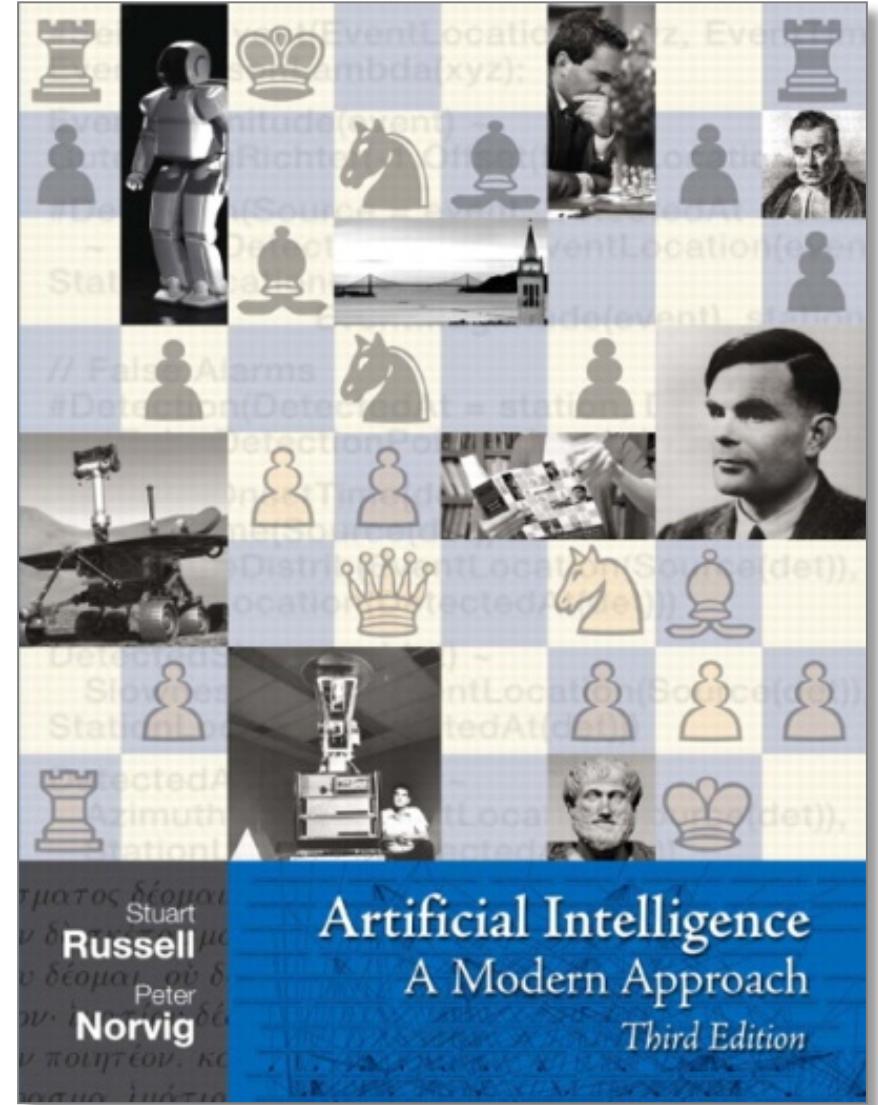
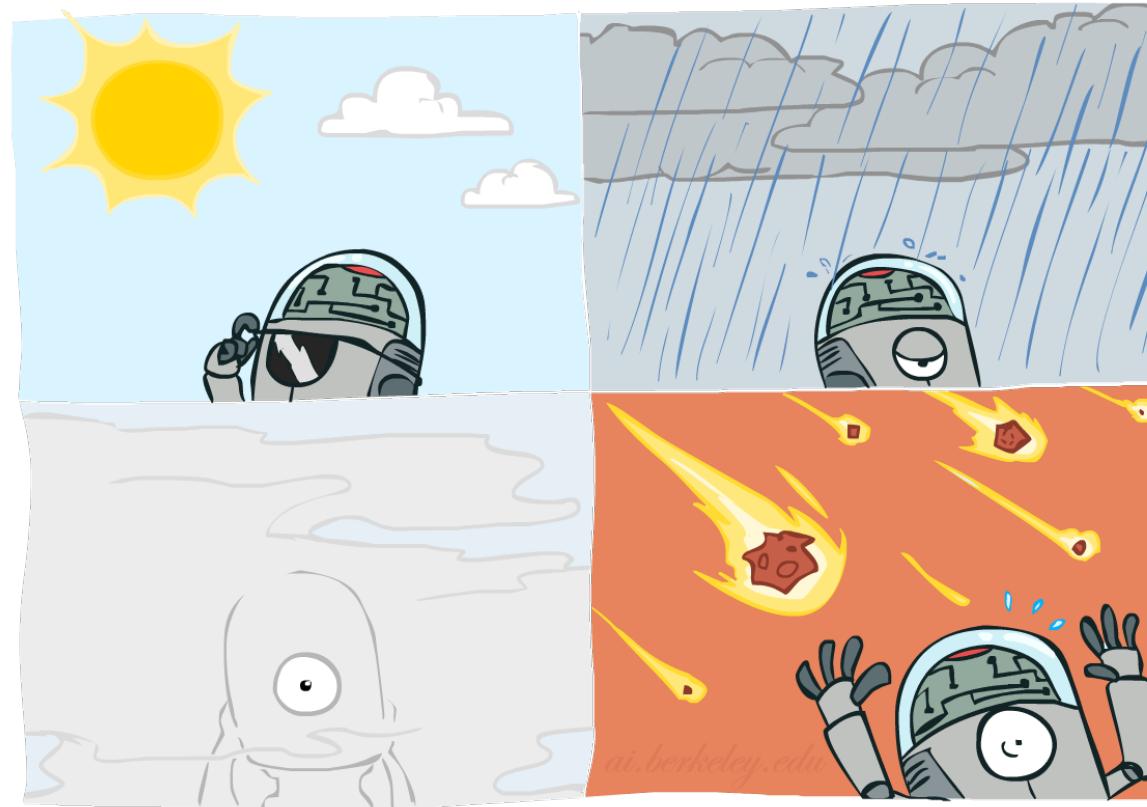


Acting Under Uncertainty

Read AIMA
Chapter 13 "Quantifying Uncertainty" (13.1-13.5)
and
Chapter 15 "Probabilistic Reasoning Over Time"
(15.2-15.5)



Probability



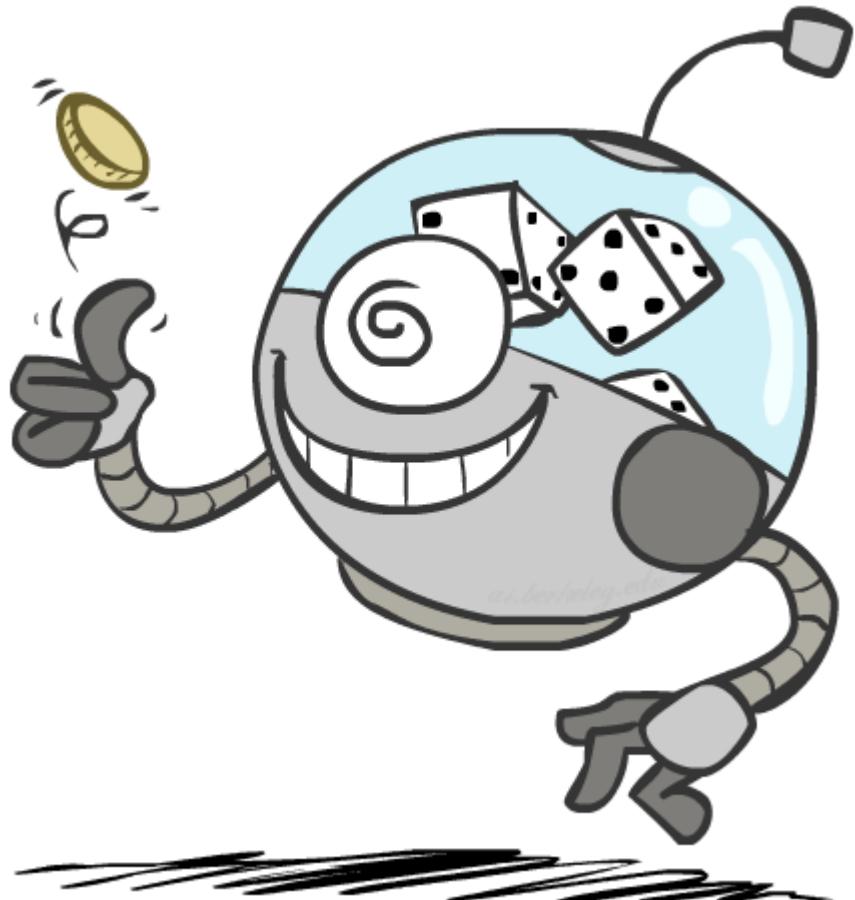
Slides courtesy of Dan Klein and Pieter Abbeel – University of California, Berkeley

[These slides were created by Dan Klein and Pieter Abbeel for CS188 Intro to AI at UC Berkeley. All CS188 materials are available at <http://ai.berkeley.edu>.]

