Pose-parameter graph optimisation

Art van Liere

Delft Center for Systems and Control (DCSC)
Delft University of Technology

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1st supervisor: prof.dr.ir. Tamás Keviczky (DCSC)

2nd supervisor: ir. Martijn Krijnen (Demcon)

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Outline

1. Introduction
2. Pose graph optimisation
3. Pose-parameter graph optimisation
   - Motivation
   - Research
   - Parameters
   - Connectivity
4. Conclusion
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Outline

Introduction

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SLAM

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Simultaneous **L**ocalisation **A**nd **M**apping

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- Localisation

Figure: Localisation from known map

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Figure: Localisation from known map

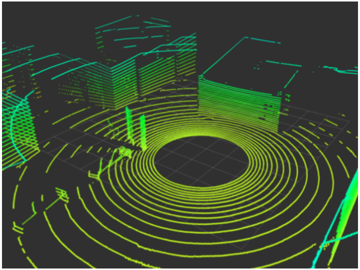

Figure: Mapping from known location

Introduction

Graph-based SLAM methods

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Discretise trajectory as set of robot poses $x_k = \left(t_x, t_y, \theta\right) \in \mathbb{R}^3$

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- Input: $\mathcal{Z}$ (set of measurements)
- Output: $\mathcal{X}^*$ (set of estimated poses)

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Solves the so-called SCLAM (**S**imultaneous **C**alibration **L**ocalisation **A**nd **M**apping) problem!

Introduction

Parameters

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- Wheel radii

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- Parameter influence might be unknown

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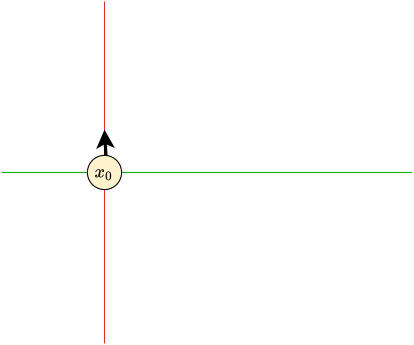
Pose graph optimisation

Pose graph optimisation

Current status

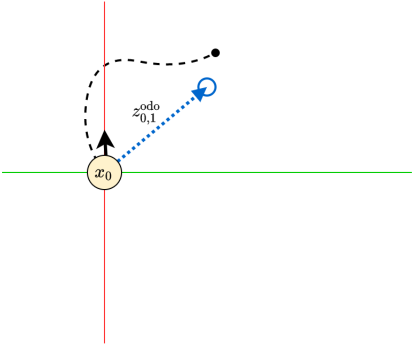
Fix first pose at origin: $x_0 = (0, 0, 0)$



Pose graph optimisation

Current status

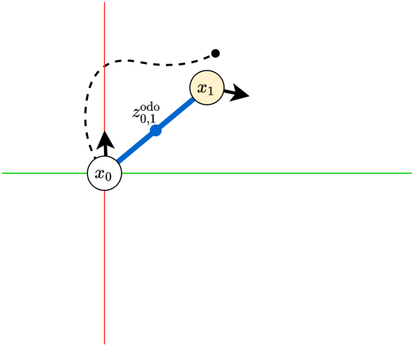
Measure relative pose transformation: $x_1 - x_0$



Pose graph optimisation

Current status

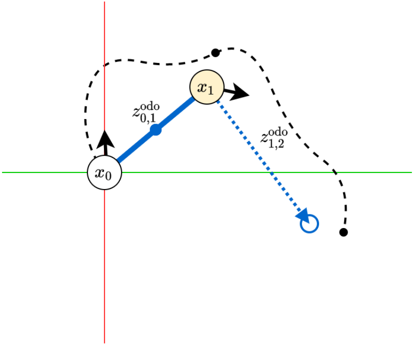
Add odometry constraint: $z_{0,1}^{\text{odo}}$



Pose graph optimisation

Current status

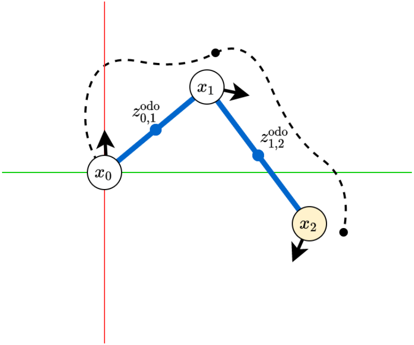
Measure relative pose transformation: $x_2 - x_1$



