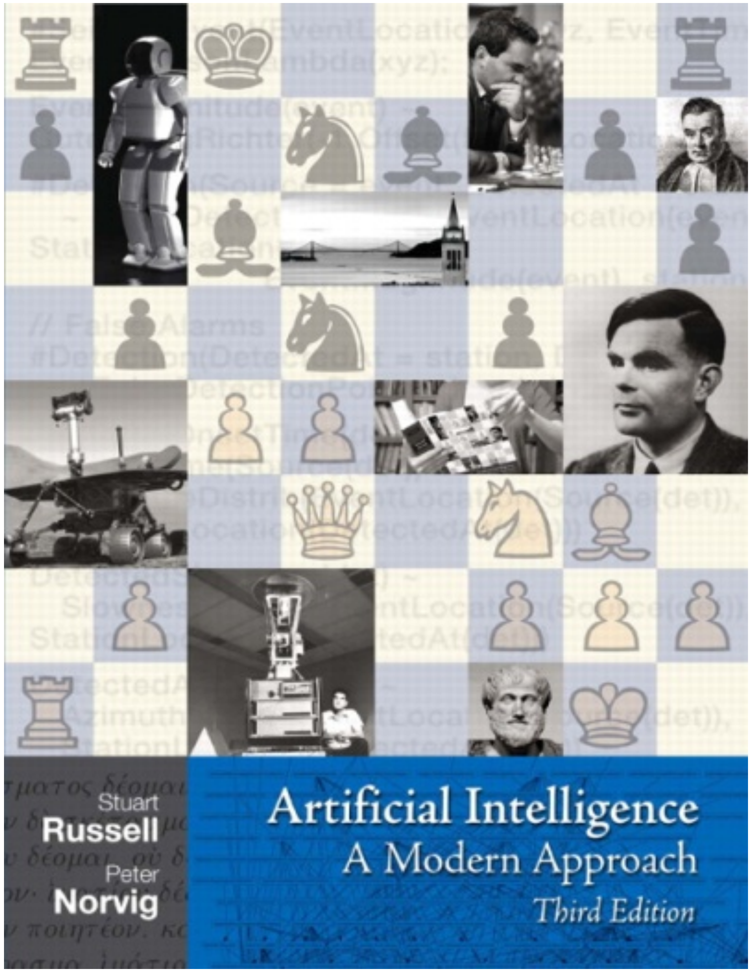
Expectimax and Utilities

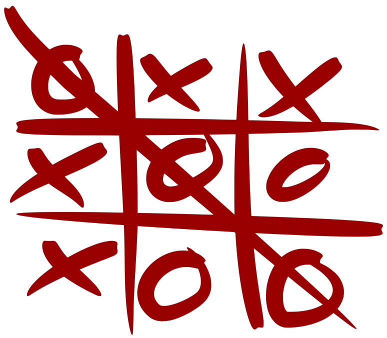
AIMA 5.1-5.5,

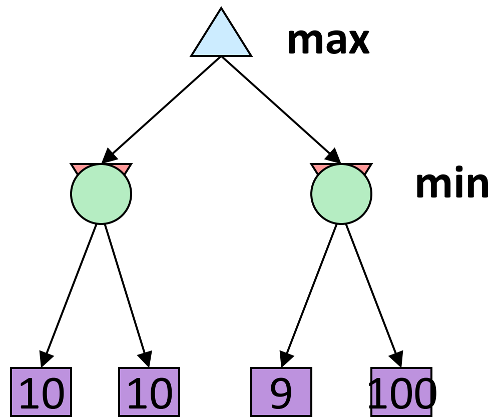
AIMA 16.1-16.3



Uncertain Outcomes

Worst-Case vs. Average Case

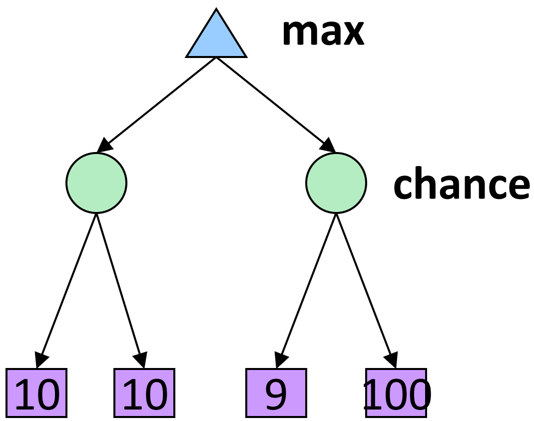




Idea: Uncertain outcomes controlled by chance, not an adversary!

