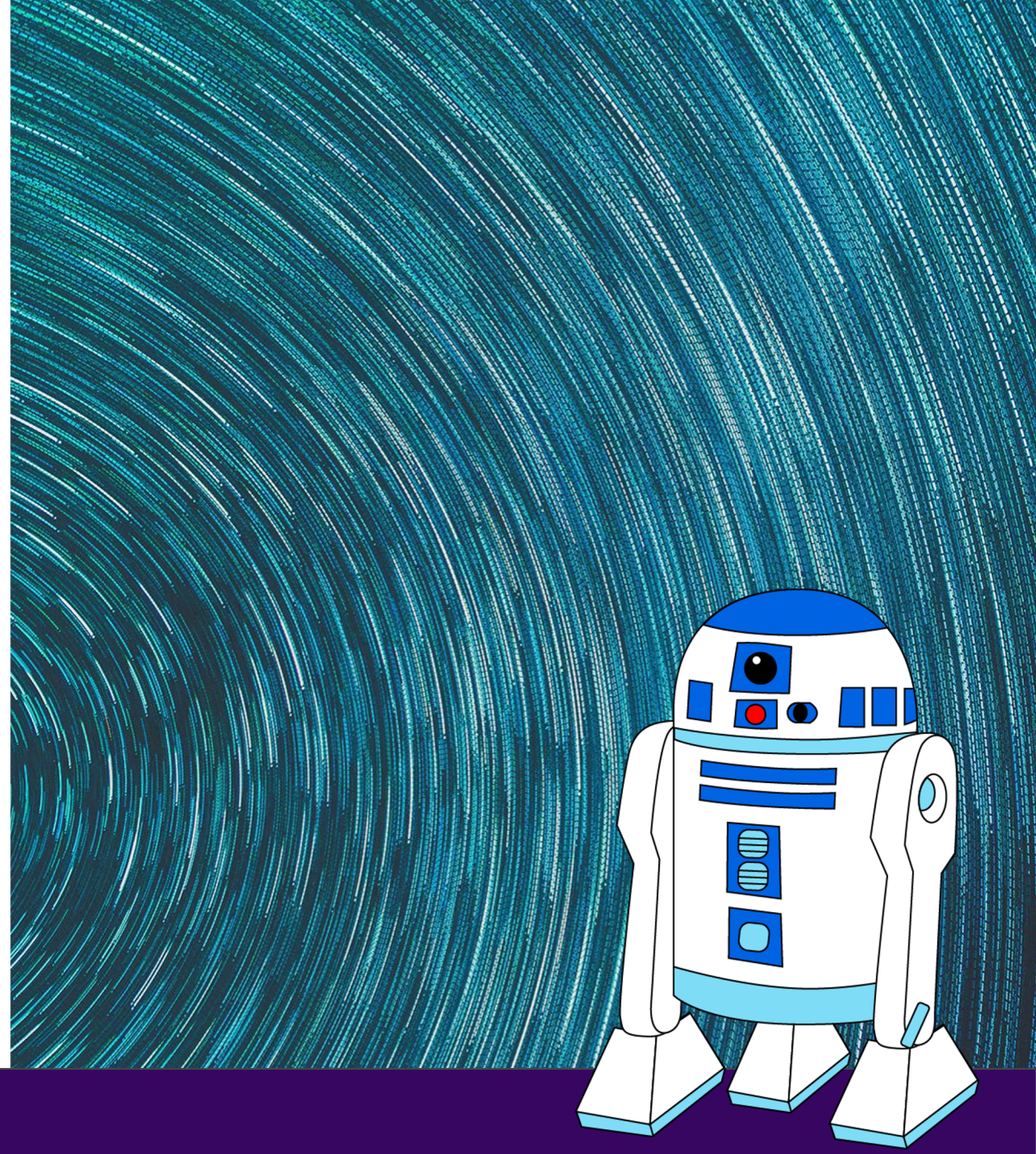
CIS 521:
ARTIFICIAL INTELLIGENCE

Expectimax and Utilities

Professor Chris Callison-Burch

Many of today's slides are courtesy of Dan Klein and Pieter Abbeel of University of California, Berkeley



Penn Engineering
University of Pennsylvania

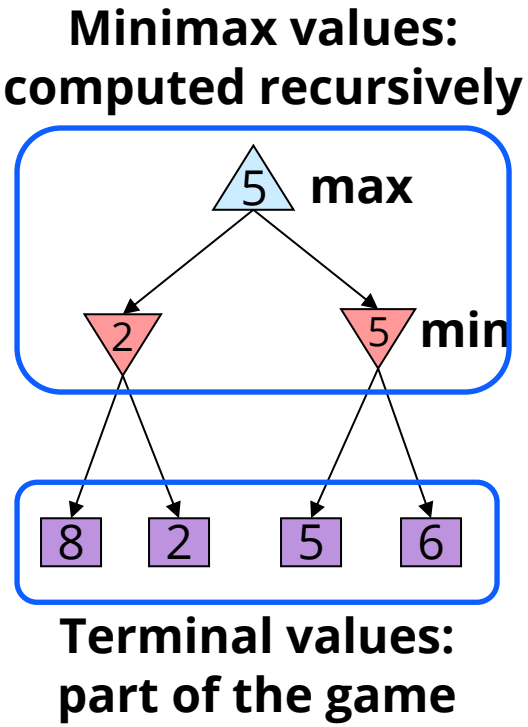
Review: Adversarial Search (Minimax)

- Minimax search:

  A state-space search tree

  Players alternate turns

  Compute each node's minimax value: the best achievable utility against a rational (optimal) adversary



Review: Minimax Implementation

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def max-value(state):
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       return the state's utility
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   for each successor of state:
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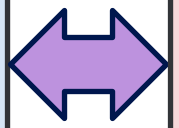

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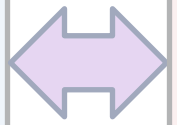

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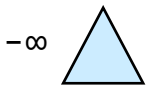


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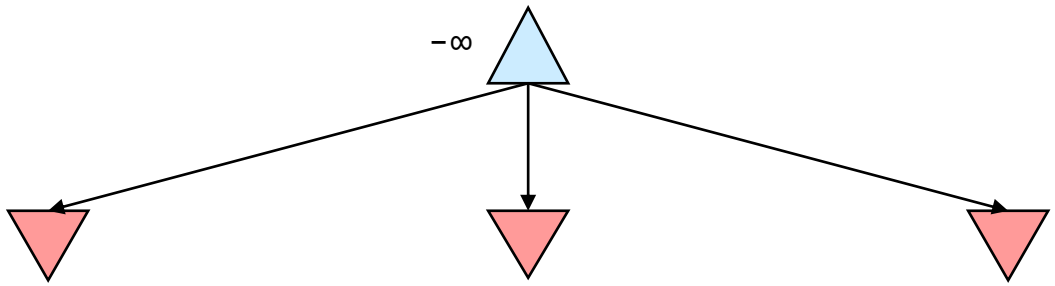


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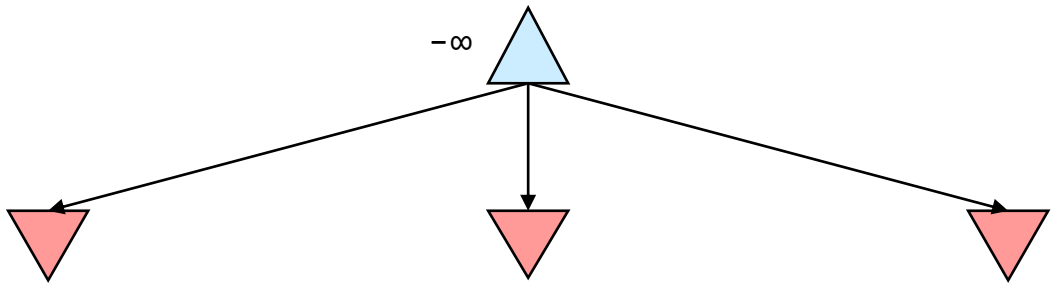


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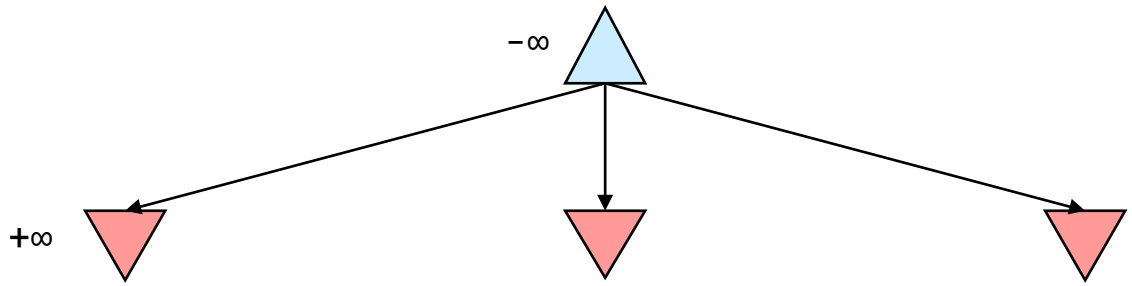


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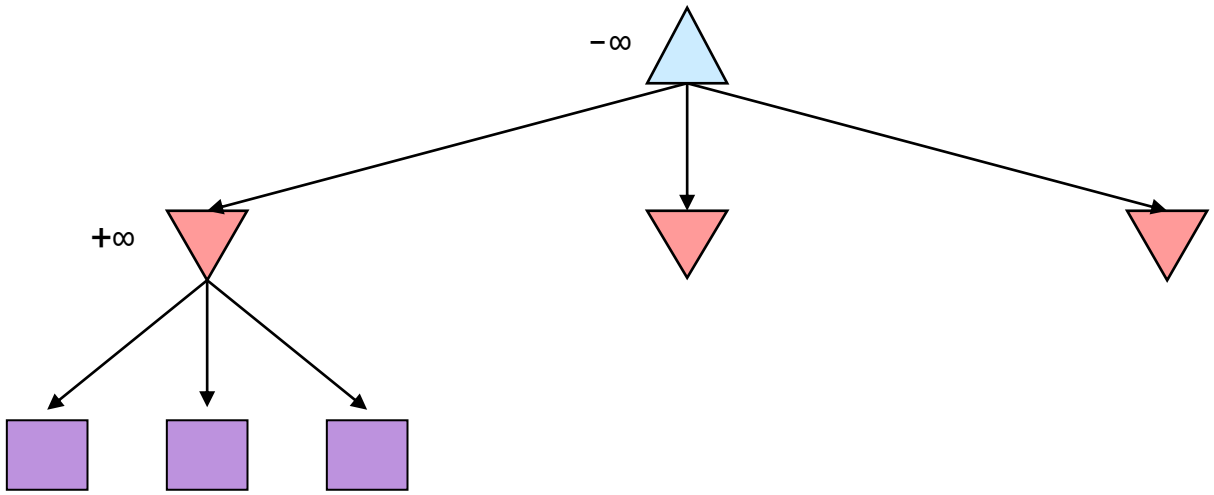


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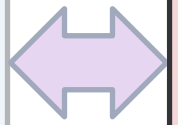

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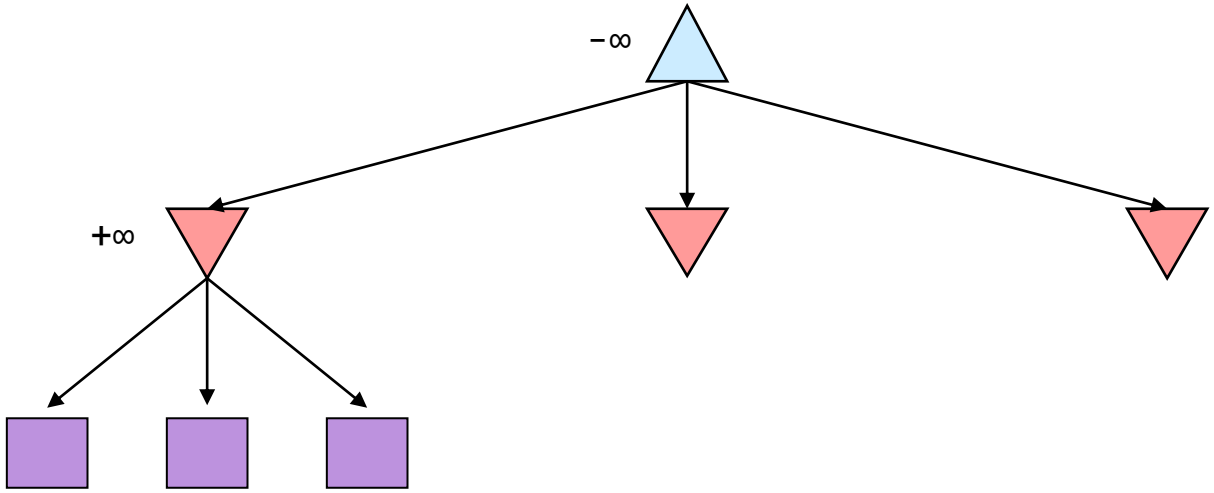


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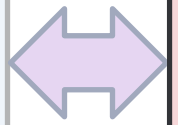

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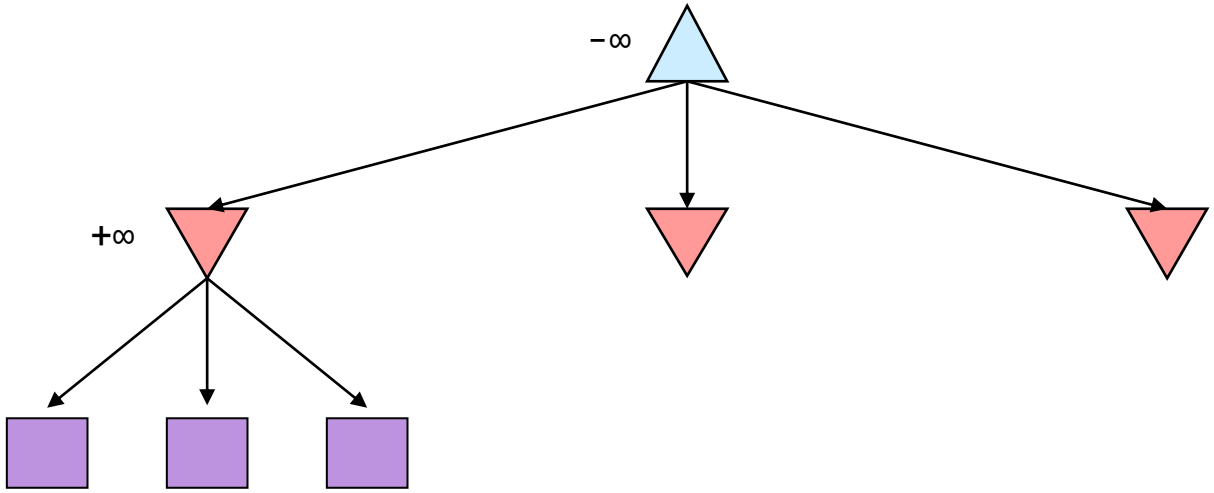


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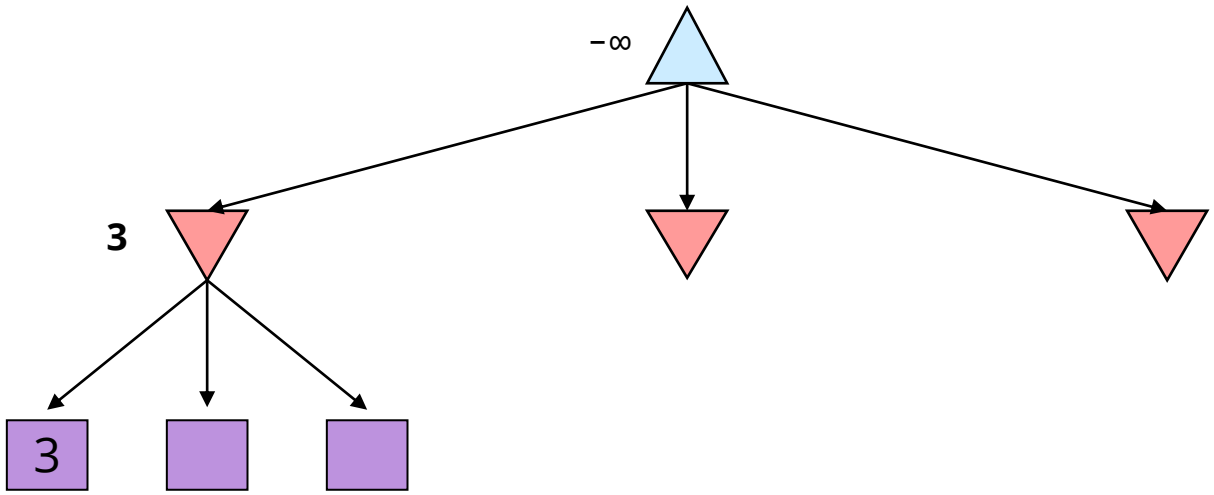


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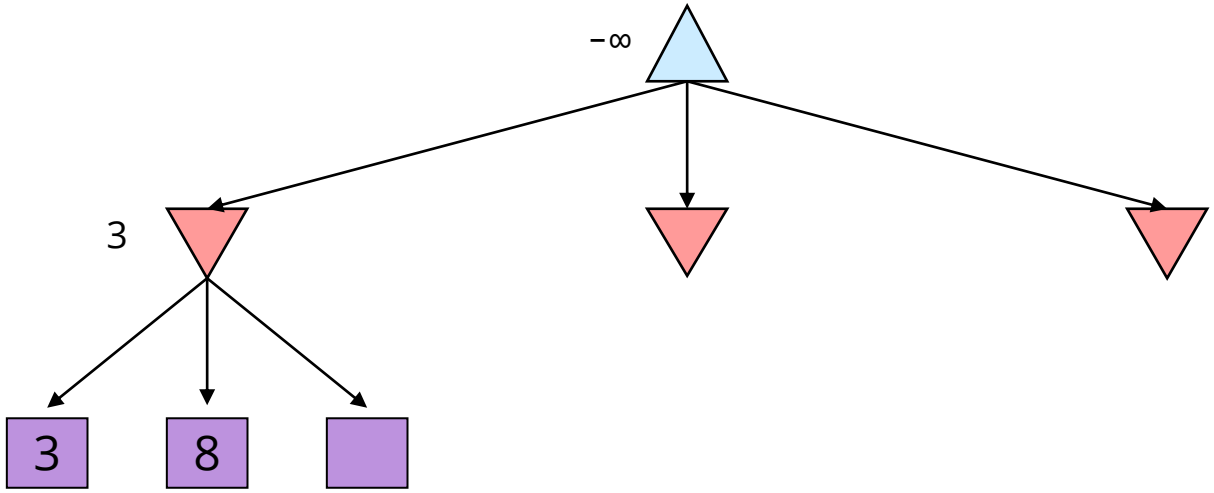


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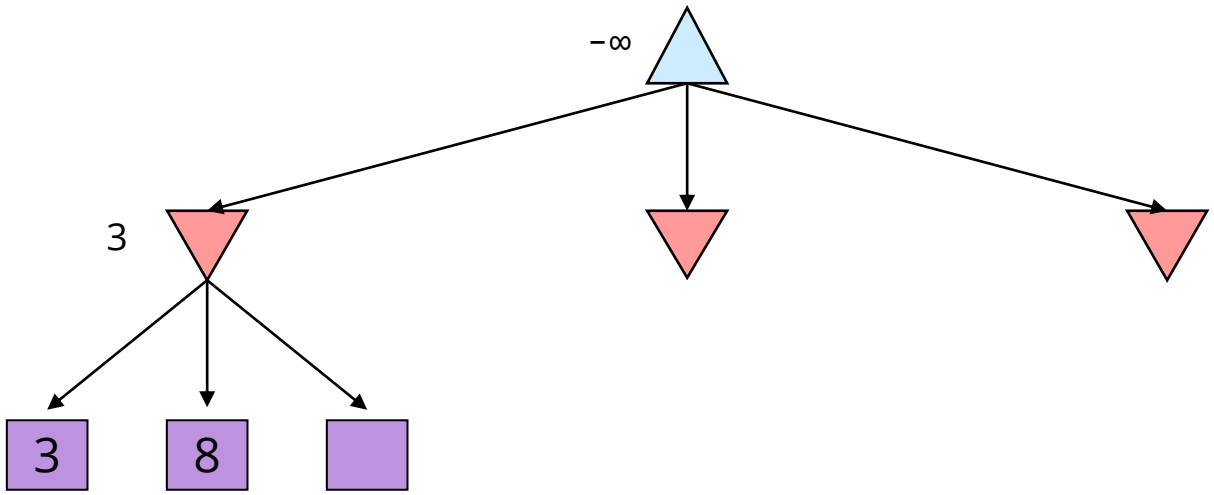


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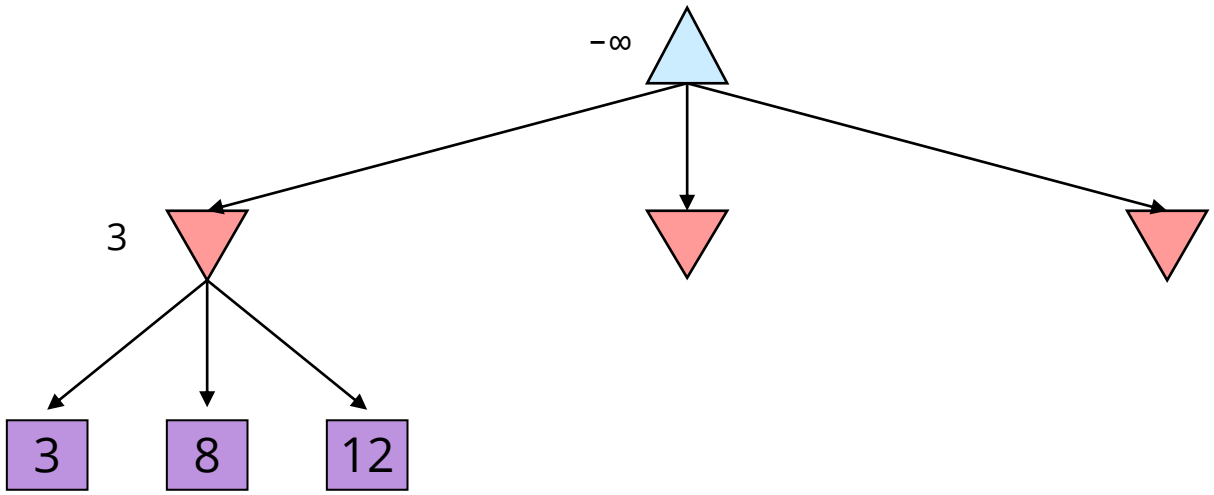


