Multivariate Time Series Models

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Motivation of Multivariate Models

- Multivariate time series analysis is used when one wants to model and explain the interactions and co-movements among a group of time series variables.
- In finance, multivariate time series analysis is used to model systems of asset returns, asset prices and exchange rates, the term structure of interest rates, asset returns/prices, and economic variables etc.
- Many of the time series concepts described previously for univariate time series carry over to multivariate time series in a natural way. Additionally, there are some important time series concepts that are particular to multivariate time series.

Motivating Example

Consider the relationship between two U.S. weekly interest rate series:

- r_{1t} : the 1-year Treasury constant maturity rate
  r_{3t} : the 3-year Treasury constant maturity rate
- This example leads naturally to the consideration of a linear regression in the form

$$r_{3t} = \alpha_1 + \beta_1 r_{1t} + e_t$$

  The least squares (LS) method is often used to estimate this model.

Motivating Example

However, it is one of the most commonly misused econometric models, because

- In practice, it is common to see that the error term e_t is serially correlated.
- Issue of Simultaneity : changes in the value of r_{3t} will impact upon the explanatory variable r_{1t}, i.e. r_{1t} is not exogenous.

$$r_{1t} = \alpha_2 + \beta_2 r_{3t} + \epsilon_t$$

- Spurious Regression : the two interest rate series are unit-root nonstationary, but they are not cointegrated. (later lectures for discussion of cointegration).
- The OLS estimates of α_1 and β_1 may not be consistent.

Two-dimensional Time Series

- Time series:

$$r_t = \begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix}$$

- Data: $\{r_1, r_2, \cdots, r_T\}$
- Some examples: (a) U.S. quarterly GDP and unemployment rate series; (b) Stock prices and dividends
- Why consider two series jointly? (a) Obtain the relationship between the series and (b) improve the accuracy of forecasts (use more information).

Weak Stationarity

- Both

$$E(r_t) = \begin{bmatrix} E(r_{1t}) \\ E(r_{2t}) \end{bmatrix} = \mu \qquad \textit{and}$$

$$Var(r_t) = \begin{bmatrix} Var(r_{1t}) & Cov(r_{1t}, r_{2t}) \\ Cov(r_{2t}, r_{1t}) & Var(r_{2t}) \end{bmatrix} = \Gamma_0$$

are time invariant.

Autocovariance Matrix : Lag-j

$$\begin{aligned}
\Gamma_j &= Cov(r_t, r_{t-j}) = E[(r_t - \mu)(r_{t-j} - \mu)'] \\
&= \begin{bmatrix} E(r_{1t} - \mu_1)(r_{1,t-j} - \mu_1) & E(r_{1t} - \mu_1)(r_{2,t-j} - \mu_2) \\ E(r_{2t} - \mu_2)(r_{1,t-j} - \mu_1) & E(r_{2t} - \mu_2)(r_{2,t-j} - \mu_2) \end{bmatrix} \\
&= \begin{bmatrix} \Gamma_{11}(j) & \Gamma_{12}(j) \\ \Gamma_{21}(j) & \Gamma_{22}(j) \end{bmatrix}
\end{aligned}$$

Not symmetric if $j \neq 0$. Consider Γ_1 :

- $\Gamma_{12}(1) = Cov(r_{1t}, r_{2,t-1})$ (r_{1t} depends on past r_{2t})
- $\Gamma_{21}(1) = Cov(r_{2t}, r_{1,t-1})$ (r_{2t} depends on past r_{1t})

Cross-Correlation Matrices

- Let the diagonal matrix D be

$$D = \begin{bmatrix} std(r_{1t}) & 0 \\ 0 & std(r_{2t}) \end{bmatrix} = \begin{bmatrix} \sqrt{\Gamma_{11}(0)} & 0 \\ 0 & \sqrt{\Gamma_{22}(0)} \end{bmatrix}$$

- Cross-Correlation Matrices :

$$\rho_l = D^{-1}\Gamma_l D^{-1}$$

  Thus, $\rho_{ij}(l)$ is the cross-correlation between r_{it} and $r_{j,t-l}$.

