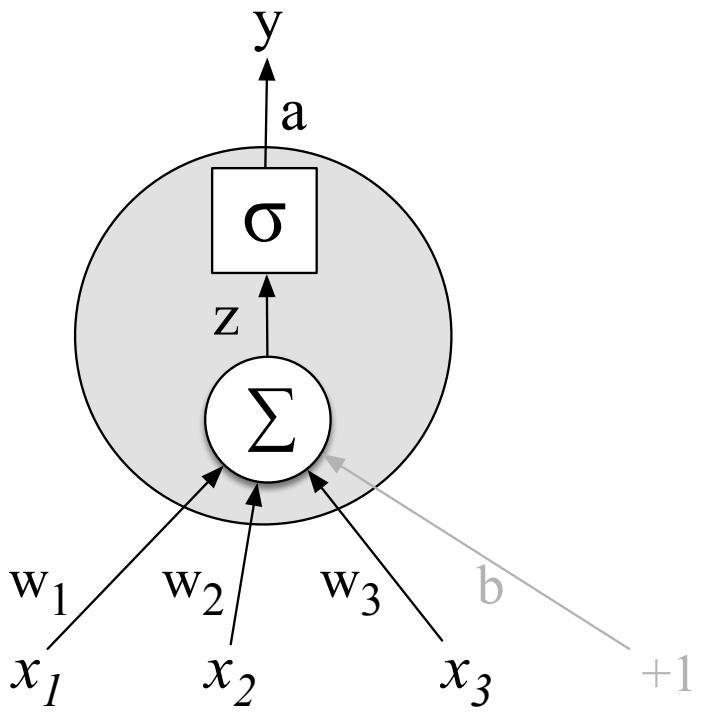
Neural Networks

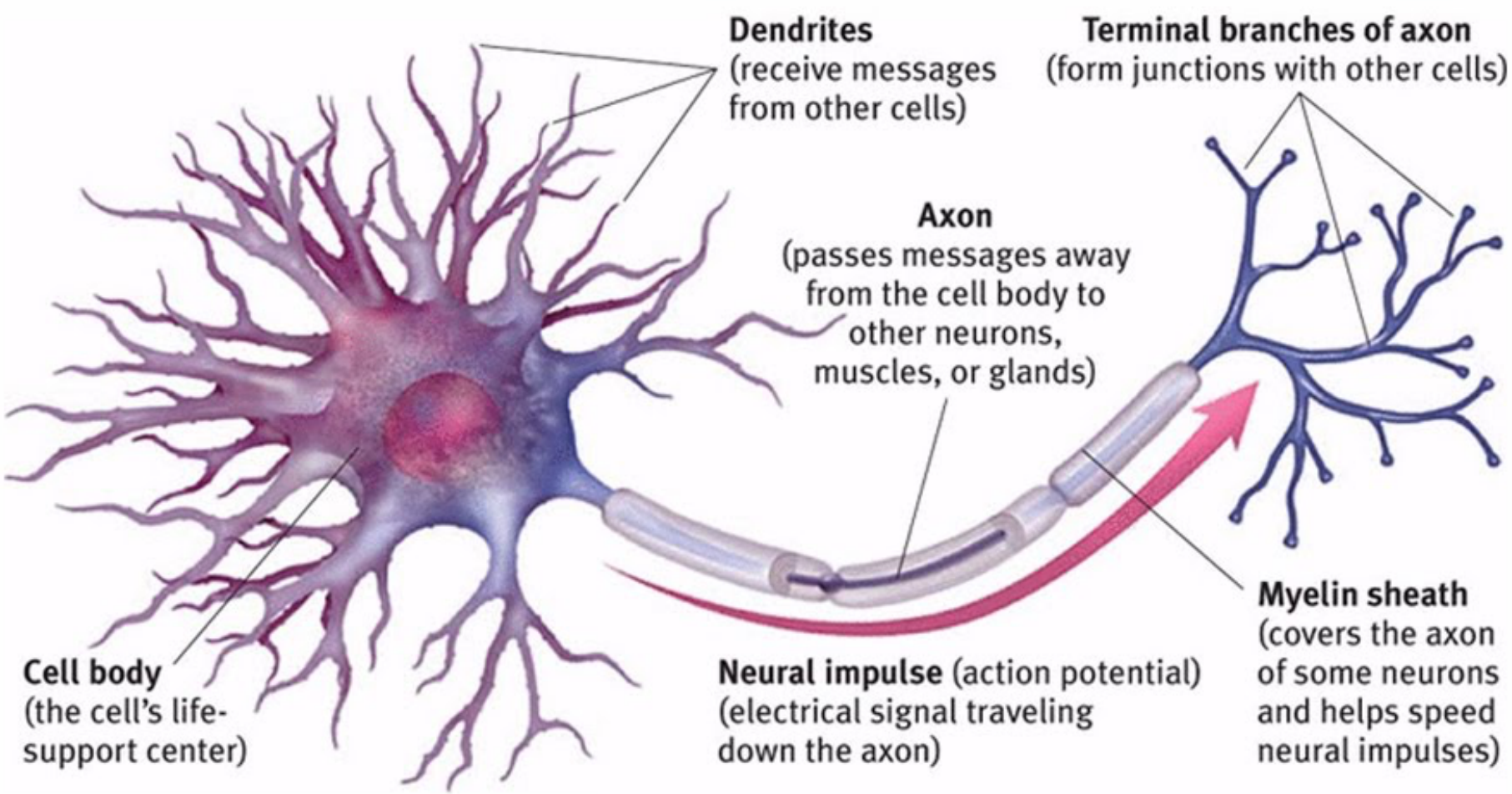
JURAFSKY AND MARTIN CHAPTER 7

Neural Networks

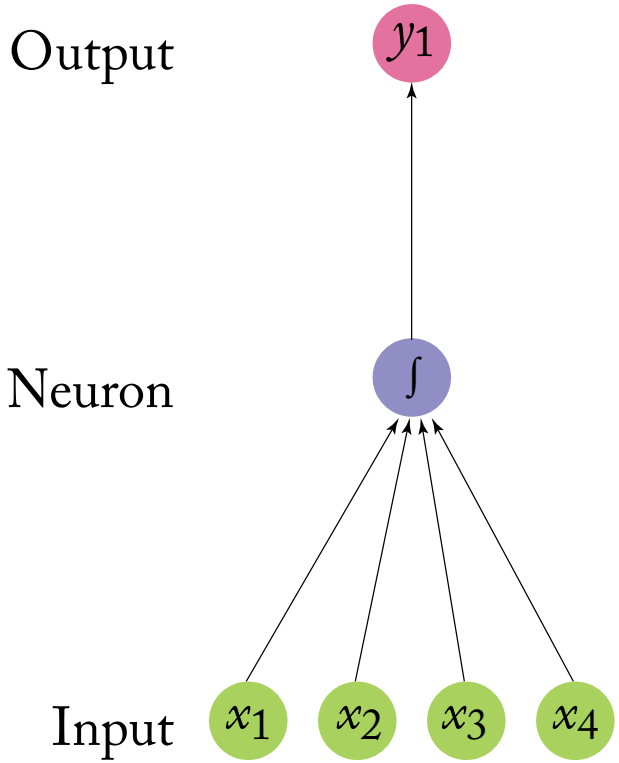
The building block of a neural network is a single computational unit. A unit takes a set of real valued numbers as input, performs some computation.

