

Reminders

- 14 days until the American election. I voted. Did you? If you haven't returned your PA mail-in ballot yet, post it today or drop it off here:
<https://www.votespa.com/Voting-in-PA/pages/drop-box.aspx>
- Midterm details:
 - * Saturday Oct 24: Practice midterm is due.
 - * Midterm available Monday Oct 26 and Tuesday Oct 27.
 - * 3 hour block. Open book, open notes, no collaboration.
- Partners on HW will be likely details after midterm.



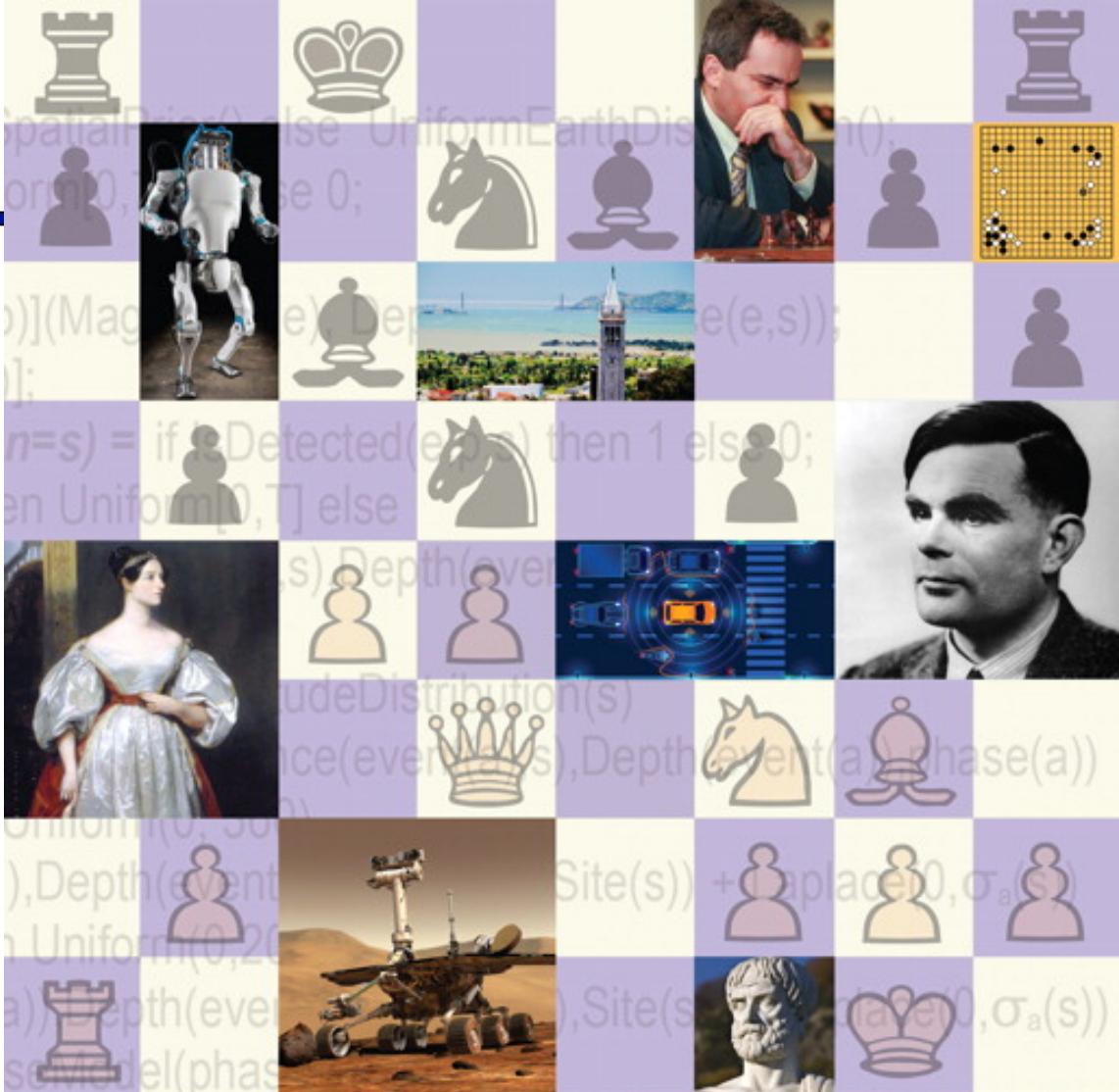
Reading

Chapter 22 – Reinforcement Learning

Sections 22.1-22.5

Chapter 17.3 – Bandit Problems

(These topics won't be on Tuesday's midterm)



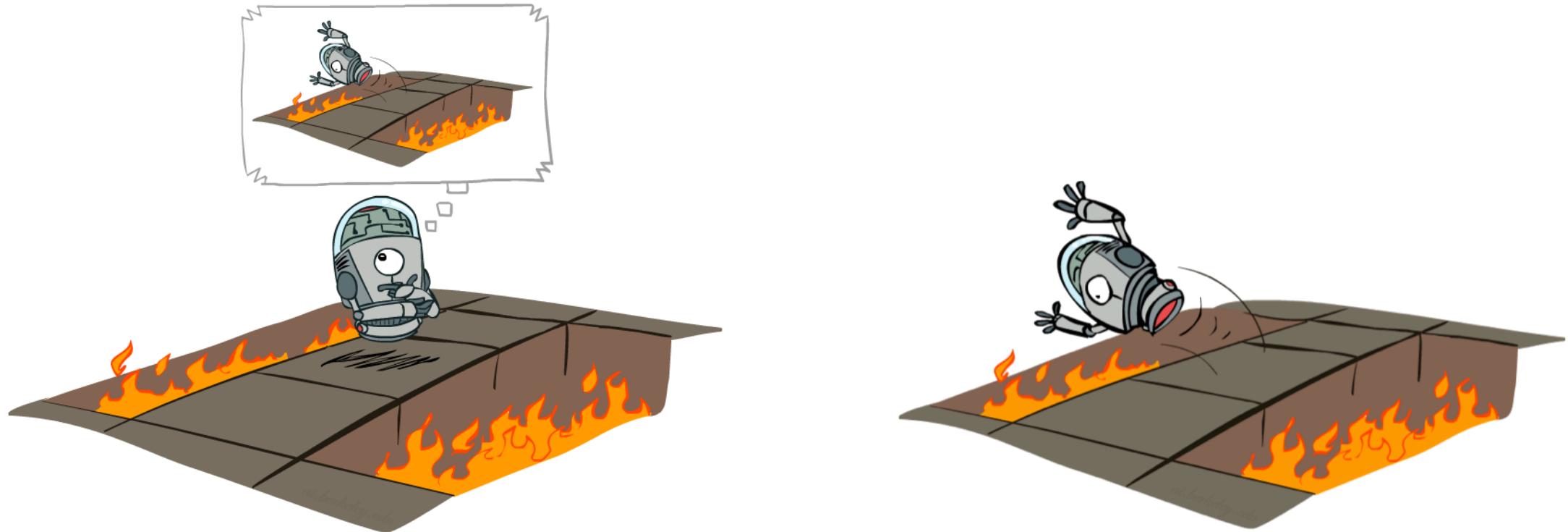
Stuart
Russell
Peter
Norvig



Artificial Intelligence

A Modern Approach

Reinforcement Learning

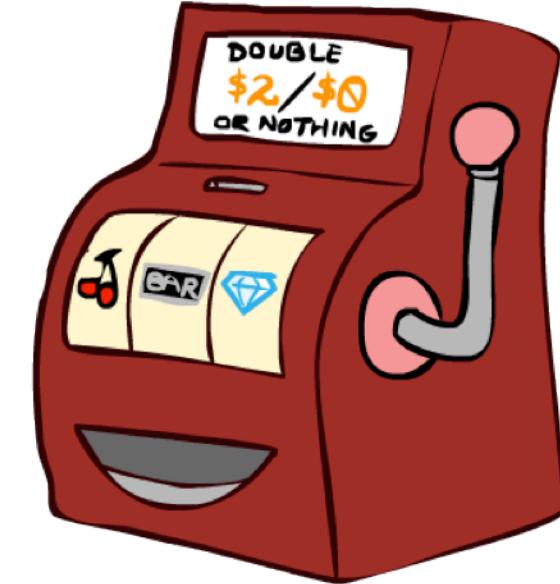
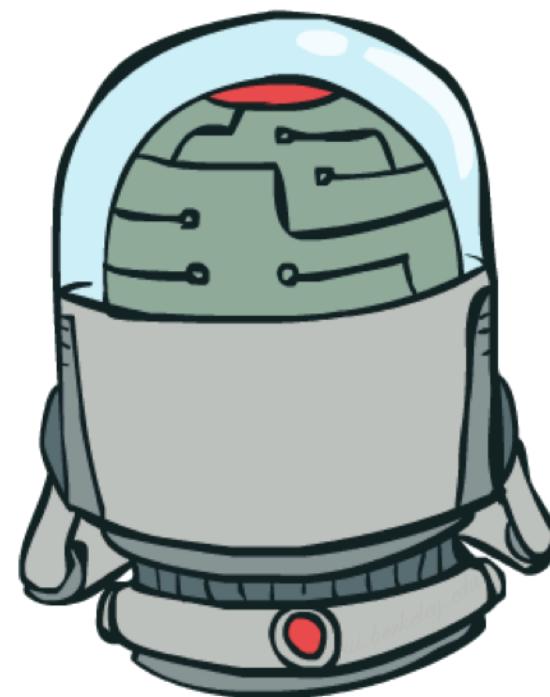