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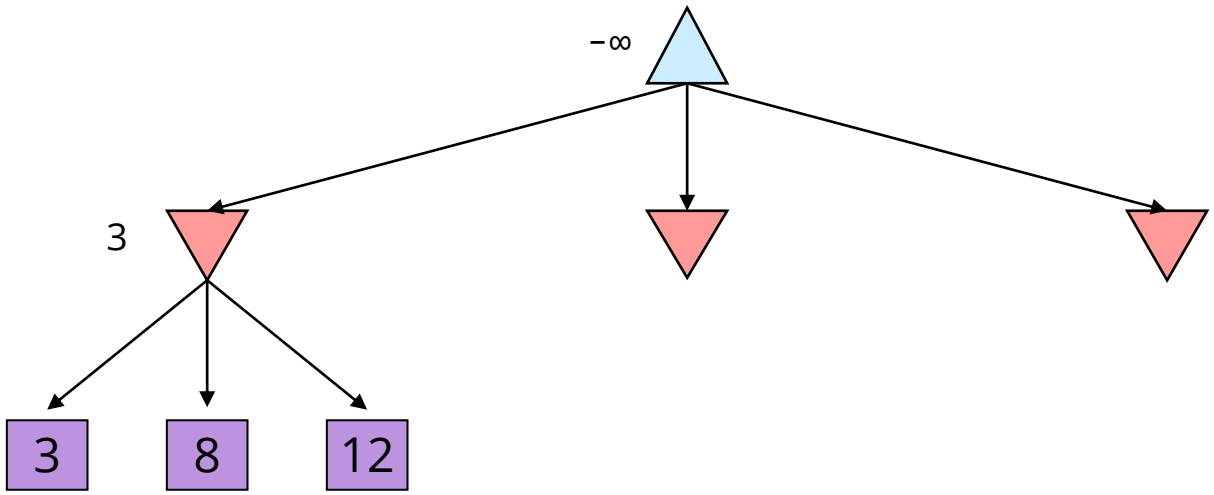


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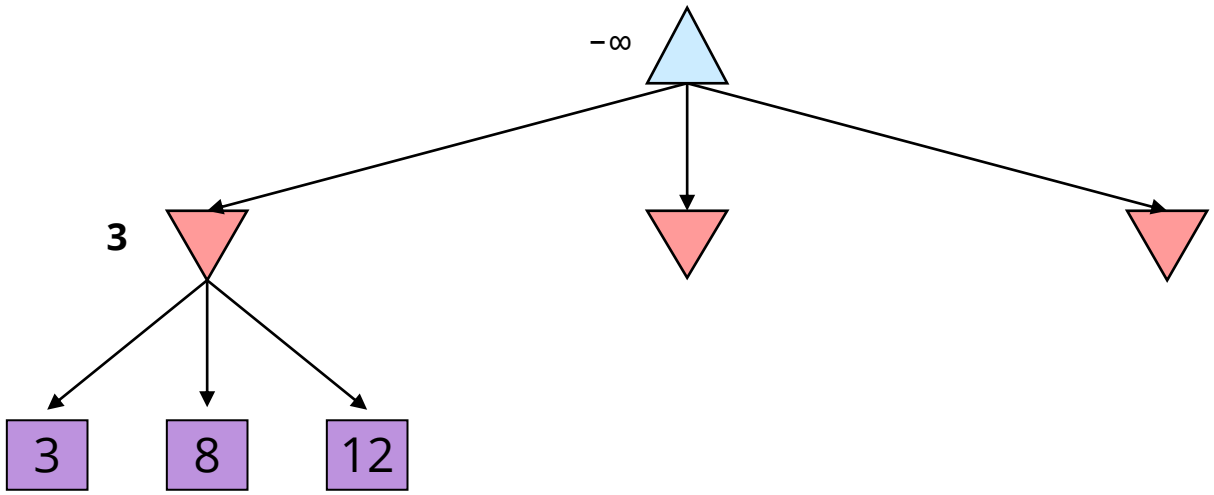


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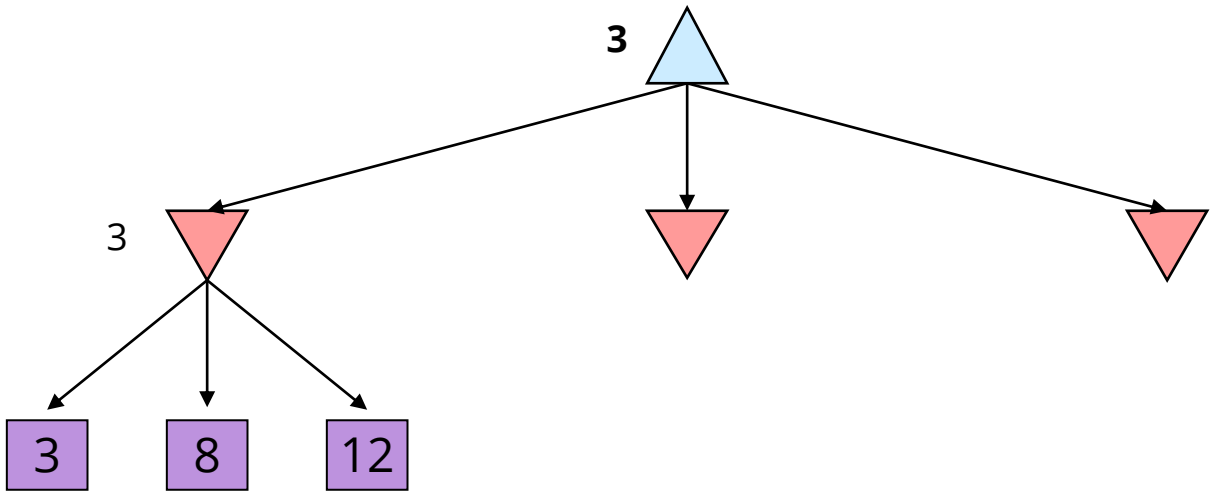


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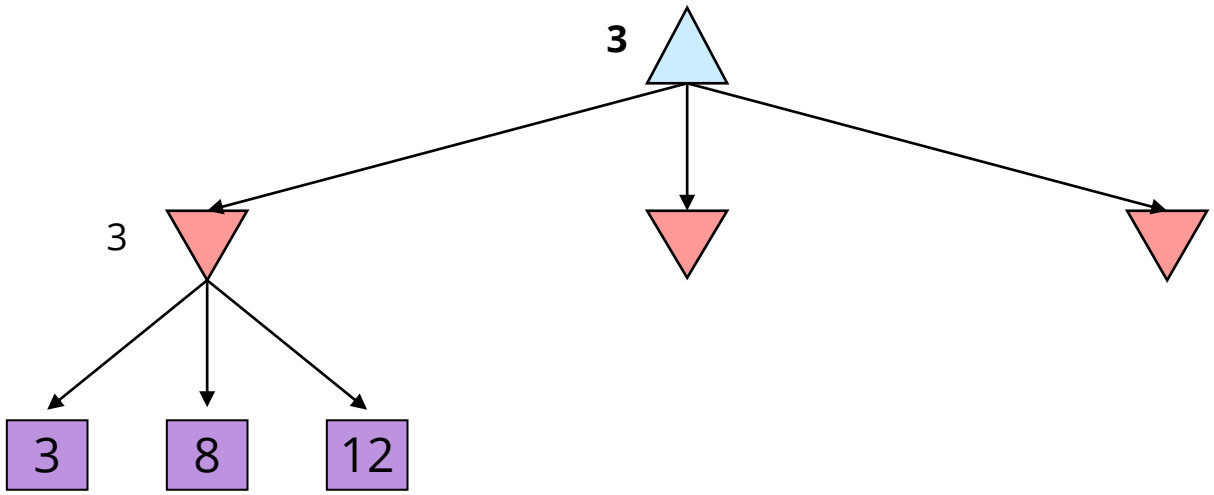


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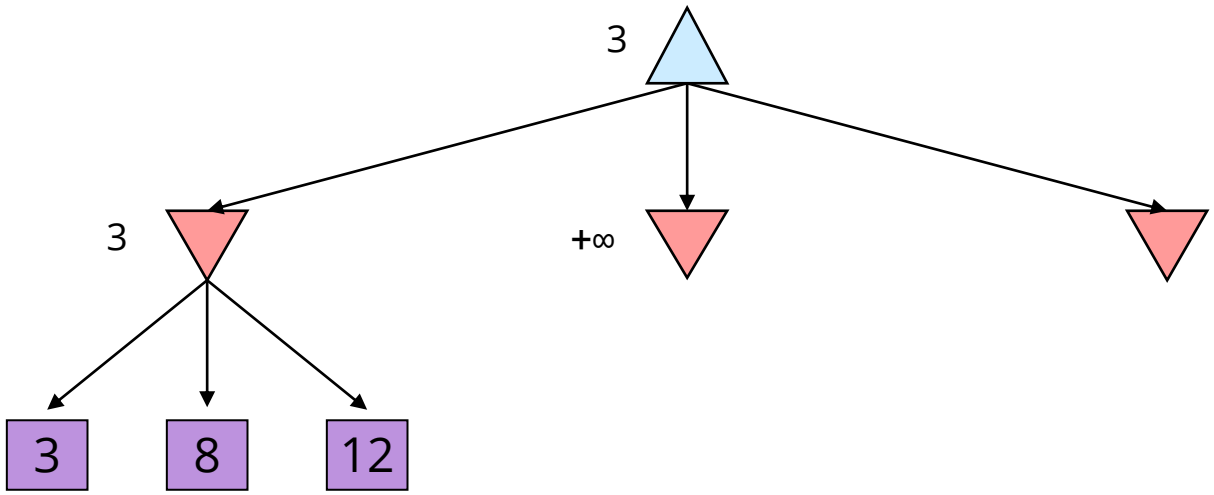


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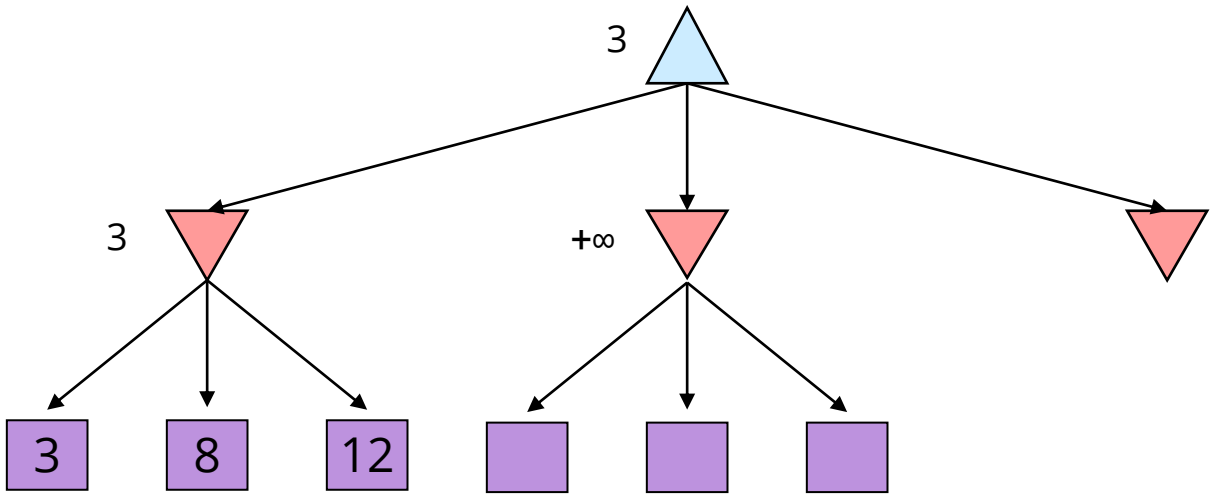


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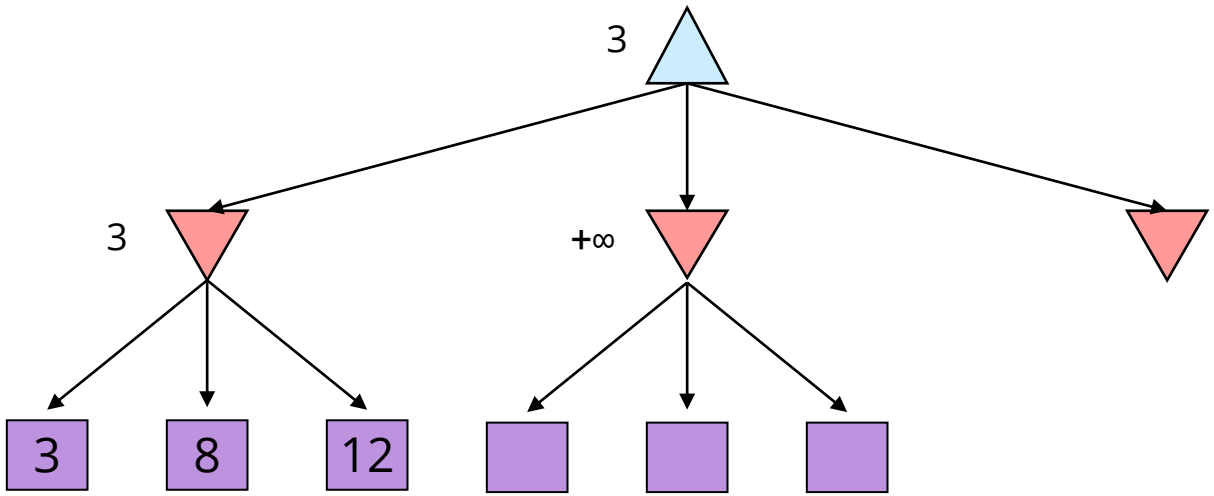


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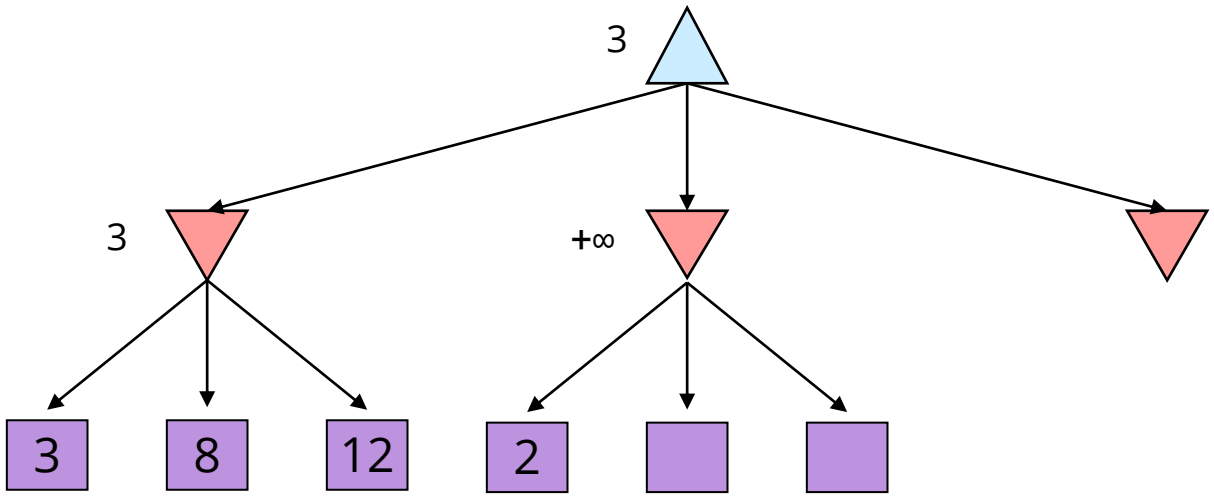


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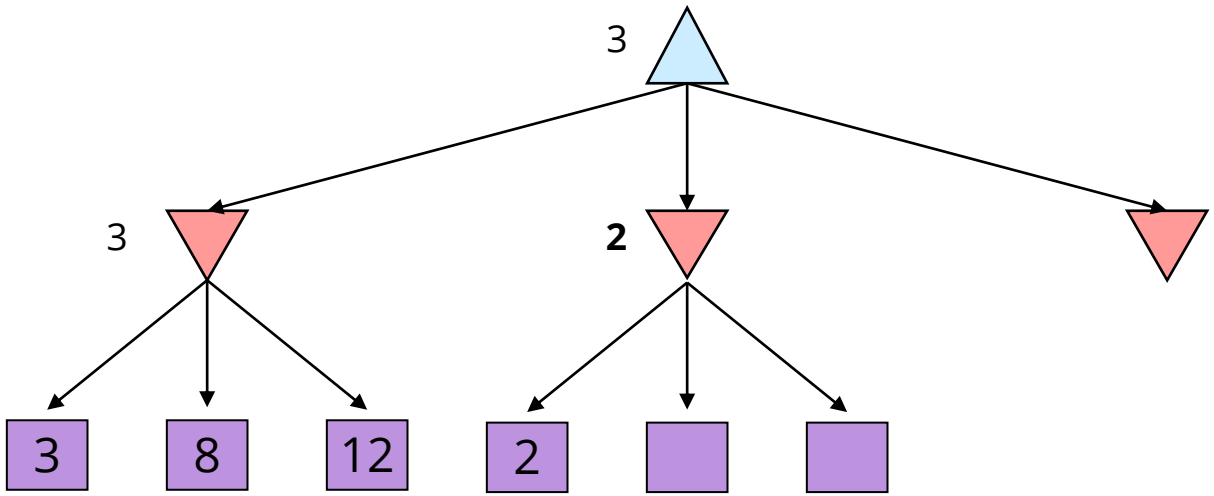
$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Example

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   if the state is a terminal state:
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   initialize v = -∞
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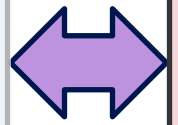

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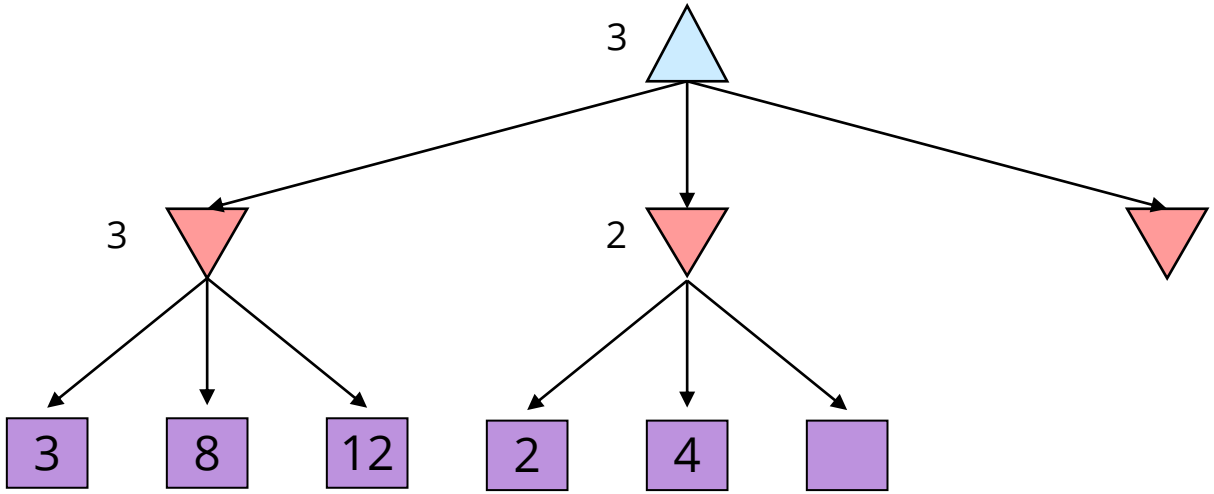


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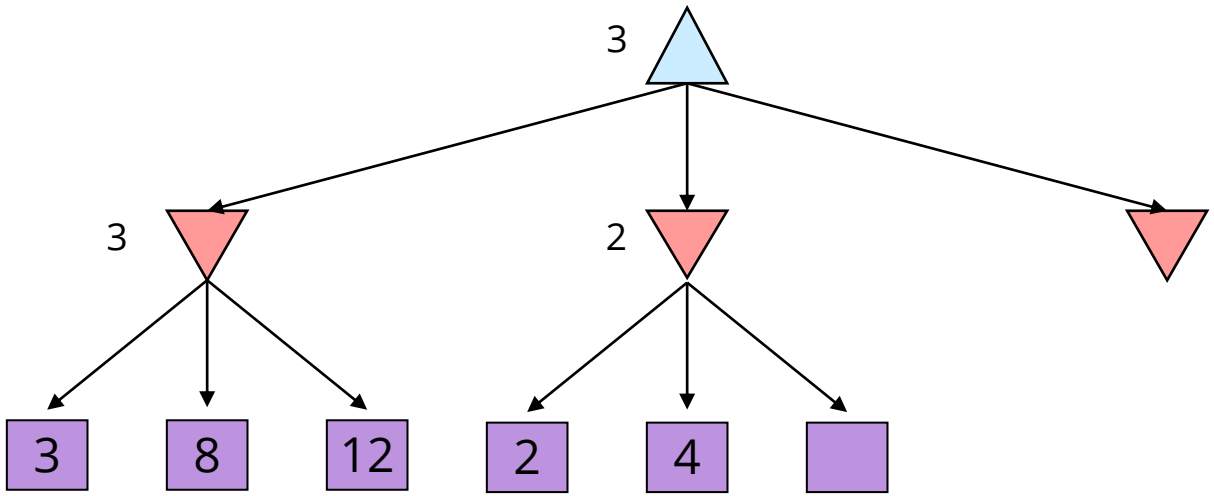


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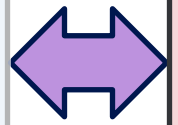

