Jet and Rocket Propulsion Equation Sheet

Mathematica Basics

Useful Commands

Notation Palette

You have to use Symbolize[▪] to create variables with Greek letters and subscripts if you find that a variable is misbehaving

```
<< Notation`
```

Simplify last output

```
In[ ]:= Simplify[%] // Chop
```

Keyboard Shortcuts

Evaluation

shift + enter - evaluate currently-selected cell
ctrl+= (ctrl++) - insert free-form input (useful for getting constants and using units easily, acts like a wolfram alpha input basically)

Mathematical Typesetting

ctrl+2 - creates square roots $\sqrt{\square}$
ctrl+6 (ctrl+^) - creates superscripts $\square^{\square}$
ctrl+7 - puts symbols over another $\overset{\square}{\square}$
ctrl+_ (ctrl+-)- creates subscripts $\square_{\square}$

Text Formatting

alt + (1 - 9) - changes text formatting
the first 6 are headers of various sizes, 7 is text, 8 is formatted code (for use in a report, for exam-

ple), and 9 is the default input style.

Constants

Gamma for air

$\gamma = 1.4$

Gas constant for air

In[ ]:= $R = 287\ \mathrm{J}/(\mathrm{kg\,K})$

Specific heat of air

$C_p = 1005\ \mathrm{J}/(\mathrm{kg\,K})$

Perfect Gas

Ideal Gas Law

$P = \rho R T$

Gas Constant

$$R = \frac{R_{universal}}{M} = \frac{(\text{universal gas constant})}{(\text{molecular weight})}$$

Specific Heat

$$C_p - C_v = R$$

$$\frac{C_p}{C_v} = \gamma$$

$$C_p = \frac{\gamma}{\gamma - 1} R$$

Gamma

$$\gamma = 1 + \frac{2}{n}$$

n is the number of degrees of freedom (translational)

$\gamma = 1.4$ for air

Isentropic Relations

In[ ]:= $$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma}$$

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma-1}$$

Helpful equations from HW1

$$u = \sqrt{\frac{2\gamma R T_0}{\gamma-1}\left(1-\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$\dot{m} = A P_0 \sqrt{\frac{2\gamma}{R T_0(\gamma-1)}\left(\left(\frac{P}{P_0}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_0}\right)^{\frac{\gamma+1}{\gamma}}\right)}$$

$$\dot{m}_{\max} = A^* P_0 \sqrt{\frac{\gamma}{R T_0}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\frac{u}{u^*} = \sqrt{\frac{\gamma+1}{\gamma-1}\left(1-\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}$$

$$\frac{A}{A^*} = \frac{\left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}{\left(\frac{P}{P_0}\right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma+1}{\gamma-1}\left(1-\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}\right)}}$$

$$\frac{T}{T^*} = \frac{\gamma+1}{2}\left(\frac{P}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

$$M = \sqrt{\frac{2}{\gamma-1}\left(\left(\frac{P_0}{P}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)}$$

$$\frac{C^*}{C_0} = \sqrt{\frac{2}{\gamma+1}}$$

$$\frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{T^*}{T_0} = \frac{2}{\gamma+1}$$

Fluid Mechanics

Total energy of a fluid

$$e + \frac{1}{2}\left(u^2 + v^2 + w^2\right)$$

Mass flux through a given area

$\dot{m} = \int \rho\, \vec{u}\, d\vec{A} = \rho\, \vec{A}\, \vec{u}$ (if velocity and density are constant)

Continuity

In[ ]:= $A_1\, \rho_1\, u_1 = A_2\, \rho_2\, u_2$

Energy flux through a control volume

$\text{Influx} = \rho_1\, u_1\, A_1\left(e_1 + \frac{1}{2}\, {u_1}^2\right)$

$\text{Outflux} = \rho_2\, u_2\, A_2\left(e_2 + \frac{1}{2}\, {u_2}^2\right)$

Enthalpy : $h = e + \frac{1}{2}\, u^2$

(For a conduit with no heat transfer or forces other than inlet and outlet pressures)

$$h_1 + \frac{1}{2}\, {u_1}^2 = h_2 + \frac{1}{2}\, {u_2}^2$$

(If heat is added)

$$\left(h_2 + \frac{1}{2}\, {u_2}^2\right) - \left(h_1 + \frac{1}{2}\, {u_1}^2\right) = \Delta q$$

$$h_{0_2} - h_{0_1} = \Delta q$$

Bernoulli Equation

(For steady, incompressible, inviscid flow)

$$P + \tfrac{1}{2}\, \rho\, u^2 = \text{constant}$$

(For isentropic, steady flow of a perfect gas)

$$\frac{\gamma}{(\gamma - 1)}\, \frac{P}{\rho} + \frac{u^2}{2} = \text{constant}$$

