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1. $F_1 = \{\phi, \Omega, \{1\}, \{2,3,4\}\}$

$F_2 = \{\phi, \Omega, \{2\}, \{1,3,4\}\}$

F_1 and F_2 are σ-algebras

$F_1 \cup F_2 = \{\phi, \Omega, \{1\}, \{2\}, \{1,3,4\}, \{2,3,4\}\}$

we have $\{1\} \in F_1 \cup F_2$, $\{2\} \in F_1 \cup F_2$

but $\{1,2\} \notin F_1 \cup F_2$

so $F_1 \cup F_2$ is not a σ-algebra

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2. ~~The set~~

We take the set $A = \{w : 1/2 < w \leq 1\} \in \mathcal{B}([0,1])$

Thus we have $\mathbb{P}(A) = \frac{1}{2} > 0$

But $\tilde{\mathbb{P}}(A) = 0$

Thus $\exists$ a set A s.t. $\tilde{\mathbb{P}}(A) = 0$ but $\mathbb{P}(A) > 0$

Thus $\tilde{\mathbb{P}}$ and $\mathbb{P}$ are not equivalent ~~po~~ probability measures.

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3. $P(A) = \liminf\limits_{n\to\infty} \frac{\#(A \cap \{1,2,\dots n\})}{n}$

$P(\Omega) = \liminf\limits_{n\to\infty} \frac{\#(\Omega \cap \{1,2,\dots n\})}{n}$

$= \liminf\limits_{n\to\infty} \frac{n}{n} = 1$

Let $A_i = \{i\}$

Thus $A_1 \dots A_n$ are disjoint

$\cancel{P(\cup A_i) =}$ $P(A_i) = \liminf\limits_{n\to\infty} \frac{\#(A_i \cap \{1,2,\dots n\})}{n}$

$= 0$

But $P(\cup A_i) = P(A) \neq 0$

Thus P is not a probability measure.

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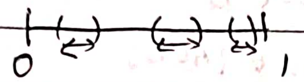
4.

a) $P(A) = 1/2$

$\therefore$ ~~Let~~ Length of all intervals in A sums to $1/2$



We have 2 cases

i) $x \in A$

In this case, since the given set is open

$\exists\, \varepsilon > 0$ s.t. $x+\varepsilon \in A$

~~Thus $f(x+\varepsilon) = 1$~~

$$\therefore P(A \cap [0, x+\varepsilon]) = P(A \cap [0, x])$$

$$\therefore f(x+\varepsilon) = f(x)$$

$$\Rightarrow f(x+\varepsilon) - f(x) < \lambda \quad \forall \lambda > 0$$

Similar is for continuity from behind

ii) $x \in A^c$

If $x \notin \partial A^c$

Then same argument as above holds

If $x \in \partial A^c$

Then there will still be continuity as function will increase from there.

b) Yes Let $A = \bigcup_{i=1}^{\infty} L_i$ where L_i are disjoint intervals which make A such that $L_i = (l_i, l_{i+1})$

For each L_i we take $K_i = \left(l_i, \frac{l_{i+1}}{2}\right)$

Let $B = \bigcup_{i=1}^{\infty} K_i$ thus $B \subset A$ and $P(B) = 1/4$

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5. We have

$$X(\omega) = \begin{cases} 1 & \omega \in A \\ -2 & \omega \in B \\ 2 & \omega \notin \{A \cup B\} \end{cases}$$

$P(A) = 1/3$

$P(B) = 1/2$

$\cancel{P(}P(\text{none}) = 1/6$

$$F_X(\cancel{\omega}) = \begin{cases} 1/6 & \text{if } \omega \notin A \text{ or } B \\ 1/2 & \text{if } \omega \in \cancel{B}A \\ 1 & \text{if } \cancel{\omega \in} \text{ otherwise} \end{cases}$$