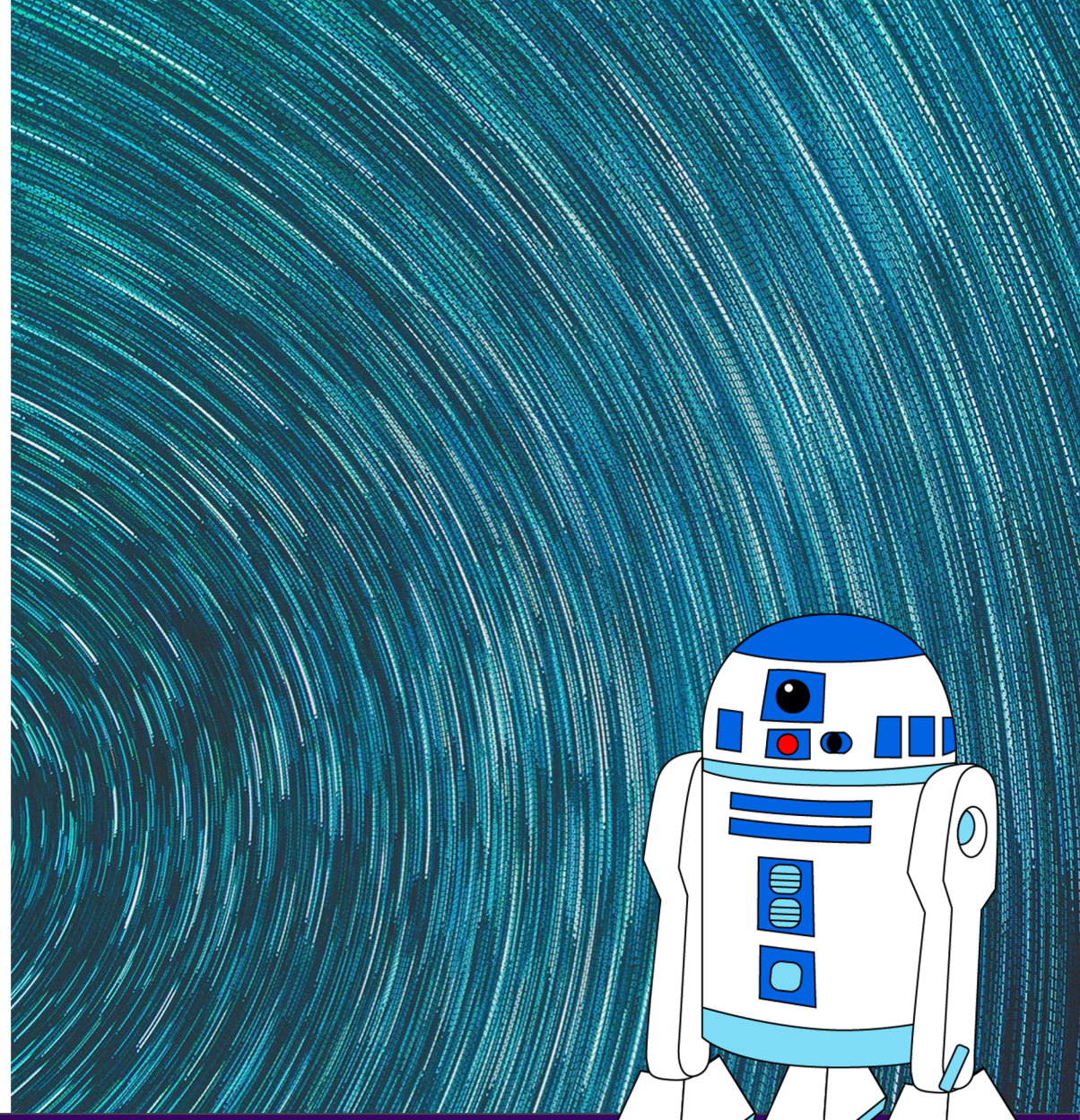
CIS 421/521:
ARTIFICIAL INTELLIGENCE

# Informed Search



Penn Engineering
University of Pennsylvania

# Review: Search problem definition

*States*: a set ***S***

An *initial state* $s_i \in S$

*Actions: a set* ***A***

$\forall s$ ***Actions(s)*** = *the set of actions that can be executed in* ***s****, that are applicable* in ***s***.

*Transition Model:* $\forall s \forall a \in$ ***Actions(s) Result(s, a)*** $\rightarrow s_r$

$s_r$ *is called a successor of* ***s***

$\{s_i\} \cup$ ***Successors***$(s_i)$***** = *state space*

*Path cost (Performance Measure):* Must be additive

e.g. sum of distances, number of actions executed, …

***c(x,a,y)*** is the step cost, assumed ≥ 0

(where action ***a*** goes from state ***x*** to state ***y****)*

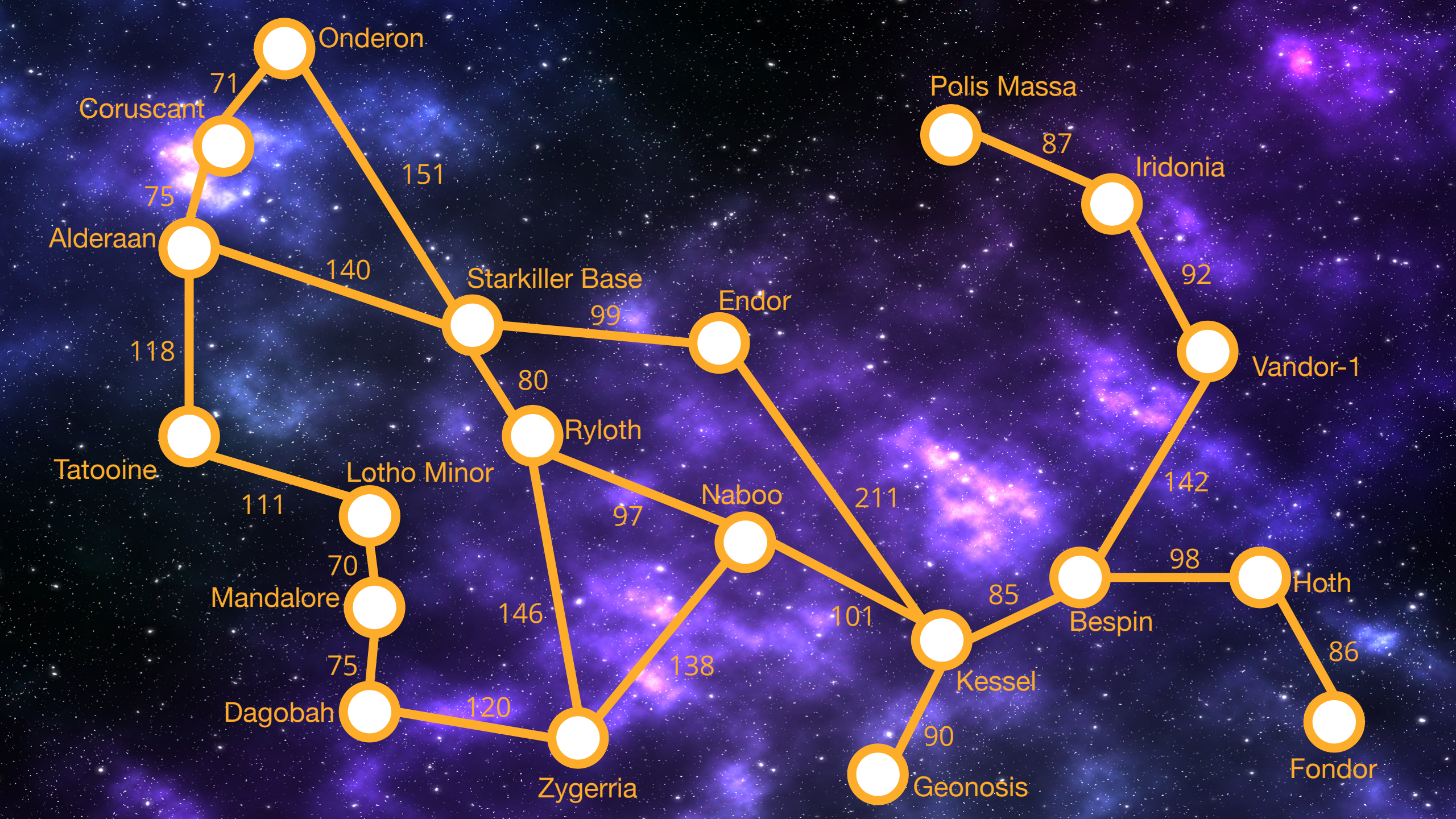
*Goal test:* ***Goal(s)***

Can be implicit, e.g. ***checkmate(s)***

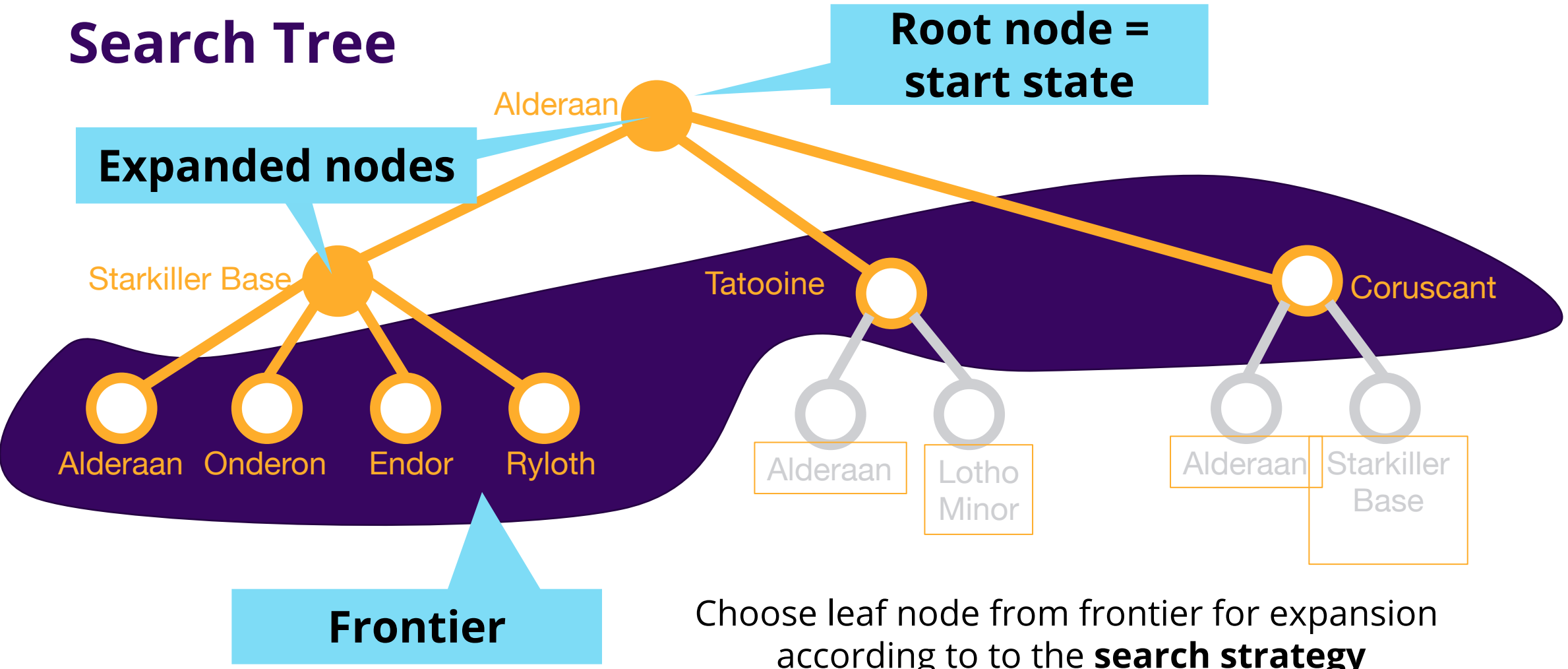
***s*** is a *goal state* if ***Goal(s)*** is *true*

# Review: Useful Concepts

- *State space*: the set of all states reachable from the initial state by *any* sequence of actions
  - *When several operators can apply to each state, this gets large very quickly*
  - *Might be a proper subset of the set of configurations*
- *Path*: a sequence of actions leading from one state $s_j$ to another state $s_k$
- *Frontier:* those states that are available for *expanding* (for applying legal actions to)
- *Solution*: a path from the initial state $s_i$ to a state $s_g$ that satisfies the goal test


Onderon
71
Coruscant
151
75
Alderaan
140
Starkiller Base
99
Endor
118
80
Ryloth
Tatooine
Lotho Minor
111
Naboo
97
211
70
Mandalore
146
101
85
Bespin
98
Hoth
138
75
86
Dagobah
120
Kessel
90
Fondor
Zygerria
Geonosis
Polis Massa
87
Iridonia
92
Vandor-1
142

# Search Tree




Choose leaf node from frontier for expansion according to to the **search strategy**

**Determines the search process**

# Review: Search Strategies

*Strategy* = order of tree expansion

- Implemented by different **queue structures** (LIFO, FIFO, priority)

Dimensions for evaluation

- *Completeness-* always find the solution?
- *Optimality* - finds a least cost solution (lowest path cost) **first**?
- *Time complexity* - # of nodes generated *(worst case)*
- *Space complexity* - # of nodes simultaneously in memory *(worst case)*

Time/space complexity variables

- *b*, *maximum branching factor* of search tree
- *d*, *depth* of the shallowest goal node
- *m*, maximum length of any path in the state space (potentially $\infty$)