- From stationarity :

$$\Gamma_l = \Gamma'_{-l}, \qquad \rho_l = \rho'_{-l}$$

  For instance, $corr(r_{1t}, r_{2,t-1}) = corr(r_{2t}, r_{1,t+1})$.

k-dimensional Time Series

- Consider k time series variables $\{r_{1t}\}, \cdots \{r_{kt}\}$. A k-dimensional time series is the $(k \times 1)$ vector time series $\{r_t\}$ where the i^{th} row of $\{r_t\}$ is $\{r_{it}\}$. That is, for any time t, $r_t = (r_{1t}, \cdots, r_{kt})'$.
- $$\begin{aligned} &E(r_t) = \mu = (\mu_1, \cdots, \mu_k)' \\ &Var(r_t) = \Gamma_0 = E[(r_t - \mu)(r_t - \mu)'] \\ &= \begin{pmatrix} Var(r_{1t}) & Cov(r_{1t}, r_{2t}) & \cdots & Cov(r_{1t}, r_{kt}) \\ Cov(r_{2t}, r_{1t}) & Var(r_{2t}) & \cdots & Cov(r_{2t}, r_{kt}) \\ \vdots & \vdots & \ddots & \vdots \\ Cov(r_{kt}, r_{1t}) & Cov(r_{kt}, r_{2t}) & \cdots & Var(r_{kt}) \end{pmatrix} \end{aligned}$$

k-dimensional Time Series

- D is a $(k \times k)$ diagonal matrix

$$D = \begin{pmatrix} \sqrt{Var(r_{1t})} & 0 & \cdots & 0 \\ 0 & \sqrt{Var(r_{2t})} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{Var(r_{kt})} \end{pmatrix}$$

- The correlation matrix of r_t is the $k \times k$ matrix

$$corr(r_t) = \rho_0 = D^{-1}\Gamma_0 D^{-1}$$

Lead-Lag Relationships : Cross-Correlation Matrices

- The lag-l cross-covariance matrix of r_t

$$\Gamma_l \equiv [\Gamma_{ij}(l)] = E[(r_t - \mu)(r_{t-l} - \mu)']$$

  The $\Gamma_{ij}(l)$ is the covariance between r_{it} and $r_{j,t-l}$. For a weakly stationary series, the cross-covariance matrix Γ_l is a function of l, not the time index t.

- The lag-l cross-correlation matrix (CCM) of r_t

$$\rho_l \equiv [\rho_{ij}(l)] = D^{-1}\Gamma_l D^{-1}$$

- $\rho_{ij}(l)$ is the correlation coefficient between r_{it} and $r_{j,t-l}$. If $\rho_{ij}(l) \neq 0$ and $l > 0$, we say that the series r_{jt} leads the series r_{it} at lag l.
- Similarly, if $\rho_{ji}(l) \neq 0$ and $l > 0$, we say that the series r_{it} leads the series r_{jt} at lag l.
- It is possible that r_{it} leads r_{jt} and vice-versa. In this case, there is said to be feedback between the two series.
- $\rho_{ii}(l)$ is simply the lag-l autocorrelation coefficient of r_{it} .

Sample Cross-Correlation Matrices

- The cross-covariance matrix Γ_l can be estimated by

$$\widehat{\Gamma}_l = \frac{1}{T} \sum_{t=l+1}^{T} (r_t - \overline{r})(r_{t-l} - \overline{r})', \quad l \geq 0$$

  where $\overline{r} = (\sum_{t=1}^{T} r_t)/T$ is the vector of sample means.
- The cross-correlation matrix ρ_l is estimated by

$$\widehat{\rho}_l = \widehat{D}^{-1} \widehat{\Gamma}_l \widehat{D}^{-1}, \quad l \geq 0$$

  where $\widehat{D}$ is the $k \times k$ diagonal matrix of the sample standard deviations of the component series.

Examples

- Example 8.1. and Example 8.2. pp 393-397
- Eviews commands
  1. Quick → Group Statistics → Correlations
  2. Quick → Group Statistics → Descriptive Statistics
  3. Quick → Group Statistics → Cross Correlogram
  4. Quick → Graph → date/10000 IBM (XY line)
  5. Quick → Graph → SP IBM (scatter)
  6. Quick → Graph → SP(-1) IBM (scatter)

Testing for Serial Dependence

Multivariate Portmanteau Tests/Ljung-Box statistics $Q(m)$:

- $H_0 : \rho_1 = \cdots = \rho_m = 0$ *v.s.* $H_a : \rho_i \neq 0$ *for some* $i \in \{1, \cdots, m\}$
- The test statistics is

$$Q_k(m) = T^2 \sum_{l=1}^{m} \frac{1}{T-l} tr\left(\widehat{\Gamma}_l{}' \widehat{\Gamma}_0{}^{-1} \widehat{\Gamma}_l \widehat{\Gamma}_0{}^{-1}\right)$$

where k is the dimension of r_t and tr is the sum of diagonal elements.

