

Fibonacci Sequences for Fun & Profit

Brian Spiering



What is a Fibonacci sequence?

$$1 + 1 = 2$$

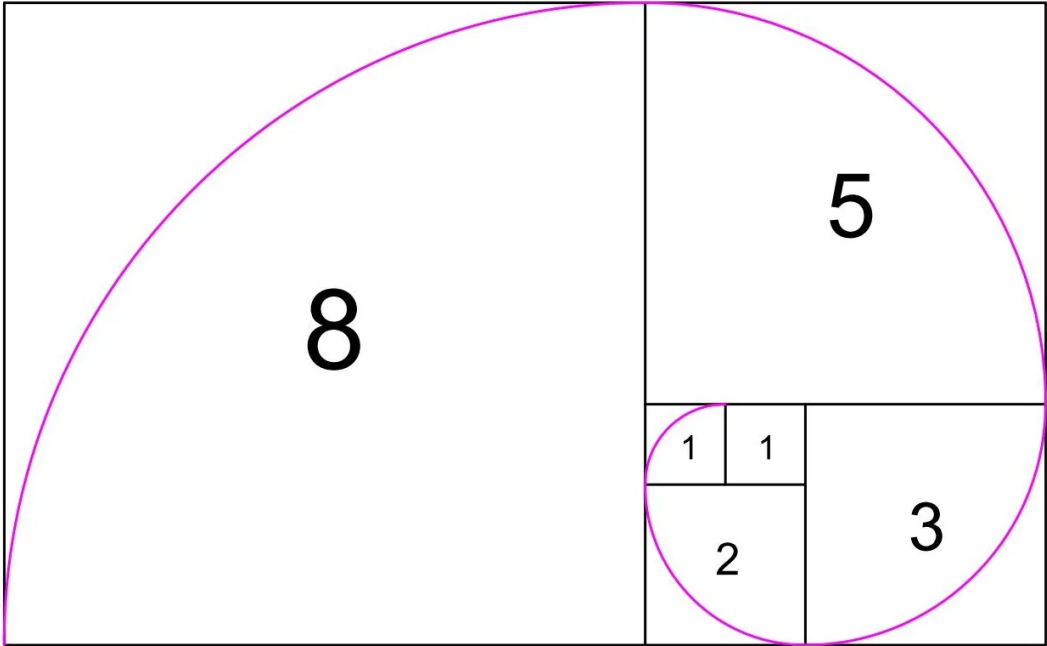
$$1 + 2 = 3$$

$$2 + 3 = 5$$

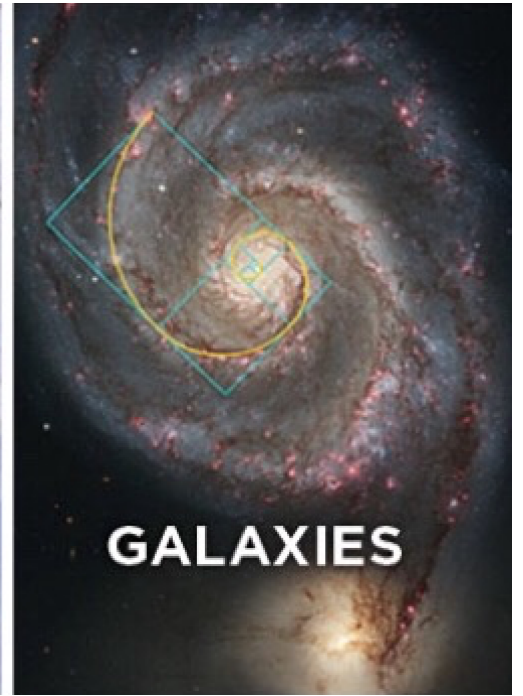
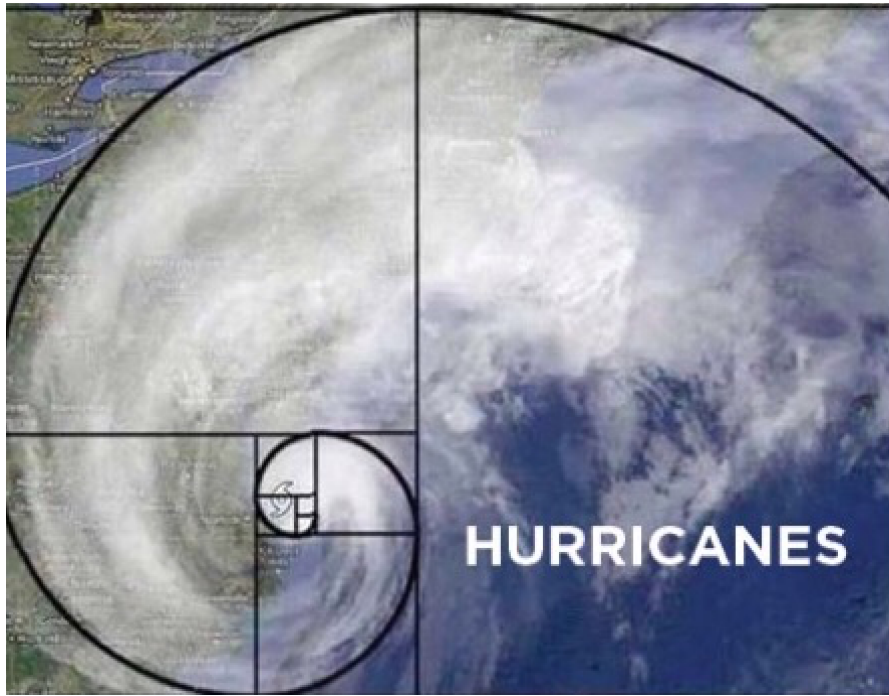
$$3 + 5 = 13$$

