

**MA323-Monte Carlo  
Simulation**

**ASSIGNMENT 2**

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# ROLL NO. 200123011

## QUS1-

a) The Linear Congruence Generator is used to

generate the first 17 values of  $U_i$  of the form :

$$x_{i+1} = (ax_i + b) \bmod m$$

$$u_{i+1} = x_{i+1}/m$$

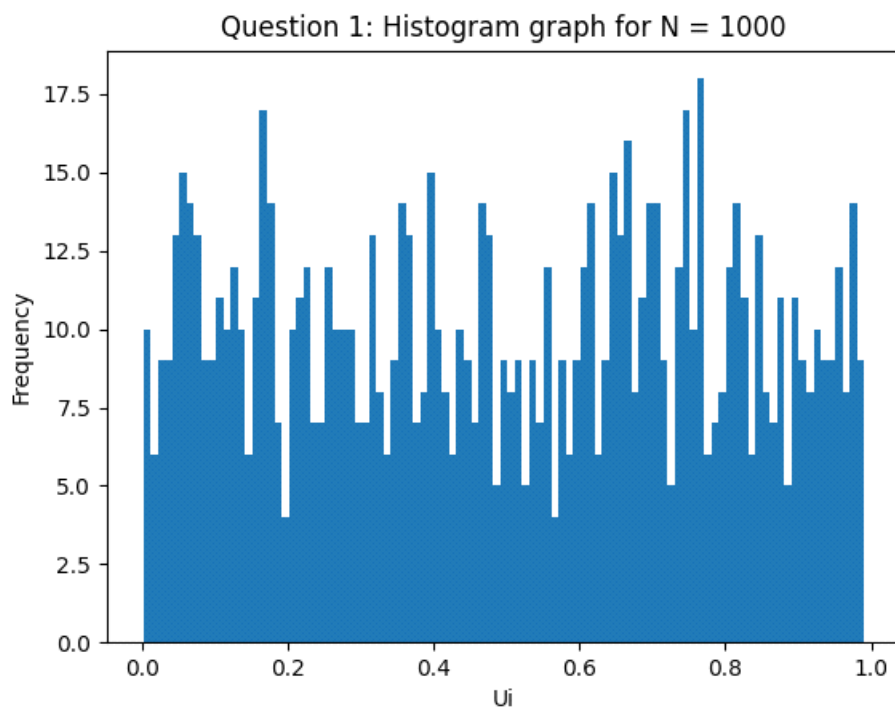
with  $a = 1229$ ,  $b = 1$ ,  $m = 2048$ .

