CS 325 HW6– Solutions

1. Shortest Paths using LP: **(7 points)** Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

max dt
subject to
    ds = 0
    dv – du $\leq$ w(u,v) for all (u,v) $\in$ E

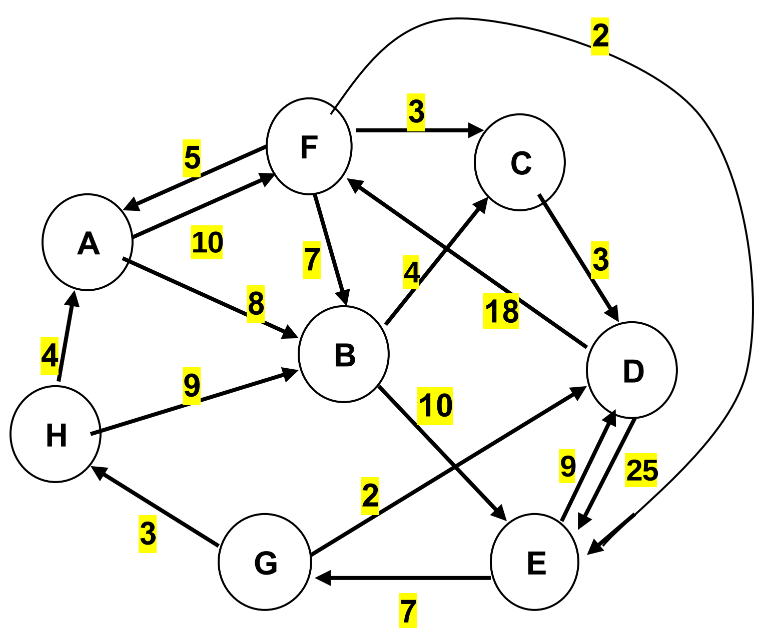
- We can compute the single-source by changing the objective function to

max $\sum_{v \in V} dv$

Use linear programming to answer the questions below. Submit a copy of the LP code and output.

a) Find the distance of the shortest path from G to C in the graph below.

b) Find the distances of the shortest paths from G to all other vertices.



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a) Shortest path from g to c is 16 (3 points)

```
max c
ST
        g = 0
        f - a <= 10
        a - f <= 5
        b - a <= 8
        a - h <= 4
        b - h <= 9
        h - g <= 3
        d - g <= 2
        g - e <= 7
        e - b <= 10
        c - b <= 4
        b - f <= 7
        c - f <= 3
        e - f <= 2
        d - c <= 3
        e - d <= 25
        d - e <= 9
        f - d <= 18
```

```
LP OPTIMUM FOUND AT STEP      6

      OBJECTIVE FUNCTION VALUE

      1)      16.00000

 VARIABLE        VALUE          REDUCED COST
        C        16.000000          0.000000
        G         0.000000          0.000000
        F        13.000000          0.000000
        A         4.000000          0.000000
        B        12.000000          0.000000
        H         3.000000          0.000000
        D         0.000000          0.000000
        E         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         1.000000          0.000000
       4)        14.000000          0.000000
       5)         0.000000          0.000000
       6)         3.000000          0.000000
       7)         0.000000          1.000000
       8)         0.000000          1.000000
       9)         2.000000          0.000000
      10)         7.000000          0.000000
      11)        22.000000          0.000000
      12)         0.000000          1.000000
      13)         8.000000          0.000000
      14)         0.000000          0.000000
      15)        15.000000          0.000000
      16)        19.000000          0.000000
      17)        25.000000          0.000000
      18)         9.000000          0.000000
      19)         5.000000          0.000000

NO. ITERATIONS=       6
```

b) Find the distances of the shortest paths from G to all other vertices. (4 points)