Animation of Graph BFS algorithm
set to music 'flight of bumble bee'

https://youtu.be/x-VTfcmrLEQ

Animation of Graph DFS algorithm
Depth First Search of Graph
set to music 'flight of bumble bee'

https://youtu.be/NUgMa5coCoE

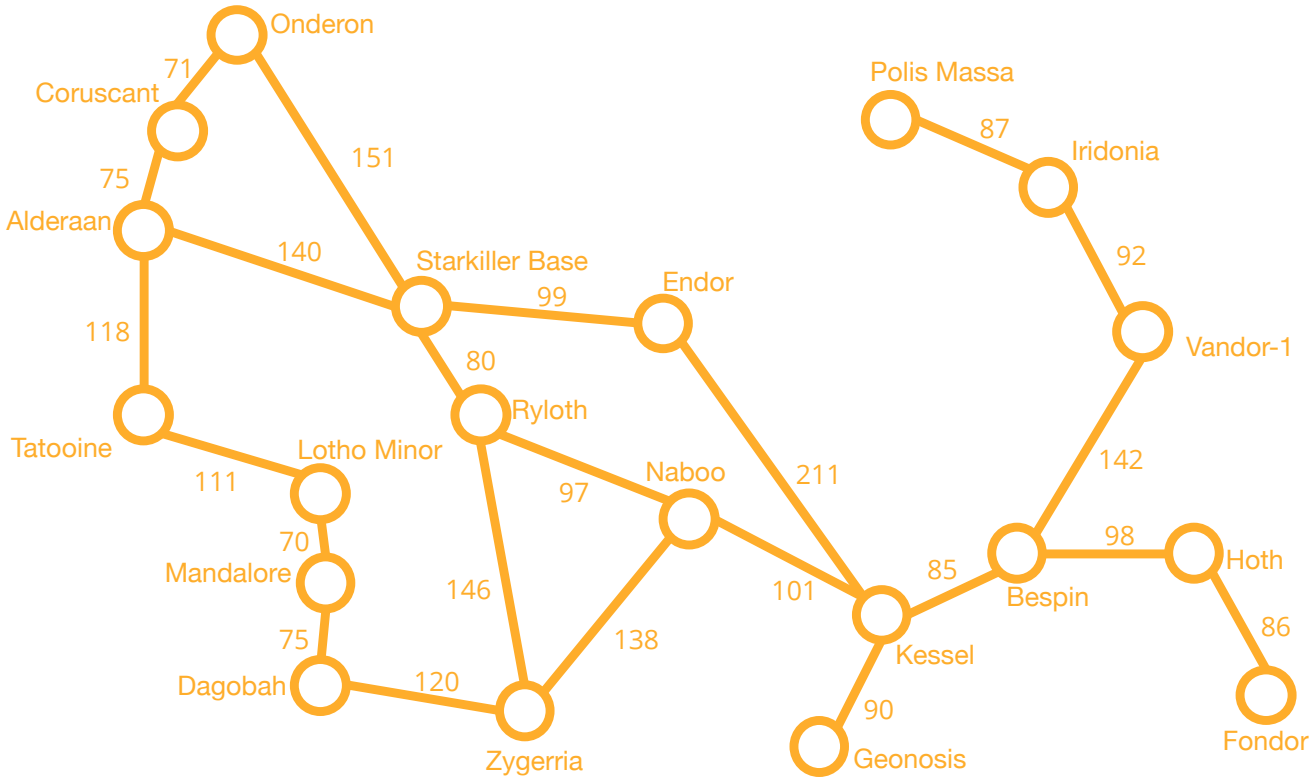
# “Uniform Cost” Search

“In computer science, *uniform-cost* search (UCS) is a tree search algorithm used for traversing or searching a *weighted* tree, tree structure, or graph.” - Wikipedia

# Motivation: Map Navigation Problems

All our search methods so far assume *step-cost = 1*

*This is only true for some problems*



# g(N): the path cost function

- **Our assumption so far: All moves equal in cost**
  - Cost = # of nodes in path-1
  - *g(N)* = *depth(N)* in the search tree

- More general: Assigning a (potentially) unique cost to each step
  - $N_0$, $N_1$, $N_2$, $N_3$ = nodes visited on path *p* from $N_0$ to $N_3$
  - *C(i,j):* Cost of going from $N_i$ to $N_j$
  - If $N_0$ the root of the search tree,

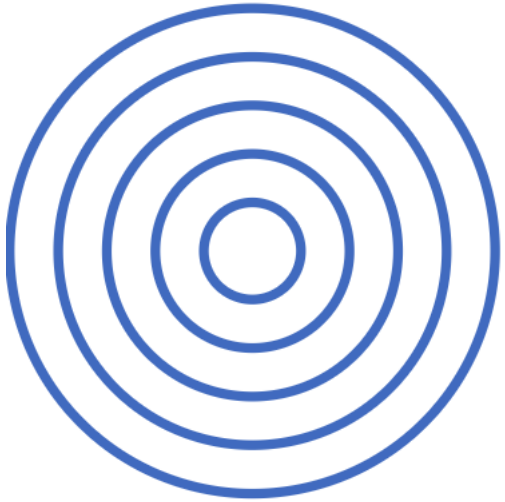
$$g(N3)=C(0,1)+C(1,2)+C(2,3)$$

# Uniform-cost search (UCS)

- Extension of BF-search:
  - **Expand node with *lowest path cost***
- Implementation:
  - *frontier* = priority queue ordered by g(n)
- Subtle but significant difference from BFS:
  - Tests if a node is a goal state when it is selected for expansion, not when it is added to the frontier.
  - Updates a node on the frontier if a better path to the same state is found.
  - So always enqueues a node *before checking whether it is a goal.*

*WHY???*

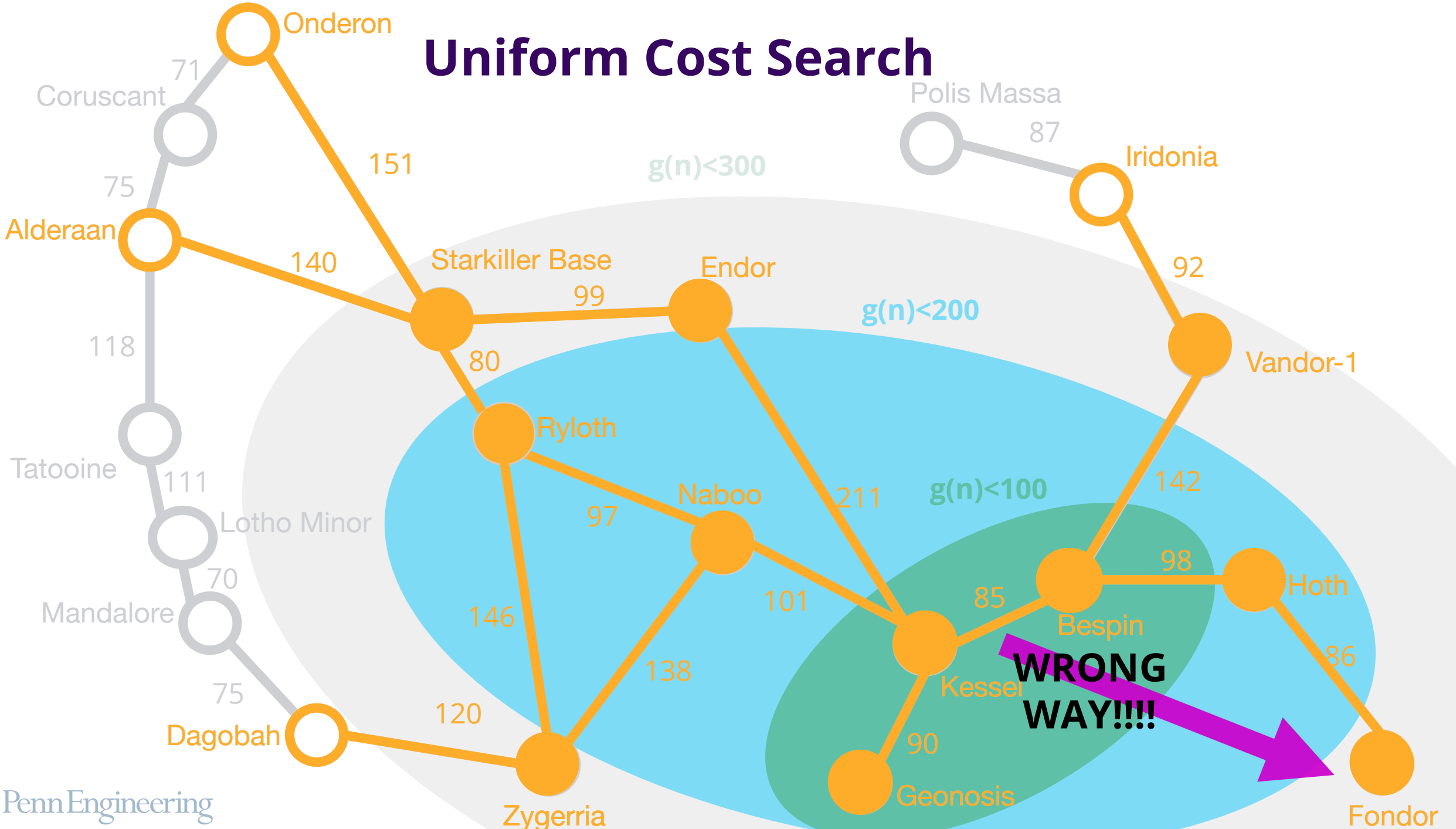
# Shape of Search



- **Breadth First Search** explores equally in all directions. Its frontier is implemented as a FIFO queue. This results in smooth contours or “plys”.

- **Uniform Cost Search** lets us prioritize which paths to explore. Instead of exploring all possible paths equally, it favors lower cost paths. Its frontier is a priority queue. This results in “cost contours”.

# Uniform Cost Search

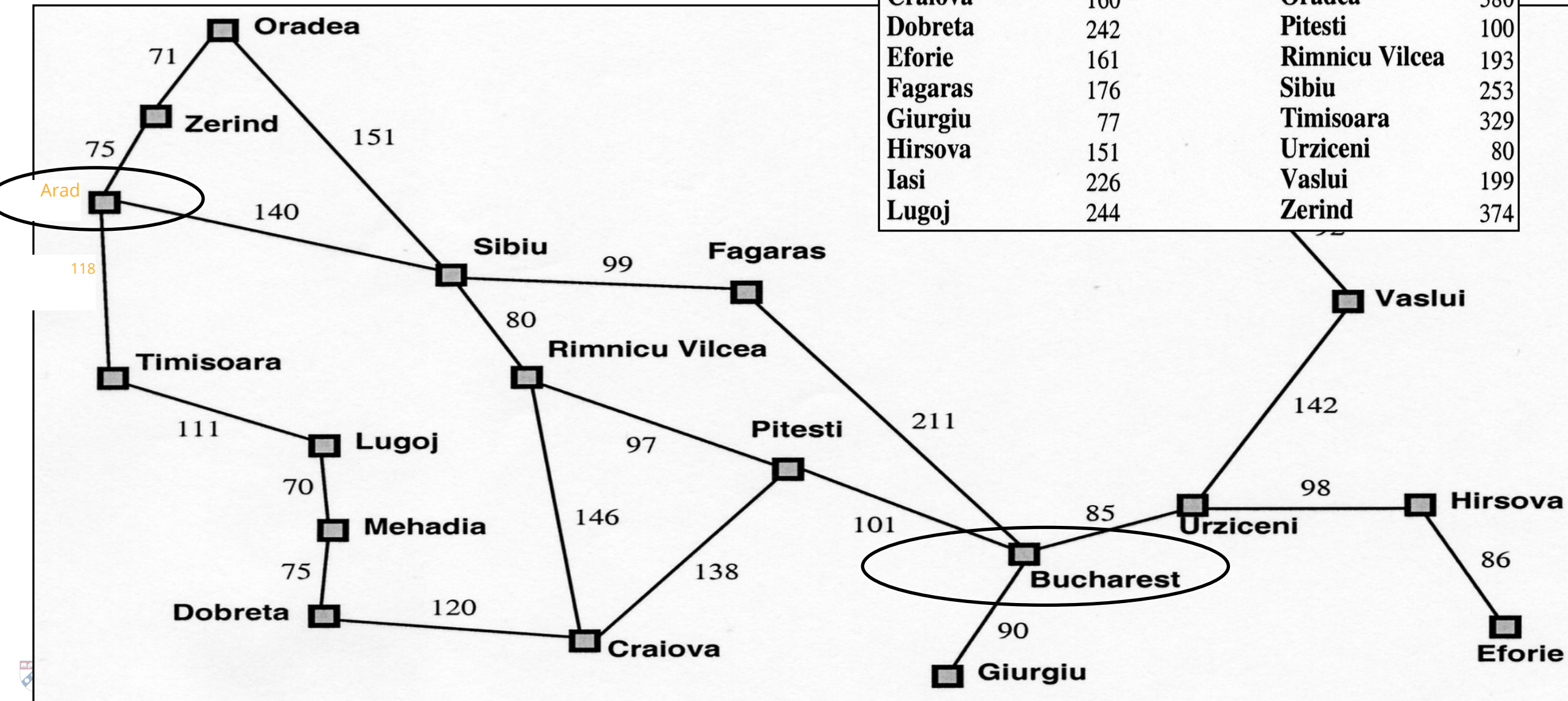


# A Better Idea…

- Node expansion based on *an estimate* which *includes distance to the goal*
- General approach of informed search:
  - *Best-first search*: node selected for expansion based on an *evaluation function* ***f(n)***
    - ✓ ***f(n)*** includes *estimate* of distance to goal ***(new idea!)***
  - Implementation: Sort frontier queue by this new ***f(n)***.
    - Special cases: **greedy search**, and ***A* search***

# Simple, useful estimate heuristic: straight-line distances

| | | | |
|---|---|---|---|
| Arad | 366 | Mehadia | 241 |
| Bucharest | 0 | Neamt | 234 |
| Craiova | 160 | Oradea | 380 |
| Dobreta | 242 | Pitesti | 100 |
| Eforie | 161 | Rimnicu Vilcea | 193 |
| Fagaras | 176 | Sibiu | 253 |
| Giurgiu | 77 | Timisoara | 329 |
| Hirsova | 151 | Urziceni | 80 |
| Iasi | 226 | Vaslui | 199 |
| Lugoj | 244 | Zerind | 374 |



# Heuristic (estimate) functions


*Heureka! ---Archimedes*

**[dictionary]*"A rule of thumb, simplification, or educated guess that reduces or limits the search for solutions in domains that are difficult and poorly understood."***

*Heuristic knowledge* is useful, but not necessarily correct.