Slides courtesy of Dan Klein and Pieter Abbeel

University of California, Berkeley

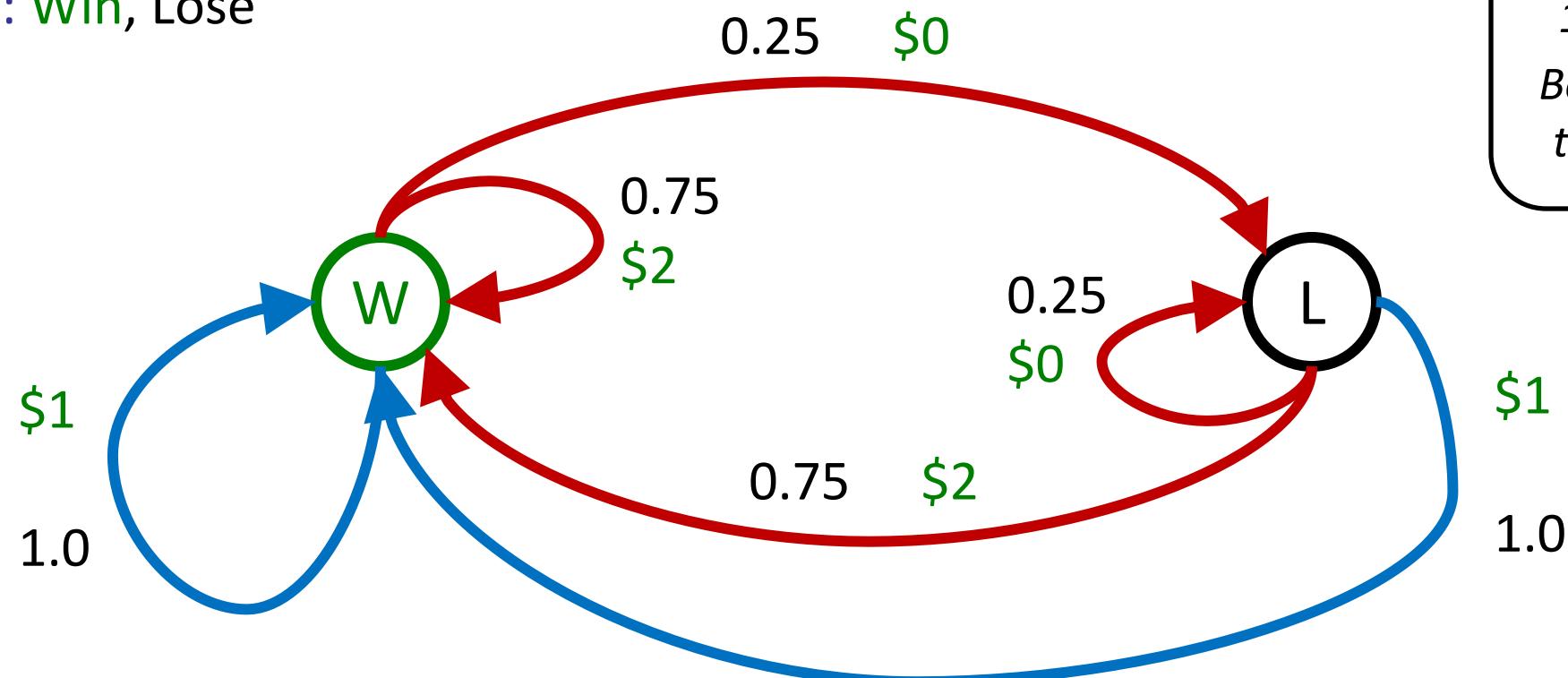
[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

Double Bandits



Double-Bandit MDP

- Actions: *Blue, Red*
- States: *Win, Lose*



*No discount
100 time steps
Both states have
the same value*

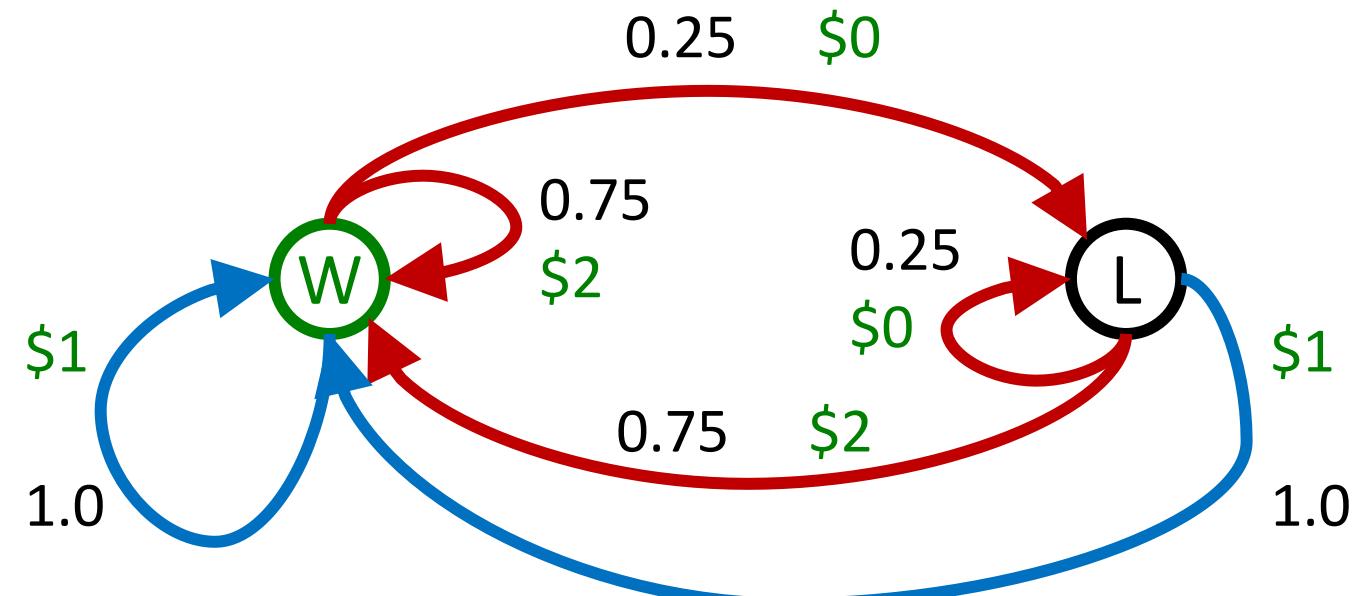
Offline Planning

- Solving MDPs is offline planning

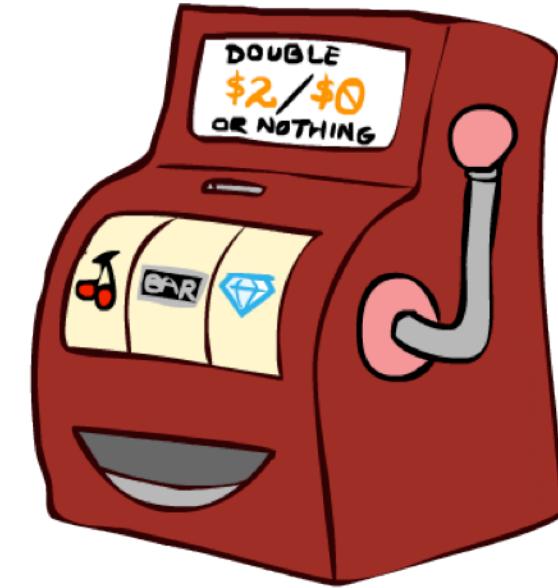
- You determine all quantities through computation
- You need to know the details of the MDP
- You do not actually play the game!

*No discount
100 time steps
Both states have
the same value*

	Value
Play Red	150
Play Blue	100



Let's Play!

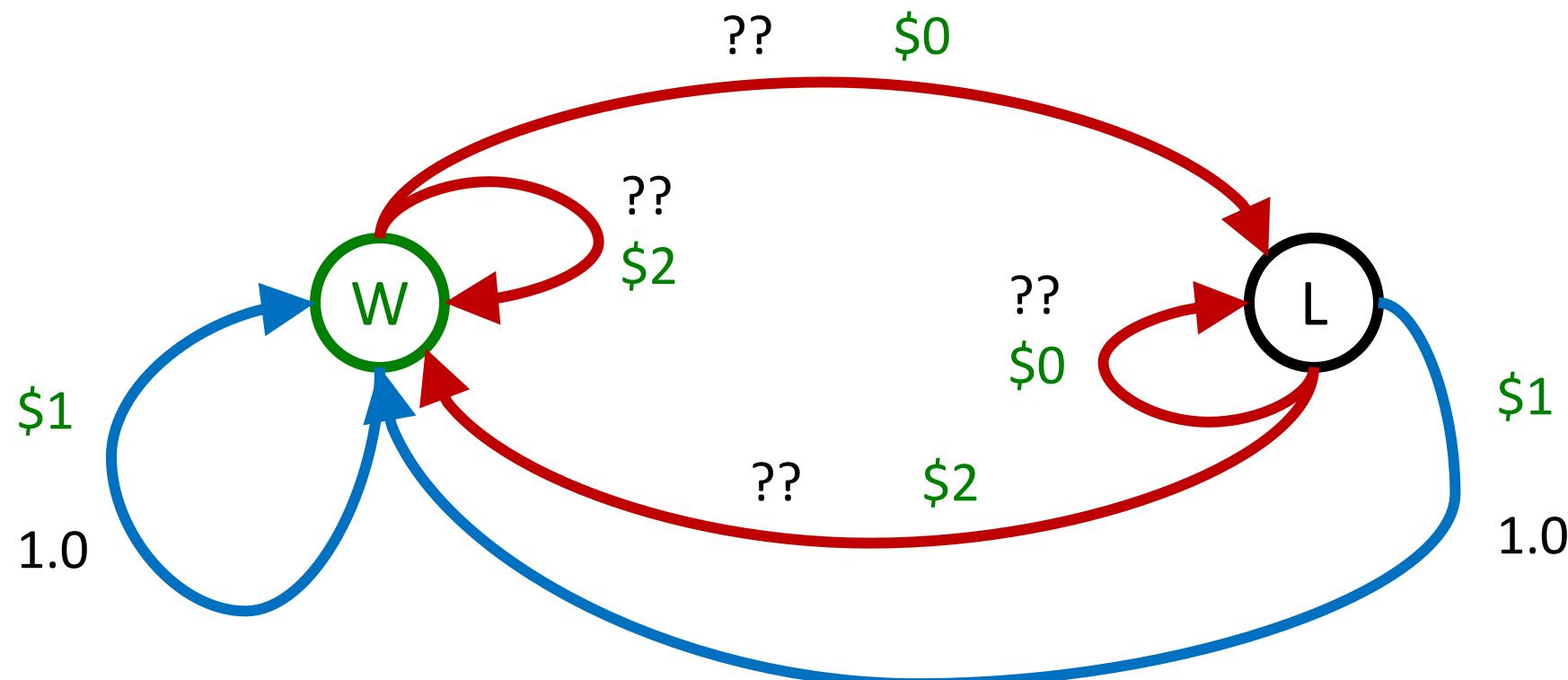


\$2 \$2 \$0 \$2 \$2

\$2 \$2 \$0 \$0 \$0

Online Planning

- Rules changed! Red's win chance is different.



Let's Play!