A	B	C	D	E	F	H
7	12	16	2	19	17	3

```
max a + b + c + d + e + f + h
ST
        g = 0
        f - a <= 10
        a - f <= 5
        b - a <= 8
        a - h <= 4
        b - h <= 9
        h - g <= 3
        d - g <= 2
        g - e <= 7
        e - b <= 10
        c - b <= 4
        b - f <= 7
        c - f <= 3
        e - f <= 2
        d - c <= 3
        e - d <= 25
        d - e <= 9
        f - d <= 18
```

```
 LP OPTIMUM FOUND AT STEP      7

        OBJECTIVE FUNCTION VALUE

        1)      76.00000

  VARIABLE        VALUE          REDUCED COST
         A         7.000000          0.000000
         B        12.000000          0.000000
         C        16.000000          0.000000
         D         2.000000          0.000000
         E        19.000000          0.000000
         F        17.000000          0.000000
         H         3.000000          0.000000
         G         0.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          7.000000
        3)         0.000000          2.000000
        4)        15.000000          0.000000
        5)         3.000000          0.000000
        6)         0.000000          3.000000
        7)         0.000000          2.000000
        8)         0.000000          6.000000
        9)         0.000000          1.000000
       10)        26.000000          0.000000
       11)         3.000000          0.000000
       12)         0.000000          1.000000
       13)        12.000000          0.000000
       14)         4.000000          0.000000
       15)         0.000000          1.000000
       16)        17.000000          0.000000
       17)         8.000000          0.000000
       18)        26.000000          0.000000
       19)         3.000000          0.000000

 NO. ITERATIONS=       7
```

2. Product Mix: **(7 points)** profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	Cost per yard	Yards available per month
Silk	$20	1,000
Polyester	$6	2,000
Cotton	$9	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	$6.70	$3.55	$4.31	$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

type	selling price	labor	material	profit per tie
silk s	6.7	0.75	2.5	3.45
poly p	3.55	0.75	0.48	2.32
blend1 b	4.31	0.75	0.75	2.81
blend2 c	4.81	0.75	0.81	3.25

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Formulate the problem as a linear program with an objective function and all constraints.

Max 3.45s + 2.32p + 2.81b + 3.25c

ST 0.125s <= 1000 : silk

0.08p + 0.05b + 0.03c <= 2000 : poly

0.05b + 0.07c <= 1250 :cotton

S >= 6000 ; S <= 7000

P >= 10,000 ; p <= 14,000

B >= 13,000; b <= 16000

C >= 6000; c <= 8500

2 points

Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output.

```
Max 3.45s + 2.32p + 2.81b + 3.25c
ST       0.125s <= 1000
         0.08p + 0.05b + 0.03c <= 2000
         0.05b + 0.07c <= 1250
s >= 6000
s <= 7000
p >= 10000
p <= 14000
b >= 13000
b <= 16000
c >= 6000
c <= 8500
```

```
 LP OPTIMUM FOUND AT STEP      4

        OBJECTIVE FUNCTION VALUE

        1)      120196.0

  VARIABLE        VALUE          REDUCED COST
         S      7000.000000          0.000000
         P     13625.000000          0.000000
         B     13100.000000          0.000000
         C      8500.000000          0.000000


       ROW   SLACK OR SURPLUS     DUAL PRICES
        2)       125.000000          0.000000
        3)         0.000000         29.000000
        4)         0.000000         27.200001
        5)      1000.000000          0.000000
        6)         0.000000          3.450000
        7)      3625.000000          0.000000
        8)       375.000000          0.000000
        9)       100.000000          0.000000
       10)      2900.000000          0.000000
       11)      2500.000000          0.000000
       12)         0.000000          0.476000

 NO. ITERATIONS=       4
```