Pose graph optimisation

Current status

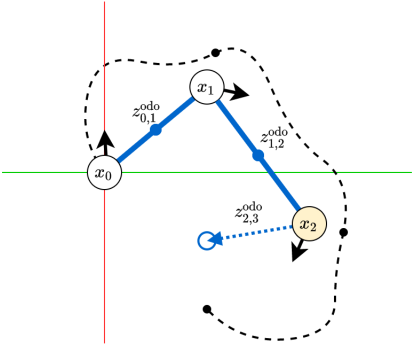
Add odometry constraint: $z_{1,2}^{\text{odo}}$



Pose graph optimisation

Current status

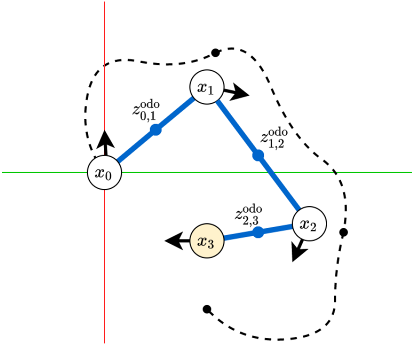
Measure relative pose transformation: $x_3 - x_2$



Pose graph optimisation

Current status

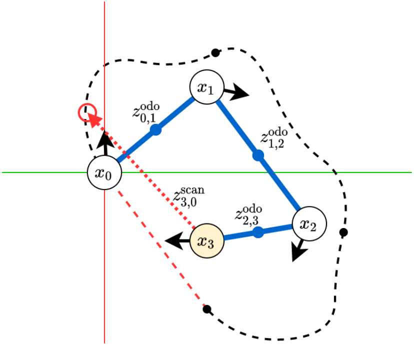
Add odometry constraint: $z^{\text{odo}}_{2,3}$



Pose graph optimisation

Current status

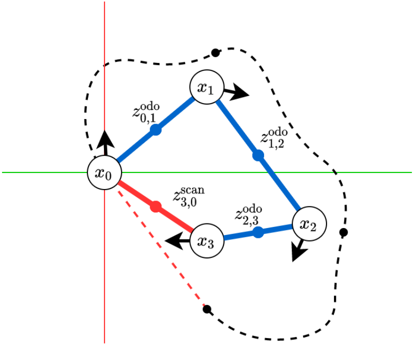
Measure relative pose transformation: $x_o - x_3$



Pose graph optimisation

Current status

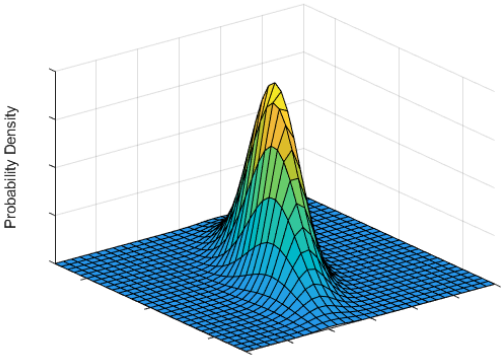
Add scan-matching constraint: $z^{\text{scan}}_{3,0}$



Pose graph optimisation

Current status

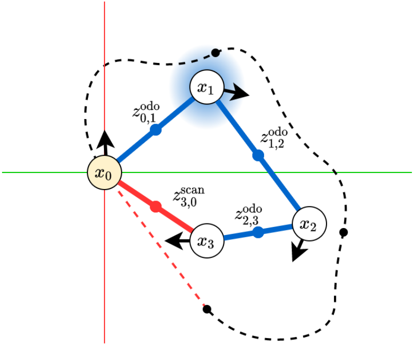
Gaussian probability distribution



Pose graph optimisation

Current status

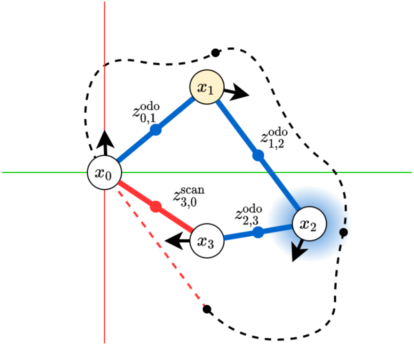
Probability of x_1 due to $z_{0,1}^{\mathrm{odo}}$



