Order effects for geometric concepts with dual structure

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Introduction

Here I take a closer look at the experiments proposed in my proposal "Expressivity and learning geometric concepts across levels of expressivity". I begin by reviewing the results of Experiments 1 and 2, which provide context for the Experiments 3 and 4. Next, I work the the logic of the two new order-based conditions in Experiment 3. Finally, I discuss the main hypothesis for this experiment as well as the two possible outcomes for the proposal. I consider the implications of each outcome as well as whether or not each would necessitate additional work.

Review of previous experiments

First, to quickly review the relevant details of the three proposed experiments. These experiments use four quadrilateral shapes found in high school geometry: square, rectangle, rhombus, and parallelogram. The proposal argues that these concepts are made up of a dual structure where one part of the structure, called the relational-level, is at a different level of expressivity than the other part, called the object-level. These levels are described in table 1 and fig. 1. One way to understand the importance of expressivity in separating these two parts of the concepts is to consider that while the object-level concepts can be learned by logically ruling out inconsistent alternative hypotheses, the relational-level concepts cannot. We wish for subjects to learn that "all squares are also parallelograms" but because we can never show the subjects an example for all possible squares, this concept cannot be deductively learned.

Experiment 1 provided a test of the idea that the two levels of structure are separated by expressivity. For one of the four geometric concepts, it made use of a batch learning procedure where training examples were presented simultaneously. Relational-level improvement was measured through a pre- and post-test. The results indicated that very little learning seems to be occurring at the relational-level despite the use of a conventional training procedure at the object-level.

Experiment 2 replaced the batch training procedure with a sequential procedure, providing the ability to measure object-level improvement as well as relational-level improvement. Those who reached criterion did show relational-level improvement above baseline and above zero. However, both those who reached criterion and those who did not demonstrated learning in the training phase. Despite this, the aggregate group did not experience relational-level improvement above baseline. Additionally, Experiment 2 allowed the measurement of the correlation between the object-level improvement and relational-level

Shape concept	Object-level definition
Square	$f_r \wedge f_e \wedge f_q$
Rectangle	$f_r \wedge f_q$
Rhombus	$f_e \wedge f_q$
Parallelogram	f_q

Table 1

Object-level definitions for geometric concepts

The four geometric concepts are defined at the object-level using the Boolean formulas above.

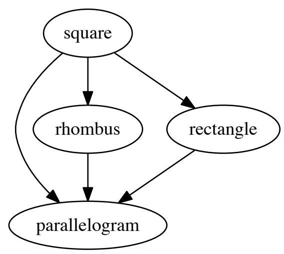


Figure 1. A directed graph representing the relational structure of the geometric concepts used. Each arrow from A to B represents the relation "all As are also Bs".

improvement. Despite both the criterion and non-criterion group experiencing learning in the training phase, object-level improvement did not predict relational-level improvement in either group nor in the combined group. The lack of such a correlation suggests that there is not transfer of learning from the object-level to the relational-level.

Experiment 3

Mathy and Feldman (2009) proposed that it is possible to provide learning examples for a concept in such a way that the order of the examples reflects the underlying representation of the concept. They found that such a representation-based order leads to quicker learning for both simple and complex concepts. Consider the form of the complex concept in this experiment:

$$f = (a \wedge b) \ \vee \ (a \wedge c) \ \vee \ (a \wedge \bar{b} \wedge c \wedge d) \ \vee \ (a \wedge b \wedge \bar{c} \wedge d) \tag{1}$$

The representation is made up of four[1] terms, each separated by $\vee$. A representation-based ordering of examples means that each new example is selected to minimize the number of terms required to represent the intermediate concept. So, in the representation-based condition, the examples corresponding to one term are all presented before moving on to another term. This is in contrast to previous theories that proposed that minimizing (or maximizing) the difference between examples. Mathy and Feldman showed that for both simple and complex concepts, ordering according to the representation resulted in quicker learning and fewer errors compared to similarity and dissimilarity-based orders.

[1]The first two terms above overlap to make the three used by Mathy and Feldman (2009).