Expectimax Search

- **Why wouldn't we know what the result of an action will be?**
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the opponent isn't optimal
  - Actions can fail: when moving a robot, wheels might slip
- **Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes**
- **Expectimax search: compute the average score under optimal play**
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children
- **Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes**



Expectimax Pseudocode

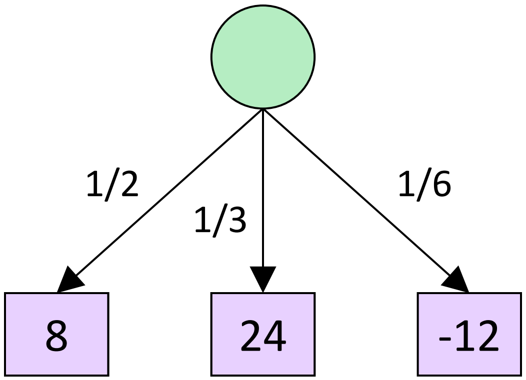
```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

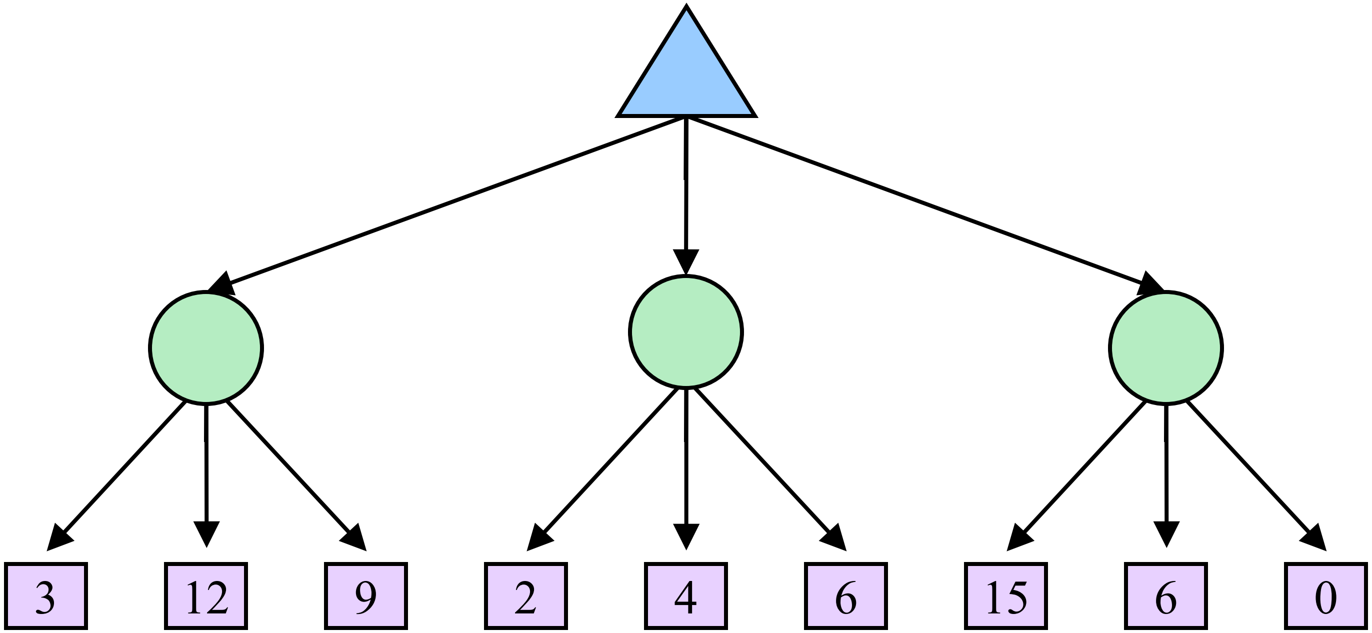
Expectimax Pseudocode

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

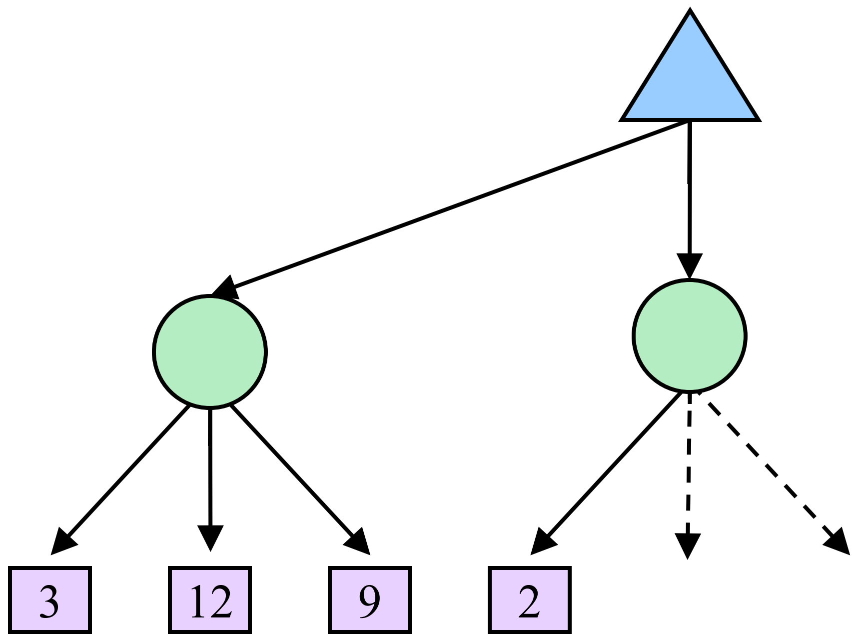


v = (1/2) (8) + (1/3) (24) + (1/6) (-12) = 10

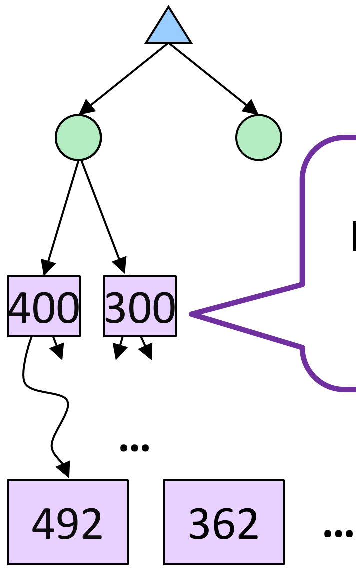
Expectimax Example



Expectimax Pruning?