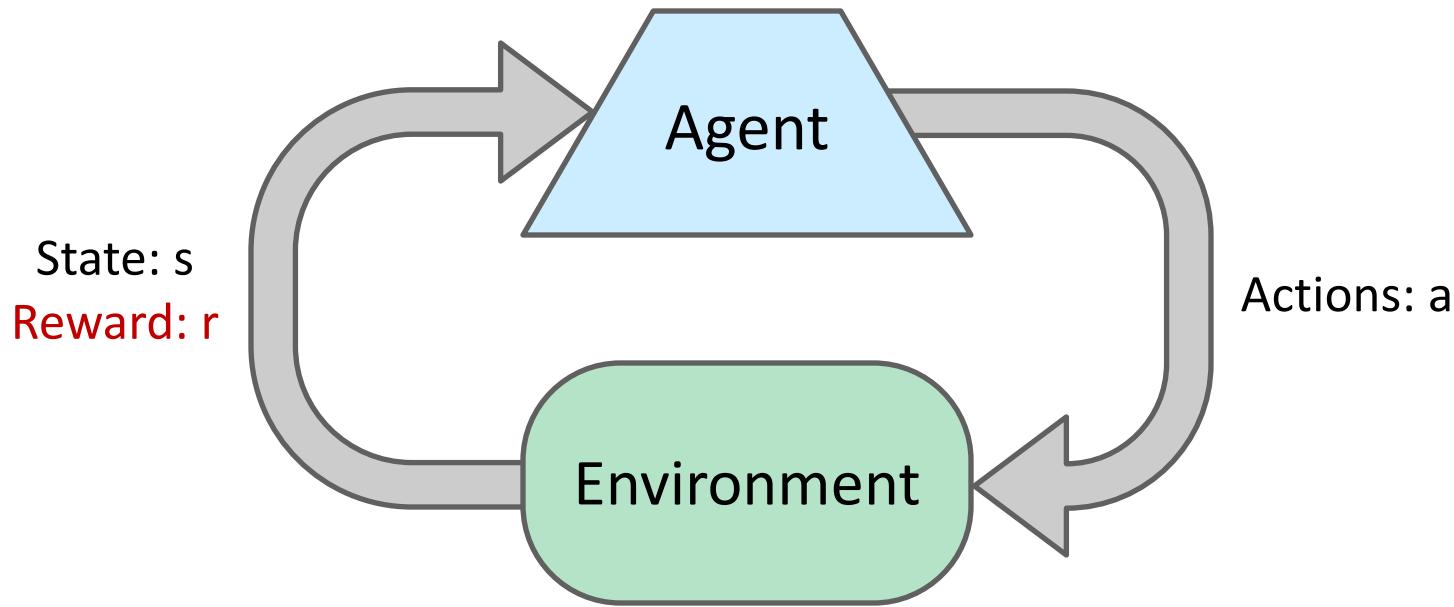
\$0	\$0	\$0	\$2	\$0
\$2	\$0	\$0	\$0	\$0

What Just Happened?

- That wasn't planning, it was learning!
 - Specifically, reinforcement learning
 - There was an MDP, but you couldn't solve it with just computation
 - You needed to actually act to figure it out
- Important ideas in reinforcement learning that came up
 - **Exploration:** you have to try unknown actions to get information
 - **Exploitation:** eventually, you have to use what you know
 - **Regret:** even if you learn intelligently, you make mistakes
 - **Sampling:** because of chance, you have to try things repeatedly
 - **Difficulty:** learning can be much harder than solving a known MDP



Reinforcement Learning



- Basic idea:
 - Receive feedback in the form of **rewards**
 - Agent's utility is defined by the reward function
 - Must (learn to) act so as to **maximize expected rewards**
 - All learning is based on observed samples of outcomes!

Reinforcement Learning

- Still assume a Markov decision process (MDP):

- A set of states $s \in S$
- A set of actions (per state) A
- A model $T(s,a,s')$
- A reward function $R(s,a,s')$

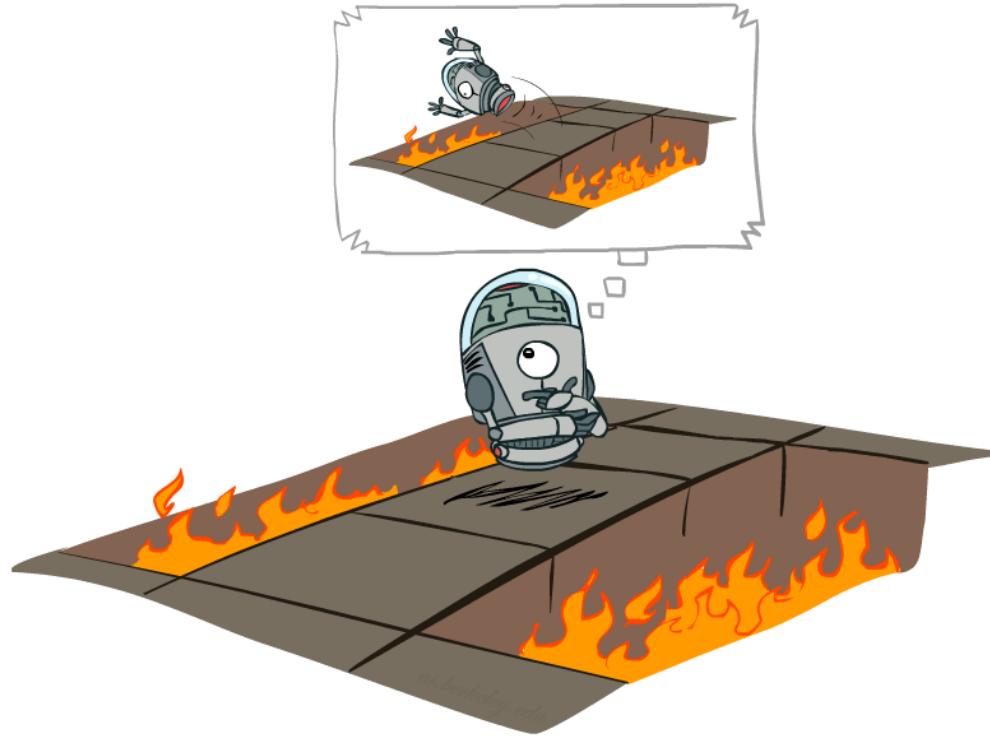
- Still looking for a policy $\pi(s)$

- New twist: don't know T or R

- I.e. we don't know which states are good or what the actions do
- Must actually try actions and states out to learn



Offline (MDPs) vs. Online (RL)

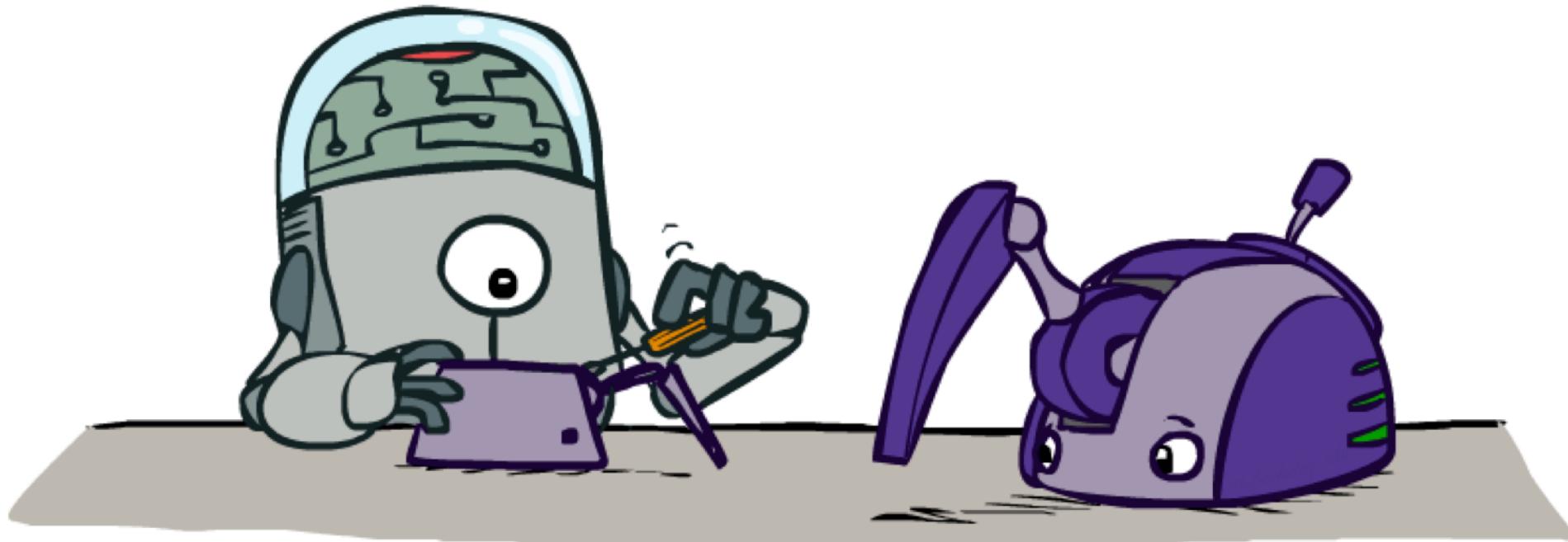


Offline Solution



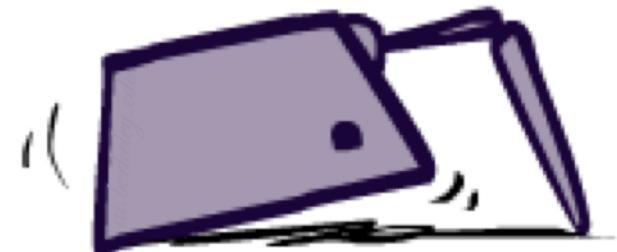
Online Learning

Model-Based Learning



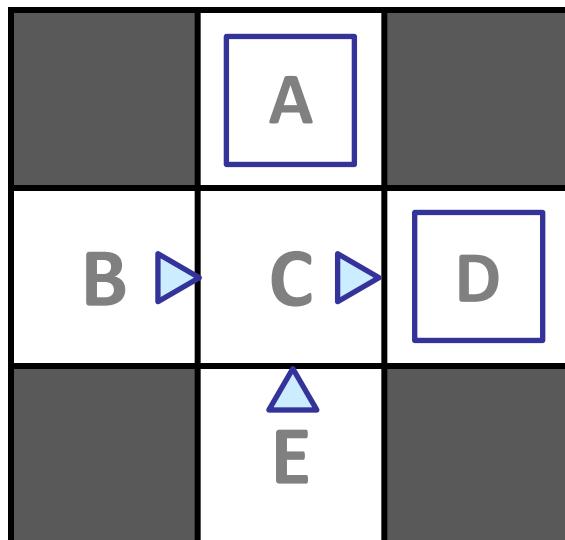
Model-Based Learning

- Model-Based Idea:
 - Learn an approximate model based on experiences
 - Solve for values as if the learned model were correct
- Step 1: Learn empirical MDP model
 - Count outcomes s' for each s, a
 - Normalize to give an estimate of $\hat{T}(s, a, s')$
 - Discover each $\hat{R}(s, a, s')$ when we experience (s, a, s')
- Step 2: Solve the learned MDP
 - For example, use value iteration, as before