*Heuristic algorithms* use heuristic knowledge to solve a problem.

A *heuristic function h(n)* takes a state *n* and returns an *estimate* of the distance from *n* to the goal.

# Greedy Best-First Search

First attempt at integrating heuristic knowledge

# Review: Best-first search

*Basic idea:*

*select node for expansion* with minimal *evaluation function* ***f(n)***

- where ***f(n)*** is some function that includes ***estimate heuristic h(n)*** of the remaining distance to goal

Implement using priority queue

Exactly UCS with *f(n)* replacing *g(n)*

# Greedy best-first search: *f(n) = h(n)*

Expands the node that *is estimated* to be closest to goal

Completely ignores *g(n):* the cost to get to *n*

In our Romanian map, $h(n) = h_{SLD}(n)$ = straight-line distance from *n* to Bucharest

In a grid, the heuristic distance can be calculated using the “Manhattan distance”:

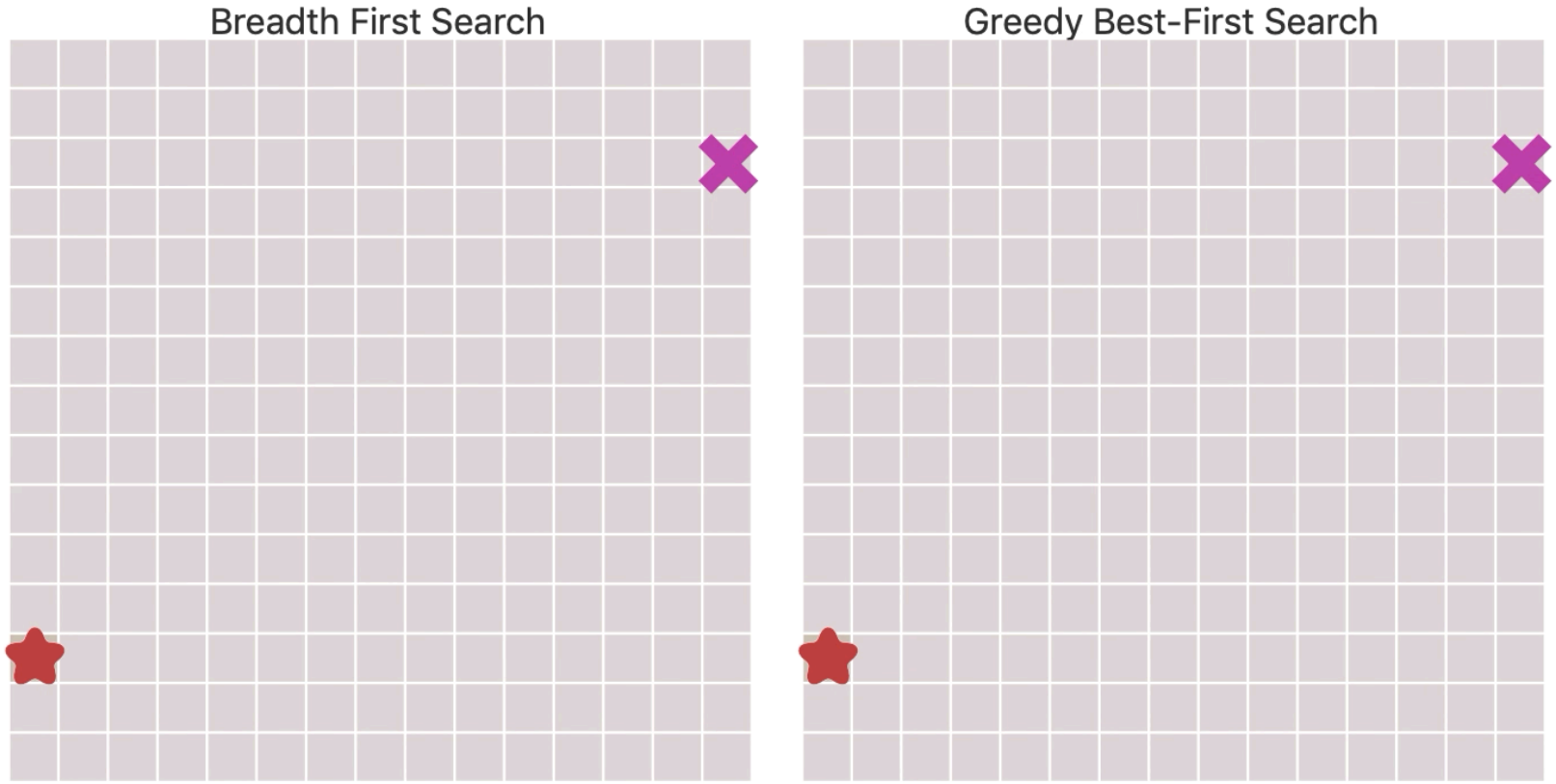
```
def heuristic(a, b):
    # Manhattan distance on a square grid
    return abs(a.x - b.x) + abs(a.y - b.y)
```

# Greedy best-first search

```
frontier = PriorityQueue()
frontier.put(start, 0)
came_from = {}
came_from[start] = None

while not frontier.empty():
   current = frontier.get()

   if current == goal:
      break

   for next in graph.neighbors(current):
      if next not in came_from:
         priority = heuristic(goal, next)
         frontier.put(next, priority)
         came_from[next] = current
```

Code from Amit Patel of Red Blob Games

# BFS v. Greedy Best-First Search




https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Greedy best-first search example

Frontier queue:

Arad 366


- **Initial State = Arad**
- **Goal State = Bucharest**

| | | | |
|---|---|---|---|
| Arad | 366 | Mehadia | 241 |
| Bucharest | 0 | Neamt | 234 |
| Craiova | 160 | Oradea | 380 |
| Dobreta | 242 | Pitesti | 100 |
| Eforie | 161 | Rimnicu Vilcea | 193 |
| Fagaras | 176 | Sibiu | 253 |
| Giurgiu | 77 | Timisoara | 329 |
| Hirsova | 151 | Urziceni | 80 |
| Iasi | 226 | Vaslui | 199 |
| Lugoj | 244 | Zerind | 374 |

# Greedy best-first search example

Frontier queue:

Sibiu 253

Timisoara 329

Zerind 374


| | | | |
|---|---|---|---|
| Arad | 366 | Mehadia | 241 |
| Bucharest | 0 | Neamt | 234 |
| Craiova | 160 | Oradea | 380 |
| Dobreta | 242 | Pitesti | 100 |
| Eforie | 161 | Rimnicu Vilcea | 193 |
| Fagaras | 176 | Sibiu | 253 |
| Giurgiu | 77 | Timisoara | 329 |
| Hirsova | 151 | Urziceni | 80 |
| Iasi | 226 | Vaslui | 199 |
| Lugoj | 244 | Zerind | 374 |

# Greedy best-first search example

Frontier queue:

Fagaras 176

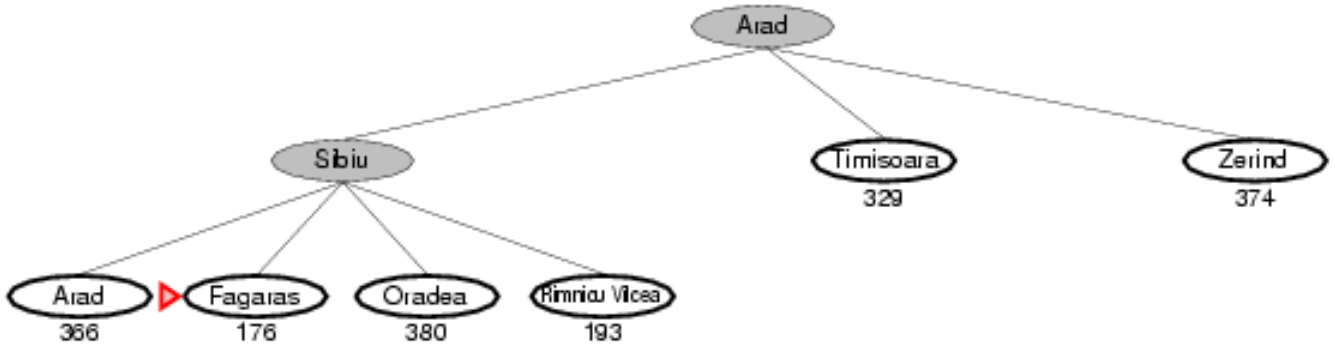
Rimnicu Vilcea 193

Timisoara 329

Arad 366

Zerind 374

Oradea 380




| | | | |
|---|---|---|---|
| Arad | 366 | Mehadia | 241 |
| Bucharest | 0 | Neamt | 234 |
| Craiova | 160 | Oradea | 380 |
| Dobreta | 242 | Pitesti | 100 |
| Eforie | 161 | Rimnicu Vilcea | 193 |
| Fagaras | 176 | Sibiu | 253 |
| Giurgiu | 77 | Timisoara | 329 |
| Hirsova | 151 | Urziceni | 80 |
| Iasi | 226 | Vaslui | 199 |
| Lugoj | 244 | Zerind | 374 |

# Greedy best-first search example

Frontier queue:

Bucharest 0

Rimnicu Vilcea 193

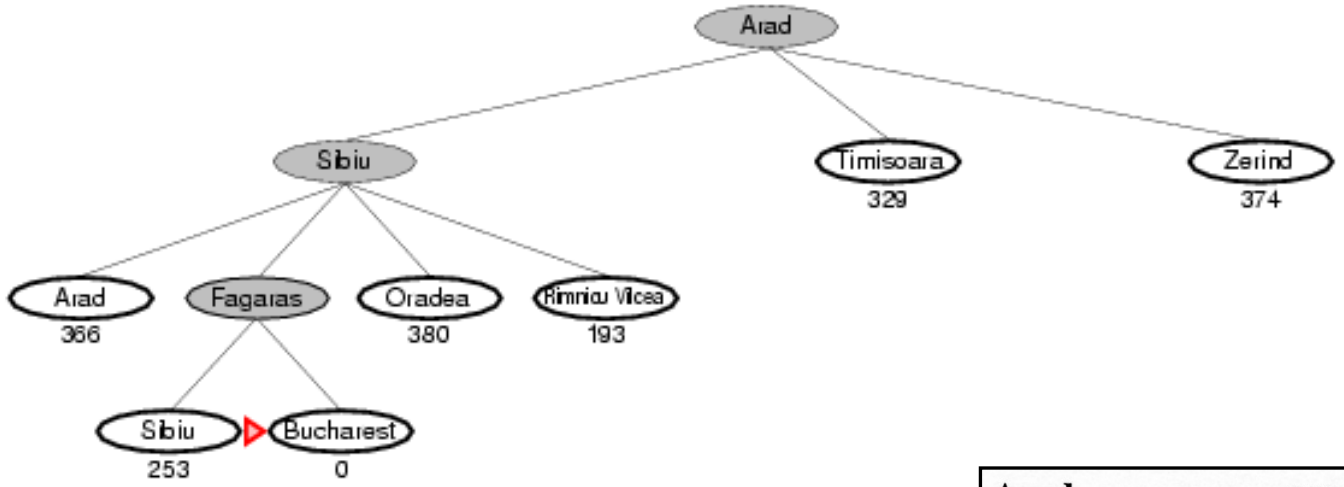
Sibiu 253

Timisoara 329

Arad 366

Zerind 374

Oradea 380



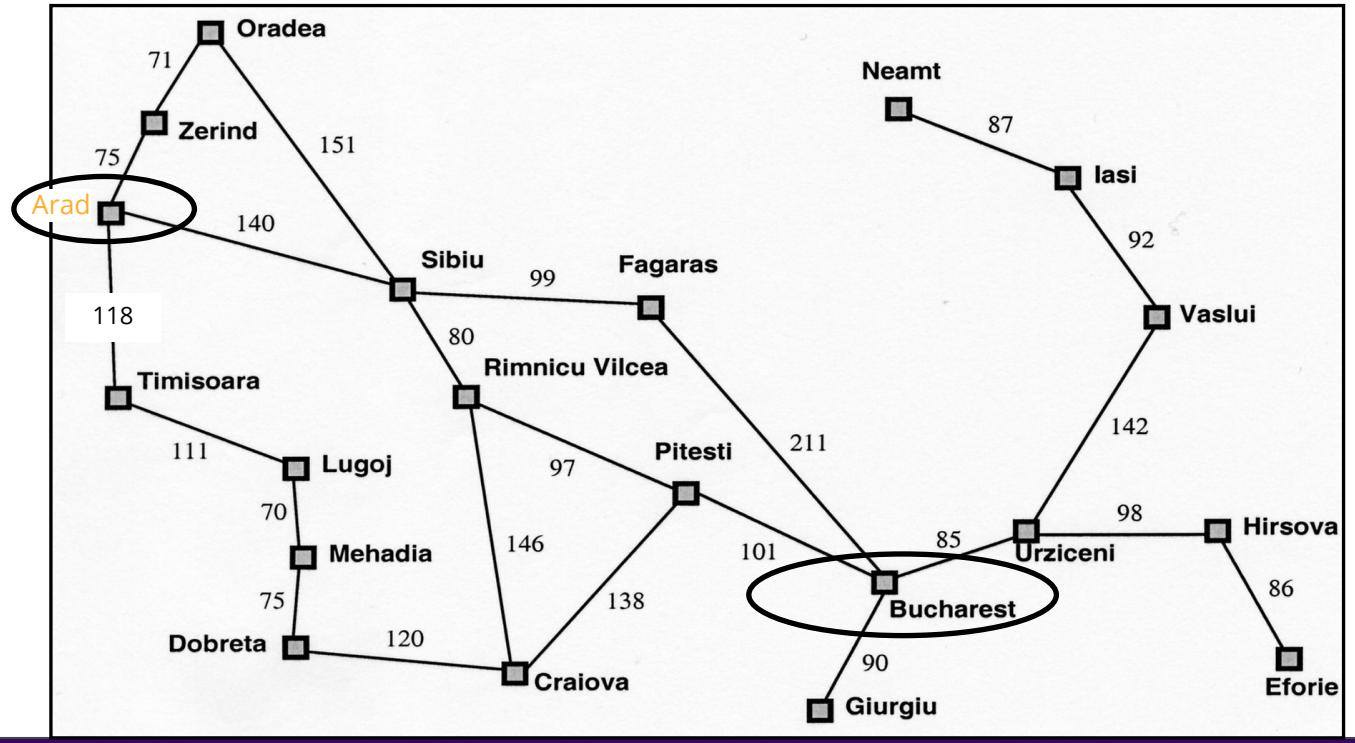

***Goal reached !!***

| | | | |
|---|---|---|---|
| Arad | 366 | Mehadia | 241 |
| Bucharest | 0 | Neamt | 234 |
| Craiova | 160 | Oradea | 380 |
| Dobreta | 242 | Pitesti | 100 |
| Eforie | 161 | Rimnicu Vilcea | 193 |
| Fagaras | 176 | Sibiu | 253 |
| Giurgiu | 77 | Timisoara | 329 |
| Hirsova | 151 | Urziceni | 80 |
| Iasi | 226 | Vaslui | 199 |
| Lugoj | 244 | Zerind | 374 |

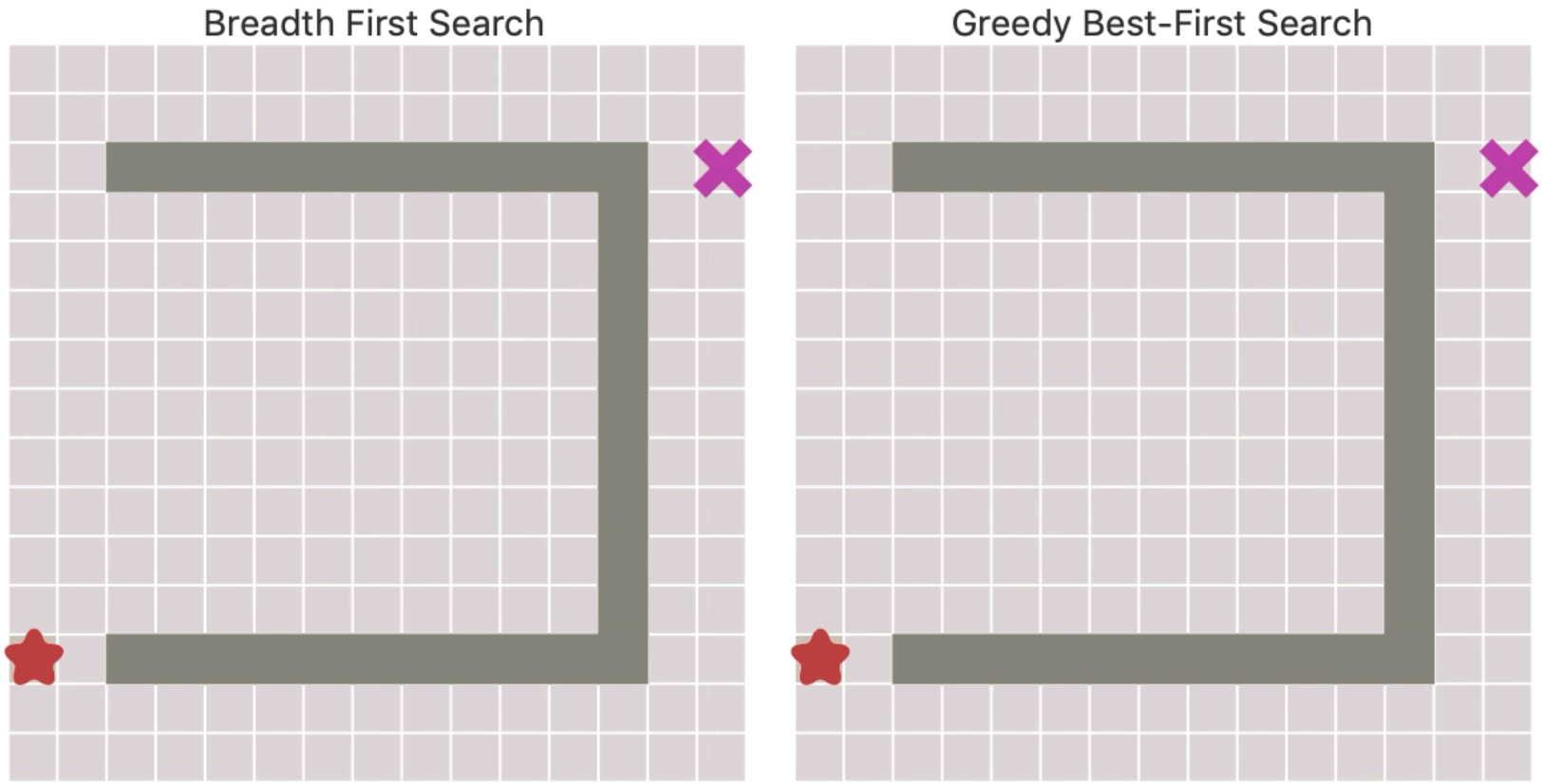
# Properties of greedy best-first search

***Optimal?***

- No!
  - Found: *Arad → Sibiu → Fagaras → Bucharest (450km)*
  - Shorter: *Arad → Sibiu → Rimnicu Vilcea → Pitesti → Bucharest (418km)*



# BFS v. Greedy Best-First Search



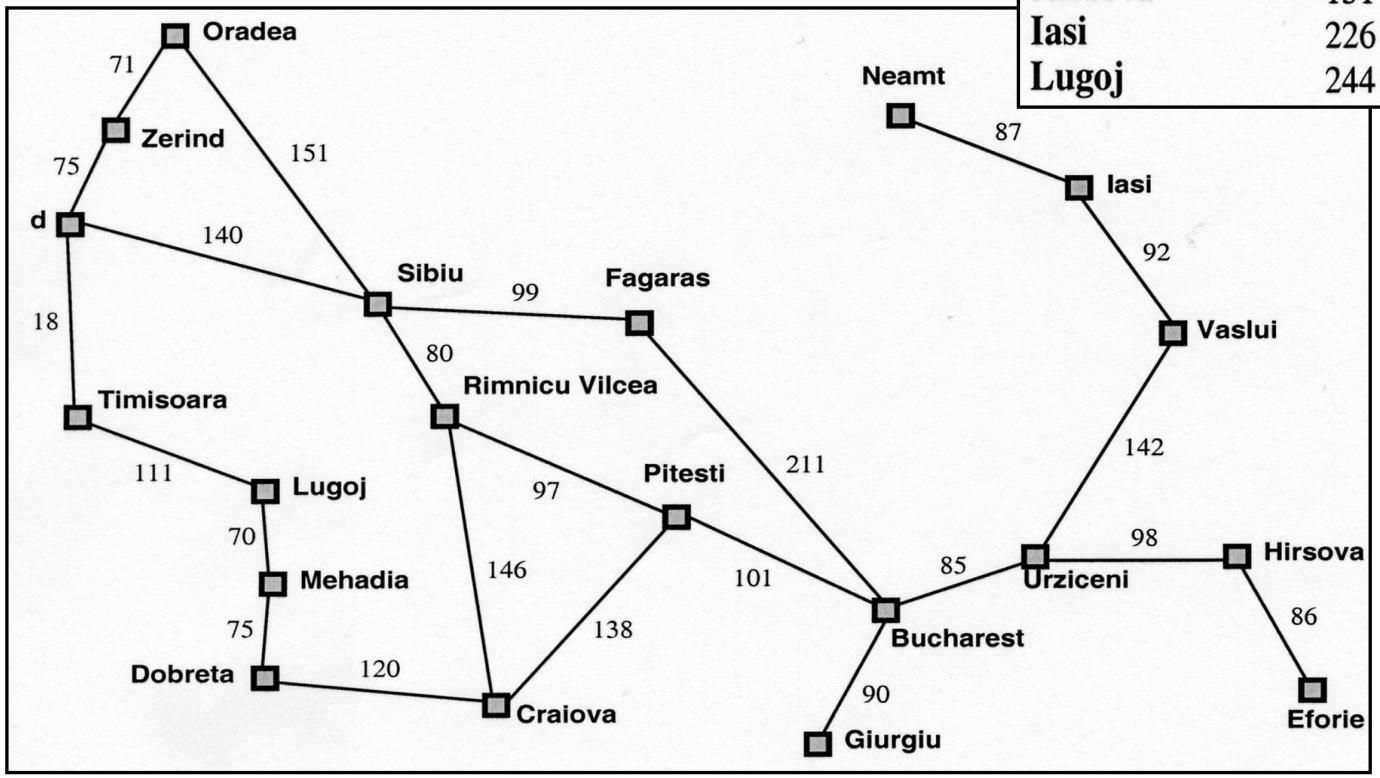

https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Properties of greedy best-first search

| | | | |
|---|---|---|---|
| Arad | 366 | Mehadia | 241 |
| Bucharest | 0 | Neamt | 234 |
| Craiova | 160 | Oradea | 380 |
| Dobreta | 242 | Pitesti | 100 |
| Eforie | 161 | Rimnicu Vilcea | 193 |
| Fagaras | 176 | Sibiu | 253 |
| Giurgiu | 77 | Timisoara | 329 |
| Hirsova | 151 | Urziceni | 80 |
| Iasi | 226 | Vaslui | 199 |
| Lugoj | 244 | Zerind | 374 |

Complete?

- No – can get stuck in loops,
- e.g., Iasi → Neamt → Iasi → Neamt →…



# A* search

AIMA 3.5

# A* search

Best-known form of best-first search.

Key Idea: avoid expanding paths that are already expensive, but expand most promising first.

*Simple idea*: *f(n)=g(n) + h(n)*

- ***g(n)*** the actual cost (so far) to *reach* the node
- ***h(n)*** estimated cost to *get from the node to the goal*
- ***f(n)*** estimated *total cost* of path through *n* to goal

Implementation: Frontier queue as priority queue by increasing *f(n)* *(as expected…)*

# Key concept: Admissible heuristics

A heuristic *h(n)* is *admissible* if it *never overestimates* the cost to reach the goal;

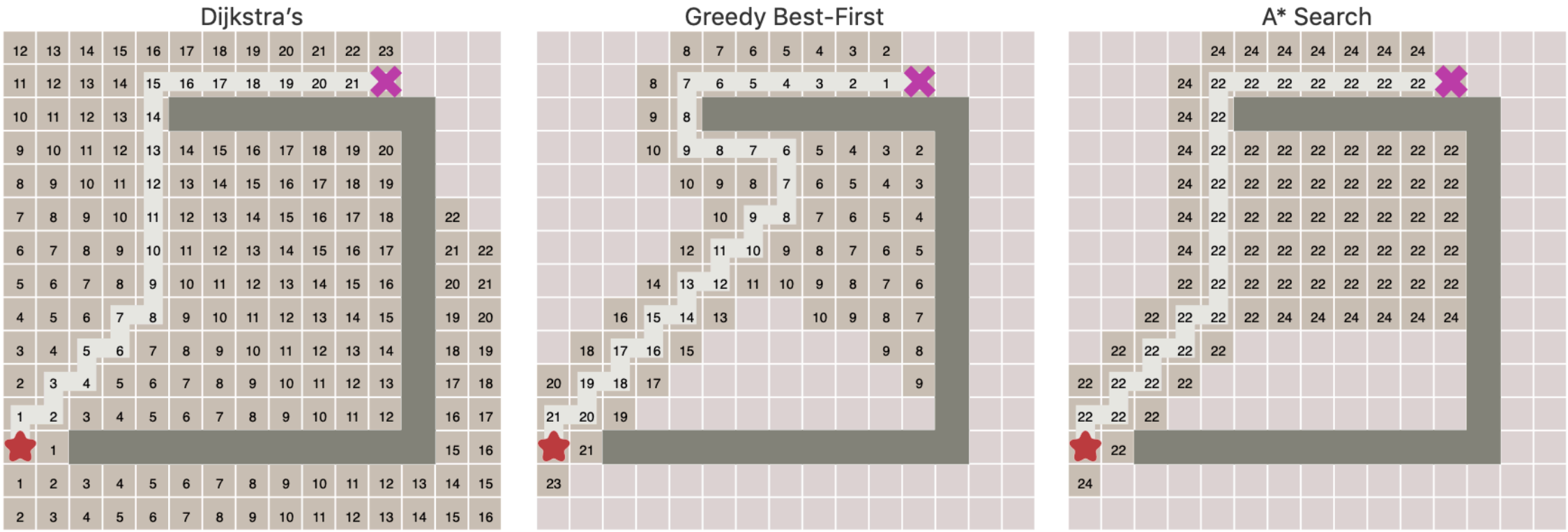
i.e. it is *optimistic*

- Formally: $\forall n$, *n* a node:
  - $h(n) \leq h^*(n)$ where $h^*(n)$ is the true cost from *n*
  - $h(n) \geq 0$ so $h(G)=0$ for any goal *G*.