Entropy Change

$$C_p \, \mathrm{dT} = \mathrm{dh} = \frac{\mathrm{dP}}{\rho} + T \, \mathrm{dS}$$

If dS = 0:

$C_p \, \mathrm{dT} + u \, \mathrm{du} = 0$

$$\Delta \mathrm{S} = C_p \log\left(\frac{T_2}{T_1}\right) - R \log\left(\frac{P_2}{P_1}\right)$$

(if u=0, natural log)

If the process is adiabatic :

$T_{02} = T_{01}$

$$\Delta \mathrm{S} = -R \log\left(\frac{P_2}{P_1}\right)$$

Since $\Delta S \geq 0$, $P_{01} \geq P_{02}$

Change in entropy $\rightarrow$ reduction in stagnation pressure P_0

Speed of sound

$$C = \sqrt{\gamma R T}$$

Mach Number

$$M = \frac{u}{C} = \frac{u}{\sqrt{\gamma R T}}$$

$$\frac{T_0}{T} = 1 + \frac{(\gamma - 1)}{2} M^2$$

$$\frac{P_0}{P} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{(\gamma - 1)}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

Flow in a Nozzle

$\rho \mathrm{uA} = \mathrm{constant}$

$$\frac{\mathrm{d}\rho}{\rho}\left(1 - \frac{1}{M^2}\right) = -\frac{\mathrm{dA}}{A} \text{ or } \frac{\mathrm{du}}{u}\left(1 - M^2\right) = -\frac{\mathrm{dA}}{A}$$

$\Rightarrow$ For $M<1$ | $A\downarrow \;\Rightarrow\; u\uparrow$ | $A\uparrow \;\Rightarrow\; u\downarrow$

$M>1$ | $A\downarrow \;\Rightarrow\; u\downarrow$ | $A\uparrow \;\Rightarrow\; u\uparrow$

For M = 1 -> dA = 0 (throat conditions are marked with *)

Compressible flow in constant diameter pipes

We do not assume that the flow is isentropic.

Pressure

$$\frac{P_1}{P_2} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2}$$

If somewhere along the pipe M=1

$$\frac{P}{P^*} = \frac{1+\gamma}{1+\gamma M^2}$$

$$\frac{P_0}{P_0{}^*} = \frac{1+\gamma}{1+\gamma M^2}\left(\frac{1+\frac{\gamma-1}{2}M^2}{\frac{\gamma+1}{2}}\right)^{\frac{\gamma}{\gamma-1}}$$

Temperature

Since $\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2}$,

$\frac{P_2}{P_1} = \frac{\rho_2\,T_2}{\rho_1\,T_1}$

Then :

$$\frac{T_2}{T_1} = \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^2 \frac{M_2^2}{M_1^2}$$

$$\frac{T}{T^*} = M^2\left(\frac{1+\gamma}{1+\gamma M^2}\right)^2$$

$$\frac{T_0}{T_0{}^*} = \frac{2\,M^2(1+\gamma)\left(1+\frac{\gamma-1}{2}M^2\right)}{\left(1+\gamma \mathrm{M}^2\right)^2}$$

Density and velocity

Similarly :

$$\frac{\rho_1}{\rho_2} = \frac{u_2}{u_1} = \frac{T_2\,P_1}{T_1\,P_2} = \frac{M_2{}^2}{M_1{}^2}\,\frac{1+\gamma\,M_1{}^2}{1+\gamma \mathrm{M}_2{}^2}$$