Eaves and Shafto (2014) used a related representation-based ordering but for relational concepts. Their findings were similar to Mathy and Feldman (2009), when subjects learn from conditions that minimize the complexity of the intermediate steps while a concept being learned, subjects learn more quickly and with fewer errors. The concept used in one experiment was based on atomic orbitals, with examples like “ver-ras”, “sed-ras”, . . . , “sed-cep”, “sto-cep”. They contrasted a vertical ordering that showed subjects all of the examples with the same ending, e.g. “_____-ras”, before moving on to examples with the next such ending. Subjects in this condition learned more quickly and with fewer errors than subjects in a condition where both nodes changed with every example.

These two examples provide insight into how object-level examples might be organized in order to promote learning of the relational-level structure. Consider the concept structure in fig. 1. The objective of the order conditions is to arrange the 12 relational examples such that they keep the intermediate concepts as simple as possible. This can be accomplished in two ways: by ordering the examples by each of the inputs to the subset relation “all _____ are also _____”. Doing so minimizes the number of nodes present in the intermediate concepts subjects learn in the training phase.

Thus, Experiment 3 is as experiment 2 with the addition of two conditions that modify the ordering of the examples presented in the sequential training phase. In the sequential training phase, the shape-order condition keeps the shapes constant while the labels are cycled in a random order until all the labels are exhausted and the training moves on to the next shape. Vice versa, the label-order condition keeps the labels constant while cycling through the different shapes. These two orders each minimize the complexity of the intermediate concepts learned by focusing on one of the two inputs to the subset relation “all _____ are also _____”. There is no a priori reason to expect that subjects would benefit from using one input more than the other so both orderings are included.

For Experiment 3, two hypotheses are of interest. The first proposes that the relational-level improvement will be greater for the two order conditions than for the random condition. The second proposes a positive correlation between improvement at the object-level and improvement at the relational-level.

The first hypothesis will use a one-way ANOVA to compare the different groups: shape-order, label-order, and random. Similar to the results of experiment 2, each condition will have a subgroup of those that reached criterion and those that did not. I will assess whether or not learning occurred in each sub-group of each condition by a correlational analysis. If the average error rate decreases as the number of “trials to go” decreases, the subgroups will be said to have demonstrated significant learning. Assuming that both criterion and non-criterion subgroups experience learning (as in experiment 2) then I will not separate the condition groups according to criterion. However, if one or more subgroups do not experience learning, “reached criterion” will be added as a factor to the (now) two-way ANOVA comparing the different condition groups.

The second hypothesis will use a regression analysis to asses the extent to which object-level improvement predicts relational-level improvement. Like the first hypothesis, the exact nature of this analysis will depend on whether or not subjects experience learning in all criterion-based subgroups.

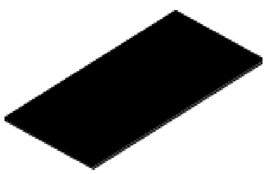
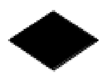
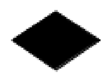
Shape-order	Label-order	Random-order
 Is this a rhombus?	 Is this a rhombus?	 Is this a rhombus?
 Is this a rectangle?	 Is this a rhombus?	 Is this a rectangle?
 Is this a square?	 Is this a rhombus?	 Is this a square?
 Is this a parallelogram?	 Is this a rhombus?	 Is this a parallelogram?
...	...	...

Table 2

Experiment 3 order conditions

Shown are examples of the three conditions planned for Experiment 3. The shape-order condition groups object-level questions by the shape presented, label-order condition groups by the label queried, and the random condition randomizes all questions.

Possible outcomes and additional steps

The first two experiments showed that two standard concept-learning approaches at the object-level result in little or no improvement at the relational-level. The second experiment in particular shows that the level of improvement at the object-level does not predict the level of improvement at the relational level. This is evidence consistent with the idea that these concepts have a dual structure and that that structure is separated by expressivity. However, these experiments leave too much room for alternative explanations. Several factors might account for the lack of learning at the relational-level other than a dual structure. Without a clear positive case of transfer of learning from the object-level to the relational-level, it is more difficult to rule out alternative explanations.

Because of the importance of a positive case of relational-level improvement, the outcome of third experiment has a large impact on the cohesiveness of the proposed dissertation. In the case where there is a clear case of transfer of learning in either the shape-order or label-order conditions, the experiments will together support a coherent explanation: The quadrilateral concepts have a dual structure separated by levels of expressivity; common concept learning strategies lead to improvement at the object-level but not the relational level; however, modifying the order of the examples subjects see such that the order reflects the underlying representation of the relational-level leads to learning at both levels and, crucially, this learning comes from a transfer across levels.