Pose graph optimisation

Current status

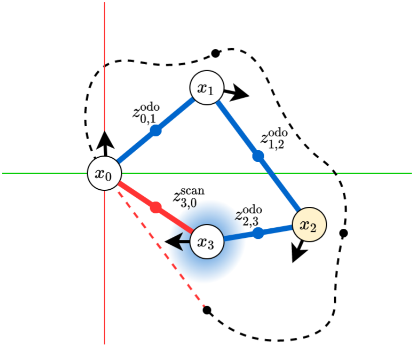
Probability of x_2 due to $z_{1,0}^{\mathrm{odo}}$



Pose graph optimisation

Current status

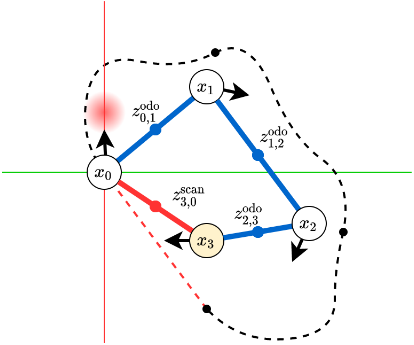
Probability of x_3 due to $z^{\text{odo}}_{2,1}$



Pose graph optimisation

Current status

Probability of x_0 due to $z_{3,\mathrm{o}}^{\mathrm{scan}}$



Pose graph optimisation

Cost function

Penalise unlikely pose configurations:

$$\mathbf{e}_{0,1}^{\mathrm{odo}}(x_0, x_1) = \underbrace{f_{0,1}^{\mathrm{odo}}(x_0, x_1)}_{\text{measurement model}} - z_{1,2}^{\mathrm{odo}}$$

Pose graph optimisation

Cost function

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Scale with inverse covariance:

$$F_{0,1}^{\mathrm{odo}}(x_0, x_1) = \|\mathbf{e}_{0,1}^{\mathrm{odo}}(x_0, x_1)\|_{\Sigma_{1,2}^{\mathrm{odo}}}^2$$

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Summate error terms over all measurements (with indices $\mathcal{I}$):

$$F_{\text{pgo}}(\mathcal{X}) = F_{0,1}^{\text{odo}}(x_0, x_1) + F_{1,2}^{\text{odo}}(x_1, x_2) + F_{2,3}^{\text{odo}}(x_2, x_3) + F_{3,0}^{\text{scan}}(x_3, x_0)$$

Pose graph optimisation

Cost function

Penalise unlikely pose configurations:

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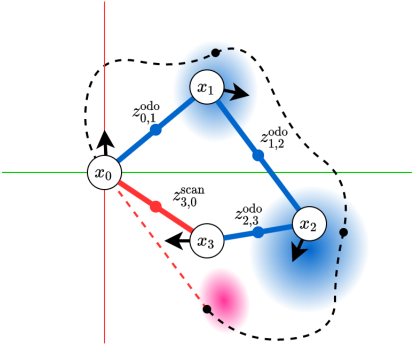
$$\begin{aligned} F_{\text{pgo}}(\mathcal{X}) &= F_{0,1}^{\text{odo}}(x_0, x_1) + F_{1,2}^{\text{odo}}(x_1, x_2) + F_{2,3}^{\text{odo}}(x_2, x_3) + F_{3,0}^{\text{scan}}(x_3, x_0) \\ \mathcal{X}^* &= \arg\min_{\mathcal{X}} F_{\text{pgo}}(\mathcal{X}) \end{aligned}$$

Pose graph optimisation

Current status

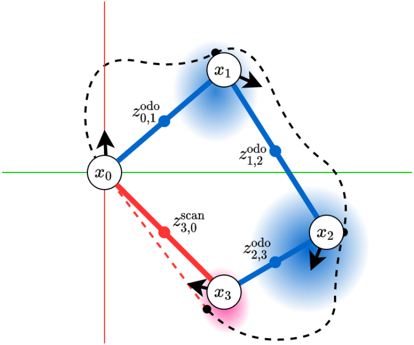
Probability of $\mathcal{X}$



Pose graph optimisation

Current status

Optimise pose graph to find $\mathcal{X}^* = \arg\min_{\mathcal{X}} F_{\text{pgo}}(\mathcal{X})$