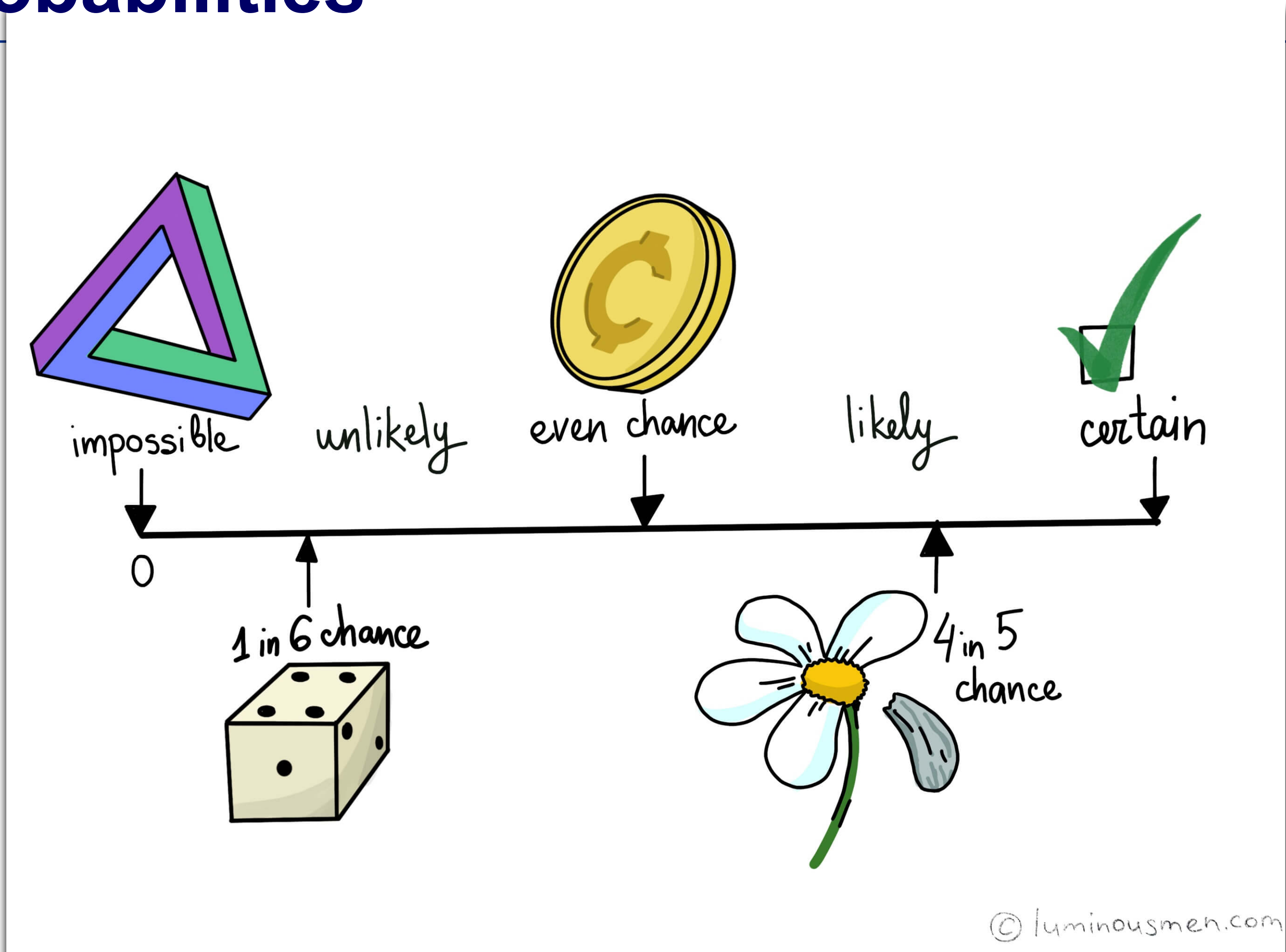


Depth-Limited Expectimax



Estimate of true expectimax value (which would require a lot of work to compute)

Probabilities



Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

- **Example: Traffic on freeway**
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

0.25

- **Some laws of probability (more later):**
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

0.50

- **As we get more evidence, probabilities may change:**
  - P(T=heavy) = 0.25, P(T=heavy | Hour=8am) = 0.60
  - We'll talk about methods for reasoning and updating probabilities later

0.25

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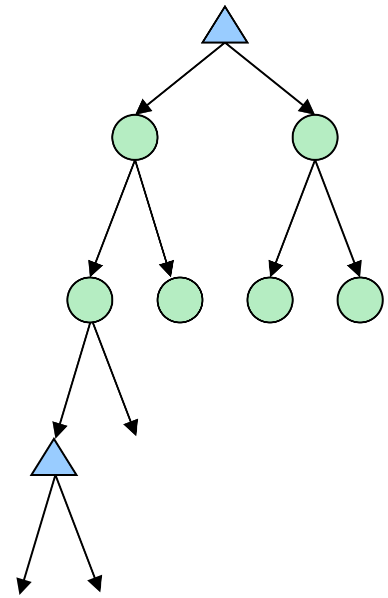
Expectations

- **The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes**
- **Example: How long to get to the airport?**

Time:	20 min		30 min		60 min		
	x	+	x	+	x	→	35 min
Probability:	0.25		0.50		0.25		

What Probabilities to Use?