In the case where neither the subjects in the shape-order condition nor those in the label-order condition show clear transfer from the object-level to relational-level, there is a risk that the outcome will provide an incomplete answer to the questions raised in the first half of the proposal. It will be clear that the subjects are not improving performance at the relational-level but the theory that this is caused by a difference in expressivity may not be the most compelling explanation.

Experiment 4

To remedy this problem, I am proposing an additional experiment to be conducted in any case where the outcomes of the first three experiments do not together support a coherent explanation of what the results of the experiments have to say about the the theory that motivated them. The first three experiments involve subjects learning at the object-level and a measurement of their improvement at the relational-level and object-level. Thus these experiments only measure transfer of learning in the object-level -> relational-level direction and they risk failing to demonstrate any learning at the relational-level at all.

The fourth experiment follows the same procedure as experiments 2 and 3 except that, instead of subjects learning at the object-level, subjects would learn at the relational-level and the pre- and post-test would be designed to measure improvement at the object-level. The sequential training phase of Experiment 4 will present subjects with the 12 relational-level questions in the pre- and posttest of previous experiments. These relational-level questions will be presented in sequence and feedback will be given after each response. Subjects will remain in the learning phase until reaching the criterion of two consecutive trials with 100% accuracy.

Experiment 4 will include three order-based conditions analogous to those in Experiment 3: a random-order condition as well as two conditions that order questions according

first-order	second-order	Random-order
Are all rhombuses also parallelograms?	Are all rhombuses also parallelograms?	Are all rhombuses also parallelograms?
Are all rhombuses also rectangles?	Are all rectangles also parallelograms?	Are all rectangles also squares?
Are all rhombuses also squares?	Are all squares also parallelograms?	Are all squares also rectangles?
Are all parallelograms also rhombuses?	Are all parallelograms also rhombuses?	Are all rhombuses also squares?
...	...	...

Table 3

Experiment 4 order conditions

Shown are examples of the three conditions planned for Experiment 4. The first-order condition groups relational-level questions by the first blank in "Are all ______ also ______ ?" while the second-order condition groups questions by the second blank. The random-order condition randomizes the order of all 12 questions.

to the first and second arguments, respectively, in the relation "Are all ______ also ______ ?". Improvement at the object-level will now be measured as the difference between the object-level post-test and pre-tests and improvement at the relational-level will now be measured by the difference between each subject's last learning trial and first learning trial. The number of subjects in each condition will be based on a power analysis of Experiment 4 with effect sizes updated to reflect the outcome of experiment three.

Analogous to Experiment 3, there are two main hypotheses for Experiment 4: First, one or both of the order conditions are expected to result in greater improvement in learning at the object-level than the random-order condition. Second, for one or both of the order conditions, improvement at the relational-level, is expected to be correlated with improvement at the object-level.

This additional experiment removes the risk that the outcome of Experiment 3 might be difficult to interpret. It is very likely to validate the concepts, stimuli, and procedure used to measure relational-level learning because, unlike Experiment 3, Experiment 4 it teaches relational-level concepts directly using a sequential training method that has been replicated across several studies, including Eaves and Shafto (2014). Provided that subjects demonstrate learning at the relational level, the experiment should isolate as the most plausible explanation that the reason subjects did not improve performance at the relational-level in experiments 1 and 2 is that they were, in fact, not transferring new information across levels of expressivity. This would remove the burden of demonstrating learning at the relational-level from Experiment 3 and allows the results to straightforwardly speak to whether or not the order conditions promote learning across levels of expressivity. Experiment 4 thus ensures that the experiments provide a coherent, interesting contribution regardless of the whether or not subjects demonstrate transfer of learning in Experiments 3 and 4.

References

Eaves, B. S. & Shafto, P. (2014). Order effects in learning relational structures. In *Proceedings of the 36th annual conference of the cognitive science society*.

Mathy, F. & Feldman, J. (2009). A rule-based presentation order facilitates category learning. *Psychonomic bulletin & review*, *16*(6), 1050–1057.