Today

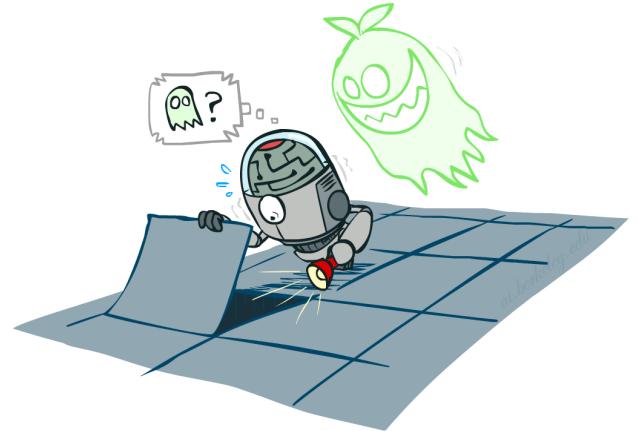
- Probability

- Random Variables
- Joint and Marginal Distributions
- Conditional Distribution
- Product Rule, Chain Rule, Bayes' Rule
- Inference
- Independence



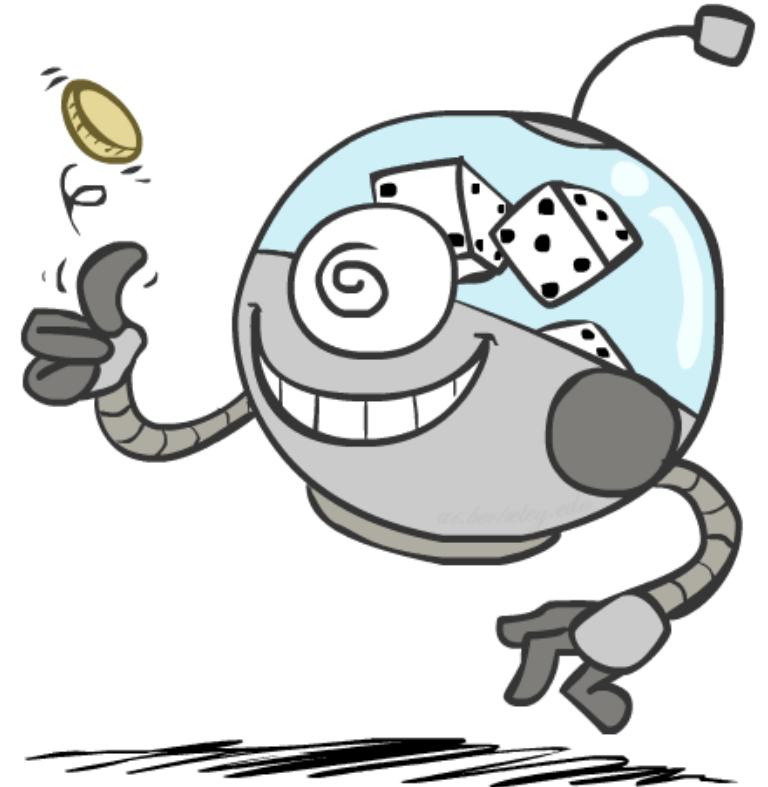
Uncertainty

- General situation:
 - **Observed variables (evidence):** Agent knows certain things about the state of the world (e.g., sensor readings or symptoms)
 - **Unobserved variables:** Agent needs to reason about other aspects (e.g. where an object is or what disease is present)
 - **Model:** Agent knows something about how the known variables relate to the unknown variables
- Probabilistic reasoning gives us a framework for managing our beliefs and knowledge



Random Variables

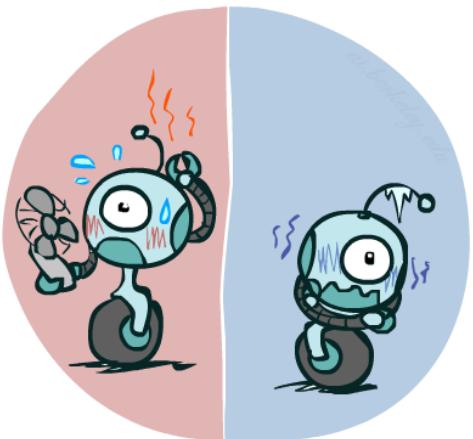
- A random variable is some aspect of the world about which we (may) have uncertainty
 - R = Is it raining?
 - T = Is it hot or cold?
 - D = How long will it take to drive to work?
 - L = Where is the ghost?
- We denote random variables with capital letters
- Like variables in a CSP, random variables have domains
 - R in {true, false} (often write as {+r, -r})
 - T in {hot, cold}
 - D in $[0, \infty)$
 - L in possible locations, maybe $\{(0,0), (0,1), \dots\}$



Probability Distributions

- Associate a probability with each value

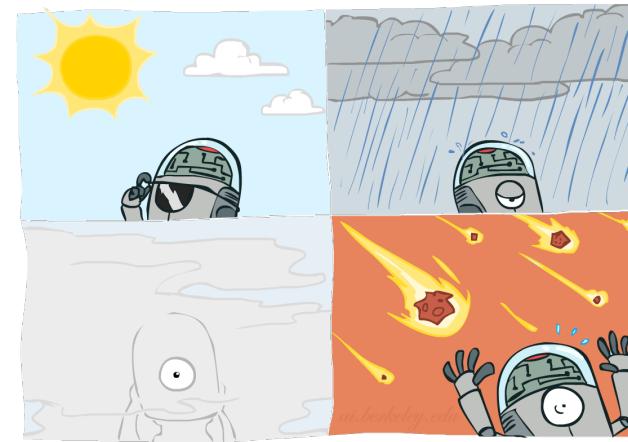
- Temperature:



$P(T)$

T	P
hot	0.5
cold	0.5

- Weather:



$P(W)$

W	P
sun	0.6
rain	0.1
fog	0.3
meteor	0.0

Probability Distributions

- Unobserved random variables have distributions

T	P
hot	0.5
cold	0.5

W	P
sun	0.6
rain	0.1
fog	0.3
meteor	0.0

- A distribution is a TABLE of probabilities of values

- A probability (lower case value) is a single number

$$P(W = \text{rain}) = 0.1$$

- Must have: $\forall x \ P(X = x) \geq 0$ and $\sum_x P(X = x) = 1$

Shorthand notation:

$$P(\text{hot}) = P(T = \text{hot}),$$

$$P(\text{cold}) = P(T = \text{cold}),$$

$$P(\text{rain}) = P(W = \text{rain}),$$

...

OK if all domain entries are unique

Joint Distributions

- A *joint distribution* over a set of random variables: X_1, X_2, \dots, X_n specifies a real number for each assignment (or *outcome*):

$$P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n)$$

$$P(x_1, x_2, \dots, x_n)$$

- Must obey: $P(x_1, x_2, \dots, x_n) \geq 0$

$$\sum_{(x_1, x_2, \dots, x_n)} P(x_1, x_2, \dots, x_n) = 1$$

$$P(T, W)$$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

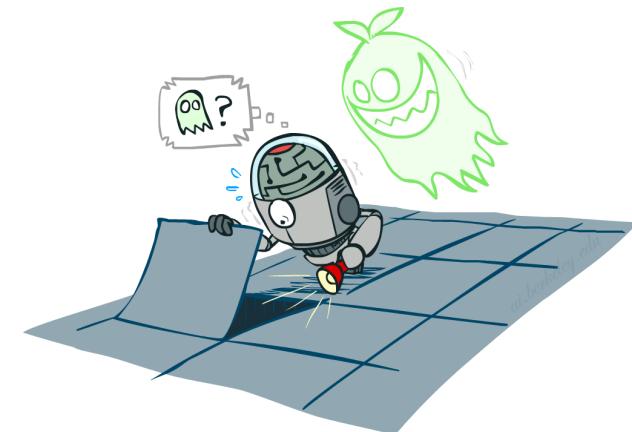
- Size of distribution if n variables with domain sizes d ?
 - For all but the smallest distributions, impractical to write out!

Probabilistic Models

- A probabilistic model is a joint distribution over a set of random variables
- Probabilistic models:
 - (Random) variables with domains
 - Assignments are called *outcomes*
 - Joint distributions: say whether assignments (outcomes) are likely
 - *Normalized*: sum to 1.0
 - Ideally: only certain variables directly interact

Distribution over T,W

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3



Events

- An *event* is a set E of outcomes

$$P(E) = \sum_{(x_1 \dots x_n) \in E} P(x_1 \dots x_n)$$

- From a joint distribution, we can calculate the probability of any event

- Probability that it's hot AND sunny?
- Probability that it's hot?
- Probability that it's hot OR sunny?