b) Generated the values of  $U_{18}, U_{19}, \dots, U_N$  for  $N =$

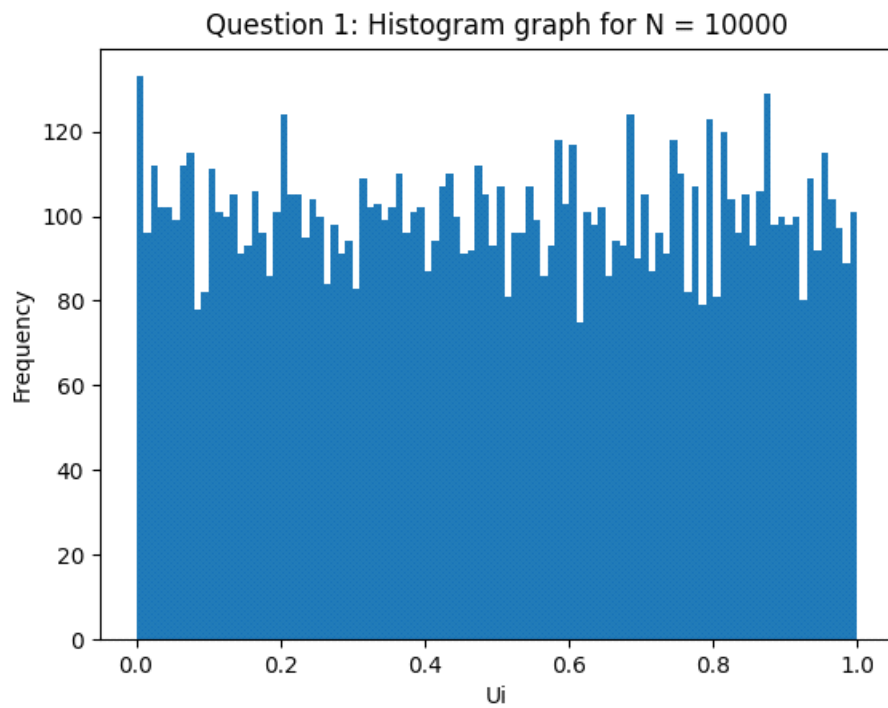
1000, 10000, and 100000

c) histogram for different n valuse-

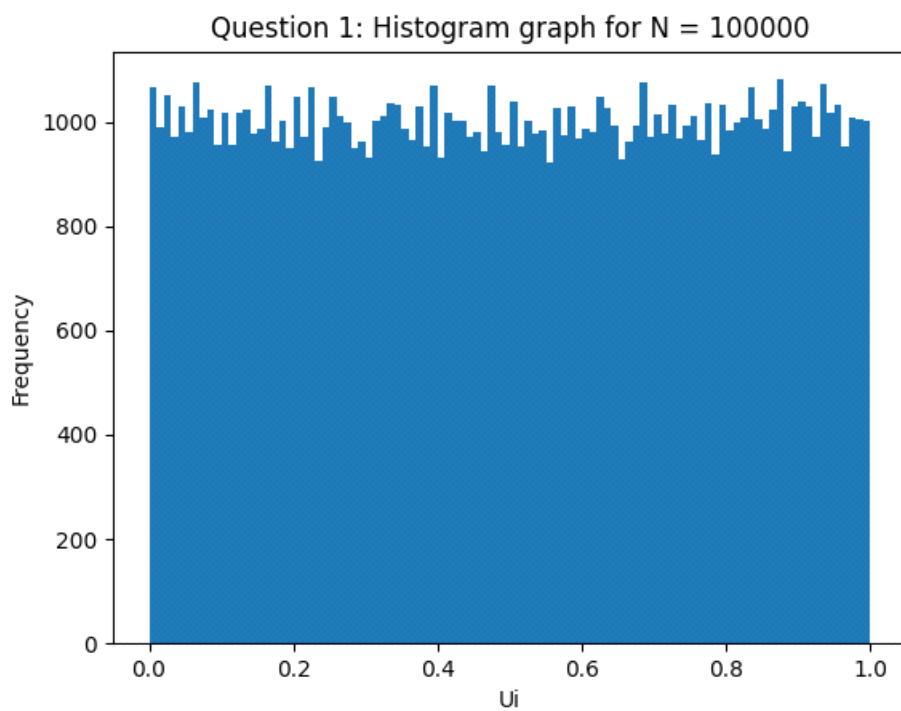
For  $N=1000$  as follow



For  $N=10000$  as follow



For  $N=100000$  as follow



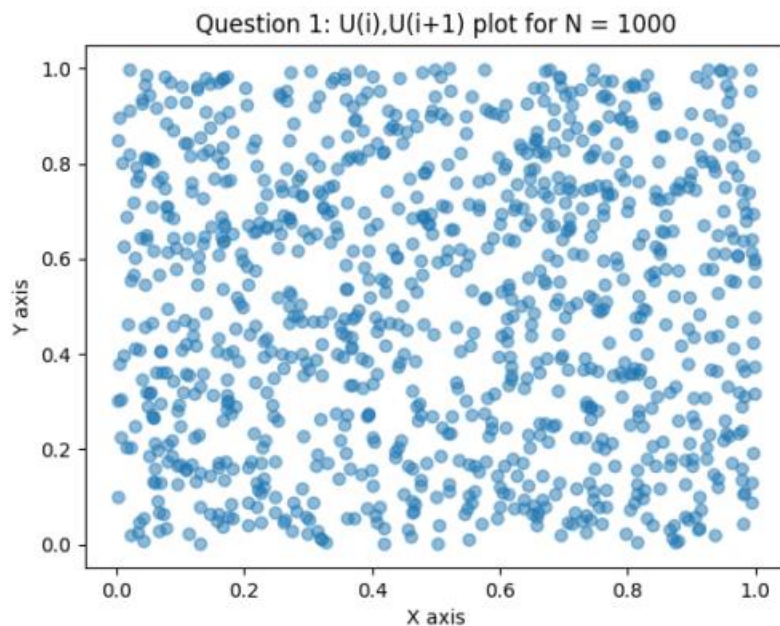
## Observations

The histogram suggest that the random generator follows the 2 properties of the ideal random generator:

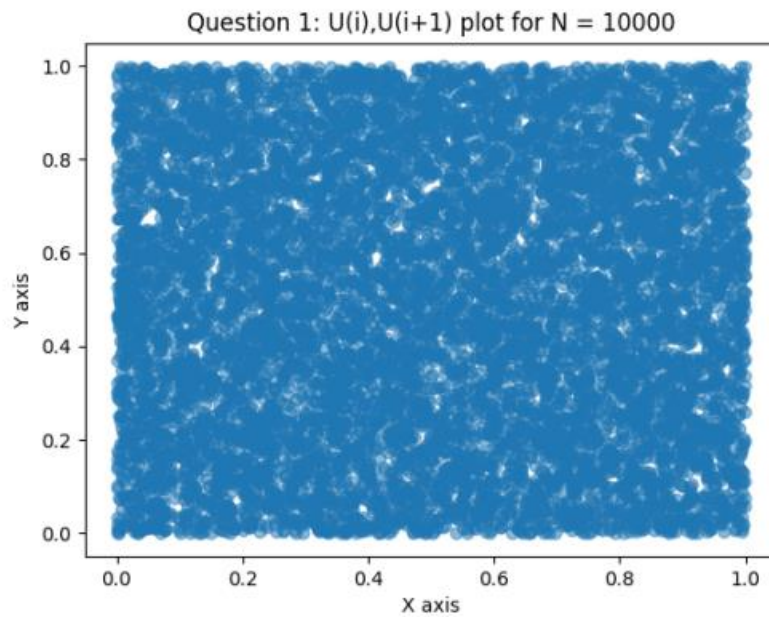
- a) Each  $U_i$  is uniformly distributed between 0 and 1.
- b) The  $U_i$  are mutually independent.