$$\frac{\rho^*}{\rho} = \frac{u}{u^*} = \frac{M^2(\gamma + 1)}{1 + \gamma M^2}$$

Entropy

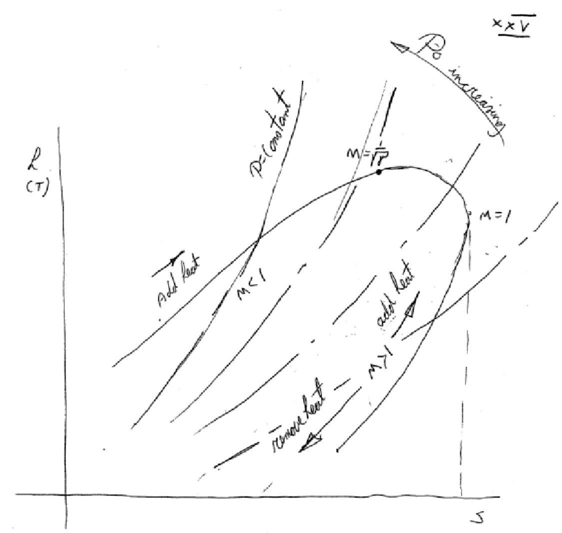
(natural log)

$$\frac{S_2 - S_1}{C_p} = \log\left(\frac{M_2^2}{M_1^2}\left(\frac{1 + \gamma M_1{}^2}{1 + \gamma M_2{}^2}\right)^{\frac{\gamma+1}{\gamma}}\right)$$

$$\frac{S_2 - S^*}{C_p} = \log\left(M^2\left(\frac{1 + \gamma}{1 + \gamma M^2}\right)^{\frac{\gamma+1}{\gamma}}\right)$$

Rayleigh Line

Constant diameter pipe with heat transfer, no friction



for $M>1$ $\quad (1-M^2)<0$

$ds>0 \Rightarrow dM<0 \quad$ Since $2(1-\gamma M^2)<0 \quad dT>0$

$ds<0 \Rightarrow dM>0 \quad$ Since $2(1-\gamma M^2)<0 \quad dT<0$

for $\underline{M<1}$

$ds>0 \Rightarrow dM>0$

$ds<0 \Rightarrow dM<0$

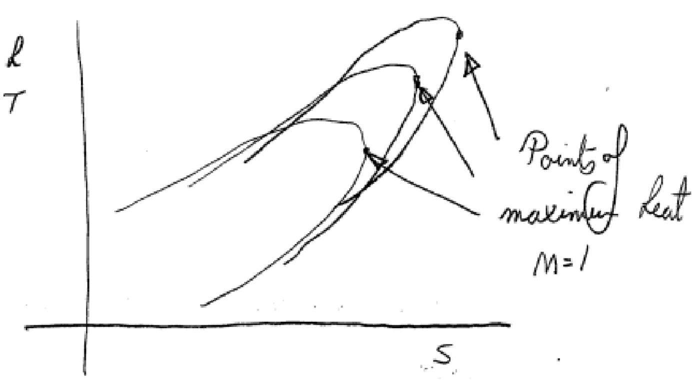
for $M<\frac{1}{\sqrt{\gamma}} \quad 1-\gamma M^2>0$

for $M>\frac{1}{\sqrt{\gamma}} \quad 1-\gamma M^2<0$

$ds>0 \,;\; M<\frac{1}{\sqrt{\gamma}} \Rightarrow dM>0 \quad \underline{dT>0}$

$ds>0 \quad M>\frac{1}{\sqrt{\gamma}} \Rightarrow dM>0 \quad dT<0 \quad ?$

No more heat can be added when $M = 1$, to add more heat, the flow rate must be changed. As the flow rate increases, the curve moves up and to the right:



Effects of Added Heat

$$\frac{dq}{dT} = \frac{\gamma R}{\gamma - 1}\left(\frac{1 - M^2}{1 - \gamma M^2}\right)$$