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

- Typically, the events we care about are *partial assignments*, like $P(T=\text{hot})$

Quiz: Events

- $P(+x, +y) ?$

$$P(X, Y)$$

- $P(+x) ?$

- $P(-y \text{ OR } +x) ?$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

Marginal Distributions

- Marginal distributions are sub-tables which eliminate variables
- Marginalization (summing out): Combine collapsed rows by adding

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3



$$P(t) = \sum_s P(t, s)$$



$$P(s) = \sum_t P(t, s)$$

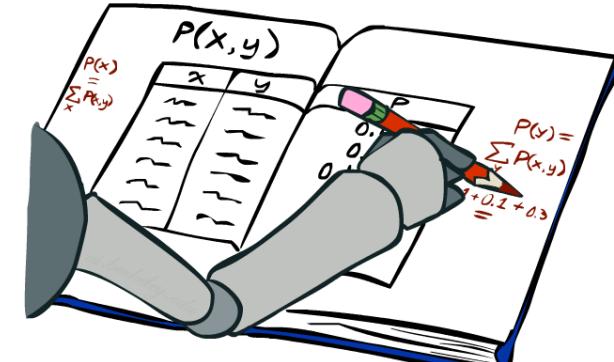
$$P(X_1 = x_1) = \sum_{x_2} P(X_1 = x_1, X_2 = x_2)$$

$P(T)$

T	P
hot	0.5
cold	0.5

$P(W)$

W	P
sun	0.6
rain	0.4



Quiz: Marginal Distributions

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

$$P(x) = \sum_y P(x, y)$$

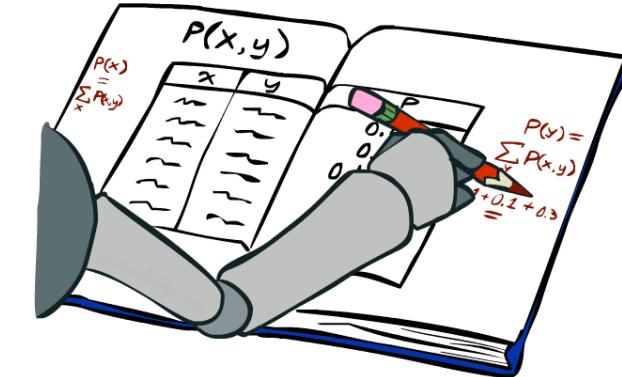
$$P(y) = \sum_x P(x, y)$$

$P(X)$

X	P
+x	
-x	

$P(Y)$

Y	P
+y	
-y	



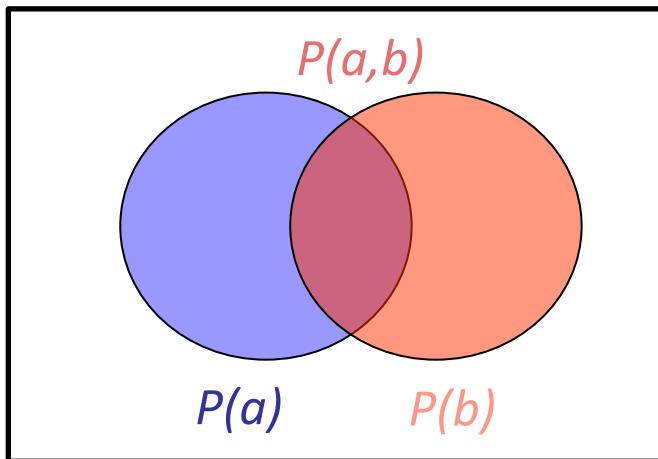
Conditional Probabilities

- A simple relation between joint and conditional probabilities
 - In fact, this is taken as the *definition* of a conditional probability

$$P(a|b) = \frac{P(a,b)}{P(b)}$$

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3



$$P(W = s | T = c) = \frac{P(W = s, T = c)}{P(T = c)} = \frac{0.2}{0.5} = 0.4$$

$$\begin{aligned} &= P(W = s, T = c) + P(W = r, T = c) \\ &= 0.2 + 0.3 = 0.5 \end{aligned}$$

Quiz: Conditional Probabilities

- $P(+x \mid +y) ?$

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

- $P(-x \mid +y) ?$

- $P(-y \mid +x) ?$

Conditional Distributions

- Conditional distributions are probability distributions over some variables given fixed values of others

Conditional Distributions

$$P(W|T)$$

$P(W T = hot)$	
W	P
sun	0.8
rain	0.2
$P(W T = cold)$	
W	P
sun	0.4
rain	0.6

Joint Distribution

$$P(T, W)$$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Normalization Trick

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

$$\begin{aligned} P(W = s|T = c) &= \frac{P(W = s, T = c)}{P(T = c)} \\ &= \frac{P(W = s, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\ &= \frac{0.2}{0.2 + 0.3} = 0.4 \end{aligned}$$



$$\begin{aligned} P(W = r|T = c) &= \frac{P(W = r, T = c)}{P(T = c)} \\ &= \frac{P(W = r, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\ &= \frac{0.3}{0.2 + 0.3} = 0.6 \end{aligned}$$

$P(W|T = c)$

W	P
sun	0.4
rain	0.6

Normalization Trick

$$\begin{aligned} P(W = s|T = c) &= \frac{P(W = s, T = c)}{P(T = c)} \\ &= \frac{P(W = s, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\ &= \frac{0.2}{0.2 + 0.3} = 0.4 \end{aligned}$$

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

SELECT the joint probabilities matching the evidence
→

T	W	P
cold	sun	0.2
cold	rain	0.3

$P(c, W)$

NORMALIZE the selection
(make it sum to one)
→

W	P
sun	0.4
rain	0.6

$P(W|T = c)$

$$\begin{aligned} P(W = r|T = c) &= \frac{P(W = r, T = c)}{P(T = c)} \\ &= \frac{P(W = r, T = c)}{P(W = s, T = c) + P(W = r, T = c)} \\ &= \frac{0.3}{0.2 + 0.3} = 0.6 \end{aligned}$$

Normalization Trick

$P(T, W)$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

SELECT the joint probabilities matching the evidence
→

$P(c, W)$

T	W	P
cold	sun	0.2
cold	rain	0.3

NORMALIZE the selection
(make it sum to one)
→

$P(W|T = c)$

W	P
sun	0.4
rain	0.6

- Why does this work? Sum of selection is $P(\text{evidence})!$ ($P(T=c)$, here)

$$P(x_1|x_2) = \frac{P(x_1, x_2)}{P(x_2)} = \frac{P(x_1, x_2)}{\sum_{x_1} P(x_1, x_2)}$$

Quiz: Normalization Trick

- $P(X | Y=-y) ?$

$P(X, Y)$

X	Y	P
+x	+y	0.2
+x	-y	0.3
-x	+y	0.4
-x	-y	0.1

SELECT the joint
probabilities
matching the
evidence



NORMALIZE the
selection
(make it sum to one)



To Normalize

- (Dictionary) To bring or restore to a **normal condition**

All entries sum to ONE

- Procedure:

- Step 1: Compute $Z = \text{sum over all entries}$
- Step 2: Divide every entry by Z

- Example 1

W	P
sun	0.2
rain	0.3

Normalize \rightarrow $Z = 0.5$

W	P
sun	0.4
rain	0.6

- Example 2

T	W	P
hot	sun	20
hot	rain	5
cold	sun	10
cold	rain	15

Normalize \rightarrow $Z = 50$

T	W	P
hot	sun	0.4
hot	rain	0.1
cold	sun	0.2
cold	rain	0.3

Probabilistic Inference

- Probabilistic inference: compute a desired probability from other known probabilities (e.g. conditional from joint)
- We generally compute conditional probabilities
 - $P(\text{on time} \mid \text{no reported accidents}) = 0.90$
 - These represent the agent's *beliefs* given the evidence
- Probabilities change with new evidence:
 - $P(\text{on time} \mid \text{no accidents, 5 a.m.}) = 0.95$
 - $P(\text{on time} \mid \text{no accidents, 5 a.m., raining}) = 0.80$
 - Observing new evidence causes *beliefs to be updated*



Inference by Enumeration

- General case:

- Evidence variables: $E_1 \dots E_k = e_1 \dots e_k$
- Query* variable: Q
- Hidden variables: $H_1 \dots H_r$

$$\left. \begin{array}{l} E_1 \dots E_k = e_1 \dots e_k \\ Q \\ H_1 \dots H_r \end{array} \right\} X_1, X_2, \dots, X_n$$

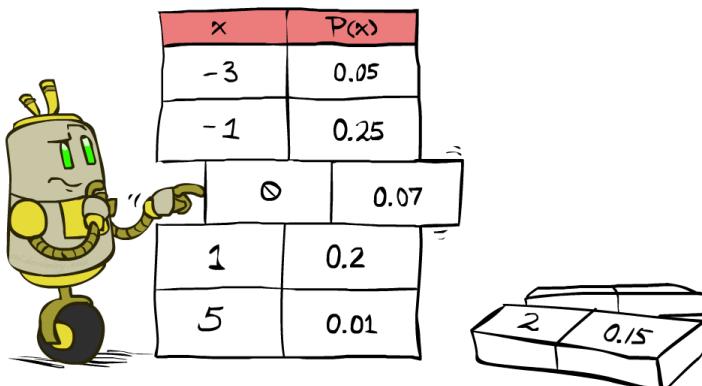
All variables

- We want:

