<u>Distribution</u> $I(2) \sim N(0, 5)$ → Variance is written here.

$J(3) = 3W(3) - W(2) - W(1)$

$Var(I(3)) = E\left[(3W(3) - W(2) - W(1))^2\right]$

$= E\left[9W^2(3) + W^2(2) + W^2(1) - 6W(3)W(2) - 6W(3)W(1) + 2W(2)W(1)\right]$

$= 27 + 2 + 1 - 12 - 6 + 2$

$= \underline{\underline{14}}$

(7) $S_\alpha(\Pi) = \sum_{j=0}^{n-1} \left[(1-\alpha)W(t_j) + \alpha W(t_{j+1})\right](D_j)$

where $D_j = W(t_{j+1}) - W(t_j)$

$W(t_j)$

$\lim_{\|\Pi\| \to 0} S_\alpha(\Pi) = \sum_{j=0}^{n-1} D_j W_j + \alpha(W_{j+1} - W_j) D_j$

$= \sum_{j=0}^{n-1} D_j W_j + \alpha D_j^2$

$\lim_{\|\Pi\| \to 0} S_\alpha(\Pi) = \lim_{\|\Pi\| \to 0} \sum D_j (W_j) + \alpha T$

W_j

$\int^T \left[\sum_{j=0}^{n-1} W_j - W(t)\right]^2 = 0$

By defn $\int_0^T W(t)\,dW(t) + \alpha T$

$= \frac{1}{2} W^2(T) + (\alpha - 1)T$

$E[S_\alpha(\Pi)] =$

8. Same as 3.(a), 3.(b)

(9). $E[S_\alpha(\Pi)] = E\left(\sum_{j=0}^{n-1} \left[\underline{(W(t_{j+1}) - W(t_j))^2} - (t_{j+1} - t_j) \right]\right)$

$\downarrow$

$= \sum \left(E[\quad] - (t_{j+1} - t_j) \right)$

$= \sum \left((t_{j+1} - t_j) - (\quad) \right)$

$= \sum 0 = 0$

$Var(S_\alpha(\Pi)) = E[S_\alpha(\Pi)^2]$

$= E\left[\left[(W(t_{j+1}) - W(t_j))^2 - (t_{j+1} - t_j) \right]\right]^2$

refer notes

$\leq 2T\|\Pi\|$

$\therefore$ $Var(S_\alpha(\Pi)) \to 0$

Therefore (in L^2) we can say that

$\lim_{\|\Pi\| \to 0} S_\alpha(\Pi) = E[S_\alpha(\Pi)] = 0$

(10) $f(t,x) = x^5 - 10tx^3 + 15t^2x$

~~$f_t = \ldots 30tx^2 + 30tx$~~

$f_t = 30tx - 10x^3$

$f_x = 5x^4 - 30tx^2 + 30t^2.$

$f_{xx} = 20x^3 - 60tx$

$\frac{1}{2}f_{xx} + f_t = 0$

$\Rightarrow f(t, W(t)) - f(0, W(0)) = \int_0^T f_x \, du$ $\leftarrow$ ITO'S INTEGRAL MARTINGALE

(11) Let $f(t,x) = \cos x$

$\Rightarrow f_t = 0$

$f_x = -\sin x$

$f_{xx} = -\cos x$

~~$\cos W(0)$~~ $\cos W(T) - \cos W(0) = \int -\sin(W)\,dW + \frac{1}{2}\int -\cos W(t)\,dt$

$\int_0^T \sin(W(t))\,dW(t) = 1 - \cos W(T) - \frac{1}{2}\int_0^T \cos W(t)\,dt$

(12)

$\cancel{\int_0 dX(t) = f(s)\,dW(s)}$

$\cancel{f(s)}$

$\cancel{f(t,x) = \frac{tx^2}{2}}$

$\cancel{f(s)W}$

$\cancel{f(s) = \sum_{i=0} \eta(s)\,\mathbb{1}_{[0,T]}}$

$\cancel{\int_0^t f(s)\,dW(s)}$

$$f(T, X(T)) - f(0, X(0)) = \int_0^T f_t\,dt + \int_0^T f_x\,\Delta(t)\,dW(t)$$

$$+ \int_0^T f_x\,\Theta(t)\,dt + \frac{1}{2}\int f_{xx}\,\Delta^2(t)\,dt$$

Here

$f_x\,\Delta(t) = f(s)X(s)$ & $\Delta(t) = f(s)$ & $\Theta(t) = 0$

$x = X(s)$

∴ Take $f = \frac{x^2}{2}$

Then $f_t = 0$, $f_x = x$, $f_{xx} = 1$

$$\frac{X(T)^2}{2} - \frac{X(0)^2}{2} = \int_0^T f(s)X(s)\,dW(s) + \frac{1}{2}\int_0^T f^2(s)\,dt$$

$$\Rightarrow \int_0^T f(s)X(s)\,dW(s) = \frac{X(T)^2 - X(0)^2 - \int_0^T f^2(s)\,dt}{2}$$

(13) $X(t) = \int_0^t f(t)[\sin(W(t)) + \cos(W(t))]\, dW(t)$

$\underline{X(0) = 0}$

$X(t)$ is a martingale

$\therefore \; E[X(t)] = X(0) = 0$

$\boxed{Var(X(t))}$ Using ITO's Isometry,

$Var(X(t)) = E[I_T(f)]^2 = \int_0^T E|f(u)|^2\, du$

$= \int_0^T E[f(t)^2[\sin W(t) + \cos W(t)]^2]\, dt$

$= \int_0^T f^2(t)\, E[1 + \sin 2W(t)]\, dt$

$f = \sin(2\pi t)$

$f_t = 2\pi \cos(2\pi t)$

$f_x = 2t \cos 2\pi t$

$f_{xx} = 4t\pi^2$

$E[\sin(2\pi t)] = \int \sin$