Outline

Pose-parameter graph optimisation — Motivation

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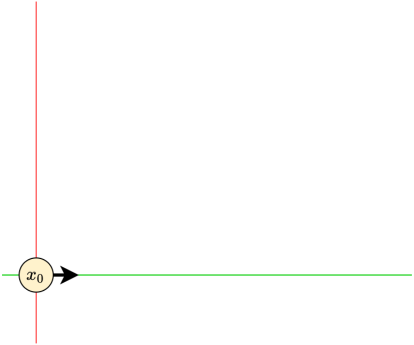
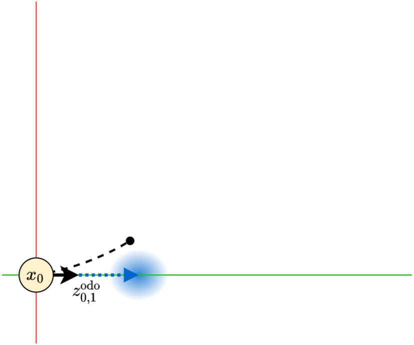
Fix first pose at origin: $x_0 = (0, 0, 0)$



Pose-parameter graph optimisation — Motivation

Current status

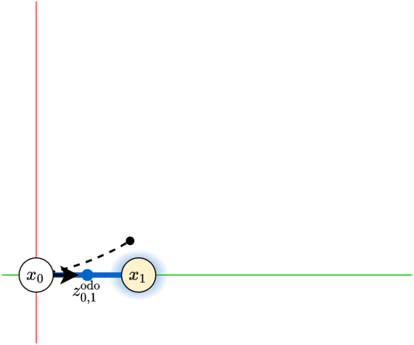
Measure relative pose transformation: $x_1 - x_0$



Pose-parameter graph optimisation — Motivation

Current status

Add odometry constraint: $z_{0,1}^{\mathrm{odo}}$



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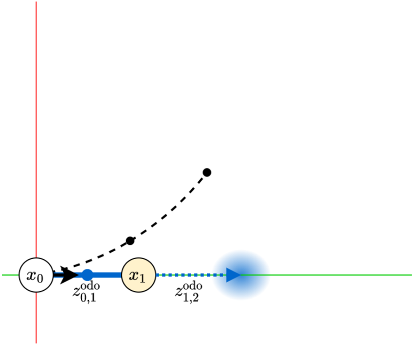
Current status

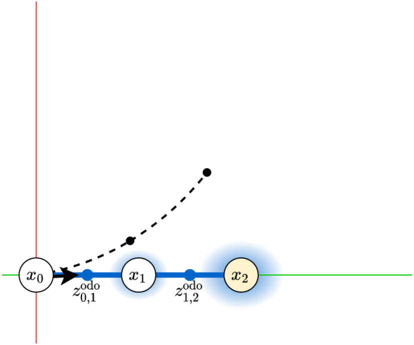
Measure relative pose transformation: $x_2 - x_1$



Pose-parameter graph optimisation — Motivation

Current status

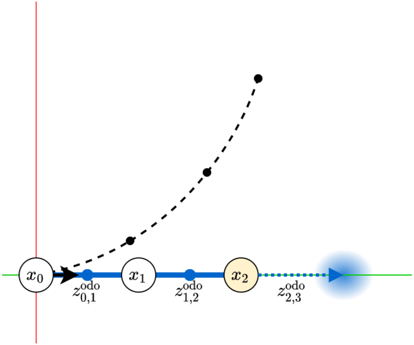
Add odometry constraint: $z^{\mathrm{odo}}_{1,2}$



Pose-parameter graph optimisation — Motivation

Current status

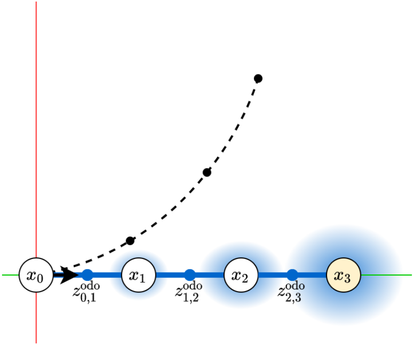
Measure relative pose transformation: $x_3 - x_2$



Pose-parameter graph optimisation — Motivation

Current status

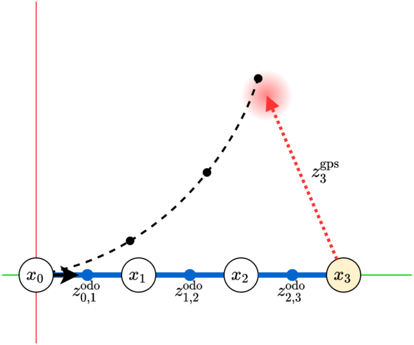
Add odometry constraint: $z_{2,3}^{\mathrm{odo}}$



