MIT 6.036 Spring 2019: Homework 3

This colab notebook provides code and a framework for problems 1-7 of the homework. You can work out your solutions here, then submit your results back on the homework page when ready.

Setup

help(tidy plot)

First, download the code distribution for this homework that contains test cases and helper functions.

Run the next code block to download and import the code for this lab.

```
In [1]:
        ! pwd
        /home/art/mydir/ref/Учеба конспекты решения/mit 6.036 intro to ml
In [2]: #!rm -rf code and data for hw3*
        #!rm -rf mnist
        #!wget --quiet https://introml oll.odl.mit.edu/6.036/static/homework/hw03/code and data for hw3.zip
        #!unzip code and data for hw3.zip
        \#!mv code and data for hw3/* .
        from code for hw3 part1 import *
        import code for hw3 part2 as hw3
        Importing code for hw03
        Imported tidy plot, plot separator, plot data, plot nonlin sep, cv, rv, y, positive, score
        Datasets: super simple separable through origin(), super simple separable(), xor(), xor more()
        Tests for part 2: test linear classifier with features, mul, make polynomial feature fun,
                          test with features
        Also loaded: perceptron, one hot internal, test one hot
        Importing code for hw03 (part 2, imported as hw3)
        Imported tidy plot, plot separator, plot data, plot nonlin sep, cv, rv, y, positive, score
                 xval learning alg, eval classifier
        Tests: test linear classifier
        Dataset tools: load auto data, std vals, standard, raw, one hot, auto data and labels
                       load review data, clean, extract words, bag of words, extract bow feature vectors
                       load mnist data, load mnist single
```

```
Help on function tidy_plot in module code_for_hw3_part1:
tidy plot(xmin, xmax, ymin, ymax, center=False, title=None, xlabel=None, ylabel=None)
```

Feature Transformation

Running Perceptron

In problems 1,2 and 3, you will have to run the Perceptron algorithm several times to obtain linear classifiers. We provide you with an implementation of the algorithm which you can use to obtain your results.

The specifications for the percept ron method provided are:

- data is a numpy array of dimension \$d\$ by \$n\$
- labels is numpy array of dimension \$1\$ by \$n\$
- params is a dictionary specifying extra parameters to this algorithm; your algorithm runs a number of iterations equal to \$T\$
- hook is either None or a function that takes the tuple (th, th0) as an argument and displays the separator graphically.

It should return a tuple of \$\theta\$ (a \$d\$ by 1 array) and \$\theta 0\$ (a 1 by 1 array).

