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NUMA41: Numerical Analysis Basic Course
Numerical Analysis, Matematikcentrum

Assignment 3

Goal: To get experience with Polynomial and Spline interpolation, Chebyshev polynomials, Runge's phenomenon

Task 1 This task is just to get experience with simple data interpolation and extrapolation. In the table below you find three columns with energy data from my house in Södra Sandby. Construct the unique polynomial which interpolates the energy consumption and another which interpolates the temperature. Plot the curves. Evaluate these polynomials to see, which energy I will consume on Tuesday, February, 8 and find out which will be the average temperature at that day.

Day	Temperature (C)	Energy (kwh)
040127	-1.9	109.26
040128	-3.7	92.4
040129	-5.77	115.33
040130	2.53	107.77
040131	4.32	61.14

Solve this task with three methods

- by using the commands `polyfit` and `polyval` from scipy.interploate,
- by using Vandermonde's approach
- by using Lagrange polynomials.

Note, you get in all three cases the same polynomial but in totally different representations. I.e. your plots should be identical in all three cases (if you made things correctly).

Task 2 When interpolating a function with a polynomial using $x_i, i = 0 : n$ as interpolation points the error has the form

$$|f(x) - p(x)| = \frac{1}{(n+1)!}|f^{(n+1)}|\,|(x-x_0)(x-x_1)\cdots(x-x_n)|.$$

Therefor, the polynomial $\omega_n(x) = (x-x_0)(x-x_1)\cdots(x-x_n)$ influences the size of the interpolation error.
Put n distinct (!) interpolation points in different locations in the interval $[-1, 1]$ and plot $\omega_n(x)$. Set $n = 5$ and try out different choices of interpolation points. Can you recommend an optimal one? Test even the case $n = 15$.

Task 3 Select in Task 2 the interpolation points as Chebyshev points and compare the resulting curve with your previous results.

Task 4 Now, we visualize the error. For this end interpolate the function

$$f(x) = \frac{1}{1 + 25x^2}$$

on an equidistant grid in the interval $[-1, 1]$ with $n = 3, 9, 15$. Construct also an interpolation polynomial on a grid with Chebyshev points.

Task 4 Let $\mathcal{P}^n$ be the space of all polynomials of max degree n and assume that $x_i, i = 1 : k$ are given points. Show that

- $(p, q) = \sum_{i=0}^{n} p(x_i)q(x_i)$ is an inner product on $\mathcal{P}^n$
- $(p, q) = \sum_{i=0}^{k} p(x_i)q(x_i)$ with $k < n$ is *no* inner product on $\mathcal{P}^n$.

Task 5 Write a program `coeff=cubspline(xint,yint)`, which takes as input the x- and y-values of $m + 1$ interpolation points and returns a $m \times 4$ coefficient matrix of the natural spline which interpolates the data. The i-th row of this matrix contains the coefficients a_i, b_i, c_i, d_i of the cubic subpolynomial s_i of the spline.
The program may be written for equidistant node-points x_i (h constant.)

Task 5 Write a program `yval=cubsplineval(coeff,xint,xval)`, which evaluates the spline at `xval`. Test both programs on a simple test problem of your choice.

Task 6 We will now discuss a wheel profile function, that is based the standard profile S1002, which is defined sectionwise by polynomials up to degree 7. The profile and its sections ar shown in Fig. 1. The polynomials are defined by

$$\begin{array}{lrcl}
\text{Section A:} & F(s) & = & a_A - b_A s \\
\text{Section B:} & F(s) & = & a_B - b_B s + c_B s^2 - d_B s^3 + e_B s^4 - f_B s^5 + g_B s^6 - h_B s^7 + i_B s^8 \\
\text{Section C:} & F(s) & = & -a_C - b_C s - c_C s^2 - d_C s^3 - e_C s^4 - f_C s^5 - g_C s^6 - h_C s^7 \\
\text{Section D:} & F(s) & = & a_D - \sqrt{b_D^2 - (s + c_D)^2} \\
\text{Section E:} & F(s) & = & -a_E - b_E s \\
\text{Section F:} & F(s) & = & a_F + \sqrt{b_F^2 - (s + c_F)^2} \\
\text{Section G:} & F(s) & = & a_G + \sqrt{b_G^2 - (s + c_G)^2} \\
\text{Section H:} & F(s) & = & a_H + \sqrt{b_H^2 - (s + c_H)^2}
\end{array}$$

and

	A	B	C	D
a	1.364323640	0.0	$4.320221063\,10^{+3}$	16.446
b	0.066666667	$3.358537058\,10^{-2}$	$1.038384026\,10^{+3}$	13.
c	–	$1.565681624\,10^{-3}$	$1.065501873\,10^{+2}$	26.210665
d	–	$2.810427944\,10^{-5}$	$6.051367875\,10^{+0}$	–
e	–	$5.844240864\,10^{-8}$	$2.054332446\,10^{-1}$	–
f	–	$1.562379023\,10^{-8}$	$4.169739389\,10^{-3}$	–
g	–	$5.309217349\,10^{-15}$	$4.687195829\,10^{-5}$	–
h	–	$5.957839843\,10^{-12}$	$2.252755540\,10^{-7}$	–
i	–	$2.646656573\,10^{-13}$	–	–
$\xi_{\min}$	32.15796	-26	-35	-38.426669071
$\xi_{\max}$	60	32.15796	-26	-35

	E	F	G	H
a	93.576667419	8.834924130	16.	9.519259302
b	2.747477419	20	12.	20.5
c	–	58.558326413	55.	49.5
$\xi_{\min}$	-39.764473993	-49.662510381	-62.764705882	-70.0
$\xi_{\max}$	-38.426669071	-39.764473993	-49.662510381	-62.764705882

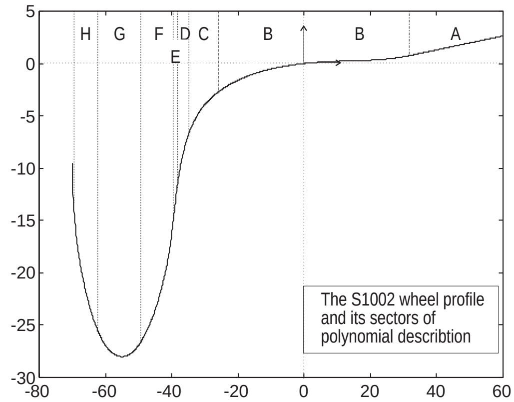

Figure 1.

Describe this wheel profile by means of a natural cubic spline. For this end, download the file `s1002.m`, which contains the above description of the s1002 wheel standard and generate from this data, which you then use to generate an interpolating spline with your programs in Task 1 and 2. Plot the resulting curve.

Lycka till!