*Example:* $h_{SLD}(n)$ never overestimates the actual road distance

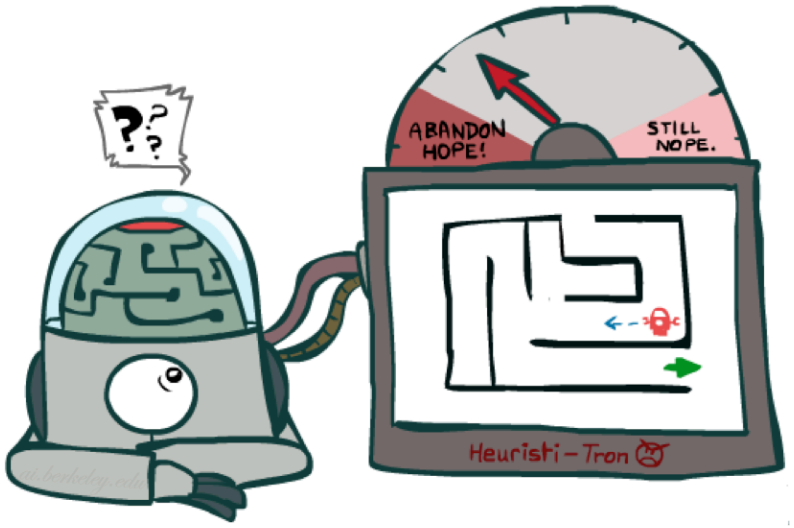
Theorem: If *h(n)* is *admissible*, A* using Tree Search is *optimal*

# A* is optimal with admissible heuristic

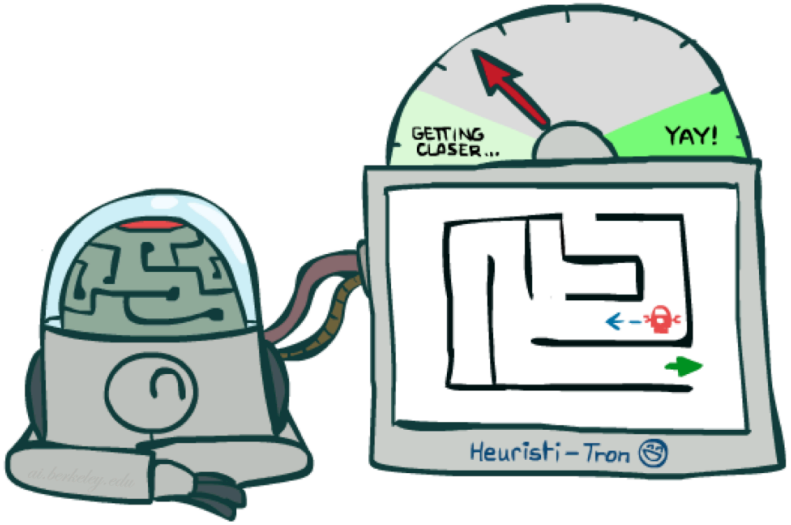



https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Idea: Admissibility




Inadmissible (pessimistic) heuristics break optimality by trapping good plans on the frontier

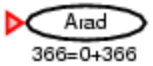



Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs

# A* search example

Frontier queue:

Arad 366



# A* search example

Frontier queue:

Sibiu 393

Timisoara 447

Zerind 449


We add the three nodes we found to the Frontier queue.

We sort them according to the **g()+h()** calculation.

# A* search example

Frontier queue:

Rimricu Vicea 413

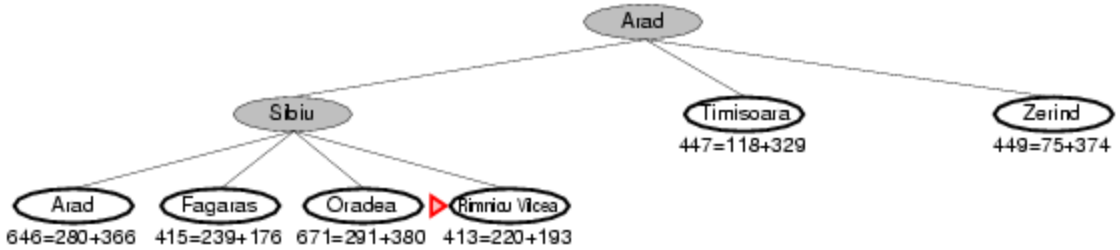
Fagaras 415

Timisoara 447

Zerind 449

Arad 646

Oradea 671




When we expand Sibiu, we run into Arad again. Note that we've already expanded this node once; but we still add it to the Frontier queue again.

# A* search example

Frontier queue:

Fagaras 415

Pitesti 417

Timisoara 447

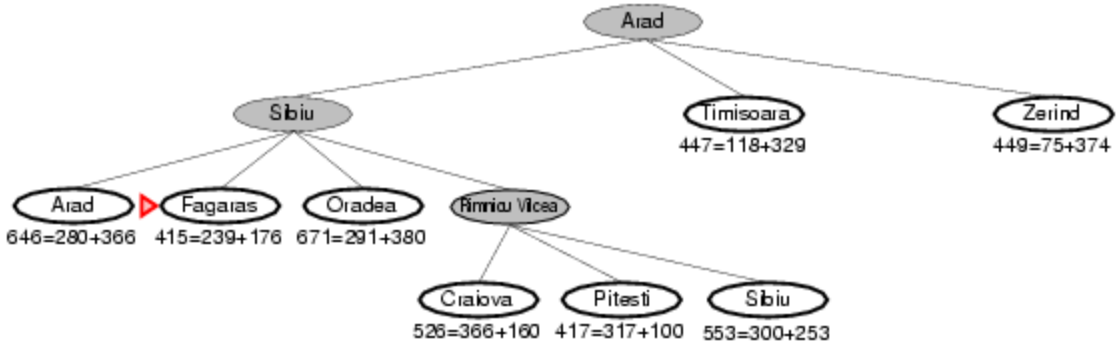
Zerind 449

Craiova 526

Sibiu 553

Arad 646

Oradea 671




We expand Rimricu Vicea.

# A* search example

Frontier queue:

Pitesti 417

Timisoara 447

Zerind 449

***Bucharest 450***

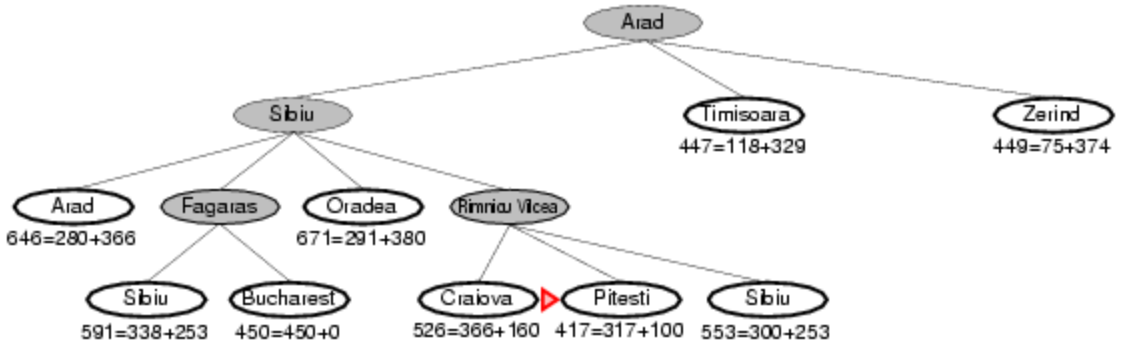
Craiova 526

Sibiu 553

Sibiu 591

Arad 646

Oradea 671




When we expand Fagaras, we find Bucharest, but we're not done. The algorithm doesn't end until we "expand" the goal node – it has to be at the top of the Frontier queue.

# A* search example

Frontier queue:

Bucharest 418

Timisoara 447

Zerind 449

***Bucharest 450***

Craiova 526

Sibiu 553

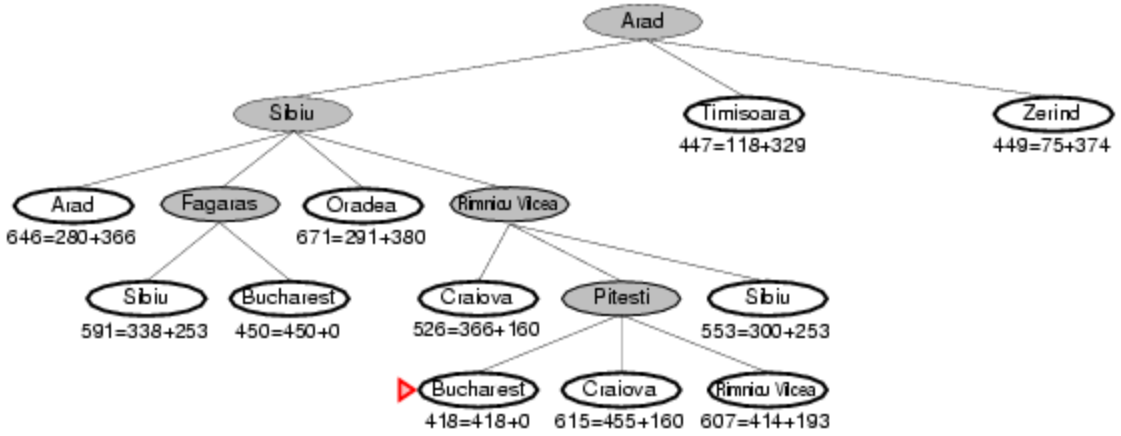
Sibiu 591

Rimricu Vicea 607

Craiova 615

Arad 646

Oradea 671




Note that we just found a better value for Bucharest!

Now we expand this better value for Bucharest since it's at the top of the queue.

We're done and we know the value found is optimal!

# Heuristic functions

For the 8-puzzle

- **Avg. solution cost is about 22 steps**
  - **(branching factor ≤ 3)**
- **(branching factor ≤ 3)**
- **A good heuristic function can reduce the search process**


Start State


Goal State

# Example Admissible heuristics

For the 8-puzzle:

$h_{oop}(n)$ = number of out of place tiles

$h_{md}(n)$ = total Manhattan distance (i.e., # of moves from desired location of each tile)

$h_{oop}(S) = 8$

$h_{md}(S) = 3+1+2+2+2+3+3+2 = 18$


Start State


Goal State

# Relaxed problems

A problem with fewer restrictions on the actions than the original is called a *relaxed problem*

***The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem***

If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_{oop}(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square,* then $h_{md}(n)$ gives the shortest solution

# Defining Heuristics: *h(n)*

Cost of an exact solution to a *relaxed* problem (fewer restrictions on operator)

Constraints on *Full* Problem:

A tile can move from square A to square B *if* A is adjacent to B *and* B is blank.