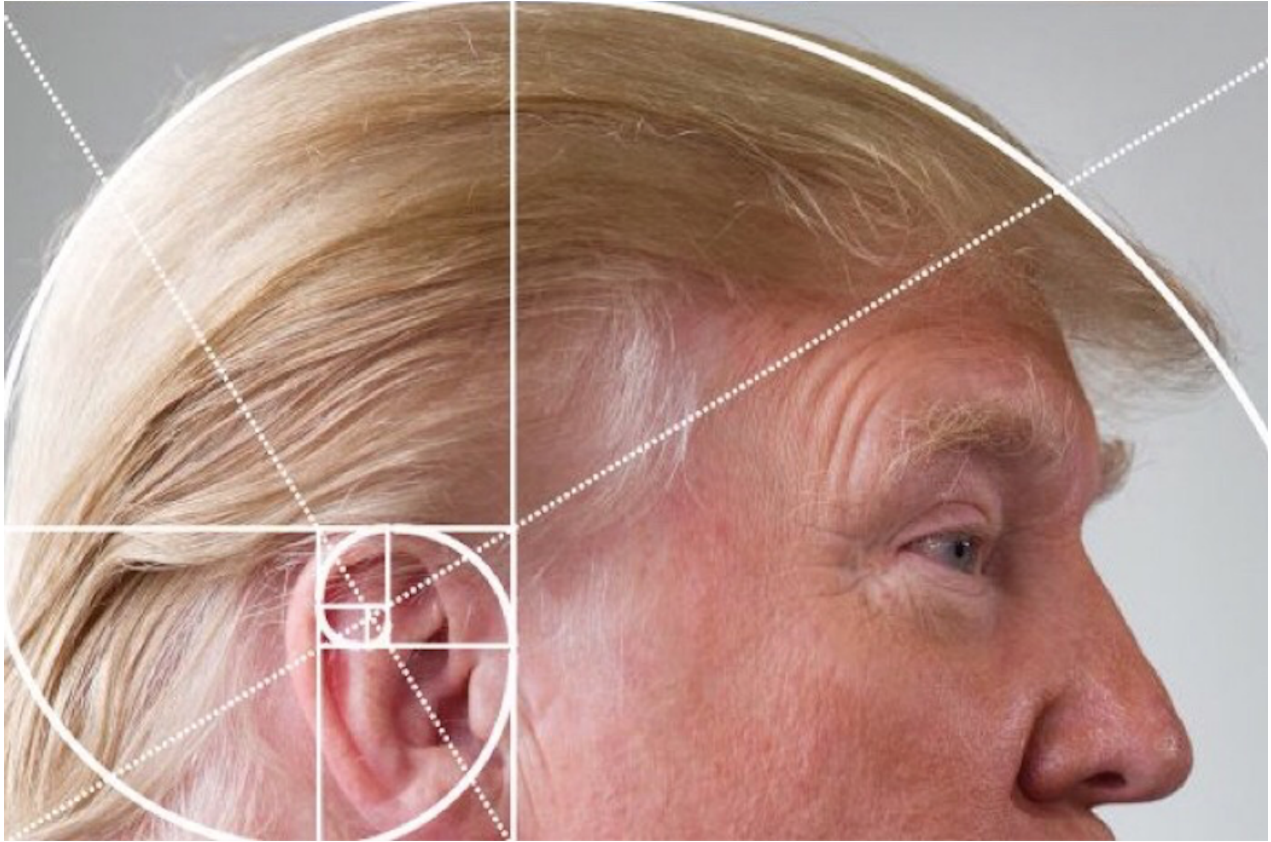
1, 1, 2, 3, 5, 8, 13

Where do Fibonacci sequences appear?



