2016 2 0 1 6

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Mathematical Relationships in Electrostatic and Magnetic Fields

by

James Clerk Maxwell

A Dissertation Submitted to the Faculty of the

Graduate Interdisciplinary Program
in Applied Mathematics

In Partial Fulfillment of the Requirements
For the Degree of

Doctor of Philosophy
With a Major in Electrical and Computer Engineering

In the Graduate College

The University of Arizona

June 1, 2023

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Statement by Author

Signed: ______________________________

I would like to express my sincere gratitude to my supervisor, Dr. Carl F. Gauss, for the suggestion of this research topic and for his guidance and support throughout this project. Thanks are also extended to Dr. Charles A. Coulomb and Dr. Conte A. Volta for their participation on the committee and for their helpful comments. My fellow graduate students, Mr. Andre M. Ampere, Mr. Georg S. Ohm, and Mr. Michael Faraday, deserve many thanks for their friendship and help during my graduate studies. I would like to thank the members of my family, who have always supported my education with their love, encouragement, patience, and support. Finally, thanks to the German Electric Power Company for funding my research assistantship.

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Abstract

[Limited to 150 words for an M.S. thesis and 350 words for a Ph.D. dissertation.] The applications of electric and magnetic fields are widespread. The goal of this work was to develop the mathematical foundations for static fields. Four basic laws are presented which describe the relationships between the electric flux density, the electric charge density, the electric field intensity, the magnetic field intensity, the electric current density, and the magnetic flux density. A detailed derivation of the corresponding equations is presented, along with a discussion of their applications. Experimental results are also shown, which confirm the validity of the theoretical work.

Mathematical Relationships in Electrostatic and Magnetic Fields

James Clerk Maxwell, Ph.D.
The University of Arizona, June 1, 2023

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Chapter 1
Introduction

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Chapter 2

Background

Background background background background background background background background background background background background background background background background background background background.

2.1 Vector Calculus

Vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus vector calculus.

2.1.1 Gradient

Gradient gradient.

2.1.2 Divergence

Divergence divergence.

2.1.3 Curl

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curl curl curl curl curl curl curl curl.

2.2 Electricity and Magnetism

Electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity electricity.

Chapter 3
A New Theory of Electromagnetics

New mathematical relationships have been derived, fundamental behavior of electrostatie and steady magnetic fields. These equations were verified by experimental results, as discussed in Chap. **??**. Portions of this work are based on earlier results by Gauss [**?**], Ampere [**?**], and Faraday [**?**].

3.1 Electric Flux Density

Eq. **??** is the first of Maxwell's equations, which describes the relationship between the electric flux density and the electric charge density.

$$\vec{\nabla} \cdot \vec{D} = \rho \tag{3.1}$$

This is derived from the point form of Gauss's Law, given in integral form as

$$\oint_S \vec{D} \cdot d\vec{S} = Q \tag{3.2}$$

3.2 Electric Field Intensity

Eq. **??** shows the second of Maxwell's equations, which involves the electric field intensity.

$$\vec{\nabla} \times \vec{E} = 0 \tag{3.3}$$

This is related to the point form of Ampere's Circuital Law,

$$\oint \vec{H} \cdot d\vec{L} = I \tag{3.4}$$

3.3 Magnetic Field Intensity

The third of Maxwell's equations describes the relationship between the magnetic field intensity and the electric current density:

$$\vec{\nabla} \times \vec{H} = \vec{J} \tag{3.5}$$

The corresponding integral formula is

$$\oint \vec{H} \cdot d\vec{L} = I \tag{3.6}$$

3.4 Magnetic Flux Density

The last of Maxwell's equations involves the magnetic flux density:

$$\vec{\nabla} \times \vec{B} = 0 \tag{3.7}$$

The corresponding integral formula is

$$\oint_S \vec{B} \cdot d\vec{S} = 0 \tag{3.8}$$

Chapter 4
Experimental Results

This chapter describes the experiments that were performed, along with the results of the experiments that were performed when they were performed. Experiments were performed experiments were performed Experiments were performed experiments were performed Experiments were performed experiments were performed.

4.1 Experimental Setup

Fig. **??** shows a nifty PostScript drawing. Experimental setup experimental setup.

4.2 Presentation of Data

Various measurements were made and data was tabulated. Table **??** gives a presentation of the data. Presentation of data presentation of data presentation of data. Presentation of data presentation of data presentation of data. Presentation of data presentation of data presentation of data. Presentation of data presentation of data presentation of data.

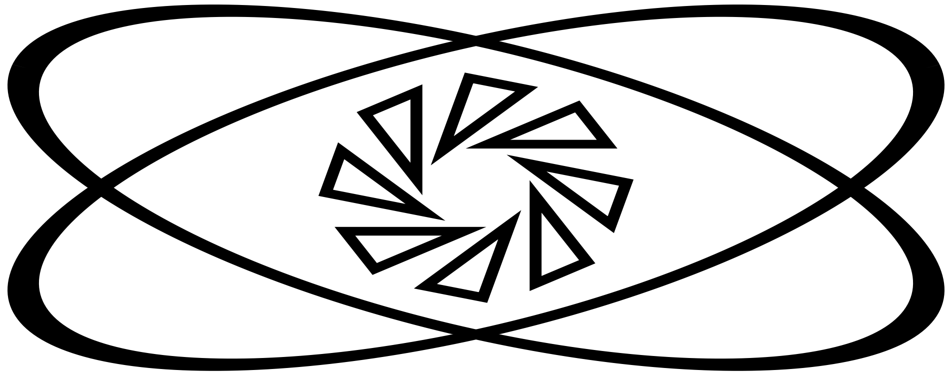

Figure 4.1. Nifty PostScript drawing.

Experiment	Measurement (units)
1	10.3
2	23.5
3	42.9

Table 4.1. Experimental Data

4.3 Analysis of Results

Several observations about the experimental results will now be made. Analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results analysis of results.

Chapter 5

Conclusion

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Appendix A

Detailed Material

Here we present the gruesome details of the derivations of some equations. Here we present the gruesome details of the derivations of some equations. Here we present the gruesome details of the derivations of some equations.

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