- **In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state**
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!
- **For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes**



Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

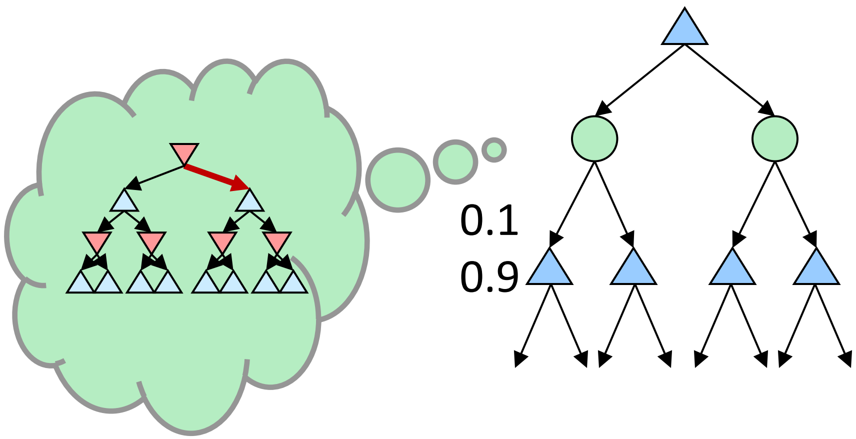
What are Probabilities?

- **Objectivist / frequentist answer:**
  - Averages over repeated *experiments*
  - E.g. empirically estimating P(rain) from historical observation
  - Assertion about how future experiments will go (in the limit)
  - New evidence changes the *reference class*
  - Makes one think of *inherently random* events, like rolling c

- **Subjectivist / Bayesian answer:**
  - Degrees of belief about unobserved variables
  - E.g. an agent's belief that it's raining, given the temperatur
  - E.g. agent's belief how an opponent will behave, given the state
  - Often *learn* probabilities from past experiences (more later
  - New evidence *updates beliefs* (more later)

Quiz: Informed Probabilities

- **Let’s say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise**
- **Question: What tree search should you use?**



- Answer: Expectimax!
  - To figure out EACH chance node’s probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

Example: Backgammon