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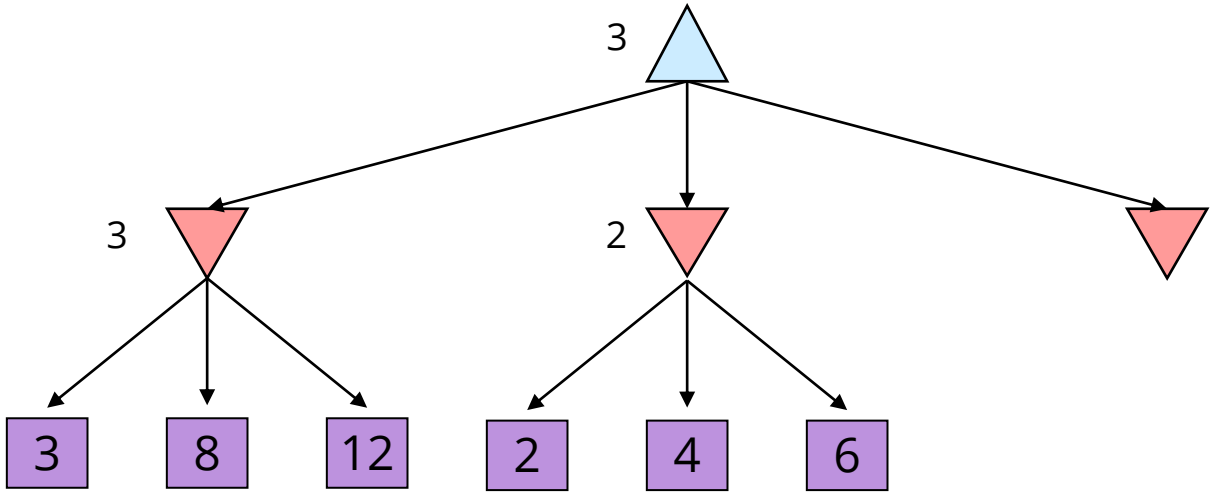


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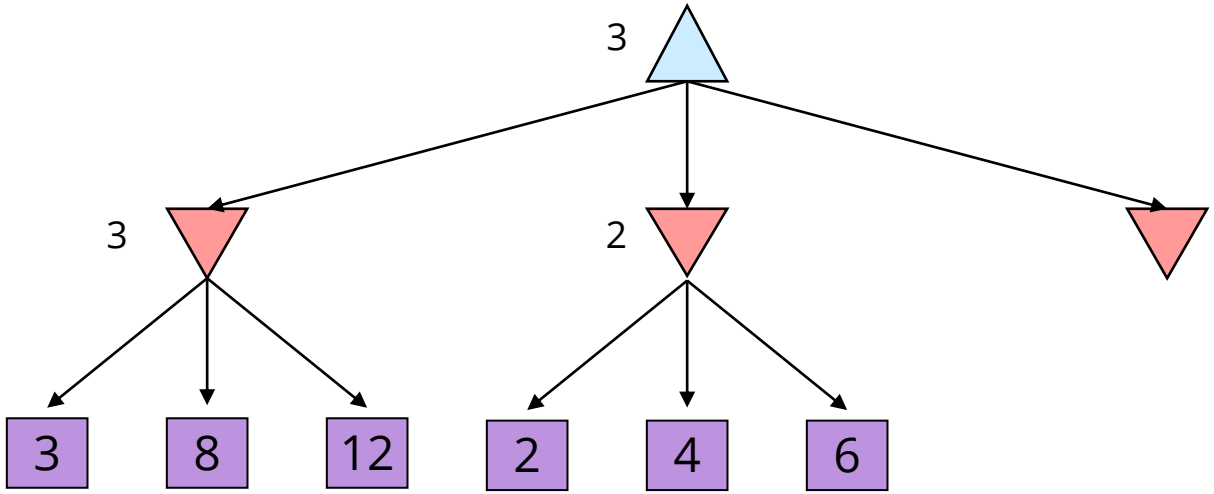


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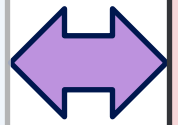

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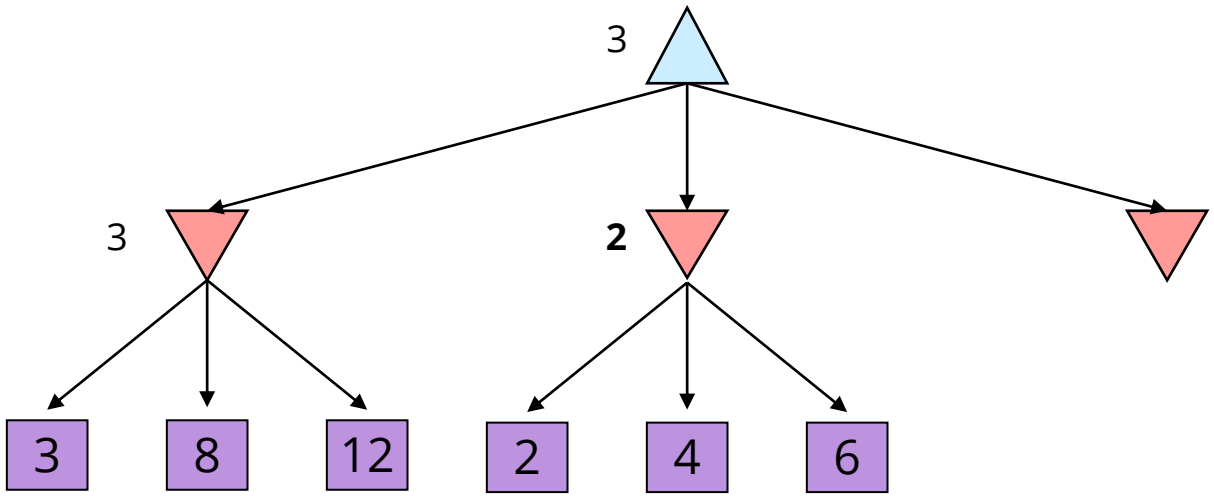


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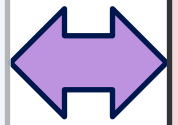

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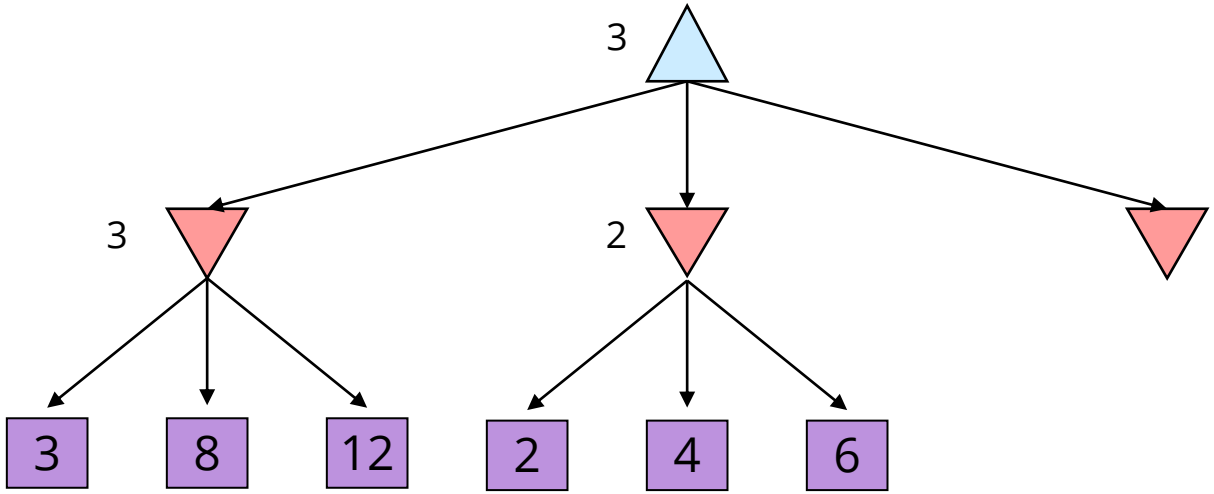


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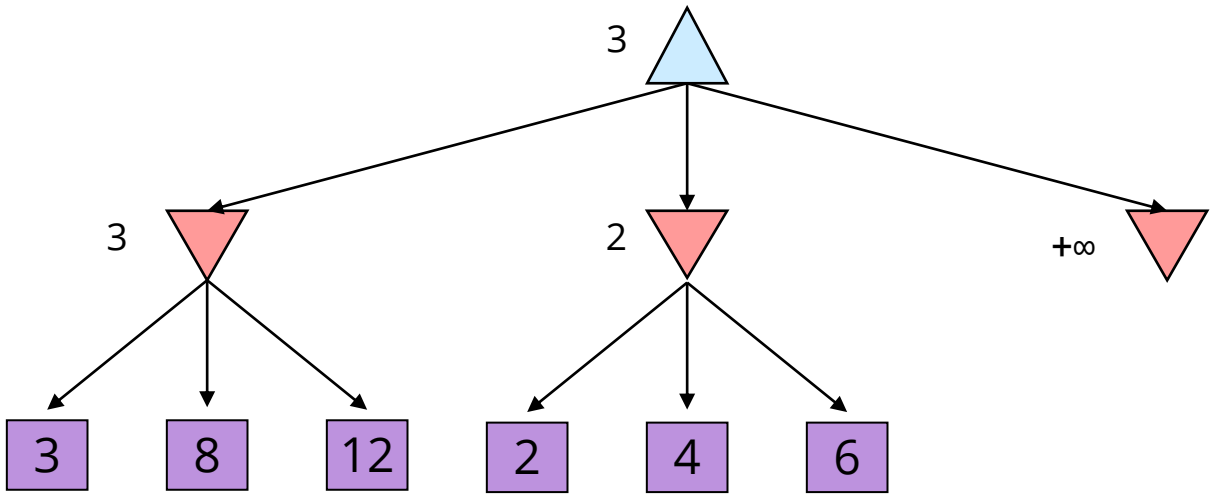


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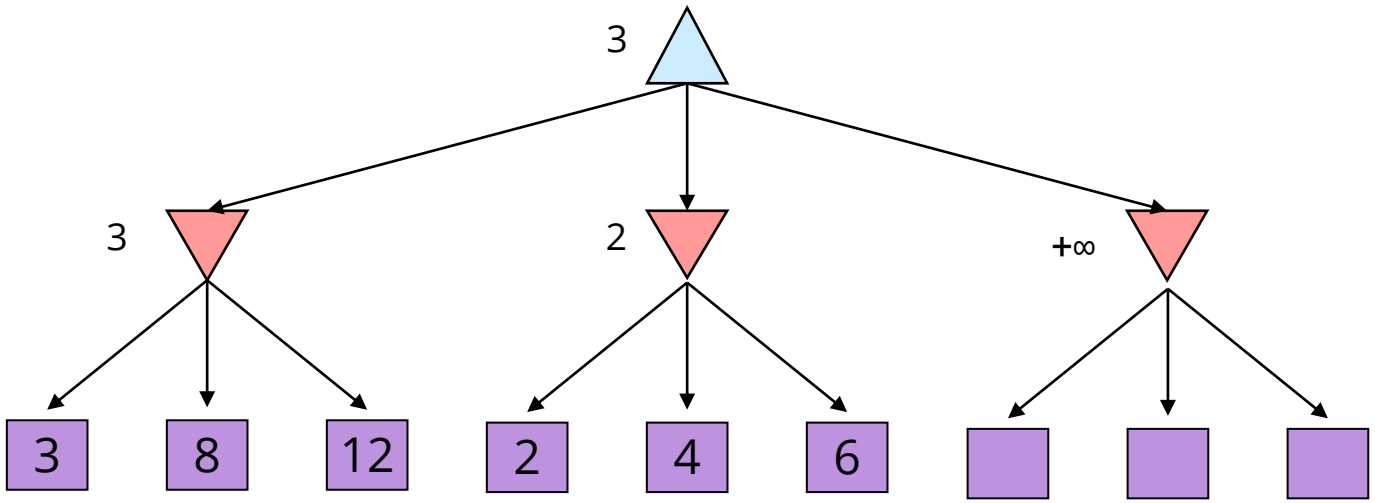


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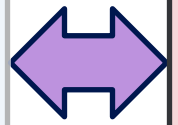

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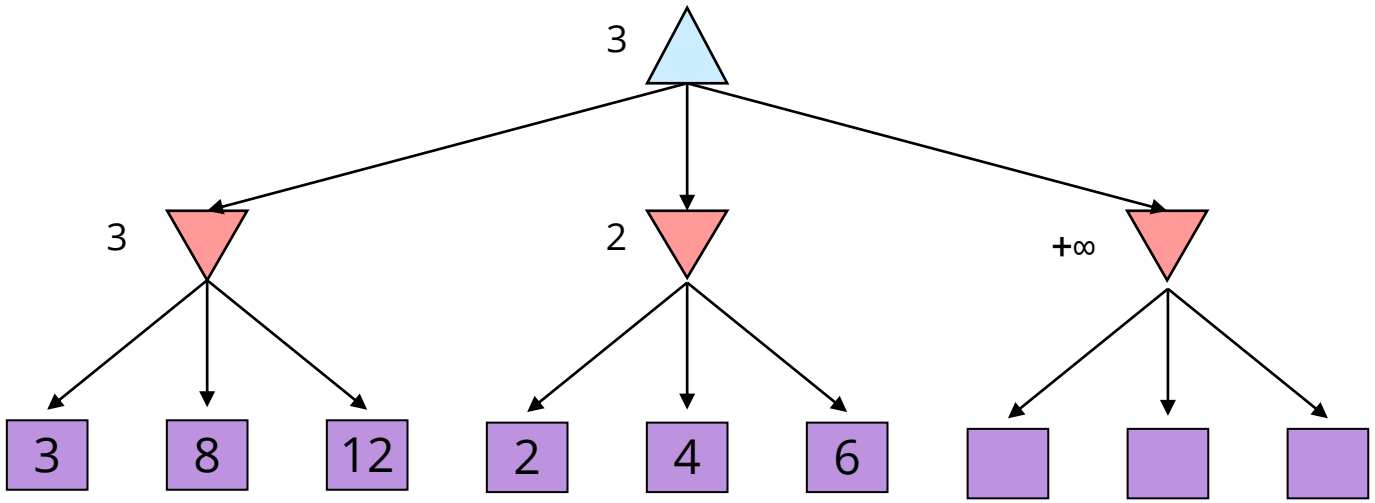


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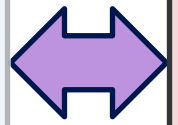

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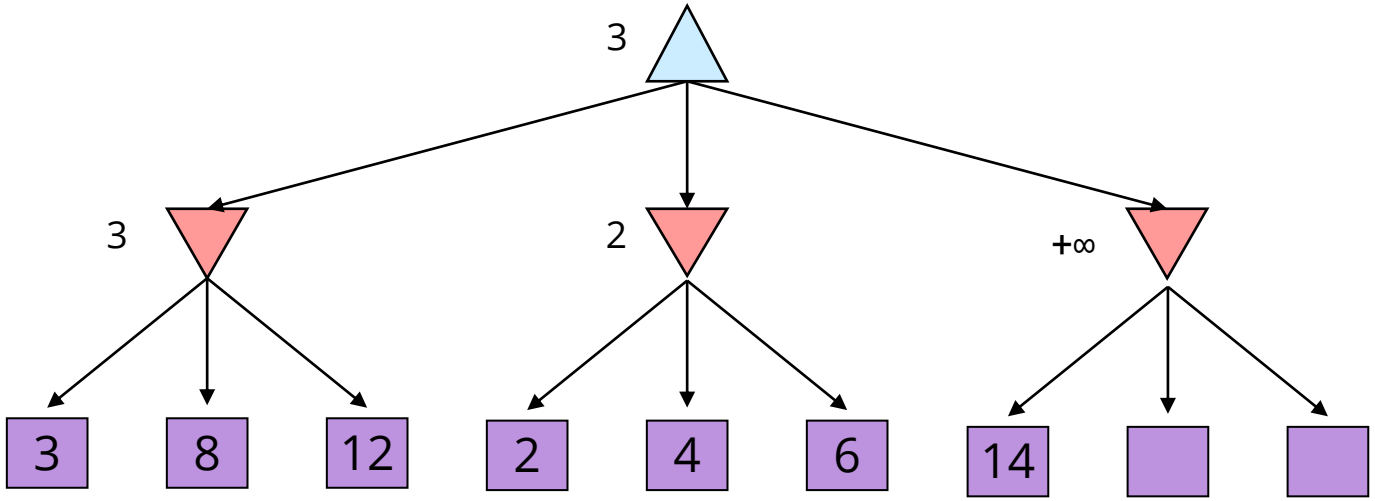


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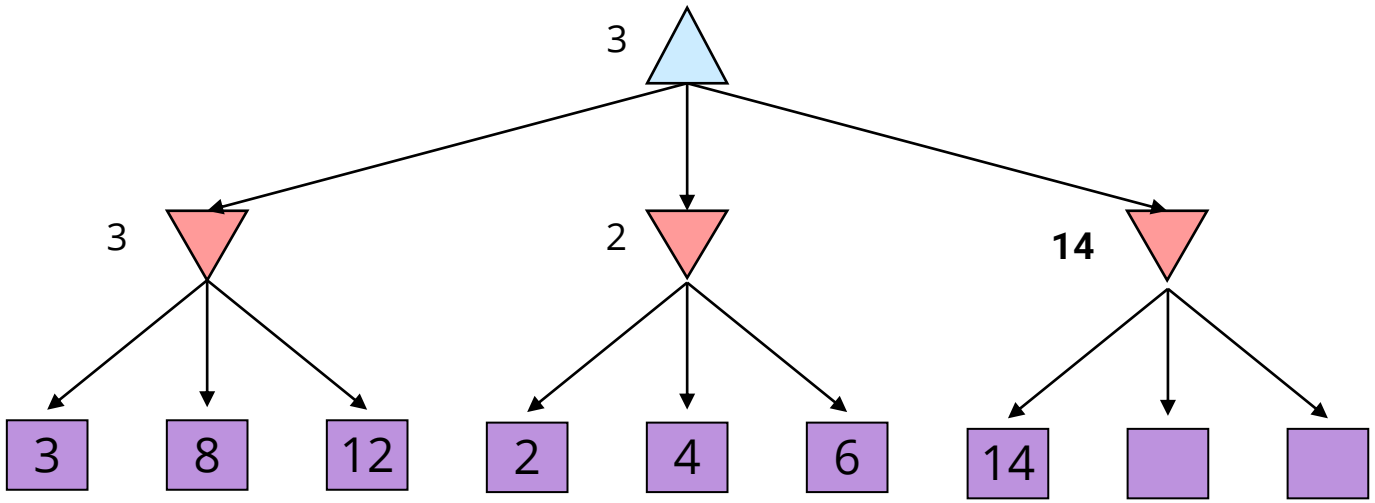


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   initialize v = -∞
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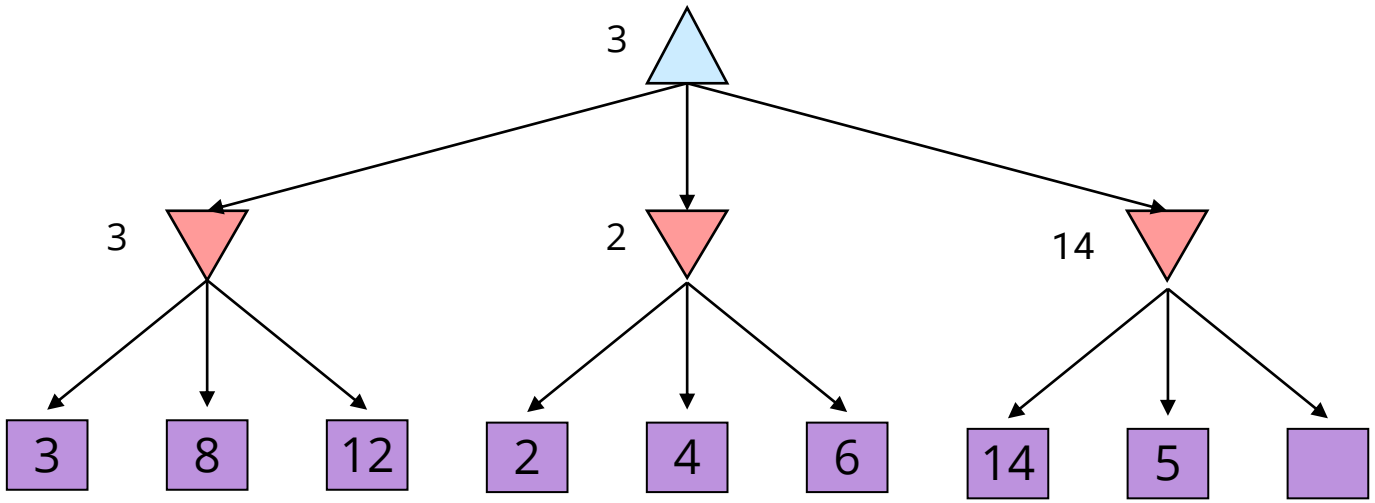


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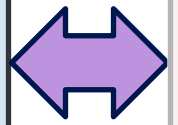

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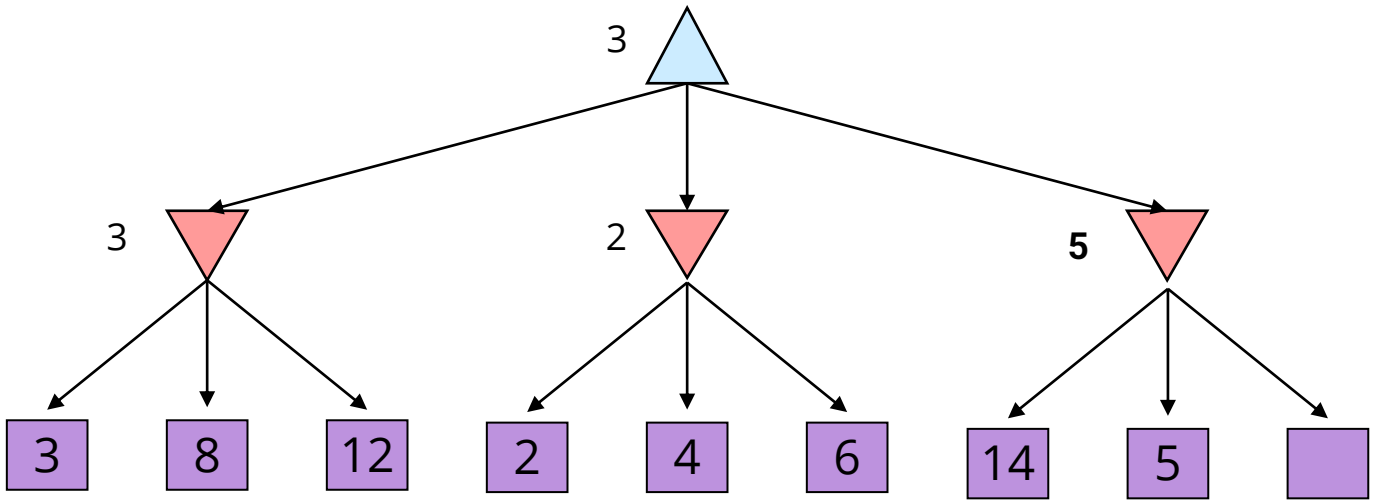