When $M < \frac{1}{\sqrt{\gamma}}$, $\frac{dq}{dT} > 0$

When $M > 1$, $\frac{dq}{dT} > 0$

When $\frac{1}{\sqrt{\gamma}} < M < 1$, $\frac{dq}{dT} < 0$

Fanno Line

Steady flow in a constant diameter pipe with friction, no heat transfer

Energy

$$\frac{dT}{T} + \frac{\gamma - 1}{2} M^2 \frac{d(u^2)}{u^2} = 0$$

Continuity

$$\frac{d\rho}{\rho} + \frac{1}{2} \frac{d(u^2)}{u^2} = 0$$

Momentum Balance

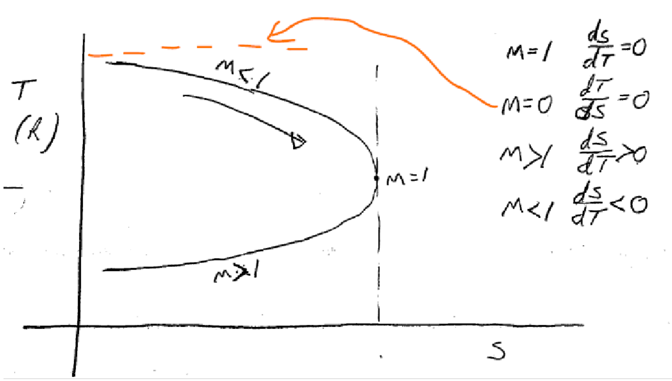
$$\frac{dP}{P} + \frac{4 C_f \, dx}{D_H} * \frac{\gamma M^2}{2} + \frac{\gamma M^2}{2} \frac{d(u^2)}{u^2} = 0$$

Entropy

$$\frac{ds}{dM} = \frac{C_p(\gamma - 1)(1 - M^2)}{\gamma M\left(1 + \frac{\gamma-1}{2} M^2\right)}$$

Temperature

$$\frac{dT}{dM} = -T^* \frac{(\gamma + 1)(\gamma - 1)}{2\left(1 + \frac{\gamma-1}{2} M^2\right)^2}$$



$$\frac{dM^2}{M^2} = \frac{1+\frac{\gamma-1}{2}M^2}{1-M^2}\cdot \gamma M^2 \cdot \frac{4C_f\,dx}{D_H}$$

$$\frac{du}{u} = \frac{\gamma M^2}{2(1-M^2)}\,\frac{4C_f\,dx}{D_H}$$

$$\frac{dT}{T} = -\frac{\gamma(\gamma-1)}{2}\,M^4\cdot\frac{1}{1-M^2}\cdot\frac{4C_f\,dx}{D_H}$$

$$\frac{dP_0}{P_0} = -\frac{\gamma M^2}{2}\cdot\frac{4C_f\,dx}{D_H}$$

$$\frac{d\rho}{\rho} = -\frac{\gamma M^2}{2(1-M^2)}\,4C_f\frac{dx}{D_H}$$

$$\frac{ds}{C_p} = \frac{\gamma-1}{2}M^2\cdot\frac{4C_f\,dx}{D_H}$$

“If the pipe is long enough, it will choke” -J. Katz

(Flow tends to reach M = 1 through friction alone)

Velocity in a pipe:

$$\frac{4\,C_f(x_2-x_1)}{D_H} = \frac{{M_2}^2-{M_1}^2}{\gamma {M_2}^2\,{M_1}^2} + \frac{\gamma+1}{2\,\gamma}\ln\left[\frac{{M_1}^2}{{M_2}^2} * \frac{\left(1+\frac{\gamma-1}{2}\,{M_2}^2\right)}{\left(1+\frac{\gamma-1}{2}\,{M_1}^2\right)}\right]$$

Maximum pipe length before choking:

$$\frac{4\,x^*\,C_f}{D_H} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\,\gamma}\ln\left[M^2\,\frac{(\gamma+1)}{2\left(1+\frac{\gamma-1}{2}\,M^2\right)}\right]$$

Normal Shocks

1. dq = 0 (no heat transfer)
2. Steady flow
3. No Friction
4. D = constant

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

$$\rho_1\,u_1 = \rho_2\,u_2$$

$$(P_1-P_2)\,A = \rho\,u\,A\,(u_2-u_1)$$

Conditions downstream of shock

Mach Number

$$M_2 = \sqrt{\frac{M_1{}^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1} M_1{}^2 - 1}}$$

Pressure

$$\frac{P_2}{P_1} = \frac{2\gamma}{\gamma+1} M_1{}^2 - \frac{\gamma-1}{\gamma+1}$$

or $\tilde{P} = \frac{P_2}{P_1} - 1 = \frac{2\gamma}{\gamma+1}\left(M_1{}^2 - 1\right)$

$$\frac{P_{02}}{P_{01}} = \left[\frac{\frac{\gamma+1}{2} M_1{}^2}{1+\frac{\gamma-1}{2} M_1{}^2}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{2\gamma}{\gamma+1} M_1{}^2 - \frac{\gamma-1}{\gamma+1}\right]^{1}\left[\frac{2\gamma}{\gamma+1} M_1{}^2 - \frac{\gamma-1}{\gamma+1}\right]^{-\frac{\gamma}{\gamma-1}}$$