3 points

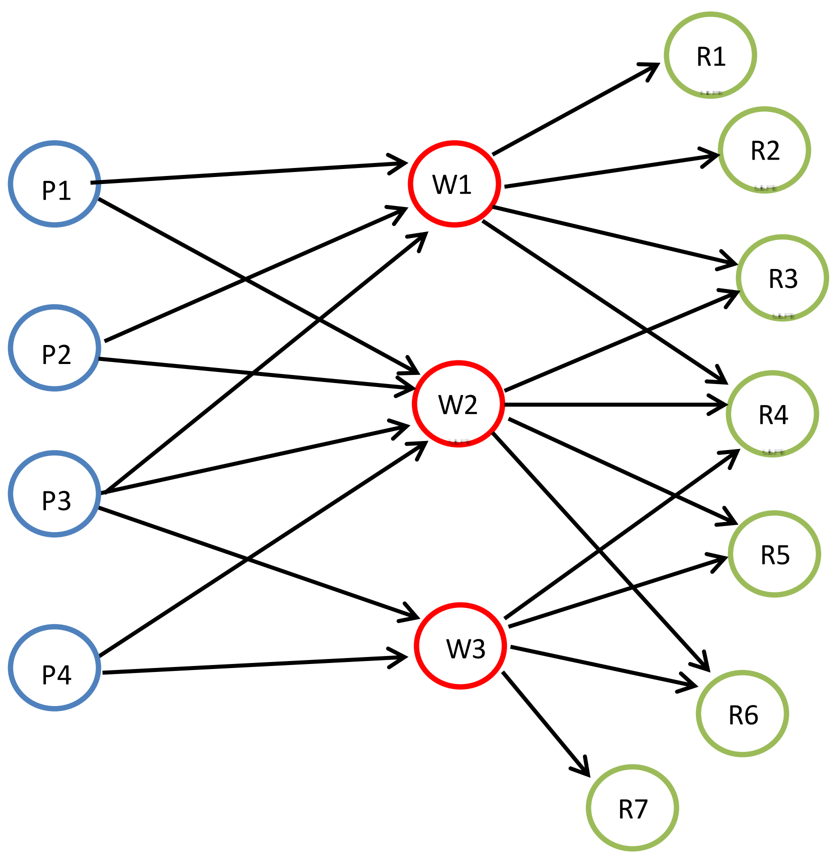
Maximum profit is $120,196 from producing 7000 silk ties, 13625 polyester ties, 13,100 blend1 and 8,500 blend 2.

3 points

3. Transshipment Model **(10 points)**

This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (p_i) must be shipped to a Warehouse (w_j) before being shipped to the Retailer (r_k). Each Plant will have an associated supply (s_i) and each Retailer will have a demand (d_k). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (p_i) to warehouse (w_j) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (w_j) to a retailer (r_k) have costs associated denoted cw(j,k).

The graph below shows the transshipment map for a manufacturer of refrigerators. Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



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(6 points) PART A) Formulate the problem as a linear program with an objective function and all constraints. Solve and provide the code.