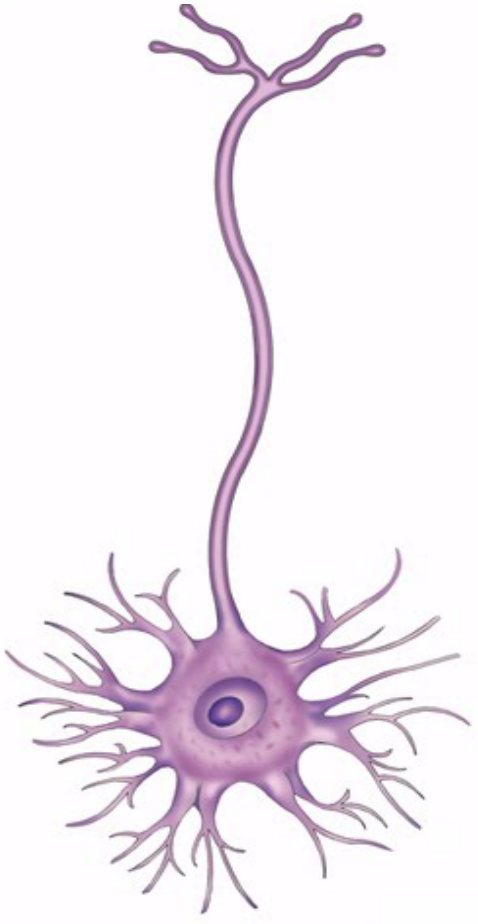


Neural Networks: A brain-inspired metaphor

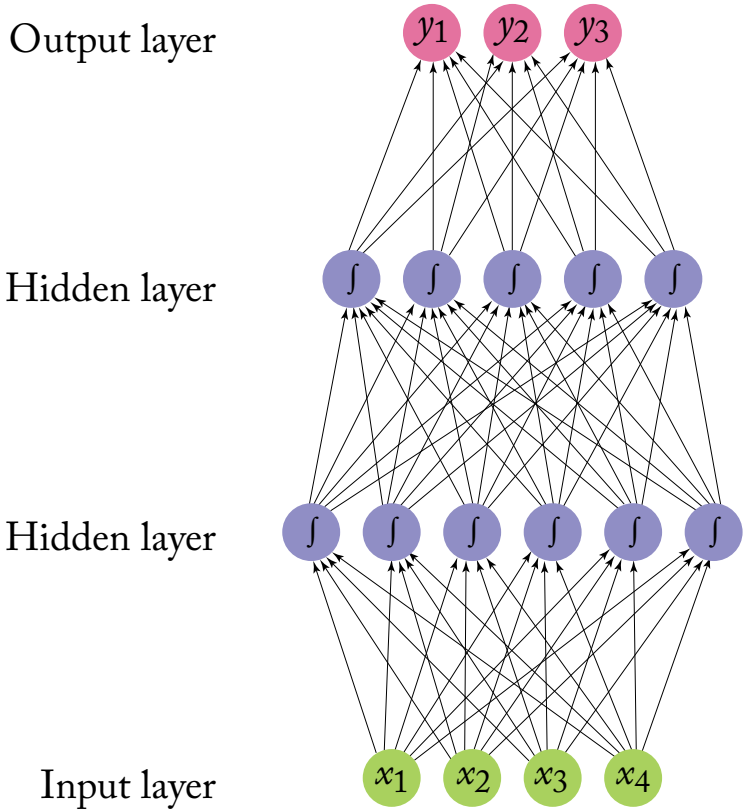


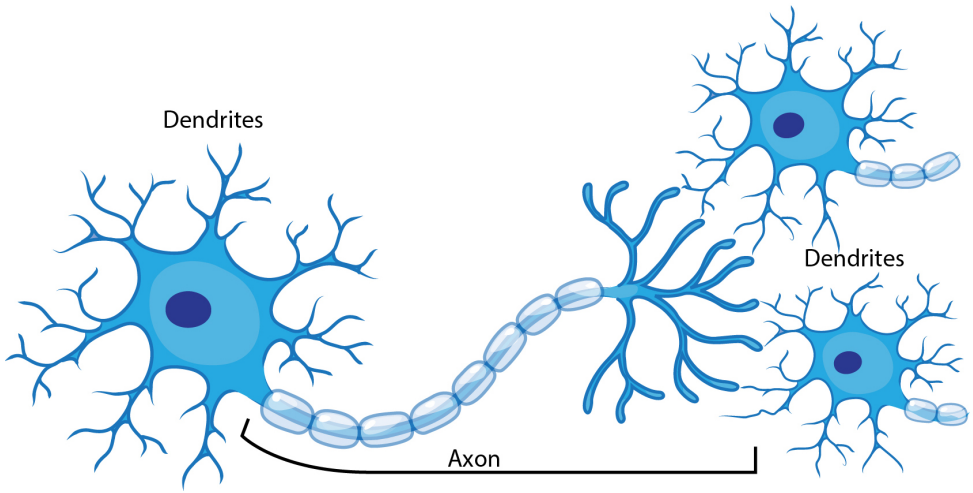
A single neuron





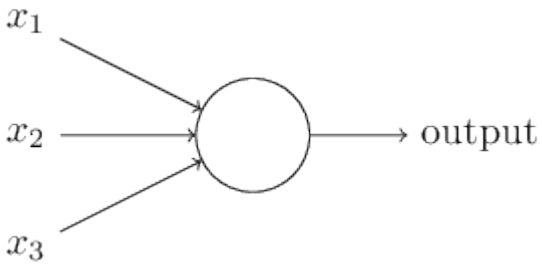
Neural networks

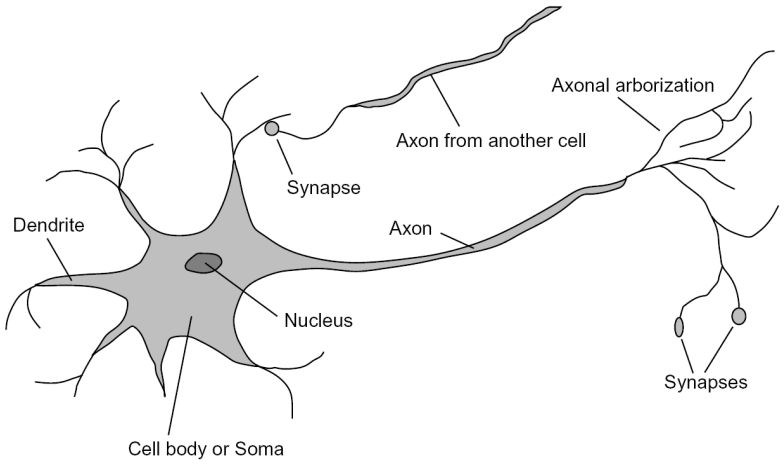




Review: Perceptron

Perceptrons were developed in the 1950s and 1960s loosely inspired by the neuron.