- Constraints on *relaxed* problems:
  - A tile can move from square A to square B *if* A is adjacent to B. ($h_{md}$)
  - A tile can move from square A to square B *if* B is blank.
  - A tile can move from square A to square B. ($h_{oop}$)

# Dominance: A metric on better heuristics

If $h_2(n) \geq h_1(n)$ for all *n* (both admissible)

- then $h_2$ *dominates* $h_1$

So $h_2$ is optimistic, but more accurate than $h_1$

- $\boldsymbol{h_2}$ is therefore better for search
- Notice: $\boldsymbol{h_{md}}$ dominates $\boldsymbol{h_{oop}}$

Typical search costs (average number of nodes expanded):

*d=12* Iterative Deepening Search = 3,644,035 nodes

$A^*(h_{oop})$ = 227 nodes, $A^*(h_{md})$ = 73 nodes

*d=24* IDS = too many nodes

$A^*(h_{oop})$ = 39,135 nodes, $A^*(h_{md})$ = 1,641 nodes

# The best and worst admissible heuristics

**h*(n) - the (unachievable) Oracle heuristic**

- h*(n) = the true distance from the n to goal

**$h_{\text{we're here already}}(n) = h_{\text{teleportation}}(n) = 0$**

**Admissible: both yes!!!**

**h*(n)** ***dominates all other heuristics***

**$h_{\text{teleportation}}(n)$** ***is dominated by all heuristics***

# Reminders

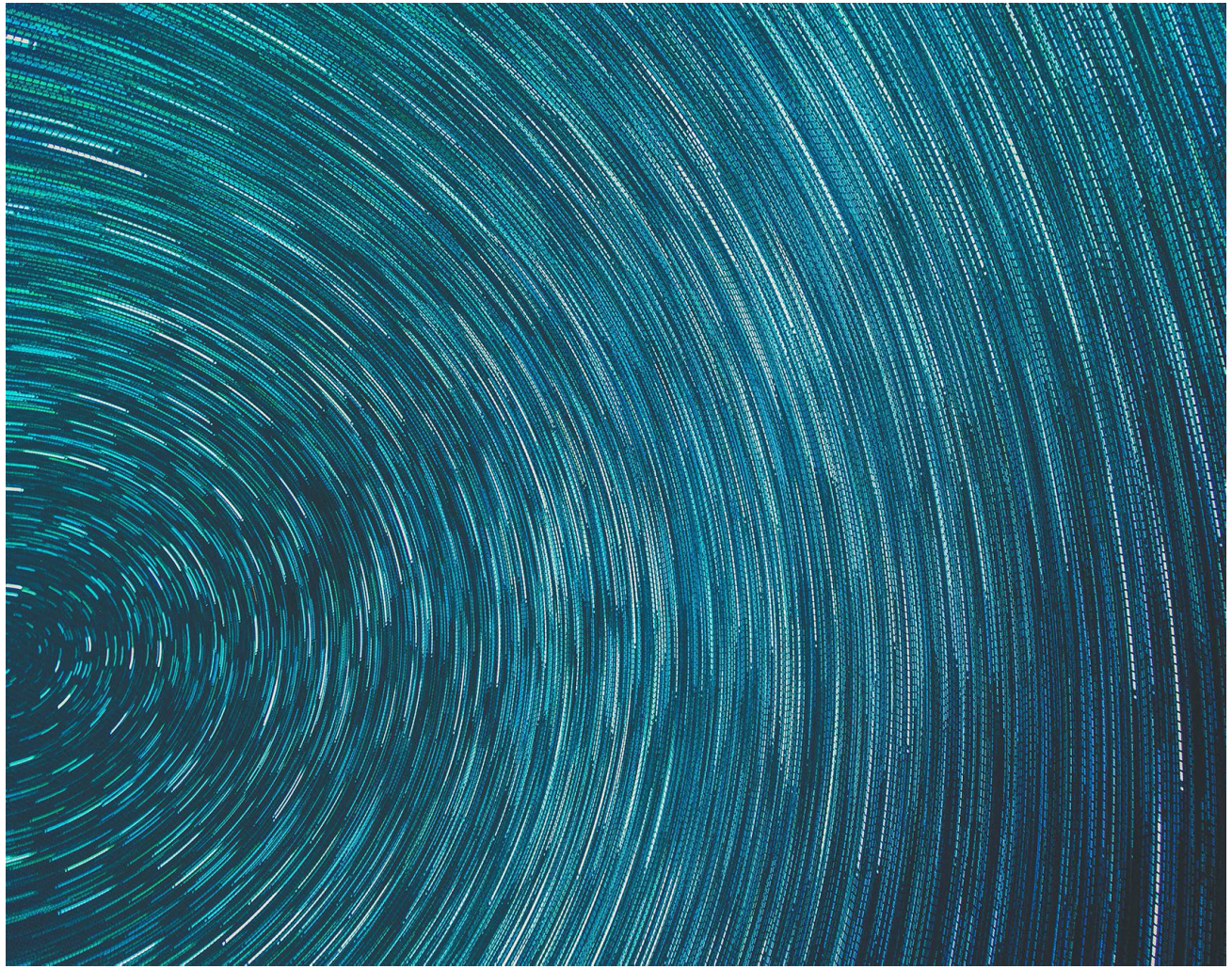
HW2 is due tonight before 11:59pm Eastern time.

HW3 has been released.

Register to vote: https://vote.gov

**Online registration and mail-in request deadline:**
Monday, October 19, 2020

Be a poll worker:
https://www.votespa.com/Resources/Pages/Be-a-Poll-Worker.aspx



# A* search is Optimal

AIMA 3.5

# Key: Admissibility



Inadmissible (pessimistic) heuristics break optimality by pushing good plans too far back on the frontier, which means they may never get expanded.

Admissible (optimistic) heuristics slow down bad plans but never outweigh true costs. That means that the true best plan will always be expanded.



Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

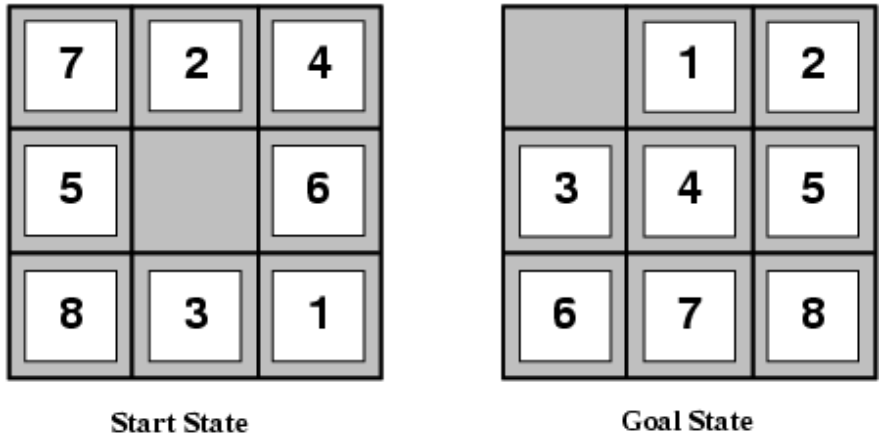
# Admissible Heuristics

A heuristic $h$ is *admissible* (optimistic) if:

$$0 \leq h(n) \leq h^*(n)$$

where $h^*(n)$ is the true cost to a nearest goal

Is Manhattan Distance admissible?




Coming up with admissible heuristics is most of what's involved in using A* in practice.

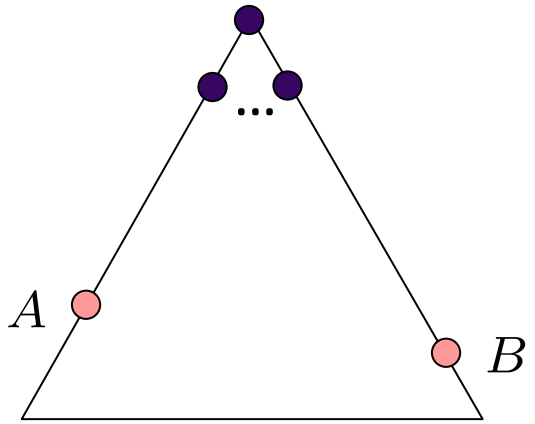
# Optimality of A* Tree Search

Assume:

A is an optimal goal node

B is a suboptimal goal node

h is admissible




Claim:

A will exit the frontier before B

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# Optimality of A* Tree Search

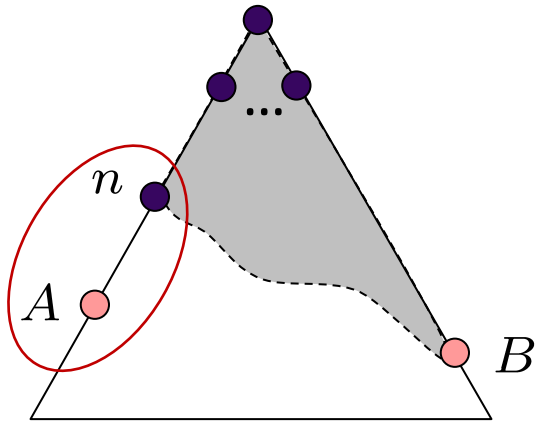
Proof:

Imagine B is on the frontier

Some ancestor *n* of A is on the frontier, too (maybe A!)

Claim: *n* will be expanded before B

- f(n) is less or equal to f(A)




$f(n) = g(n) + h(n)$ Definition of f-cost

$f(n) \leq g(A)$ Admissibility of h

$g(A) = f(A)$ h = 0 at a goal

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# Optimality of A* Tree Search

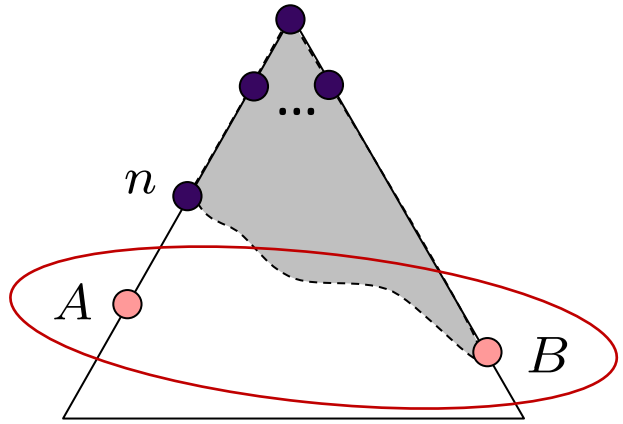
Proof:

Imagine B is on the frontier

Some ancestor *n* of A is on the frontier, too (maybe A!)

Claim: *n* will be expanded before B

- f(n) is less or equal to f(A)
- f(A) is less than f(B)




$g(A) < g(B)$ B is suboptimal

$f(A) < f(B)$ h = 0 at a goal

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# Optimality of A* Tree Search

Proof:

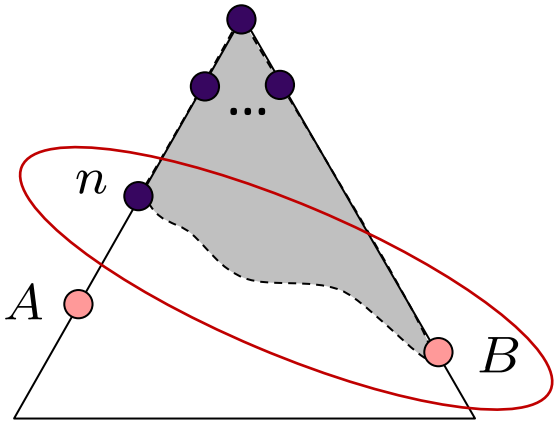
- Imagine B is on the frontier
- Some ancestor *n* of A is on the frontier, too (maybe A!)
- Claim: *n* will be expanded before B

    f(n) is less or equal to f(A)

    f(A) is less than f(B)

    *n* expands before B

- All ancestors of A expand before B
- A expands before B
- A* search is optimal




$$f(n) \leq f(A) < f(B)$$

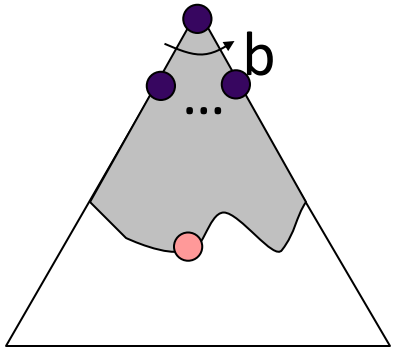
Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# Properties of A*

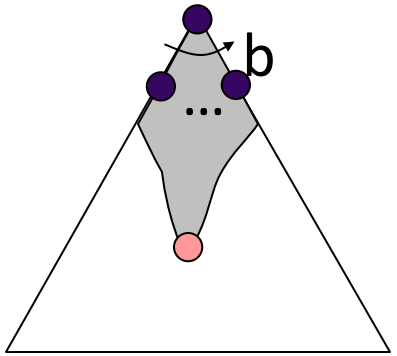
Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# Properties of A*

Uniform-Cost



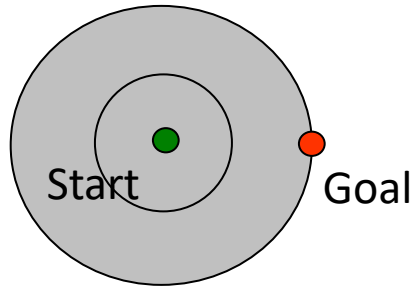

A*



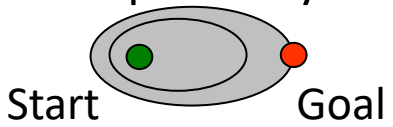

Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# UCS vs A* Contours

Uniform-cost expands equally in all “directions”




A* expands mainly toward the goal, but does hedge its bets to ensure optimality




Slide credit: Dan Klein and Pieter Abbeel
http://ai.berkeley.edu

# A* Applications

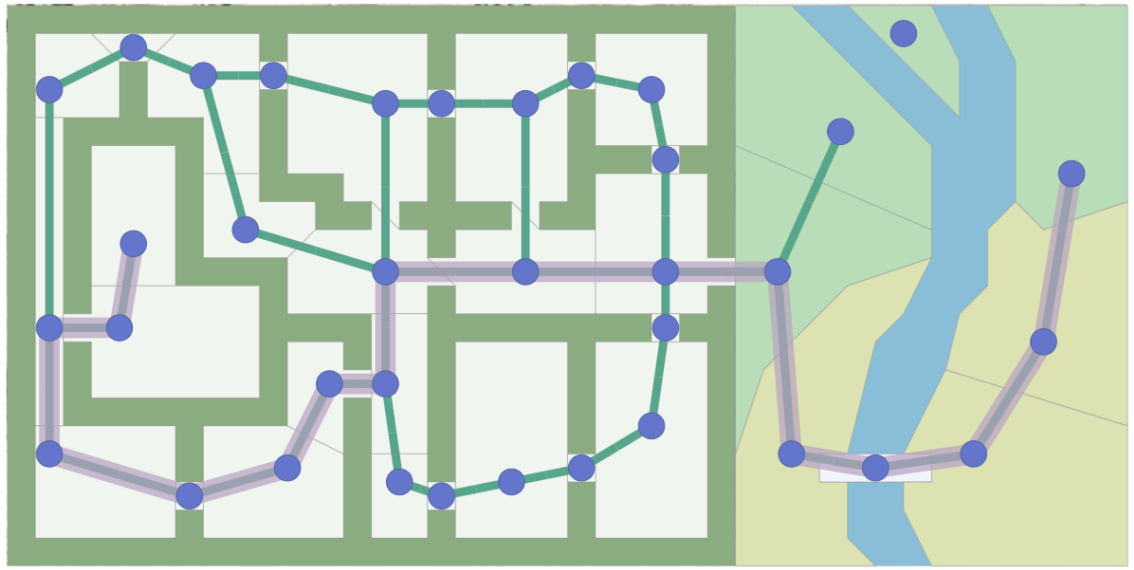
Pathing / routing problems (A* is in your GPS!)

