CS 325 HW 6

You may solve the problems using your choice of software, state which software package/language(s) you used and provide the code or spreadsheet. There is no submission to TEACH this week.

1. <u>Shortest Paths using LP</u>: **(7 points)**

Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

  max dt
  subject to
  ds = 0
  dv – du ≤ w(u,v) for all (u,v)∈E

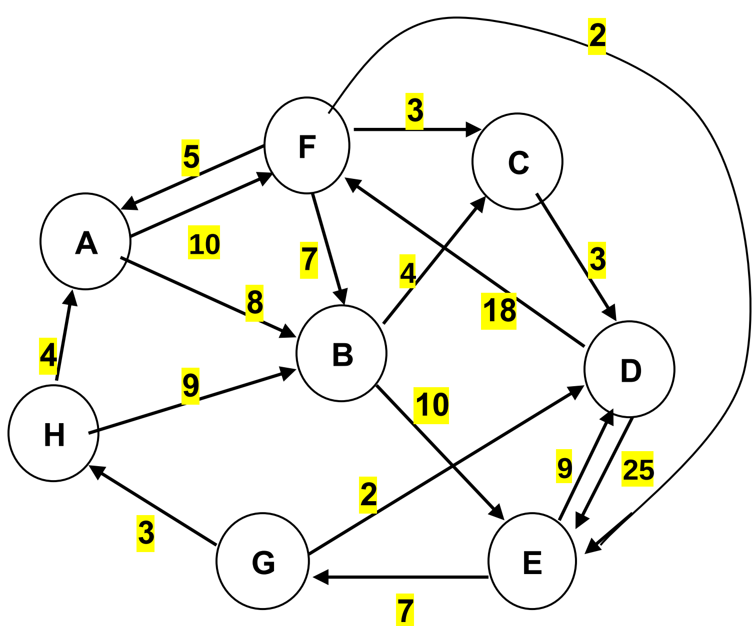
- We can compute the single-source by changing the objective function to

  max $\sum_{v \in V} dv$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

a) Find the distance of the shortest path from G to C in the graph below.

b) Find the distances of the shortest paths from G to all other vertices.



CS 325 HW 6

2. Product Mix: **(7 points)**
Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	**Cost per yard**	**Yards available per month**
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	**Type of Tie**			
	Silk = s	**Poly = p**	**Blend1 = b**	**Blend2 = c**
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	**Type of Tie**			
	Silk	**Polyester**	**Blend 1 (50/50)**	**Blend 2 (30/70)**
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

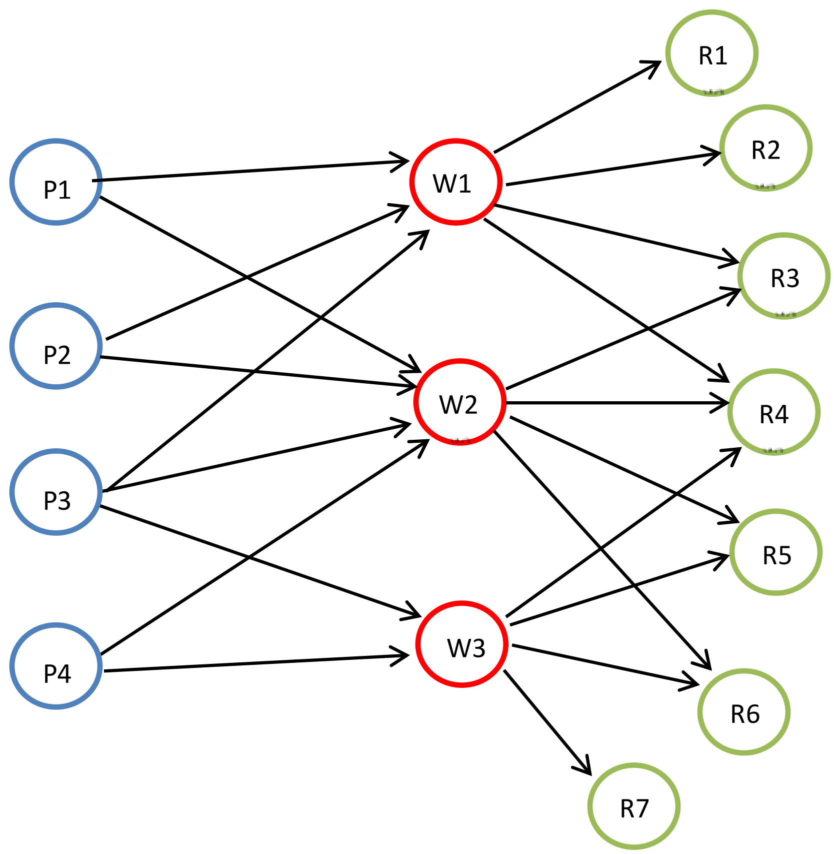
Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

CS 325 HW 6

3. Transshipment Model **(10 points)**

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (p_i) to warehouse (w_j) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (w_j) to a retailer (r_k) have costs associated denoted cw(j,k).

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer. If it is impossible to ship between the two locations an X is placed in the table.

cost	W1	W2	W3
P1	\$10	\$15	X
P2	\$11	\$8	X
P3	\$13	\$8	\$9
P4	X	\$14	\$8

cost	R1	R2	R3	R4	R5	R6	R7
W1	$5	$6	$7	$10	X	X	X
W2	X	X	$12	$8	$10	$14	X
W3	X	X	X	$14	$12	$12	$6

The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Part A: Determine the number of refrigerators to be shipped from the plants to the warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost?

Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?

Part C: Instead of closing Warehouse 2 management has decide to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?

Note: *Include a copy of the code for all parts of the problem.*

4. Making Change**(6 points)**

Given coins of denominations (value) $1 = v_1 < v_2 < \ldots < v_n$, we wish to make change for an amount A using as few coins as possible. Assume that v_i's and A are integers. Since $v_1 = 1$ there will always be a solution. Solve the coin change using integer programming. For each the following denomination sets and amounts formulate the problem as an integer program with an objective function and constraints, determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

a) V = [1, 5, 10, 25] and A = 202.
b) V = [1, 3, 7, 12, 27] and A = 293