Example: Model-Based Learning

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

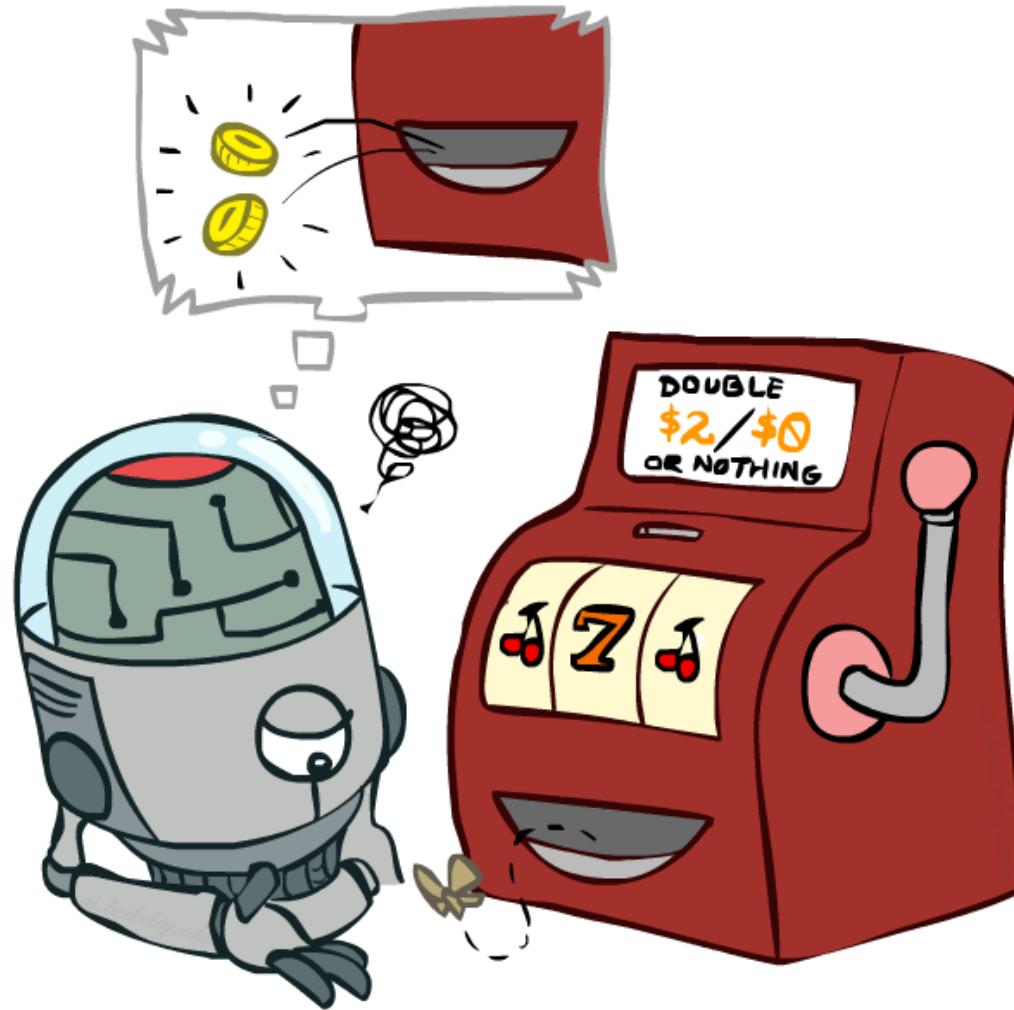
$$\hat{T}(s, a, s')$$

$T(B, \text{east}, C) = 1.00$
 $T(C, \text{east}, D) = 0.75$
 $T(C, \text{east}, A) = 0.25$
...

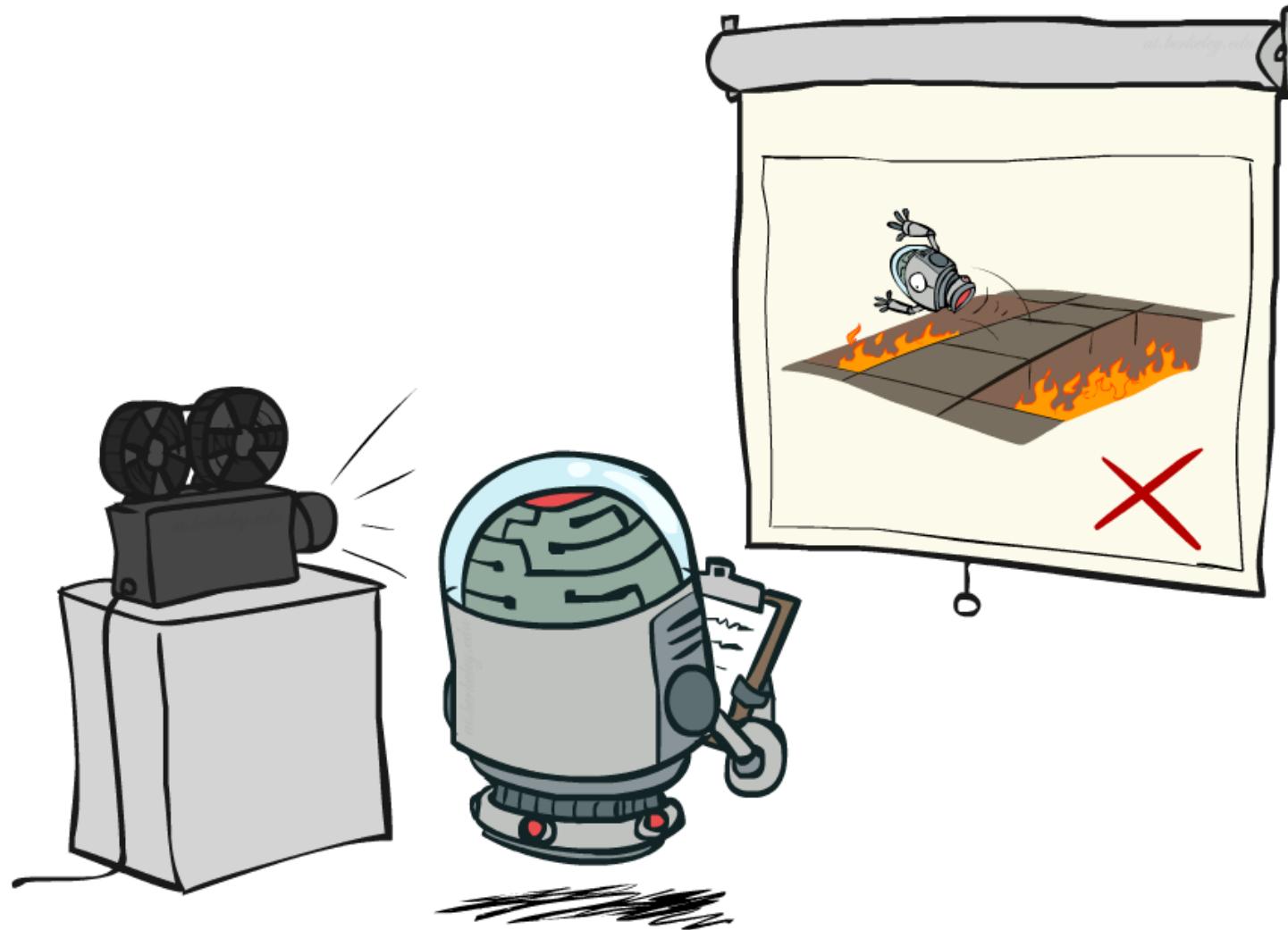
$$\hat{R}(s, a, s')$$

$R(B, \text{east}, C) = -1$
 $R(C, \text{east}, D) = -1$
 $R(D, \text{exit}, x) = +10$
...

Model-Free Learning



Passive Reinforcement Learning



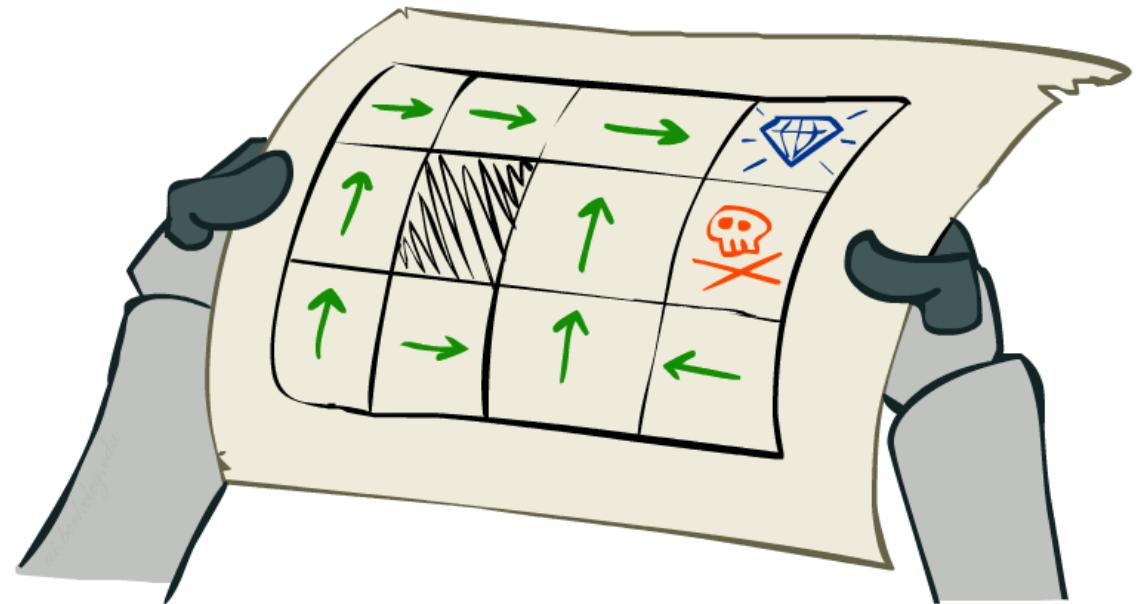
Passive Reinforcement Learning

- Simplified task: policy evaluation

- Input: a fixed policy $\pi(s)$
- You don't know the transitions $T(s,a,s')$
- You don't know the rewards $R(s,a,s')$
- Goal: learn the state values

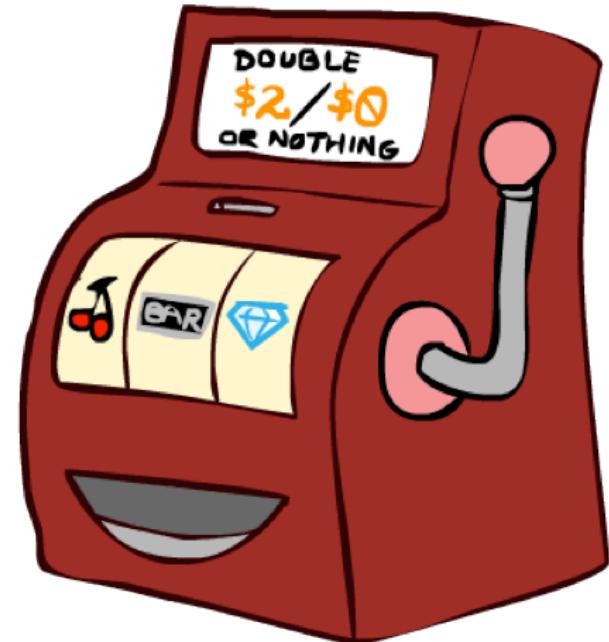
- In this case:

- Learner is “along for the ride”
- No choice about what actions to take
- Just execute the policy and learn from experience
- This is NOT offline planning! You actually take actions in the world.