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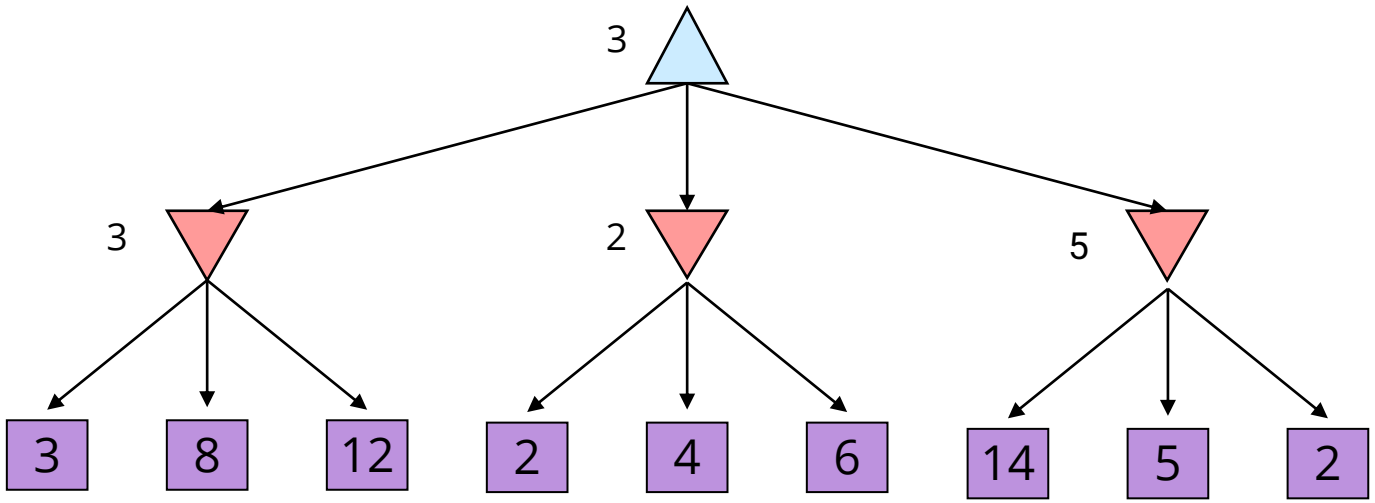


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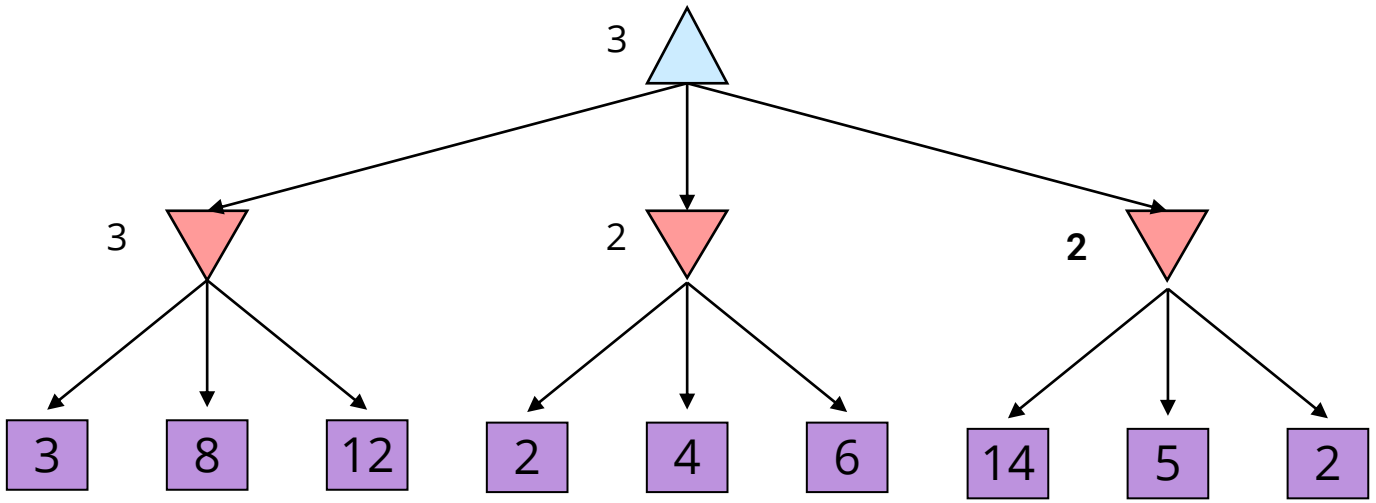


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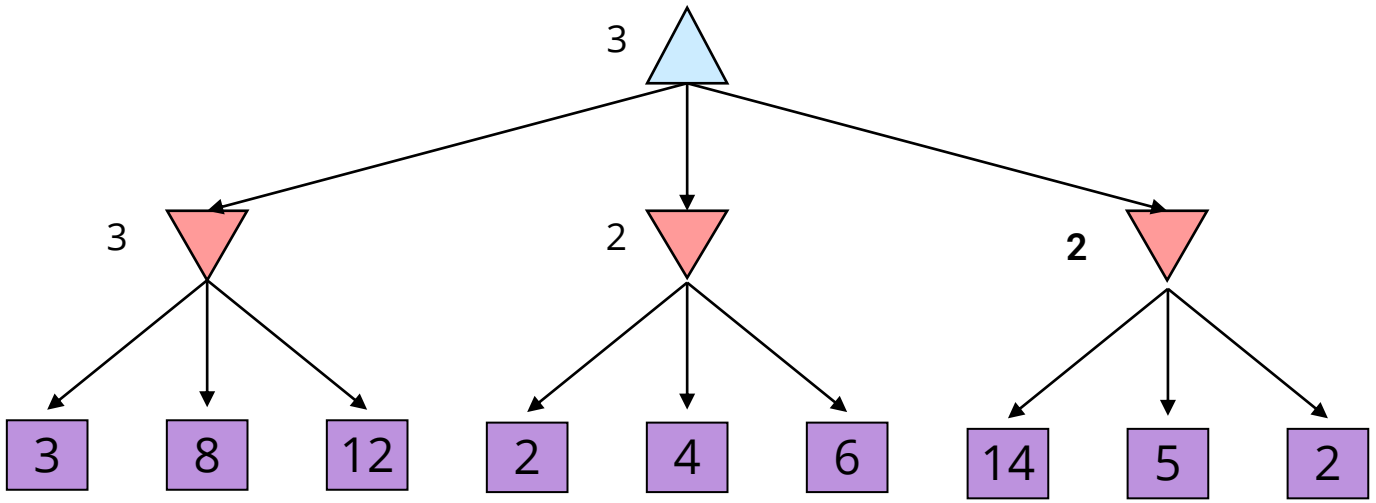


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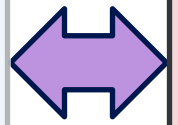

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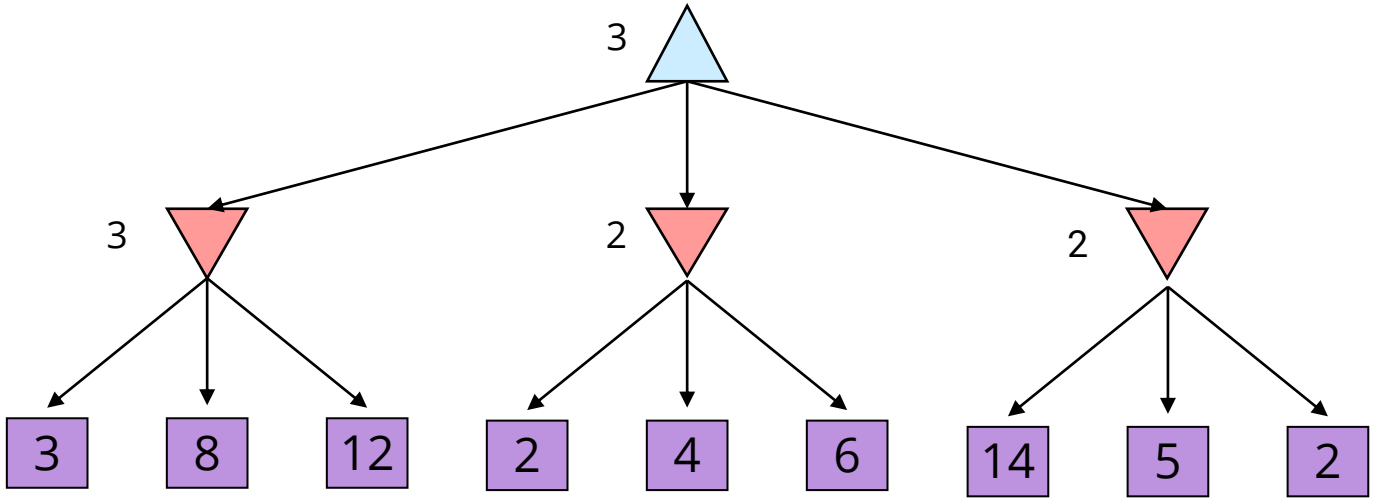


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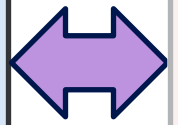

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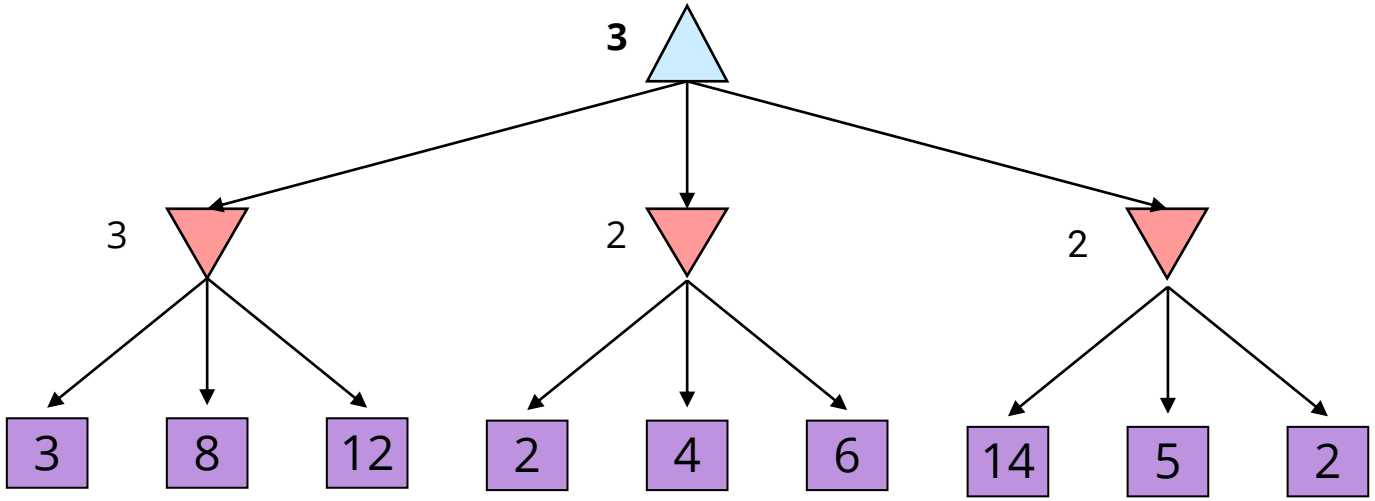


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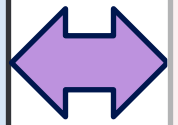
   return v
```

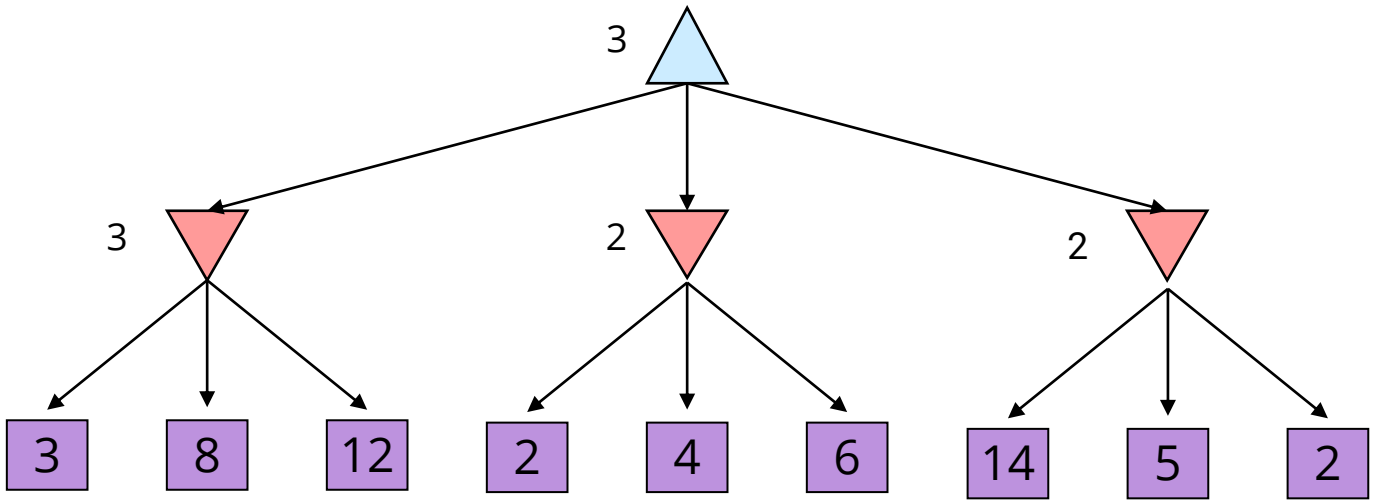


```
def min-value(state):
   if the state is a terminal state:
      return the state's utility
   initialize v = +∞
   for each successor of state:
      v = min(v, max-value(successor))
   return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Minimax Example




```
def max-value(state):
  if the state is a terminal state:
     return the state's utility
  initialize v = -∞
  for each successor of state:
    v = max(v, min-value(successor))
  return v
```

```
def min-value(state):
  if the state is a terminal state:
     return the state's utility
  initialize v = +∞
  for each successor of state:
    v = min(v, max-value(successor))
  return v
```

$$V(s) = \max_{s' \in \text{successors}(s)} V(s')$$

$$V(s') = \min_{s \in \text{successors}(s')} V(s)$$

Alpha-Beta Pruning

- During Minimax, keep track of two additional values:

  α: MAX's current *lower* bound on MAX's outcome

  β: MIN's current *upper* bound on MIN's outcome

- MAX will never allow a move that could lead to a worse score (for MAX) than α
- MIN will never allow a move that could lead to a better score (for MAX) than β

- Therefore, stop evaluating a branch whenever:

  When evaluating a MAX node: a value $v \geq \beta$ is backed-up
  - MIN will never select that MAX node

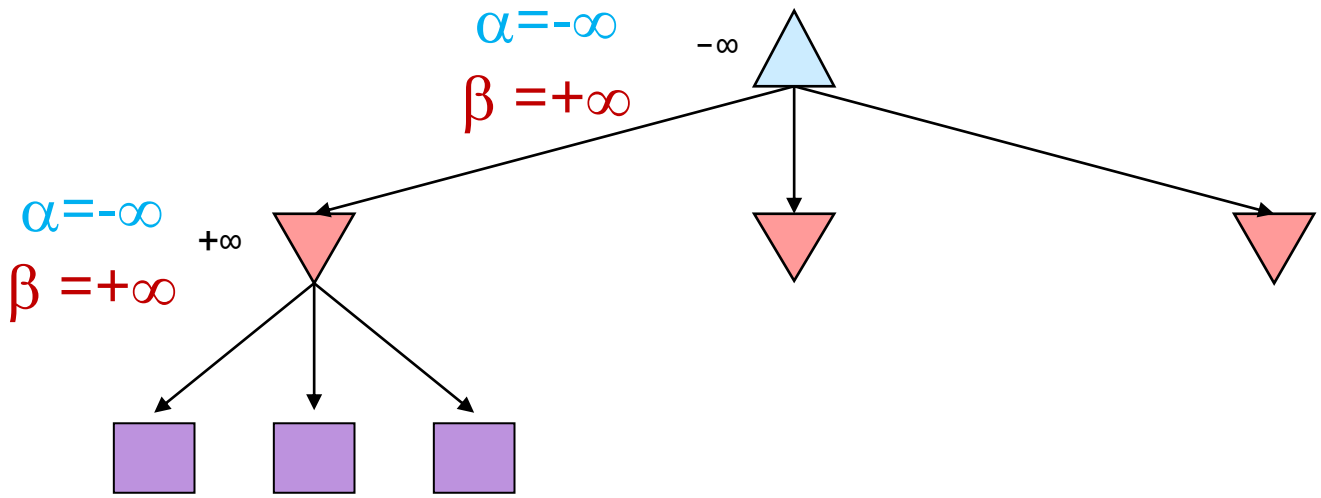
  When evaluating a MIN node: a value $v \leq \alpha$ is found
  - MAX will never select that MIN node