- **Dice rolls increase *b*: 21 possible rolls with 2 dice**
  - Backgammon ≈ 20 legal moves
  - Depth 2 = 20 x (21 x 20)^3 = 1.2 x 10^9
- **As depth increases, probability of reaching a given search node shrinks**
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- **Historic AI: TDGammon uses depth-2 search + very good evaluation function + reinforcement learning: world-champion level play**
- **1st AI world champion in any game!**

Image: Wikipedia

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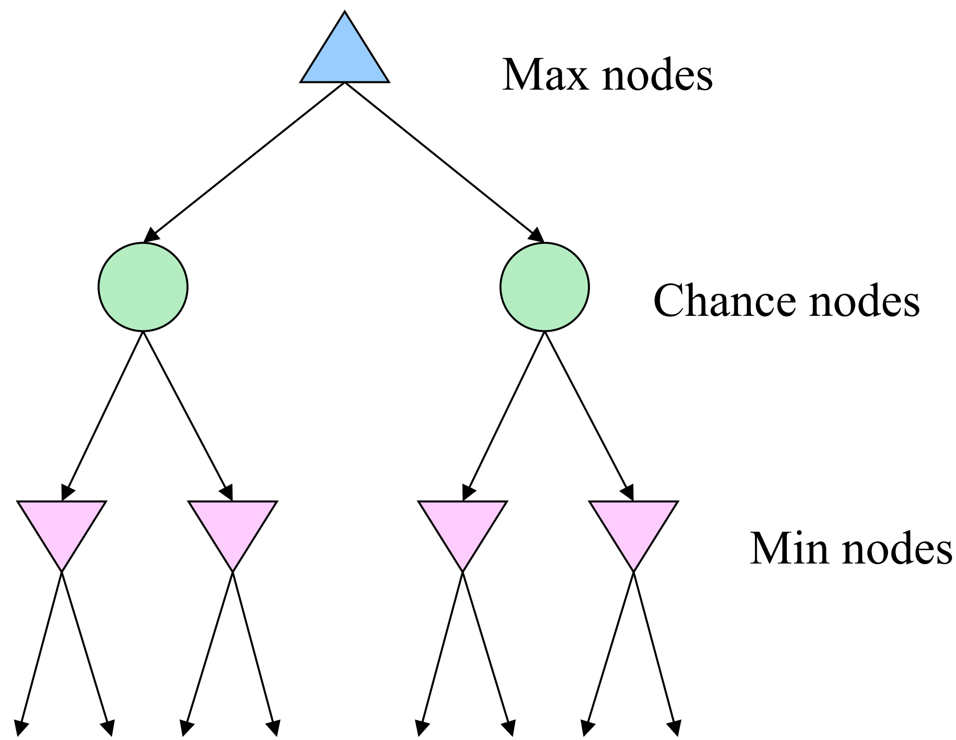
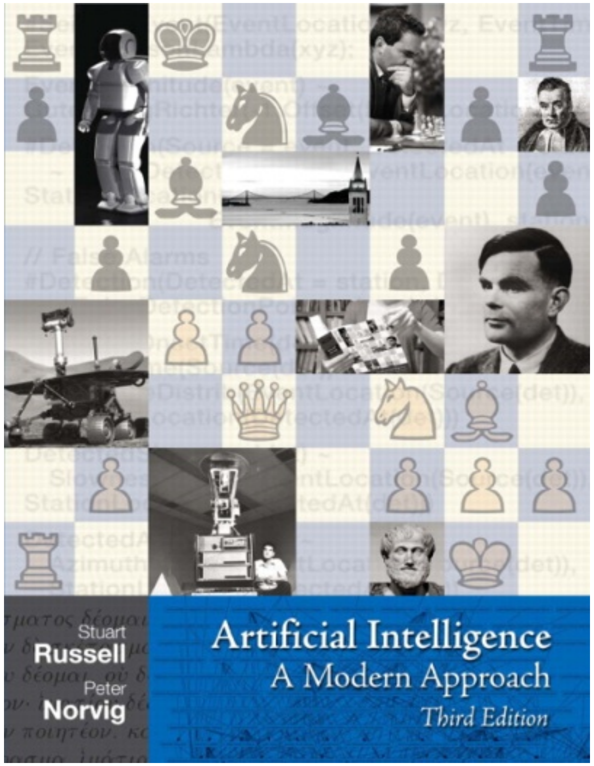
Mixed Layer Types

- **E.g. Backgammon**
- **Expectiminimax**
  - Environment is an extra "random agent" player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



Utilities

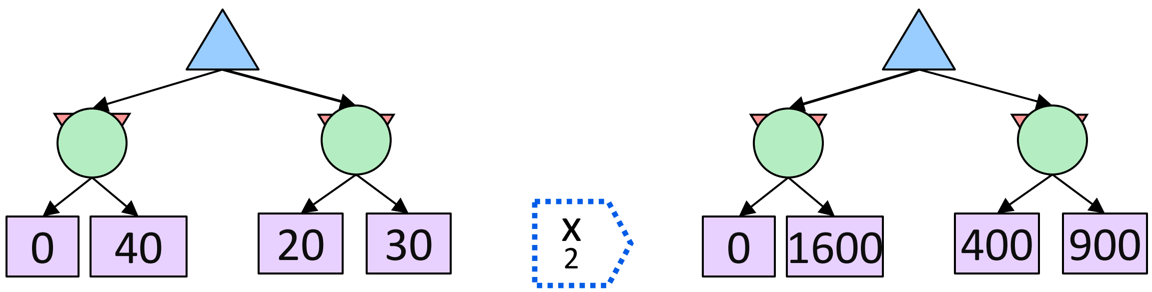
AIMA Chapter
16.1-16.3



Maximum Expected Utility

- **Why should we average utilities? Why not minimax?**
- **Principle of maximum expected utility:**
  - A rational agent should chose the action that maximizes its expected utility, given its knowledge
- **Questions:**
  - Where do utilities come from?
  - How do we know such utilities even exist?
  - How do we know that averaging even makes sense?
  - What if our behavior (preferences) can't be described by utilities?