__Golden Spiral__ 🐚





What are the ways to caculate a Fib sequence in Python🐍?

- Recursion
- Dynamic Programming
- Imperative
- Closed form
- Functional

```
In [2]: def test_fib(f, test_large=False):
        "Test that a Fibonacci function returns the correct value"
        fib_sequence = {0: 0, # Index -> Expeced Fib value
                          1: 1,
                          2: 1,
                          3: 2,
                          9: 34,
                          25: 75_025,
                          }

        for key, fib_value in fib_sequence.items():
            assert f(key) == fib_value

        if test_large:
            assert f(32) == 21_78_309

        print('tests pass 😊')
```

```
In [3]: def fib_recursive(n):
        "Calculate nth Fibonacci number using recursion"
        if n == 0: return 0
        if n == 1: return 1
        return fib_recursive(n-1) + fib_recursive(n-2)
```

```
In [4]: # Take a peek to be sure it is correct 👁👁
        [fib_recursive(n) for n in range(10)]
```

```
Out[4]: [0, 1, 1, 2, 3, 5, 8, 13, 21, 34]
```

```
In [5]: # Let's test it
        test_fib(fib_recursive)
```

```
tests pass 😊
```

```
In [6]: # Recursion does not scale in Python ⌚
        test_fib(fib_recursive, test_large=True)
```

```
tests pass 😊
```

Those who cannot remember the past are condemned to repeat it.

-Dynamic Programming

```
In [7]: from functools import lru_cache as memo

@memo(maxsize=32)
def fib_cache(n):
    "Calculate nth Fibonacci number using dynamic programming"
    if n == 0: return 0
    if n == 1: return 1
    return fib_cache(n-1) + fib_cache(n-2)

test_fib(fib_cache)

tests pass 😊
```

```
In [8]: test_fib(fib_cache, test_large=True)

tests pass 😊
```

```
In [9]: def fib_subproblems(n):
    "Calculate nth Fibonacci number by storing the previous values"
    fib_seq = [0, 1]
    for i in range(n):
        fib_seq.append(fib_seq[-1]+fib_seq[-2])
    return fib_seq[-2]

test_fib(fib_subproblems, test_large=True)

tests pass 😊
```

Big O Analysis of Imperative Fib Function

Time complexity is $O(n)$

Space complexity is $O(n)$ 🙅🏻🧠

```
In [10]: def fib_swap(n):  
          "Calculate nth Fibonacci number only keeping the needed values"  
          a, b = 0, 1  
          for _ in range(n):  
              a, b = b, a+b  
          return a  
  
test_fib(fib_swap, test_large=True)  
  
tests pass 😊
```

What is the best Big O for time complexity?

$O(1)$ - Constant time

Constant time for mathematical operations are the closed-form solutions

Binet's Formula

$$F_n = \frac{1}{\sqrt{5}} \left(\frac{1 + \sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1 - \sqrt{5}}{2} \right)^n$$

MathOps FTW 🙌!

```
In [11]: from math import sqrt

def fib_binet(n):
    "Calculate nth Fibonacci number using Binet's formula"
    first_term = (1/sqrt(5))*((1+sqrt(5))/2)**n
    second_term = (1/sqrt(5))*((1-sqrt(5))/2)**n
    return round(first_term - second_term)

test_fib(fib_binet, test_large=True)

tests pass 😊
```

Fibonacci the Functional Way 🐢

```
In [12]: def fib_gen():
    "A Fibonacci sequence generator"
    a, b = 0, 1
    while True:
        a, b = b, a+b
        yield a # Replace return statement
```



```
In [13]: from itertools import islice

n = 100_000
big_fib = next(islice(fib_gen(), n-1, n))
print(f"{big_fib:,}")
```