Alpha-Beta Pruning Example

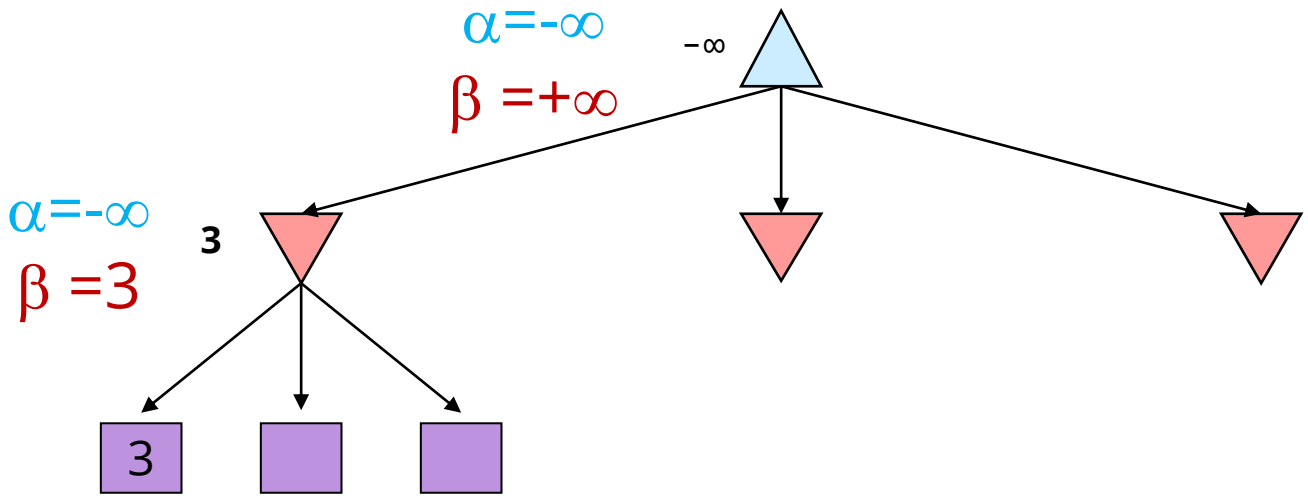
$\alpha = -\infty$

$\beta = +\infty$

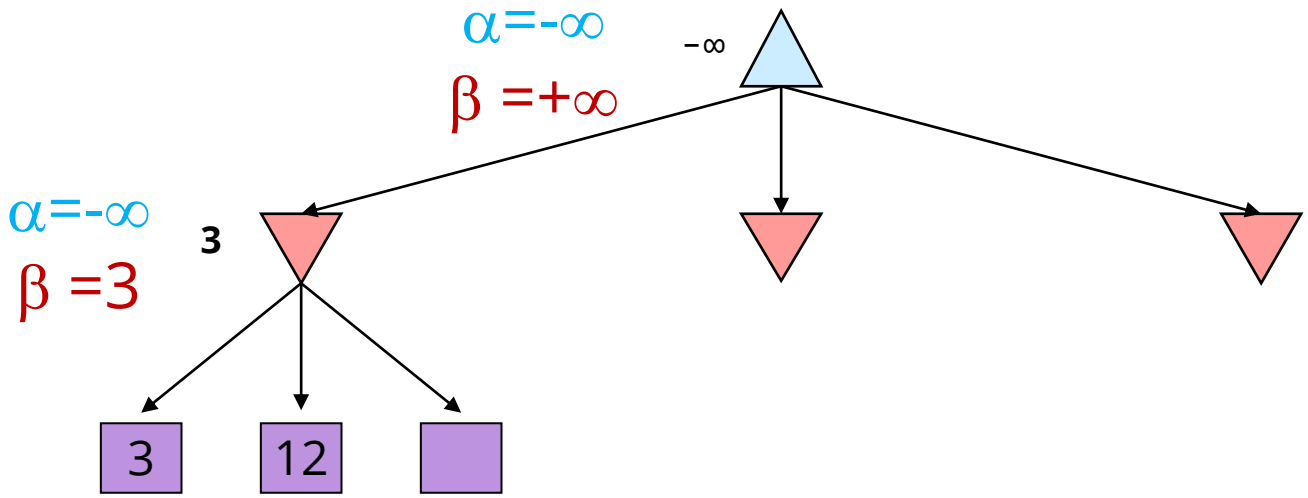
Alpha-Beta Pruning Example



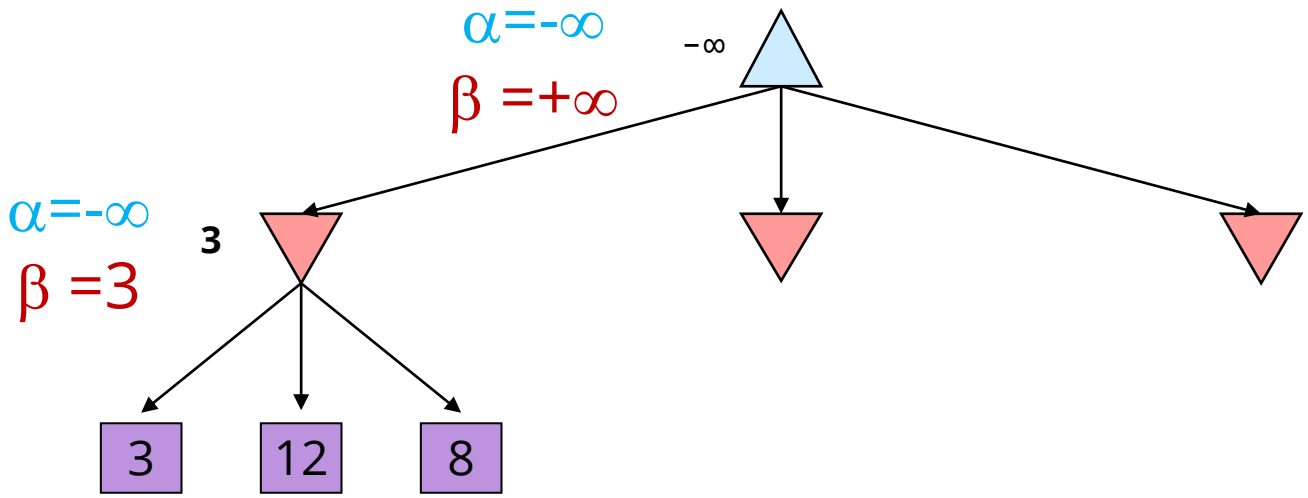
Alpha-Beta Pruning Example



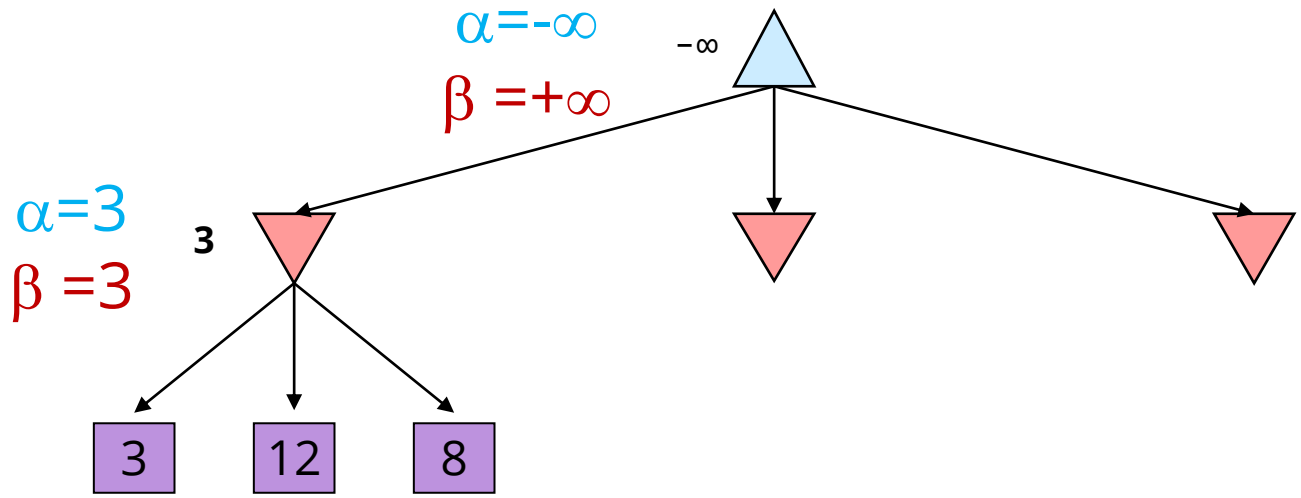
Alpha-Beta Pruning Example



Alpha-Beta Pruning Example



Alpha-Beta Pruning Example



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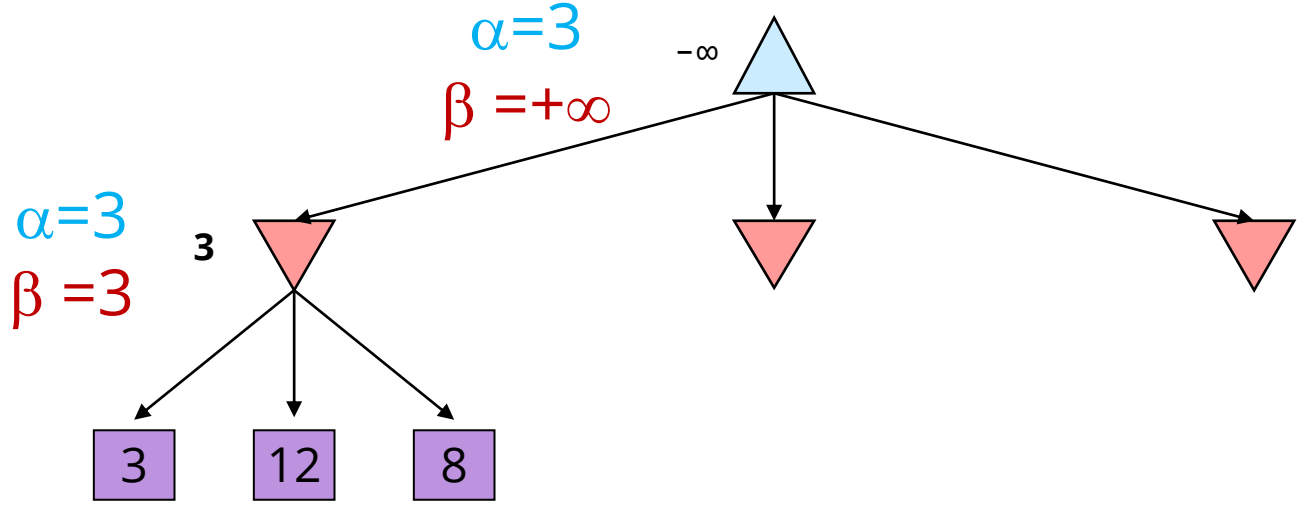

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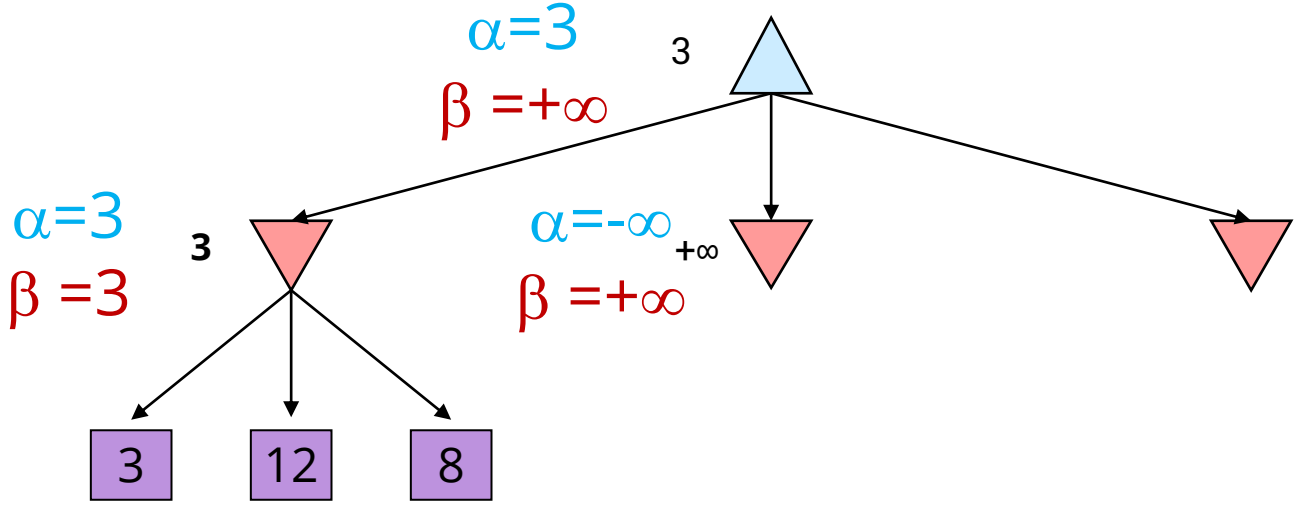
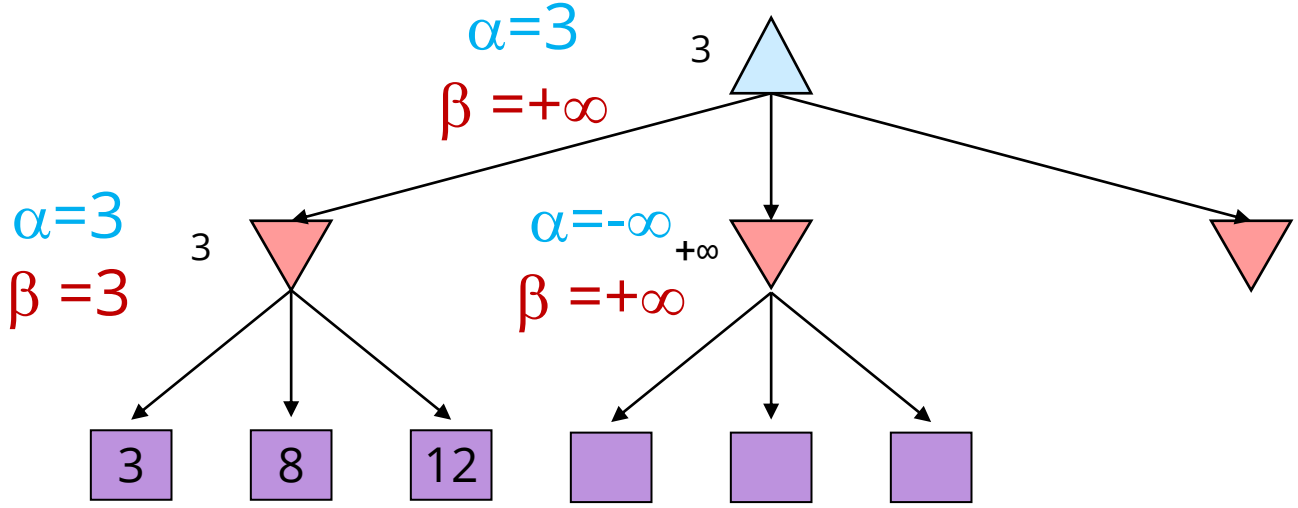
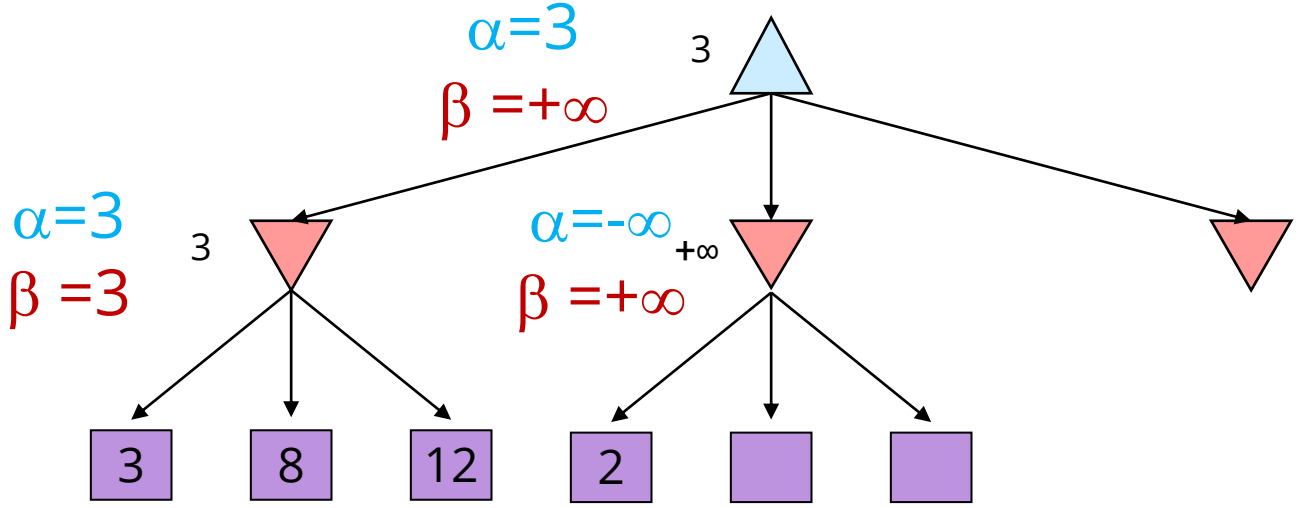


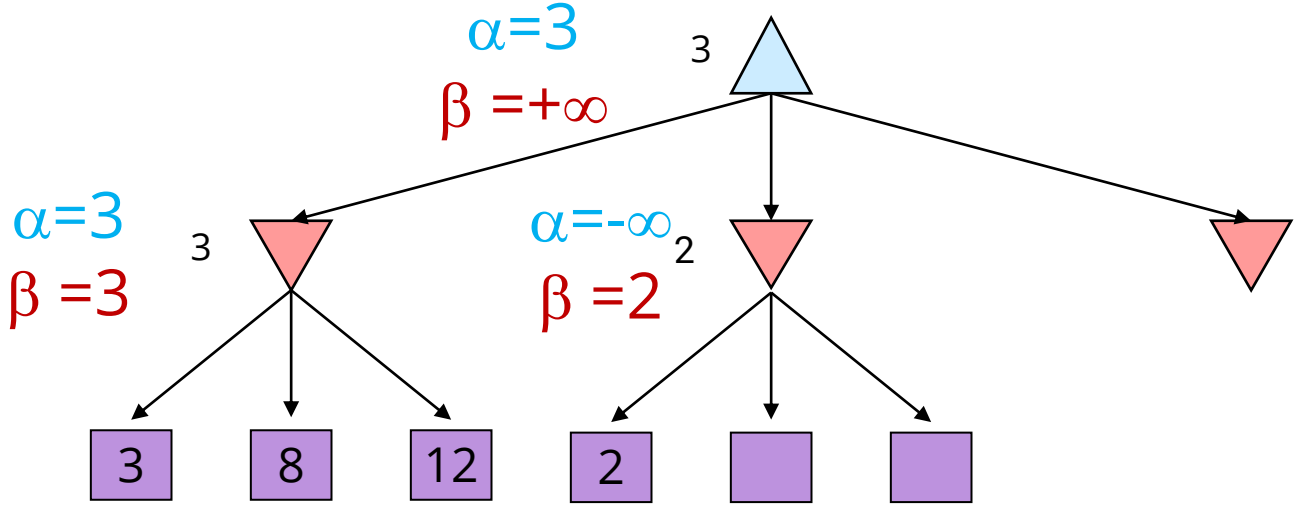
Alpha-Beta Pruning Example



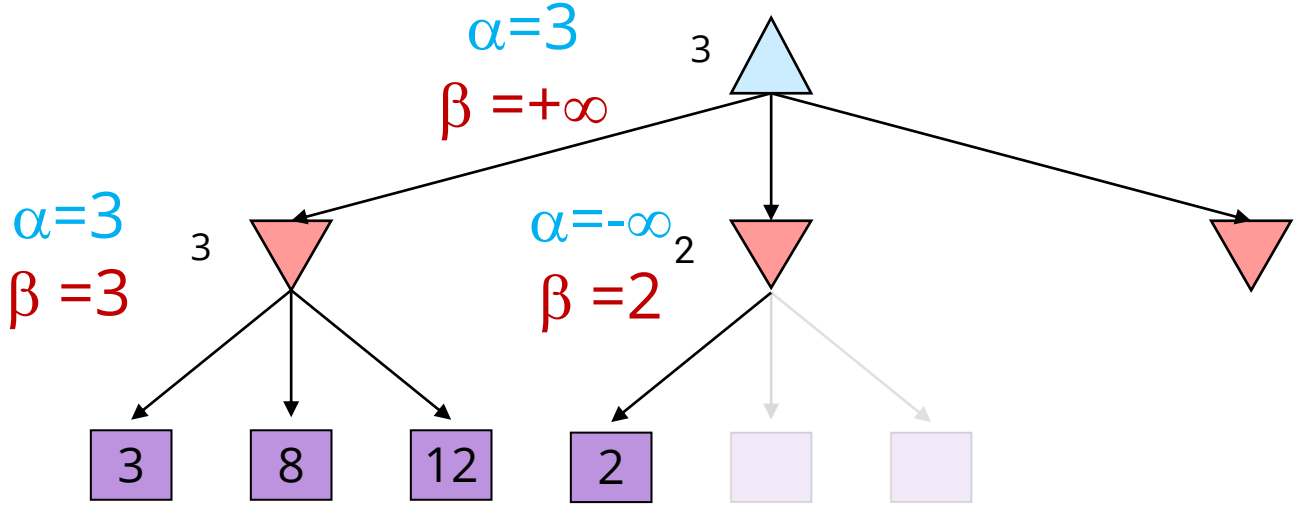
Alpha-Beta Pruning Example



Alpha-Beta Pruning Example



Alpha-Beta Pruning Example



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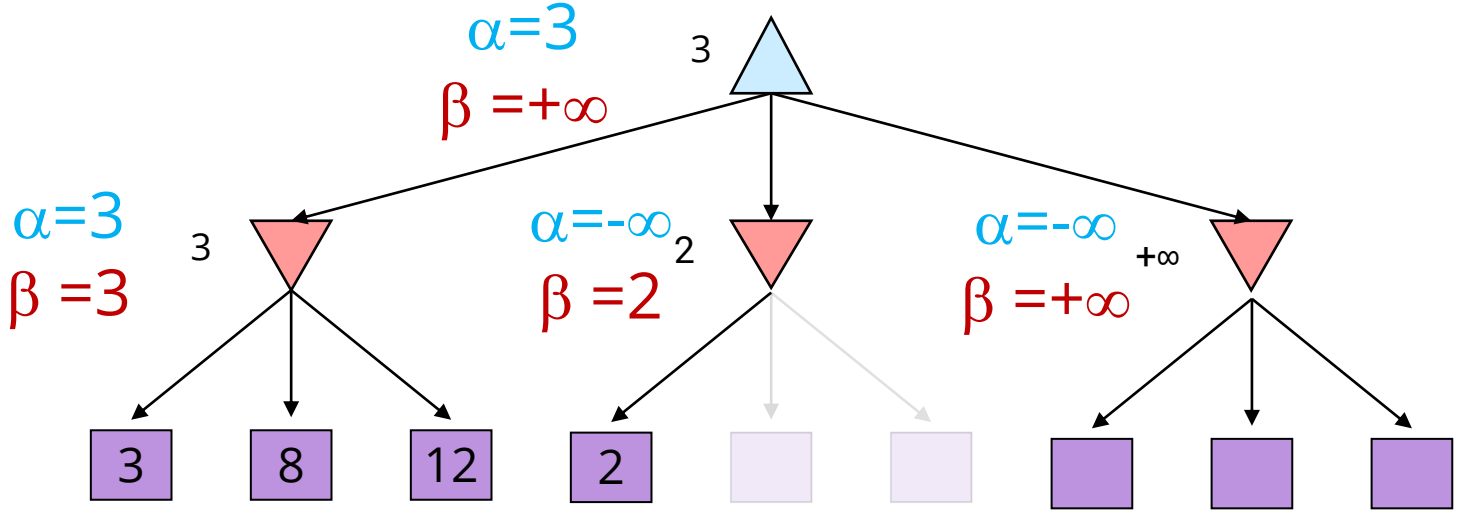
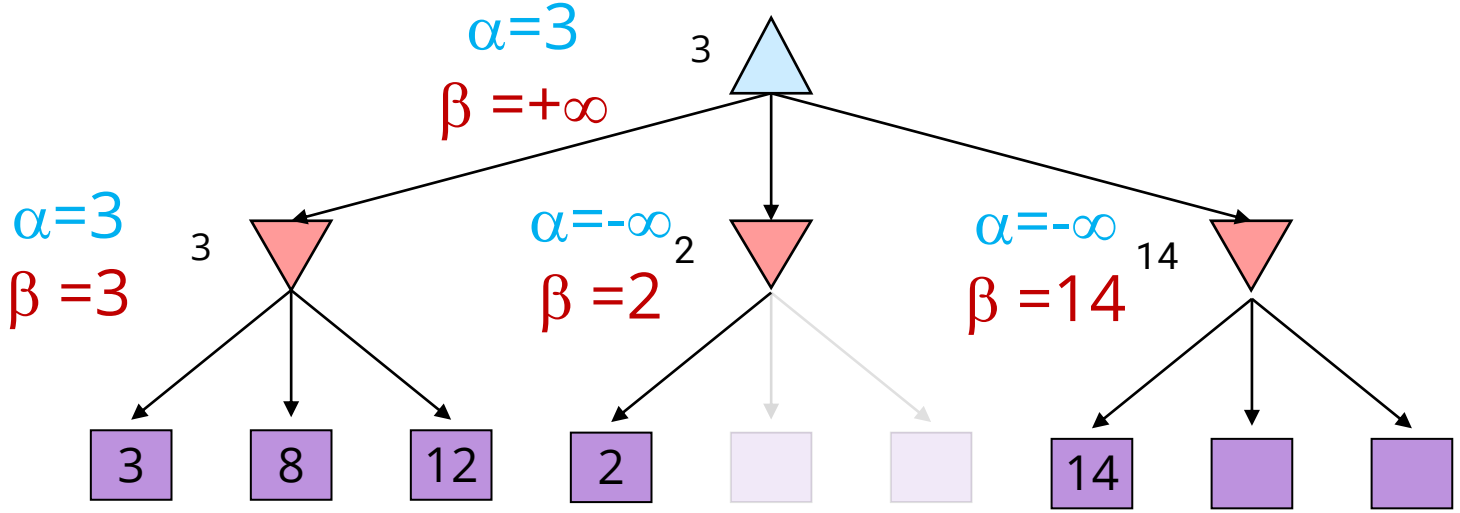