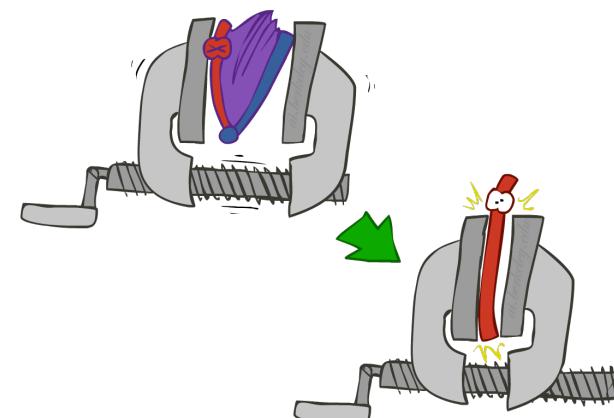
$$P(Q|e_1 \dots e_k)$$

* Works fine with multiple query variables, too

- Step 1: Select the entries consistent with the evidence



- Step 2: Sum out H to get joint of Query and evidence



- Step 3: Normalize

$$\times \frac{1}{Z}$$

$$P(Q, e_1 \dots e_k) = \sum_{h_1 \dots h_r} P(\underbrace{Q, h_1 \dots h_r}_{X_1, X_2, \dots, X_n}, e_1 \dots e_k)$$

$$Z = \sum_q P(Q, e_1 \dots e_k)$$

$$P(Q|e_1 \dots e_k) = \frac{1}{Z} P(Q, e_1 \dots e_k)$$

Inference by Enumeration

- $P(W)?$
- $P(W | \text{winter})?$
- $P(W | \text{winter, hot})?$

S	T	W	P
summer	hot	sun	0.30
summer	hot	rain	0.05
summer	cold	sun	0.10
summer	cold	rain	0.05
winter	hot	sun	0.10
winter	hot	rain	0.05
winter	cold	sun	0.15
winter	cold	rain	0.20

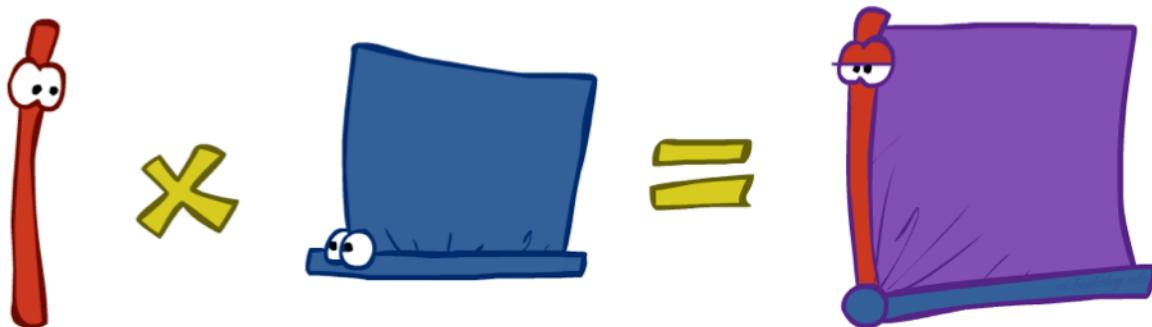
Inference by Enumeration

- Obvious problems:
 - Worst-case time complexity $O(d^n)$
 - Space complexity $O(d^n)$ to store the joint distribution

The Product Rule

- Sometimes have conditional distributions but want the joint

$$P(y)P(x|y) = P(x, y) \quad \longleftrightarrow \quad P(x|y) = \frac{P(x, y)}{P(y)}$$



The Product Rule

$$P(y)P(x|y) = P(x, y)$$

- Example:

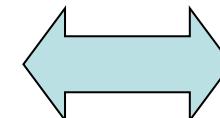
R	P
sun	0.8
rain	0.2

$$P(D|W)$$

D	W	P
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3

$$P(D, W)$$

D	W	P
wet	sun	
dry	sun	
wet	rain	
dry	rain	



The Chain Rule

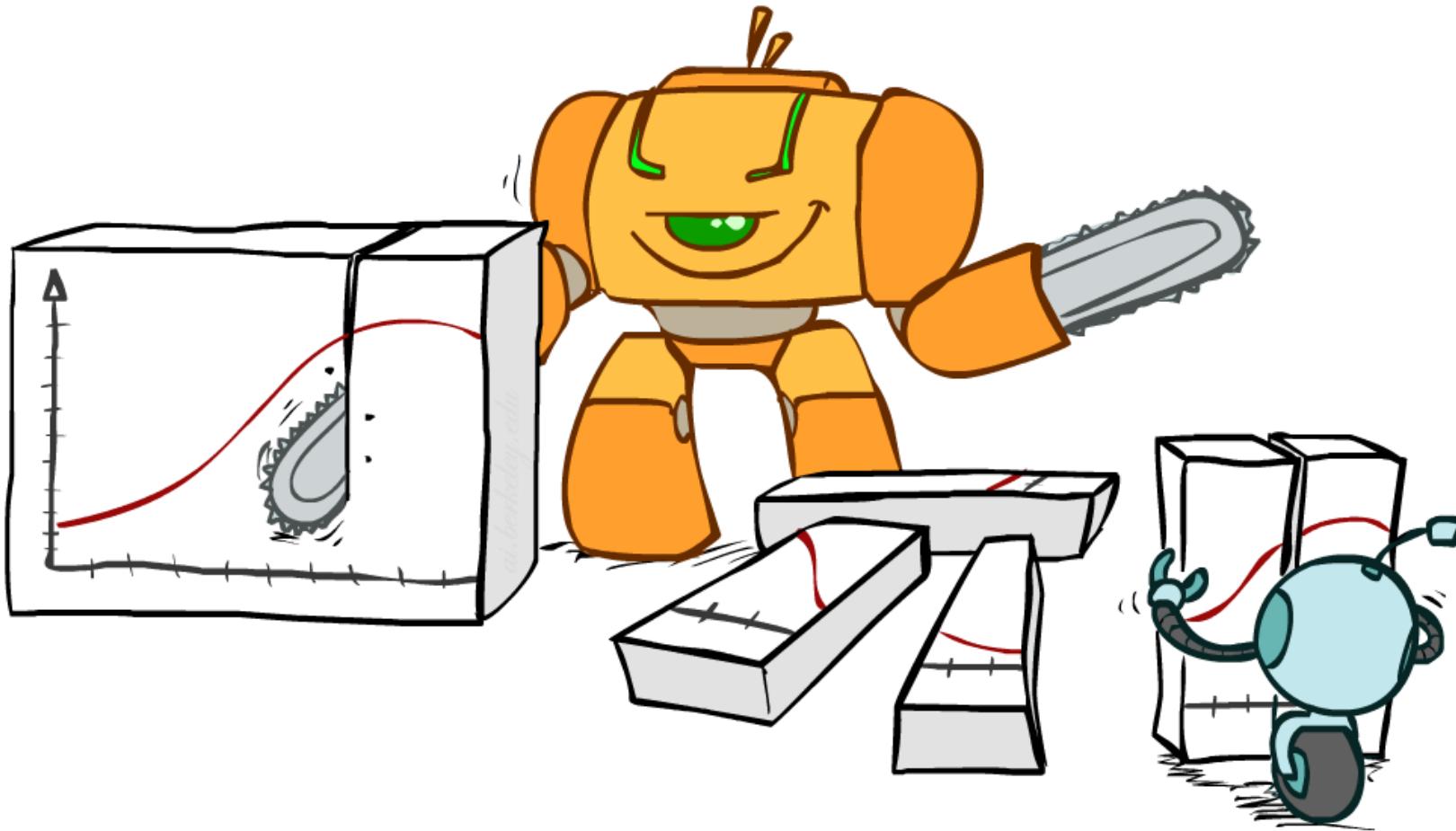
- More generally, can always write any joint distribution as an incremental product of conditional distributions

$$P(x_1, x_2, x_3) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2)$$

$$P(x_1, x_2, \dots, x_n) = \prod_i P(x_i|x_1 \dots x_{i-1})$$

- Why is this always true?

Bayes Rule



Bayes' Rule

- Two ways to factor a joint distribution over two variables:

$$P(x, y) = P(x|y)P(y) = P(y|x)P(x)$$

- Dividing, we get:

$$P(x|y) = \frac{P(y|x)}{P(y)}P(x)$$

- Why is this at all helpful?

- Lets us build one conditional from its reverse
- Often one conditional is tricky but the other one is simple
- Foundation of many systems we'll see later (e.g. ASR, MT)



- In the running for most important AI equation!

Inference with Bayes' Rule

- Example: Diagnostic probability from causal probability:

$$P(\text{cause}|\text{effect}) = \frac{P(\text{effect}|\text{cause})P(\text{cause})}{P(\text{effect})}$$

- Example:

- M: meningitis, S: stiff neck

$$\left. \begin{array}{l} P(+m) = 0.0001 \\ P(+s|m) = 0.8 \\ P(+s|-m) = 0.01 \end{array} \right\} \text{Example givens}$$

$$P(+m|s) = \frac{P(+s|m)P(+m)}{P(+s)} = \frac{P(+s|m)P(+m)}{P(+s|m)P(+m) + P(+s|-m)P(-m)} = \frac{0.8 \times 0.0001}{0.8 \times 0.0001 + 0.01 \times 0.999}$$

- Note: posterior probability of meningitis still very small
 - Note: you should still get stiff necks checked out! Why?

Quiz: Bayes' Rule

- Given:

$$P(W)$$

R	P
sun	0.8
rain	0.2

$$P(D|W)$$

D	W	P
wet	sun	0.1
dry	sun	0.9
wet	rain	0.7
dry	rain	0.3

- What is $P(W | \text{dry})$?

