

MA 323 – Monte Carlo Simulation

Assignment – 3

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Qus1-

(A)

The smallest value of c is 1.4815 for $g(x) = 2(1-x)$ such that

$$f(x) \leq cg(x).$$

● $c = 1.4815$

a) Average no. of iterations is 1.4818 that is needed to generate a random number. We can clearly see that the calculated average value is very close to the theoretical value of c . The closeness increases as the count of random numbers generated increases.

b) Generate 10000 random numbers from the distribution:

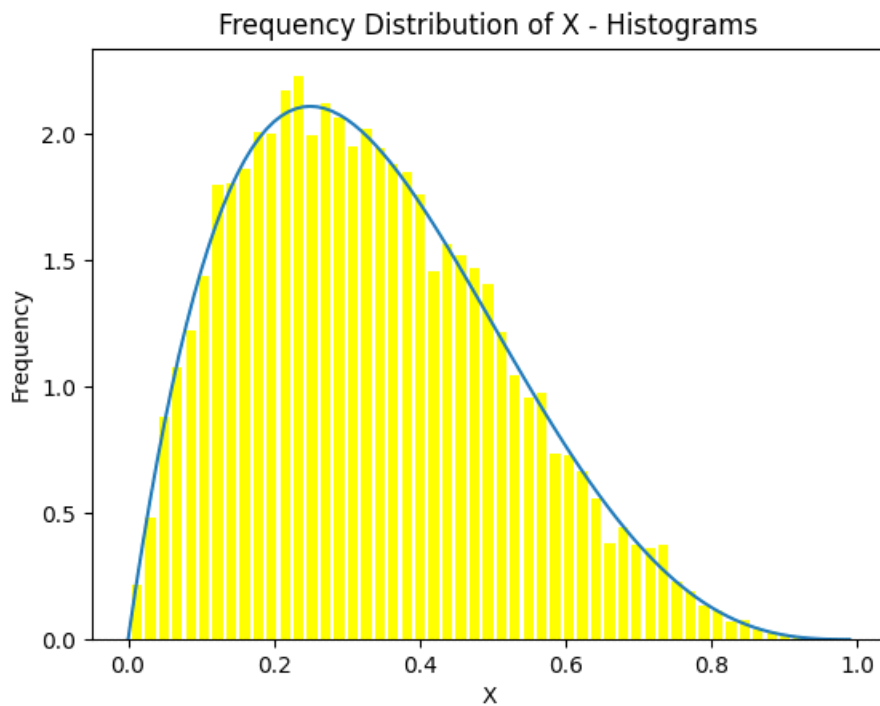
Mean of Accepted no. = 0.33272318192267114

c) the approximate value of $P(0.25 \leq X \leq 0.75)$ based on the generated sample in the part (b)

Probability of accepted is: 0.2645260202761339

d) Total no of Iteration for finding 10000 random variables is: 14805

(e)



● $c = 3$

a) Average no. of iterations is: 3.007

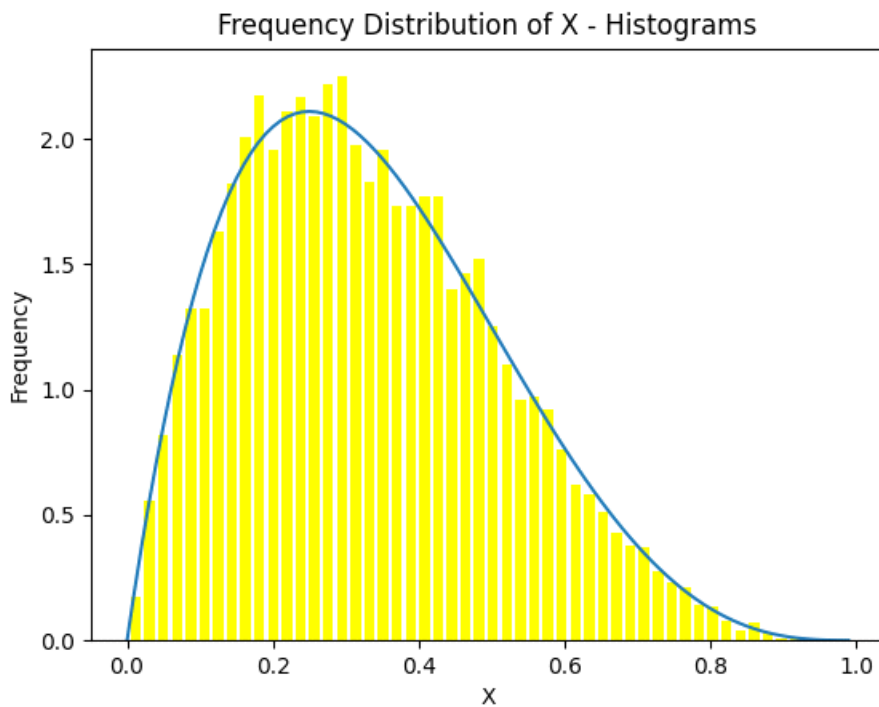
b) Mean of Accepted no. = 0.3347359615704076

c) the approximate value of $P(0.25 \leq X \leq 0.75)$ based on the generated sample in the part (b)