Pose-parameter graph optimisation — Motivation

Current status

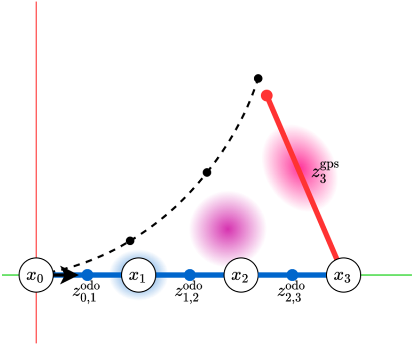
Measure absolute pose transformation: x_3



Pose-parameter graph optimisation — Motivation

Current status

Add GPS constraint: z_3^{gps}



Pose-parameter graph optimisation — Motivation

Example

Example of error due to rotation bias

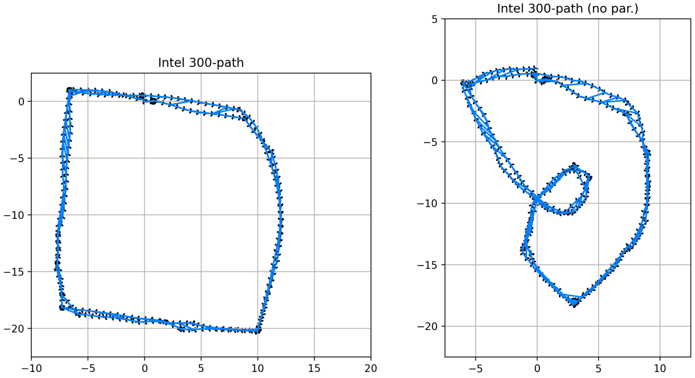


Figure: Intel dataset without and with rotation bias

Pose-parameter graph optimisation — Motivation

Possible solution

1. Add bias component to measurement model

Pose-parameter graph optimisation — Motivation

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2. Include bias parameter as a graph node

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Proposed method: *Pose-parameter graph optimisation* (PPGO)

Pose-parameter graph optimisation — Motivation

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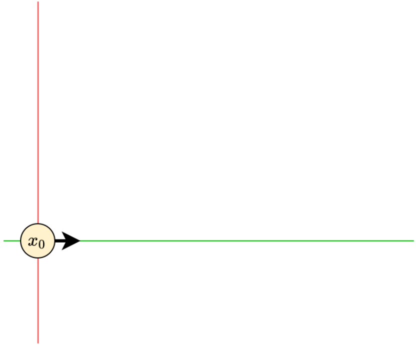
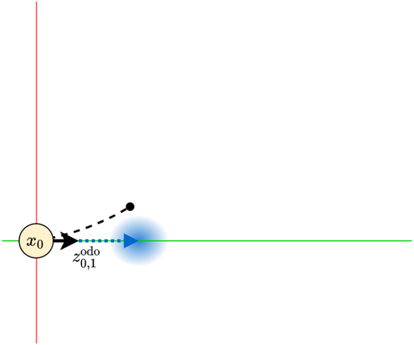


Pose-parameter graph optimisation — Motivation

Current status

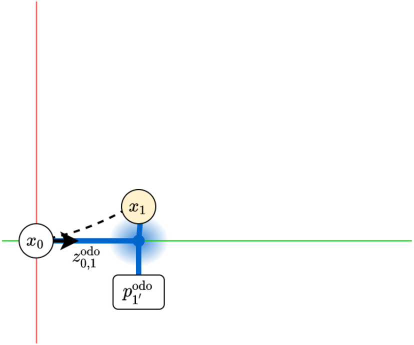
Measure relative pose transformation: $x_1 - x_0$



Pose-parameter graph optimisation — Motivation

Current status

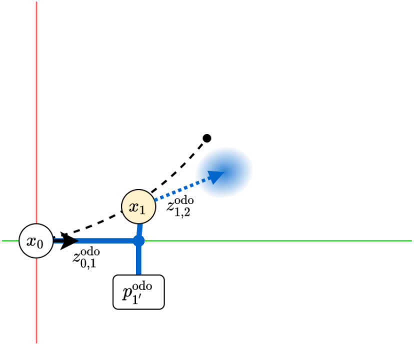
Add odometry constraint: $z_{0,1}^{\text{odo}}$



Pose-parameter graph optimisation — Motivation

Current status

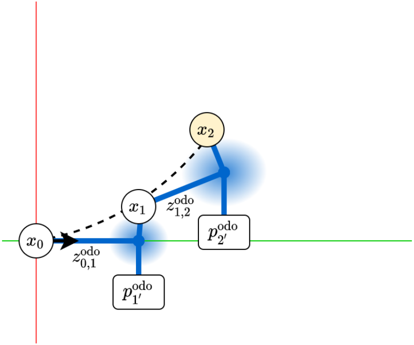
Measure relative pose transformation: $x_2 - x_1$



