MA 323 - Monte Carlo Simulation Assignment - 1

(Mathematics and Computing)

Submitted By:

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Question 1:

Generate the sequence of numbers xi for a = 6, b = 0, m =11, and x0 ranging from 0 to 10. Also, generate the sequence of numbers xi for a = 3, b = 0, m = 11, and x0 ranging from 0 to 10. Observe the sequence of numbers generated and observe the repetition of values.

Tabulate these for each group of values. How many distinct values appear before repetitions? Which, in your opinion, are the best choices and why?

Outputs:

Case1: a = 6, b = 0, m = 11

Sequence : 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...

Repetition Period = 1

X0 = 1

Sequence : 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, …

Repetition Period = 10

X0 = 2

Sequence : 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, ...

Repetition Period = 10

X0 = 3

Sequence : 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, ...

Repetition Period = 10

X0 = 4

Sequence : 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, ...

Repetition Period = 10

X0 = 5

Sequence : 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, ...

Repetition Period = 10

X0 = 6

Sequence : 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, …

Repetition Period = 10

X0 = 7

Sequence: 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, ...

Repetition Period = 10

X0 = 8

Sequence: 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, ...

Repetition Period = 10

X0 = 9

Sequence: 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, 7, ...

Repetition Period = 10

X0 = 10

Sequence: 10, 5, 8, 4, 2, 1, 6, 3, 7, 9, 10, 5, 8, 4, 2, 1, 6, 3, ...

Repetition Period = 10

Case 2: a = 3, b = 0, m = 11

$X0 = 0$

Sequence: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, ...

Repetition Period = 1

$X0 = 1$

Sequence: 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, …

Repetition Period = 5

$X0 = 2$

Sequence: 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, ...

Repetition Period = 5

$X0 = 3$

Sequence: 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, ...

Repetition Period = 5

$X0 = 4$

Sequence: 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, ...

Repetition Period = 5

$X0 = 5$

Sequence: 5, 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, 1, ...

Repetition Period = 5

X0 = 6

Sequence: 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, …

Repetition Period = 5

X0 = 7

Sequence: 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, ...

Repetition Period = 5

X0 = 8

Sequence: 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, 6, ...

Repetition Period = 5

X0 = 9

Sequence: 9, 5, 4, 1, 3, 9, 5, 4, 1, 3, 9, 5, 4, ...

Repetition Period = 5

X0 = 10

Sequence: 10, 8, 2, 6, 7, 10, 8, 2, 6, 7, 10, 8, 2, ...

Repetition Period = 5

Observations:

1) In Case 1- ($a = 6, b = 0, m = 11$) has a period of 1 for $X0 = 0$ and a period of 10 for $X0 \neq 0$.

2) In Case 2- ($a = 3, b = 0, m = 11$) has a period of 1 for $X0 = 0$ and a period of 5 for $X0 \neq 0$.

3) The largest possible period length of linear congruence generator is $m - 1$. This value is achieved when $a = 6$ (full period), while period length for $a = 3$ is only 5. So the linear congruence generator with $a = 6$ is preferred over $a = 3$ as it has higher period length. This is because there will be more randomness in the generated numbers as there are more numbers in the sequence.

Question 2-

Generate a sequence u_i, $i = 1, 2, \ldots, 10000$ with $m = 244944$, $a = 1597, 51749$ (choosing x_0 as per your choice). Then group the values in the ranges [0, 0.05), [0.05, 0.10), [0.10, 0.15), . . . , [0.95, 1)and observe their frequencies (i.e., the number of values falling in each group). For 5 different x_0 values, tabulate the frequencies in each case, draw the bar diagrams for these data and put in your observations.

Outputs:

5 distinct values of X0 are generated randomly and first 100000 elements of the

sequence are generated and the frequencies between various ranges are plotted as a histogram.