2,597,406,934,722,172,416,615,503,402,127,591,541,488,048,538,651,769,6
58,472,477,070,395,253,454,351,127,368,626,555,677,283,671,674,475,463,
758,722,307,443,211,163,839,947,387,509,103,096,569,738,218,830,449,30
5,228,763,853,133,492,135,302,679,278,956,701,051,276,578,271,635,608,0
73,050,532,200,243,233,114,383,986,516,137,827,238,124,777,453,778,337,
299,916,214,634,050,054,669,860,390,862,750,996,639,366,409,211,890,12
5,271,960,172,105,060,300,350,586,894,028,558,103,675,117,658,251,368,3
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991,330,062,204,363,035,213,756,525,421,823,998,690,848,556,374,080,17
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277,707,554,446,794,723,709,030,979,019,014,860,432,846,819,857,961,01
5,951,001,850,608,264,919,234,587,313,399,150,133,919,932,363,102,301,8
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051,187,957,316,766,834,979,901,682,011,849,907,756,686,456,845,066,28
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855,573,864,080,579,813,028,266,640,270,354,294,412,104,919,995,803,13
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255,814,045,517,000,250,299,764,322,193,462,165,366,210,841,876,745,42
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09,374,715,485,131,027,220,817,305,432,264,866,949,630,987,914,714,362,
925,554,252,624,043,999,615,326,979,876,807,510,646,819,068,792,118,29
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746,695,651,879,126,916,993,244,564,817,673,322,257,149,314,967,763,34
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057,055,248,158,884,051,779,418,192,905,216,769,576,608,748,815,567,86
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8,287,264,284,501,725,332,048,648,319,457,892,039,984,893,823,636,745,6
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778,891,798,913,667,325,190,623,247,118,037,280,173,921,572,390,822,76
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21,519,554,793,578,923,866,762,502,745,370,104,660,909,382,449,626,626,
935,321,303,744,538,892,479,216,161,188,889,702,077,910,448,563,199,51
4,826,630,802,879,549,546,453,583,866,307,344,423,753,319,712,279,158,8
61,707,289,652,090,149,848,305,435,983,200,771,326,653,407,290,662,016,
775,706,409,690,183,771,201,306,823,245,333,477,966,660,525,325,490,87
3,601,961,480,378,241,566,071,271,650,383,582,257,289,215,708,209,369,5
10,995,890,132,859,490,724,306,183,325,755,201,208,090,007,175,022,022,
949,742,801,823,445,413,711,916,298,449,914,722,254,196,594,682,221,46
8,260,644,961,839,254,249,670,903,104,007,581,488,857,971,672,246,322,8
87,016,438,403,908,463,856,731,164,308,169,537,326,790,303,114,583,680,
575,021,119,639,905,615,169,154,708,510,459,700,542,098,571,797,318,01