d) Plot  $(U_i, U_{i+1})$  for all  $N$  values

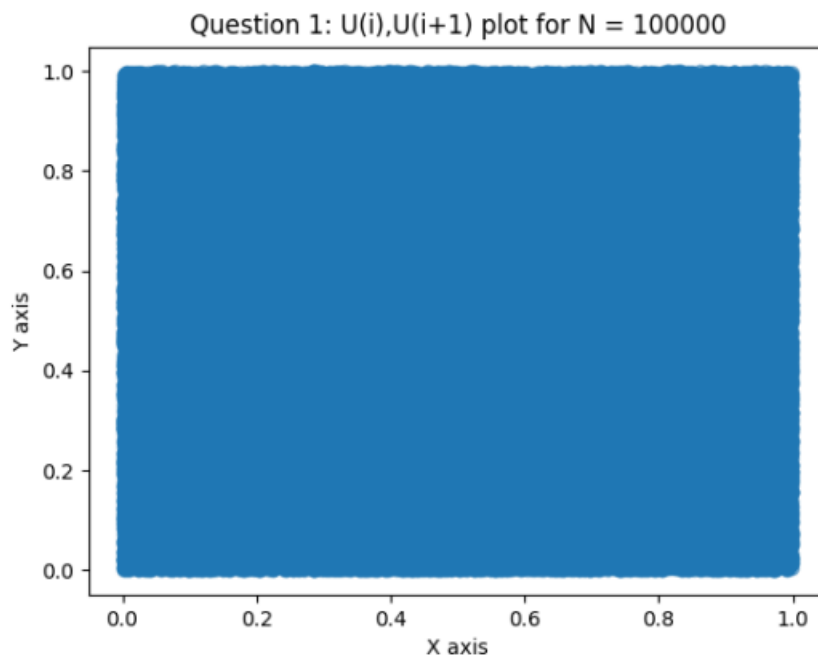
for  $N=1000$ , attached below-



for  $N=10000$ , attached below-



for  $N=100000$ , attached below-



Observations:

The  $(U_i, U_{i+1})$  plot suggests that the  $U_i$  do not follow

any particular/specific pattern. Hence, they are almost completely random.