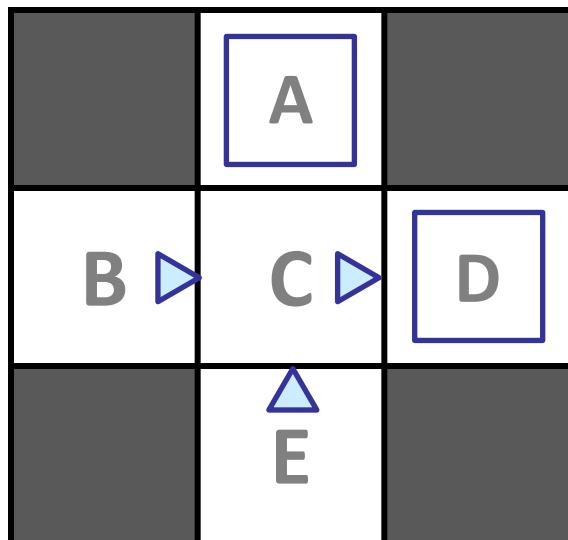
Direct Evaluation

- Goal: Compute values for each state under π
- Idea: Average together observed sample values
 - Act according to π
 - Every time you visit a state, write down what the sum of discounted rewards turned out to be
 - Average those samples
- This is called direct evaluation



Example: Direct Evaluation

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Output Values

	-10	
A	+4	+10
B	+8	
C		D
E	-2	

Problems with Direct Evaluation

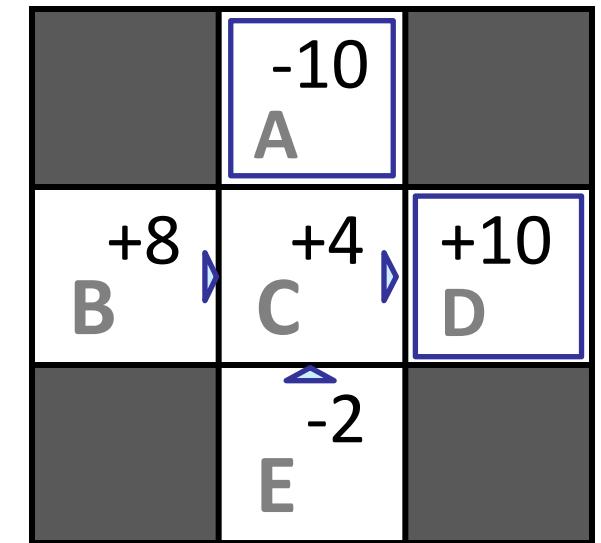
- What's good about direct evaluation?

- It's easy to understand
- It doesn't require any knowledge of T, R
- It eventually computes the correct average values, using just sample transitions

- What bad about it?

- It wastes information about state connections
- Each state must be learned separately
- So, it takes a long time to learn

Output Values



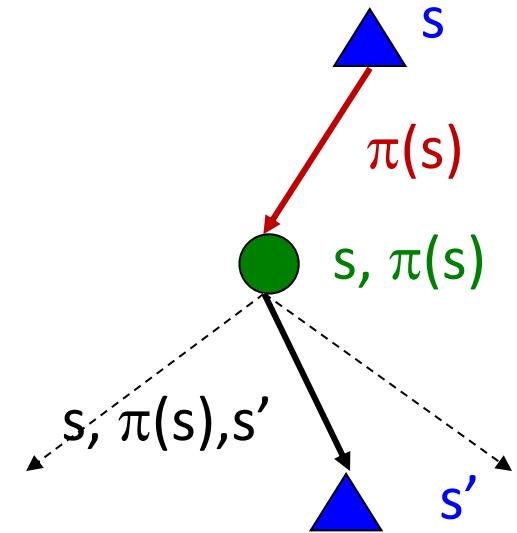
If B and E both go to C under this policy, how can their values be different?

Why Not Use Policy Evaluation?

- Simplified Bellman updates calculate V for a fixed policy:
 - Each round, replace V with a one-step-look-ahead layer over V

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^\pi(s')]$$



- This approach fully exploited the connections between the states
- Unfortunately, we need T and R to do it!
- Key question: how can we do this update to V without knowing T and R ?
 - In other words, how to we take a weighted average without knowing the weights?

Sample-Based Policy Evaluation?

- We want to improve our estimate of V by computing these averages:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} T(s, \pi(s), s')[R(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

- Idea: Take samples of outcomes s' (by doing the action!) and average

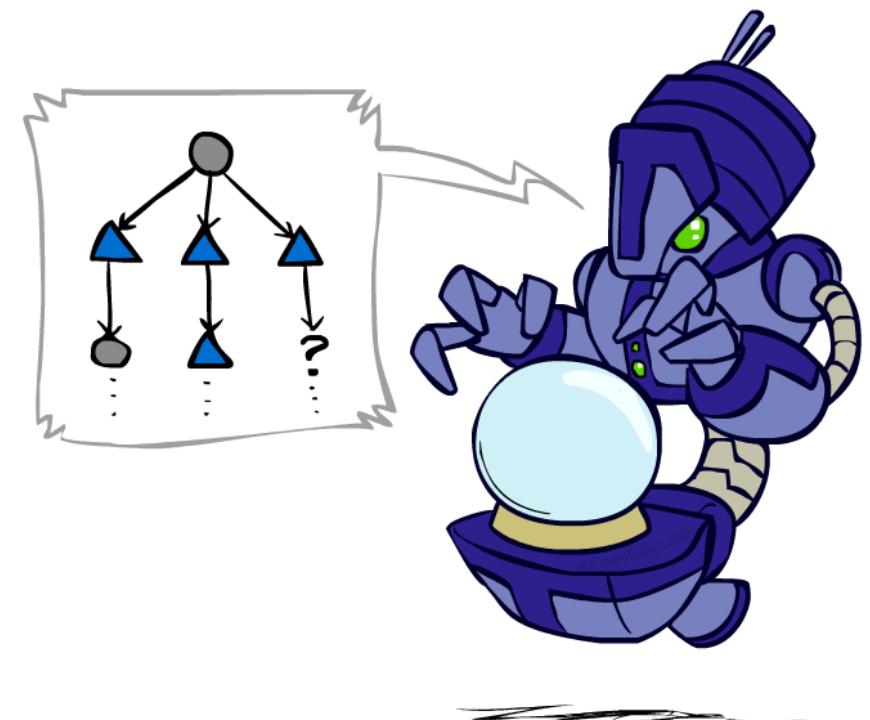
$$sample_1 = R(s, \pi(s), s'_1) + \gamma V_k^{\pi}(s'_1)$$

$$sample_2 = R(s, \pi(s), s'_2) + \gamma V_k^{\pi}(s'_2)$$

...

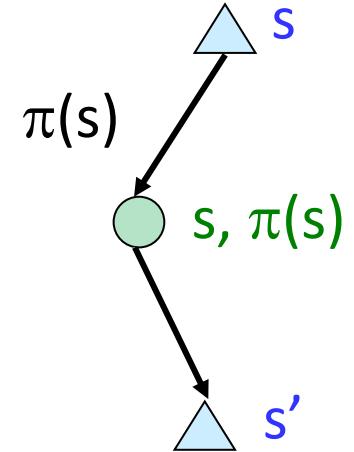
$$sample_n = R(s, \pi(s), s'_n) + \gamma V_k^{\pi}(s'_n)$$

$$V_{k+1}^{\pi}(s) \leftarrow \frac{1}{n} \sum_i sample_i$$



Temporal Difference Learning

- Big idea: learn from every experience!
 - Update $V(s)$ each time we experience a transition (s, a, s', r)
 - Likely outcomes s' will contribute updates more often
- Temporal difference learning of values
 - Policy still fixed, still doing evaluation!
 - Move values toward value of whatever successor occurs: running average



Sample of $V(s)$: $sample = R(s, \pi(s), s') + \gamma V^\pi(s')$

Update to $V(s)$: $V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + (\alpha)sample$

Same update: $V^\pi(s) \leftarrow V^\pi(s) + \alpha(sample - V^\pi(s))$

Exponential Moving Average

- Exponential moving average

- The running interpolation update: $\bar{x}_n = (1 - \alpha) \cdot \bar{x}_{n-1} + \alpha \cdot x_n$
- Makes recent samples more important:

$$\bar{x}_n = \frac{x_n + (1 - \alpha) \cdot x_{n-1} + (1 - \alpha)^2 \cdot x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + \dots}$$