fig. 1-

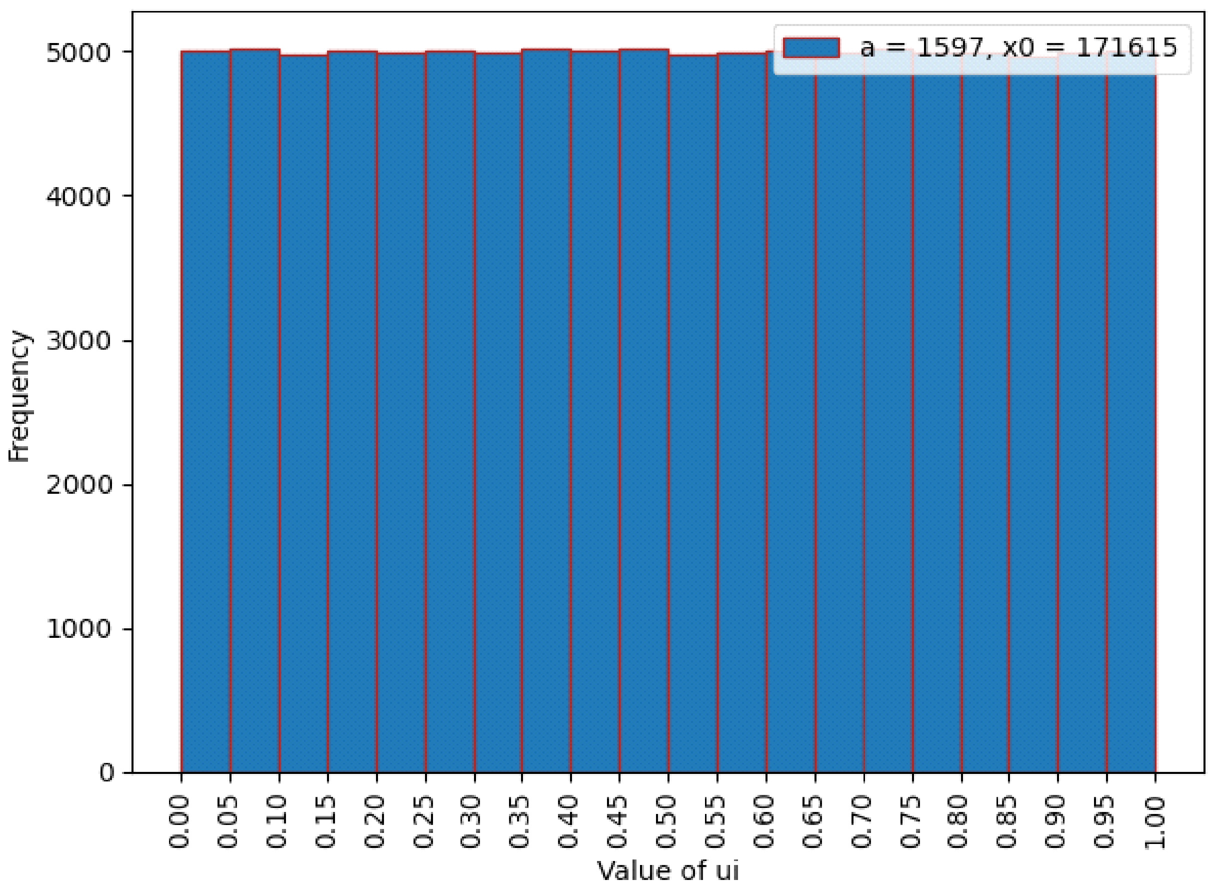


fig. 2-

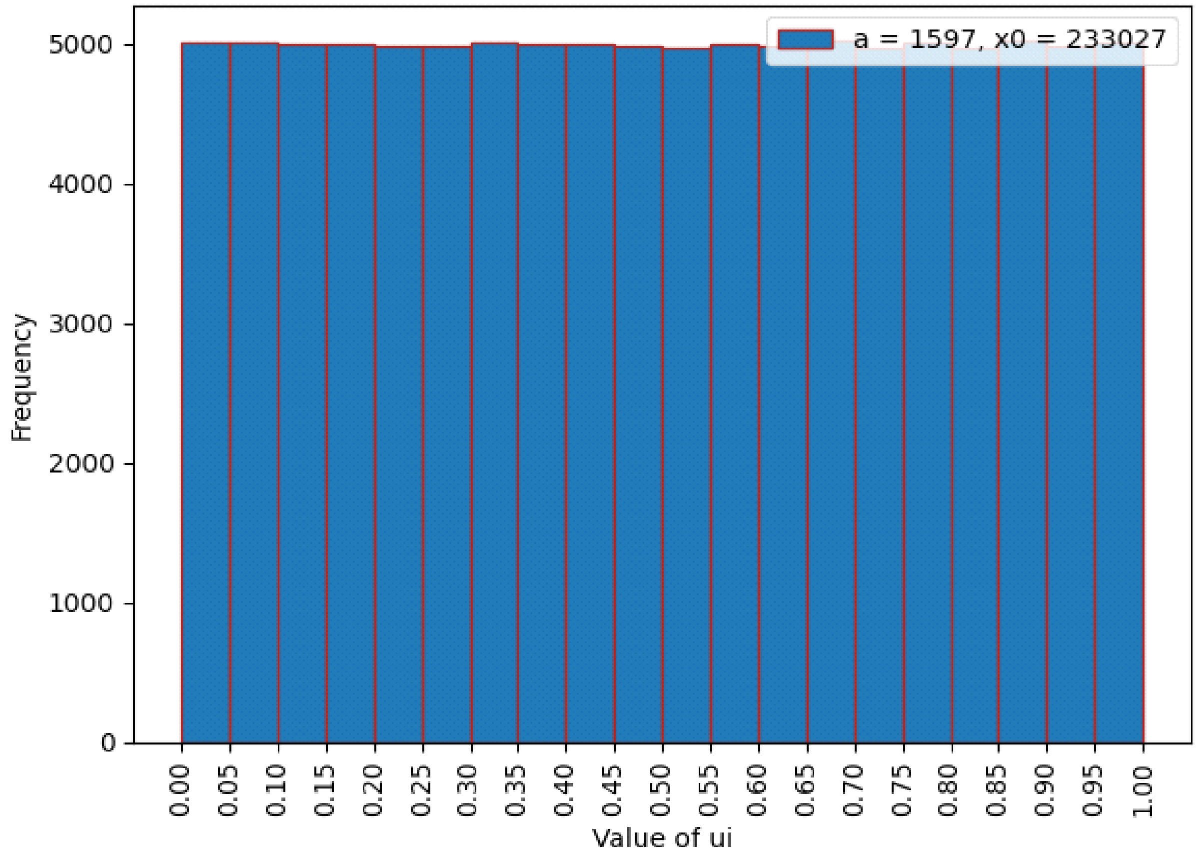
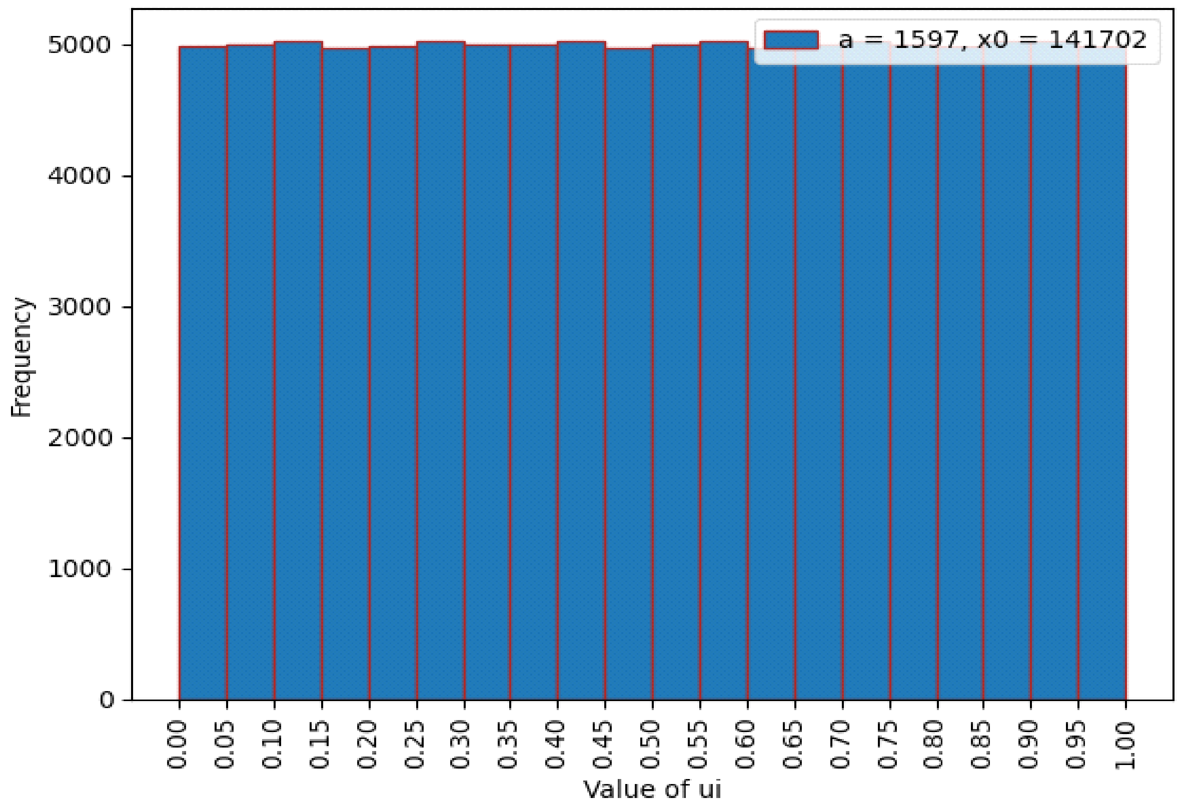

a = 1597, x0 = 233027
Frequency
5000
4000
3000
2000
1000
0
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Value of ui

fig. 3-



a = 1597, x0 = 141702
Frequency
5000
4000
3000
2000
1000
0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui

fig. 4-

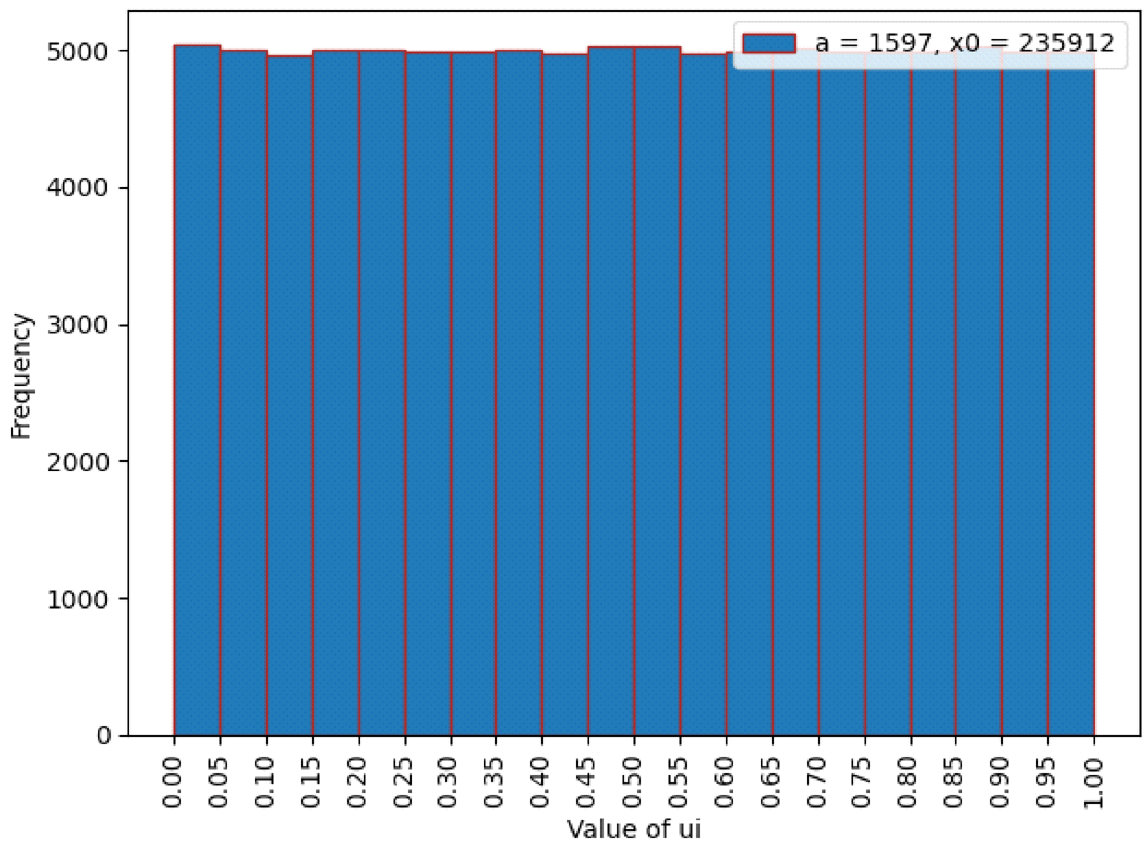

a = 1597, x0 = 235912
Frequency
5000
4000
3000
2000
1000
0
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Value of ui

fig. 5-

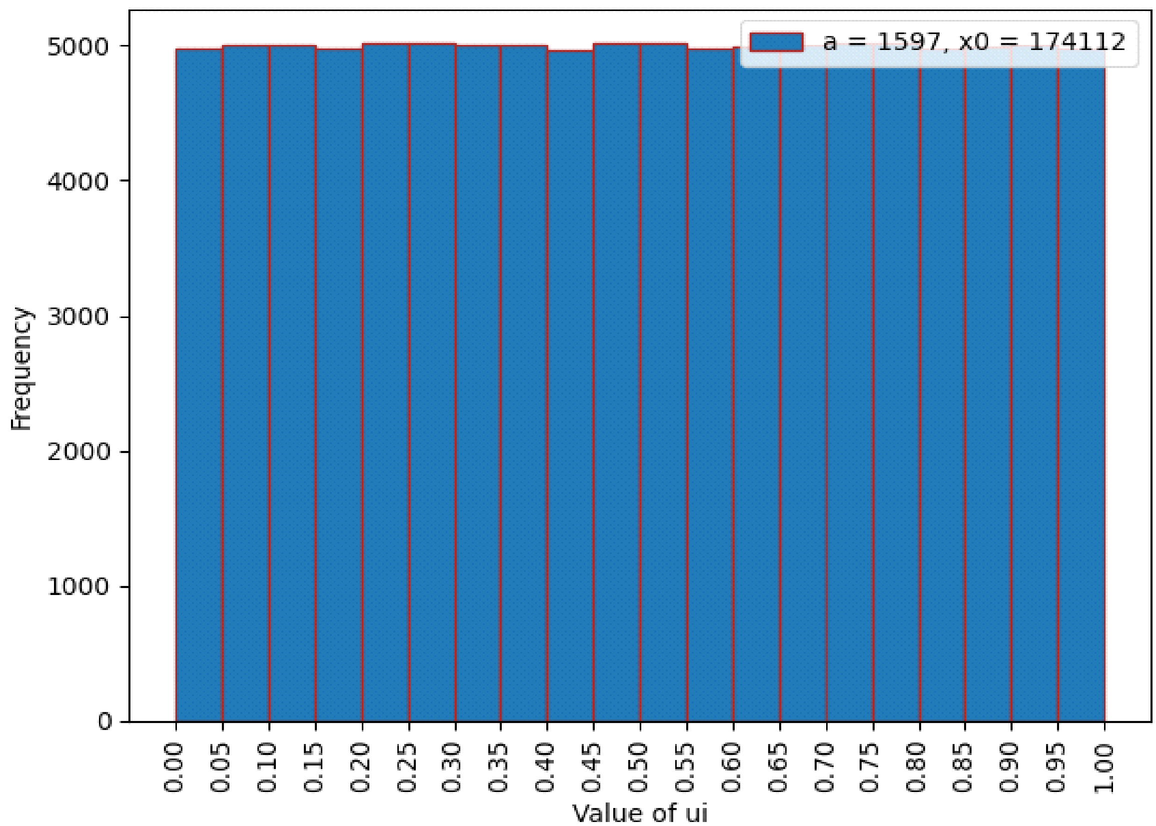

a = 1597, x0 = 174112
Frequency
Value of ui
0
1000
2000
3000
4000
5000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00