Pose-parameter graph optimisation — Motivation

Current status

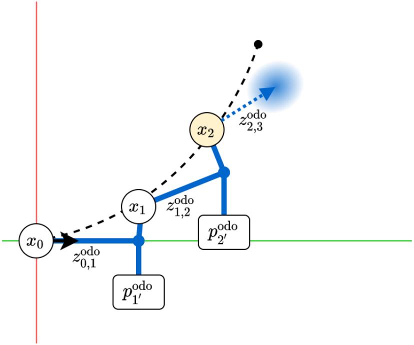
Add odometry constraint: $z_{1,2}^{\text{odo}}$



Pose-parameter graph optimisation — Motivation

Current status

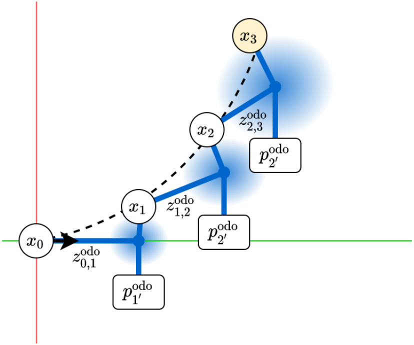
Measure relative pose transformation: $x_3 - x_2$



Pose-parameter graph optimisation — Motivation

Current status

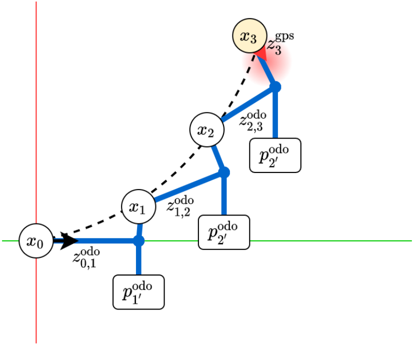
Add odometry constraint: $z_{2,3}^{\mathrm{odo}}$



Pose-parameter graph optimisation — Motivation

Current status

Measure absolute pose transformation: x_3



Pose-parameter graph optimisation — Motivation

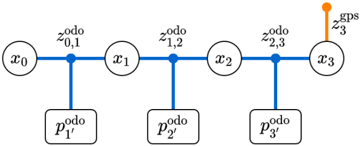


Figure: Schematic pose-parameter graph

Pose-parameter graph optimisation — Motivation

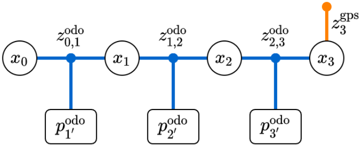


Figure: Schematic pose-parameter graph

Under-constrained problem

- 4 equations (constraints/measurements)
- 6 unknowns (non-fixed nodes)

Pose-parameter graph optimisation — Motivation

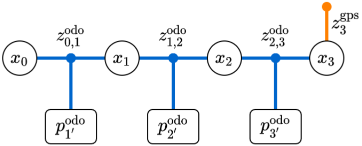


Figure: Schematic pose-parameter graph

Under-constrained problem

- 4 equations (constraints/measurements)
- 6 unknowns (non-fixed nodes)

We need alternative *connectivity strategies*!

Pose-parameter graph optimisation — Motivation

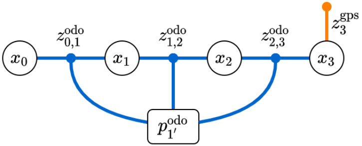


Figure: Schematic pose-parameter graph

Feasible problem

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- 4 unknowns (non-fixed nodes)

Pose-parameter graph optimisation — Motivation

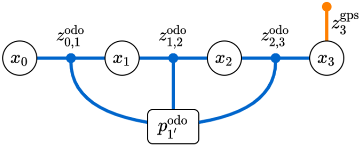

Figure: Schematic pose-parameter graph

Feasible problem

- 4 equations (constraints/measurements)
- 4 unknowns (non-fixed nodes)

Static connectivity strategy for *constant* parameters

Pose-parameter graph optimisation — Research

Aim of this thesis

- Develop simulation framework and pose-parameter graph analysis framework
- Investigate parameter implementations
- Investigate connectivity strategies for:
  - Time-dependent parameters
  - Space-dependent parameters