Electronic 'Brain' Teaches Itself

The Navy last week demonstrated the embryo of an electronic computer named the Perceptron which, when completed in about a year, is expected to be the first non-living mechanism able to "perceive, recognize and identify its surroundings without human training or control." Navy officers demonstrating a preliminary form of the device in Washington said they hesitated to call it a machine because it is so much like a "human being without life."

Dr. Frank Rosenblatt, research psychologist at the Cornell Aeronautical Laboratory, Inc., Buffalo, N. Y., designer of the Perceptron, conducted the demonstration. The machine, he said, would be the first electronic device to think as the human brain. Like humans, Perceptron will make mistakes at first, "but it will grow wiser as it gains experience," he said.

The first Perceptron, to cost about $100,000, will have about 1,000 electronic "association cells" receiving electrical impulses from an eyelike scanning device with 400 photocells. The human brain has ten billion

recognize the difference between right and left, almost the way a child learns.

When fully developed, the Perceptron will be designed to remember images and information it has perceived itself, whereas ordinary computers remember only what is fed into them on punch cards or magnetic tape.

Later Perceptrons, Dr. Rosenblatt said, will be able to recognize people and call out their names. Printed pages, longhand letters and even speech commands are within its reach. Only one more step of development, a difficult step, he said, is needed for the device to hear speech in one language and instantly translate it to speech or writing in another language.

Self-Reproduction

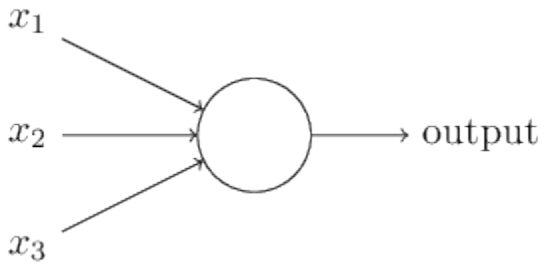
In principle, Dr. Rosenblatt said, it would be possible to build Perceptrons that could reproduce themselves on an assembly line and which would be "conscious" of their existence.

Perceptron, it was pointed out, needs no "priming." It is not nec-

Review: Perceptron

Perceptron has inputs, $x_1, x_2, .. x_N$, and weights $w_1, w_2, .. w_N$

The perceptron outputs 0 or 1, based on the weighted sum is less than or greater than a threshold value

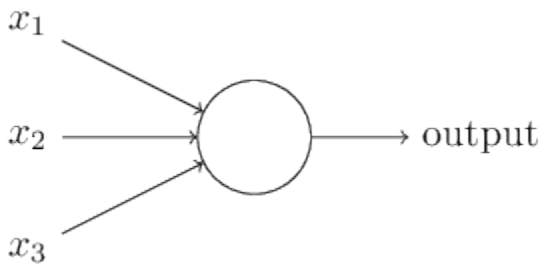


$$\text{output} = \begin{cases} 0 & \text{if } \sum_j w_j x_j \leq \text{threshold} \\ 1 & \text{if } \sum_j w_j x_j > \text{threshold} \end{cases}$$

Perceptrons for decision making

We can think about the perceptron or the sigmoid neuron as a device that makes decisions by weighing up evidence.

Example: Suppose there's a cheese festival in your town. You like cheese.



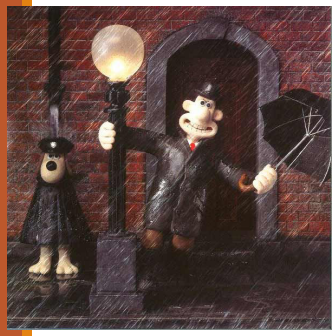
Example from Michael Nielsen's book Neural Networks and Deep Learning

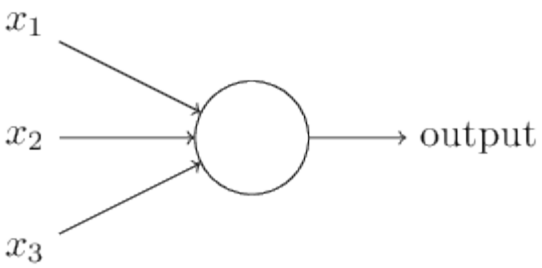
Perceptrons for decision making

You might use 3 factors to decide whether to go.

1. Is the weather good?
2. Can your loyal companion come with you?
3. Is the festival near public transit?

These can be the binary input values to a perceptron



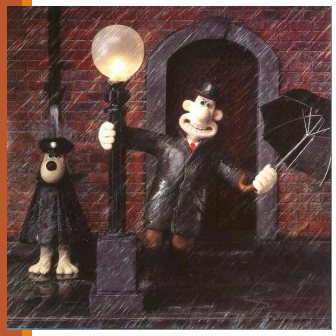


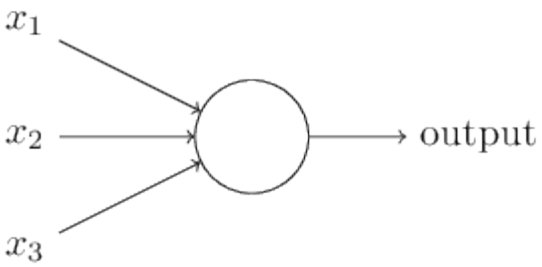
Perceptrons for decision making

By varying weights and the threshold we get different models of decision making

Example 1: $w_1 = 6$ $w_2 = 2$ $w_3 = 2$, threshold = 5

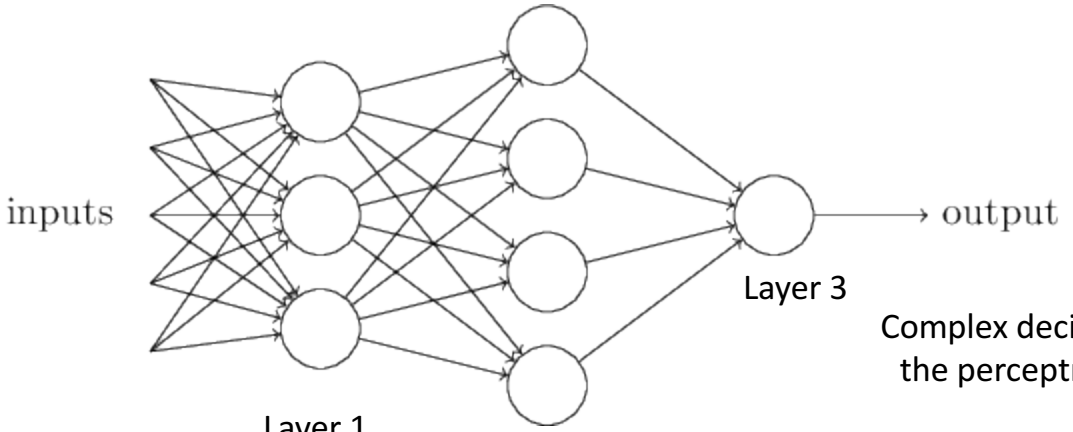
Example 2: $w_1 = 6$ $w_2 = 2$ $w_3 = 2$, threshold = 3





