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Read the questions carefully. Please clearly state any assumptions that you make that are not explicitly stated in the question.

Please answer all questions in the space provided. Use the back of pages for scratch work. There are 9 pages to this exam. Note that (x) denotes the question is worth x points.

1. (4) Group the following into sets that are equal.

- (a) Ø
- (b) $\{\emptyset\}$
- (c) $\{0\}$
- (d) {{}}
- (e) {}

2. (4) Show that there are sets A, B, C such that $A \cap B = A \cap C$ and $B \neq C$.

3. In each of the following give an example of a relation R on the set $A = \{1, 2, 3\}$ such that:

(a) (2) R is both symmetric and antisymmetric.

- (b) (2) R is neither symmetric nor antisymmetric.
- (c) (2) R is reflexive.

- (d) (2) R is a function.
- (e) (2) R is an invertible function.

4. Let R be the relation on the natural numbers defined by

$$R = \{(x, y) : x, y \in \mathbb{N}, x^2 + y^2 \le 8\}.$$

- (a) (2) Write out the elements of R as a set of ordered pairs.
- (b) (2) Is R an equivalence relation, and explain why or why not?
- (c) (2) Is R a partial order and explain why or why not?

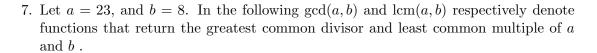
5. The Division Algorithm Theorem can be written as:

$$a = bq + r$$
 and $0 \le r < |b|$

For each of the following statements state whether it is true or false, and if it is false correct it.

- (a) (2) $a, b \in \mathbb{R}$
- (b) (2) q and r are unique.
- (c) (2) $q, r \in \mathbb{Z}$
- 6. Find the quotient q and remainder r, as given by the Division Algorithm theorem for the following examples.
 - (a) (2) a = 23, b = 8
 - (b) (2) a = -23, b = 8
 - (c) (2) a = -23, b = -8
 - (d) (2) a = 23, b = -8

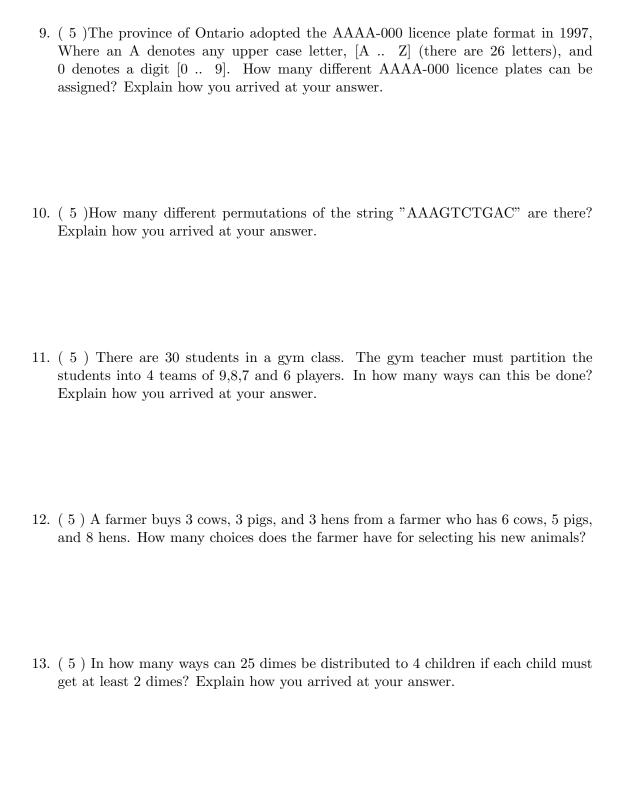
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- (a) (4) Find $g = \gcd(a, b)$. Show the steps used by Euclid's algorithm to find $\gcd(a, b)$.
- (b) (4) Find integers m and n such that g = ma + nb, and show the steps that you used.
- (c) (4) Find lcm(a, b), and show the steps that you used.

- 8. Let a, b, c be Integers.
 - (a) (4) Prove that if a|b and a|c, then a|(b+c)
 - (b) (4) Prove that if a|b then for any integer n, a|bn.

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14. Consider the following logical expressions. Determine whether the expression is true or false, and justify your answer using truth tables.

(a) (4)
$$p \rightarrow q \equiv \neg p \rightarrow \neg q$$

(b) (4)
$$p \rightarrow q \equiv q \rightarrow p$$

15. (5) Use induction to prove $1+3+5+\cdots+2n-1=n^2$, for all $n\in\mathbb{N}, n\geq 1$.

- 16. Consider the recursive function given by $a_1 = 1$ and $a_n = 3a_{n-1}$.
 - (a) (2) Find a_2 , a_3 , and a_4 .
 - (b) (5) Using the values of a_2 , a_3 , and a_4 guess the value of a_n and prove that it is true using mathematical induction.