Testing for Serial Dependence

Multivariate Portmanteau Tests/Ljung-Box statistics $Q(m)$:

- Under the null hypothesis and some regularity conditions, $Q_k(m)$ follows asymptotically a $\chi^2(k^2m)$.
- The $Q_k(m)$ statistic is a joint test for checking the first m cross-correlation matrices of r_t being zero. If it rejects the null hypothesis, then we build a multivariate model for the series to study the lead-lag relationships between the component series.
- Estimate VAR : regress on a constant $\rightarrow$ residual tests $\rightarrow$ Portmanteau Autocorrelation Test

Vector Autoregressive Models (VAR)

- The vector autoregressive (VAR) model is one of the most successful, flexible, and easy to use models for the analysis of multivariate time series.
- It is a natural extension of the univariate autoregressive model to dynamic multivariate time series.
- Has proven to be especially useful for describing the dynamic behavior of economic and financial time series and for forecasting.
- It often provides superior forecasts to those from univariate time series models and elaborate theory-based simultaneous equations models.

VAR(1) Model for Two Return Series

$$\begin{bmatrix} r_{1t} \\ r_{2t} \end{bmatrix} = \begin{bmatrix} \phi_{10} \\ \phi_{20} \end{bmatrix} + \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \begin{bmatrix} r_{1,t-1} \\ r_{2,t-1} \end{bmatrix} + \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}$$

where $a_t = (a_{1t}, a_{2t})'$ is a sequence of iid bivariate normal random vectors with mean zero and covariance matrix

$$Cov(a_t) = \Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \quad \sigma_{12} = \sigma_{21}$$

VAR(1) Model for Two Return Series

Rewrite the model as

$$
\begin{aligned}
r_{1t} &= \phi_{10} + \phi_{11} r_{1,t-1} + \phi_{12} r_{2,t-1} + a_{1t} \\
r_{2t} &= \phi_{20} + \phi_{21} r_{1,t-1} + \phi_{22} r_{2,t-1} + a_{2t}
\end{aligned}
$$

Thus, ϕ_{11} and ϕ_{12} denotes the dependence of r_{1t} on the past returns $r_{1,t-1}$ and $r_{2,t-1}$, respectively.

VAR(1) Model for Two Return Series : Unidirectional Dependence

For the VAR(1) model, if $\phi_{12} = 0$, but $\phi_{21} \neq 0$, then

- r_{1t} does not depend on $r_{2,t-1}$, but
- r_{2t} depends on $r_{1,t-1}$,

implying that knowing $r_{1,t-1}$ is helpful in predicting r_{2t}, but $r_{2,t-1}$ is not helpful in forecasting r_{1t}.
Here $\{r_{1t}\}$ is an **input**, $\{r_{2t}\}$ is the **output** variable. This is an example of **Granger** causality relation.

VAR(1) Model for Two Return Series : Feedback Relationship

For the VAR(1) model, if $\phi_{12} \neq 0$, and $\phi_{21} \neq 0$, then

- r_{1t} depends on $r_{2,t-1}$ in the presence of $r_{1,t-1}$
- r_{2t} depends on $r_{1,t-1}$ in the presence of its own past

implying that knowing $r_{1,t-1}$ is helpful in predicting r_{2t}, and $r_{2,t-1}$ is also helpful in forecasting r_{1t}.

VAR(1) Model for Two Return Series : Uncoupled

For the VAR(1) model, if $\phi_{12} = \phi_{21} = 0$, then

- r_{1t} does not depend on $r_{2,t-1}$ in the presence of $r_{1,t-1}$
- r_{2t} does not depend on $r_{1,t-1}$ in the presence of $r_{2,t-1}$

What is the implication?

VAR(1) Model

- Stationarity condition : write the VAR(1) model as (generalization of 1-dimensional case)

$$r_t = \Phi_0 + \Phi r_{t-1} + a_t$$

  $\{r_t\}$ is stationary if solutions of $det(I - \Phi z) = 0$ are all greater than 1 in modulus.

- $det(I - \Phi z) = 0$ for bivariate VAR(1) model becomes

$$(1 - \phi_{11} z)(1 - \phi_{22} z) - \phi_{12}\phi_{21} z^2 = 0$$

  1. Stability condition involves cross terms ϕ_{12} and ϕ_{21}.
  2. If $\phi_{12} = \phi_{21} = 0$ (diagonal VAR) then bivariate stability condition reduces to univariate stability conditions for each equation.

