Queen's University School of Computing

CISC 203: Discrete Mathematics for Computing II
Problem Set 2
Due October 8, 2021 by 11:59pm (Kingston time)

Show all your work/steps and explain all your solutions.

Your solutions must be prepared in LaTeX. If we notice that you have submitted handwritten solutions, depending on when we notice we will either: 1) ask you to type up and resubmit your work, incurring a late penalty if applicable, or 2) mark your submission with a 33% penalty applied.

You may use the Overleaf template below, which contains some useful sample code:

I. Log in using your NetID, by clicking the "Log in through your institution" button at the bottom of this page: https://www.overleaf.com/edu/queensu

II. Access the template here: https://www.overleaf.com/read/ncdhzmmxtkzj

III. Click "Menu" on the top left corner and then on "Copy Project" to create your copy of the template.

IV. When you are finished writing your solutions, download your PDF and submit it on onQ.

Problem set collaboration policy:

1. This problem set is an individual assessment that is meant to replace an in-class test, i.e., you are expected to solve the problems and write your solutions individually. However, it is open book and you are being given an extended period of time to help better manage your learning workload.

2. Do not ask the TAs or other students for help on directly solving the problem set questions. However, you may identify examples or exercises in the course notes that refer to similar concepts or require a similar strategy to solve (identifying such connections between questions demonstrates individual effort in solving the problem), and may ask for help during office hours or on the discussion board to understand those examples.

3. Do not publicly post (e.g., on the discussion board or Microsoft Teams) that "Question X in the problem set is similar to Question Y in the course notes". It is important that each student becomes familiarized enough with the material to observe such connections on their own.

4. If you encounter any explanations on the discussion board or during office hours that help you to solve a question, add a sentence to your solution(s) (and provide the URL if applicable) to give credit. Failure to do so may result in penalty (when in doubt, no need to ask—just give credit).

5. Questions about clarifying any aspect of the problem set (e.g., to clarify the wording of a question) should be posted on the discussion board topic, so that all students can benefit equally from the answer: https://discourse.caslab.queensu.ca/c/cisc-203-f21/33.

Note: 2 marks (out of 68) in Problem Set 2 are reserved for formatting, which consists of correctly entering your name into the LaTeX template, removing the sample text/images from it, and formatting your solutions in a way that is readable by the TAs (please be considerate of them), e.g., by inserting line breaks where appropriate. Everybody should be able to get these 2 marks!

[13 marks] 1. Prove that $3 \mid (2n^2+1)$ if and only if $3 \nmid n$, for all $n \in \mathbb{Z}$. Experiment with the different techniques **that were covered in class** and use what you find easiest.

[13 marks] 2. Does there exist any $a \in \mathbb{Z}$ such that $a \equiv 5 \bmod 14$ **and** $a \equiv 3 \bmod 21$? Make a conjecture. Then, use an appropriate technique covered in class to prove your conjecture.

[13 marks] 3. Let $n \in \mathbb{N}$. Give a proof by smallest counterexample that $5 \mid (n^5 - n)$.

[13 marks] 4. Consider the sequence defined by

$$a_n = 2a_{n-1} - a_{n-3}$$

for $n \geq 4$ and with initial terms $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$. Prove by induction that $a_n = a_{n-1} + a_{n-2}$ for all $n \geq 3$.

[14 marks] 5. Consider the sequence defined by

$$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3}$$

for $n \geq 4$ and with initial terms $a_1 = 3, a_2 = 7$, and $a_3 = 13$.

(a) Calculate a_4, a_5, a_6, and a_7 by hand.

(b) Using a script (e.g., Python) or spreadsheet, calculate a few hundred terms and plot the values on an xy-plane.

(c) Based on your experimentation/observations above, conjecture a closed-form expression for the sequence and briefly explain how you arrived at your conjecture.

(d) Prove by induction the correctness of your closed-form expression.

Note: After your solution to this question, include evidence of how you generated the plotted values (e.g., code and/or screenshots) from part (b) above.