CHAIN RULE

1. EXAMPLE :-

$$f(x) = \frac{1}{1+e^{-x}} \qquad \frac{df}{dx} = ?$$

Let $\underbrace{p = -x}_{p(x)}$ $\qquad$ $q = 1+e^{-x} = 1+e^{p}$ $\rightarrow$ $q(p)$

$$f(q) = \frac{1}{q} \qquad f(q(p(x))) = \frac{1}{1+e^{(-x)}}$$

$$\boxed{\frac{df}{dx} = \frac{df}{dq} \cdot \frac{dq}{dp} \cdot \frac{dp}{dx}}$$

$$\frac{df}{dq} = -\frac{1}{q^2}$$

formula $x^n = nx^{n-1}$

$$\frac{dq}{dp} = \frac{d(1+e^{p})}{dp} = 0 + e^{p}$$

$e^x \rightarrow e^x$

$$\Rightarrow \frac{dq}{dp} = e^{p}$$

$$\frac{dp}{dx} = -1$$

$$= -\frac{1}{q^2} \cdot e^{p} \cdot (-1)$$

$$\frac{df}{dx} = \frac{e^{p}}{q^2}$$

$$\frac{df}{dx} = \frac{e^{-x}}{(1+e^{-x})^2}$$

$$= \frac{1+e^{-x}-1}{(1+e^{-x})^2}$$

$$= \frac{\cancel{(1+e^{-x})}}{(1+e^{-x})^{\cancel{2}}} - \frac{1}{(1+e^{-x})^2}$$

$$= \frac{1}{(1+e^{-x})} - \frac{1}{(1+e^{-x})^2} = \frac{1}{1+e^{-x}}\left\{1 - \frac{1}{1+e^{-x}}\right\}$$

$$\frac{df}{dx} = f(x)\,[1 - f(x)]$$