Perceptrons for decision making

- A complex network of perceptrons could make quite subtle decisions:



1st layer makes 3 simple decisions by weighing the input evidence

2nd layer makes a decision by weighing up the results from the 1st layer of decision-making

Complex decisions can be made by the perceptron in the third layer

Weights, bias and dot products

- Two notational changes simplify the way that perceptrons are described.
- The first change is to replace the weighted sum as a dot product

$$w \cdot x \equiv \sum_j w_j x_j$$

- The second change is to move the threshold to the other side of the inequality, and to replace it by a *bias*, b≡−threshold

$$\text{output} = \begin{cases} 0 & \text{if } \sum_j w_j x_j \leq \text{threshold} \\ 1 & \text{if } \sum_j w_j x_j > \text{threshold} \end{cases}$$

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

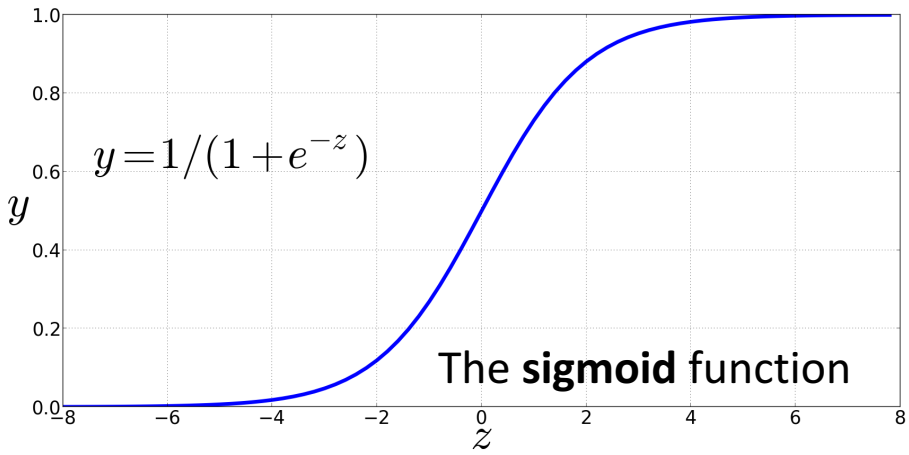
Activation Functions

Instead of directly outputting $z = w \cdot x + b$, which is a linear function of x, neuron units apply a non-linear function f to z.

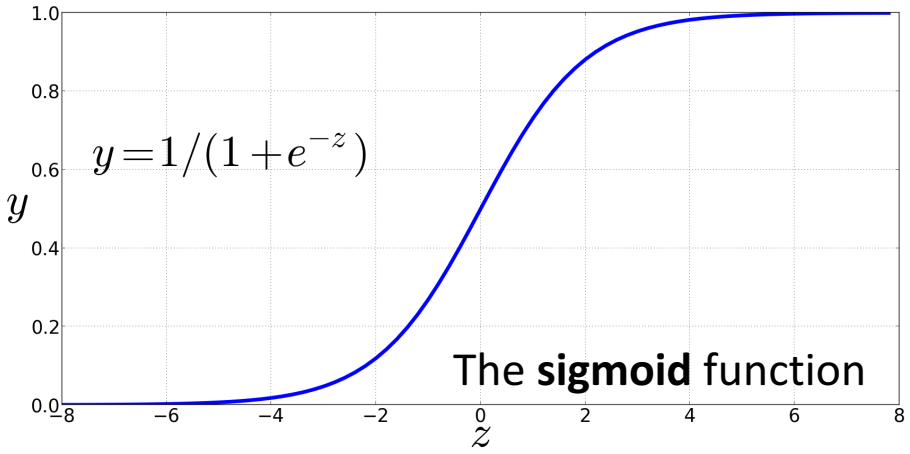
The output of this function is called the **activation value** for the unit, represented by the variable **a**. The output of a neural network is called **y,** so if the activation of a node is the final output of a network then

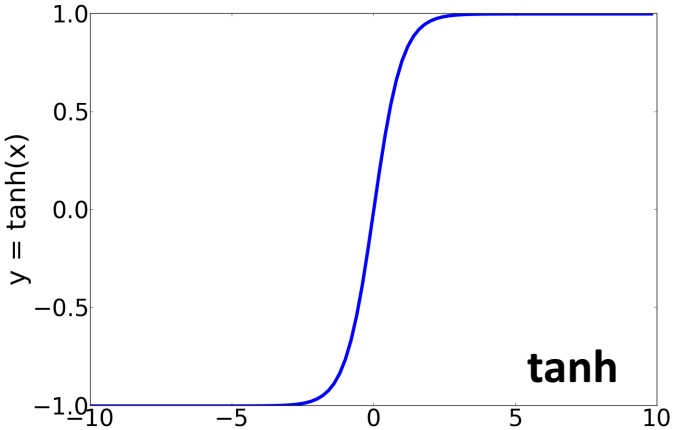
$$y = a = f(z)$$

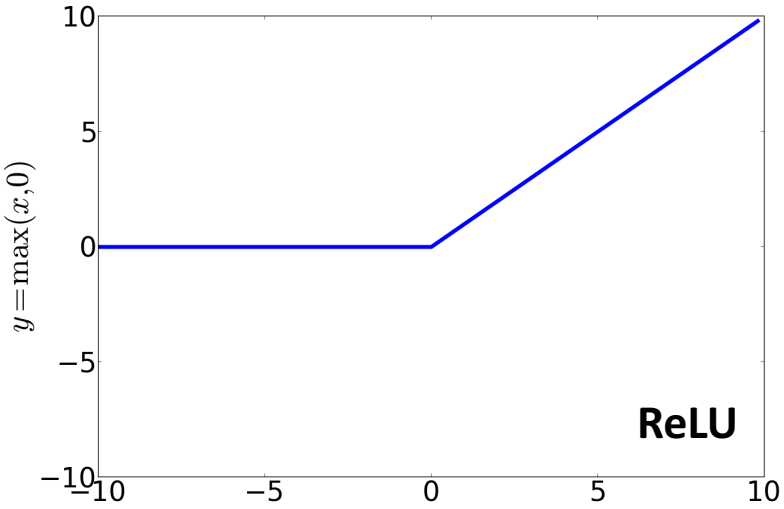
There are 3 commonly used non-linear functions used for *f:*
The **sigmoid** function
The **tanh** function
The **rectified linear unit ReLU**