(I'm not transcribing that unless I have to use it lmao)

I did lol, and it simplifies to:

$$\frac{P_{0_2}}{P_{0_1}} = \left(\frac{\frac{\gamma+1}{2} M_1^2}{1+\frac{\gamma-1}{2} M_1^2}\right)^{\frac{\gamma}{\gamma-1}} \left(\frac{2\gamma}{\gamma+1} M_1^2 - \frac{\gamma-1}{\gamma+1}\right)^{-1}$$

Temperature

$$\frac{T_2}{T_1} = \frac{\left(1+\frac{\gamma-1}{2} M_1{}^2\right)\left(\frac{2\gamma}{\gamma-1} M_1{}^2 - 1\right)}{\frac{(\gamma+1)^2}{2(\gamma-1)} M_1{}^2}$$

Density and Velocity

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{1}{\left(\frac{1}{\gamma+1}\right)\left(\gamma + \frac{2}{M_1^2} - 1\right)}$$

Entropy Change

$$S_2 - S_1 = -R \log\left(\frac{P_{0,2}}{P_{0,1}}\right)$$

Rankine Hugoniot Relations

Pressure

$$\frac{P_2}{P_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_2}{\rho_1}}$$

Maximum Density Ratio

$$\frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}$$

Entropy Change

$$\frac{S_2 - S_1}{C_v} = \ln\left[1 + \tilde{P}\right] - \gamma \ln\left[1 + \frac{\gamma+1}{2\,\gamma}\tilde{P}\right] + \gamma \ln\left[1 + \frac{\gamma-1}{2\,\gamma}\tilde{P}\right]$$

Performing a Taylor series expansion, the important terms simplify to:

$$\frac{S_2 - S_1}{C_v} = \frac{2}{3}\frac{\gamma}{(\gamma+1)^3}\left({M_1}^2 - 1\right)^3$$

Important takeaway: entropy increases in proportion to ΔP^3, so weak shocks are preferable.

Shock Wave as a Compressor

Efficiency:

$$\eta_c = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_2}{T_1} - 1}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} * \frac{\rho_1}{\rho_2} = \frac{P_2}{P_1} * \frac{1 + \frac{\gamma-1}{2\,\gamma}\left(\frac{P_2}{P_1} - 1\right)}{1 + \frac{\gamma+1}{2\,\gamma}\left(\frac{P_2}{P_1} - 1\right)}$$

Oblique Shocks

Shock angle

Use == and Solve[] for this

$$\tan(\theta) = \frac{2\left(M_1^2 \sin^2(\beta) - 1\right)}{\tan(\beta)\left(M_1^2\left(\cos(2\,\beta) + \gamma\right) + 2\right)}$$

^Two solutions; weak solution is usually the correct one.

In all cases:

$$\sin^{-1}\left(\frac{1}{M_1}\right) \leq \beta \leq \frac{\pi}{2}$$

Conditions downstream of shock

$$M_2 = \sqrt{\frac{\frac{1}{2}(\gamma - 1) M_1^2 \sin^2(\beta_1) + 1}{\sin^2(\beta_1 - \theta)\left(\gamma M_1^2 \sin^2(\beta_1) - \frac{\gamma-1}{2}\right)}}$$

$$P_2 = \frac{2\gamma P_1 \left(M_1^2 \sin^2(\beta_1) - 1\right)}{\gamma + 1} + P_1$$

$$\rho_2 = \frac{\rho_1(\gamma + 1) {M_1}^2 \sin^2(\beta)}{(\gamma - 1) {M_1}^2 \sin^2(\beta) + 2}$$

$$T_2 = T_1\left(\frac{2(\gamma - 1)\left(M_1^2 \sin^2(\beta) - 1\right)\left(\gamma M_1^2 \sin^2(\beta) + 1\right)}{(\gamma + 1)^2 \left(M_1^2 \sin^2(\beta)\right)} + 1\right)$$

Weak shock approximation

$$\theta - \frac{2\left(M_1^2 \sin^2(\beta) - 1\right)}{\tan(\beta)\left((\gamma + 1) M_1^2\right)} = 0$$

$$\frac{\Delta \mathrm{P}}{P_1} = \frac{\gamma \mathrm{M_1}^2}{\sqrt{{M_1}^2 - 1}}\,\theta$$

This means ΔS is proportional to θ^3.

