Lab 02: The Node Voltage Method

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Coded in LaTeX

Abstract

The node voltage method is the backbone of dc electrical circuit analysis. It offers a precise way to map out a circuit and create references to each node with regard to another. The course starts out detailing the Branch Current method, which will always work, however once complicated circuits are introduced, this method begins to show its limitations. Additionally, many times the inner workings of a circuit are of no importance and the relationship between the voltage at the output terminals compared to the input terminals is all that is desired. With branch current, the entire circuit must be "solved" which is simply not practical or feasable from a cost perspective. Time is money and an engineer can prove their worth by making quick and accurate calculations which do not necessarily include all the tiny branch currents and voltage drops where not important.

Introduction

The *Node Voltage Method* is a way to formulate the different nodes in which three or more different components are connected to and reference them to each other or a common reference point, commonly denoted as a digital ground.

This experiment starts off with some napkin math on the calculation of a few simple circuits which demonstrate the versatility of the method. The practicality of the application is then enforced by building the circuits in hardware utilizing the Protoboard and resistors included in the experiment kit.

The experimental work concludes with calculations within the PSpice simulation suite. This is not the first time that this is used in the course, however as more components are introduced, more features and capabilities of the software suite are shown.

Theory

The Node Voltage Method

As stated in the abstract of this document, a node voltage is the potential difference between any given node and some other node that has been denoted as the *reference node*. The reference node is usually denoted as the *ground*.

Current always flows from the node with the higher potential to the node with the lower potential. Utilzing the passive sign convention and knowledge gained from previous experimental work, components can be assigned polarities which correspond to the purpose that they serve within the circuit.

For example, a DC voltage source which is supplying power to a circuit has current pointing in towards the positive terminal, while a DC voltage source that is absorbing power will have the current flowing the other direction. The overall direction of the current flow, as stated earlier, has to do with the magnitude of the potential of the nodes surrounding the component.

Taking a step back, one may realize that this will change depending on what ground the circuit analyzer chooses and rush to point out that this will lead to problems in the analysis. In actuality, and only once one convinces themselves by repeated experimentation, the specific location of the reference node does in fact not matter in the grand scheme of the analysis as long as the rules are sound and the laws are obeyed, with a consistent use of *discipline* in the passive sign convention, the results will turn out equal.

The *Node Voltage Method* relys on heavy use of Kirchoff's Current Law which was heavily explained in Lab 01's lab report, hence I will cover it briefly here.

$$\sum i_{total} = i_1 + ... + i_p = 0$$

Each node gets its own equation set to 0, set in reference to all other nodes in the circuit. This results in an amount of equations that is equal to the number of essential nodes minus one, the reference node.

Recall that current, i, is equivalent to $\frac{v}{R}$ and a large portion of learning comes from the realization that when utilizing the node voltage method, one solves for missing voltages, although the KCL equation is a sum of currents. Confusing at first, but with with some repeated reps, this becomes painfully simple, which is good.

There are additional cases in which this may become even more simple, for example with a dedicated voltage source in series with nothing else within a branch, the node voltage value is simply the value of the source itself. Additionally, if there is a voltage source inbetween two nodes, the nodes may be combined to form a single node, as long as the missing current is handled accordingly.

Eventually, one finds the voltages utilizing linear algebra and the circuit can be solved for a modicum of values, or as seen later on in analysis, simplified to an equivalent simpler circuit.

Experiemntial Procedures

Part One

This portion requires finding the expected node voltage, V_1, with the assumption that the bottom node is the reference node for the circuit. Refer to Figure 1.

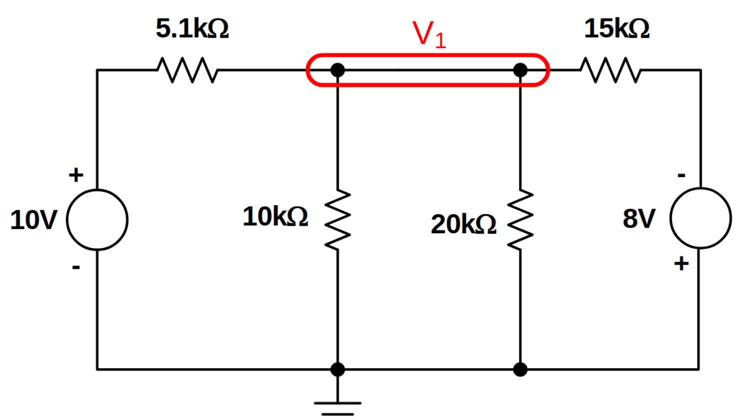

Figure 1: A Simple DC Resistive Circuit

Analytically, there are two essential nodes. Hence, we have one equation and one unknown, V_1. The equations are setup in units of $k\Omega$, V, mA, and mW to save complexity and obfuscation of the math.

$$\frac{V_1 - 10}{5.1} + \frac{V_1}{10} + \frac{V_1}{20} + \frac{V_1 + 8}{15} = 0$$
$$V_1 = 3.458Volts$$

The results and discussion portion elaborates on the experimental findings of building this circuit.