Note that you are free to modify the method. For example, a useful modification for this homework would be to make the method return the number of mistakes made on the input data, while it runs.

```
In [4]: # Perceptron algorithm with offset.
        # data is dimension d by n
        # labels is dimension 1 by n
        # T is a positive integer number of steps to run
        def perceptron(data, labels, params = {}, hook = None):
            # if T not in params, default to 100
            T = params.get('T', 50)
            (d, n) = data.shape
            theta = np.zeros((d, 1)); theta 0 = np.zeros((1, 1))
            for t in range(T):
                for i in range(n):
                    x = data[:,i:i+1]
                    y = labels[:,i:i+1]
                    if y * positive(x, theta, theta 0) \leftarrow 0.0:
                         theta = theta + y * x
                         theta 0 = theta 0 + y
                         if hook: hook((theta, theta 0))
            return theta, theta 0
        def averaged perceptron(data, labels, params = {}, hook = None):
            T = params.get('T', 50)
            (d, n) = data.shape
            theta = np.zeros((d, 1)); theta 0 = np.zeros((1, 1))
            theta sum = theta.copy()
            theta 0 sum = theta 0.copy()
            for t in range(T):
                for i in range(n):
                    x = data[:,i:i+1]
                    y = labels[:,i:i+1]
                     if y * positive(x, theta, theta 0) <= 0.0:</pre>
                         theta = theta + y * x
                         theta 0 = theta 0 + y
                         if hook: hook((theta, theta 0))
                    theta sum = theta sum + theta
                    theta 0 sum = theta 0 sum + theta 0
            theta avg = theta sum / (T*n)
            theta 0 avg = theta 0 sum / (T*n)
            if hook: hook((theta avg, theta 0 avg))
            return theta avg, theta 0 avg
        def eval classifier(learner, data train, labels train, data test, labels test):
            th, th0 = learner(data train, labels train)
            return score(data test, labels test, th, th0)/data test.shape[1]
        def positive(x, th, th0):
```

```
return np.sign(th.T@x + th0)
def score(data, labels, th, th0):
   return np.sum(positive(data, th, th0) == labels)
def xval learning alg(learner, data, labels, k):
   _{n} = data.shape
   idx = list(range(n))
   np.random.seed(0)
   np.random.shuffle(idx)
   data, labels = data[:,idx], labels[:,idx]
   score sum = 0
   s data = np.array split(data, k, axis=1)
   s labels = np.array split(labels, k, axis=1)
   for i in range(k):
       data train = np.concatenate(s data[:i] + s data[i+1:], axis=1)
       labels train = np.concatenate(s labels[:i] + s labels[i+1:], axis=1)
       data test = np.array(s data[i])
       labels test = np.array(s labels[i])
       score sum += eval classifier(learner, data train, labels train,
                                              data test, labels test)
   return score sum/k
```

```
In [ ]: perceptron(data, labels, params = {'T':100}, hook = None)
```

```
In [41]: def my perceptron(data, labels, params={}, hook=None):
             # if T not in params, default to 100
             T = params.get('T', 100)
             d = data.shape[0]
             n = data.shape[1]
             theta = np.transpose([[0.0] * d])
             theta 0 = 0.0
             \#ax = plot data(data, labels)
             num = 0
             for test in range(T):
                 founderror = False
                 for i in range(n):
                     xi = data[:,i:i+1]
                     yi = labels[0, i]
                     if yi * np.sign(theta.T @ xi + theta 0) <= 0:</pre>
                          num += 1
                          founderror = True
                         theta += yi * xi
                         theta 0 += yi
                         if hook:
                              hook(theta, theta 0)
                 if not founderror:
                      break
             #plot separator(ax=ax, th=theta, th 0=theta 0)
             return (theta, np.array([[theta 0]]), num)
In [62]: data = np.array([
             [200, 800, 200, 800],
             [0.2, 0.2, 0.8, 0.8],
             [1, 1, 1, 1]
         ])
         labels = np.array([[-1, -1, 1, 1]])
         th = np.array([[0, 1, -0.5]])
         gamma = np.abs(np.min(labels.T * (th @ data) / np.linalg.norm(th) ))
         r = np.max(np.linalg.norm(data, axis=0))
         r, gamma, (r / gamma)**2
         (800.0010249993434, 0.2683281572999748, 8888911.666666666)
Out[62]:
In [43]: data = np.array([
             [200, 800, 200, 800],
             [0.2, 0.2, 0.8, 0.8]
         ])
         labels = np.array([[-1, -1, 1, 1]])
```

```
In [44]: perceptron(data, labels, params={'T':1000})
         (array([[ 0.],
Out[44]:
                 [600.]]),
          array([[0.]]),
          2000)
         perceptron(data, labels, params={'T':10000})
In [45]:
         (array([[ 0.],
Out[45]:
                 [6000.]]),
          array([[0.]]),
          20000)
         perceptron(data, labels, params={'T':100000})
In [46]:
         (array([[ 600.
Out[46]:
                 [69997.80000031]]),
          array([[0.]]),
          233326)
         perceptron(data, labels, params={'T':1000000})
         (array([[-2.000000e+02],
Out[47]:
                 [ 2.000068e+05]]),
          array([[-4.]]),
          666696)
         data = np.array([
In [63]:
             [200, 800, 200, 800],
             [0.2, 0.2, 0.8, 0.8],
             [1, 1, 1, 1]
         ])
         data[0:2] *= 0.001
         labels = np.array([[-1, -1, 1, 1]])
         th = np.array([[0, 1, -0.0005]])
         gamma = np.abs(np.min(labels.T * (th @ data) / np.linalg.norm(th) ))
         data, gamma
         r = np.max(np.linalg.norm(data, axis=0))
         r, gamma, (r / gamma)**2
         (1.2806250973645645, 0.00029999996250000706, 18222233.88889067)
Out[63]:
```

```
In [64]: data = np.array([
             [200, 800, 200, 800],
             [0.2, 0.2, 0.8, 0.8],
             [1, 1, 1, 1]
         1)
         data[0:1] *= 0.001
         labels = np.array([[-1, -1, 1, 1]])
         th = np.array([[0, 1, -0.5]])
         gamma = np.abs(np.min(labels.T * (th @ data) / np.linalg.norm(th) ))
         data, gamma
         r = np.max(np.linalg.norm(data, axis=0))
         r, gamma, (r / gamma)**2
         (1.50996688705415, 0.2683281572999748, 31.666666666666664)
Out[641:
         perceptron(data, labels, params={'T':1000000})
In [65]:
         (array([[-0.2],
Out [651:
                 [ 2.8],
                 [-1.]]),
          array([[-1.]]),
          11)
```

2D) Encoding Discrete Values

It is common to encode sets of discrete values, for machine learning, not as a single multi-valued feature, but using a one hot encoding. So, if there are \$k\$ values in the discrete set, we would transform that single multi-valued feature into \$k\$ binary-valued features, in which feature \$i\$ has value \$+1\$ if the original feature value was \$i\$ and has value \$0\$ (or \$-1\$) otherwise.

