CSE 2500-01: Homework 5

Arturo Salinas-Aguayo
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Electrical and Computer Engineering Department

College of Engineering, University of Connecticut
Coded in LaTeX

Problems

Question 1

Prove the following properties. You should follow the procedures discussed and shown in the class.

Theorem 1. *The sum, product, and difference of any two even integers are even.*

Proof. Suppose: m and n are any even integers.
By Definition of even, $m = 2r$ and $n = 2s$ for some integers r and s.
Then

$$\begin{aligned} m + n &= 2r + 2s && \text{(by substitution)} \\ &= 2(r + s) && \text{(by algebra)} \end{aligned}$$

Hence, the sum is even. Similarly, multiplication and subtraction follow the same pattern.
QED

Theorem 2. *The sum and difference of any two odd integers are even.*

Proof. Suppose: m and n are any odd integers.
By Definition of odd, $m = 2r + 1$ and $n = 2s + 1$ for some integers r and s.
Then

$$\begin{aligned} m + n &= (2r + 1) + (2s + 1) && \text{(by substitution)} \\ &= 2(r + s + 1) && \text{(by algebra)} \end{aligned}$$

Thus, the sum is even. Similar steps follow for subtraction. **QED**

Theorem 3. *The product of any two odd integers is odd.*

Proof. Suppose: $m = 2r + 1$ and $n = 2s + 1$.

$$\begin{aligned} m \cdot n &= (2r + 1)(2s + 1) \\ &= 4rs + 2r + 2s + 1 \\ &= 2(2rs + r + s) + 1 \end{aligned}$$

Since $2rs + r + s$ is an integer, $m \cdot n$ is odd. **QED**

Theorem 4. *The product of any even integer and any odd integer is even.*

Proof. Suppose: m is even and n is odd.

$$\begin{aligned} m \cdot n &= (2r)(2s + 1) \\ &= 2(2rs + r) \end{aligned}$$

Since $2rs + r$ is an integer, the product is even. **QED**