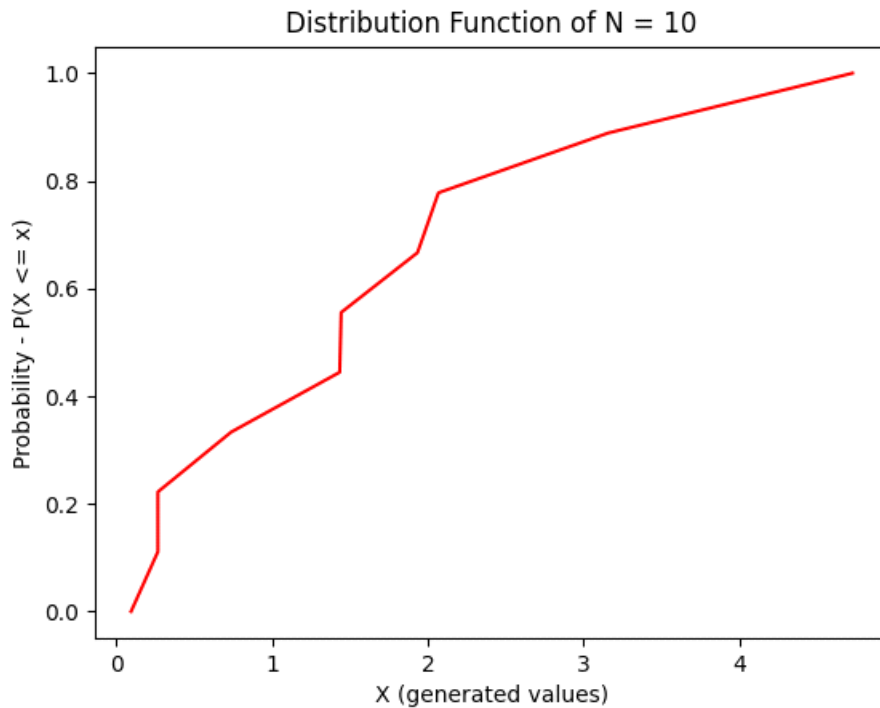
## QUS 2-

I assumed that ,the Mean ( $\theta$ ) =  $\pi / 2$

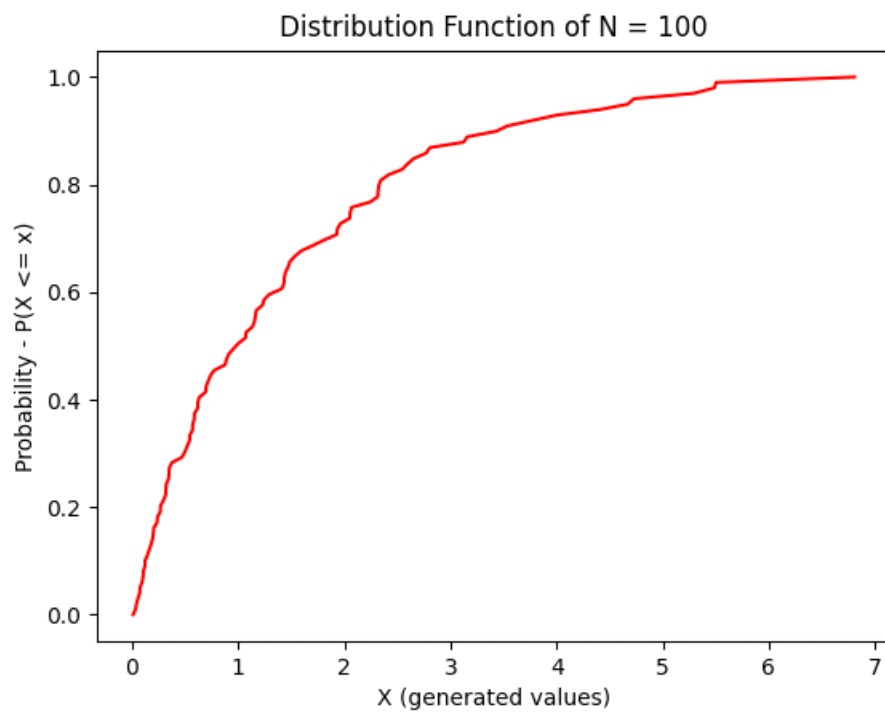
so for different values of N that is given below

N = 10, 100, 1000, 10000, 100000.