Activation Functions



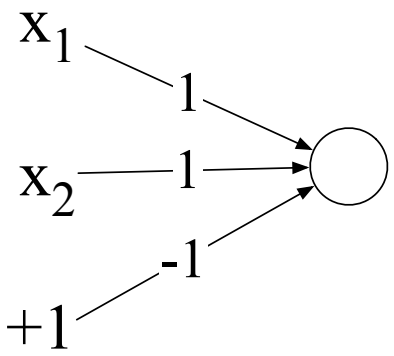


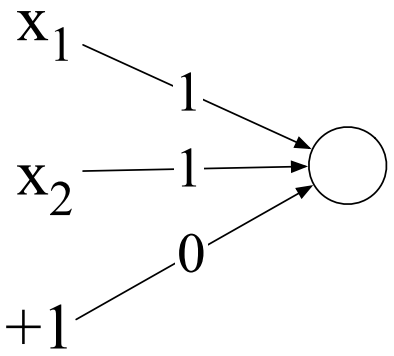


Decision making OR logical functions

Perceptrons can be used is to compute logical functions like AND, OR and NAND

Example:



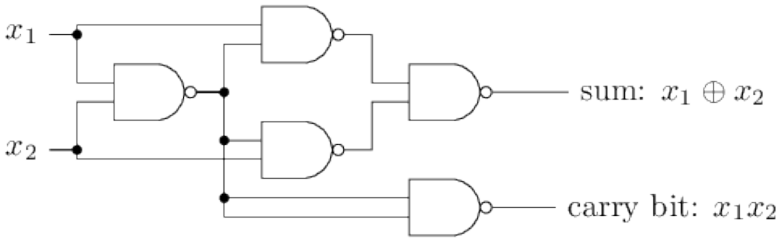


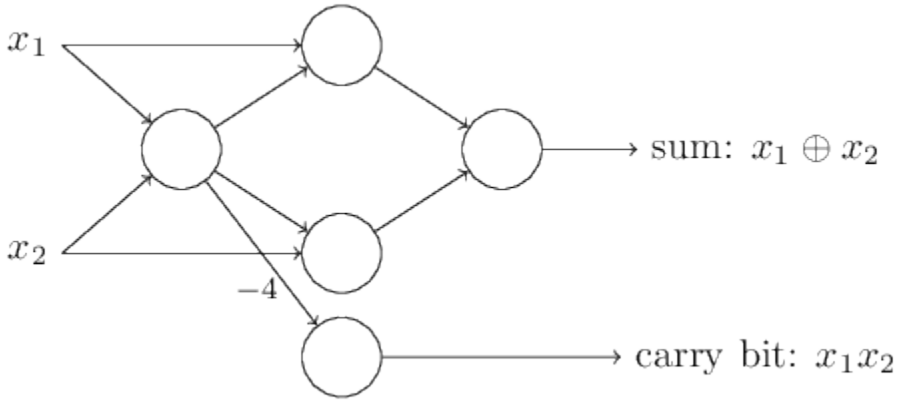
Logical functions

Networks of perceptrons to compute *any* logical function

We can build any computation up out of NAND gates.

For example, a circuit which adds two bits x_1 and x_2





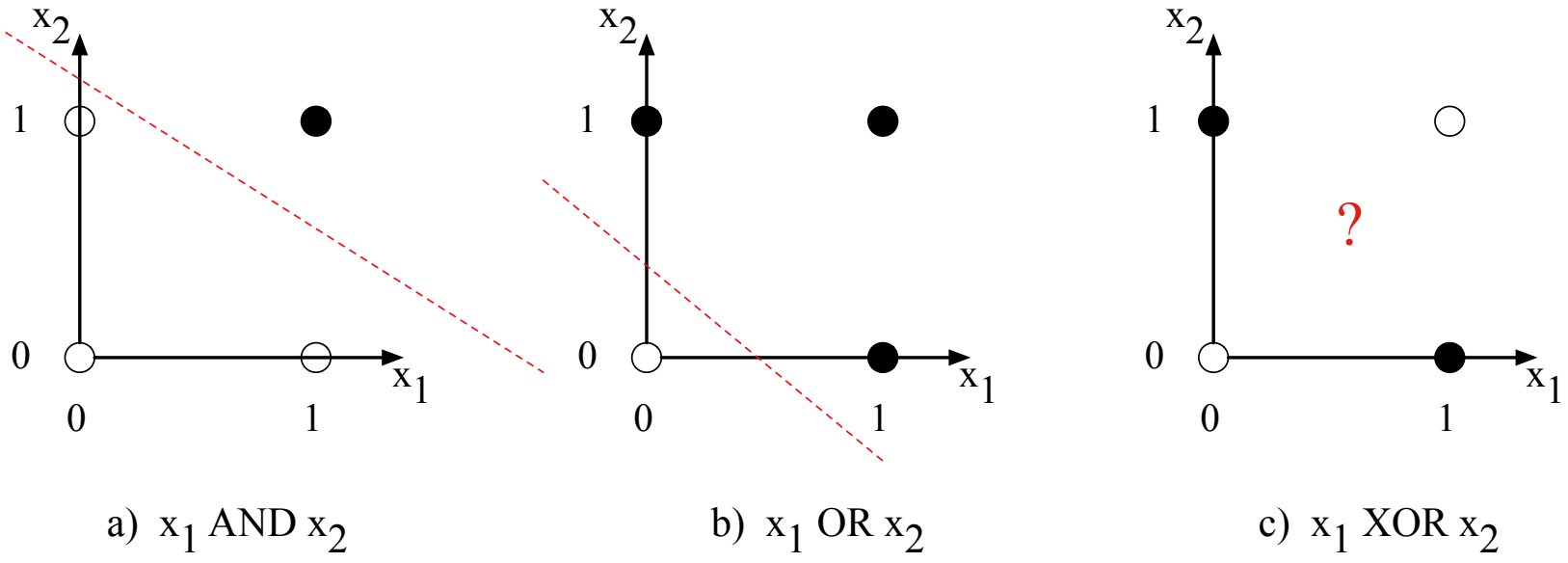
All unlabeled weights are -2, all biases =3.