Language Modeling with N-Grams

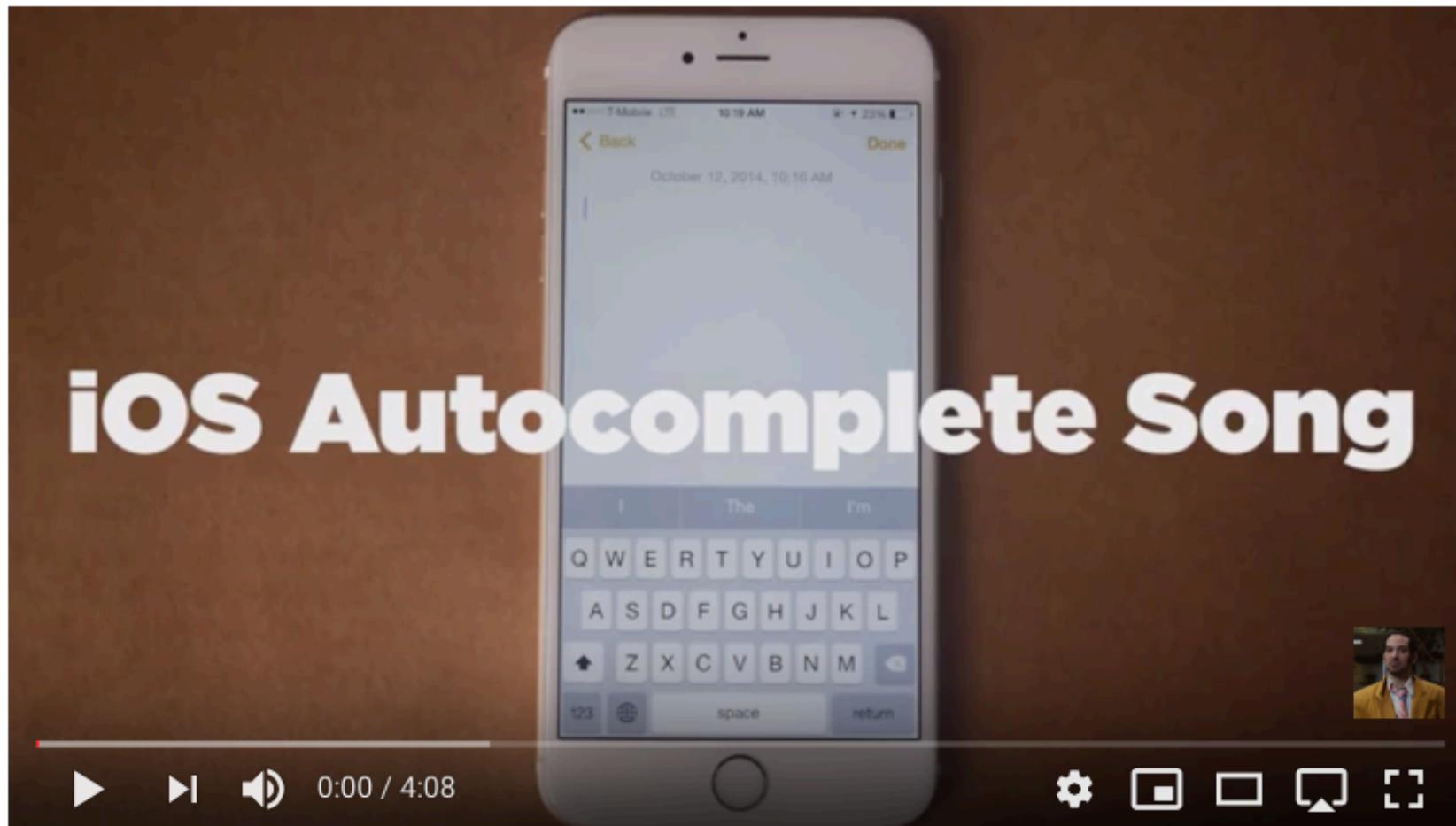
Speech and Language Processing (3rd Edition draft)

[Chapter 3 “Language Modeling with N-Grams”](#)



YouTube

Search



🎵 iOS Autocomplete Song | Song A Day #2110

<https://www.youtube.com/watch?v=M8MJFrdfGe0>

Probabilistic Language Models

- Today's goal: assign a probability to a sentence
- Autocomplete for texting
- Machine Translation
- Spelling Correction
- Speech Recognition
- Other NLG tasks: summarization, question-answering, dialog systems

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words
- Related task: probability of an upcoming word
- A model that computes either of these
- Better: **the grammar** But **language model** or **LM** is standard

Probabilistic Language Modeling

- Goal: compute the probability of a sentence or sequence of words

$$P(W) = P(w_1, w_2, w_3, w_4, w_5 \dots w_n)$$

- Related task: probability of an upcoming word

$$P(w_5 | w_1, w_2, w_3, w_4)$$

- A model that computes either of these

$P(W)$ or $P(w_n | w_1, w_2 \dots w_{n-1})$ is called a **language model**.

- Better: **the grammar** But **language model** or **LM** is standard

How to compute $P(W)$

- How to compute this joint probability:
 - $P(\text{the, underdog, Philadelphia, Eagles, won})$
- Intuition: let's rely on the Chain Rule of Probability

The Chain Rule

The Chain Rule

- Recall the definition of conditional probabilities

$$p(B|A) = P(A,B)/P(A) \quad \text{Rewriting: } P(A,B) = P(A)P(B|A)$$

- More variables:

$$P(A,B,C,D) = P(A)P(B|A)P(C|A,B)P(D|A,B,C)$$

- The Chain Rule in General

$$P(x_1, x_2, x_3, \dots, x_n) = P(x_1)P(x_2|x_1)P(x_3|x_1, x_2)\dots P(x_n|x_1, \dots, x_{n-1})$$

The Chain Rule applied to compute joint probability of words in sentence

The Chain Rule applied to compute joint probability of words in sentence

$$P(w_1 w_2 \dots w_n) = \prod_i P(w_i | w_1 w_2 \dots w_{i-1})$$

$P(\text{"the underdog Philadelphia Eagles won"}) =$

$$\begin{aligned} & P(\text{the}) \times P(\text{underdog} | \text{the}) \times P(\text{Philadelphia} | \text{the underdog}) \\ & \quad \times P(\text{Eagles} | \text{the underdog Philadelphia}) \\ & \quad \times P(\text{won} | \text{the underdog Philadelphia Eagles}) \end{aligned}$$

How to estimate these probabilities

- Could we just count and divide?

How to estimate these probabilities

- Could we just count and divide? Maximum likelihood estimation (MLE)

$$P(\text{won} \mid \text{the underdog team}) = \frac{\text{Count}(\text{the underdog team won})}{\text{Count}(\text{the underdog team})}$$

- Why doesn't this work?

Simplifying Assumption = Markov Assumption

Simplifying Assumption = Markov Assumption

- $P(\text{won} | \text{the underdog team}) \approx P(\text{won} | \text{team})$
- Only depends on the previous k words, not the whole context
- $\approx P(\text{won} | \text{underdog team})$
- $\approx P(w_i | w_{i-2} w_{i-1})$
- $P(w_1 w_2 w_3 w_4 \dots w_n) \approx \prod_i^n P(w_i | w_{i-k} \dots w_{i-1})$
- K is the number of context words that we take into account

How much history should we use?

unigram	no history	$\prod_i^n p(w_i)$	$p(w_i) = \frac{\text{count}(w_i)}{\text{all words}}$
bigram	1 word as history	$\prod_i^n p(w_i w_{i-1})$	$\begin{aligned} p(w_i w_{i-1}) \\ = \frac{\text{count}(w_{i-1} w_i)}{\text{count}(w_{i-1})} \end{aligned}$
trigram	2 words as history	$\prod_i^n p(w_i w_{i-2} w_{i-1})$	$\begin{aligned} p(w_i w_{i-2} w_{i-1}) \\ = \frac{\text{count}(w_{i-2} w_{i-1} w_i)}{\text{count}(w_{i-2} w_{i-1})} \end{aligned}$
4-gram	3 words as history	$\prod_i^n p(w_i w_{i-3} w_{i-2} w_{i-1})$	$\begin{aligned} p(w_i w_{i-3} w_{i-2} w_{i-1}) \\ = \frac{\text{count}(w_{i-3} w_{i-2} w_{i-1} w_i)}{\text{count}(w_{i-3} w_{i-2} w_{i-1})} \end{aligned}$

Historical Notes

1913	Andrei Markov counts 20k letters in <i>Eugene Onegin</i>	
1948	Claude Shannon uses n-grams to approximate English	
1956	Noam Chomsky decries finite-state Markov Models	
1980s	Fred Jelinek at IBM TJ Watson uses n-grams for ASR, think about 2 other ideas for models: (1) MT, (2) stock market prediction	
1993	Jelinek at team develops statistical machine translation $\operatorname{argmax}_e p(e f) = p(e) p(f e)$	
	Jelinek left IBM to found CLSP at JHU Peter Brown and Robert Mercer move to Renaissance Technology	

Simplest case: Unigram model

$$P(w_1 w_2 \dots w_n) \approx \prod_i P(w_i)$$

Some automatically generated sentences from a unigram model

fifth an of futures the an incorporated a a the
inflation most dollars quarter in is mass

thrift did eighty said hard 'm july bullish

that or limited the

Bigram model

- Condition on the previous word:

$$P(w_i | w_1 w_2 \dots w_{i-1}) \approx P(w_i | w_{i-1})$$

texaco rose one in this issue is pursuing growth in a boiler house said mr. gurria mexico 's motion control proposal

without permission from five hundred fifty five yen

outside new car parking lot of the agreement reached

this would be a record november

N-gram models

- We can extend to trigrams, 4-grams, 5-grams
- In general this is an insufficient model of language
 - because language has **long-distance dependencies**:
- “The computer(s) which I had just put into the machine room on the fifth floor is (are) crashing.”
- But we can often get away with N-gram models

Language Modeling

Estimating N-gram Probabilities

Estimating bigram probabilities

- The Maximum Likelihood Estimate

$$P(w_i | w_{i-1}) = \frac{\text{count}(w_{i-1}, w_i)}{\text{count}(w_{i-1})}$$

$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

An example

$$P(w_i | w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

< s > I am Sam < /s >

< s > Sam I am < /s >

< s > I do not like green eggs and ham < /s >

$$P(\text{I} | \text{<s>}) = \frac{2}{3} = .67$$

$$P(\text{</s>} | \text{Sam}) = \frac{1}{2} = 0.5$$

$$P(\text{Sam} | \text{<s>}) = \frac{1}{3} = .33$$

$$P(\text{Sam} | \text{am}) = \frac{1}{2} = .5$$

$$P(\text{am} | \text{I}) = \frac{2}{3} = .67$$

$$P(\text{do} | \text{I}) = \frac{1}{3} = .33$$

Problems for MLE

- Zeros

Train	Test
denied the allegations	denied the memo
denied the reports	
denied the claims	
denied the requests	

- $P(\text{memo} \mid \text{denied the}) = 0$
- And we also assign 0 probability to all sentences containing it!