For N=10 as follow-

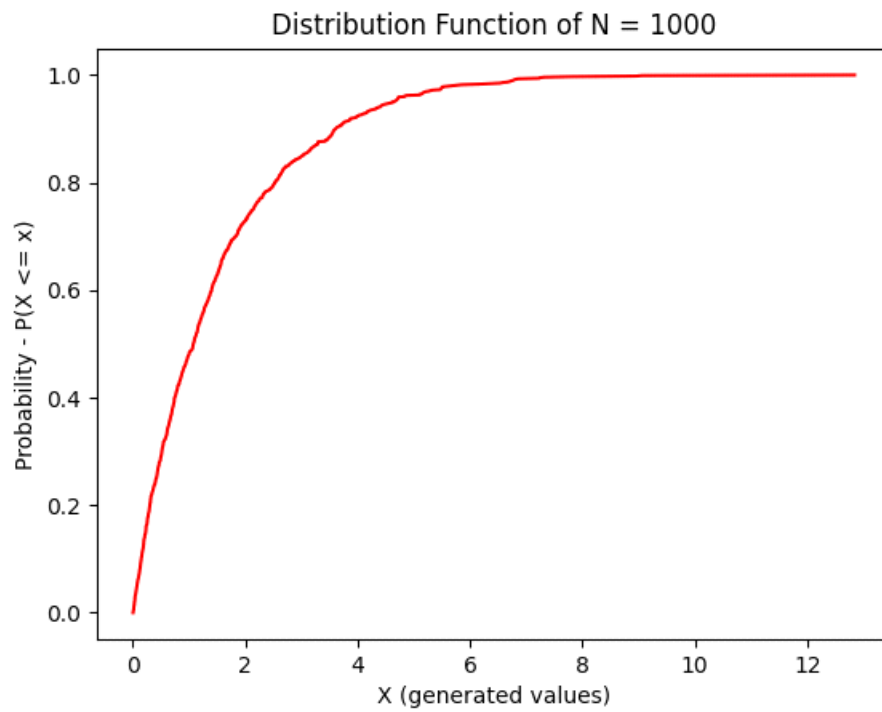


For N=100 as follow,

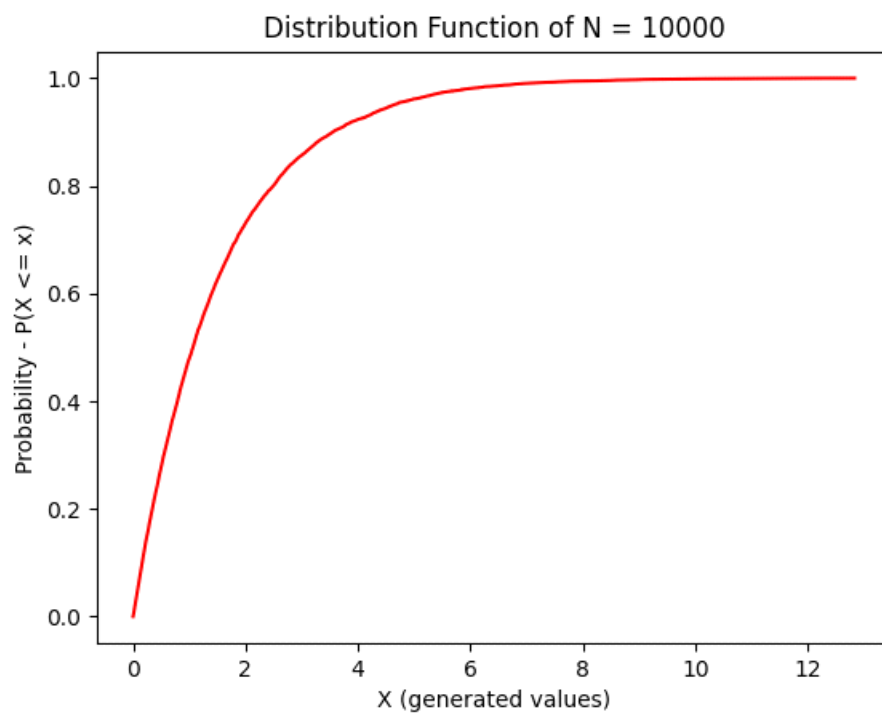


For  $N=1000$  as follow

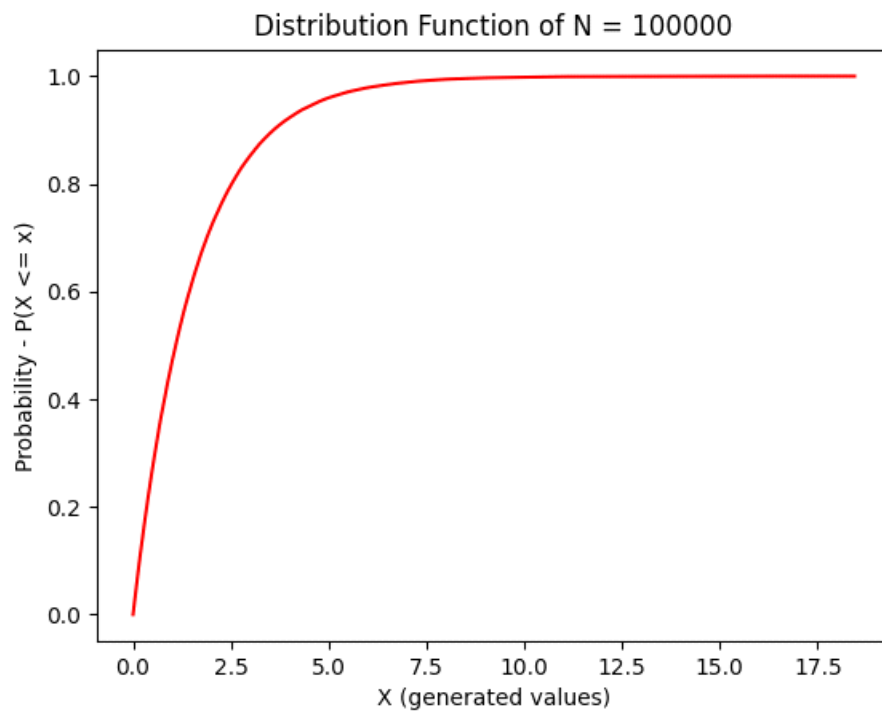




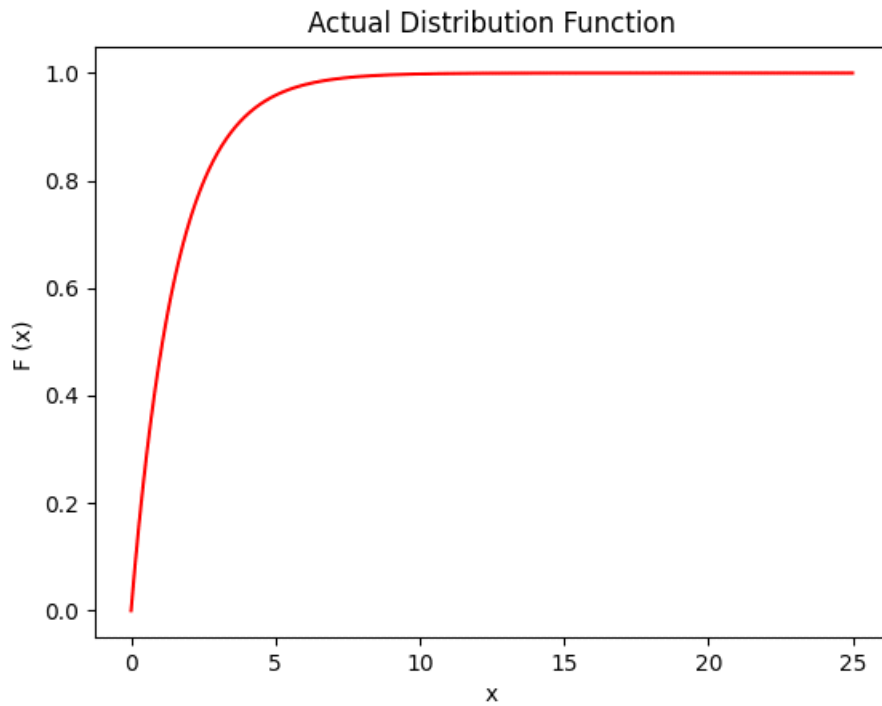
For  $N=10000$  as follow



For  $N=10000$  as follow



Actual Distribution function as follows-



Actual Mean = 1.5707963267948966

Actual Variance = 2.4674011002723395

Table below-

N	Mean	Variable
10	1.613160826095664	1.9168020190182393
100	1.4368828596650938	2.066560680583305
1000	1.527608770849963	2.3312738492502976
10000	1.5354517674732717	2.342475561946338