Write a function one_hot that takes as input \$x\$, a single feature value (between \$1\$ and \$k\$), and \$k\$, the total possible number of values this feature can take on, and transform it to a numpy column vector of \$k\$ binary features using a one-hot encoding (remember vectors have zero-based indexing).

```
In [71]: data = np.array([[2, 3, 4, 5]])
    labels = np.array([[1, 1, -1, -1]])
    th, th0 = perceptron(data, labels)

th, th0, labels * (th @ data + th0), th @ np.array([[1, 6]]) + th0

Out[71]: (array([[-2.]]), array([[7.]]), array([[3., 1., 1., 3.]]), array([[ 5., -5.]]))
```

```
In [84]: a = np.zeros((1, 10))
         a[:, 1] = 1
         а
         array([[0., 1., 0., 0., 0., 0., 0., 0., 0., 0.]])
Out[84]:
 In [8]: def one hot(x, k):
             a = np.zeros((k, 1))
             a[x-1,0] = 1.0
             return a
 In [9]: test one hot(one hot)
         Passed!
In [16]: data =
                  [[2, 3, 4, 5]]
         data[0]
Out[16]: [2, 3, 4, 5]
In [47]: data = np.array([[2, 3, 4, 5]])
         labels = np.array([[1, 1, -1, -1]])
         data = np.concatenate(
             [one hot(e, 6) for e in data[0]],
             axis=1
         data, labels
         (array([[0., 0., 0., 0.],
Out[47]:
                 [1., 0., 0., 0.],
                 [0., 1., 0., 0.],
                 [0., 0., 1., 0.],
                 [0., 0., 0., 1.],
                 [0., 0., 0., 0.]]),
          array([[ 1, 1, -1, -1]]))
In [49]: th, th0 = perceptron(data, labels)
         th.T, th0
         (array([[ 0., 2., 1., -2., -1., 0.]]), array([[0.]]))
Out[49]:
In [71]: samsung = one hot(1, 6)
         nokia = one hot(6, 6)
         samsung, nokia, th.T
```

```
(array([[1.],
Out[71]:
                 [0.],
                 [0.],
                 [0.],
                 [0.],
                 [0.]]),
          array([[0.],
                 [0.],
                 [0.],
                 [0.],
                 [0.],
                 [1.]]),
          array([[ 0., 2., 1., -2., -1., 0.]]))
In [70]: [ th.T @ xi + th0 for xi in (samsung, nokia)]
         [array([[0.]]), array([[0.]])]
Out[70]:
         [ (th.T @ xi + th0) / np.linalg.norm(th) for xi in (samsung, nokia)]
In [72]:
         [array([[0.]]), array([[0.]])]
Out[72]:
In [75]:
                  [[1, 2, 3, 4, 5, 6]]
         data =
         labels = np.array([[1, 1, -1, -1, 1, 1]])
         data = np.concatenate(
             [one hot(e, 6) for e in data[0]],
             axis=1
         data, labels
         (array([[1., 0., 0., 0., 0., 0.],
Out[75]:
                 [0., 1., 0., 0., 0., 0.],
                 [0., 0., 1., 0., 0., 0.],
                 [0., 0., 0., 1., 0., 0.],
                 [0., 0., 0., 0., 1., 0.],
                 [0., 0., 0., 0., 0., 1.]]),
          array([[ 1, 1, -1, -1, 1, 1]]))
In [76]: th, th0 = perceptron(data, labels)
         th.T, th0
         (array([[ 1., 1., -2., -2., 1., 1.]]), array([[0.]]))
Out[76]:
```

3) Polynomial Features

[0.]]

One systematic way of generating non-linear transformations of your input features is to consider the polynomials of increasing order. Given a feature vector $x = [x_1, x_2, ..., x_d]^T$, we can map it into a new feature vector that contains all the factors in a polynomial of order \$d\$. For example, for $x = [x_1, x_2, ..., x_d]^T$ and order 2, we get \$\phi(x) = [1, x_1, x_2, x_1x_2, x_1^2, x_2^2]^T\$ and for order 3, we get \$\phi(x) = [1, x_1, x_2, x_1^2, x_2^2, x_1^2, x_1^2, x_2^2, x_1^2, x_1^2, x_2^2, x_1^2, x_1^2, x_2^2, x_1^2, x_1