Problems for MLE

- Out of vocabulary items (OOV)
- <unk> to deal with OOVs
- Fixed lexicon L of size V
- Normalize training data by replacing any word not in L with <unk>
- Avoid zeros with smoothing

Practical Issues

- We do everything in log space
 - Avoid underflow
 - (also adding is faster than multiplying)

$$\log(p_1 \times p_2 \times p_3 \times p_4) = \log p_1 + \log p_2 + \log p_3 + \log p_4$$

Language Modeling Toolkits

- SRILM
 - <http://www.speech.sri.com/projects/srilm/>
- KenLM
 - <https://kheafield.com/code/kenlm/>

Google N-Gram Release, August 2006

AUG

3

All Our N-gram are Belong to You

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word [n-gram models](#) for a variety of R&D projects,

...

That's why we decided to share this enormous dataset with everyone. We processed 1,024,908,267,229 words of running text and are publishing the counts for all 1,176,470,663 five-word sequences that appear at least 40 times. There are 13,588,391 unique words, after discarding words that appear less than 200 times.

Google N-Gram Release

- serve as the incoming 92
- serve as the incubator 99
- serve as the independent 794
- serve as the index 223
- serve as the indication 72
- serve as the indicator 120
- serve as the indicators 45
- serve as the indispensable 111
- serve as the indispensable 40
- serve as the individual 234

<http://googleresearch.blogspot.com/2006/08/all-our-n-gram-are-belong-to-you.html>

Google Book N-grams

- <http://ngrams.googlecode.com/>

Language Modeling

Evaluation and Perplexity

Evaluation: How good is our model?

- Does our language model prefer good sentences to bad ones?
 - Assign higher probability to “real” or “frequently observed” sentences
 - Than “ungrammatical” or “rarely observed” sentences?
- We train parameters of our model on a **training set**.
- We test the model’s performance on data we haven’t seen.
 - A **test set** is an unseen dataset that is different from our training set, totally unused.
 - An **evaluation metric** tells us how well our model does on the test set.

Training on the test set

- We can't allow test sentences into the training set
- We will assign it an artificially high probability when we set it in the test set
- “Training on the test set”
- Bad science!
- And violates the honor code

Extrinsic evaluation of N-gram models



Difficulty of extrinsic (task-based) evaluation of N-gram models

- Extrinsic evaluation
 - Time-consuming; can take days or weeks
- So
 - Sometimes use **intrinsic** evaluation: **perplexity**
 - Bad approximation
 - unless the test data looks **just** like the training data
 - So **generally only useful in pilot experiments**
 - But is helpful to think about.

Intuition of Perplexity

- The Shannon Game:
 - How well can we predict the next word?

I always order pizza with cheese and _____

Intuition of Perplexity

- The Shannon Game:

- How well can we predict the next word?

I always order pizza with cheese and _____

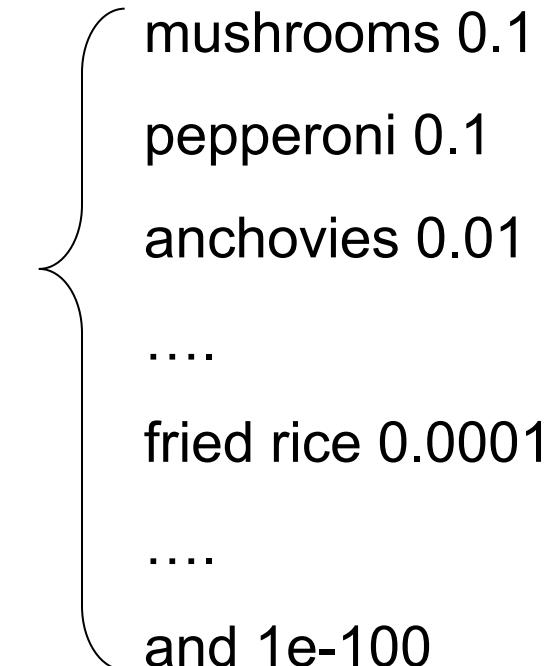
The 33rd President of the US was _____

I saw a _____

- Unigrams are terrible at this game. (Why?)

- A better model of a text

- is one which assigns a higher probability to the word that actually occurs



Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest $P(\text{sentence})$

Perplexity is the inverse probability of the test set, normalized by the number of words

Minimizing perplexity is the same as maximizing probability

Perplexity

The best language model is one that best predicts an unseen test set

- Gives the highest $P(\text{sentence})$

Perplexity is the inverse probability of the test set, normalized by the number of words

$$PP(W) = P(w_1 w_2 \dots w_N)^{-\frac{1}{N}}$$

$$= \sqrt[N]{\frac{1}{P(w_1 w_2 \dots w_N)}}$$

Chain rule:

$$PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_1 \dots w_{i-1})}}$$

For bigrams:

$$PP(W) = \sqrt[N]{\prod_{i=1}^N \frac{1}{P(w_i | w_{i-1})}}$$

Minimizing perplexity is the same as maximizing probability

Perplexity as branching factor

- Let's suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign $P=1/10$ to each digit?

Perplexity as branching factor

- Let's suppose a sentence consisting of random digits
- What is the perplexity of this sentence according to a model that assign $P=1/10$ to each digit?

$$\begin{aligned} \text{PP}(W) &= P(w_1 w_2 \dots w_N)^{-\frac{1}{N}} \\ &= \left(\frac{1}{10}\right)^N)^{-\frac{1}{N}} \\ &= \frac{1}{10}^{-1} \\ &= 10 \end{aligned}$$

Lower perplexity = better model

Minimizing perplexity is the same as maximizing probability

- Training 38 million words, test 1.5 million words,
WSJ

N-gram Order	Unigram	Bigram	Trigram
Perplexity	962	170	109

Language Modeling

Generalization and zeros

The Shannon Visualization Method

- Choose a random bigram ($\langle s \rangle, w$) according to its probability
- Now choose a random bigram (w, x) according to its probability
- And so on until we choose $\langle /s \rangle$
- Then string the words together

$\langle s \rangle$ I
I want
want to
to eat
eat Chinese
Chinese food
food $\langle /s \rangle$

I want to eat Chinese food

Approximating Shakespeare

- | | |
|-----------|---|
| 1
gram | –To him swallowed confess hear both. Which. Of save on trail for are ay device and
rote life have |
| | –Hill he late speaks; or! a more to leg less first you enter |
| 2
gram | –Why dost stand forth thy canopy, forsooth; he is this palpable hit the King Henry. Live
king. Follow. |
| | –What means, sir. I confess she? then all sorts, he is trim, captain. |
| 3
gram | –Fly, and will rid me these news of price. Therefore the sadness of parting, as they say,
'tis done. |
| | –This shall forbid it should be branded, if renown made it empty. |
| 4
gram | –King Henry. What! I will go seek the traitor Gloucester. Exeunt some of the watch. A
great banquet serv'd in; |
| | –It cannot be but so. |

Shakespeare as corpus

- $V=29,066$ types, $N=884,647$ tokens

Shakespeare as corpus

- $N=884,647$ tokens, $V=29,066$
- Shakespeare produced 300,000 bigram types out of $V^2= 844$ million possible bigrams.
 - So 99.96% of the possible bigrams were never seen (have zero entries in the table)
- 4-grams worse: What's coming out looks like Shakespeare because it *is* Shakespeare

The wall street journal is not shakespeare (no offense)

1
gram

Months the my and issue of year foreign new exchange's september were recession exchange new endorsed a acquire to six executives

2
gram

Last December through the way to preserve the Hudson corporation N. B. E. C. Taylor would seem to complete the major central planners one point five percent of U. S. E. has already old M. X. corporation of living on information such as more frequently fishing to keep her

3
gram

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and Brazil on market conditions

Can you guess the author of these random 3-gram sentences?

They also point to ninety nine point six billion dollars from two hundred four oh six three percent of the rates of interest stores as Mexico and gram Brazil on market conditions

This shall forbid it should be branded, if renown made it empty.

“You are uniformly charming!” cried he, with a smile of associating and now and then I bowed and they perceived a chaise and four to wish for.