The XOR problem

A single neural unit cannot be used to compute the XOR function

AND			OR			XOR		
x1	x2	y	x1	x2	y	x1	x2	y
0	0	0	0	0	0	0	0	0
0	1	0	0	1	1	0	1	1
1	0	0	1	0	1	1	0	1
1	1	1	1	1	1	1	1	0

The XOR Problem

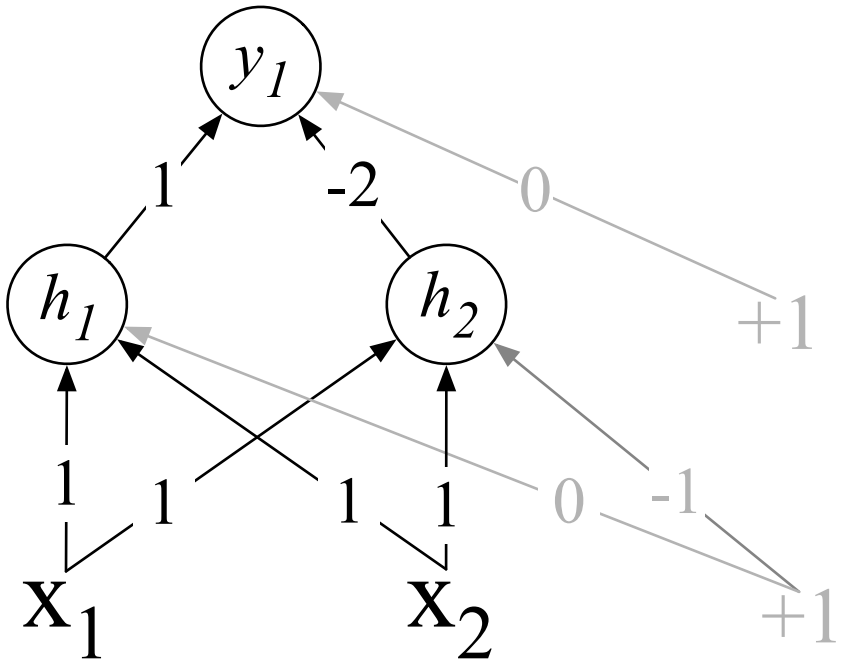


a) x_1 AND x_2

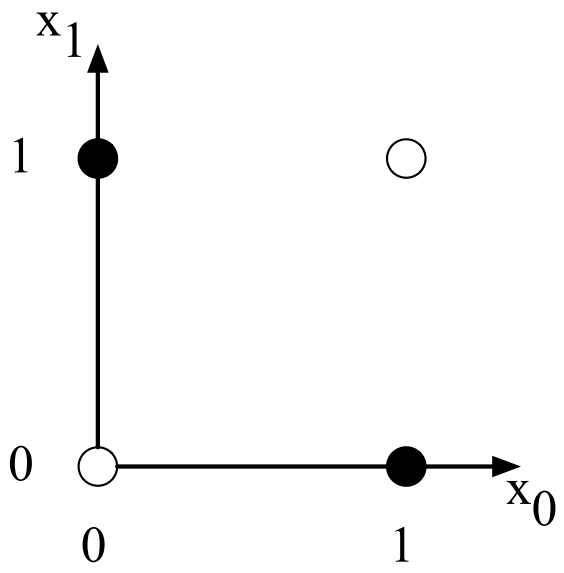
b) x_1 OR x_2

c) x_1 XOR x_2

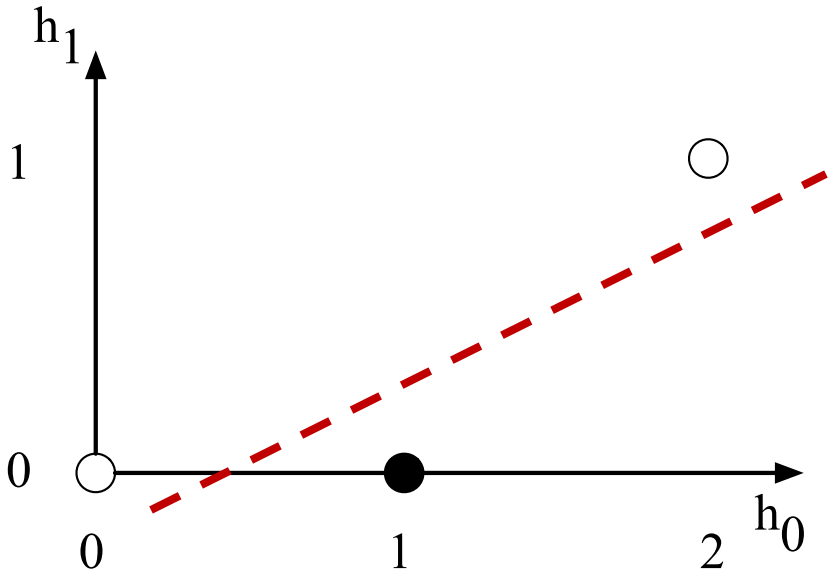
The XOR Solution



The XOR Solution



a) The original *x* space



b) The new *h* space

Power of Perceptrons

Networks of Perceptrons are universal for computation, like NAND gates

Perceptrons can be as powerful as any other computing device!

We can devise *learning algorithms* to automatically tune the weights and biases of a network of artificial neurons

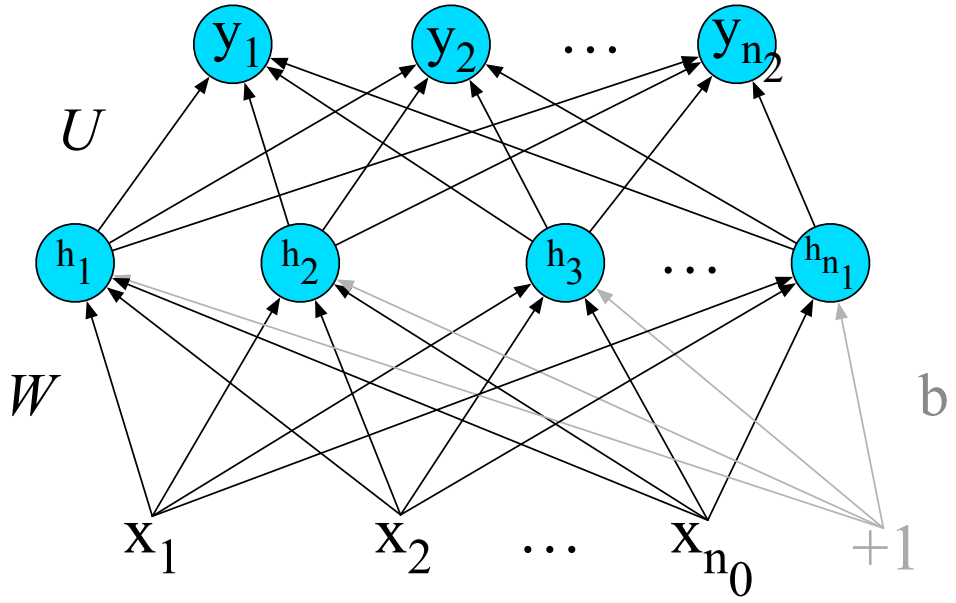
Instead of laying out a circuit of NAND and other gates, neural networks can simply learn to solve problems

Feed-Forward Neural Network

The simplest kind of is the **Feed-Forward Neural Network**

Multilayer network, all units are usually **fully-connected**, and **no cycles**.

The outputs from each layer are passed to units in the next higher layer, and no outputs are passed back to lower layers.