In the code that has been loaded, we have defined make_polynomial_feature_fun that, given the order, returns a feature transformation function (analogous to \$\phi\$ in the description). You should use it in doing this problem.

```
In [77]: ## For example, make polynomial feature fun could be used as follows:
         import numpy as np
         # Data
         data = np.zeros((5,1))
         # Generate transformation of order 2
         transformation = make polynomial feature fun(2)
         # Use transformation on data
         print(transformation(data))
         [[1.]
          [0.]
          [0.]
           [0.]
          [0.]
          [0.]
          [0.]
          [0.]
          [0.]
          [0.]
           [0.]
           [0.]
          [0.]
          [0.]
          [0.]
           [0.]
           [0.]
           [0.]
          [0.]
          [0.]
```

```
In [79]: # Enter a list of 6 integers indicating the number of polynomial features of degrees [1, 10, 20, 30, 40, 50] for a 2-dimension
         \# (x1, x2)
         # 1: 2
         # 10:
         # 1, 1
         # 2 11 2
         # 3 21 12 3
         # 4 31 22 13 4
         # 5 41 32 23 14 5
         # 6 51 42 33 24 15 6
         # 10 91 82 73 64 55 46 37 28 19 10
              1 + 2 + 3 + 4 + \dots
         # 20: 2 + ... + 21 = 23 * 20 / 2 = 230
         # 30: 2 + .. + 31 = 33 * 30 / 2 = 330 + 165 = 495
         # 40: 43 * 40 / 2 = 860
         # 50: 53 * 50 / 2 = 5300 / 4 = 1325
         # 2, 65, 230, 495, 860, 1325; +1
         [1 + (n+3) * n // 2  for n  in (10, 20, 30, 40, 50)]
```

Out[79]: [66, 231, 496, 861, 1326]

Note that iterative animations, which update a plot within a loop, don't work the same way in colab, as with a local python console installation. One workaround for colab to be able to show such plot iterations is to show all the plots, and this can be done for the test code using this patched function:

```
In [162... def test linear classifier with features(dataFun, learner, feature fun,
                                       draw = True, refresh = True, pause = True):
              raw data, labels = dataFun()
              data = feature fun(raw data) if feature fun else raw data
              if draw:
                  def hook(params):
                      ax = plot data(raw data, labels) # create plot axis on each iteration
                      (th, th0) = params
                      predictor = lambda \times 1, \times 2: int(positive(feature fun(cv([x1, x2])), th, th0))
                      plot nonlin sep(
                          predictor,
                          ax = ax
                      plot data(raw data, labels, ax)
                      plt.show()
                                                          # force plot to push to the colab notebook and be displayed
                      print('th', th.T, 'th0', th0)
                      if pause: input('press enter here to continue:')
              else:
                  hook = None
              th, th0 = learner(data, labels, hook = hook)
              if hook: hook((th, th0))
              print("Final score", int(score(data, labels, th, th0)))
              print("Params", np.transpose(th), th0)
          def test with features(dataFun, order = 2, draw=True, pause=True, learner=perceptron):
              test linear classifier with features(
                  dataFun,
                                                   # data
                  learner,
                                                # learner
                  make polynomial feature fun(order), # feature maker
                  draw=draw,
                  pause=pause)
```

Here's a test you can run to see plots:

```
In [163... def perceptron_with_params(T=100):
    myparams = {'T': T}
    def f(data, labels, params = {}, hook = None):
        return perceptron(data, labels, myparams, hook)
    return f
In [144... print(super_simple_separable_through_origin()[0].shape)
test with features(super simple separable through origin, order=2, draw=False, pause=False)
```

```
(2, 4)
Final score 4
Params [[ 2. 4. 17. -46. 59. 107.]] [[2.]]
```

```
In [143... print(super simple separable()[0].shape)
         test with features(super simple separable, order=2, draw=False, pause=False, learner=perceptron with params(T=1000))
         (2, 4)
         Final score 4
         Params [[ -11. -26. 11. -190. 140. 235.]] [[-11.]]
         print(xor()[0].shape)
In [115...
         test with features (xor, order=2, draw=False, pause=False, learner=perceptron with params(T=1000))
         (2, 4)
         Final score 4
         Params [[ 1. -1. -1. -5. 11. -5.]] [[1.]]
         print(xor more())
In [165...
         test with features (xor more, order=3, draw=False, pause=False, learner=perceptron with params(T=10000))
         (array([[1, 2, 1, 2, 2, 4, 1, 3],
                [1, 2, 2, 1, 3, 1, 3, 3]]), array([[ 1, 1, -1, -1, 1, 1, -1, -1]]))
         2202
         Final score 8
                                     72. 248. -19. 76. -522. 476. -153.]] [[-78.]]
         Params [[ -78.
                          28. -39.
```

We know that a better way to do this exists (eg using colab plot animations) - if you are willing to contribute some nice code which lets our plotting functions do this, please do share!