VAR(1) Model

- Mean of r_t satisfies

$$(I - \Phi)\mu = \Phi_0, \quad or$$

$$\mu = (I - \Phi)^{-1}\Phi_0$$

  if the inverse exists.

- Covariance matrices of VAR(1) models :

$$Cov(r_t) = \sum_{i=0}^{\infty} \Phi^i \Sigma (\Phi^i)'$$

VAR(1) Model

- The lag-l cross-covariance matrix of r_t:

$$\Gamma_l = \Phi\Gamma_{l-1} \quad \textit{for } l > 0$$

  Can be generalized to higher order models.

- The lag-l cross-correlation matrix of r_t:

$$\rho_l = \Upsilon\rho_{l-1} \quad \textit{for } l > 0$$

  where $\Upsilon = D^{-1/2}\Phi D^{1/2}$.

Does the VAR Include Contemporaneous Terms?

- The concurrent relationship between r_{1t} and r_{2t} is shown by the off-diagonal element σ_{12} of the covariance matrix Σ of a_t. If $\sigma_{12} = 0$, then r_{1t} and r_{2t} are not concurrently correlated.
- In the econometric literature, the VAR(1) model is called a reduced-form model because it does not show explicitly the concurrent dependence between the component series.

Structural VAR

- Structural VAR

$$B_0 r_t = c + B_1 r_{t-1} + u_t$$

where

$$u_t \sim^{i.i.d.} N\left(0, \left[\begin{array}{cc} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{array}\right]\right)$$

- Rewrite the model as

$$\begin{aligned} r_{1t} - b_{12}^{(0)} r_{2t} &= c_{10} + b_{11}^{(1)} r_{1,t-1} + b_{12}^{(1)} r_{2,t-1} + u_{1t} \\ r_{2t} - b_{21}^{(0)} r_{1t} &= c_{20} + b_{21}^{(1)} r_{1,t-1} + b_{22}^{(1)} r_{2,t-1} + u_{2t} \end{aligned}$$

Reduced Form VAR

- Reduced form VAR : we can then pre-multiply both sides by $B_0^{-1} = \frac{1}{1-b_{12}^{(0)}b_{21}^{(0)}}\begin{bmatrix} 1 & b_{12}^{(0)} \\ b_{21}^{(0)} & 1 \end{bmatrix}$ to give

$$r_t = B_0^{-1}c + B_0^{-1}B_1 r_{t-1} + B_0^{-1}u_t$$

- or

$$r_t = \Phi_0 + \Phi_1 r_{t-1} + a_t$$

with

$$a_t \sim^{i.i.d.} N(0, \Sigma) \quad \Sigma = B_0^{-1}\begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} B_0^{-1\prime}$$

Does the VAR Include Contemporaneous Terms?

The structural form of the VAR model, is not identified. Why? Check the order condition.

- We impose a restriction to achieve the identification. The choice of restrictions is ideally made on theoretical grounds.
- It is also very common to estimate only a reduced form VAR. It is enough if the purpose of producing the VAR is to make forecasts.

Does the VAR Include Contemporaneous Terms?

- If we could identify B_0, then we can retrieve c and B_1.
- Restriction (for the purpose of identification) : one of the coefficients on the contemporaneous terms is zero. i.e. B_0 is a lower triangular matrix.
- Cholesky decomposition: $\Sigma = LGL'$, where L is a lower triangular matrix with unit diagonal elements and G is a diagonal matrix.
- Reduced form VAR : $r_t = \Phi_0 + \Phi r_{t-1} + a_t$
  Structural form :

$$L^{-1} r_t = L^{-1} \Phi_0 + L^{-1} \Phi r_{t-1} + L^{-1} a_t = \Phi_0^* + \Phi^* r_{t-1} + u_t$$

Does the VAR Include Contemporaneous Terms?

- Example 8.3. pp 400
- It is very common to estimate only a reduced form VAR, for two reasons. The first reason is ease in estimation. The second and main reason is that the concurrent correlations cannot be used in forecasting.