A continuous curve (effectively infinite weak shocks) can be treated as isentropic.

Expansion waves, where $\theta < 0$, are also isentropic

Velocity change through an oblique shock

General case:

$$\frac{w_2}{w_1} = \frac{\cos(\beta)}{\cos(\beta - \theta)}$$

For a weak shock:

$$\frac{w_2 - w_1}{w_1} = \frac{-\theta}{\sqrt{{M_1}^2 - 1}}$$

For a continuous curve or expansion wave:

$$\frac{\text{dw}}{w} = \frac{-\text{d}\theta}{\sqrt{M^2 - 1}}$$

Prandtl-Meyer Function

Only applies for weak shock assumptions or expansion waves

$$v(M) = \left\{ \sqrt{\frac{\gamma + 1}{\gamma - 1}} \tan^{-1}\left(\sqrt{\frac{\gamma - 1}{\gamma + 1} (M^2 - 1)} \right) - \tan^{-1}\left(\sqrt{M^2 - 1} \right) \right\}$$

v (M) = c - θ

(Where c is some constant)

v(M) is defined solely by Mach number, and can be calculated using changes in angle:

$v(M_2) = v(M_1) - (\theta_2 - \theta_1)$

Values of v and their corresponding Mach numbers are tabulated below.

TABLE V

MACH NUMBER AND MACH ANGLE VERSUS PRANDTL-MEYER FUNCTION

ν (deg)	M	μ (deg)	ν (deg)	M	μ (deg)
0.0	1.000	90.000	17.5	1.689	36.293
0.5	1.051	72.099	18.0	1.706	35.874
1.0	1.082	67.574	18.5	1.724	35.465
1.5	1.108	64.451	19.0	1.741	35.065
2.0	1.133	61.997	19.5	1.758	34.673
2.5	1.155	59.950	20.0	1.775	34.290
3.0	1.177	58.180	20.5	1.792	33.915
3.5	1.198	56.614	21.0	1.810	33.548
4.0	1.218	55.205	21.5	1.827	33.188
4.5	1.237	53.920	22.0	1.844	32.834
5.0	1.256	52.738	22.5	1.862	32.488
5.5	1.275	51.642	23.0	1.879	32.148
6.0	1.294	50.619	23.5	1.897	31.814
6.5	1.312	49.658	24.0	1.915	31.486
7.0	1.330	48.753	24.5	1.932	31.164
7.5	1.348	47.896	25.0	1.950	30.847
8.0	1.366	47.082	25.5	1.968	30.536
8.5	1.383	46.306	26.0	1.986	30.229
9.0	1.400	45.566	26.5	2.004	29.928
9.5	1.418	44.857	27.0	2.023	29.632
10.0	1.435	44.177	27.5	2.041	29.340
10.5	1.452	43.523	28.0	2.059	29.052
11.0	1.469	42.894	28.5	2.078	28.769
11.5	1.486	42.287	29.0	2.096	28.491
12.0	1.503	41.701	29.5	2.115	28.216
12.5	1.520	41.134	30.0	2.134	27.945
13.0	1.537	40.585	30.5	2.153	27.678
13.5	1.554	40.053	31.0	2.172	27.415
14.0	1.571	39.537	31.5	2.191	27.155
14.5	1.588	39.035	32.0	2.210	26.899
15.0	1.605	38.547	32.5	2.230	26.646
15.5	1.622	38.073	33.0	2.249	26.397
16.0	1.639	37.611	33.5	2.269	26.151
16.5	1.655	37.160	34.0	2.289	25.908
17.0	1.672	36.721	34.5	2.309	25.668

TABLE V (*Continued*)