3 points

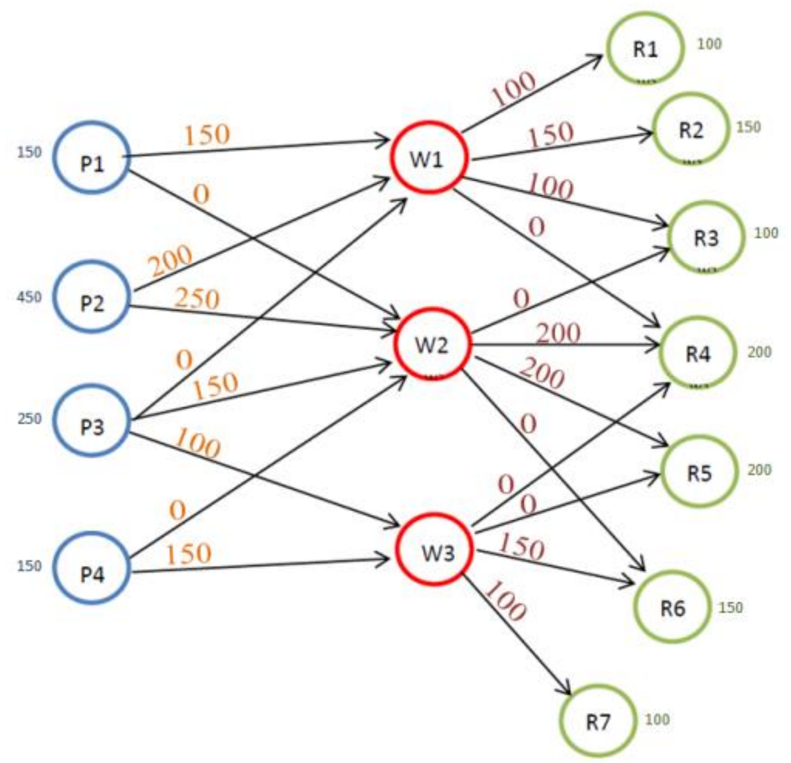
```
! Objective to minimize shipping costs
min 10p1w1 + 15p1w2 + 11 p2w1 + 8 p2w2 + 13 p3w1 + 8 p3w2 + 9 p3w3 + 14 p4w2
+ 8 p4w3 + 5 w1r1 + 6 w1r2 + 7 w1r3 + 10 w1r4 + 12 w2r3 + 8 w2r4 + 10 w2r5
+ 14 w2r6 + 14 w3r4 + 12 w3r5 + 12 w3r6 + 6 w3r7

ST
  ! Constraints on plants supply

        p1w1 + p1w2 <= 150
        p2w1 + p2w2 <= 450
        p3w1 + p3w2 + p3w3 <= 250
        p4w2 + p4w3 <= 150

  ! Constraints on retailers demand

        w1r1 >= 100
        w1r2 >= 150
        w1r3 + w2r3 >= 100
        w1r4 + w2r4 + w3r4 >= 200
        w2r5 + w3r5 >= 200
        w2r6 + w3r6 >= 150
        w3r7 >= 100

  ! Constraints on warehouses

        w1r1 + w1r2 + w1r3 + w1r4 - p1w1 - p2w1 -p3w1 = 0
        w2r3 + w2r4 + w2r5 + w2r6 - p1w2 - p2w2 - p3w2 - p4w2 = 0
        w3r4 + w3r5 + w3r6 + w3r7 - p3w3 - p4w3 = 0
```

```
LP OPTIMUM FOUND AT STEP      0

      OBJECTIVE FUNCTION VALUE

      1)      17100.00

 VARIABLE        VALUE          REDUCED COST
     P1W1       150.000000          0.000000
     P1W2         0.000000          8.000000
     P2W1       200.000000          0.000000
     P2W2       250.000000          0.000000
     P3W1         0.000000          2.000000
     P3W2       150.000000          0.000000
     P3W3       100.000000          0.000000
     P4W2         0.000000          7.000000
     P4W3       150.000000          0.000000
     W1R1       100.000000          0.000000
     W1R2       150.000000          0.000000
     W1R3       100.000000          0.000000
     W1R4         0.000000          5.000000
     W2R3         0.000000          2.000000
     W2R4       200.000000          0.000000
     W2R5       200.000000          0.000000
     W2R6         0.000000          1.000000
     W3R4         0.000000          7.000000
     W3R5         0.000000          3.000000
     W3R6       150.000000          0.000000
     W3R7       100.000000          0.000000
```

3 points

What are the optimal shipping routes and minimum cost. Minimal value is $17,100

Route	Quantity	Price/Item	$ Cost
P1W1	150	10	1500
P2W1	200	11	2200
P2W2	250	8	2000
P3W2	150	8	1200
P3W3	100	9	900
P4W3	150	8	1200
W1R1	100	5	500
W1R2	150	6	900
W1R3	100	7	700
W2R4	200	8	1600
W2R5	200	10	2000
Total	**2000**		**$17,100**



(2 points) Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not? NOT FEASIBLE or No Solution

(2 points) Part C: Instead of closing Warehouse 2 management has decide to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?

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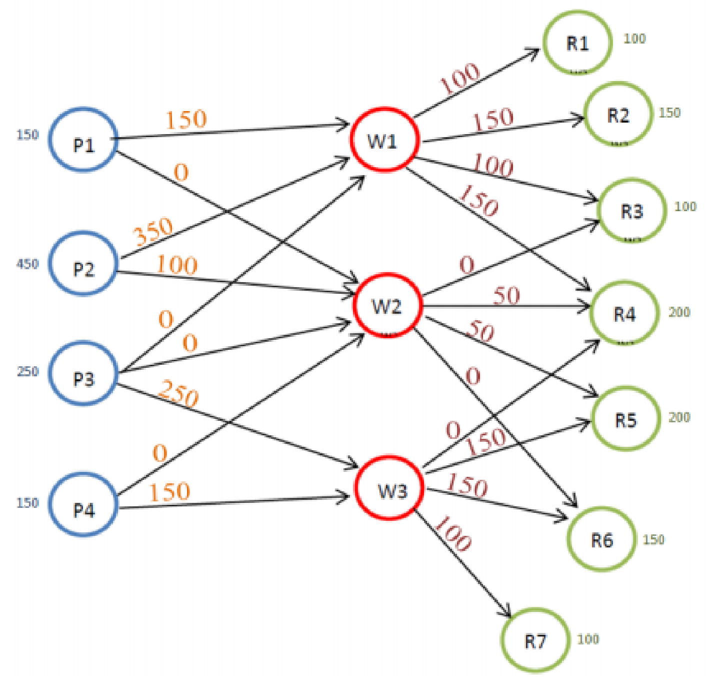
Add the constraint: w2r3 + w2r4 + w2r5 + w2r6 <= 100