Video games

Robot motion planning

Resource planning problems

…



# Supplemental Reading

I recommend this A* tutorial by Amit Patel of Red Blob Games

https://www.redblobgames.com/pathfinding/a-star/introduction.html

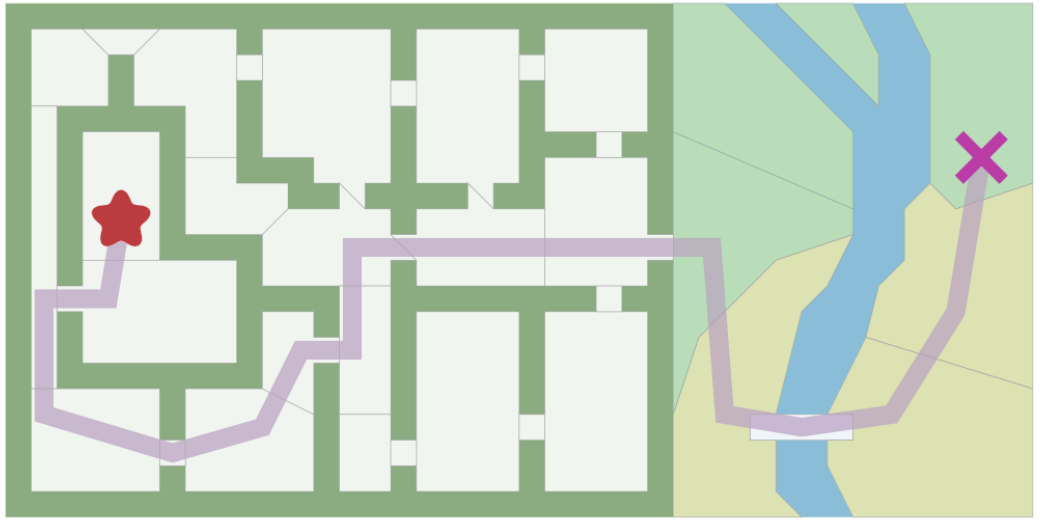
## Introduction to the A* Algorithm

from Red Blob Games

Home Blog Links Twitter About Search

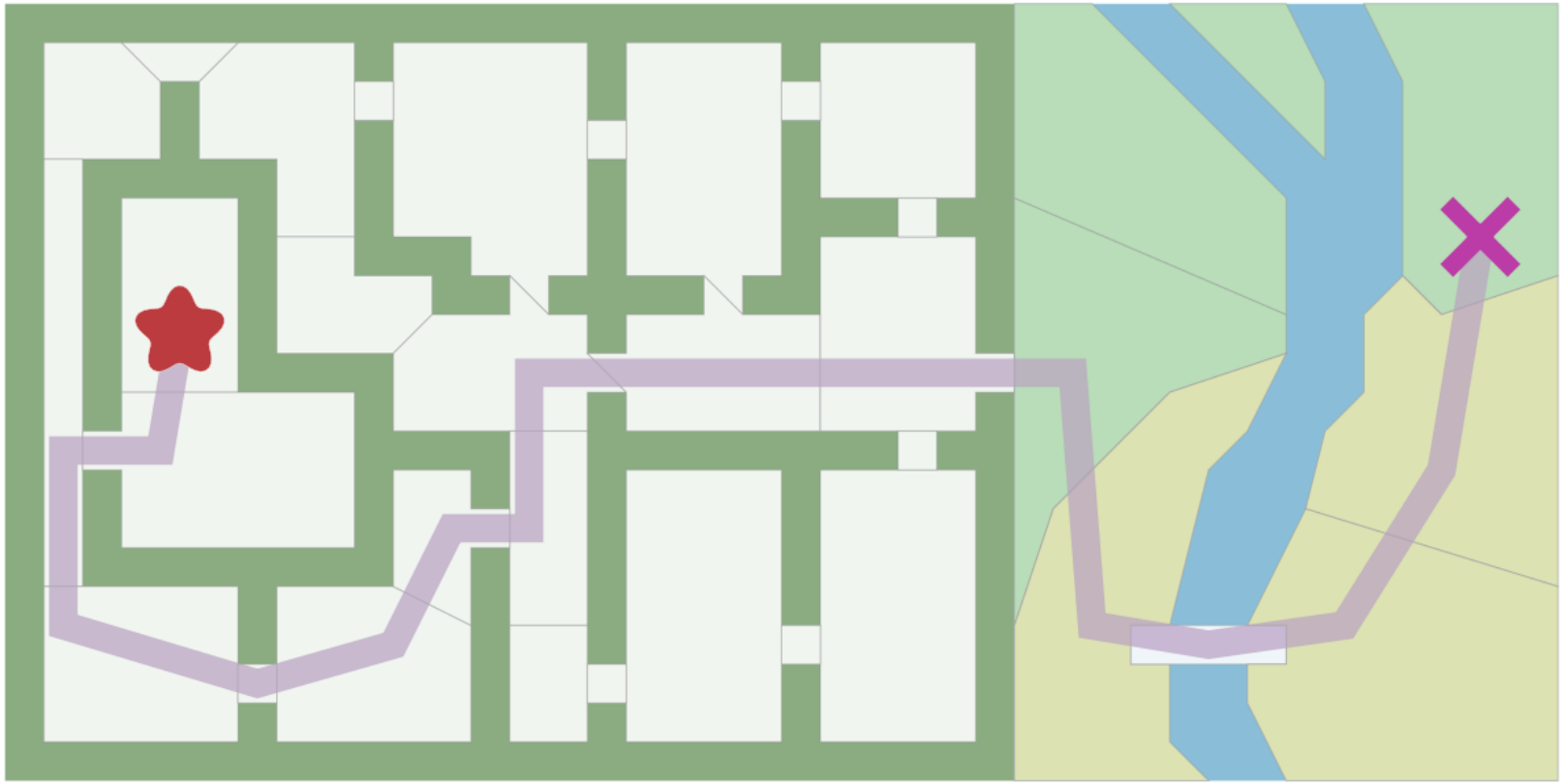
*Created 26 May 2014, updated Aug 2014, Feb 2016, Jun 2016*

In games we often want to find paths from one location to another. We're not only trying to find the shortest distance; we also want to take into account travel time. Move the blob (start point) and cross (end point) to see the shortest path.



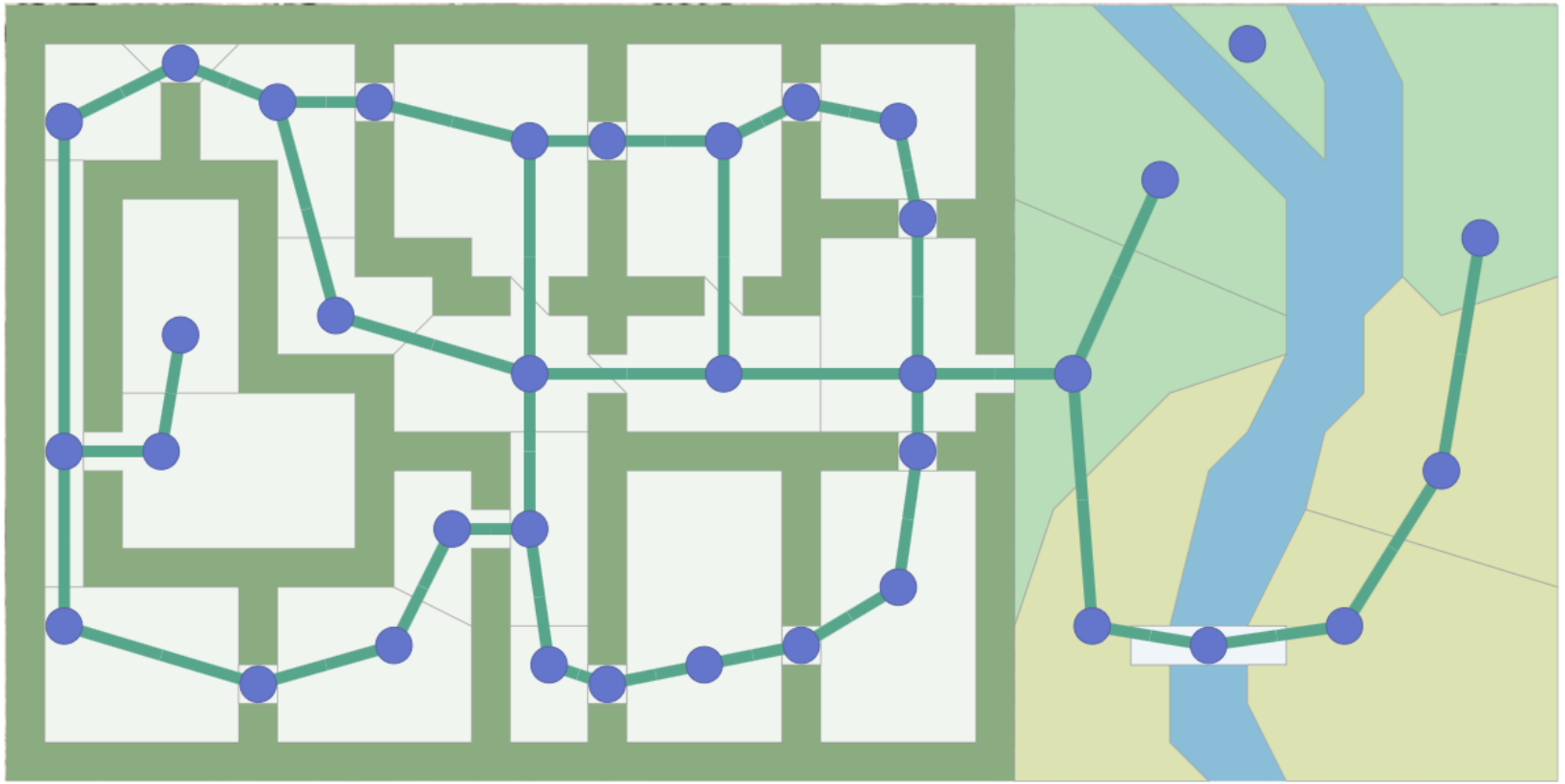
To find this path we can use a *graph search* algorithm, which works when the map is represented as a graph. **A*** is a popular choice for graph search. **Breadth First Search** is the simplest of the graph search algorithms, so let's start there, and we'll work our way up to A*.

# Pathfinding in Games



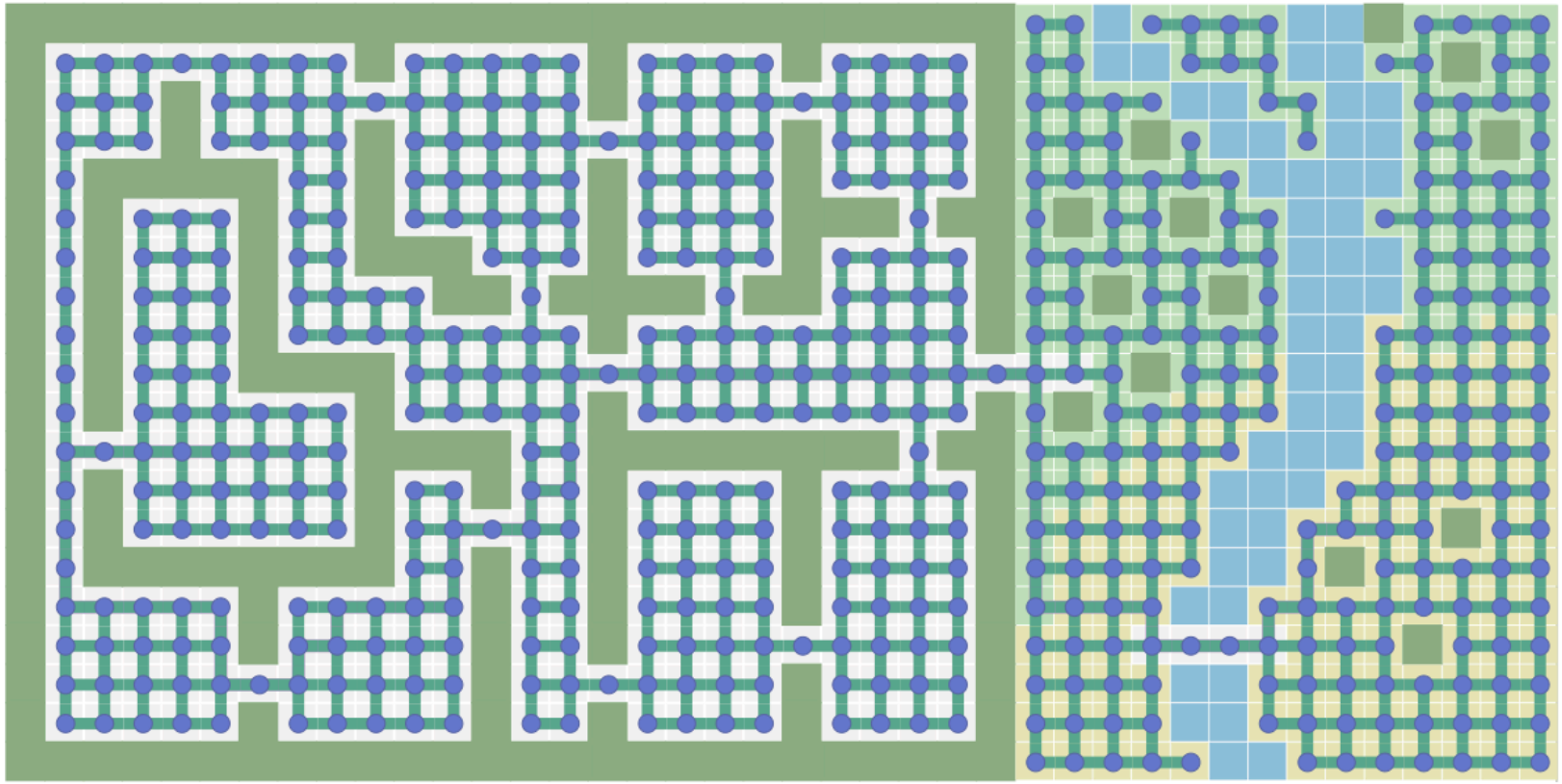
https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Pathfinding in Games