Experiments

4) Evaluating algorithmic and feature choices for AUTO data

We now want to build a classifier for the auto data, focusing on the numeric data. In the code file for this part of the assignment, we have supplied you with the load_auto_data function, that can be used to read the relevant .tsv file. It will return a list of dictionaries, one for each data item.

We then have to specify what feature function to use for each column in the data. The file hw3_part2_main.py has an example for constructing the data and label arrays using raw feature function for all the columns. Look at the definition of features in hw3_part2_main.py, this indicates a feature name to use and then a feature function, there are three defined in the code_for_hw3_part2.py file (raw, standard and one_hot). raw just uses the original value, standard subtracts out the mean value and divides by the standard deviation and one_hot does the encoding described in the notes.

The function auto_data_and_labels will process the dictionaries and return data, labels where data are arrays of dimension \$(d, 392)\$, with \$d\$ the total number of features specified, and labels is of dimension \$(1, 392)\$. The data in the file is sorted by class, but it will be shuffled when you read it in.

```
In [166... # Returns a list of dictionaries. Keys are the column names, including mpg.
         auto data all = hw3.load auto data('auto-mpg.tsv')
         # The choice of feature processing for each feature, mpg is always raw and
         # does not need to be specified. Other choices are hw3.standard and hw3.one hot.
         # 'name' is not numeric and would need a different encoding.
         features = [('cylinders', hw3.raw),
                     ('displacement', hw3.raw),
                      ('horsepower', hw3.raw),
                      ('weight', hw3.raw),
                      ('acceleration', hw3.raw),
                     ## Drop model year by default
                     ## ('model year', hw3.raw),
                      ('origin', hw3.raw)]
         # Construct the standard data and label arrays
         auto data, auto labels = hw3.auto data and labels(auto data all, features)
         print('auto data and labels shape', auto data.shape, auto labels.shape)
         avg and std {}
         entries in one hot field {}
         auto data and labels shape (6, 392) (1, 392)
 In [ ]:
In [172... hw3.xval learning alg(
             lambda data, labels: perceptron(data, labels, {"T": 1}),
             auto data,
             auto labels,
              10)
         97
         97
         95
         90
         98
         94
         99
         97
         92
         94
          0.6526282051282052
Out[1721:
```

```
In [173...
        hw3.xval learning alg(
             lambda data, labels: averaged perceptron(data, labels, {"T": 1}),
             auto data.
             auto labels,
             10)
          0.8441025641025641
Out[173]:
In [174... | features2 = [('cylinders', hw3.one hot),
                     ('displacement', hw3.standard),
                     ('horsepower', hw3.standard),
                     ('weight', hw3.standard),
                     ('acceleration', hw3.standard),
                     ## Drop model year by default
                     ## ('model year', hw3.raw),
                     ('origin', hw3.one hot)]
         # Construct the standard data and label arrays
         auto data2, auto labels2 = hw3.auto data and labels(auto data all, features2)
         print('auto data and labels shape', auto data2.shape, auto labels2.shape)
         avg and std {'displacement': (388.3482142857143, 302.0458123396403), 'horsepower': (509.3545918367347, 333.6521151716361), 'wei
         ght': (2977.5841836734694, 848.3184465698365), 'acceleration': (15.541326530612228, 2.7553429127509963)}
         entries in one hot field {'cylinders': [3.0, 4.0, 5.0, 6.0, 8.0], 'origin': [1.0, 2.0, 3.0]}
         auto data and labels shape (12, 392) (1, 392)
         hw3.xval learning alg(
In [188...
             lambda data, labels: perceptron(data, labels, {"T": 1}),
             auto data2,
             auto labels2,
             10), hw3.xval learning alg(
             lambda data, labels: averaged perceptron(data, labels, {"T": 1}),
             auto data2,
             auto labels2,
             10)
          Out[188]:
In [182...
         hw3.xval learning alg(
             lambda data, labels: perceptron(data, labels, {"T": 10}),
             auto data,
             auto labels,
             10), hw3.xval learning alg(
             lambda data, labels: averaged perceptron(data, labels, {"T": 10}),
             auto data,
             auto labels,
             10)
```

```
(0.7423076923076924, 0.8366025641025641)
Out[182]:
In [177... hw3.xval learning alg(
             lambda data, labels: perceptron(data, labels, {"T": 10}),
              auto data2,
             auto labels2,
             10), hw3.xval learning alg(
             lambda data, labels: averaged perceptron(data, labels, {"T": 10}),
             auto data2,
             auto labels2,
              10)
         545
         540
         563
         529
         546
         540
         531
         495
         532
         547
          (0.8061538461538461, 0.8979487179487181)
Out[177]:
In [183... hw3.xval learning alg(
             lambda data, labels: perceptron(data, labels, {"T": 50}),
             auto data,
             auto labels,
             10), hw3.xval learning alg(
             lambda data, labels: averaged perceptron(data, labels, {"T": 50}),
             auto data,
             auto labels,
              10)
          (0.6909615384615384, 0.8366025641025641)
Out[183]:
In [187... hw3.xval learning alg(
             lambda data, labels: perceptron(data, labels, {"T": 50}),
              auto data2,
             auto labels2,
             10), hw3.xval learning alg(
             lambda data, labels: averaged perceptron(data, labels, {"T": 50}),
             auto data2,
             auto labels2,
             10)
```

```
(0.8060256410256409, 0.9005128205128207)
Out[187]:
In [189... th, th0 = averaged perceptron(auto data2, auto labels2, params={'T':50})
          th, th0
          (array([[-1.98173469],
Out[189]:
                   [ 0.34622449],
                   [ 0.51530612],
                   [-0.95596939],
                   [ 2.80469388],
                   [-1.46206452],
                   [ 0.27203955],
                   [-6.55860703],
                   [ 0.83288456],
                   [-0.10352041],
                   [ 1.1647449 ],
                   [-0.33270408]]),
           array([[0.72852041]]))
```

