Exercises

In [1]:
```
import numpy as np
# Exercies

xy = [
     ((3, 2), +1), ((1, 1), -1), ((4, 2), -1)
]
th = (1,1)
th0 =-4

for xi, yi in xy:
    print(xi,yi, yi* ( xi[0] * th[0] + xi[1] * th[1] + th0) / (th[0]**2+t
```

```
(3, 2) 1 0.7071067811865475
(1, 1) -1 1.414213562373095
(4, 2) -1 -1.414213562373095
```

In [ ]:
```
# 0.7071067811865475, 1.414213562373095, -1.414213562373095
```

In [7]:
```
data = np.array([[1, 1, 3, 3],[3, 1, 4, 2]])
labels = np.array([[-1, -1, 1, 1]])
th = np.array([[0, 1]]).T
th0 = -3

labels * (th.T @ data + th0) / np.linalg.norm(th)
```

Out[7]:
```
array([[ 0.,  2.,  1., -1.]])
```

Lab

1)

In [ ]:
```

```

```
(2 x + 3) ** 2
2 * ( 2x + 3) * 2 = 0
x = -3/2

|1 - 8*e|  < 1
-1 < 1 - 8*e < 1
-2 < -8*e < 0
1/4 > e > 0

1 - 8*e = 0
1 - 8/8 = 0

x(k)+3/2=(1-8η)**k(x(0)+3/2).

1 - 8 * 10 / 100
(100 - 8 * 10 ) ^ k / 100 ^ k

1 - 8 * 11 / 100
1 - 8 * 12 / 100

x = 10..15
xk + 3/2 = (x0 + 3/2) * (1 - 8*x/100)^k
```

<u>Week 4 . 8 lab</u>

1/7/22-6
ML, MIT 6.036

1D) $x^{(k+1)} = x^{(k)} - \eta \nabla_x f(x^{(k)})$

$z^{(k+1)} = \alpha z^{(k)}$

$z = x - C$

$x^{(k+1)} - C = \alpha(x^{(k)} - C)$

$x^{(k+1)} = \alpha x^{(k)} + C(1-\alpha)$

$- \quad f(x) = (2x+3)^2 \qquad \nabla_x f^2 = 2(2x+3)\cdot 2$

$x^{(k+1)} + \frac{3}{2} = (1-8\eta)(x^{(k)} + \frac{3}{2})$

$x^{(k+1)} = x^{(k)} - \eta \cdot 4(2x^{(k)}+3)$
$\qquad 8(x^{(k)} + \frac{3}{2})$

$+\frac{3}{2} - 12\eta$
12

$\frac{3}{2} + x^{(k+1)} = x^{(k)} - 8\eta(x^{(k)} + \frac{3}{2}) = x^{(k)}(1-8\eta) - 4\cdot 3\eta + \frac{3}{2} =$

$- \frac{3}{2}\cdot 12\eta$

$\frac{3}{2} - 12\eta = \frac{3}{2}(1-8\eta)$

$= (1-8\eta)\left(x^{(k)} + \frac{3}{2}\right)$

$\underbrace{x^{(k)} + \frac{3}{2}}_{z^{(k)}} = \underbrace{(1-8\eta)^k}_{\alpha} \cdot \underbrace{(x^{(0)} + \frac{3}{2})}_{z^{(0)}}$

$\alpha > 1 : z \to \infty$
$\alpha = 1 : z^{(k)} = z^{(0)}$
$1 > \alpha > 0 ; \ z \to 0$
$\alpha = 0 :$
$0 > \alpha > -1 : \pm z \to 0$
$\alpha = -1 : z^{(k)} = \pm z^{(0)}$
$-1 > \alpha : z \to \pm\infty$

1.H) $\eta = 0,1$

$\lim_{k\to\infty} (1 - 8 \cdot 0,1)^k \qquad \left(1 - \frac{80}{100}\right)^k = \left(\frac{20}{100}\right)^k$

$\eta = 0,11 \Rightarrow \left(\frac{12}{100}\right)^k \qquad \eta = 0,12 \Rightarrow \left(\frac{4}{100}\right)^k$

$\eta = 0{,}13 \Rightarrow \left(\frac{-4}{100}\right)^k$ $\eta = 0{,}14 \Rightarrow \left(\frac{-12}{100}\right)^k$ $\left(\frac{4}{100} \frac{100}{\cdot 20}\right)^k$!

$\eta = 0{,}15 \Rightarrow \left(\frac{-20}{100}\right)^k$

2) Where to meet?

> 2A) Pose this problem as an (unconstrained) optimization problem. Assume there are nnn friends and the iii-th friend is located at lil_ili. Denote the location of the party by ppp. What is the objective as a function of ppp? Write it down.

```
l_i - location of i-th friend
p - party location
```

$$J = \sum_{i=1}^{n} (e_i - p)^2$$

$$\nabla J = \begin{bmatrix} 2(e_1 - p) \cdot 1 \\ \vdots \\ 2(e_n - p) \end{bmatrix}$$

> 2B) Compute the gradient (write down/show your computation). Where is it zero?

$$\nabla J = 0 \Rightarrow \begin{matrix} e_1 = p \\ \vdots \\ e_n = p \end{matrix}$$