CSE 2500-01: Homework 1

Arturo Salinas-Aguayo
Spring 2025
Electrical and Computer Engineering Department

College of Engineering, University of Connecticut
Coded in LaTeX

1. (10 points) Restate the statement "Every positive number has a positive square root" by filling in the blank:

   (a) All positive numbers have have a positive square root.

   (b) For a positive number e, there is a positive square root for e.

   (c) For all positive numbers e, there is a positive number r such that its square is positive..

2. (20 points)

   (a) Is $2 \in \{2\}$?

   Yes, 2 is an element in the set.

   (b) How many elements are in the set $\{2, 2, 2, 2\}$?

   There is one element in the set.

   (c) How many elements are in the set $\{0, \{0\}\}$?

   There are two elements in this set.

   (d) Is $\{0\} \in \{\{0\}, \{1\}\}$?

   Yes the set of 0 is in the set.

   (e) Is $0 \in \{\{0\}, \{1\}\}$?

   No, the element 0 is not in the set.

3. (15 points) Let $S = \{1, 3, 10, 20\}$ and $T = \{1, 10\}$.

   (a) Is $\emptyset \subseteq S$?

   Yes, the empty set is in the set S.

   (b) Is $\emptyset \subseteq T$?

   Yes, the empty set in in the set T.

   (c) Is $S \subseteq T$?

   No, the set S is not a subset of set T.

   (d) Is $T \subseteq S$?

   Yes, the set T is a subset of S.

   (e) Is T a proper subset of S? How is this question different from part d)?

   A Proper Subset is a term which defines the case in which every element of the compared set is in the target set, however, there is at least one element that is not in the compared set that is in the target set.

   Yes, the set T is a proper subset of S.

4. (10 points) Let $S = \{2, 4, 6\}$ and $T = \{1, 3, 5\}$. Use the set-roster notation to write the following sets:

(a) $S \times T$

$$S \times T = \{(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3),(6,5)\}$$

(b) $S \times S$

$$S \times S = \{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}$$

5. (25 points) Let $G = \{-2, 0, 2\}$ and $H = \{4, 6, 8\}$, and define a relation V from G to H as follows: for all $(x, y) \in G \times H$, $(x, y) \in V$ if $\frac{x-y}{4}$ is an integer. Answer the following:

   (a) Is $2\,V\,6$?

   This is equivalent to $\frac{2-6}{4}$ and checks that the outcome is an integer.
   $\frac{2-6}{4} = -1$, -1 is an integer, therefore this is 2 is related to 6 by V.

   (b) Is $(-2)\,V\,(6)$?

   This is asking if $\frac{-2-6}{4}$ outputs an integer.
   The outcome is $\frac{-2-6}{4} = -2$ which is an integer, therefore -2 is related to 6 by V.

   (c) Is $(0, 6) \in V$?

   This is asking if 0 and 6 are related by V.
   The outcome is $\frac{0-6}{4} = \frac{-3}{2}$ this is not an integer and therefore 0 and 6 are not related by V

   (d) Is $(2, 4) \in V$?

   This is asking if 2 and 4 are related by 4.
   The outcome is $\frac{2-4}{4} = \frac{-1}{2}$ which is not an integer, therefore 2 and 4 are not related by V.

   (e) Write V as a set of ordered pairs.

   (f) Write the domain and co-domain of V.

(g) Draw an arrow diagram for V.

6. (10 points) Let $X = \{2, 4, 5\}$ and $Y = \{1, 2, 4, 6\}$. For each of the following relations, draw an arrow diagram and say whether the relation is a function:

   (a) $R = \{(2, 6), (4, 2), (5, 2)\}$

   (b) $V = \{(2, 4), (4, 1), (4, 2), (5, 6)\}$

7. (10 points) Define functions H and K from $\mathbb{R}$ to $\mathbb{R}$ by the following formulae: for all $x \in \mathbb{R}$,

$$H(x) = (x - 2)^2$$

$$K(x) = (x - 1)(x - 3) + 1$$

   (a) Are these the same function? Why or why not?