Alpha-Beta Pruning Example



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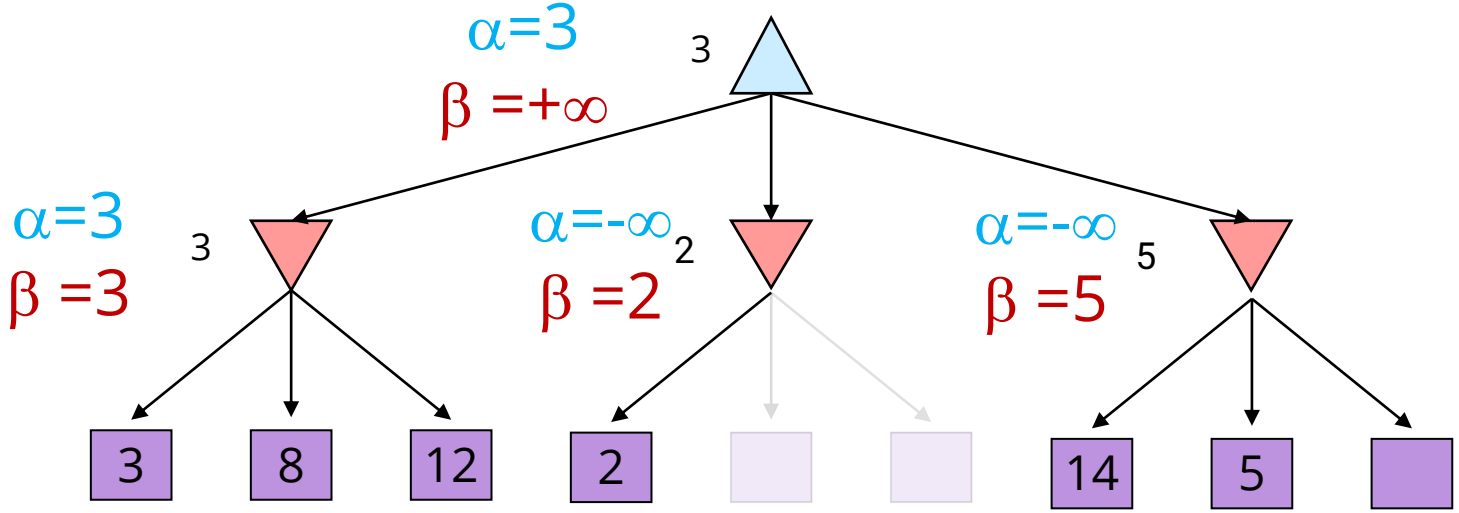

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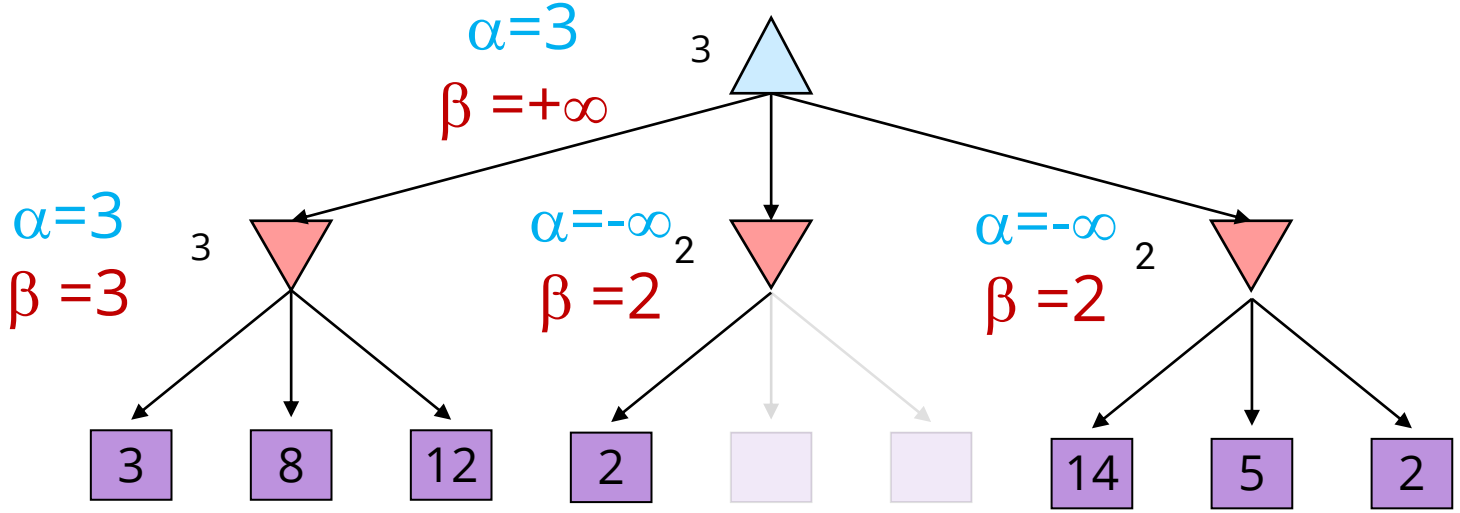

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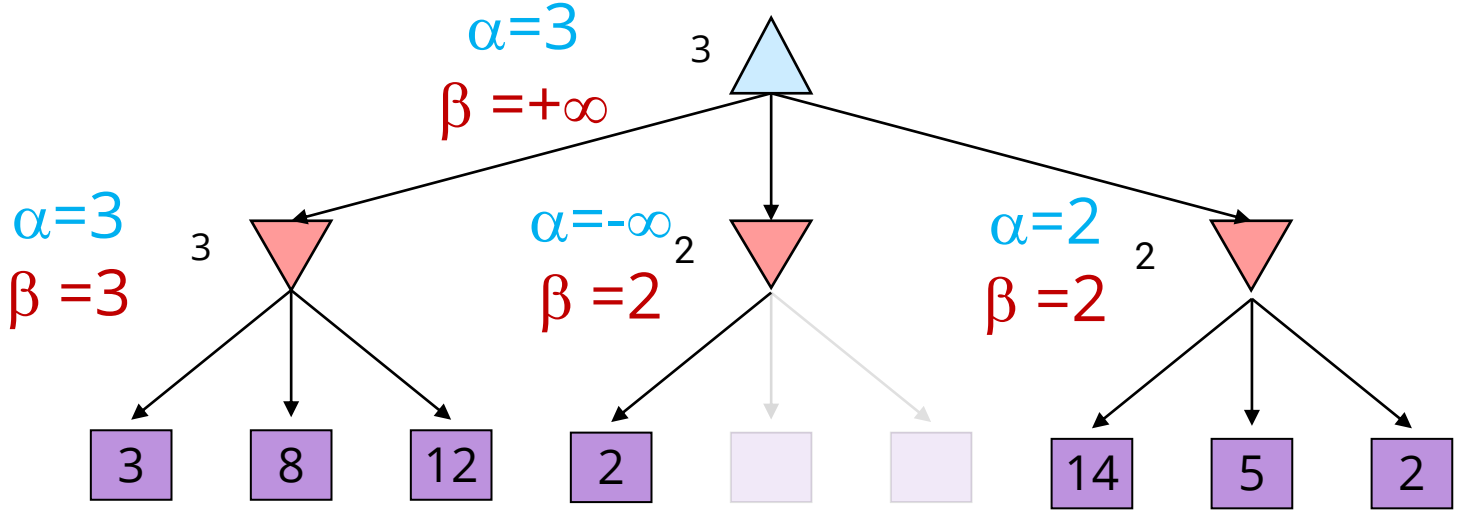

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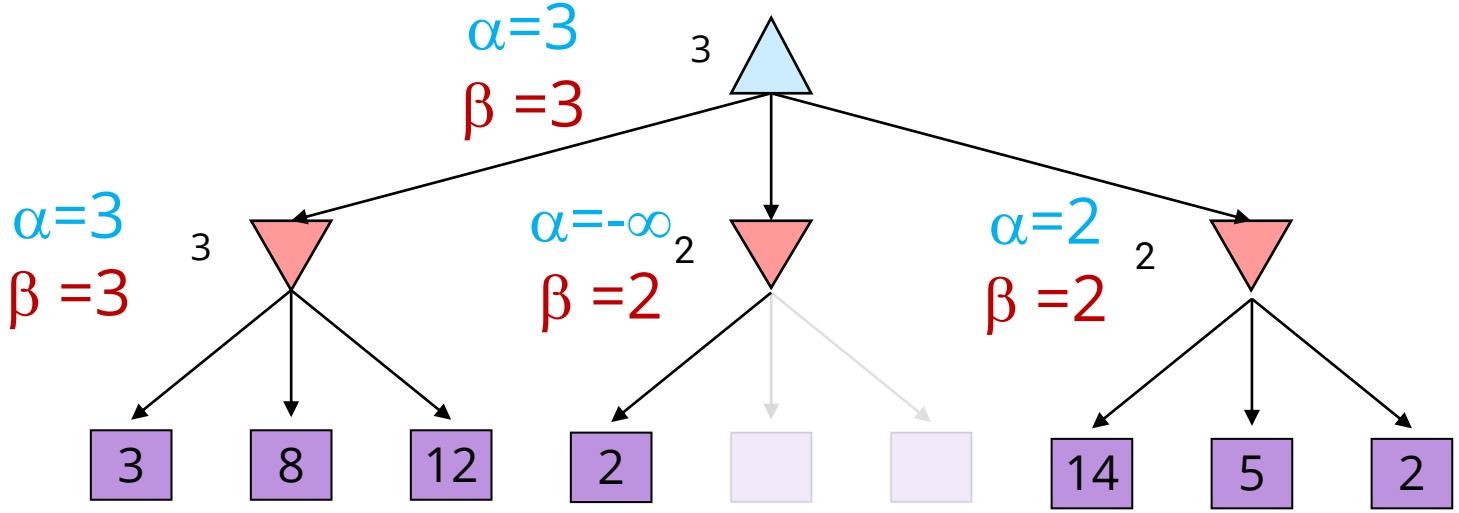
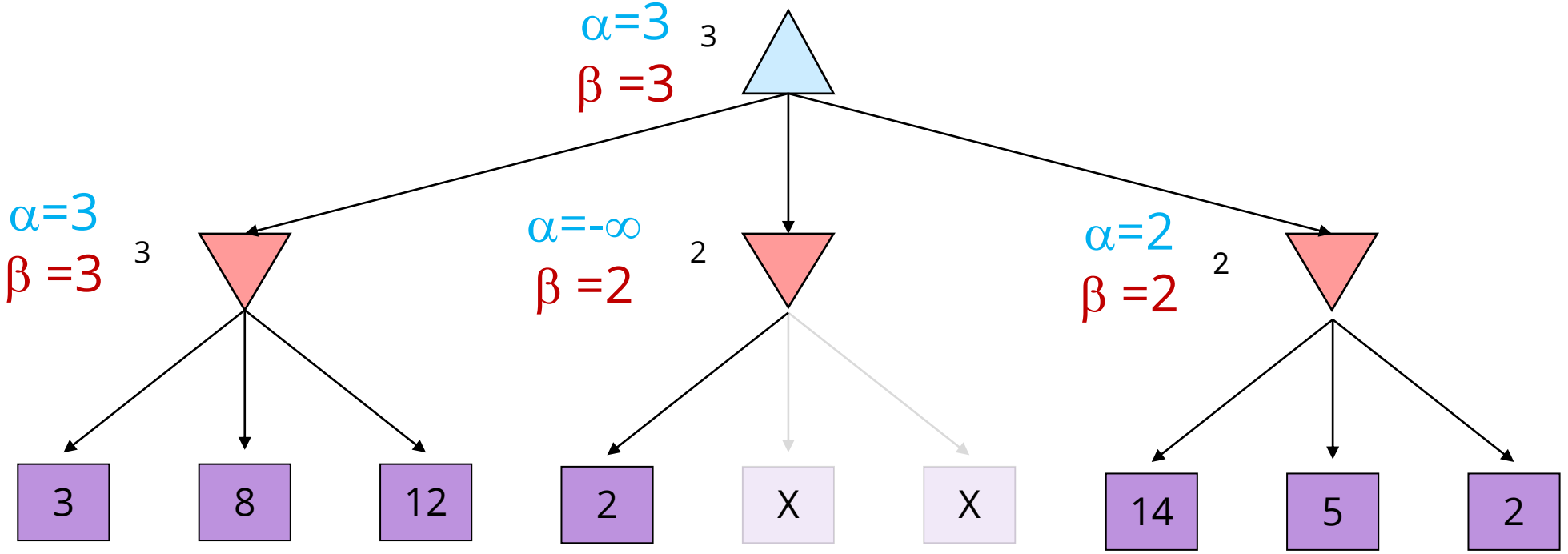


Alpha-Beta Pruning



α: MAX's current lower bound on MAX's outcome
β: MIN's current upper bound on MIN's outcome
α: MAX's best option on path to root
β: MIN's best option on path to root

Review: Evaluation functions

- Evaluates how good a 'board position' is

  Based on *static features* of that board alone

- Zero-sum assumption lets us use one function to describe goodness for both players.

  $f(n) > 0$ if MAX is winning in position *n*

  $f(n) = 0$ if position *n* is tied

  $f(n) < 0$ if MIN is winning in position *n*

- Build using expert knowledge,

  Tic-tac-toe: $f(n) = (\#\text{ of 3 lengths open for MAX}) - (\#\text{ open for MIN})$

(AIMA 5..1)

Review: Chess Evaluation Functions

- Chess needs an evaluation function since it is impossible to search the game tree deeply enough to reach the terminal nodes
- $f(n) = (sum\ of\ A's\ piece\ values) - (sum\ of\ B's\ piece\ values)$
- More complex: weighted sum of positional features:

$$\sum w_i \cdot \text{feature}_i\ (n)$$

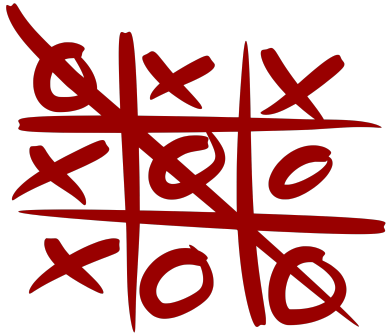
- $f(n)$ can be a **weighted linear function**

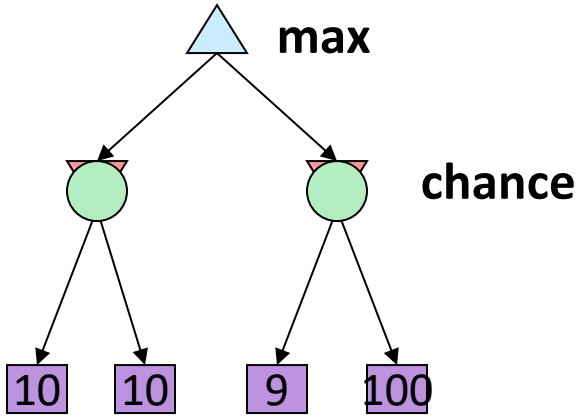
Pawn	**1.0**
Knight	**3.0**
Bishop	**3.25**
Rook	**5.0**
Queen	**9.0**

Pieces values for a simple evaluation function often taught to novice chess players

Uncertain Outcomes

Title

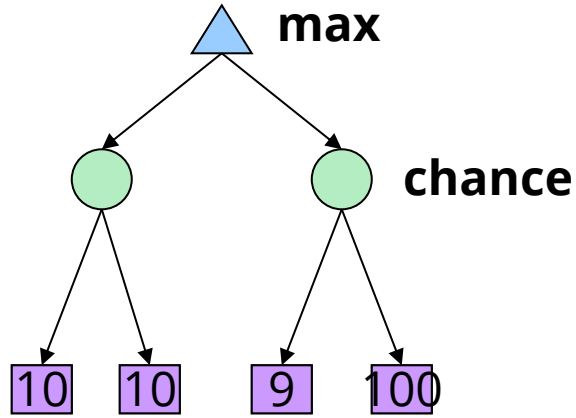




Idea: Uncertain outcomes controlled by chance, not an adversary!

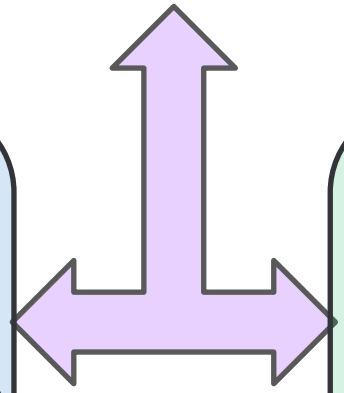
Expectimax Search

- Why wouldn't we know what the result of an action will be?
  - Explicit randomness: rolling dice
  - Unpredictable opponents: the opponent isn't optimal
  - Actions can fail: when moving a robot, wheels might slip
- Values should now reflect average-case (expectimax) outcomes, not worst-case (minimax) outcomes
- Expectimax search: compute the average score under optimal play
  - Max nodes as in minimax search
  - Chance nodes are like min nodes but the outcome is uncertain
  - Calculate their expected utilities
  - I.e. take weighted average (expectation) of children
- Later, we'll learn how to formalize the underlying uncertain-result problems as Markov Decision Processes



Expectimax Pseudocode

```
def value(state):
    if the state is a terminal state: return the state's utility
    if the next agent is MAX: return max-value(state)
    if the next agent is EXP: return exp-value(state)
```

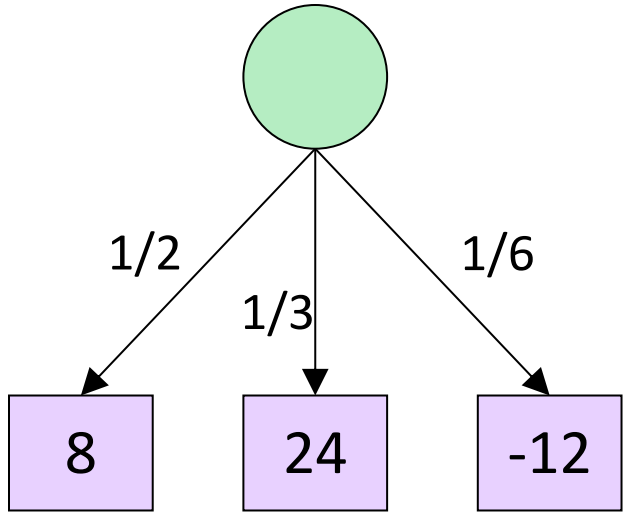


```
def max-value(state):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor))
    return v
```

```
def exp-value(state):
    initialize v = 0
    for each successor of state:
        p = probability(successor)
        v += p * value(successor)
    return v
```

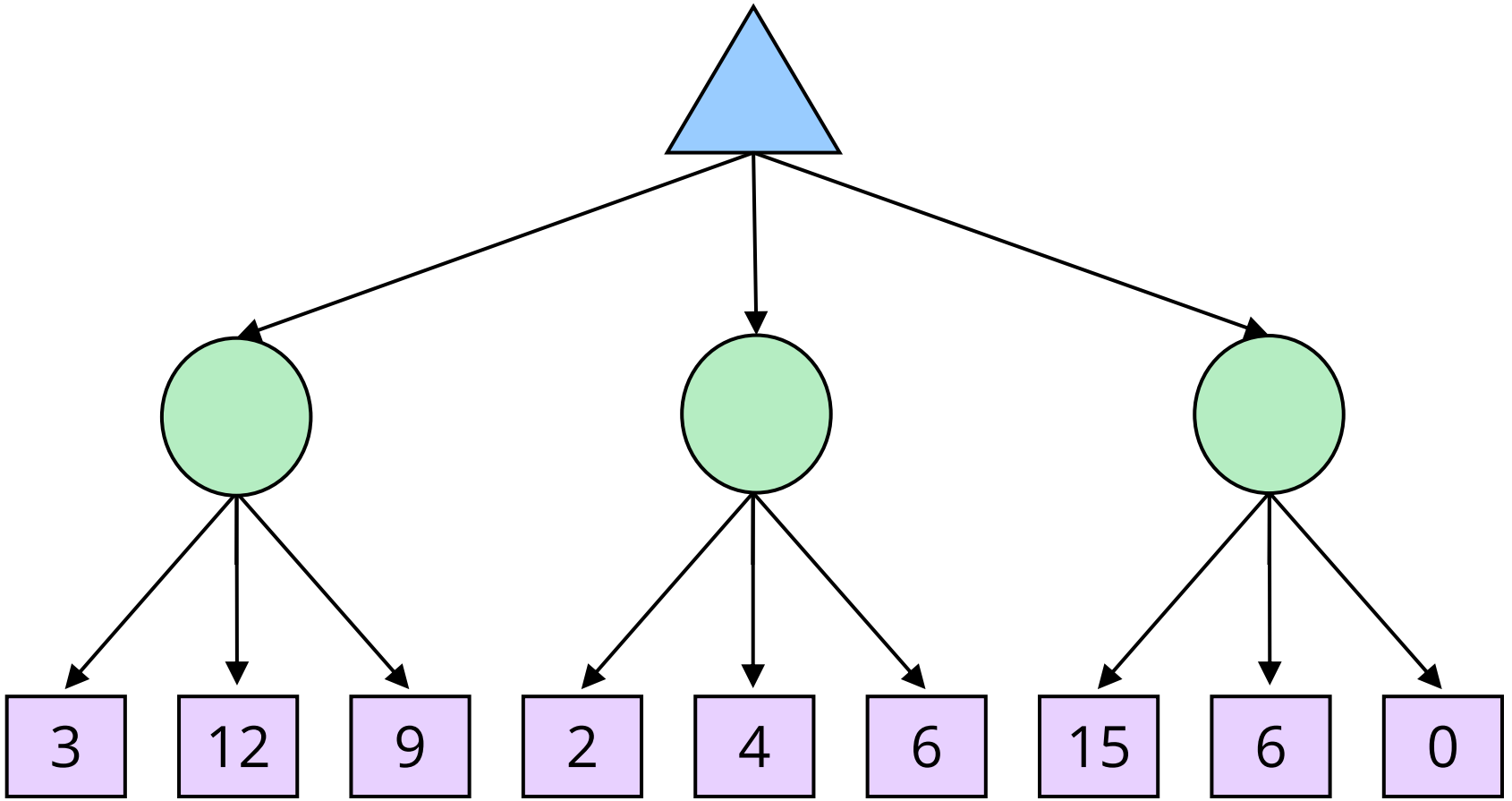
Expectimax Pseudocode

```
def exp-value(state):
  initialize v = 0
  for each successor of state:
      p =
   probability(successor)
      v += p *
   value(successor)
  return v
```