5) Evaluating algorithmic and feature choices for review data

We have supplied you with the load_review_data function, that can be used to read a .tsv file and return the labels and texts. We have also supplied you with the bag_of_words function, which takes the raw data and returns a dictionary of unigram words. The resulting dictionary is an input to extract_bow_feature_vectors which computes a feature matrix of ones and zeros that can be used as the input for the classification algorithms. The file hw3_part2_main.py has code for constructing the data and label arrays. Using these arrays and our implementation of the learning algorithms, you will be able to compute \$\theta\$ and \$\theta_0\$. You will need to add your (or the one written by staff) implementation of perceptron and averaged perceptron.

```
In [5]: # Returns lists of dictionaries. Keys are the column names, 'sentiment' and 'text'.
# The train data has 10,000 examples
review_data = hw3.load_review_data('reviews.tsv')

# Lists texts of reviews and list of labels (1 or -1)
review_texts, review_label_list = zip(*((sample['text'], sample['sentiment']) for sample in review_data))

# The dictionary of all the words for "bag of words"
dictionary = hw3.bag_of_words(review_texts)

# The standard data arrays for the bag of words
review_bow_data = hw3.extract_bow_feature_vectors(review_texts, dictionary)
review_labels = hw3.rv(review_label_list)
print('review_bow_data and labels shape', review_bow_data.shape, review_labels.shape)
review bow data and labels shape (19945, 10000) (1, 10000)
```

```
In [6]: import time
In [7]: for T in (1, 10, 50):
             print('T', T)
             start = time.time()
             print('P', xval learning alg(
                 lambda data, labels: perceptron(data, labels, {"T": T}),
                 review bow data, review labels, 10))
             print(time.time() - start)
             start = time.time()
             print('AP', xval learning alg(
                 lambda data, labels: averaged perceptron(data, labels, {"T": T}),
                 review bow data, review labels, 10))
             print(time.time() - start)
         T 1
         P 0.7672000000000001
         7.181673765182495
         AP 0.812099999999998
         7.737758159637451
         T 10
         P 0.7871
         31.119680643081665
         AP 0.8237
         38.83753275871277
         T 50
         P 0.8036
         130.36860251426697
         AP 0.8157
         179.88108468055725
In [14]: th, th0 = averaged perceptron( review bow data, review labels, {"T": 10})
         th, th0
         (array([[ 0.15984],
Out[14]:
                 [-2.74048],
                 [-1.23668],
                 . . . ,
                 [ 0.
                       ],
                 [-1.2001],
                         ]]),
                 [ 0.
          array([[-1.72795]]))
In [16]: mysorted = sorted((e, i) for i,e in enumerate(th))
         rdict = hw3.reverse dict(dictionary)
         [rdict[i] for (e, i) in mysorted[:10]], [rdict[i] for (e, i) in mysorted[-10:]]
```

```
(['worst',
Out[16]:
            'awful'.
            'unfortunately',
            'horrible',
            'stuck',
            'changed',
            'disappointment',
            'bland',
            'poor',
            'formula'],
           ['great',
            'individually',
            'bright',
            'yummy',
            'skeptical',
            'perfect',
            'easily',
            'satisfied',
            'delicious',
            'excellent'])
```

6) Evaluating features for MNIST data

This problem explores how well the perceptron algorithm works to classify images of handwritten digits, from the well-known ("MNIST") dataset, building on your thoughts from lab about extracting features from images. This exercise will highlight how important feature extraction is, before linear classification is done, using algorithms such as the perceptron.