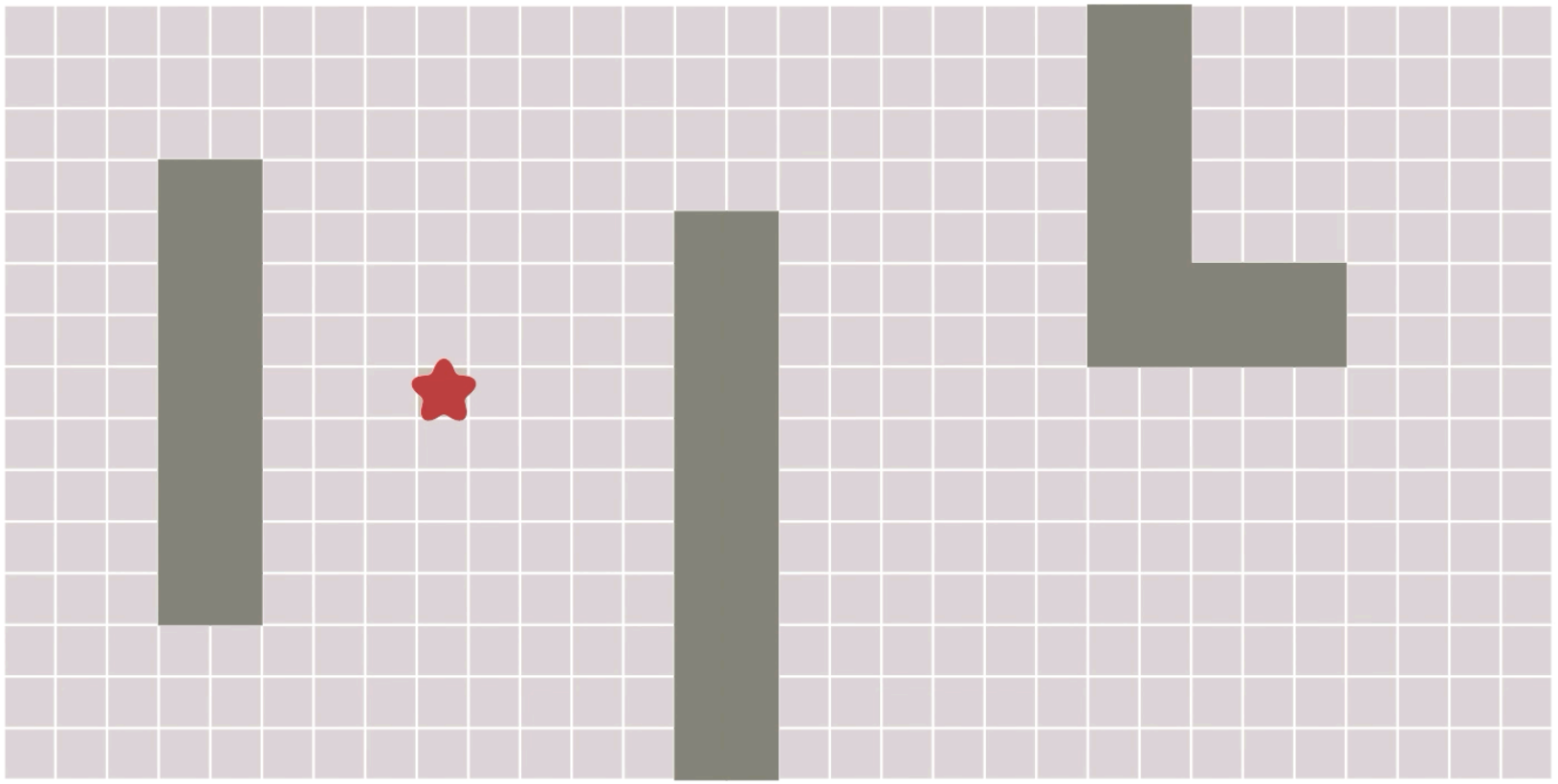
https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Pathfinding in Games



https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Breadth First Search



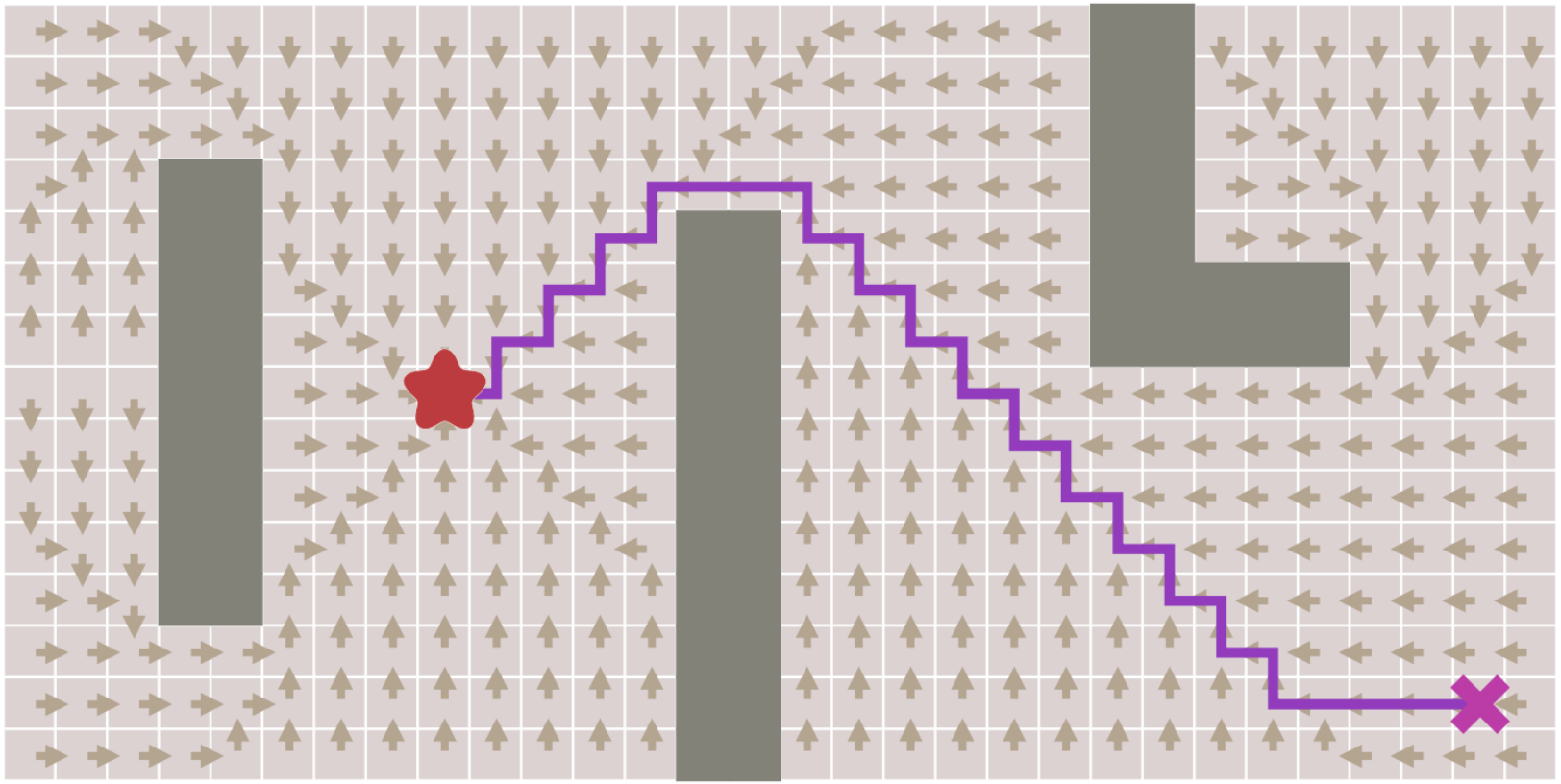
https://www.redblobgames.com/pathfinding/a-star/introduction.html

# BFS in 10 lines of Python

```
frontier = Queue()
frontier.put(start)
visited = {}
visited[start] = True

while not frontier.empty():
    current = frontier.get()
    for next in graph.neighbors(current):
        if next not in visited:
            frontier.put(next)
            visited[next] = True
```

https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Finding the shortest path



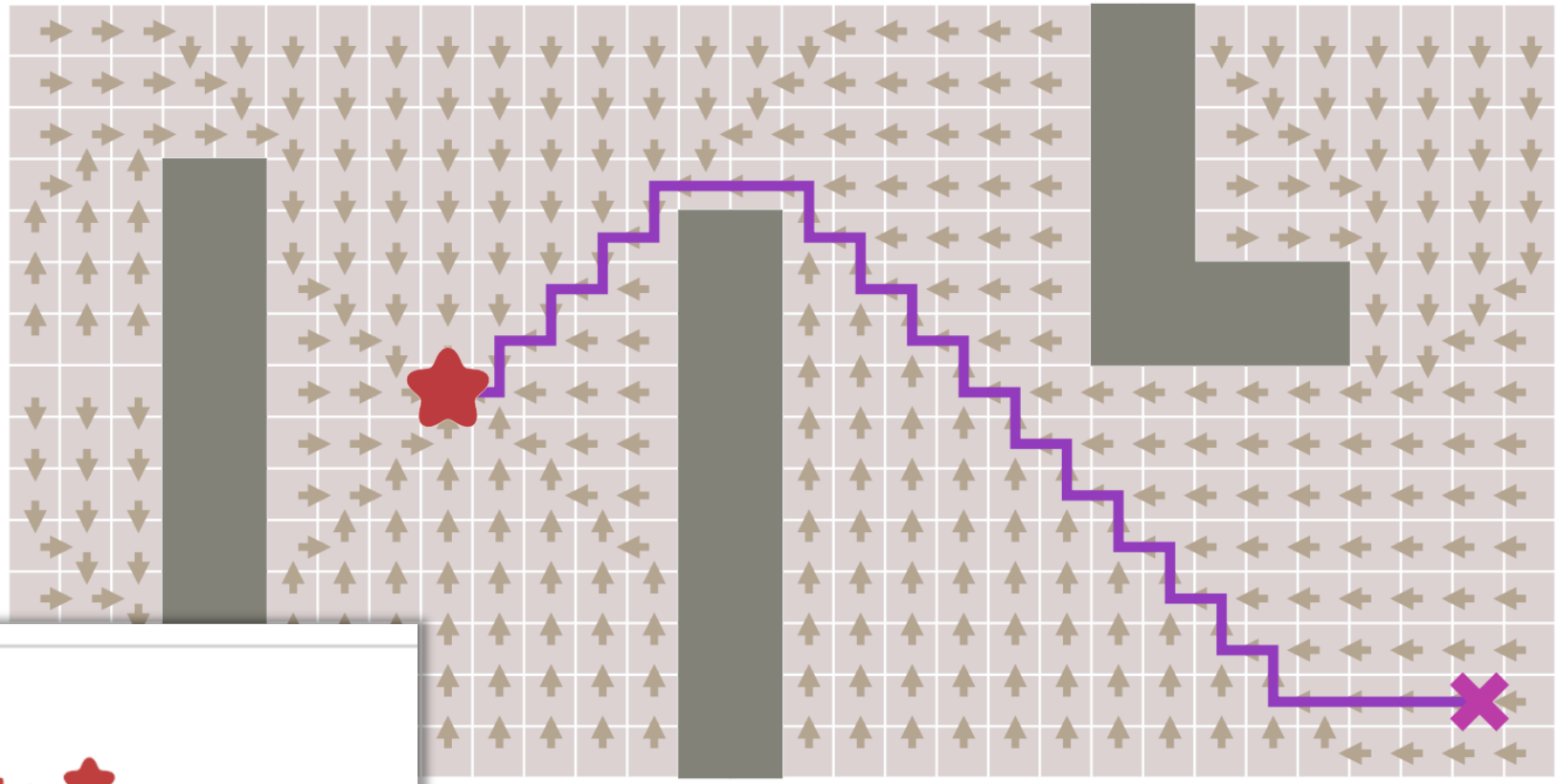
https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Finding the shortest path

```
frontier = Queue()
frontier.put(start)
came_from = {}
came_from[start] = None

while not frontier.empty():
    current = frontier.get()
    for next in graph.neighbors(current):
        if next not in came_from:
            frontier.put(next)
            came_from[next] = current
```

https://www.redblobgames.com/pathfinding/a-star/introduction.html

# Finding the shortest path



```
current = goal
path = []
while current != start:
    path.append(current)
    current = came_from[current]
path.append(start) # optional
path.reverse() # optional
```