5,564,741,406,172,334,145,847,111,268,547,929,892,443,001,391,468,289,1
03,679,179,216,978,616,582,489,007,322,033,591,376,706,527,676,521,307,
143,985,302,760,988,478,056,216,994,659,655,461,379,174,985,659,739,22
7,379,416,726,495,377,801,992,098,355,427,866,179,123,126,699,374,730,7
77,730,569,324,430,166,839,333,011,554,515,542,656,864,937,492,128,687,
049,121,754,245,967,831,132,969,248,492,466,744,261,999,033,972,825,67
4,873,460,201,150,442,228,780,466,124,320,183,016,108,232,183,908,654,7
71,042,398,228,531,316,559,685,688,005,226,571,474,428,823,317,539,456,
543,881,928,624,432,662,503,345,388,199,590,085,105,211,383,124,491,86
1,802,624,432,195,540,433,985,722,841,341,254,409,411,771,722,156,867,0
86,291,742,124,053,110,620,522,842,986,199,273,629,406,208,834,754,853,
645,128,123,279,609,097,213,953,775,360,023,076,765,694,208,219,943,03
4,648,783,348,544,492,713,539,450,224,591,334,374,664,937,701,655,605,7
63,384,697,062,918,725,745,426,505,879,414,630,176,639,760,457,474,311,
081,556,747,091,652,708,748,125,267,159,913,793,240,527,304,613,693,96
1,169,892,589,808,311,906,322,510,777,928,562,071,999,459,487,700,611,8
01,002,296,132,304,588,294,558,440,952,496,611,158,342,804,908,643,860,
880,796,440,557,763,691,857,743,754,025,896,855,927,252,514,563,404,38
5,217,825,890,599,553,954,627,451,385,454,452,916,761,042,969,267,970,8
93,580,056,234,501,918,571,489,030,418,495,767,400,819,359,973,218,711,
957,496,357,095,967,825,171,096,264,752,068,890,806,407,651,445,893,13
2,870,767,454,169,607,107,931,692,704,285,168,093,413,311,046,353,506,2
42,209,810,363,216,771,910,420,786,162,184,213,763,938,194,625,697,286,
781,413,636,389,620,123,976,910,465,418,956,806,197,323,148,414,224,55
0,071,617,215,851,321,302,030,684,176,087,215,892,702,098,879,108,938,0
81,045,903,397,276,547,326,416,916,845,445,627,600,759,561,367,103,584,
575,649,094,430,692,452,532,085,003,091,068,783,157,561,519,847,567,56
9,191,284,784,654,692,558,665,111,557,913,461,272,425,336,083,635,131,3
42,183,905,177,154,511,228,464,455,136,016,013,513,228,948,543,271,504,
760,839,307,556,100,908,786,096,663,870,612,278,690,274,831,819,331,60
6,701,484,957,163,004,705,262,228,238,406,266,818,448,788,374,548,131,9
94,380,387,613,830,128,859,885,264,201,992,286,188,208,499,588,640,888,
521,352,501,457,615,396,482,647,451,025,902,530,743,172,956,899,636,49
9,615,707,551,855,837,165,935,367,125,448,515,089,362,904,567,736,630,0
35,562,457,374,779,100,987,992,499,146,967,224,041,481,601,289,530,944,
015,488,942,613,783,140,087,804,311,431,741,858,071,826,185,149,051,13
8,744,831,358,439,067,228,949,408,258,286,021,650,288,927,228,387,426,4
32,786,168,690,381,960,530,155,894,459,451,808,735,197,246,008,221,529,
343,980,828,254,126,128,257,157,209,350,985,382,800,738,560,472,910,94
1,184,006,084,485,235,377,833,503,306,861,977,724,501,886,364,070,344,9
73,366,473,100,602,018,128,792,886,991,861,824,418,453,968,994,777,259,
482,169,137,133,647,470,453,172,979,809,245,844,361,129,618,997,595,69
6,240,971,845,564,020,511,432,589,591,844,724,920,942,930,301,651,488,7
13,079,802,102,379,065,536,525,154,780,298,059,407,529,440,513,145,807,
551,537,794,861,635,879,901,158,192,019,808,879,694,967,187,448,224,15
6,836,463,534,326,160,242,632,934,761,634,458,163,890,163,805,123,894,1
84,523,973,421,841,496,889,262,398,489,648,642,093,409,816,681,494,771,
155,177,009,562,669,029,850,101,513,537,599,801,272,501,241,971,119,87
1,526,593,747,484,778,935,488,777,815,192,931,171,431,167,444,773,882,9
41,064,615,028,751,327,709,474,504,763,922,874,890,662,989,841,540,259,
350,834,035,142,035,136,168,819,248,238,998,027,706,666,916,342,133,42
4,312,054,507,359,388,616,687,691,188,185,776,118,135,771,332,483,965,2
09,882,085,982,391,298,606,386,822,804,754,362,408,956,522,921,410,859,
852,037,330,544,625,953,261,340,234,864,689,275,060,526,893,755,148,40
3,298,542,086,991,221,052,597,005,628,576,707,702,567,695,300,978,970,0
46,408,920,009,852,106,980,295,419,699,802,138,053,295,798,159,478,289,
934,443,245,491,565,327,845,223,840,551,240,445,208,226,435,420,656,31