Probability of accepted no is: 0.26546681458026

d) Total no of Iteration for finding 10000 random variables is: 30070

(e)



c=10

a) Average no. of iterations: 10.1765

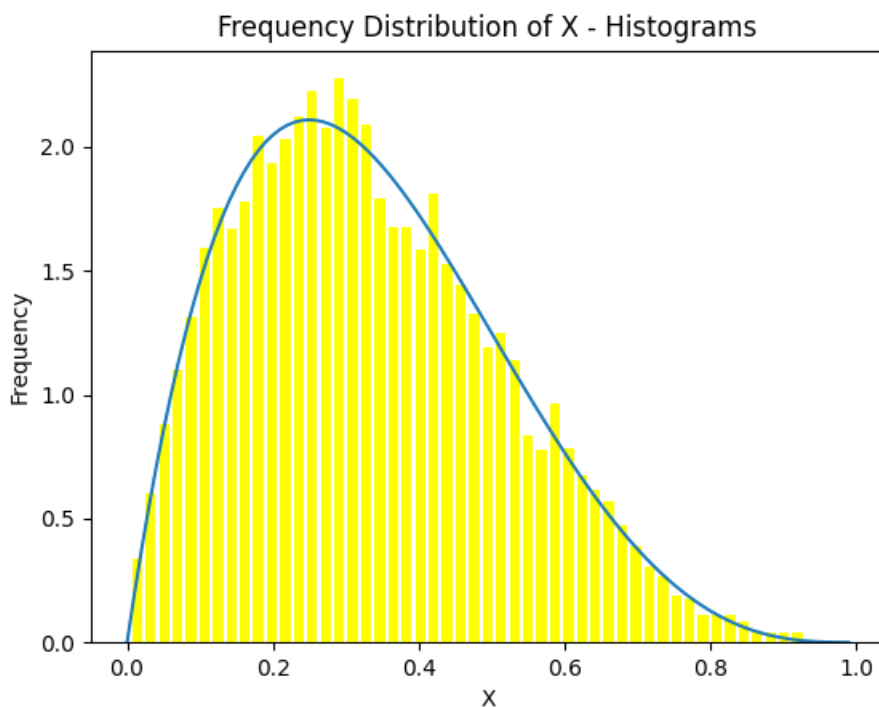
b) Mean of Accepted no.= 0.3309973439408394

c) the approximate value of $P(0.25 \leq X \leq 0.75)$ based on the generated sample in the part (b)

Probability of accepted no is: 0.26158054398952785

d) Total no of Iteration for finding 10000 random variables is: 101765

(e)



Observations:

hence we can see,

when the value of c deviates from the smallest value, the errors increase in the random numbers, hence it deviates from the distribution for which we are generating them.

This deviation is more prominent when the distance of c is

more from its smallest value.

(B)

sample mean of distribution is 0.3347

The Expectation of PDF f is

$$\int_0^1 x f(x) dx = \int_0^1 20x^2(1-x)^3 dx = \frac{1}{3}$$

After calculating we can see the values are close to each other.

(C)

The approximate values generated from samples is 0.6233

The actual values is

$$\int_{0.25}^{0.75} f(x) dx = \int_{0.25}^{0.75} 20x(1-x)^3 dx \approx 0.6171875$$

After calculating we can see the values are close to each other.

(D)

Avg. no of iterations needed we have to calculate each od random number in part(b) is always around the value of C. I have taken 3 different values of C for the experiment and then calculate dthe value of average

iteration for finding random no. mention above.

We can see the values are close to the value obtained in part (a).

Qus2-

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Mean 1 = 0.8139296153234593
Variance 1 = 0.147575740291671

Mean 2 = 2.970515542426607
Variance 2 = 3.014212262167067

Mean 3 = 3.795243788964131
Variance 3 = 3.206820545802355
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Note that $f(x) \propto e^{-x} (x^{\alpha-1})$

So let $f(x) = k (e^{-x} (x^{\alpha-1}))$

We can find the value of k by using the fact that $\int_0^1 f(x) dx$ is 1

$$\int_0^1 f(x) dx = \int_0^1 k e^{-x} x^{\alpha-1} dx = k(\Gamma(0, \alpha) - \Gamma(1, \alpha)) = 1$$

This implies $k = 1/(\Gamma(0, \alpha) - \Gamma(1, \alpha))$

I choose the dominating PDF g as $g(x) = \alpha x^{\alpha-1}$.

Thus I get $G(x) = \int_0^x g(x) dx = x^\alpha$

And hence $G^{-1}(x) = x^{1/\alpha}$

The rejection constant c can be any value greater than or equal to the maximum value of $f(x)/g(x)$ in $(0, 1)$

$$f(x)/g(x) = k e^{-x} / \alpha$$

So maximum value of $f(x)/g(x)$ in $(0, 1)$ is k/α

So I choose $c = k/\alpha$