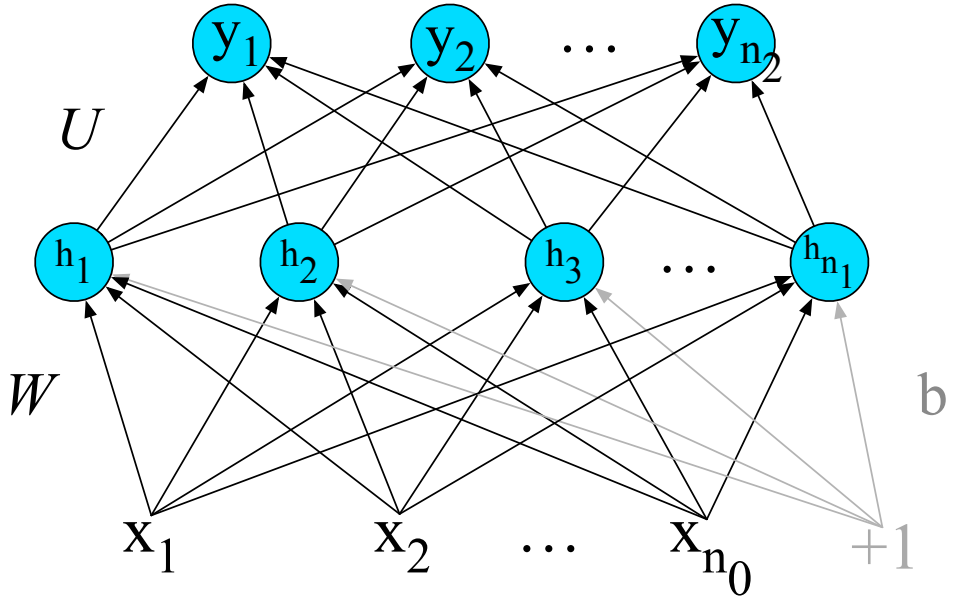


Layer 2 (output layer)

Layer 1 (hidden layer)

Layer 0 (input layer)

Equations for a feedforward network



A single hidden unit has parameters ***w*** (the weight vector) and ***b*** (the bias scalar).

We represent the parameters for the **entire hidden layer** by combining the weight vector $\mathbf{w}_i$ and bias $\boldsymbol{b}_i$ for each unit ***i*** into a single weight matrix ***W*** and a single bias vector **b** for the whole layer.

Equations for a feedforward network

The advantage of using a single matrix ***W*** for the weights of the entire layer is the hidden layer computation can be done efficiently with simple matrix operations.

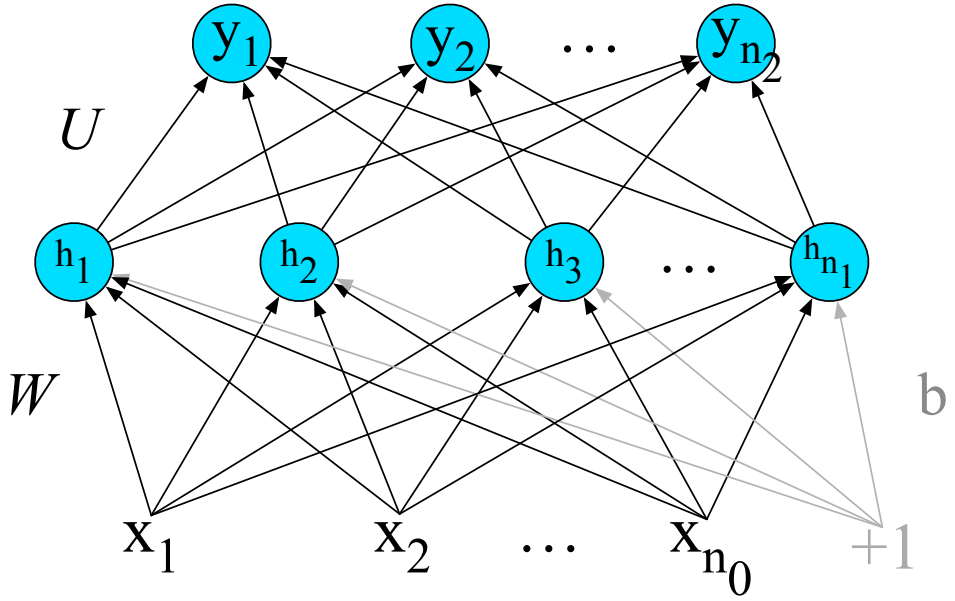
The computation has three steps:

1. multiplying the weight matrix by the input vector *x*,
2. adding the bias vector *b*, and
3. applying the activation function *g* (such as Sigmoid)

The output of the hidden layer, the vector *h*, is thus the following, using the sigmoid function σ:

$$h = \sigma(Wx+b)$$

Equations for a feedforward network



Like the hidden layer, the output layer has a weight matrix ***U***.
Its weight matrix is multiplied by its input vector (***h***) to produce the intermediate output ***z***.

$$z = Uh$$

Equations for a feedforward network

Here are the final equations for a feedforward network with a single hidden layer, which takes an input vector *x*, outputs a probability distribution *y*, and is parameterized by weight matrices *W* and *U* and a bias vector *b*:

$h = \sigma(Wx+b)$
$z = Uh$
$y = \text{softmax}(z)$

Like with logistic regression, softmax normalizes the output and turns it into a probability distribution.

Next time: training Neural Nets

And other Neural Net Architectures!

Learning to classify digits

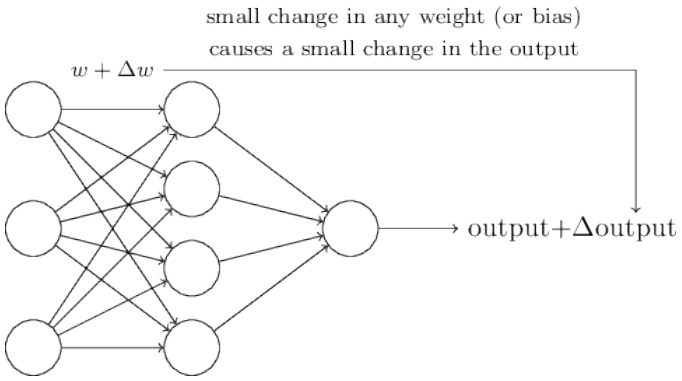
Input: image encoded input as a vector of intensities:

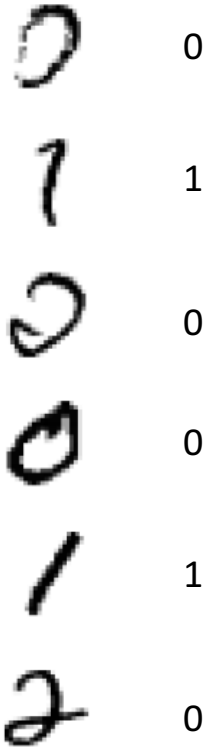
 $= \langle 0.0 \;\; 0.0 \;\; 0.3 \;\; 0.8 \;\; 0.7 \;\; 0.1 \ldots 0.0 \rangle$

Output: is this a one or not?