$$v = \frac{1}{2} \cdot (8) + \frac{1}{3} \cdot (24) + \frac{1}{6} \cdot (-12)$$

Title



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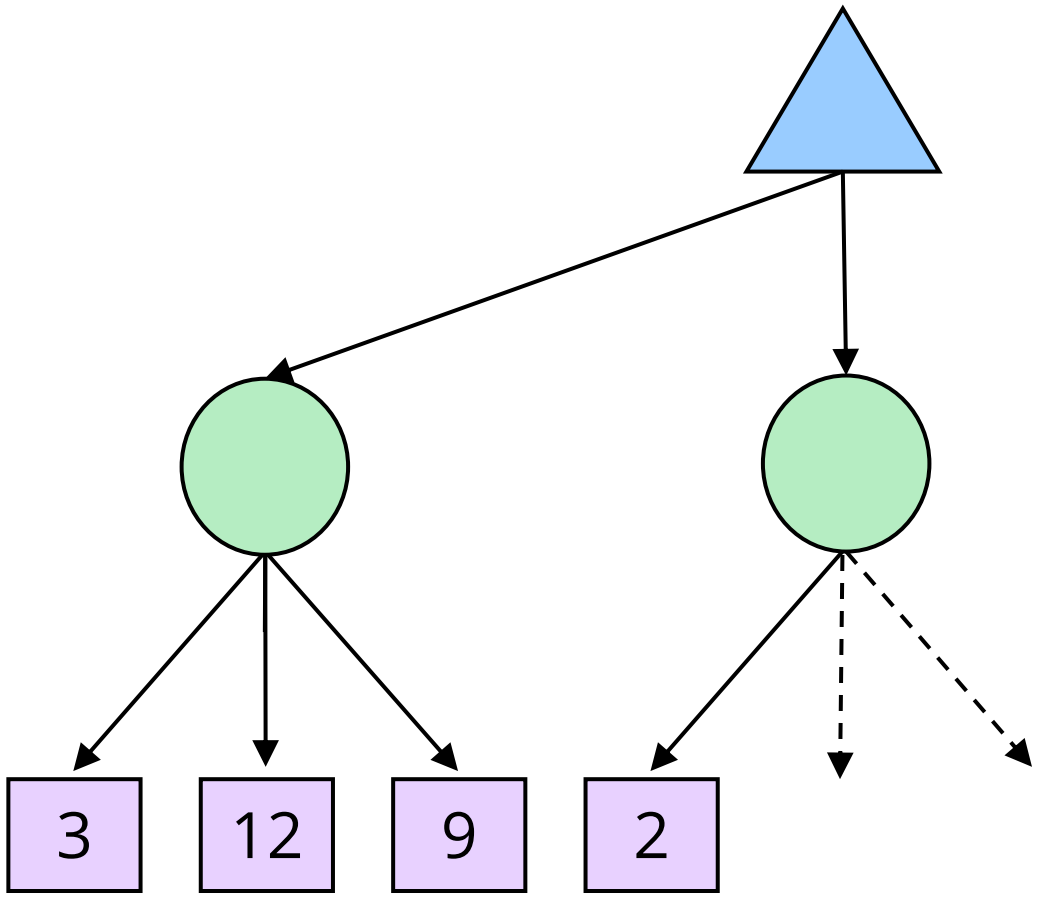

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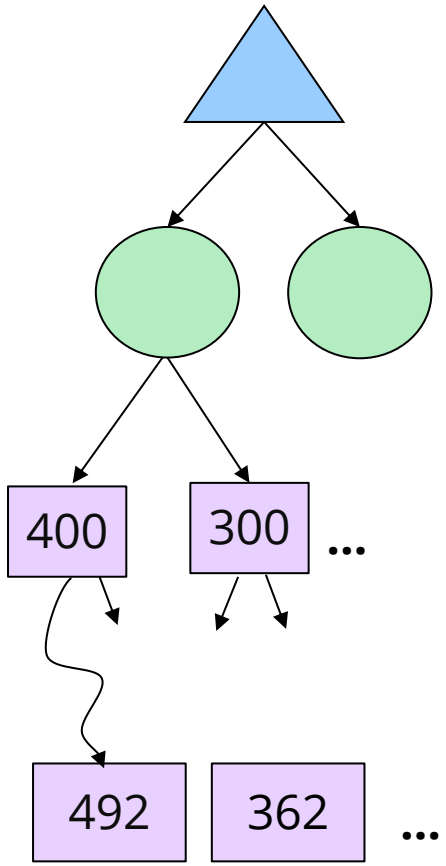
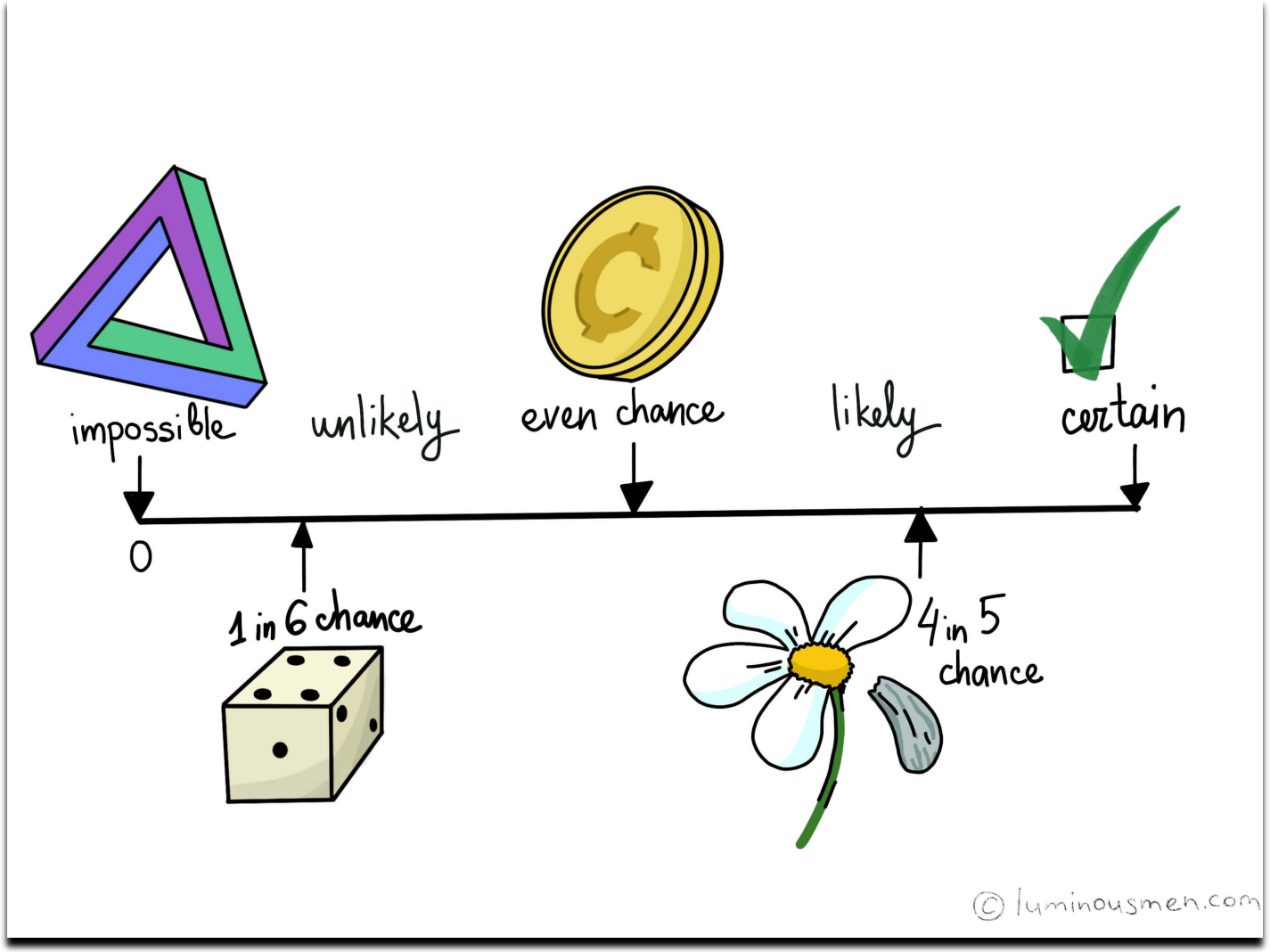


Estimate of true expectimax value (which would require a lot of work to compute)

Probabilities



Probabilities

- A random variable represents an event whose outcome is unknown
- A probability distribution is an assignment of weights to outcomes

- Example: Traffic on freeway
  - Random variable: T = whether there's traffic
  - Outcomes: T in {none, light, heavy}
  - Distribution: $P(T = \text{none}) = 0.25$, $P(T = \text{light}) = 0.50$, $P(T = \text{heavy}) = 0.25$

- Some laws of probability (more later):
  - Probabilities are always non-negative
  - Probabilities over all possible outcomes sum to one

- As we get more evidence, probabilities may change:
  - $P(T = \text{heavy}) = 0.25$, $P(T = \text{heavy} \mid \text{Hour} = 8\text{am}) = 0.60$
  - We'll talk about methods for reasoning and updating probabilities later

0.25

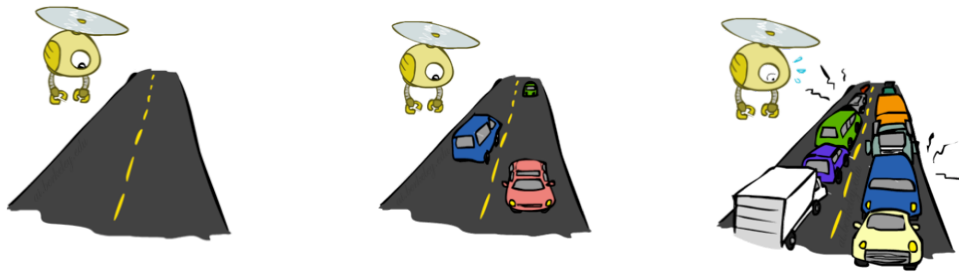
0.50

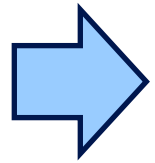
0.25

Probabilities

- The expected value of a function of a random variable is the average, weighted by the probability distribution over outcomes
- Example: How long to get to the airport?

Time:	20 min	x	+	30 min	x	+	60 min	x
Probability:	0.25			0.50			0.25	



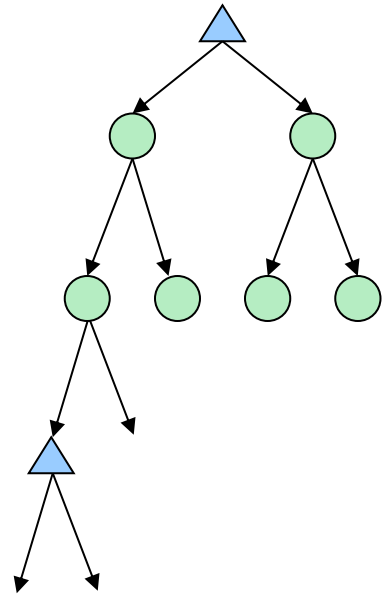


35 min

What Probabilities to Use?

- **In expectimax search, we have a probabilistic model of how the opponent (or environment) will behave in any state**
  - Model could be a simple uniform distribution (roll a die)
  - Model could be sophisticated and require a great deal of computation
  - We have a chance node for any outcome out of our control: opponent or environment
  - The model might say that adversarial actions are likely!

- **For now, assume each chance node magically comes along with probabilities that specify the distribution over its outcomes**



Having a probabilistic belief about another agent's action does not mean that the agent is flipping any coins!

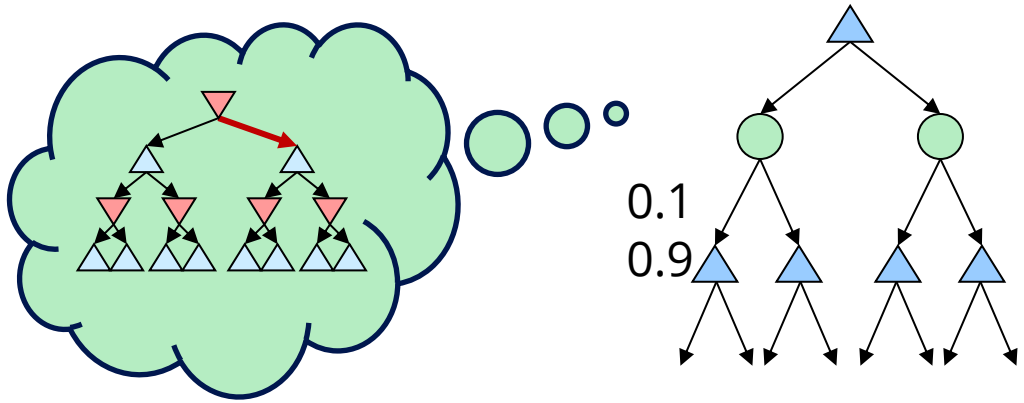
Title

- **Objectivist / frequentist answer:**
  - Averages over repeated *experiments*
  - E.g. empirically estimating P(rain) from historical observation
  - Assertion about how future experiments will go (in the limit)
  - New evidence changes the *reference class*
  - Makes one think of *inherently random* events, like rolling dice

- **Subjectivist / Bayesian answer:**
  - Degrees of belief about unobserved variables
  - E.g. an agent's belief that it's raining, given the temperature
  - E.g. agent's belief how an opponent will behave, given the state
  - Often *learn* probabilities from past experiences (more later)
  - New evidence *updates beliefs* (more later)

Quiz: Informed Probabilities

- Let's say you know that your opponent is actually running a depth 2 minimax, using the result 80% of the time, and moving randomly otherwise
- Question: What tree search should you use?



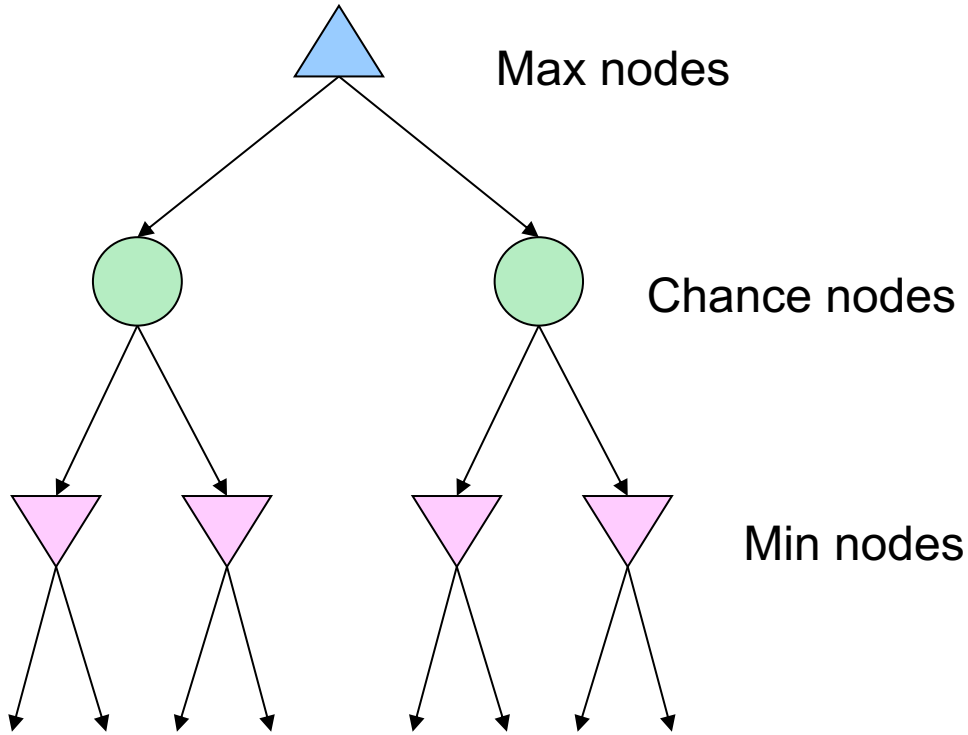
- Answer: Expectimax!
  - To figure out EACH chance node's probabilities, you have to run a simulation of your opponent
  - This kind of thing gets very slow very quickly
  - Even worse if you have to simulate your opponent simulating you…
  - … except for minimax, which has the nice property that it all collapses into one game tree

Title

- **Dice rolls increase *b*: 21 possible rolls with 2 dice**
  - Backgammon ≈ 20 legal moves
  - Depth 2 → $20\times(21\times20)^3 = 1.2\times10^9$
- **As depth increases, probability of reaching a given search node shrinks**
  - So usefulness of search is diminished
  - So limiting depth is less damaging
  - But pruning is trickier…
- **Historic AI: TDGammon uses depth-2 search + very good evaluation fuanction + reinforcement learning → world-champion level play**
- **1st AI world champion in any game!**

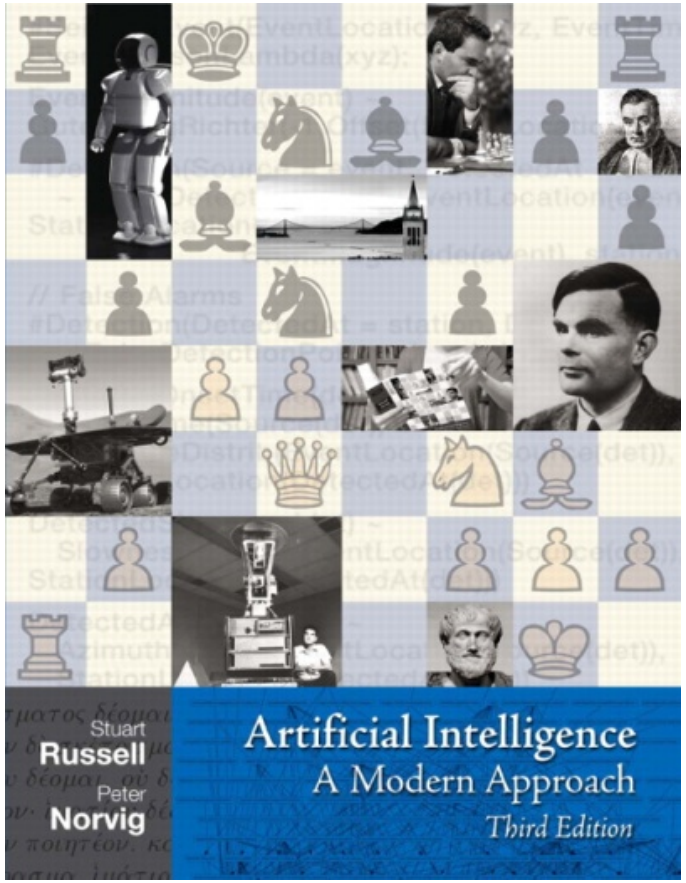
Title

- **E.g. Backgammon**
- **Expectiminimax**
  - Environment is an extra “random agent” player that moves after each min/max agent
  - Each node computes the appropriate combination of its children



Utilities

AIMA Chapter
16.1-16.3



Maximum Expected Utility

- Why should we **average** utilities? Why not minimax?

- **Principle of maximum expected utility:**

  A rational agent should choose the action that maximizes its expected utility, given its knowledge

- Questions:

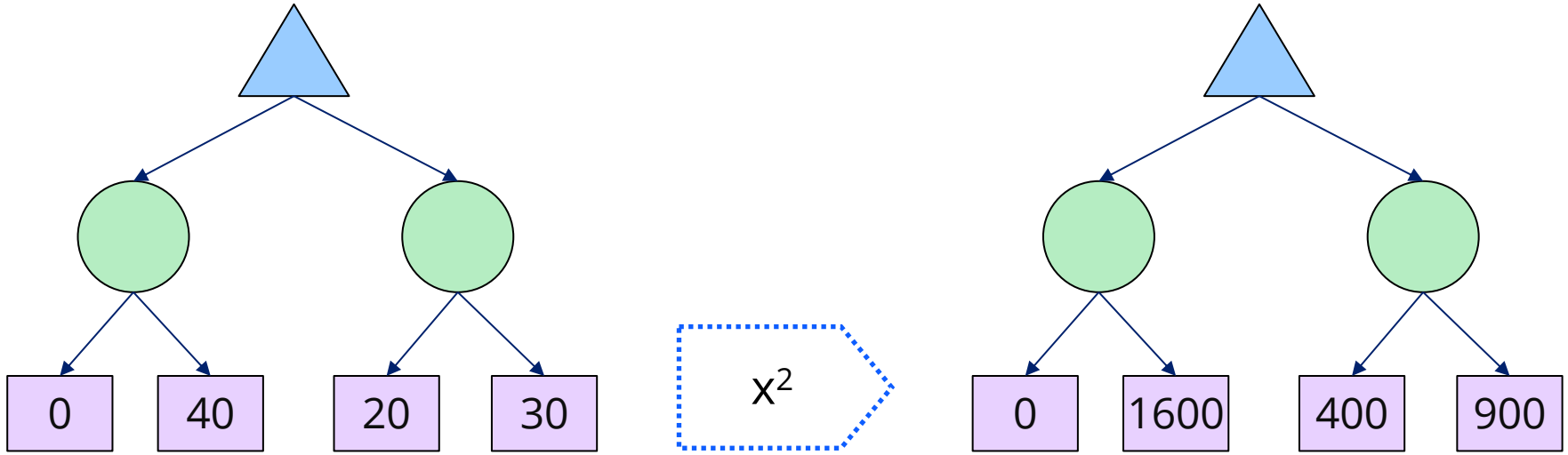
  Where do utilities come from?

  How do we know such utilities even exist?

  How do we know that averaging even makes sense?

  What if our behavior (preferences) can't be described by utilities?

