CSE 2500-01: Homework 5

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Coded in LaTeX

Problems

Question 1

Prove the following properties. You should follow the procedures discussed and shown in the class.

Theorem 1

The sum, product, and difference of any two even integers are even.

Proof. Suppose: m and n are any even integers.
By Definition of even, $m = 2r$ and $n = 2s$ for some integers r and s.
Then

$$\begin{aligned} m + n &= 2r + 2s && \text{(by substitution)} \\ &= 2(r + s) && \text{(by algebra)} \end{aligned}$$

$$\begin{aligned} m \cdot n &= 2r \cdot 2s && \text{(by substitution)} \\ &= 2(r \cdot s) && \text{(by algebra)} \end{aligned}$$

$$\begin{aligned} m - n &= 2r - 2s && \text{(by substitution)} \\ &= 2(r - s) && \text{(by algebra)} \end{aligned}$$

Let $t = r + s$, $u = r \cdot s$, and $v = r - s$.
Note: t, u, and v are integers because t is a sum of integers, u is a product of integers, and v is the difference between integers which are all integers.
Hence:

$$\begin{aligned} m + n &= 2t \\ m \cdot n &= 2u \\ m - n &= 2v \end{aligned}$$

where t, u, and v are some integers.
It follows by definition of even that $m + n$, $m \cdot n$, and $m - n$ are even. **QED**

Theorem 2

The sum and difference of any two odd integers are even.

Proof. Suppose: m and n are any odd integers.
By Definition of odd, $m = 2r + 1$ and $n = 2s + 1$ for some integers r and s.

Then

$$\begin{aligned} m+n &= (2r+1)+(2s+1) && \text{(by substitution)}\\ &= 2r+2s+2 \\ &= 2(r+s+1) && \text{(by algebra)} \end{aligned}$$

$$\begin{aligned} m-n &= (2r+1)-(2s+1) && \text{(by substitution)}\\ &= 2r-2s \\ &= 2(r-s) && \text{(by algebra)} \end{aligned}$$

Let $t = r+s+1$ and $u = r-s$.
Note: t and u are integers because t is a sum of integers, u is a difference of integers which are integers.
Hence:

$$m+n = 2t$$
$$m-n = 2u$$

where t and u are some integers.
It follows by definition of even that $m+n$ and $m-n$ are even. **QED**

Theorem 3

The product of any two odd integers is odd.

Proof. Suppose: m and n are any odd integers.
By Definition of odd, $m = 2r+1$ and $n = 2s+1$, for some integers r and s.
Then

$$\begin{aligned} m\cdot n &= (2r+1)\cdot(2s+1) && \text{(by substitution)}\\ &= 2(2rs)+2s+2r+1 \\ &= 2(2rs+s+r)+1 && \text{(by algebra)} \end{aligned}$$

Let $t = 2rs+s+r$
Note t is the product and sum of integers, which is an integer.
Hence $m\cdot n = 2t+1$ where t is some integer.
It follows by definition of odd that $m\cdot n$ is odd. **QED**

Theorem 4

The product of any even integer and any odd integer is even.

Proof. Suppose: m is any even integer, and n is any odd integer.
By definition of even and odd, $m = 2r$ and $n = 2s + 1$, for some integers r and s.
Then

$$\begin{aligned} m \cdot n &= (2r) \cdot (2s+1) && \text{(by substitution)} \\ &= 2(2rs) + 2r \\ &= 2(2rs + r) && \text{(by algebra)} \end{aligned}$$

Let $t = 2rs + r$.
Note that t is an integer because it is a sum and product of integers.
Hence $m \cdot n = 2t$ where t is some integer.
It follows by definition of even that $m \cdot n$ is even. **QED**

Theorem 5

The sum of any odd integer and any even integer is odd.

Proof. Suppose: m is any odd integer, and n is any even integer.
By definition of even and odd, $m = 2r + 1$ and $n = 2s$, for some integers r and s.
Then

$$\begin{aligned} m + n &= (2r+1) + 2s && \text{(by substitution)} \\ &= 2r + 2s + 1 \\ &= 2(r + s) + 1 && \text{(by algebra)} \end{aligned}$$

Let $t = r + s$
Note that t is an integer since it is the product and sum of integers, which is an integer.
Hence $m + n = 2t + 1$, where t is some integer.
It follows by definition of odd, that $m + n$ is odd. **QED**

Theorem 6

The difference of any odd integer minus any even integer is odd.

Proof. Suppose: m is any odd integer, and n is any even integer.
By definition of even and odd, $m = 2r + 1$ and $n = 2s$, for some integers r and s.
Then

$$\begin{aligned} m - n &= (2r+1) - 2s && \text{(by substitution)} \\ &= 2r - 2s + 1 \\ &= 2(r - s) + 1 && \text{(by algebra)} \end{aligned}$$

Let $t = r - s$.
Note t is an integer as it is the difference of two integers.
Hence $m - s = 2t + 1$, where t is some integer.
It follows by definition of odd that $m - n$ is odd. **QED**

Theorem 7

The difference of any even integer minus any odd integer is odd.

Proof. Suppose: m is any even integer and n is any odd integer.
By definition of even and odd, $m = 2r$ and $n = 2s + 1$, for some integers r and s.
Then

$$\begin{aligned} m - n &= (2r) - (2s + 1) && \text{(by substitution)} \\ &= 2r - 2s - 1 \\ &= 2(r - s - 1) + 1 && \text{(by algebra)} \end{aligned}$$

Let $t = r - s - 1$
Note t is an integer since the difference of integers are integers.
Hence $m - n = 2t + 1$, where t is some integer.
It follows by definition of odd that $m - n$ is odd. **QED**