Goal: set the parameters of the network to correctly classify the digits

Learning = changing the weights an biases in the network.





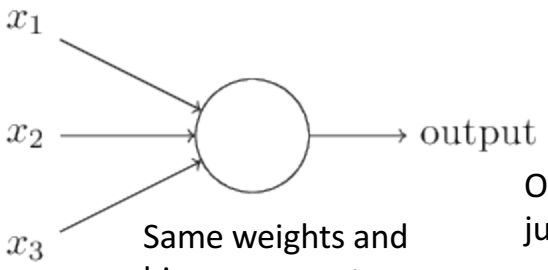
Sigmoid neurons

Problem: a small change in the weights or bias of any single perceptron in the network can causes the output to completely flip from 0 to 1.

Solution: sigmoid neuron

Perceptron

$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$



Same weights and bias as perceptron

Output is no longer just 1 or 0.

Sigmoid neuron

output = $\sigma(w \cdot x + b)$

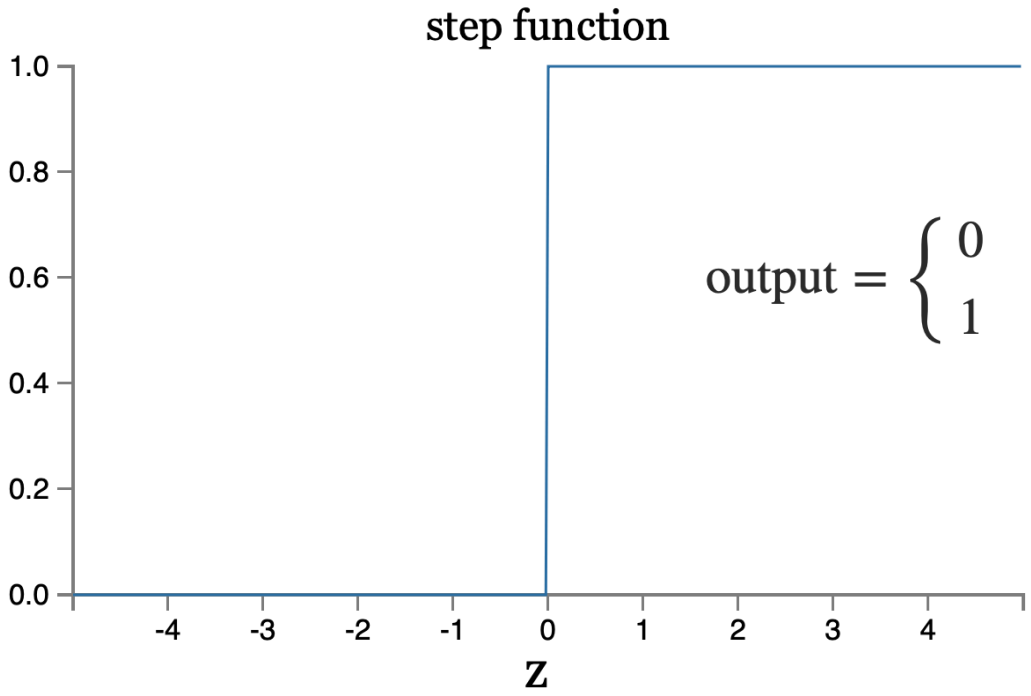
Sigmoid function:

$$\sigma(z) \equiv \frac{1}{1 + e^{-z}}$$

Inputs: any real-valued number

Perceptron

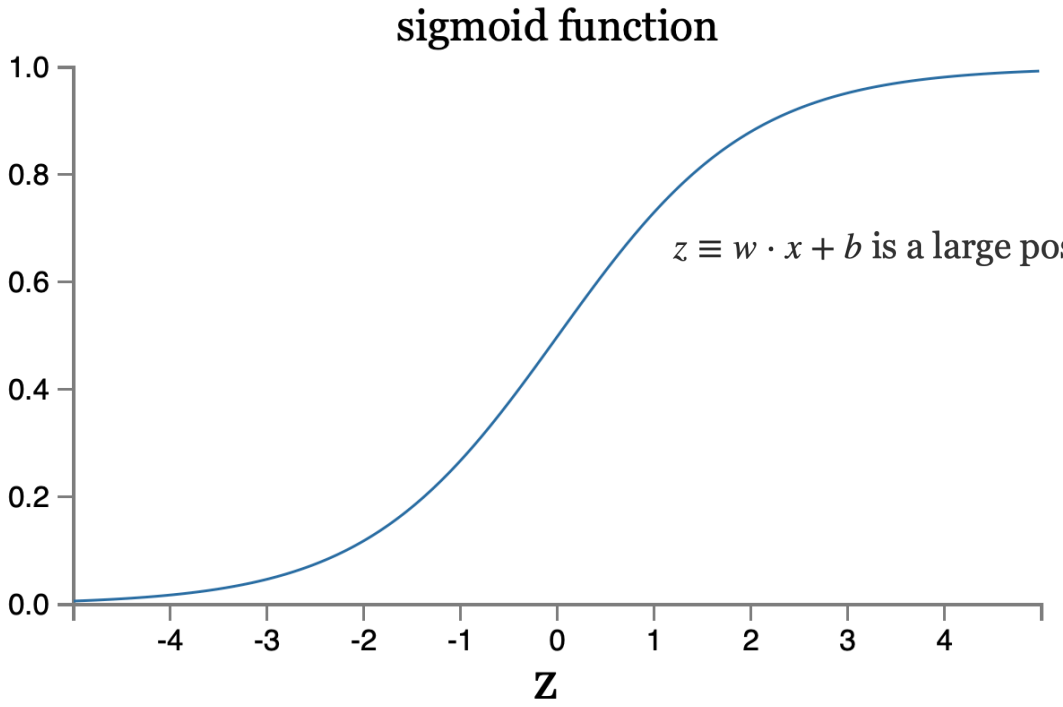
$z \equiv w \cdot x + b$



$$\text{output} = \begin{cases} 0 & \text{if } w \cdot x + b \leq 0 \\ 1 & \text{if } w \cdot x + b > 0 \end{cases}$$

Sigmoid neuron

$z \equiv w \cdot x + b$



$z \equiv w \cdot x + b$ is a large positive number. Then $e^{-z} \approx 0$

$z = w \cdot x + b$ is very negative. Then $e^{-z} \to \infty$, and $\sigma(z) \approx 0$

$$\sigma(z) \equiv \frac{1}{1 + e^{-z}}$$

Smoothness is crucial

Smoothness of σ means that small changes in the weights w_j and in the bias b will produce a small change the output from the neuron

$$\Delta \text{output} \approx \sum_j \frac{\partial \, \text{output}}{\partial w_j} \Delta w_j + \frac{\partial \, \text{output}}{\partial b} \Delta b$$

Δoutput is a *linear function* of the changes Δw_j and Δb

This makes it easy to choose small changes in the weights and biases to achieve any desired small change in the output