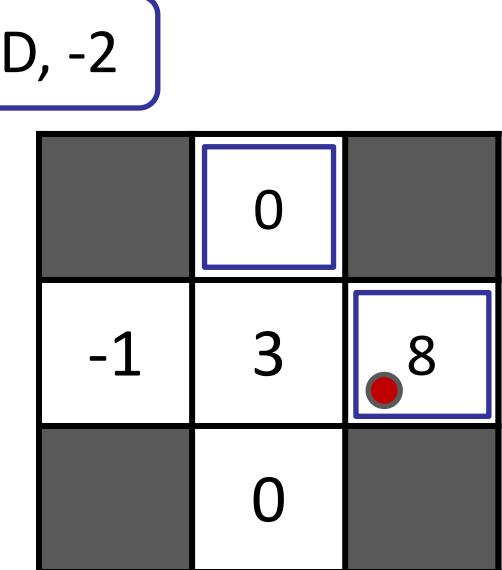
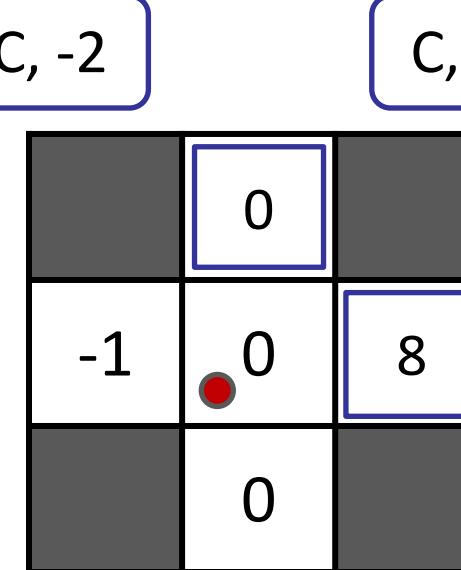
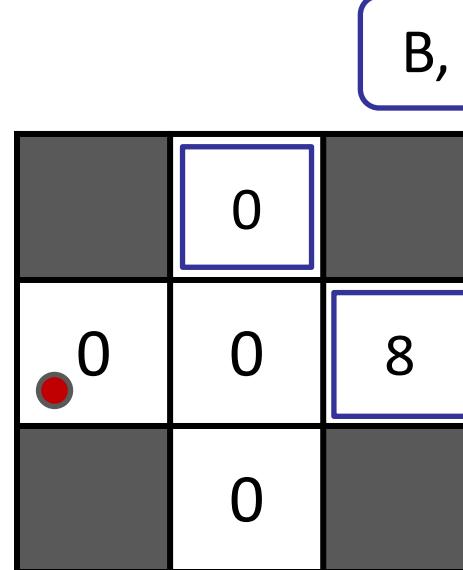
- Forgets about the past (distant past values were wrong anyway)
- Decreasing learning rate (alpha) can give converging averages

Example: Temporal Difference Learning

States

	A	
B	C	D
	E	

Observed Transitions



Assume: $\gamma = 1$, $\alpha = 1/2$

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

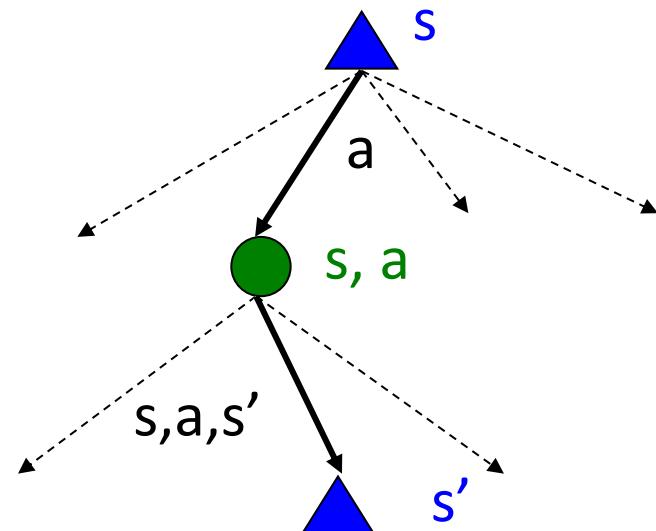
Problems with TD Value Learning

- TD value learning is a model-free way to do policy evaluation, mimicking Bellman updates with running sample averages
- However, if we want to turn values into a (new) policy, we're sunk:

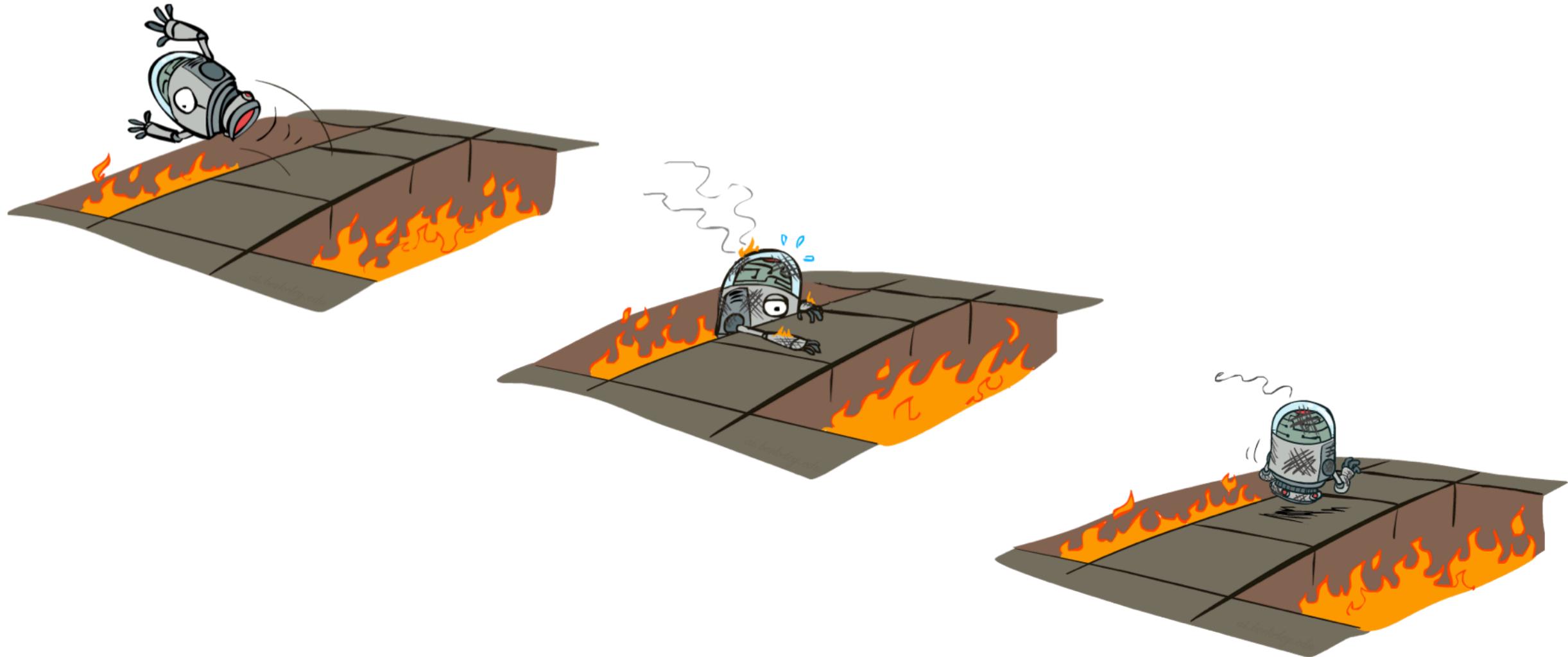
$$\pi(s) = \arg \max_a Q(s, a)$$

$$Q(s, a) = \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V(s')]$$

- Idea: learn Q-values, not values
- Makes action selection model-free too!

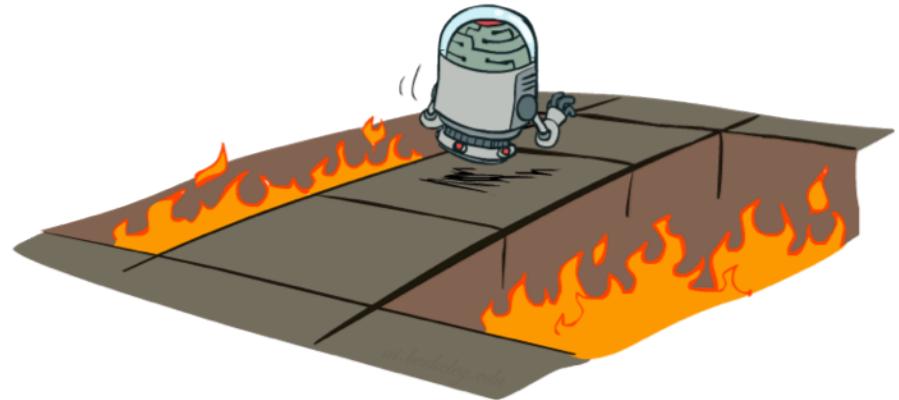


Active Reinforcement Learning



Active Reinforcement Learning

- Full reinforcement learning: optimal policies (like value iteration)
 - You don't know the transitions $T(s,a,s')$
 - You don't know the rewards $R(s,a,s')$
 - You choose the actions now
 - Goal: learn the optimal policy / values
- In this case:
 - Learner makes choices!
 - Fundamental tradeoff: exploration vs. exploitation
 - This is NOT offline planning! You actually take actions in the world and find out what happens...



Detour: Q-Value Iteration

- Value iteration: find successive (depth-limited) values

- Start with $V_0(s) = 0$, which we know is right
- Given V_k , calculate the depth $k+1$ values for all states:

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma V_k(s')]$$

- But Q-values are more useful, so compute them instead

- Start with $Q_0(s, a) = 0$, which we know is right
- Given Q_k , calculate the depth $k+1$ q-values for all q-states:

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') [R(s, a, s') + \gamma \max_{a'} Q_k(s', a')]$$

Q-Learning

- Q-Learning: sample-based Q-value iteration

$$Q_{k+1}(s, a) \leftarrow \sum_{s'} T(s, a, s') \left[R(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