Part Two

This portion requires solving for multiple unknowns. Referring to Figure 2, there are four unknown essential branch voltages, and therefore will require 3 KCL equations to find 3 unknown voltages. $V_x = 5V$.

The equations are setup in units of $k\Omega$, V, mA, and mW to save complexity and obfuscation of the math.

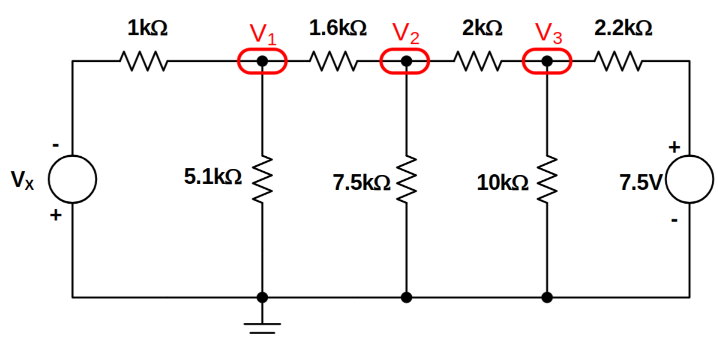

Figure 2: A Slightly More Complex DC Resistive Circuit

$$\frac{V_1 + 5}{1} + \frac{V_1}{5.1} + \frac{V_1 - V_2}{1.6} = 0$$
$$\frac{V_2 - V_1}{1.6} + \frac{V_2}{7.5} + \frac{V_2 - V_3}{2} = 0$$
$$\frac{V_3 - V_2}{2} + \frac{V_3}{10} + \frac{V_3 - 7.5}{2.2} = 0$$

Rearranging and putting each equation into general form:

$$1.821V_1 - .625V_2 + 0V_3 = -5$$
$$-0.625V_1 + 1.258V_2 - 0.5V_3 = 0$$
$$0V_1 - 0.5V_2 + 1.055V_3 = 3.41$$

$$\begin{bmatrix} 1.821 & -.625 & 0 \\ -0.625 & 1.285 & -0.5 \\ 0 & -0.5 & 1.055 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 3.41 \end{bmatrix}$$

$$V_1 = -2.787V$$
$$V_2 = -0.120V$$
$$V_3 = 3.175V$$

Part Three

This portion of the experiment introduces a strange schematic that is shaped like a cube. Utilizing symmetry however, the cube can be widdled away utilizing something called *intuition*.

Looking at Figure 3 there are 26 branch currents, and 8 essential nodes. This is quite a feat to figure out, and manual hashing out of each equation will produce the correct answer, however, as noted in the begining of the report, an engineer is not paid for time wasted like that.

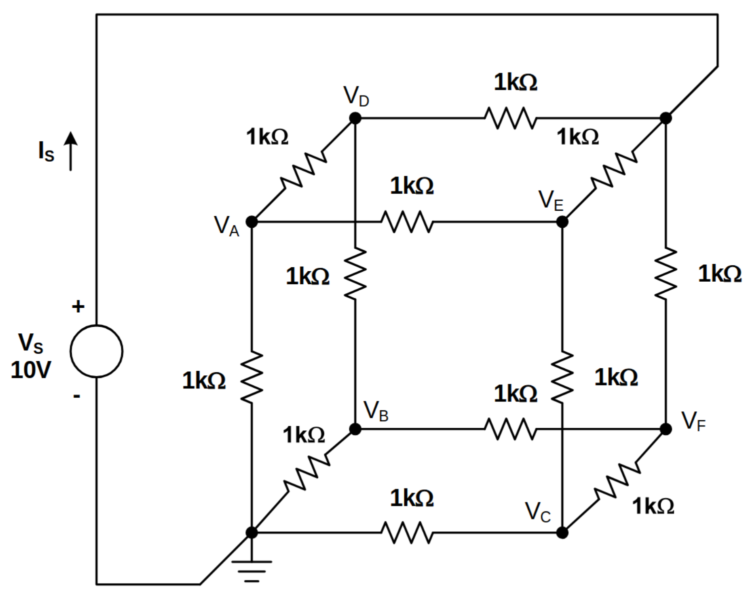

Figure 3: Cube.

As a result of symmetry, the currents flowing to V_A, V_B, and V_C are all identical, as are the currents flowing from V_E, V_D, and V_F.

Without even lifting up a calculator, one can see that the at the node entering the cube at the top right, I_s is split evenly three ways, meaning that each current leading to V_D, V_E, and V_F is simply

$$\frac{I_s}{3}$$

On the other corner of the cube, we see the same exact situation played in reverse for V_A, V_B, and V_C

By utilizing one of the special cases due to only the single 10V source connected at the top right junction, we know the potential at that node is 10V.

So what is the voltage drop between three identical $1k\Omega$ resistors in parallel?

Results and Discussion