3,310,702,940,722,371,552,770,504,263,482,073,984,454,889,589,248,861,3
97,657,079,145,414,427,653,584,572,951,329,719,091,947,694,411,910,966,
797,474,262,675,590,953,832,039,169,673,494,261,360,032,263,077,428,68
4,105,040,061,351,052,194,413,778,158,095,005,714,526,846,009,810,352,1
09,249,040,027,958,050,736,436,961,021,241,137,739,717,164,869,525,493,
114,805,040,126,568,351,268,829,598,413,983,222,676,377,804,500,626,50
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58,030,178,877,405,615,167,159,731,927,320,479,376,247,375,505,855,052,
839,660,294,566,992,522,173,600,874,081,212,014,209,071,041,937,598,57
1,721,431,338,017,425,141,582,491,824,710,905,084,715,977,249,417,049,3
20,254,165,239,323,233,258,851,588,893,337,097,136,310,892,571,531,417,
761,978,326,033,750,109,026,284,066,415,801,371,359,356,529,278,088,45
6,305,951,770,081,443,994,114,674,291,850,360,748,852,366,654,744,869,9
28,083,230,516,815,711,602,911,836,374,147,958,492,100,860,528,981,469,
547,750,812,338,896,943,152,861,021,202,736,747,049,903,930,417,035,17
1,342,126,923,486,700,566,627,506,229,058,636,911,882,228,903,170,510,3
05,406,882,096,970,875,545,329,369,434,063,981,297,696,478,031,825,451,
642,178,347,347,716,471,058,423,238,594,580,183,052,756,213,910,186,99
7,604,305,844,068,665,712,346,869,679,456,044,155,742,100,039,179,758,3
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305,684,668,420,775,409,821,781,247,053,354,523,194,797,398,953,320,17
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01,857,497,335,249,965,013,509,368,925,958,021,863,811,725,906,506,436,
882,127,156,815,751,021,712,900,765,992,750,370,228,283,963,962,915,97
3,251,173,418,586,721,023,497,317,765,969,454,283,625,519,371,556,009,1
43,680,329,311,962,842,546,628,403,142,444,370,648,432,390,374,906,410,
811,300,792,848,955,767,243,481,200,090,309,888,457,270,907,750,873,63
8,873,299,642,555,050,473,812,528,975,962,934,822,878,917,619,920,725,1
38,309,388,288,292,510,416,837,622,758,204,081,918,933,603,653,875,284,
116,785,703,720,989,718,832,986,921,927,816,629,675,844,580,174,911,80
9,119,663,048,187,434,155,067,790,863,948,831,489,241,504,300,476,704,5
27,971,283,482,211,522,202,837,062,857,314,244,107,823,792,513,645,086,
677,566,622,804,977,211,397,140,621,664,116,324,756,784,216,612,961,47
7,109,018,826,094,677,377,686,406,176,721,484,293,894,976,671,380,122,7
88,941,309,026,553,511,096,118,347,012,565,197,540,807,095,384,060,916,
863,936,906,673,786,627,209,429,434,264,260,402,902,158,317,345,003,72
7,462,588,992,622,049,877,121,178,405,563,348,492,490,326,003,508,569,0
99,382,392,777,297,498,413,565,614,830,788,262,363,322,368,380,709,822,
346,012,274,241,379,036,473,451,735,925,215,754,757,160,934,270,935,19
2,901,723,954,921,426,490,691,115,271,523,338,109,124,042,812,102,893,7
38,488,167,358,953,934,508,930,697,715,522,989,199,698,903,885,883,275,
409,044,300,321,986,834,003,470,271,220,020,159,699,371,690,650,330,54
7,577,095,398,748,580,670,024,491,045,504,890,061,727,189,168,031,394,5
28,036,165,633,941,571,334,637,222,550,477,547,460,756,055,024,108,764,
382,121,688,848,916,940,371,258,901,948,490,685,379,722,244,562,009,48
3,819,491,532,724,502,276,218,589,169,507,405,794,983,759,821,006,604,4
81,996,519,360,110,261,576,947,176,202,571,702,048,684,914,616,894,068,
404,140,833,587,562,118,319,210,838,005,632,144,562,018,941,505,945,78
0,025,318,747,471,911,604,840,677,997,765,414,830,622,179,069,330,853,8
75,129,298,983,009,580,277,554,145,435,058,768,984,944,179,136,535,891,
620,098,725,222,049,055,183,554,603,706,533,183,176,716,110,738,009,78
6,625,247,488,691,476,077,664,470,147,193,074,476,302,411,660,335,671,7
65,564,874,440,577,990,531,996,271,632,972,009,109,449,249,216,456,030,
618,827,772,947,750,764,777,446,452,586,328,919,159,107,444,252,320,08
2,918,209,518,021,083,700,353,881,330,983,215,894,608,680,127,954,224,7
52,071,924,134,648,334,963,915,094,813,097,541,433,244,209,299,930,751,
481,077,919,002,346,128,122,330,161,799,429,930,618,800,533,414,550,63