VAR(p) Models

- The basic p-lag vector autoregressive model has the form

$$r_t = \Phi_0 + \Phi_1 r_{t-1} + \cdots + \Phi_p r_{t-p} + a_t, \quad p > 0$$

  where Φ_0 is a k-dimensional vector, Φ_j are $k \times k$ matrices, and $\{a_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix Σ.
  Or

$$(I_k - \Phi_1 B - \cdots - \Phi_p B^p) r_t = \Phi_0 + a_t$$

  where I_k is the $k \times k$ identity matrix.

- Stationarity condition : the roots of $det\,(I_k - \Phi_1 z - \cdots - \Phi_p z^p) = 0$ lie outside the complex unit circle.

VAR(p) Models

If r_t is weakly stationary, then we have

- $\mu = E(r_t) = (I - \Phi_1 - \cdots - \Phi_p)^{-1}\Phi_0$
- $Cov(r_t, a_t) = \Sigma$, the covariance matrix of a_t
- $Cov(r_{t-l}, a_t) = 0$ for $l > 0$
- $\Gamma_l = \Phi_1\Gamma_{l-1} + \cdots + \Phi_p\Gamma_{l-p}$ for $l > 0$.
- $\rho_l = \Upsilon_1\rho_{l-1} + \cdots + \Upsilon_p\rho_{l-p}$ for $l > 0$, where $\Upsilon_i = D^{-1/2}\Phi_i D^{1/2}$.

VAR(p) Models: Estimation

- k equations : each equation has the same regressors.
- Endogeneity is avoided by using lagged values.
- The VAR(p) model is just a seemingly unrelated regression (SUR) model with lagged variables and deterministic terms as common regressors.

VAR(p) Models: Estimation

Assume there are no restrictions on the parameters of the model. In SUR notation, each equation in the VAR(p) may be written as

$$r_i = Z\pi_i + e_i, \quad i = 1, 2, \cdots, k$$

- r_i is a $T \times 1$ vector of observations on the i^{th} equation (the return series of the i^{th} stock)
- Z is a $T \times (kp+1)$ matrix with t^{th} row given by $Z_t' = [1, r_{t-1}', r_{t-2}', \cdots, r_{t-p}']$ (the lagged p values of all k stock returns)
- π_i is a $(kp+1) \times 1$ vector of parameters and e_i is a $T \times 1$ error with covariance matrix $\sigma_i^2 I_T$.

VAR(p) Models: Estimation

- Since each equation contains exactly the same set of regressors, the FGLS estimator in SUR model turns out to be numerically identical to OLS estimates following from Kruskal's theorem.
- We estimate each equation separately by ordinary least squares. Let

$$\widehat{\Pi} = [\hat{\pi}_1, \cdots, \hat{\pi}_k]$$

denote the $(kp + 1) \times k$ matrix of least squares coefficients for the k equations.

VAR Models : Model Selection

- The included variables in a VAR model are selected according to the relevant economic or financial theory. The selected variables must have economic influences on each other. In other terms, there must be causality between them.
- The overparameterization and loss of degrees of freedom problems must be avoided to capture the important information in the system.
- If the lag length is too small, the model will be misspecified; if it is too large, the degrees of freedom will be lost.

VAR Models : Order Specification

- Fit VAR(p) models with orders $p = 0, 1, \cdots, p_{max}$ and choose the value of p which minimizes some model selection criteria
- Under VAR(p) model, the residual is $\widehat{a_t}^{(p)} = r_t - \widehat{\Pi}' Z_t$
- The ML estimate of Σ is $\widehat{\Sigma}_p = \frac{1}{T}\sum_{t=p+1}^{T} \widehat{a_t}^{(p)} \left[\widehat{a_t}^{(p)}\right]'$
- Under the normality assumption,

$$\begin{aligned}
AIC(p) &= \log\left(|\widehat{\Sigma}_p|\right) + \frac{2k^2 p}{T} \\
BIC(p) &= \log\left(|\widehat{\Sigma}_p|\right) + \frac{k^2 p \log(T)}{T} \\
HQIC(p) &= \log\left(|\widehat{\Sigma}_p|\right) + \frac{2k^2 p \log[\log(T)]}{T}
\end{aligned}$$

VAR Models : Granger Causality Tests

One of the main uses of VAR models is forecasting. The following intuitive notion of a variable's forecasting ability is due to Granger (1969).