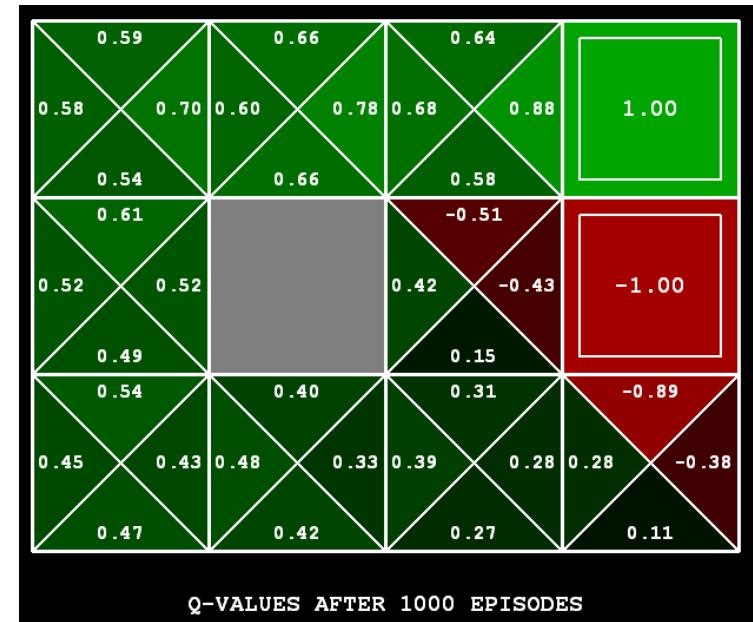
- Learn $Q(s,a)$ values as you go

- Receive a sample (s, a, s', r)
 - Consider your old estimate: $Q(s, a)$
 - Consider your new sample estimate:

$$sample = R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

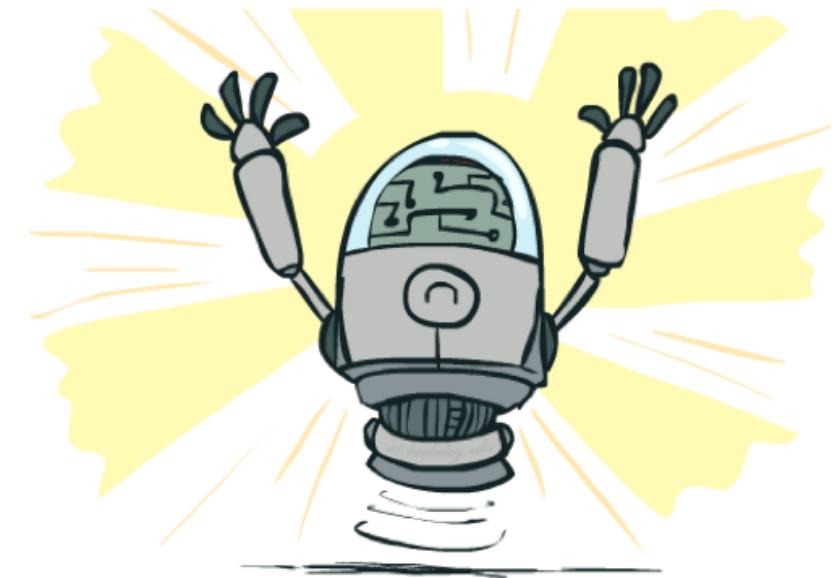
- Incorporate the new estimate into a running average:

$$Q(s, a) \leftarrow (1 - \alpha)Q(s, a) + (\alpha) [sample]$$

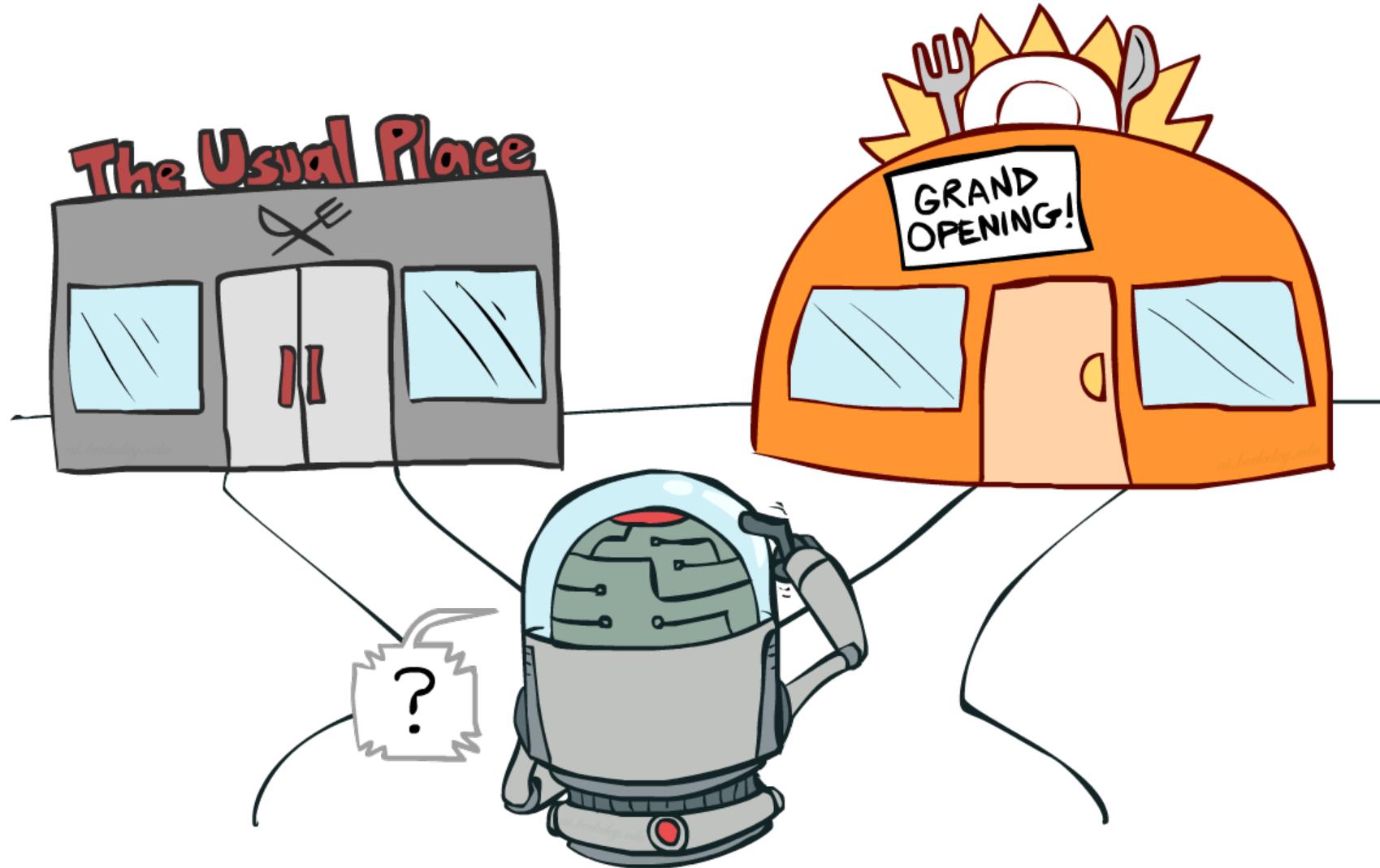


Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called **off-policy learning**
- Caveats:
 - You have to explore enough
 - You have to eventually make the learning rate small enough
 - ... but not decrease it too quickly
 - Basically, in the limit, it doesn't matter how you select actions (!)



Exploration vs. Exploitation



How to Explore?

- Several schemes for forcing exploration
 - Simplest: random actions (ε -greedy)
 - Every time step, flip a coin
 - With (small) probability ε , act randomly
 - With (large) probability $1-\varepsilon$, act on current policy



How to Explore?

- Several schemes for forcing exploration
 - Simplest: random actions (ε -greedy)
 - Every time step, flip a coin
 - With (small) probability ε , act randomly
 - With (large) probability $1-\varepsilon$, act on current policy
 - Problems with random actions?
 - You do eventually explore the space, but keep thrashing around once learning is done
 - One solution: lower ε over time
 - Another solution: exploration functions



Exploration Functions

- When to explore?

- Random actions: explore a fixed amount
- Better idea: explore areas whose badness is not (yet) established, eventually stop exploring

- Exploration function

- Takes a value estimate u and a visit count n , and returns an optimistic utility, e.g. $f(u, n) = u + k/n$

Regular Q-Update:
$$Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} Q(s', a')$$

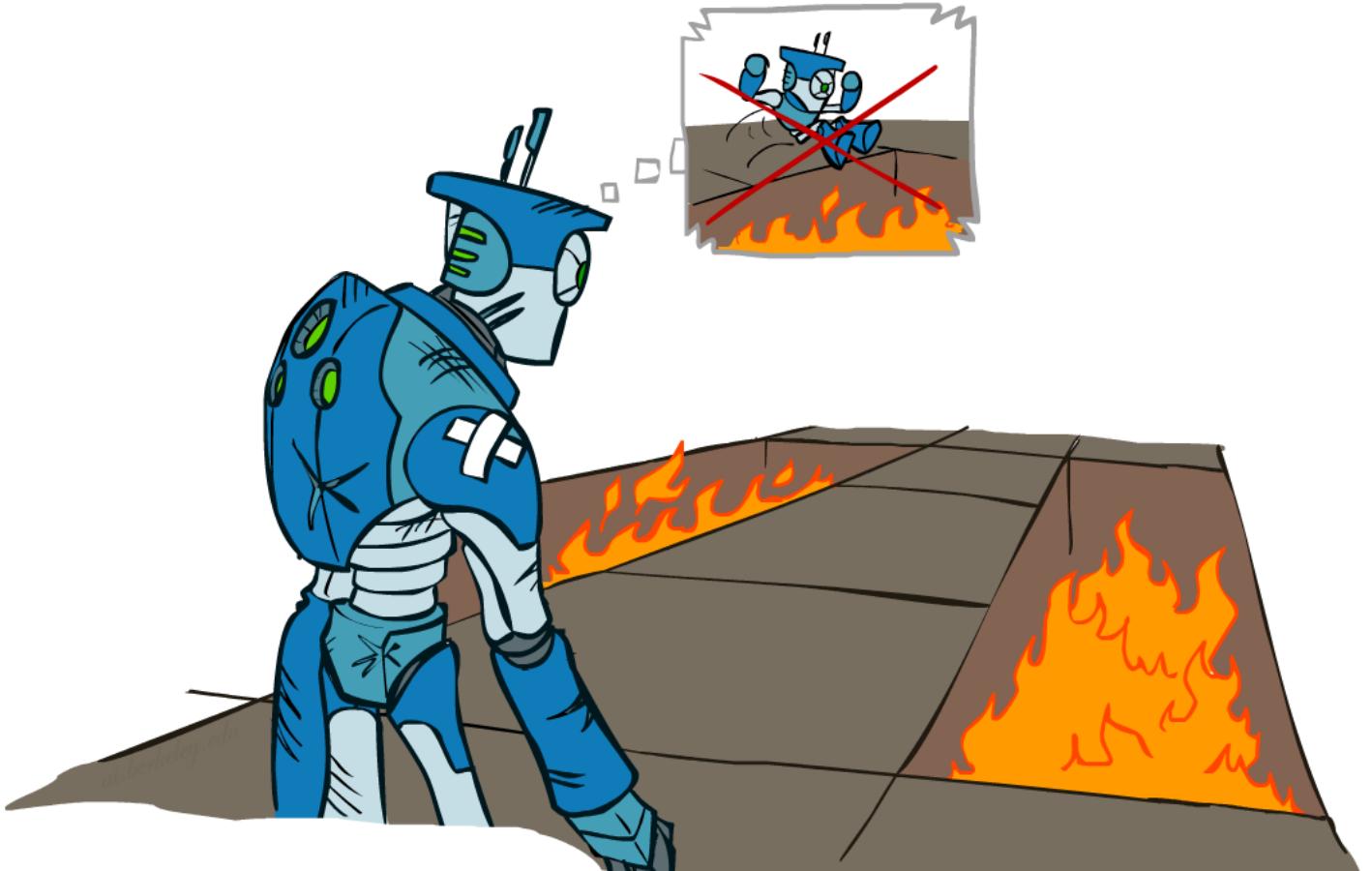
Modified Q-Update:
$$Q(s, a) \leftarrow_{\alpha} R(s, a, s') + \gamma \max_{a'} f(Q(s', a'), N(s', a'))$$