The perils of overfitting

- N-grams only work well for word prediction if the test corpus looks like the training corpus
 - In real life, it often doesn't
 - We need to train robust models that generalize!
 - One kind of generalization: Zeros!
 - Things that don't ever occur in the training set
 - But occur in the test set

Zero probability bigrams

- Bigrams with zero probability
 - mean that we will assign 0 probability to the test set!
- And hence we cannot compute perplexity (can't divide by 0)!

Language Modeling

Smoothing: Add-one (Laplace) smoothing

The intuition of smoothing (from Dan Klein)

When we have sparse statistics:

$P(w | \text{denied the})$

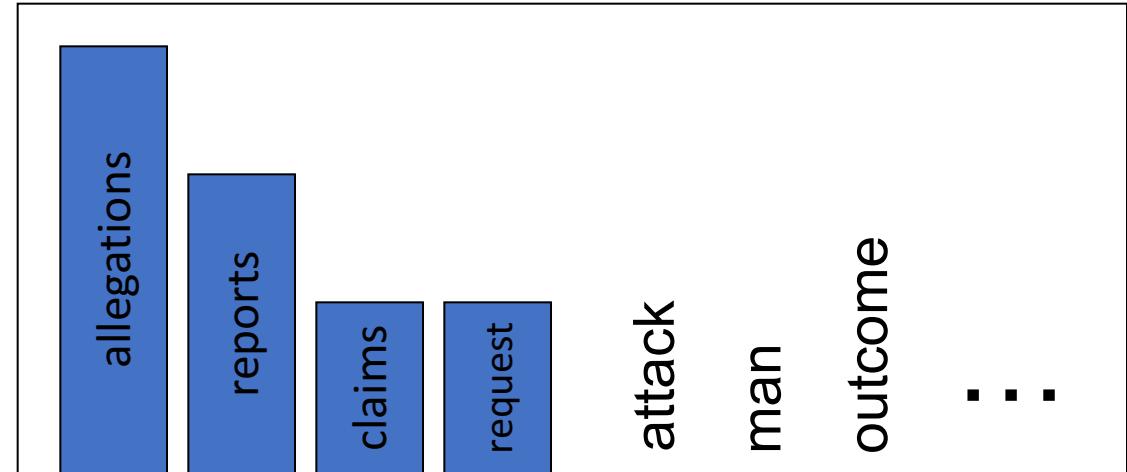
3 allegations

2 reports

1 claims

1 request

7 total



Steal probability mass to generalize better

$P(w | \text{denied the})$

2.5 allegations

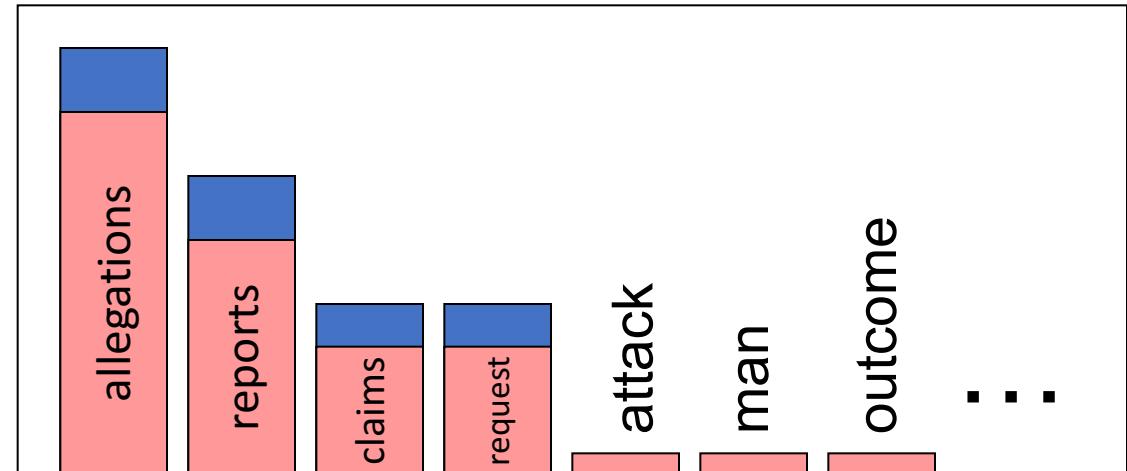
1.5 reports

0.5 claims

0.5 request

2 other

7 total



Add-one estimation

Also called Laplace smoothing

Pretend we saw each word one more time than we did

Just add one to all the counts!

MLE estimate:

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c(w_{i-1})}$$

Add-1 estimate:

$$P_{Add-1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c(w_{i-1}) + V}$$

Maximum Likelihood Estimates

- The maximum likelihood estimate
 - of some parameter of a model M from a training set T
 - maximizes the likelihood of the training set T given the model M
- Suppose the word “bagel” occurs 400 times in a corpus of a million words
- What is the probability that a random word from some other text will be “bagel”?
- MLE estimate is $400/1,000,000 = .0004$
- This may be a bad estimate for some other corpus
 - But it is the **estimate** that makes it **most likely** that “bagel” will occur 400 times in a million word corpus.

Add-1 estimation is a blunt instrument

- So add-1 isn't used for N-grams:
 - We'll see better methods
- But add-1 is used to smooth other NLP models
 - For text classification
 - In domains where the number of zeros isn't so huge.

Language Modeling

Interpolation, Backoff, and Web-Scale LMs

Backoff and Interpolation

Sometimes it helps to use **less** context

Condition on less context for contexts you haven't learned much about

Backoff:

use trigram if you have good evidence,
otherwise bigram, otherwise unigram

Interpolation:

mix unigram, bigram, trigram

Interpolation works better

Linear Interpolation

Simple interpolation

$$\begin{aligned}\hat{P}(w_n | w_{n-2} w_{n-1}) &= \lambda_1 P(w_n | w_{n-2} w_{n-1}) \\ &\quad + \lambda_2 P(w_n | w_{n-1}) \\ &\quad + \lambda_3 P(w_n)\end{aligned}$$

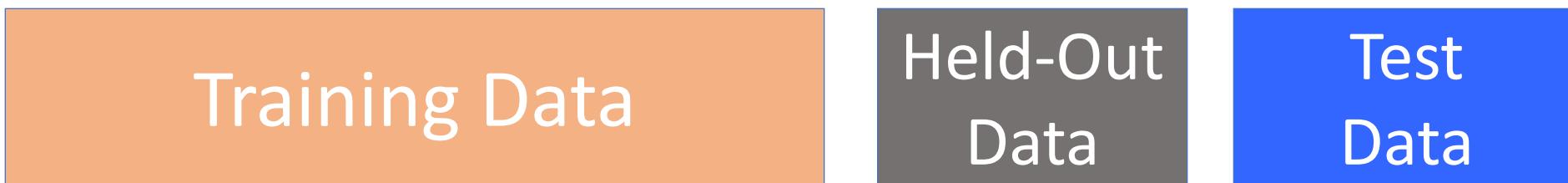
$$\sum_i \lambda_i = 1$$

Lambdas conditional on context:

$$\begin{aligned}\hat{P}(w_n | w_{n-2} w_{n-1}) &= \lambda_1(w_{n-2}^{n-1}) P(w_n | w_{n-2} w_{n-1}) \\ &\quad + \lambda_2(w_{n-2}^{n-1}) P(w_n | w_{n-1}) \\ &\quad + \lambda_3(w_{n-2}^{n-1}) P(w_n)\end{aligned}$$

How to set the lambdas?

- Use a **held-out** corpus



- Choose λ s to maximize the probability of held-out data:
 - Fix the N-gram probabilities (on the training data)
 - Then search for λ s that give largest probability to held-out set:

$$\log P(w_1 \dots w_n | M(\lambda_1 \dots \lambda_k)) = \sum_i \log P_{M(\lambda_1 \dots \lambda_k)}(w_i | w_{i-1})$$

Unknown words: Open versus closed vocabulary tasks

- If we know all the words in advanced
 - Vocabulary V is fixed
 - Closed vocabulary task
- Often we don't know this
 - **Out Of Vocabulary** = OOV words
 - Open vocabulary task
- Instead: create an unknown word token <UNK>
 - Training of <UNK> probabilities
 - Create a fixed lexicon L of size V
 - At text normalization phase, any training word not in L changed to <UNK>
 - Now we train its probabilities like a normal word
 - At decoding time
 - If text input: Use UNK probabilities for any word not in training

Huge web-scale n-grams

- How to deal with, e.g., Google N-gram corpus
- Pruning
 - Only store N-grams with count > threshold.
 - Remove singletons of higher-order n-grams
 - Entropy-based pruning
- Efficiency
 - Efficient data structures like tries
 - Bloom filters: approximate language models
 - Store words as indexes, not strings
 - Use Huffman coding to fit large numbers of words into two bytes
 - Quantize probabilities (4-8 bits instead of 8-byte float)

Smoothing for Web-scale N-grams

- “Stupid backoff” (Brants *et al.* 2007)
- No discounting, just use relative frequencies

$$S(w_i | w_{i-k+1}^{i-1}) = \begin{cases} \frac{\text{count}(w_{i-k+1}^i)}{\text{count}(w_{i-k+1}^{i-1})} & \text{if } \text{count}(w_{i-k+1}^i) > 0 \\ 0.4S(w_i | w_{i-k+2}^{i-1}) & \text{otherwise} \end{cases}$$

$$S(w_i) = \frac{\text{count}(w_i)}{N}$$

N-gram Smoothing Summary

- Add-1 smoothing:
 - OK for text categorization, not for language modeling
- The most commonly used method:
 - Extended Interpolated Kneser-Ney
- For very large N-grams like the Web:
 - Stupid backoff

