1$   
(c) (2)  $a = -23$ ,  $b = -8$   
 $-23 = (3)(-8) + 1$   
(d) (2)  $a = 23$ ,  $b = -8$   
 $23 = (2)(-8) + 7$   
 $33 = (2)(-8) + 7$   
 $33 = (2)(-8) + 7$ 

Student ID Page 5 of 9

7. Let a=23, and b=8. In the following  $\gcd(a,b)$  and  $\operatorname{lcm}(a,b)$  respectively denote functions that return the greatest common divisor and least common multiple of a and b.

(a) (4) Find 
$$g = \gcd(a, b)$$
. Show the steps used by Euclid's algorithm to find  $\gcd(a, b)$ .

Ci) 
$$23 = (2)(8) + 7$$
  
(2)  $8 = (1)(7) + 1$   $50 \text{ gcd}(23,8) = 1$   
(3)  $7 = (7)(1) + 0$ 

(b) (4) Find integers m and n such that g = ma + nb, and show the steps that you used.

out used.
$$1 = 8 - 7 = 8 - (23 - (2)(8))$$

$$= (3)8 - 23$$

(c) (4) Find lcm(a, b), and show the steps that you used.

$$\int c m (a,b) = \underbrace{\frac{1}{3} (x|b|)}_{3cd(a,b)} = \underbrace{\frac{23 \times 8}{1}}_{1}$$

8. Let a, b, c be Integers.

(a) (4) Prove that if 
$$a|b$$
 and  $a|c$ , then  $a|(b+c)$ 

$$b = g \text{ Or where } g \in \mathbb{Z}$$

$$C = p \text{ Or } \text{ or } p \in \mathbb{Z}$$

$$b + C = a \text{ (b+c)}$$

$$a \text{ (b+c)}$$

(b) ( 4 ) Prove that if a|b then for any integer n,a|bn.

$$b=ga$$
 where  $g \in Z$   
 $bn=nga$   $ng \in Z$   
 $So$   $a[bn]$ 

Student ID Page 6 of 9

9. (5) The province of Ontario adopted the AAAA-000 licence plate format in 1997, Where an A denotes any upper case letter, [A .. Z] (there are 26 letters), and 0 denotes a digit [0 .. 9]. How many different AAAA-000 licence plates can be assigned? Explain how you arrived at your answer.

3) 26 letters in deplatent

10. (5) How many different permutations of the string "AAAGTCTGAC" are there? Explain how you arrived at your answer.

11. (5) There are 30 students in a gym class. The gym teacher must partition the students into 4 teams of 9,8,7 and 6 players. In how many ways can this be done?

Explain how you arrived at your answer.

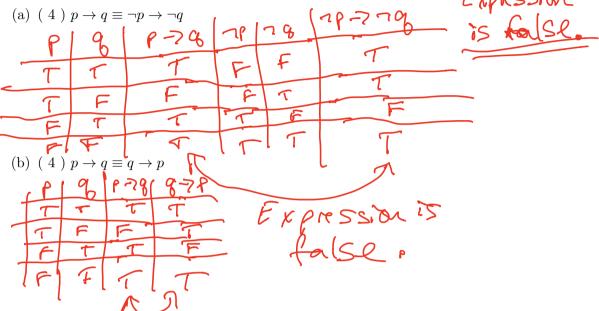
12. (5) A farmer buys 3 cows, 3 pigs, and 3 hens from a farmer who has 6 cows, 5 pigs, and 8 hens. How many choices does the farmer have for selecting his new animals?

13. (5) In how many ways can 25 dimes be distributed to 4 children if each child must get at least 2 dimes? Explain how you arrived at your answer.

le pre-allocate 2 doines to ach child leaving 25-8=17 dines

Student ID Page 7 of 9

14. Consider the following logical expressions. Determine whether the expression is true or false, and justify your answer using truth tables.



- 16. Consider the recursive function given by  $a_1 = 1$  and  $a_n = 3a_{n-1}$ .
  - (a) (2) Find  $a_2$ ,  $a_3$ , and  $a_4$ .  $a_2 = 3a_1 = 3$ ,  $a_3 = 3a_2 = 9$ a4=393=27
  - (b) (5) Using the values of  $a_2$ ,  $a_3$ , and  $a_4$  guess the value of  $a_n$  and prove that it