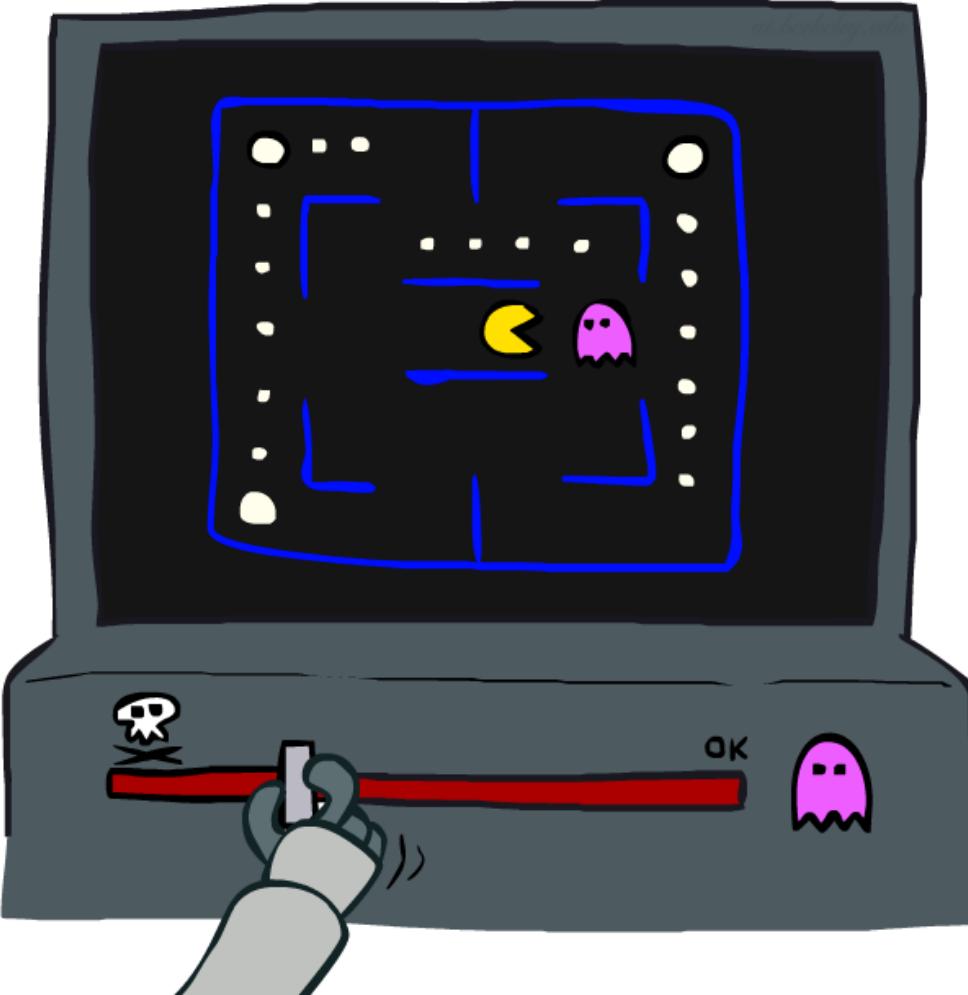
- Note: this propagates the “bonus” back to states that lead to unknown states as well!

Regret

- Even if you learn the optimal policy, you still make mistakes along the way
- Regret is a measure of your total mistake cost: the difference between your (expected) rewards, including youthful suboptimality, and optimal (expected) rewards
- Minimizing regret goes beyond learning to be optimal – it requires optimally learning to be optimal
- Example: random exploration and exploration functions both end up optimal, but random exploration has higher regret

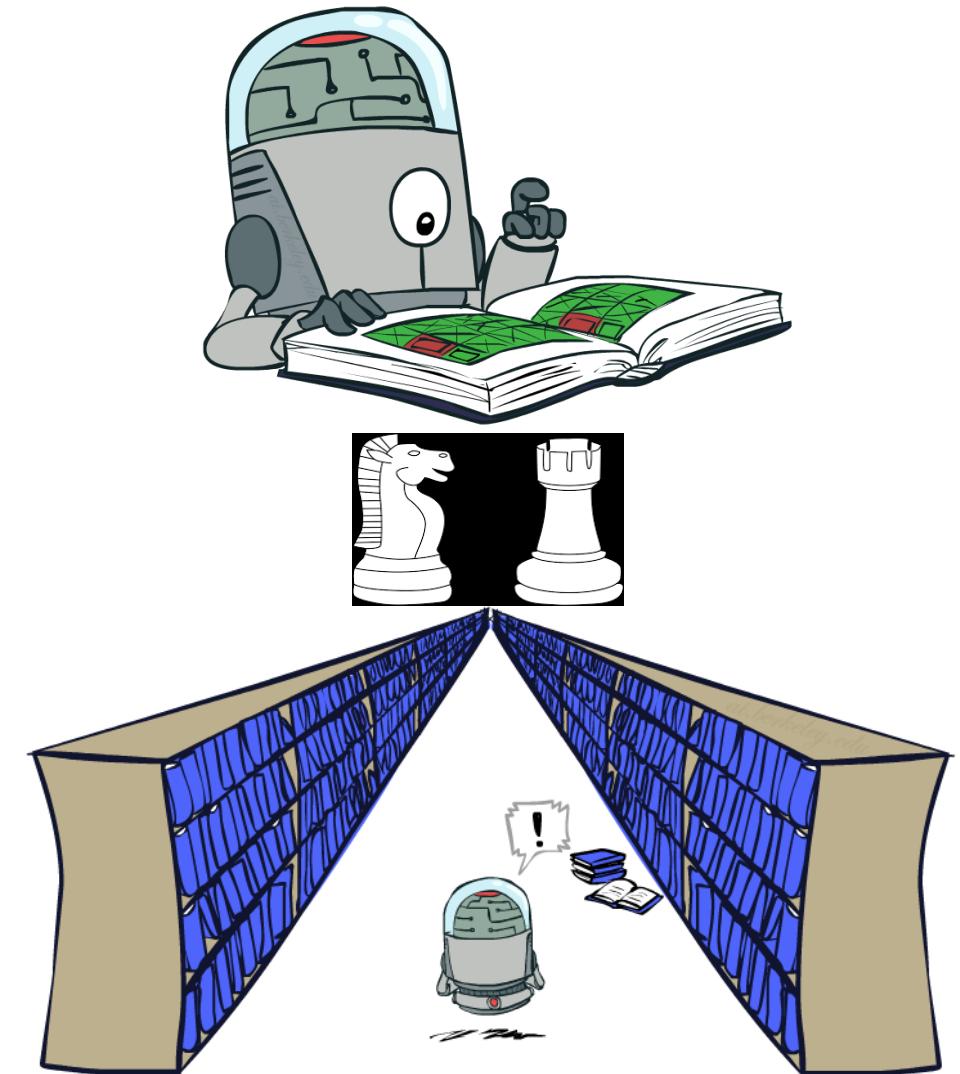


Approximate Q-Learning



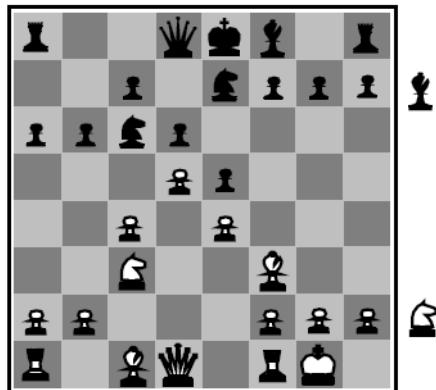
Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
 - Too many states to visit them all in training
 - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
 - Learn about some small number of training states from experience
 - Generalize that experience to new, similar situations
 - This is a fundamental idea in machine learning, and we'll see it over and over again

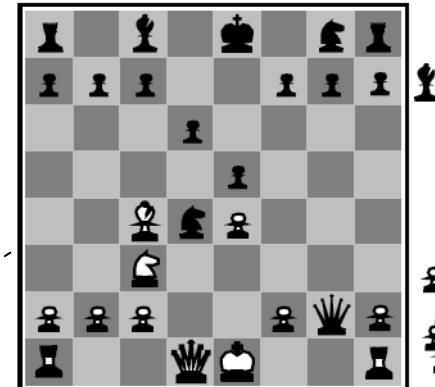
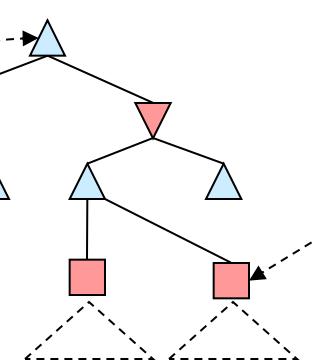


Flashback: Evaluation Functions

- Evaluation functions score non-terminals in depth-limited search



Black to move
White slightly better



White to move
Black winning

- Ideal function: returns the actual minimax value of the position
- In practice: typically weighted linear sum of features:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

- e.g. $f_1(s) = (\text{num white queens} - \text{num black queens})$, etc.

Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:

$$V(s) = w_1 f_1(s) + w_2 f_2(s) + \dots + w_n f_n(s)$$

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!

Approximate Q-Learning

$$Q(s, a) = w_1 f_1(s, a) + w_2 f_2(s, a) + \dots + w_n f_n(s, a)$$

- Q-learning with linear Q-functions:

transition = (s, a, r, s')

difference = $[r + \gamma \max_{a'} Q(s', a')] - Q(s, a)$

$Q(s, a) \leftarrow Q(s, a) + \alpha \text{ [difference]}$ Exact Q's

$w_i \leftarrow w_i + \alpha \text{ [difference]} f_i(s, a)$ Approximate Q's

- Intuitive interpretation:

- Adjust weights of active features
- E.g., if something unexpectedly bad happens, blame the features that were on:
disprefer all states with that state's features

- Formal justification: online least squares