Advanced Language Modeling

- Discriminative models:
 - choose n-gram weights to improve a task, not to fit the training set
- Parsing-based models
- Caching Models
 - Recently used words are more likely to appear

$$P_{CACHE}(w | history) = \lambda P(w_i | w_{i-2}w_{i-1}) + (1 - \lambda) \frac{c(w \in history)}{|history|}$$

Language Modeling

Advanced:

Kneser-Ney Smoothing

Absolute discounting: just subtract a little from each count

- Suppose we wanted to subtract a little from a count of 4 to save probability mass for the zeros
- How much to subtract ?
- Church and Gale (1991)'s clever idea
- Divide up 22 million words of AP Newswire
 - Training and held-out set
 - for each bigram in the training set
 - see the actual count in the held-out set!
- It sure looks like $c^* = (c - .75)$

Bigram count in training	Bigram count in heldout set
0	.0000270
1	0.448
2	1.25
3	2.24
4	3.23
5	4.21
6	5.23
7	6.21
8	7.21
9	8.26

Absolute Discounting Interpolation

- Save ourselves some time and just subtract 0.75 (or some d)!

$$P_{\text{AbsoluteDiscounting}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) - d}{c(w_{i-1})} + \lambda(w_{i-1}) P(w)$$

discounted bigram Interpolation weight

- (Maybe keeping a couple extra values of d for counts 1 and 2)
- But should we really just use the regular unigram $P(w)$?

Kneser-Ney Smoothing I

- Better estimate for probabilities of lower-order unigrams!
 - Shannon game: *I can't see without my reading _____* *Franisco*?
 - “Francisco” is more common than “glasses”
 - ... but “Francisco” always follows “San”
- The unigram is useful exactly when we haven’t seen this bigram!
- Instead of $P(w)$: “How likely is w ”
- $P_{\text{continuation}}(w)$: “How likely is w to appear as a novel continuation?
 - For each word, count the number of bigram types it completes
 - Every bigram type was a novel continuation the first time it was seen

$$P_{\text{CONTINUATION}}(w) \propto |\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

Kneser-Ney Smoothing II

- How many times does w appear as a novel continuation:

$$P_{CONTINUATION}(w) \propto |\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- Normalized by the total number of word bigram types

$$|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}|$$

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{|\{(w_{j-1}, w_j) : c(w_{j-1}, w_j) > 0\}|}$$

Kneser-Ney Smoothing III

- Alternative metaphor: The number of # of word types seen to precede w

$$|\{w_{i-1} : c(w_{i-1}, w) > 0\}|$$

- normalized by the # of words preceding all words:

$$P_{CONTINUATION}(w) = \frac{|\{w_{i-1} : c(w_{i-1}, w) > 0\}|}{\sum |\{w'_{i-1} : c(w'_{i-1}, w') > 0\}|}$$

- A frequent word (Francisco) occurring in only one context (San) will have a low continuation probability

Kneser-Ney Smoothing IV

$$P_{KN}(w_i | w_{i-1}) = \frac{\max(c(w_{i-1}, w_i) - d, 0)}{c(w_{i-1})} + \lambda(w_{i-1}) P_{CONTINUATION}(w_i)$$

λ is a normalizing constant; the probability mass we've discounted

$$\lambda(w_{i-1}) = \frac{d}{c(w_{i-1})} |\{w : c(w_{i-1}, w) > 0\}|$$

the normalized discount

The number of word types that can follow w_{i-1}
= # of word types we discounted
= # of times we applied normalized discount

Kneser-Ney Smoothing: Recursive formulation

$$P_{KN}(w_i | w_{i-n+1}^{i-1}) = \frac{\max(c_{KN}(w_{i-n+1}^i) - d, 0)}{c_{KN}(w_{i-n+1}^{i-1})} + \lambda(w_{i-n+1}^{i-1}) P_{KN}(w_i | w_{i-n+2}^{i-1})$$

$$c_{KN}(\bullet) = \begin{cases} \text{count}(\bullet) & \text{for the highest order} \\ \text{continuation count}(\bullet) & \text{for lower order} \end{cases}$$

Continuation count = Number of unique single word contexts for \bullet

Vector Space Semantics

Read Chapter 6 in the draft 3rd edition of Jurafsky and Martin

What does ongchoi mean?

Suppose you see these sentences:

Ong choi is delicious **sautéed with garlic**.

Ong choi is superb **over rice**

Ong choi **leaves** with salty sauces

And you've also seen these:

...spinach **sautéed with garlic over rice**

Chard stems and **leaves** are **delicious**

Collard greens and other **salty** leafy greens

Conclusion:

Ongchoi is a leafy green like spinach, chard, or collard greens

Ong choi: *Ipomoea aquatica* "Water Spinach"



Yamaguchi, Wikimedia Commons, public domain

Distributional Hypothesis

If we consider **optometrist** and **eye-doctor** we find that, as our corpus of utterances grows, these two occur in almost the same environments. In contrast, there are many sentence environments in which **optometrist** occurs but **lawyer** does not...

It is a question of the relative frequency of such environments, and of what we will obtain if we ask an informant to substitute any word he wishes for **oculist** (not asking what words have the same meaning).

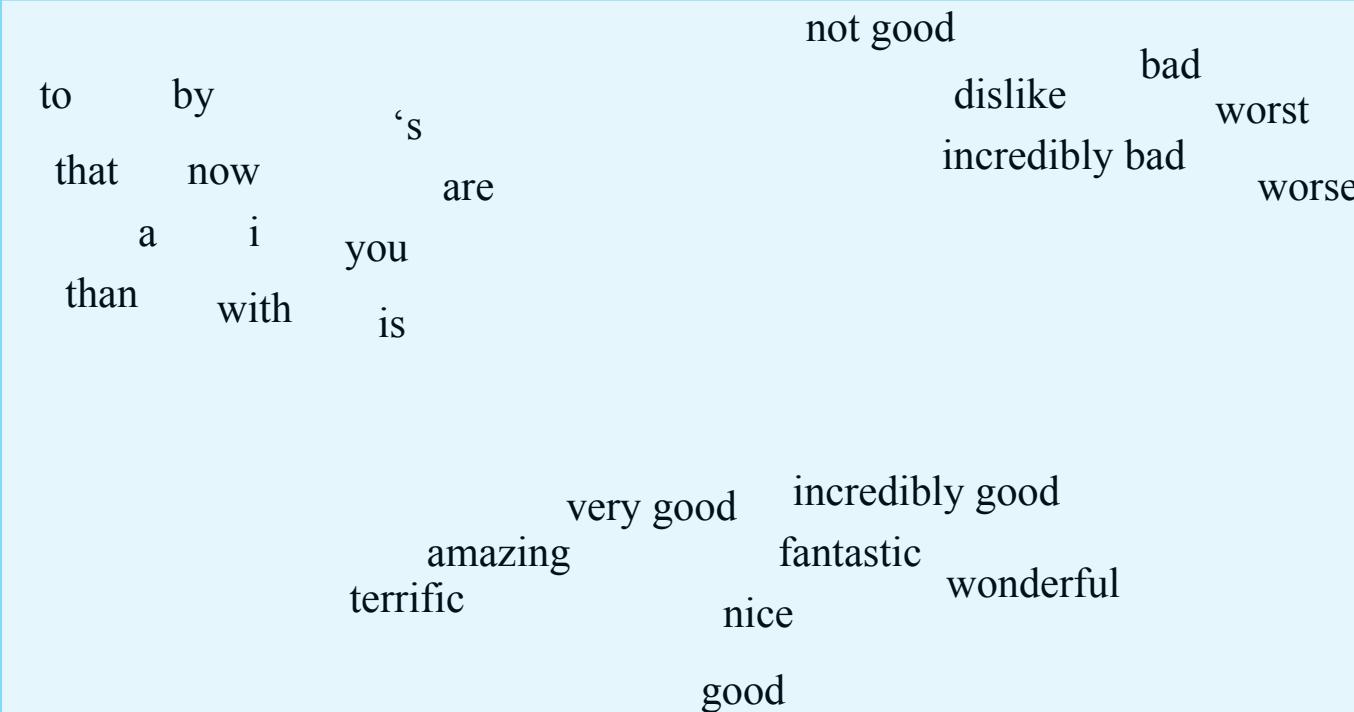
These and similar tests all measure the probability of particular environments occurring with particular elements... If A and B have almost identical environments we say that they are synonyms.

—Zellig Harris (1954)



We'll build a new representation of words that encodes their similarity

- Each word = a vector
- Similar words are "nearby in space"



We define a word as a vector

- Called an "embedding" because it's embedded into a space
- The standard way to represent meaning in NLP
- Fine-grained model of meaning for similarity
 - NLP tasks like sentiment analysis
 - With words, requires **same** word to be in training and test
 - With embeddings: ok if **similar** words occurred!!!
 - Question answering, conversational agents, etc

We'll introduce 2 kinds of embeddings

- Tf-idf
 - A common baseline model
 - Sparse vectors
 - Words are represented by a simple function of the counts of nearby words
- Word2vec
 - Dense vectors
 - Representation is created by training a classifier to distinguish nearby and far-away words