100000	1.5644662674427838	2.4328267137230175

Observations:

when we increase the number of generated values, the corresponding mean and variance of the generated values converge to the actual mean and variance.

It is also evident from the distribution function of the  $X$  for different values of sample count( $N$ ) which approaches the plot of  $F(x)$  as sample count increases continuously. It follows the Law of Large Numbers.

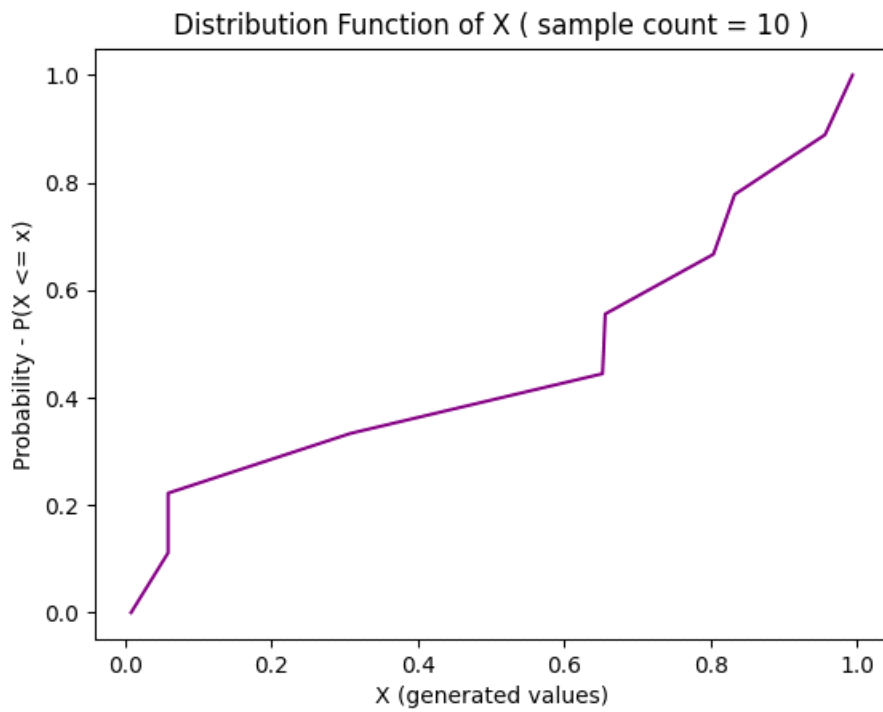
## **QUS 3-**

**a)** Generated  $X_1, X_2, X_3, X_4 \dots, X_N$  from the above distribution for different  $N$  values as given below-