fig. 6-

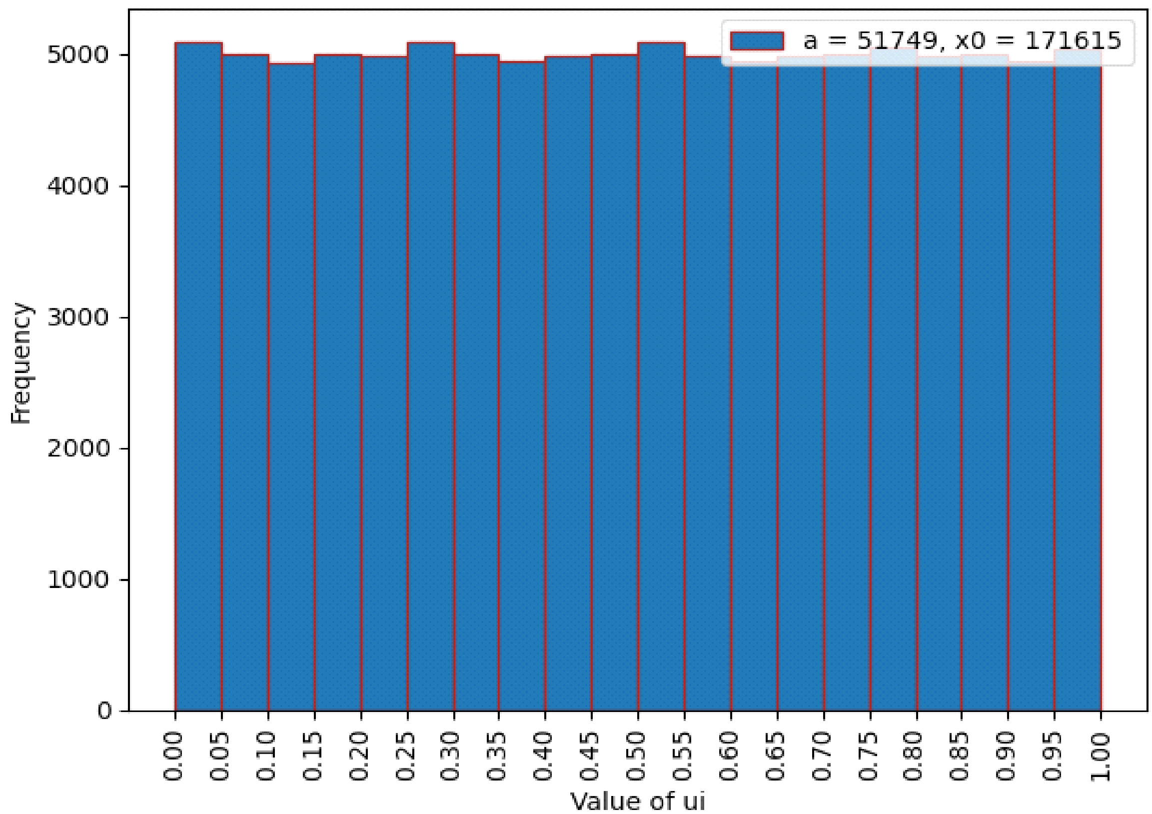

a = 51749, x0 = 171615
5000
4000
3000
2000
1000
0
Frequency
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Value of ui