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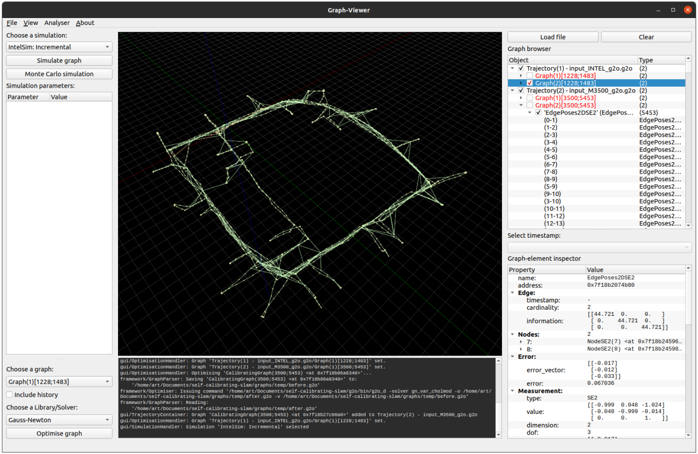

Figure: Software framework GUI

Pose-parameter graph optimisation — Research

Performance metrics

- *Absolute Trajectory Error* (ATE): average absolute translation error over all poses
- *Relative Pose Error* (RPE): average relative translation/rotation error over all poses

Pose-parameter graph optimisation — Parameters

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Bias parameter

$$f^{\mathrm{bias}}_{i,j,k}(x_i, x_j, p_k) = f_{i,j}(x_i, x_j) + p_k$$

Pose-parameter graph optimisation — Parameters

Bias parameter

$$f_{i,j,k}^{\text{bias}}(x_i, x_j, p_k) = f_{i,j}(x_i, x_j) + p_k$$

Scaling factor parameter

$$f_{i,j,k}^{\text{scale}}(x_i, x_j, p_k) = p_k \odot f_{i,j}(x_i, x_j)$$

Pose-parameter graph optimisation — Parameters

Bias parameter

$$f^{\text{bias}}_{i,j,k}(x_i, x_j, p_k) = f_{i,j}(x_i, x_j) + p_k$$

Scaling factor parameter

$$f^{\text{scale}}_{i,j,k}(x_i, x_j, p_k) = p_k \odot f_{i,j}(x_i, x_j)$$

The combination can model any affine parameter influence!

Pose-parameter graph optimisation — Connectivity

Pose-parameter graph optimisation — Connectivity

Time-dependent parameters

- *Sliding window* connectivity strategy
- *Timely batch* connectivity strategy

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Connect only recent constraints a single parameter-node

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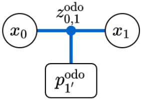

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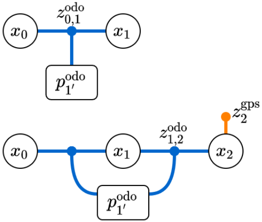

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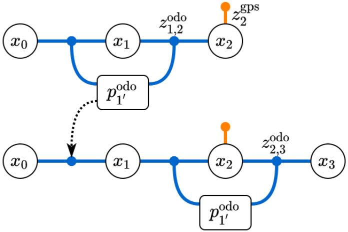

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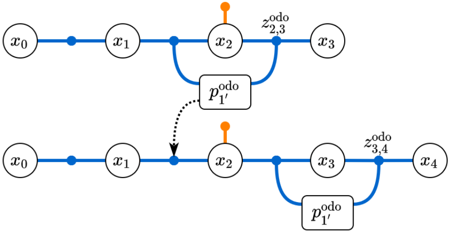

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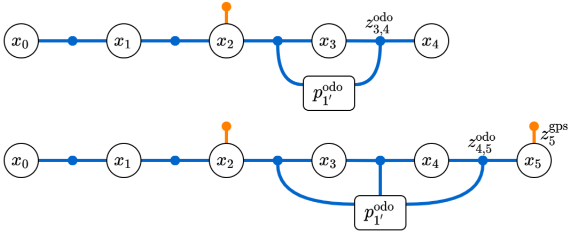

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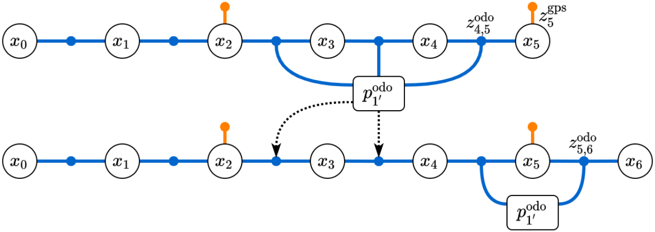