3,932,139,339,646,861,616,416,955,220,216,447,995,417,243,171,165,744,4
71,364,197,733,204,899,365,074,767,844,149,929,548,073,025,856,442,942,
381,787,641,506,492,878,361,767,978,677,158,510,784,235,702,640,213,38
8,018,875,601,989,234,056,868,423,215,585,628,508,645,525,258,377,010,6
20,532,224,244,987,990,625,263,484,010,774,322,488,172,558,602,233,302,
076,399,933,854,152,015,343,847,725,442,917,895,130,637,050,320,444,91
7,797,752,370,871,958,277,976,799,686,113,626,532,291,118,629,631,164,6
85,159,934,660,693,460,557,545,956,063,155,830,033,697,634,000,276,685,
151,293,843,638,886,090,828,376,141,157,732,003,527,565,158,745,906,56
7,025,439,437,931,104,838,571,313,294,490,604,926,582,363,108,949,535,0
90,082,673,154,497,226,396,648,088,618,041,573,977,888,472,892,174,618,
974,189,721,700,770,009,862,449,653,759,012,727,015,227,634,510,874,90
6,948,012,210,684,952,063,002,519,011,655,963,580,552,429,180,205,586,9
04,259,685,261,047,412,834,518,466,736,938,580,027,700,252,965,356,366,
721,619,883,672,428,226,933,950,325,930,390,994,583,168,665,542,234,65
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99,895,366,470,106,557,900,532,129,541,336,924,472,492,212,436,324,523,
042,895,188,461,779,122,338,069,674,233,980,694,887,270,587,503,389,22
8,395,095,135,209,123,109,258,159,006,960,395,156,367,736,067,109,050,5
66,299,603,571,876,423,247,920,752,836,160,805,597,697,778,756,476,767,
210,521,222,327,184,821,484,446,631,261,487,584,226,092,608,875,764,33
1,731,023,263,768,864,822,594,691,211,032,367,737,558,122,133,470,556,8
05,958,008,310,127,481,673,962,019,583,598,023,967,414,489,867,276,845,
869,819,376,783,757,167,936,723,213,081,586,191,045,995,058,970,991,06
4,686,919,463,448,038,574,143,829,629,547,131,372,173,669,836,184,558,1
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001,864,788,050,061,351,603,144,350,076,189,213,441,602,488,091,741,05
1,232,290,357,179,205,497,927,970,924,502,479,940,842,696,158,818,442,6
16,163,780,044,759,478,212,240,873,204,124,421,169,199,805,572,649,118,
243,661,921,835,714,762,891,425,805,771,871,743,688,000,324,113,008,70
4,819,373,962,295,017,143,090,098,476,927,237,498,875,938,639,942,530,5
95,331,607,891,618,810,863,505,982,444,578,942,799,346,514,915,952,884,
869,757,488,025,823,353,571,677,864,826,828,051,140,885,429,732,788,19
7,765,736,966,005,727,700,162,592,404,301,688,659,946,862,983,717,270,5
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484,305,467,611,034,654,748,067,290,674,399,763,612,592,434,637,719,99
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69,463,293,113,276,849,329,777,201,646,641,727,587,259,122,354,784,480,
813,433,328,050,087,758,855,264,686,119,576,962,172,239,308,693,795,75
7,165,821,852,416,204,341,972,383,989,932,734,803,429,262,340,722,338,1
55,102,209,101,262,949,249,742,423,271,698,842,023,297,303,260,161,790,
575,673,111,235,465,890,298,298,313,115,123,607,606,773,968,998,153,81
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34,215,901,479,188,632,194,659,637,483,482,564,291,616,278,532,948,239,
313,229,440,231,043,277,288,768,139,550,213,348,266,388,687,453,259,28
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03,999,026,132,015,872,678,637,873,095,678,109,625,311,008,054,489,418,
857,983,565,902,063,680,699,643,165,033,912,029,944,327,726,770,869,30
5,240,718,416,592,070,096,139,286,401,966,725,750,087,012,218,149,733,1
33,695,809,600,369,751,764,951,350,040,285,926,249,203,398,111,014,953,
227,533,621,844,500,744,331,562,434,532,484,217,986,108,346,261,345,89
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19,886,390,369,784,542,622,566,912,542,747,056,097,567,984,857,136,623,
679,023,878,478,161,201,477,982,939,080,513,150,258,174,523,773,529,51
0,165,296,934,562,786,122,241,150,783,587,755,373,348,372,764,439,838,0
82,000,667,214,740,034,466,322,776,918,936,967,612,878,983,488,942,094,
688,102,308,427,036,452,854,504,966,759,697,318,836,044,496,702,853,19