Title



- **For worst-case minimax reasoning, terminal function scale doesn't matter**
  We just want better states to have higher evaluations (get the ordering right)
  We call this insensitivity to monotonic transformations

- **For average-case expectimax reasoning, we need *magnitudes* to be meaningful**

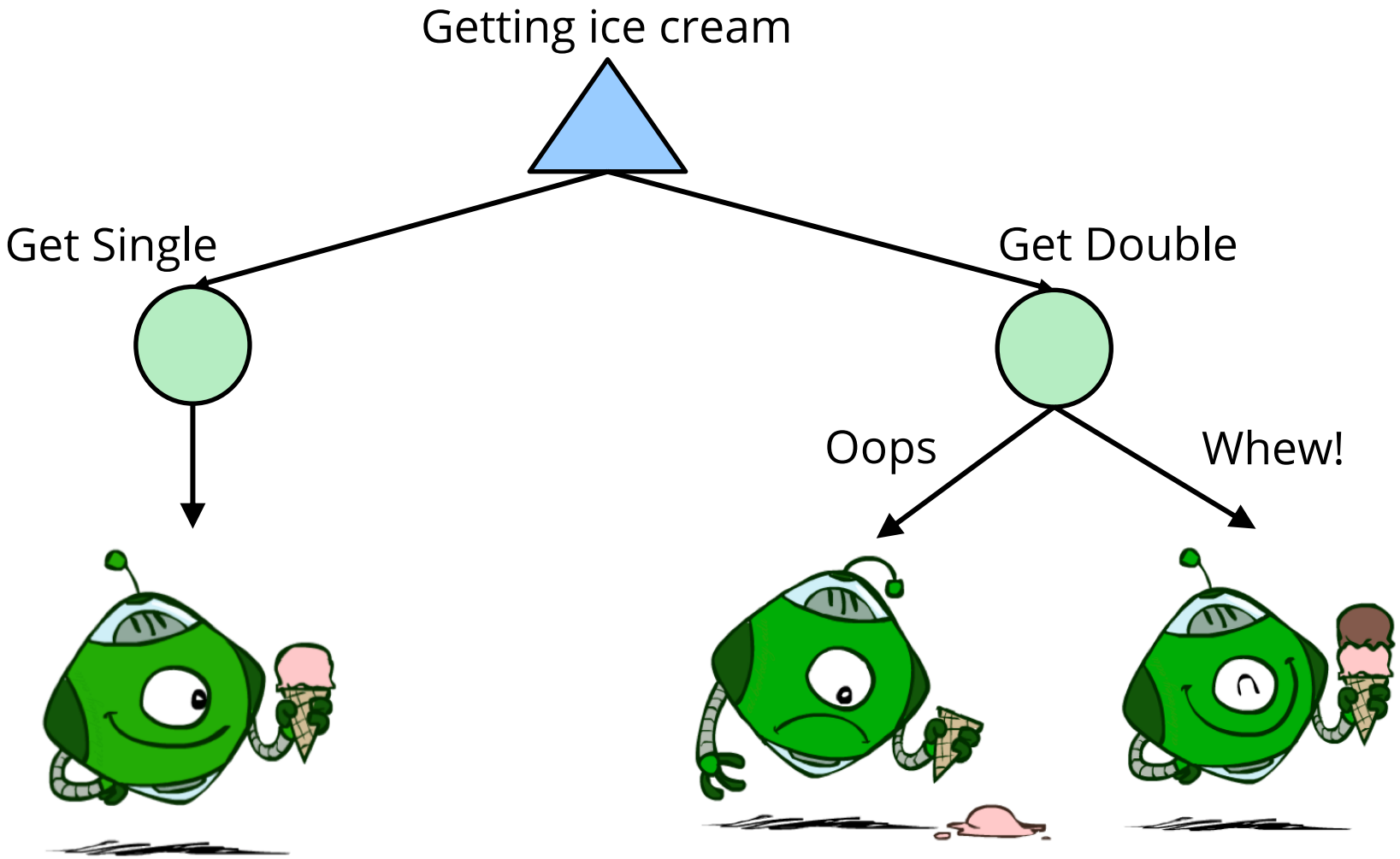
Title

- Utilities are functions from outcomes (states of the world) to real numbers that describe an agent's preferences

- Where do utilities come from?
  - In a game, may be simple (+1/-1)
  - Utilities summarize the agent's goals
  - Theorem: any "rational" preferences can be summarized as a utility function

- We hard-wire utilities and let behaviors emerge
  - Why don't we let agents pick utilities?
  - Why don't we prescribe behaviors?

Utilities: Uncertain Outcomes



Title

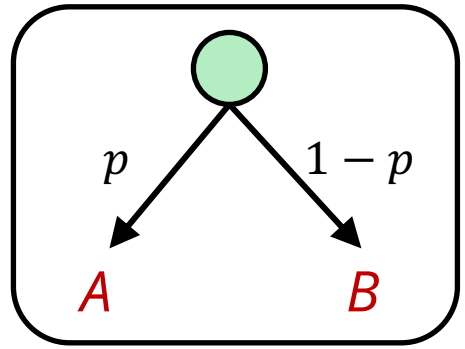
- An agent must have preferences among:
  - Prizes: $A,\ B$, etc.
  - Lotteries: situations with uncertain prizes

$$L = [p, A;\ (1-p), B]$$

- Notation:
  - Preference: $A \succ B$
  - Indifference: $A \sim B$

A Prize

A Lottery



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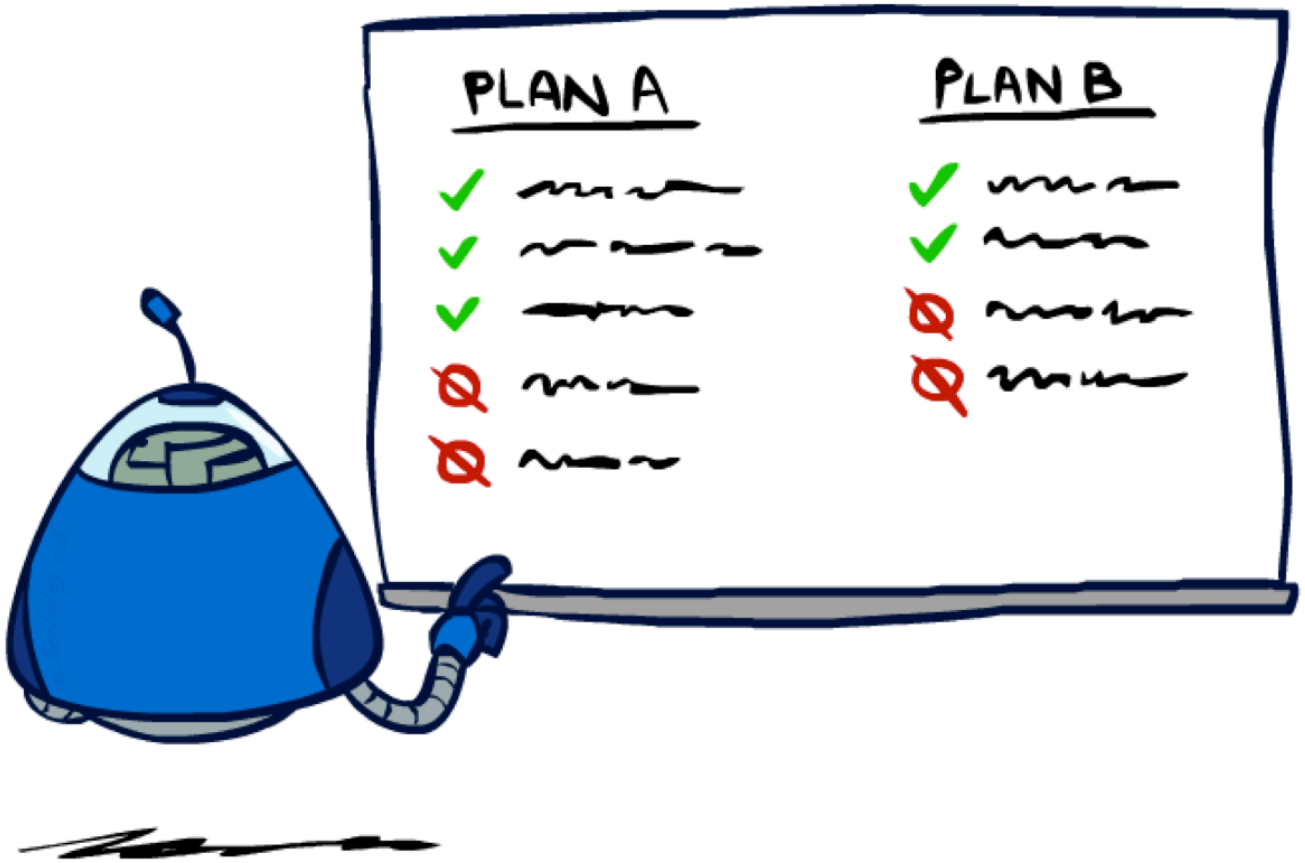


Title

- **We want some constraints on preferences before we call them rational, such as:**

Axiom of Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$

- **For example: an agent with intransitive preferences can be induced to give away all of its money**
  - If B > C, then an agent with C would pay (say) 1 cent to get B
  - If A > B, then an agent with B would pay (say) 1 cent to get A
  - If C > A, then an agent with A would pay (say) 1 cent to get C

Title

The Axioms of Rationality

Orderability

$$(A \succ B) \vee (B \succ A) \vee (A \sim B)$$

Transitivity

$$(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$$

Continuity

$$A \succ B \succ C \Rightarrow \exists p\ [p, A;\ 1-p, C] \sim B$$

Substitutability

$$A \sim B \Rightarrow [p, A;\ 1-p, C] \sim [p, B; 1-p, C]$$

Monotonicity

$$A \succ B \Rightarrow$$
$$(p \geq q \Leftrightarrow [p, A;\ 1-p, B] \succeq [q, A;\ 1-q, B])$$

Theorem: Rational preferences imply behavior describable as maximization of expected utility

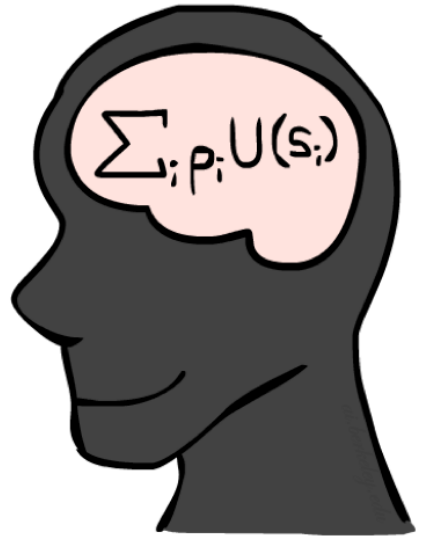
MEU Principle

- **Theorem [Ramsey, 1931; von Neumann & Morgenstern, 1944]**

  Given any preferences satisfying these constraints, there exists a real-valued function U such that:

  $$U(A) \geq U(B) \Leftrightarrow A \succcurlyeq B$$
  $$U([p_1, S_1; \ldots; p_n, S_n]) = \sum_i p_i U(S_i)$$

  I.e. values assigned by U preserve preferences of both prizes and lotteries!



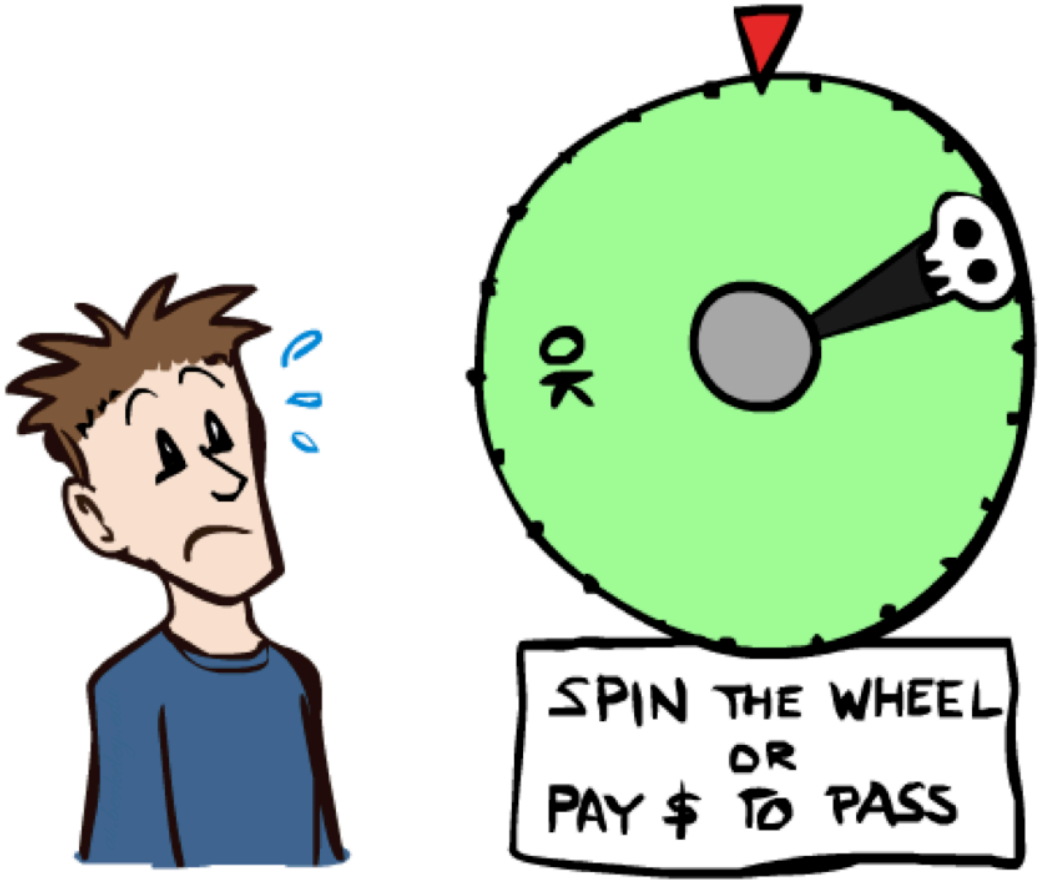
- **Maximum expected utility (MEU) principle:**

  Choose the action that maximizes expected utility

  Note: an agent can be entirely rational (consistent with MEU) without ever representing or manipulating utilities and probabilities

  E.g., a lookup table for perfect tic-tac-toe, a reflex vacuum cleaner

Title



Utility Scales

- Normalized utilities: $u_+ = 1.0, u_- = 0.0$
- Micromorts: one-millionth chance of death, useful for paying to reduce product risks, etc.
- QALYs: quality-adjusted life years, useful for medical decisions involving substantial risk
- Note: behavior is invariant under positive linear transformation

$$U'(x) = k_1 U(x) + k_2 \text{ where } k_1 \succ 0$$

- With deterministic prizes only (no lottery choices), only ordinal utility can be determined, i.e., total order on prizes

Micromort examples

Death from	Micromorts per exposure
Scuba diving	5 per dive
Skydiving	7 per jump
Base-jumping	430 per jump
Climbing Mt. Everest	38,000 per ascent

1 micromort	
Train travel	6000 miles
Jet	1000 miles
Car	230 miles
Walking	17 miles
Bicycle	10 miles
Motorbike	6 miles

Title

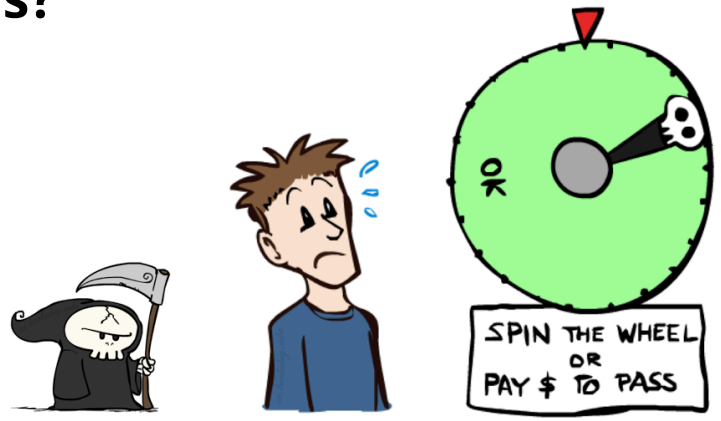
- **Utilities map states to real numbers. Which numbers?**
- **Standard approach to assessment (elicitation) of human utilities:**

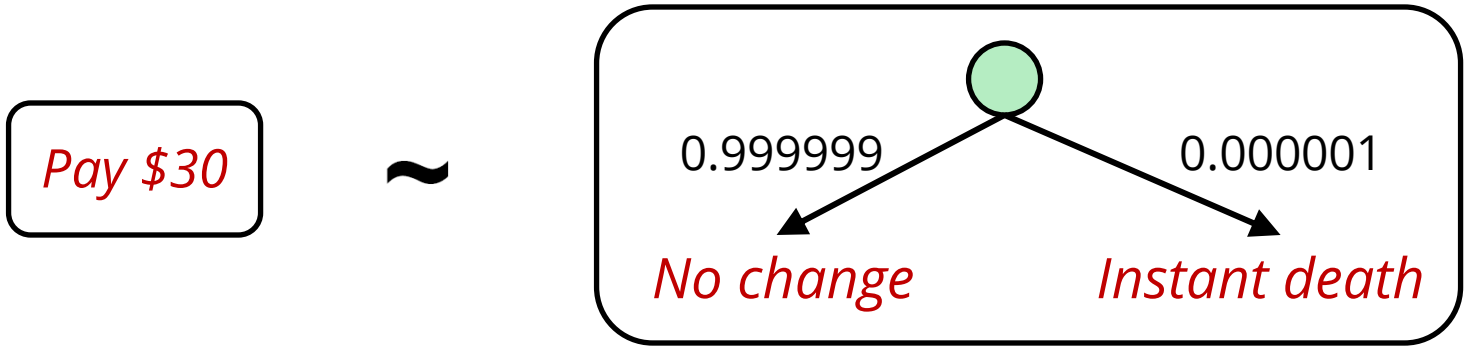
  Compare a prize A to a standard lottery L_p between

  - "best possible prize" u_+ with probability p
  - "worst possible catastrophe" u_- with probability $1-p$

  Adjust lottery probability p until indifference: $A - L_p$

  Resulting p is a utility in $[0,1]$





Money

- **Money does not behave as a utility function, but we can talk about the utility of having money (or being in debt)**
- **Given a lottery $L = [p, \$X; (1-p), \$Y]$**

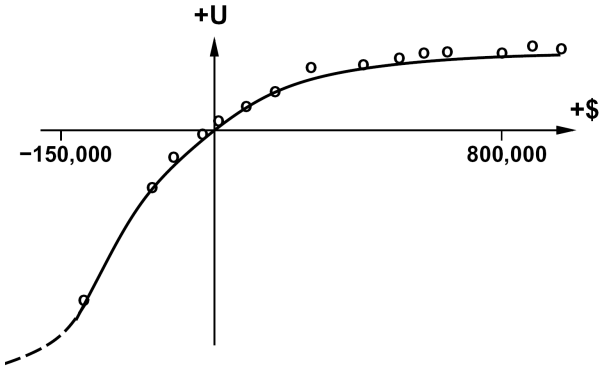
  The expected monetary value $EMV(L) = p \cdot X + (1-p) \cdot Y$

  $U(L) = p \cdot U(\$X) + (1-p) \cdot U(\$Y)$

  Typically, $U(L) < U(EMV(L))$

  In this sense, people are risk-averse

  When deep in debt, people are risk-prone



Example: Insurance

- **Consider the lottery** $[0.5, \$1000; 0.5, \$0]$

  What is its expected monetary value? ($500)

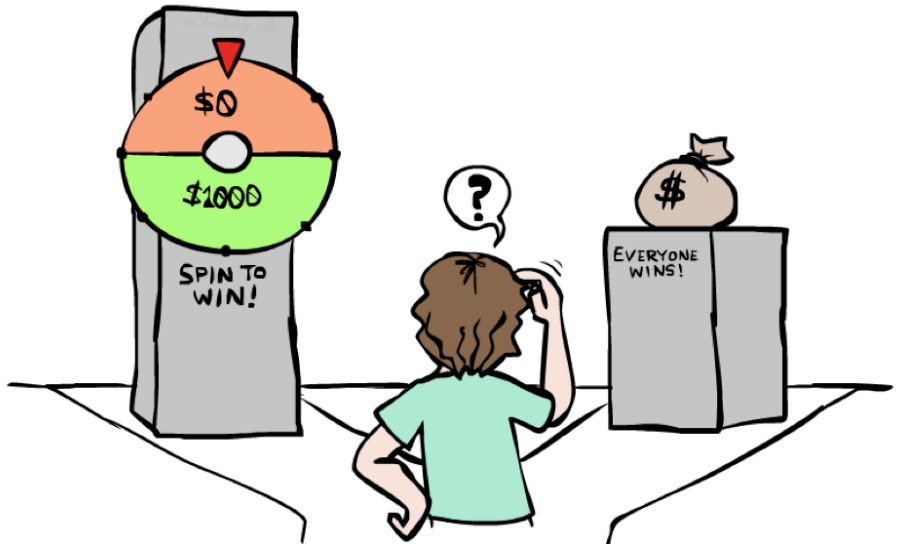
  What is its certainty equivalent?

  - Monetary value acceptable in lieu of lottery
  - $400 for most people

  Difference of $100 is the insurance premium

  - There's an insurance industry because people will pay to reduce their risk
  - If everyone were risk-neutral, no insurance needed!

  It's win-win: you'd rather have the $400 and the insurance company would rather have the lottery (their utility curve is linear and they have many lotteries)



Example: Human Rationality?

- **Famous example of Allais (1953)**

  A: $[0.8, \$4\text{k};\ \ 0.2\ \$0]$
  B: $[1.0, \$3\text{k};\ \ 0.0, \$0]$
  C: $[0.2, \$4\text{k};\ \ 0.8, \$0]$
  D: $[0.2, \$3\text{k};\ \ 0.75, \$0]$

- **Most people prefer $B > A$, $C > D$**

- **But if $U(\$0) = 0$, then**

  $B > A \implies U(\$3\text{k}) > 0.8 \cdot U(\$4\text{k})$
  $C > D \implies 0.8 \cdot U(\$4\text{k}) > U(\$3\text{k})$