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- The above conditions are called the **FJ (Fritz John)** conditions and the point $(\mathbf{x}^*, \mathbf{u})$ (or $\mathbf{x}^*$) is called a **Fritz John**, or an **FJ**, point.
- $\mathbf{x}^*$ is said to satisfy **KKT** condition if there exists non negative constants u_i, $i \in I$, such that
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- **Remark:** Hence from **Theorem 8** it is clear that if $(\mathbf{x}^*, \mathbf{u})$ (or $\mathbf{x}^*$) is an **FJ** point and if $\nabla g_i(\mathbf{x}^*)$'s for $i \in I$ are **LI** then $(\mathbf{x}^*, \mathbf{u})$ (or $\mathbf{x}^*$) is a **KKT** point.

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  Hence $[2, 1]^T$ satisfies the KKT condition.

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- So **KKT** condition is **not** a necessary condition for a **local minimum**, although FJ conditions are **necessary conditions** for a **local minimum.**

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- $\nabla g_i(\mathbf{x}^*)$'s, $i \in I$ are **LD** at $\mathbf{x}^* = [0, 0]^T$ but $G_0 \neq \phi$.
- **Remark:** If $\nabla g_i(\mathbf{x}^*)$'s, $i \in I$ are **LI** then $G_0 \neq \phi$ but the converse is **not** true.

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