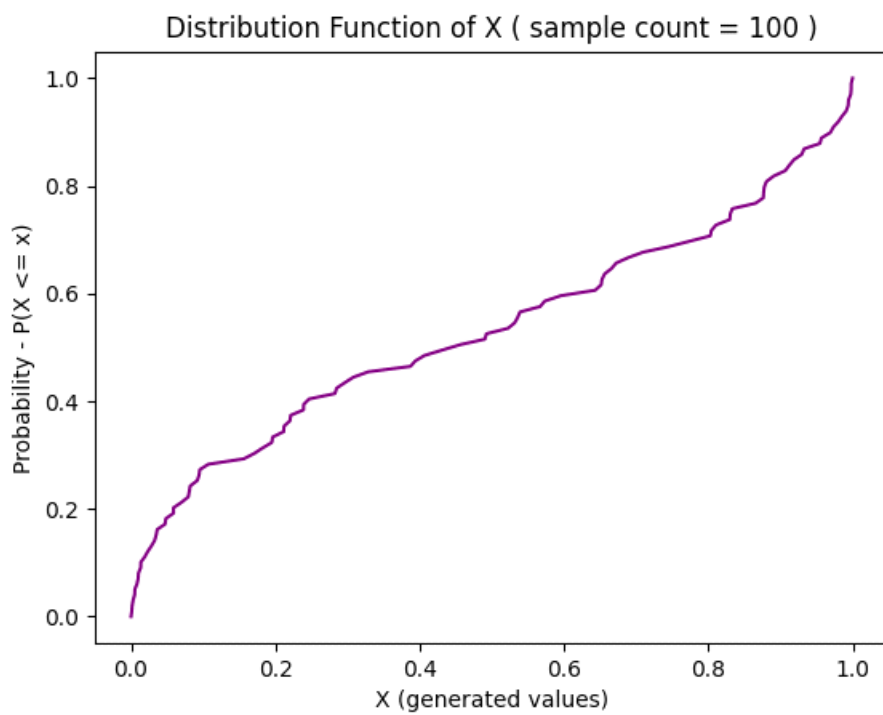
$N = 10, 100, 1000, 10000, 100000.$

**b)** Plot distribution functions for all  $N$ -

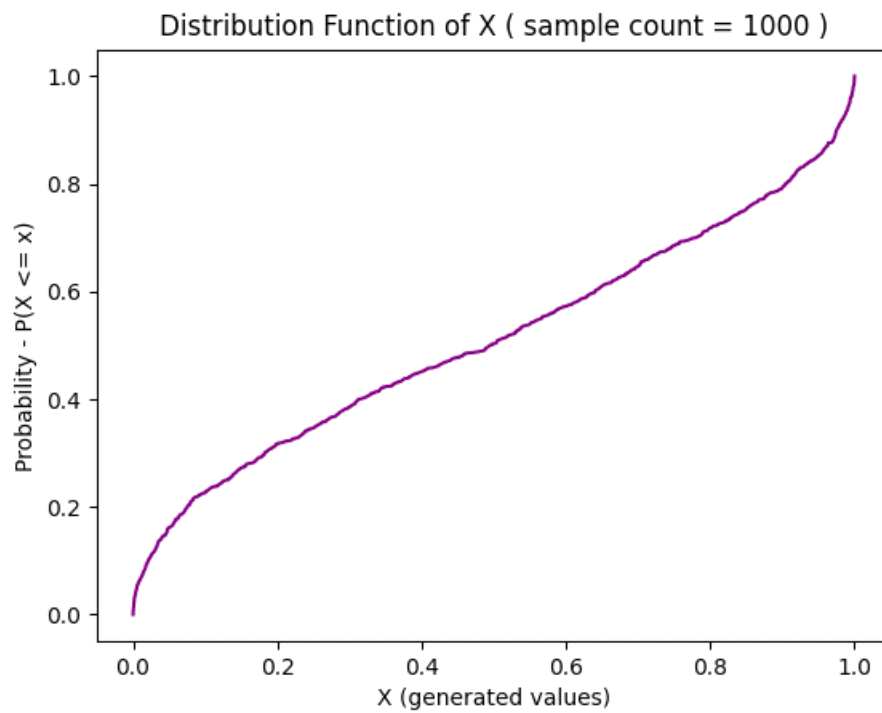
For  $N=10$  as follow,



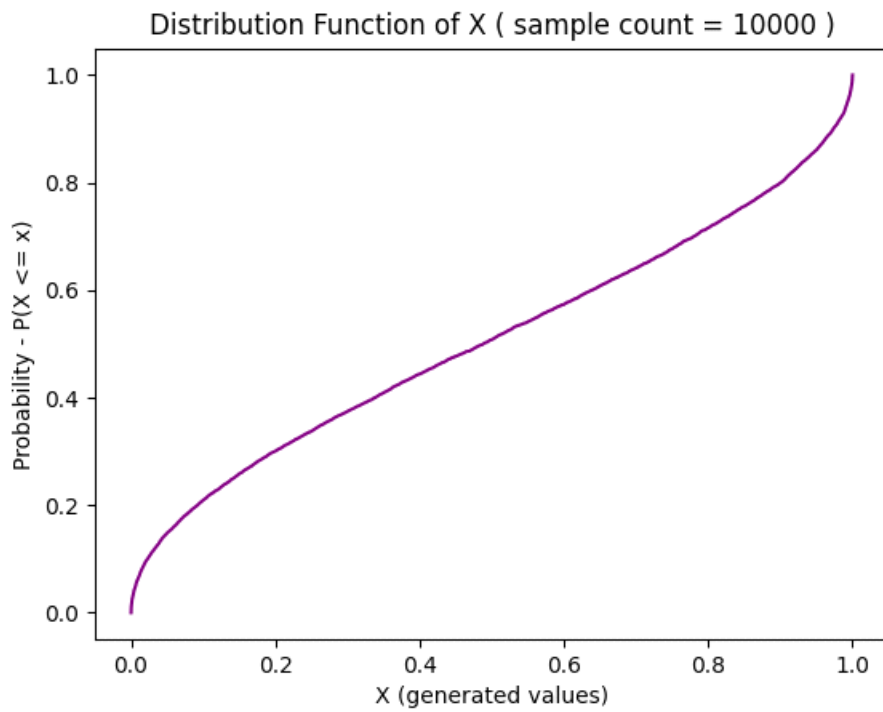
For N=100 as follow,



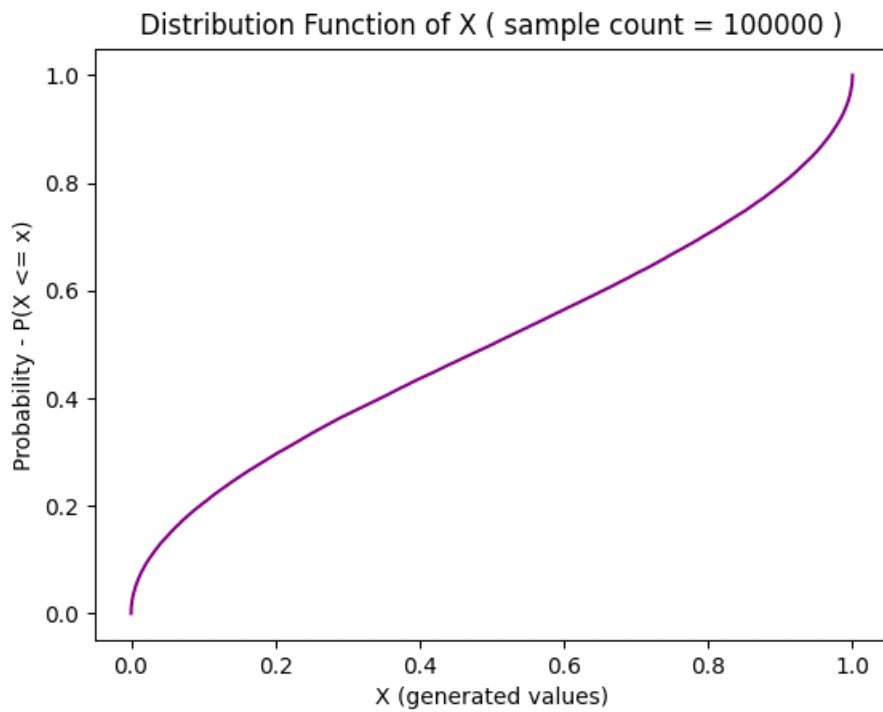
For  $N=1000$  as follow,



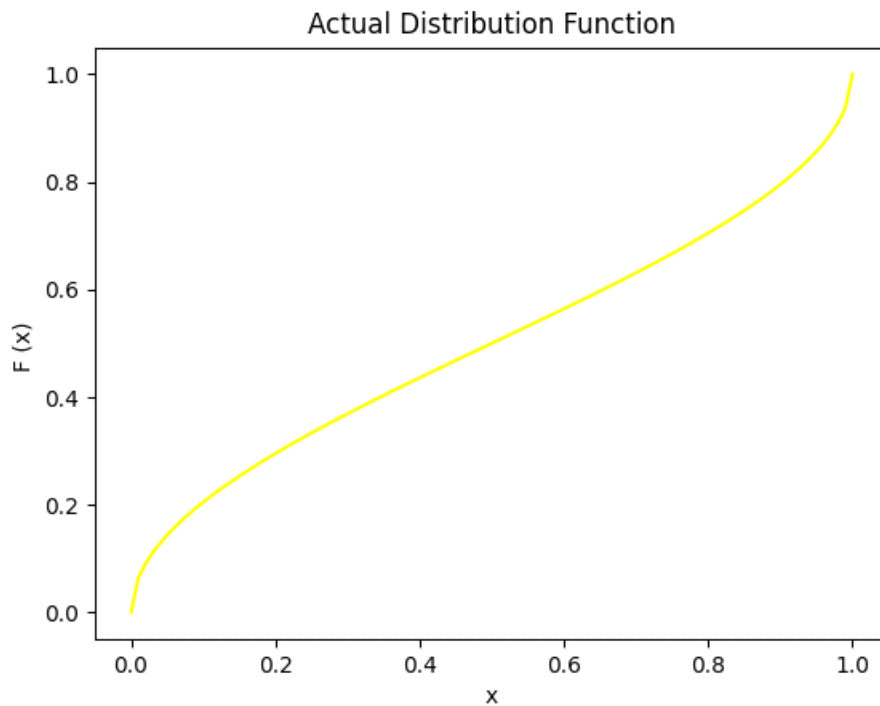
For  $N=10000$  as follow,



For  $N=100000$  as follow



Actual Distribution function plot as follows-



**c) Table Below-**

N	Mean	Variable
10	0.5330218847871611	0.13603351783787718
100	0.4649330111462657	0.13098678304171854
1000	0.4883304419967129	0.12727982689751888
10000	0.4928587988765897	0.12416196616477684
100000	0.4995406384332094	0.1248815445370180

Observations:

1. The distribution function of X is identical to the cdf  $F(x)$  in which



random variable  $X$  was generated. because  $F(x)$  is a continuous strictly increasing function and  $U$  is a uniform distribution function on  $[0, 1]$ . so,  $F^{-1}(U)$  will be a sample from  $F$ .

2. The distribution function of  $X$  approaches the plot of  $F(x)$  because the sample count increases continuously.

## **QUS 4-**

- Generated random variables from a discrete distribution, generate 100000 random numbers from a discrete uniform distribution on  $\{1, 3, 5, \dots, 9999\}$ .

Frequency graph for  $N=100000$  as follows,