What Utilities to Use?



- **For worst-case minimax reasoning, terminal function scale doesn't matter**
  - We just want better states to have higher evaluations (get the ordering right)
  - We call this insensitivity to monotonic transformations
- **For average-case expectimax reasoning, we need *magnitudes* to be meaningful**

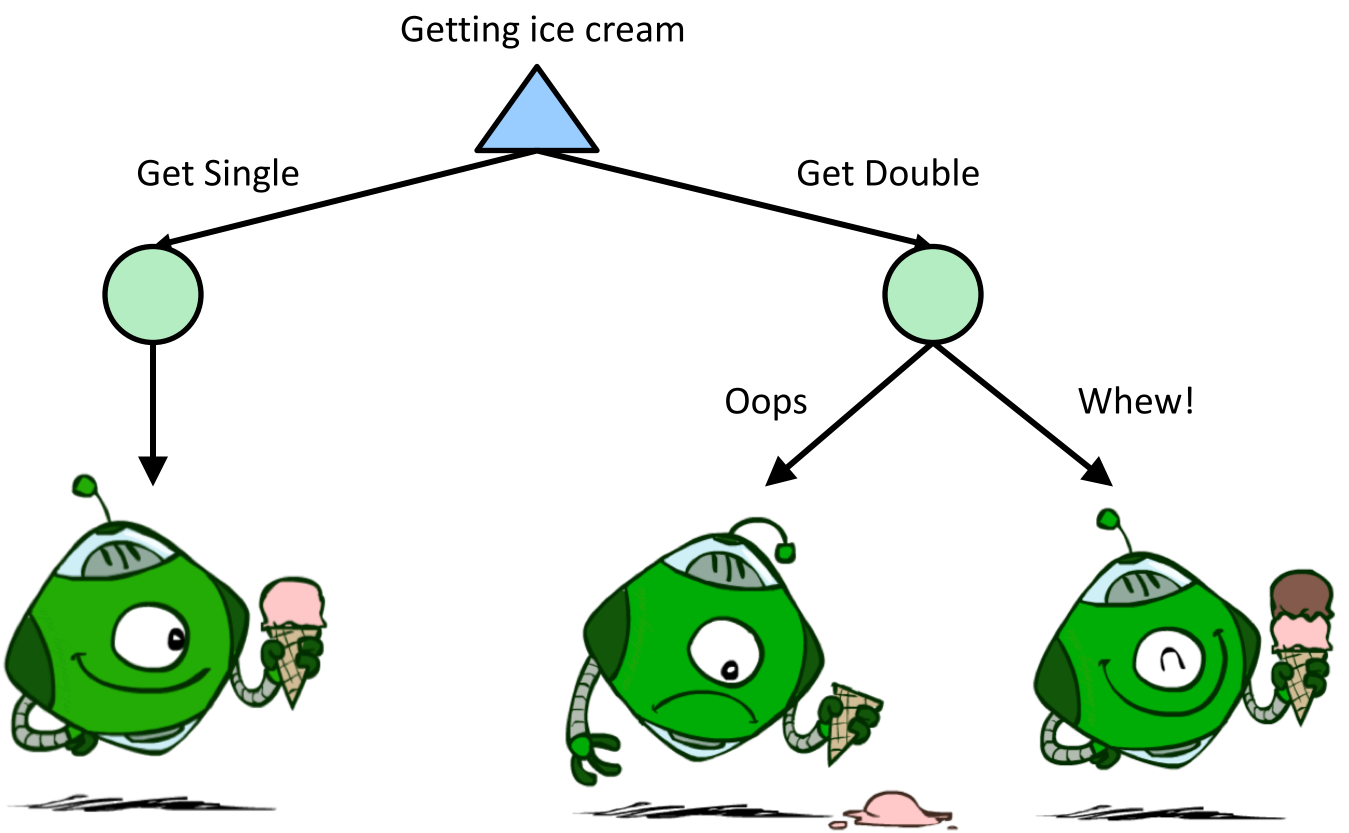
Utilities

- **Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences**

- **Where do utilities come from?**
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function

- **We hard-wire utilities and let behaviors emerge**
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?

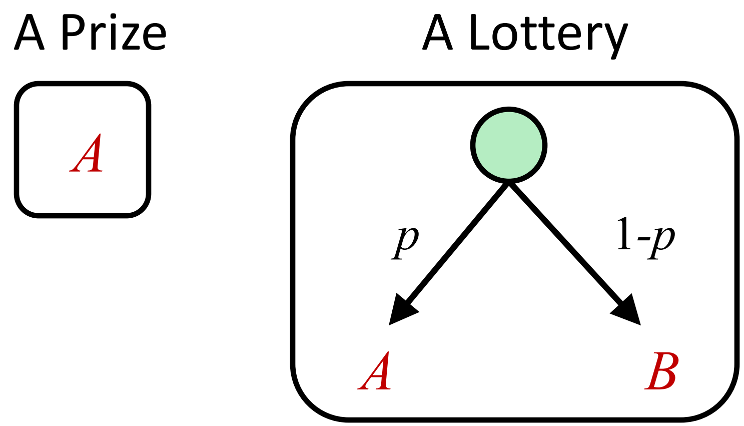
Utilities: Uncertain Outcomes



Preferences

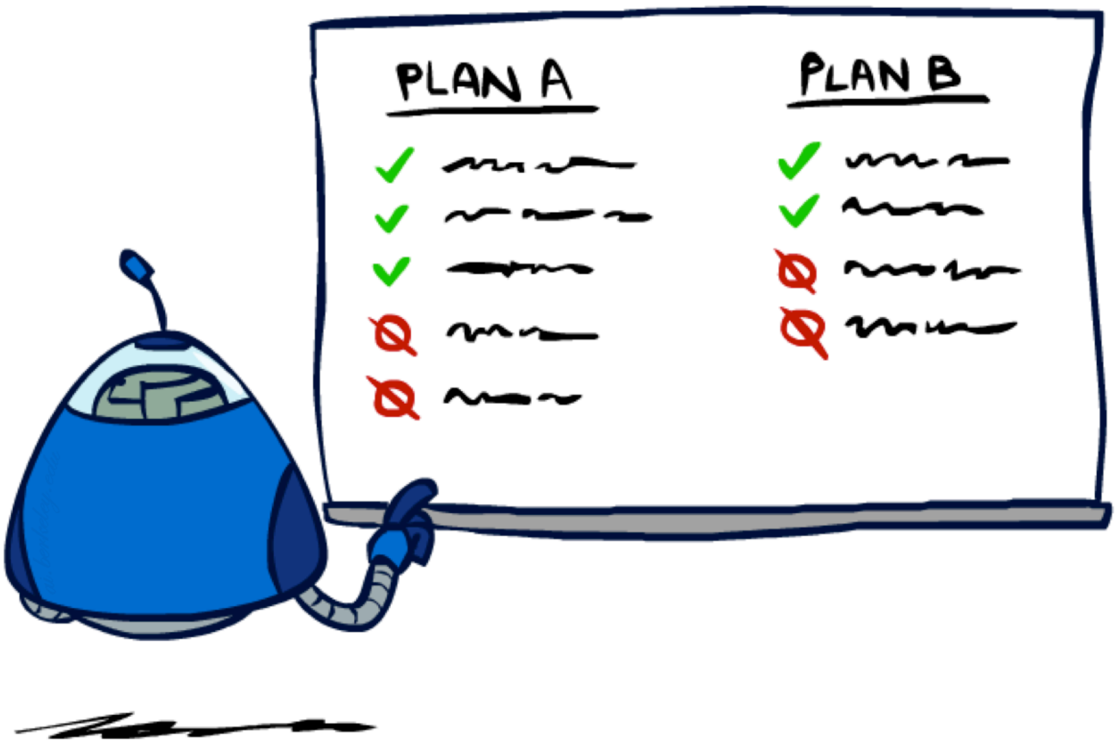
- **An agent must have preferences among:**
  - Prizes: A, B, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A; \ \ (1-p), B]$$



- **Notation:**
  - Preference: $A \succ B$
  - Indifference: $A \sim B$

Rationality

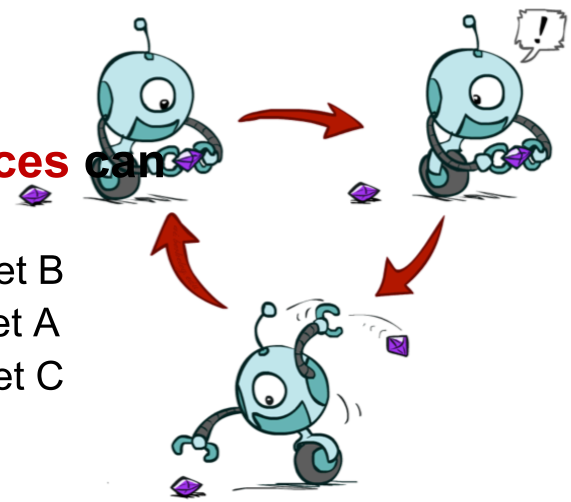


Rational Preferences

- **We want some constraints on preferences before we call them rational, such as:**

Axiom of Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

- **For example: an agent with intransitive preferences can be induced to give away all of its money**