0,637,396,916,357,980,928,865,719,935,397,723,495,486,787,180,416,401,4
15,281,489,443,785,036,291,071,517,805,285,857,583,987,711,145,474,240,
156,416,477,194,116,391,354,935,466,755,593,592,608,849,200,546,384,68
5,403,028,080,936,417,250,583,653,368,093,407,225,310,820,844,723,570,2
26,809,826,951,426,162,451,204,040,711,501,448,747,856,199,922,814,664,
565,893,938,488,028,643,822,313,849,852,328,452,360,667,045,805,113,67
9,663,751,039,248,163,336,173,274,547,275,775,636,810,977,344,539,275,8
27,560,597,425,160,705,468,689,657,794,530,521,602,315,939,865,780,974,
801,515,414,987,097,778,078,705,357,058,008,472,376,892,422,189,750,31
2,758,527,140,173,117,621,279,898,744,958,406,199,843,913,365,680,297,7
21,208,751,934,988,504,499,713,914,285,158,032,324,823,021,340,630,312,
586,072,624,541,637,765,234,505,522,051,086,318,285,359,658,520,708,17
3,392,709,566,445,011,404,055,106,579,055,037,417,780,393,351,658,360,9
04,543,047,721,422,281,816,832,539,613,634,982,525,215,232,257,690,920,
254,216,409,657,452,618,066,051,777,901,592,902,884,240,599,998,882,75
3,691,957,540,116,954,696,152,270,401,280,857,579,766,154,722,192,925,6
55,963,991,820,948,894,642,657,512,288,766,330,302,133,746,367,449,217,
449,351,637,104,725,732,980,832,812,726,468,187,759,356,584,218,383,59
4,702,792,013,663,907,689,741,738,962,252,575,782,663,990,809,792,647,0
11,407,580,367,850,599,381,887,184,560,094,695,833,270,775,126,181,282,
015,391,041,773,950,918,244,137,561,999,937,819,240,362,469,558,235,92
4,171,478,702,779,448,443,108,751,901,807,414,110,290,370,706,052,085,1
62,975,798,361,754,251,041,642,244,867,577,350,756,338,018,895,379,263,
183,389,855,955,956,527,857,227,926,155,524,494,739,363,665,533,904,52
8,656,215,464,288,343,162,282,921,123,290,451,842,212,532,888,101,415,8
84,061,619,939,195,042,230,059,898,349,966,569,463,580,186,816,717,074,
818,823,215,848,647,734,386,780,911,564,660,755,175,385,552,224,428,52
4,049,468,033,692,299,989,300,783,900,020,690,121,517,740,696,428,573,9
30,196,910,500,988,278,523,053,797,637,940,257,968,953,295,112,436,166,
778,910,585,557,213,381,789,089,945,453,947,915,927,374,958,600,268,23
7,844,486,872,037,243,488,834,616,856,290,097,850,532,497,036,933,361,9
42,439,802,882,364,323,553,808,208,003,875,741,710,969,289,725,499,878,
566,253,048,867,033,095,150,518,452,126,944,989,251,596,392,079,421,45
2,606,508,516,052,325,614,861,938,282,489,838,000,815,085,351,564,642,7
61,700,832,096,483,117,944,401,971,780,149,213,345,335,903,336,672,376,
719,229,722,069,970,766,055,482,452,247,416,927,774,637,522,135,201,71
6,231,722,137,632,445,699,154,022,395,494,158,227,418,930,589,911,746,9
31,773,776,518,735,850,032,318,014,432,883,916,374,243,795,854,695,691,
221,774,098,948,611,515,564,046,609,565,094,538,115,520,921,863,711,51
8,684,562,543,275,047,870,530,006,998,423,140,180,169,421,109,105,925,4
93,596,116,719,457,630,962,328,831,271,268,328,501,760,321,771,680,400,
249,657,674,186,927,113,215,573,270,049,935,709,942,324,416,387,089,24
2,427,584,407,651,215,572,676,037,924,765,341,808,984,312,676,941,110,3
13,165,951,429,479,377,670,698,881,249,643,421,933,287,404,390,485,538,
222,160,837,088,907,598,277,390,184,204,138,197,811,025,854,537,088,58
6,701,450,623,578,513,960,109,987,476,052,535,450,100,439,353,062,072,4
39,709,976,445,146,790,993,381,448,994,644,609,780,957,731,953,604,938,
734,950,026,860,564,555,693,224,229,691,815,630,293,922,487,606,470,87
3,431,166,384,205,442,489,628,760,213,650,246,991,893,040,112,513,103,8
35,085,621,908,060,270,866,604,873,585,849,001,704,200,923,929,789,193,
938,125,116,798,421,788,115,209,259,130,435,572,321,635,660,895,603,51
4,383,883,939,018,953,166,274,355,609,970,015,699,780,289,236,362,349,8
95,374,653,428,746,875

Takeaways

- There are often many ways to compute values (with tradeoffs)
- A little bit of math helps
- Python has speed tricks

The End 

github.com/brianspiering/fibonacci_sequences (https://github.com/brianspiering/fibonacci_sequences)