Output from Lindo:

Reports Window

```
LP OPTIMUM FOUND AT STEP      14

       OBJECTIVE FUNCTION VALUE

       1)      18300.00
```

VARIABLE	VALUE	REDUCED COST
P1W1	150.000000	0.000000
P1W2	0.000000	8.000000
P2W1	350.000000	0.000000
P2W2	100.000000	0.000000
P3W1	0.000000	4.000000
P3W2	0.000000	2.000000
P3W3	250.000000	0.000000
P4W2	0.000000	9.000000
P4W3	150.000000	0.000000
W1R1	100.000000	0.000000
W1R2	150.000000	0.000000
W1R3	100.000000	0.000000
W1R4	150.000000	0.000000
W2R3	0.000000	7.000000
W2R4	50.000000	0.000000
W2R5	50.000000	0.000000
W2R6	0.000000	4.000000
W3R4	0.000000	4.000000
W3R5	150.000000	0.000000
W3R6	150.000000	0.000000
W3R7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	2.000000
5)	0.000000	3.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-21.000000
10)	0.000000	-23.000000
11)	0.000000	-23.000000
12)	0.000000	-17.000000
13)	0.000000	11.000000
14)	0.000000	8.000000
15)	0.000000	11.000000
16)	0.000000	0.000000
17)	0.000000	5.000000
18)	150.000000	0.000000
19)	0.000000	0.000000
20)	350.000000	0.000000
21)	100.000000	0.000000
22)	0.000000	0.000000
23)	0.000000	0.000000
24)	250.000000	0.000000
25)	0.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	150.000000	0.000000
29)	100.000000	0.000000
30)	150.000000	0.000000
31)	0.000000	0.000000
32)	50.000000	0.000000
33)	50.000000	0.000000
34)	0.000000	0.000000
35)	0.000000	0.000000
36)	150.000000	0.000000
37)	150.000000	0.000000
38)	100.000000	0.000000

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Problem 4: Making Change

a) (3 points)

```
<untitled>
min V1+V2+V3+V4
ST
    1V1+5V2+10V3+25V4 = 202
    V1>=0
    V2>=0
    V3>=0
    V4>=0
END
GIN V1
GIN V2
GIN V3
GIN V4
```

The minimum number of coins is 10.
2 of 1 and 8 of 25 coins are used.

```
LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      10.00000

 VARIABLE        VALUE          REDUCED COST
       V1         2.000000          1.000000
       V2         0.000000          1.000000
       V3         0.000000          1.000000
       V4         8.000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         2.000000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          0.000000
       6)         8.000000          0.000000

NO. ITERATIONS=      32
BRANCHES=    6 DETERM.=  1.000E    0
```

Part b)

```
<untitled>
min V1+V2+V3+V4+V5
ST
    1V1+3V2+7V3+12V4+27V5 = 293
    V1>=0
    V2>=0
    V3>=0
    V4>=0
    V5>=0
END
GIN V1
GIN V2
GIN V3
GIN V4
GIN V5
```

The minimum number of coins is 14.

2 of 7, 3 of 12 and 9 of 27 coins are used.

```
LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      14.00000

 VARIABLE        VALUE          REDUCED COST
       V1         0.000000          1.000000
       V2         0.000000          1.000000
       V3         2.000000          1.000000
       V4         3.000000          1.000000
       V5         9.000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)         2.000000          0.000000
       6)         3.000000          0.000000
       7)         9.000000          0.000000

NO. ITERATIONS=      98
BRANCHES=   34 DETERM.=  1.000E    0
```