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C



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Rational Preferences

The Axioms of Rationality

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Rightarrow$$
$$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$$

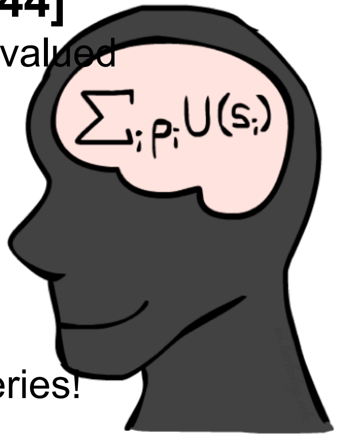
Theorem: Rational preferences imply behavior describable as maximization of expected utility

MEU Principle

- **Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]**
  - Given any preferences satisfying these constraints, there exists a real-valued function U such that:

$$U(A) \geq U(B) \;\Leftrightarrow\; A \succeq B$$

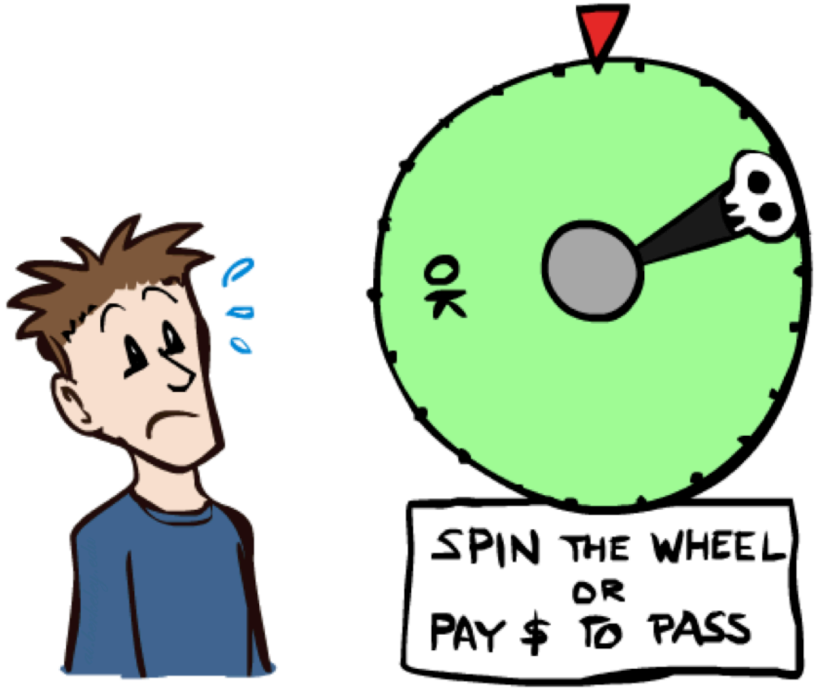
$$U([p_1, S_1;\ \ldots\ ;\ p_n, S_n]) = \textstyle\sum_i p_i U(S_i)$$



  - I.e. values assigned by U preserve preferences of both prizes and lotteries!

- **Maximum expected utility (MEU) principle:**
  - Choose the action that maximizes expected utility
  - Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities
  - E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

Human Utilities



Utility Scales

- **Normalized utilities**: $u_+ = 1.0$, $u_- = 0.0$
- **Micromorts**: one-millionth chance of death, useful for paying to reduce product risks, etc.