fig. 7-

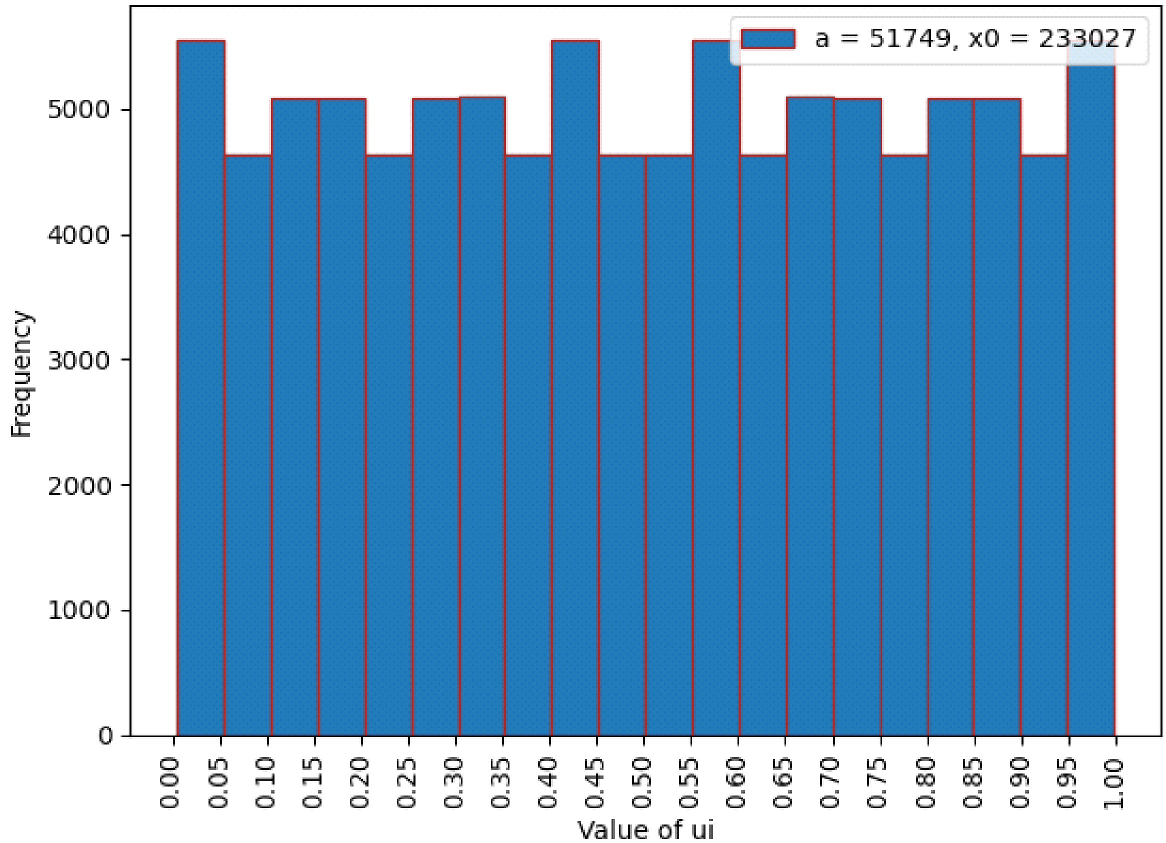

a = 51749, x0 = 233027
Frequency
0
1000
2000
3000
4000
5000
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
Value of ui

fig. 8-

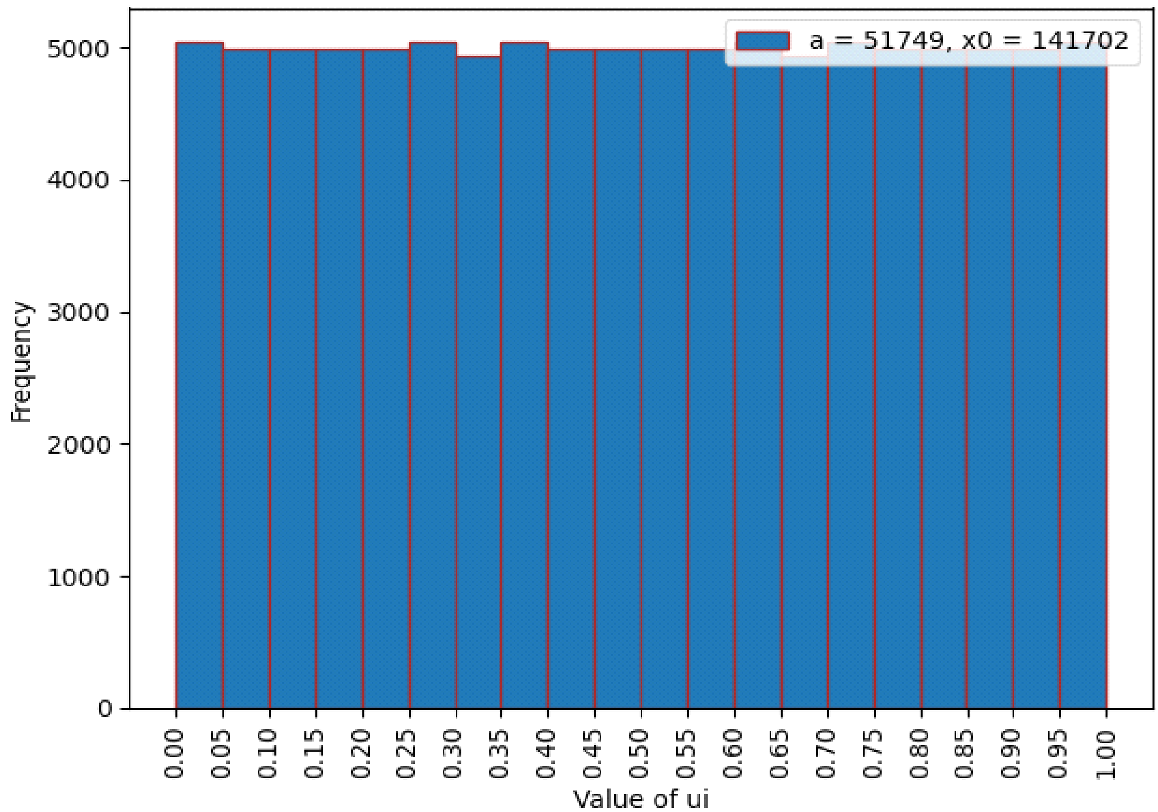

a = 51749, x0 = 141702
Frequency
Value of ui
0
1000
2000
3000
4000
5000
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00