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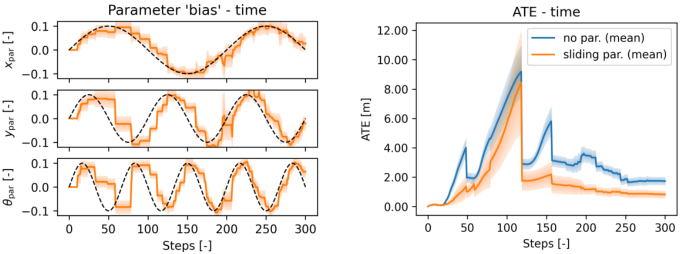

Figure: Sinusoidal bias parameter on Intel data set

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Allocate constraints in time-related batches

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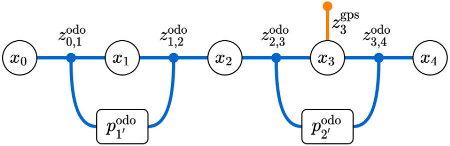

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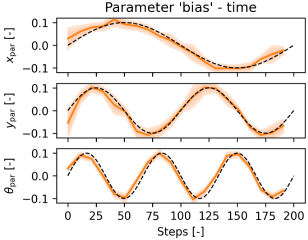

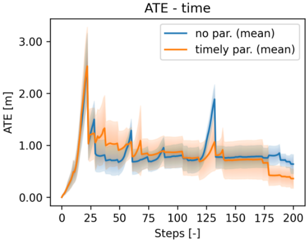

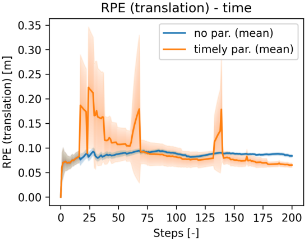

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Spatial batch connectivity strategy

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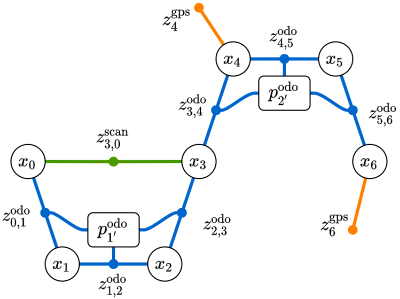

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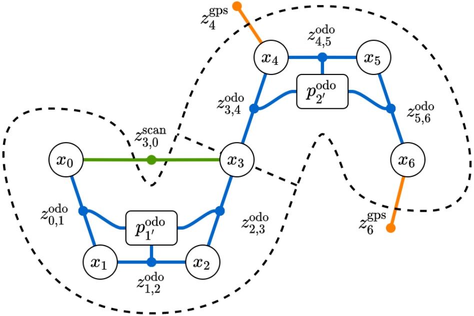

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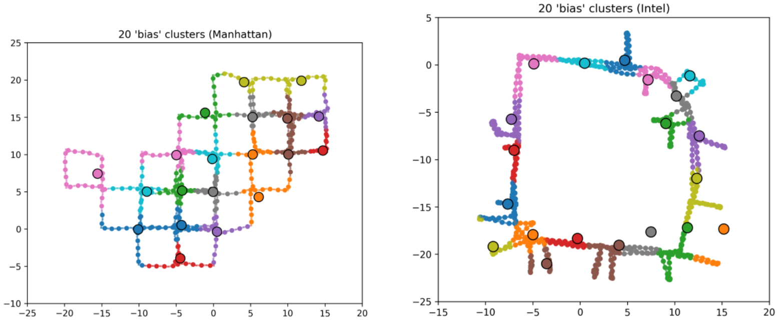

Figure: Constraint-to-batch allocation on Manhattan grid and Intel data set

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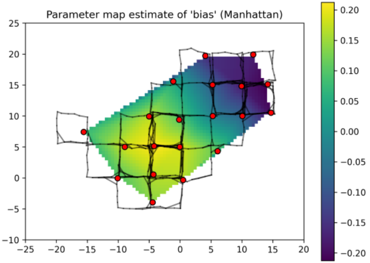

Figure: Sinusoidal parameter interpolated estimate on Manhattan grid

Outline

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Parameter implementations

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Spatial batch connectivity strategy is able to reconstruct space-dependent parameters and improve accuracy

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Solve the under-constrained optimisation problem by relating parameter-nodes with random-walk constraints.

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Investigate combinations of parameters

Investigate e.g. the combination of bias and scaling factor

Questions?