- **QALYs**: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

$$U'(x) = k_1 U(x) + k_2 \quad \text{where } k_1 > 0$$

- With deterministic prizes only (no lottery choices), only **ordinal utility** can be determined, i.e., total order on prizes

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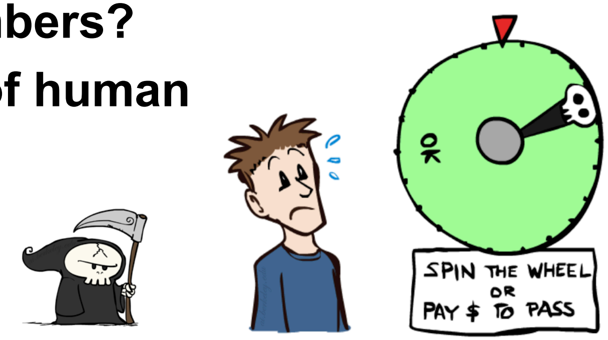
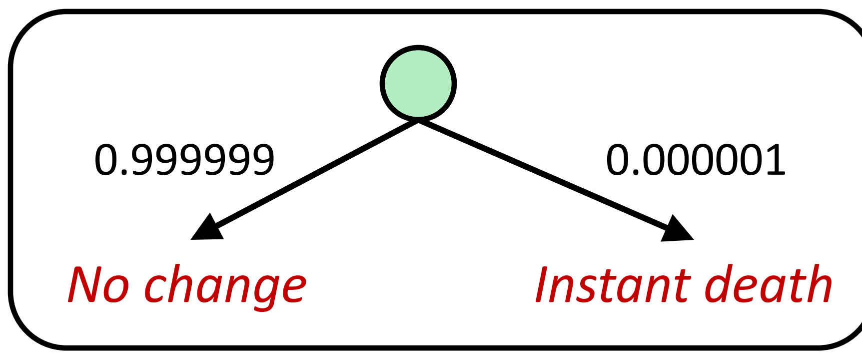
Human Utilities

- **Utilities map states to real numbers. Which numbers?**
- **Standard approach to assessment (elicitation) of human**
  - Compare a prize A to a standard lottery L_p between
    - "best possible prize" u_+ with probability p
    - "worst possible catastrophe" u_- with probability 1-p
  - Adjust lottery probability p until indifference: $A \sim L_p$
  - Resulting p is a utility in [0,1]

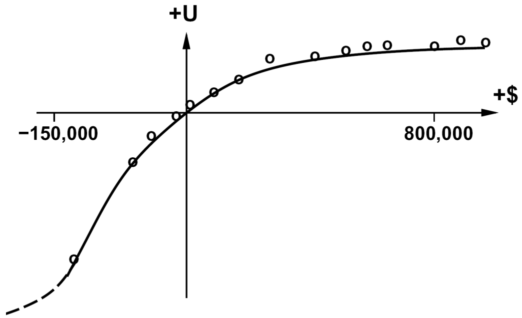


Pay $30 ~



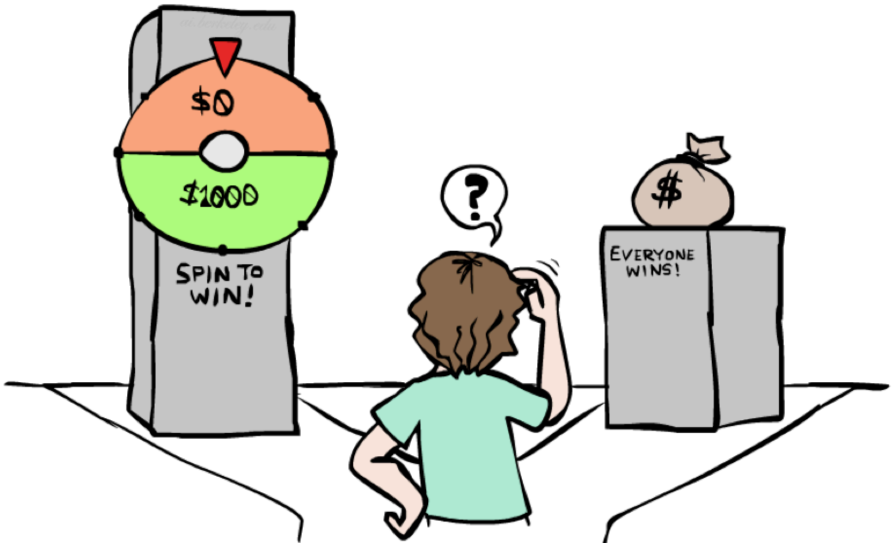
Money

- **Money <u>does not</u> behave as a utility function, but we can talk about the utility of having money (or being in debt)**
- **Given a lottery L = [p, $X; (1-p), $Y]**
  - The expected monetary value EMV(L) is p*X + (1-p)*Y
  - U(L) = p*U($X) + (1-p)*U($Y)
  - Typically, U(L) < U(EMV(L))
  - In this sense, people are risk-averse
  - When deep in debt, people are risk-prone



Example: Insurance

- **Consider the lottery [0.5, $1000; 0.5, $0]**
  - What is its expected monetary value? ($500)
  - What is its certainty equivalent?
    - — Monetary value acceptable in lieu of lottery
    - — $400 for most people
  - Difference of $100 is the insurance premium
    - — There's an insurance industry because people will pay to reduce their risk
    - — If everyone were risk-neutral, no insurance needed!
  - It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is linear and they have many lotteries)



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Example: Human Rationality?

- **Famous example of Allais (1953)**
  - A: [0.8, $4k; 0.2, $0]
  - B: [1.0, $3k; 0.0, $0]
  - C: [0.2, $4k; 0.8, $0]
  - D: [0.25, $3k; 0.75, $0]

- **Most people prefer B > A, C > D**

- **But if U($0) = 0, then**
  - B > A $\Rightarrow$ U($3k) > 0.8 U($4k)
  - C > D $\Rightarrow$ 0.8 U($4k) > U($3k)