# MONTE-CARLO SIMULATION ASSIGNMENT-8

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## Question 1-

## By method of Conditioning:

$X = Y + 2 * Z$

Where $Y \sim Exp(1)$ and $Z \sim N(0,1)$

Thus,

$P(X > 1) = E[I(X > 1)] = EE[I(X > 1)|Y = y] = E[P(X > 1| Y = y)]$

$\Rightarrow Estimator(W) = P(X > 1 \mid Y = y) = P((X - y)/2 > (1 - y)/2| Y = y) = \phi((y - 1)/2)$

$\Rightarrow Estimate = E[W]$

## By Antithetic Variables:

## Using the above result,

$W = (\phi((- ln(u) - 1)/2) + \phi((- ln(1 - u) - 1)/2))/2$

Where $u \sim U(0,1)$

Estimator = W

Estimate = E[W]

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PS C:\Users\User\Desktop\monte assignment 8> & C:/Users/User/AppData/Local/Programs/Python/Python310/python.exe "c:/Users/User/De
sktop/monte assignment 8/q1.py"
For the given procedure, expected value is 0.5090426398000001 and variance is 0.02671972401899545
For the given procedure, expected value is 0.5107177238 and variance is 0.0030671539936009133
The Percentage varinace reduction is 88.52101170124203
PS C:\Users\User\Desktop\monte assignment 8>
```

## Question 2-

Given Y~Exp(1) and we need to estimate E[Y] = $\int_{0}^{\infty} exp(-x)dx = \int_{[0,\infty)} f(x)\,p(x)\,dx$

Where f(x) = 1, p(x) = $e^{-x}$

Given,

$$q(x) = Esscher\ transform\ on\ p(x) = \frac{exp(hx)\,p(x)}{\int_{-\infty}^{\infty} exp(hx)\,p(x)} = (1-h)exp(hx)\,p(x) \quad if\ h < 1$$

$$= 0 \qquad if\ h \geq 1$$

Thus, we take h<1 (in this case it is taken as 0.5)

Domain for q(x) is [0,∞) also $q(x) > 0\ whenever\ f(x)p(x) \neq 0$

$$\therefore E[\frac{f(x)\,p(x)}{q(x)}] = \int_{0}^{\infty} \frac{f(x)\,p(x)}{q(x)} q(x)dx = E[\frac{p(x)}{(1-h)exp(hx)\,p(x)}] = E[\frac{exp(-hx)}{1-h}]$$

$$=> Estimator = exp(-hx)/(1-h) = 2exp(-0.5x)$$

Where x~Esscher transform on p(x)

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PS C:\Users\User\Desktop\monte assignment 8> & C:/Users/User/AppData/Local/Programs/Python/Python310/python.exe "c:/Users/User/Desktop/monte assignment 8/q2.py"
Estimate generated using Inverse Transform method =  1.0057356887320557
Variance of estimator generated using Inverse Transform method =  0.9267926304875953

Estimate generated using Importance Sampling method =  1.0080699927525723
Variance of estimator generated using Importance Sampling method =  0.3352648848328021

Amount of reduction in variance =  66.95461534344803
PS C:\Users\User\Desktop\monte assignment 8>
```

As seen above, the variance has reduced by Importance sampling