Dataset setup

Often, it may be easier to work with a vector whose spatial orientation is preserved. In previous parts, we have represented features as one long feature vector. For images, however, we often represent a \$m\$ by \$n\$ image as a (m,n) array, rather than a (mn,1) array (as the previous parts have done).

In the code file, we have supplied you with the load_mnist_data function, which will read from the provided image files and populate a dictionary, with image and label vectors for each numerical digit from 0 to 9. These images are already shaped as (m,n) arrays.

```
In [55]: mnist data all = hw3.load mnist data(range(10))
         print('mnist data all loaded. shape of single images is', mnist data all[0]["images"][0].shape)
         # HINT: change the [0] and [1] if you want to access different images
         def get data lables(leftd, rightd):
             d0 = mnist data all[leftd]["images"]
             d1 = mnist data all[rightd]["images"]
             y0 = np.repeat(-1, len(d0)).reshape(1,-1)
             y1 = np.repeat(1, len(d1)).reshape(1,-1)
             # data goes into the feature computation functions
             data = np.vstack((d0, d1))
             # labels can directly go into the perceptron algorithm
             labels = np.vstack((y0.T, y1.T)).T
             return data, labels
         mnist data all loaded. shape of single images is (28, 28)
In [15]:
         np.array([np.average([[1, 2, 3], [4, 5, 6], [7, 8, 9]], axis=0)]).T
         array([[4.],
Out[15]:
                [5.],
                [6.]])
In [104...] arr = np.arange(24).reshape(2,3,4)
         print(arr.ndim)
         print(arr)
         out = np.array([
             np.apply along axis(
                 lambda a: np.average(a),
                 axis=0,
                 arr=item
             for item
             in arr
         ])
         out
         [[0 1 2 3]
           [4567]
           [ 8 9 10 11]]
          [[12 13 14 15]
           [16 17 18 19]
           [20 21 22 23]]]
          array([[ 4., 5., 6., 7.],
Out[104]:
                 [16., 17., 18., 19.]])
```

```
In [174... # change these implementations to support whole datasets
          def raw mnist features(x):
               \operatorname{Qparam} \times (\operatorname{n samples}, \operatorname{m}, \operatorname{n}) \text{ array with values in } (0,1)
               @return (m*n,n samples) reshaped array where each entry is preserved
              n = x.shape
               return x.reshape((n samples, n*m, 1))
          def row average features(x):
              This should either use or modify your code from the tutor questions.
               \operatorname{Qparam} \times (\operatorname{n samples}, \operatorname{m}, \operatorname{n}) \text{ array with values in } (0,1)
               @return (m,n samples) array where each entry is the average of a row
               return np.apply along axis(
                   lambda a: [np.average(a)],
                   axis=1,
                   arr=x
          def col average features(x):
              This should either use or modify your code from the tutor questions.
               \operatorname{Oparam} \times (n \text{ samples,m,n}) \text{ array with values in } (0,1)
               @return (n,n samples) array where each entry is the average of a column
               return np.apply along axis(
                   lambda a: [np.average(a)],
                   axis=0,
                   arr=x
              ) .T
          def top bottom features(x):
              This should either use or modify your code from the tutor questions.
               (0,1)
               @return (2,n samples) array where the first entry of each column is the average of the
               top half of the image = rows 0 to floor(m/2) [exclusive]
               and the second entry is the average of the bottom half of the image
               = rows floor(m/2) [inclusive] to m
               n, m = x.shape
```

```
return cv([np.average(x[0:n//2,:]), np.average(a=x[n//2:,:])])
In [175... #Your Code Here
          ans=row average features(np.array([[1,2,3],[3,9,2]])).tolist()
          [[2.0], [4.66666666666667]]
Out[175]:
In [176... ans=col average features(np.array([[1,2,3],[3,9,2],[2,1,9]])).tolist()
          ans
          [[2.0], [4.0], [4.666666666666667]]
Out[176]:
In [177... top bottom features(np.arange(12).reshape((3,4))), np.arange(12).reshape((3,4))
          (array([[1.5],
Out[177]:
                  [7.5]]),
           array([[ 0, 1, 2, 3],
                  [4, 5, 6, 7],
                  [8, 9, 10, 11]]))
         import time
In [57]:
         # use this function to evaluate accuracy
         #print( data.shape, raw mnist features(data).shape, raw mnist features(data).T[0].shape )
          for l, r in ((0, 1), (2, 4), (6, 8), (9, 0)):
             start = time.time()
             data, labels = get data lables(l, r)
             raw data = raw mnist features(data).T[0]
             acc = hw3.get classification accuracy(raw data, labels)
             print('Done', time.time() - start)
             print('left', l)
             print('right', r)
             print(acc)
```

```
Done 0.46145176887512207
left 0
right 1
0.975
Done 0.47538232803344727
left 2
right 4
0.864166666666665
Done 0.4565155506134033
left 6
right 8
0.9479166666666667
Done 0.500361442565918
left 9
right 0
0.6470833333333333
```