- If a variable, or a group of variables, r_1 is found to be helpful for predicting another variable, or group of variables, r_2, then r_1 is said to **Granger-cause** r_2; otherwise it is said to fail to Granger-cause r_2.
- Formally, r_1 fails to Granger-cause r_2 if for all $s > 0$ the MSE of a forecast of $r_{2,t+s}$ based on $(r_{2,t}, r_{2,t-1}, \cdots)$ is the same as the MSE of a forecast of $r_{2,t+s}$ based on $(r_{2,t}, r_{2,t-1}, \cdots)$ and $(r_{1,t}, r_{1,t-1}, \cdots)$.
- The notion of Granger-causality does not imply true causality. It only implies forecasting ability.

VAR Models : Granger Causality Tests

- In a bivariate VAR(p) model, r_2 fails to Granger-cause r_1 if all of the p VAR coefficient matrices $\Phi_1, \cdots, \Phi_p$ are lower triangular:

$$\begin{aligned} \begin{pmatrix} r_{1t} \\ r_{2t} \end{pmatrix} &= \begin{pmatrix} \phi_{10} \\ \phi_{20} \end{pmatrix} + \begin{pmatrix} \phi_{11}^1 & 0 \\ \phi_{21}^1 & \phi_{22}^1 \end{pmatrix} \begin{pmatrix} r_{1,t-1} \\ r_{2,t-1} \end{pmatrix} + \cdots \\ &+ \begin{pmatrix} \phi_{11}^p & 0 \\ \phi_{21}^p & \phi_{22}^p \end{pmatrix} \begin{pmatrix} r_{1,t-p} \\ r_{2,t-p} \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \epsilon_{2t} \end{pmatrix} \end{aligned}$$

- If r_2 fails to Granger-cause r_1 and r_1 fails to Granger-cause r_2, then the VAR coefficient matrices $\Phi_1, \cdots, \Phi_p$ are diagonal. And we can model r_1 and r_2 separately.

VAR Models : Granger Causality Tests

In the bivariate model, testing H_0: r_2 does not Granger-cause r_1 reduces to a testing H_0: $\phi_{12}^1 = \phi_{12}^2 = \cdots = \phi_{12}^p = 0$ from the linear regression

$$\begin{aligned} r_{1t} &= \phi_{10} + \phi_{11}^1 r_{1,t-1} + \cdots + \phi_{11}^p r_{1,t-p} \\ &+ \phi_{12}^1 r_{2,t-1} + \cdots + \phi_{12}^p r_{2,t-p} + \epsilon_{1t} \end{aligned}$$

The test statistic is a simple F-statistic.

VAR Models : Model checking and Forecasting

- The $Q_k(m)$ statistic can be applied to the residual series to check the assumption that there are no serial or cross correlations in the residuals.
- For a fitted $VAR(p)$ model, the $Q_k(m)$ statistic of the residuals is asymptotically a $\chi^2(k^2m - g)$, where g is the number of estimated parameters in the AR coefficient matrices
- Forecasting : similar to the univariate case.

VAR Models : Example 8.4.

- simple return $\rightarrow$ log return : $logibm = 100 * log(1 + ibm)$
  $logsp = 100 * log(1 + sp)$.
- lag length selection : Quick $\rightarrow$ Estimate VAR;
  in the equation window : View $\rightarrow$ Lag Structure $\rightarrow$ Lag Length Criteria
  AIC $\rightarrow$ $VAR(5)$; BIC $\rightarrow$ $VAR(0)$; HQIC $\rightarrow$ $VAR(0)$
- system equation estimation : Object $\rightarrow$ New Object $\rightarrow$ System
  logibm=c(1)+c(2)*logsp(-1)+c(3)*logsp(-2)+c(4)*logsp(-5)
  logsp=c(5)+c(6)*logsp(-1)+c(7)*logsp(-3)+c(8)*logsp(-5)

VAR Models : Example 8.4.

- Model checking:
  1. View $\rightarrow$ residual tests $\rightarrow$ Portmanteau Autocorrelation Test
  2. degrees of freedom $= k^2m - g$, where $k = 2$ and $g = 6$ in this fitted model. In particular, $Q_2(4) \xrightarrow{\mathcal{D}} \chi^2(10)$ and $Q_2(8) \xrightarrow{\mathcal{D}} \chi^2(26)$.
  3. $p - value = P\left(\chi^2(k^2m - g) > Q_k(m)\right)$.
- Forecast:
  1. Workfile: Proc $\rightarrow$ Resize (1926M01 2009M06)
  2. System : Proc $\rightarrow$ Make Model $\rightarrow$ Solve
     Solution sample : 2009M01 2009M06

Assignment 4

- Tsay pp 462 : 8.1.