MACH NUMBER AND MACH ANGLE VERSUS PRANDTL-MEYER FUNCTION

ν (deg)	M	μ (deg)	ν (deg)	M	μ (deg)
35.0	2.329	25.430	52.5	3.146	18.532
35.5	2.349	25.196	53.0	3.174	18.366
36.0	2.369	24.965	53.5	3.202	18.200
36.5	2.390	24.736	54.0	3.230	18.036
37.0	2.410	24.510	54.5	3.258	17.873
37.5	2.431	24.287	55.0	3.287	17.711
38.0	2.452	24.066	55.5	3.316	17.551
38.5	2.473	23.847	56.0	3.346	17.391
39.0	2.495	23.631	56.5	3.375	17.233
39.5	2.516	23.418	57.0	3.406	17.076
40.0	2.538	23.206	57.5	3.436	16.920
40.5	2.560	22.997	58.0	3.467	16.765
41.0	2.582	22.790	58.5	3.498	16.611
41.5	2.604	22.585	59.0	3.530	16.458
42.0	2.626	22.382	59.5	3.562	16.306
42.5	2.649	22.182	60.0	3.594	16.155
43.0	2.671	21.983	60.5	3.627	16.005
43.5	2.694	21.786	61.0	3.660	15.856
44.0	2.718	21.591	61.5	3.694	15.708
44.5	2.741	21.398	62.0	3.728	15.561
45.0	2.764	21.207	62.5	3.762	15.415
45.5	2.788	21.017	63.0	3.797	15.270
46.0	2.812	20.830	63.5	3.832	15.126
46.5	2.836	20.644	64.0	3.868	14.983
47.0	2.861	20.459	64.5	3.904	14.840
47.5	2.886	20.277	65.0	3.941	14.698
48.0	2.910	20.096	65.5	3.979	14.557
48.5	2.936	19.916	66.0	4.016	14.417
49.0	2.961	19.738	66.5	4.055	14.278
49.5	2.987	19.561	67.0	4.094	14.140
50.0	3.013	19.386	67.5	4.133	14.002
50.5	3.039	19.213	68.0	4.173	13.865
51.0	3.065	19.041	68.5	4.214	13.729
51.5	3.092	18.870	69.0	4.255	13.593
52.0	3.119	18.701	69.5	4.297	13.459

TABLE V (*Continued*)

MACH NUMBER AND MACH ANGLE VERSUS PRANDTL-MEYER FUNCTION

ν (deg)	M	μ (deg)	ν (deg)	M	μ (deg)
70.0	4.339	13.325	87.5	6.390	9.003
70.5	4.382	13.191	88.0	6.472	8.888
71.0	4.426	13.059	88.5	6.556	8.774
71.5	4.470	12.927	89.0	6.642	8.660
72.0	4.515	12.795	89.5	6.729	8.546
72.5	4.561	12.665	90.0	6.819	8.433
73.0	4.608	12.535	90.5	6.911	8.320
73.5	4.655	12.406	91.0	7.005	8.207
74.0	4.703	12.277	91.5	7.102	8.095
74.5	4.752	12.149	92.0	7.201	7.983
75.0	4.801	12.021	92.5	7.302	7.871
75.5	4.852	11.894	93.0	7.406	7.760
76.0	4.903	11.768	93.5	7.513	7.649
76.5	4.955	11.642	94.0	7.623	7.538
77.0	5.009	11.517	94.5	7.735	7.428
77.5	5.063	11.392	95.0	7.851	7.318
78.0	5.118	11.268	95.5	7.970	7.208
78.5	5.174	11.145	96.0	8.092	7.099
79.0	5.231	11.022	96.5	8.218	6.989
79.5	5.289	10.899	97.0	8.347	6.881
80.0	5.348	10.777	97.5	8.480	6.772
80.5	5.408	10.656	98.0	8.618	6.664
81.0	5.470	10.535	98.5	8.759	6.556
81.5	5.532	10.414	99.0	8.905	6.448
82.0	5.596	10.294	99.5	9.055	6.340
82.5	5.661	10.175	100.0	9.210	6.233
83.0	5.727	10.056	100.5	9.371	6.126
83.5	5.795	9.937	101.0	9.536	6.019
84.0	5.864	9.819	101.5	9.708	5.913
84.5	5.935	9.701	102.0	9.885	5.806
85.0	6.006	9.584			
85.5	6.080	9.467			
86.0	6.155	9.350			
86.5	6.232	9.234			
87.0	6.310	9.119			

Numerical values taken from *Publication No.* 26, Jet Propulsion Laboratory, California Institute of Technology.