

MA323-Monte Carlo

Simulation

Assignment-7

ARTI SAHU
200123011

Qus1-

The exact value of I is 2 .

(a)for m=100

value of $I_m = 2.079135081051508$

Difference between I and $I_m=0.079135$

Variance of $y : 0.21236100167750255$

The 95 percent confidence interval for the $I : (1.770729647781824 , 2.3875405143211923)$

(b)for m=1000

value of $I_m = 2.002145439820333$

Difference between I and $I_m=0.002145$

Variance of $y : 0.19262551196703145$

The 95 percent confidence interval for the $I : (1.9045687531885325 , 2.0997221264521335)$

© for m=10000

value of $I_m = 1.99419469094522$

Difference between I and $I_m=0.005805$

Variance of $y : 0.19575112712429008$

The 95 percent confidence interval for the $I : (1.9636311162246156 , 2.0247582656658243)$

(d)m=100000

value of $I_m = 1.9979772880940911$
Difference between I and $I_m = 0.002023$
Variance of $y : 0.19618727823338508$
The 95 percent confidence interval for the $I : (1.9882879670111016 , 2.0076666091770807)$

OBSERVATION: Value of I_m is almost equal to the exact value of I .

Qus2-

using antithetic variates: $k = (\text{np.exp(np.sqrt(x))} + \text{np.exp(np.sqrt(1-x))})/2$

(a) For $m=100$

value of $I_m = 1.9952075557520905$
Variance of $y : 0.0015820757647553745$
Old Variance of $y : 0.21191991210417105$
The 95 percent confidence interval for the I is
(1.698697301578235 , 2.291717809925946)
Percent of variance reduction : 99.2534558225101%

(b) for $m=1000$

value of $I_m = 2.001614639955897$
Variance of $y : 0.0009217842548573342$
Old Variance of $y : 0.1865127443694157$
The 95 percent confidence interval for the I is
(1.909250024099212 , 2.0939792558125823)
Percent of variance reduction : 99.5057794801241%

©for $m=10000$

value of $I_m = 2.0004910711842983$
Variance of $y : 0.0010196606488633657$
Old Variance of $y : 0.1920575131765153$
The 95 percent confidence interval for the I is
(1.9710599203144699 , 2.0299222220541266)
Percent of variance reduction: 99.4690857795674%

(d) for m=100000

value of $I_m = 1.9997964614522294$

Variance of y : 0.00108308271081363

Old Variance of y : 0.19556893075523127

The 95 percent confidence interval for the I is

(1.9904906599664443 , 2.009102262938015)

Percent of variance reduction : 99.44618876493772%

OBSERVATION: Antithetic variable approach achieved about 99.45% reduction.

Qus3-

Using control variates - \sqrt{U} :

```
mcv=np.mean(X2)           #mean of sqrt(U)
vcv=np.var(X2)             #var of sqrt(U)
a=covariance(X2,Y2)/vcv    # cov(X2,Y2)/vcv
k=(np.exp(np.sqrt(x)) - a*(np.sqrt(x)-mcv))    #control variante
```

(a)For m=100

value of $I_m = 1.9800817056419182$

The 95 percent confidence interval for the I is

(1.6757136365237328 , 2.2844497747601036)

Variance of y : 0.002833731895532407

Old Variance of y : 0.20411584962267854

Percent of variance reduction : 98.61170413724817%

(b)For m=1000

value of $I_m = 2.0001693158586904$

The 95 percent confidence interval for the I is

(1.9074950663351053 , 2.0928435653822755)

Variance of y : 0.002801667583691633

Old Variance of y : 0.19273678547059026

Percent of variance reduction : 98.54637630442417%

© For m=10000

value of $I_m = 2.0012930959856394$

The 95 percent confidence interval for the μ is
(1.9717800494429947 , 2.030806142528284)
Variance of y : 0.0027301807769369055
Old Variance of y : 0.19435764090352226
Percent of variance reduction : 98.59527993638689%

(d) For $m=100000$

value of $\bar{y}_m = 1.9984792226953934$
The 95 percent confidence interval for the μ is
(1.9891675353582121 , 2.0077909100325746)
Variance of y : 0.002716417772714766
Old Variance of y : 0.19460269298661054
Percent of variance reduction : 98.60412118094293%

Observations: The control variate approach achieved about 98.60% reduction.

Thank you