In []: (0.975, 0.86416666666665, 0.947916666666667, 0.6470833333333333)

```
In [178... data, labels = get_data_lables(0, 1)
          print(data.shape)
          rfd = np.concatenate(
             list(
                  row average features(xi).T
                 for xi
                 in data
          ) . T
          cfd = np.concatenate(
             list(
                  col average features(xi).T
                 for xi
                 in data
          ) . T
          tbfd = np.concatenate(
             list(
                 top bottom features(xi).T
                 for xi
                 in data
          ) .T
          print(rfd.shape)
          print(cfd.shape)
          print(tbfd.shape)
          np.concatenate(
             (rfd, cfd, tbfd),
             axis=0
          ).shape
          (160, 28, 28)
          (28, 160)
          (28, 160)
         (2, 160)
```

Out[178]: (58, 160)

```
for l, r in ((0, 1), (2, 4), (6, 8), (9, 0)):
In [183...
             data, labels = get data lables(l, r)
             rfd = np.concatenate(
                 list(
                      row average features(xi).T
                      for xi
                      in data
             ) .T
             cfd = np.concatenate(
                 list(
                     col average features(xi).T
                     for xi
                      in data
             ) . T
             tbfd = np.concatenate(
                 list(
                     top bottom features(xi).T
                      for xi
                      in data
             ) . T
             res = []
             for fdata in (rfd, cfd, tbfd):
                 acc = hw3.get classification accuracy(fdata, labels)
                 res += [acc]
             print(repr(res))
         [0.48125, 0.6375, 0.48125]
         [0.775416666666666, 0.497499999999994, 0.49749999999999]
         [0.92125, 0.52125, 0.56500000000000001]
         [0.497499999999994, 0.50416666666667, 0.4974999999999994]
         0.48125,
 In [ ]:
         0.6375,
         0.48125
```

6.2F) (Optional) What does it mean if a binary classification accuracy is below 0.5, if your dataset is balanced (same number from each class)? Are these datasets balanced?

Means it is worse than randomly picking up labels.

```
In [188... # 6.2G) (Optional) Feel free to classify other images from each other. Which combinations perform the best, and which perform t
                          res = []
                          for l in range(10):
                                    for r in range(10):
                                               if l == r:
                                                           continue
                                               start = time.time()
                                               data, labels = get data lables(l, r)
                                               raw data = raw mnist features(data).T[0]
                                               acc = hw3.get classification accuracy(raw data, labels)
                                               res += [(acc, l, r)]
                          sres = sorted(res)
                          print(sres[:10])
                          print(sres[-10:])
                          [(0.4825000000000004, 5, 8), (0.4841666666666667, 4, 9), (0.507916666666667, 4, 6), (0.525, 1, 8), (0.542916666666666, 9, 1)
                         5)]
                          [(0.9737500000000001, 9, 2), (0.975, 0, 1), (0.975, 1, 0), (0.975, 9, 1), (0.98083333333333, 7, 5), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.98125, 1, 4), (0.981
                         5, 5, 7), (0.9875, 4, 1), (0.9875, 4, 3), (0.99375, 8, 0)]
In [194... print('Best 10:')
                          print('\n'.join('{{{}} vs {{}}: {{}}'.format(l, r, a) for a,l,r in reversed(sres[-10:])))
                          print('Worst 10:')
                          print('\n'.join('{{}} vs {{}}'.format(l, r, a) for a,l,r in sres[:10]))
```

```
Best 10:
8 vs 0: 0.99375
4 vs 3: 0.9875
4 vs 1: 0.9875
5 vs 7: 0.98125
1 vs 4: 0.98125
7 vs 5: 0.9808333333333333
9 vs 1: 0.975
1 vs 0: 0.975
0 vs 1: 0.975
9 vs 2: 0.973750000000001
Worst 10:
5 vs 8: 0.48250000000000004
4 vs 9: 0.484166666666667
4 vs 6: 0.507916666666667
1 vs 8: 0.525
9 vs 8: 0.542916666666666
8 vs 5: 0.5487500000000001
5 vs 3: 0.575
3 vs 5: 0.5758333333333334
7 vs 3: 0.591249999999999
0 vs 5: 0.595416666666667
```