fig. 9-

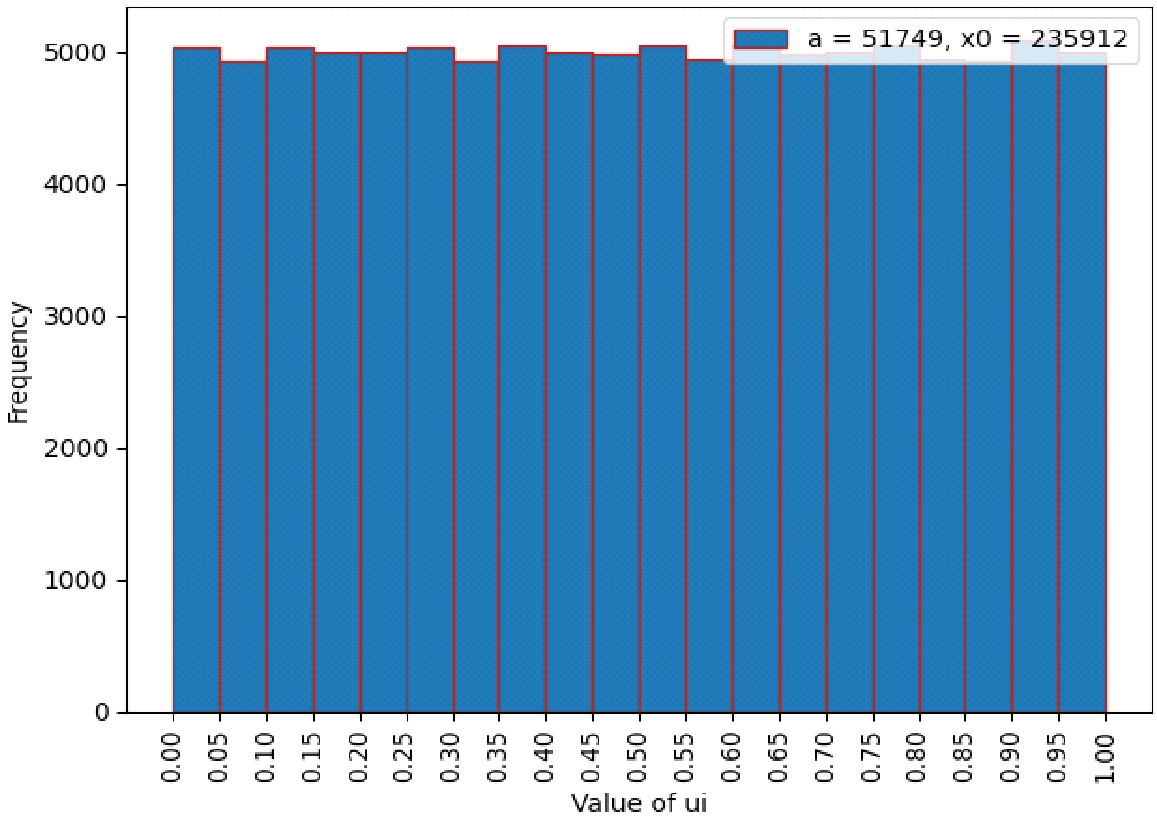
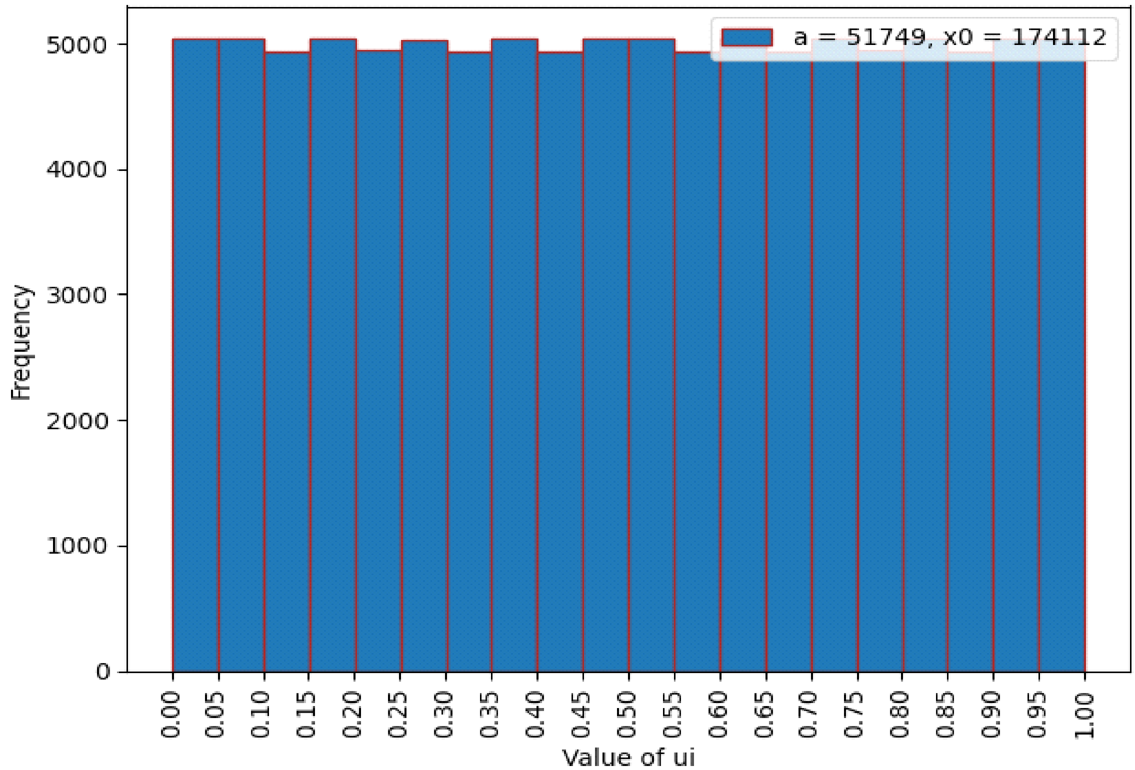

a = 51749, x0 = 235912
5000
4000
3000
2000
1000
0
Frequency
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60
0.65
0.70
0.75
0.80
0.85
0.90
0.95
1.00
Value of ui

fig. 10-



Observations:

1. The numbers are uniformly generated between 0 – 1. The frequency of different numbers lying in same length intervals are almost same. So, the random number generator follows the property of generation of numbers uniformly.

2. For different value of seed (x0), the frequencies are almost identical, and so the nature of bar graphs is same.

3. When a = 1597, b = 1, m = 244944, the Linear Congruence Generator has its full period, i.e. m – 1. But when a = 51749, b = 1, m = 244944, the Linear Congruence Generator does not achieve its full period.

Question 3:

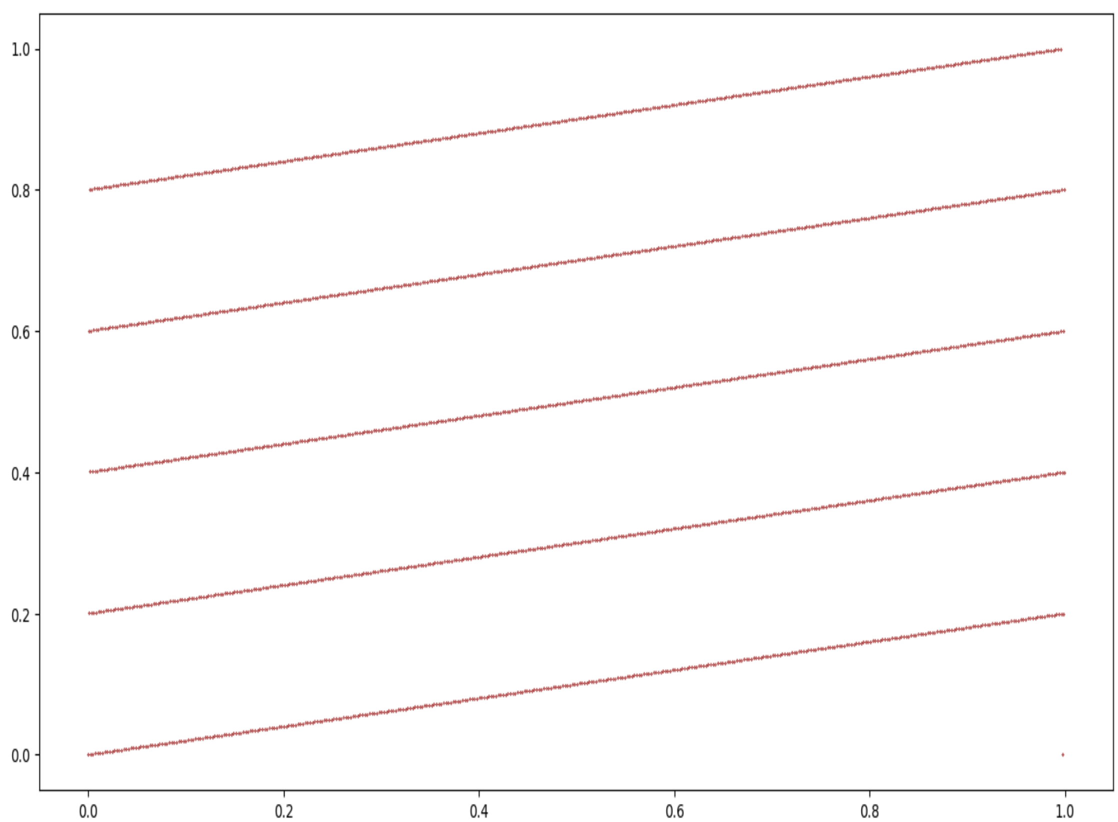
Generate a sequence ui, i = 1, 2, . . . , 10000 with a = 1229,

b = 1, m = 2048. Plot in a two- dimensional graph the points (ui**−1, ui), i.e., the points (u1, u2), (u2, u3), (u3, u4), ...**

Outputs:

The first 10000 elements of the sequence **(**un**)** is generated for

a= 1229**,** b **=** 1**,** m **=** 2048**,** and X0 chosen randomly and the values of **(**ui**-**1**,** ui**)** are plotted on a scatter plot**.** The result obtained is **-**



Observations:

1) The points (ui-1, ui) lie on parallel lines.

2) This shows the fact that the numbers generated by a linear congruence generator are not completely random (had it been completely random the plot would be uniformly scattered over the 2-D plan.

3). There is an outlier present at x = 1.0 (approx.). I believe this is present due to the precision issues while taking modulus in Python code (which is a bit different from the standard notion of modulus operation in other programming languages).