Introduction to Linear Programming

LP Model Formulation
A Maximization Example

Resource Availability: 40 hrs of labor per day
120 lbs of clay

Decision Variables: x_1 = number of bowls to produce per day
x_2 = number of mugs to produce per day

Objective Function: Maximize $Z = \$40x_1 + \$50x_2$
Where Z = profit per day

Resource Constraints: $1x_1 + 2x_2 \leq 40$ hours of labor
$4x_1 + 3x_2 \leq 120$ pounds of clay

Non-Negativity Constraints: $x_1 \geq 0$; $x_2 \geq 0$

LP Model Formulation
A Maximization Example

Complete Linear Programming Model:

Maximize $Z = \$40x_1 + \$50x_2$

subject to:
$$1x_1 + 2x_2 \leq 40$$
$$4x_1 + 3x_2 \leq 120$$
$$x_1, x_2 \geq 0$$

Feasible Solutions

A **feasible solution** does not violate **any** of the constraints:

Example: $x_1 = 5$ bowls
$x_2 = 10$ mugs
$Z = \$40x_1 + \$50x_2 = \$700$

Labor constraint check: $1(5) + 2(10) = 25 < 40$ hours
Clay constraint check: $4(5) + 3(10) = 50 < 120$ pounds

Infeasible Solutions

An **infeasible solution** violates **at least one** of the constraints:

Example: $x_1 = 10$ bowls
$x_2 = 20$ mugs
$Z = \$40x_1 + \$50x_2 = \$1400$

Labor constraint check: $1(10) + 2(20) = 50 > 40$ hours

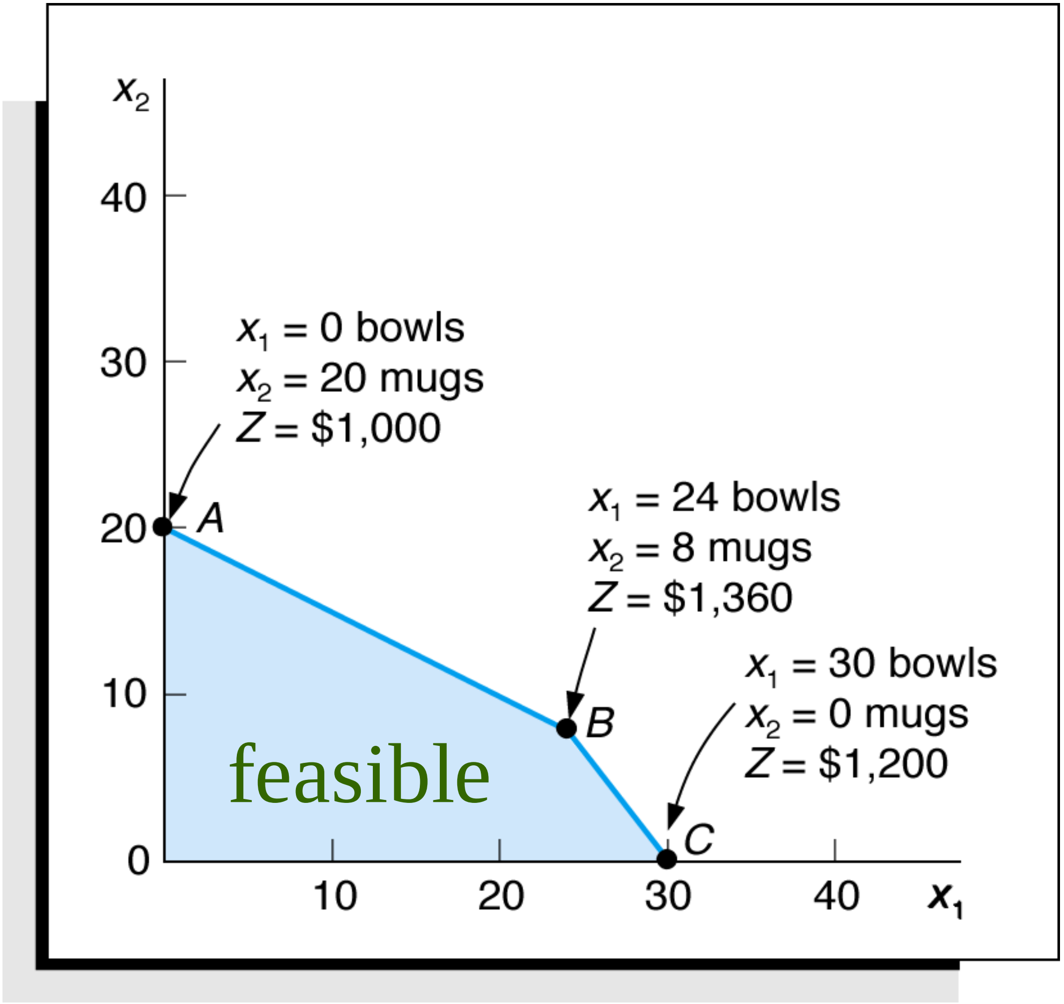
Extreme (Corner) Point Solutions
Graphical Solution of Maximization Model

Maximize $Z = \$40x_1 + \$50x_2$

subject to: $1x_1 + 2x_2 \leq 40$

$4x_1 + 3x_2 \leq 120$

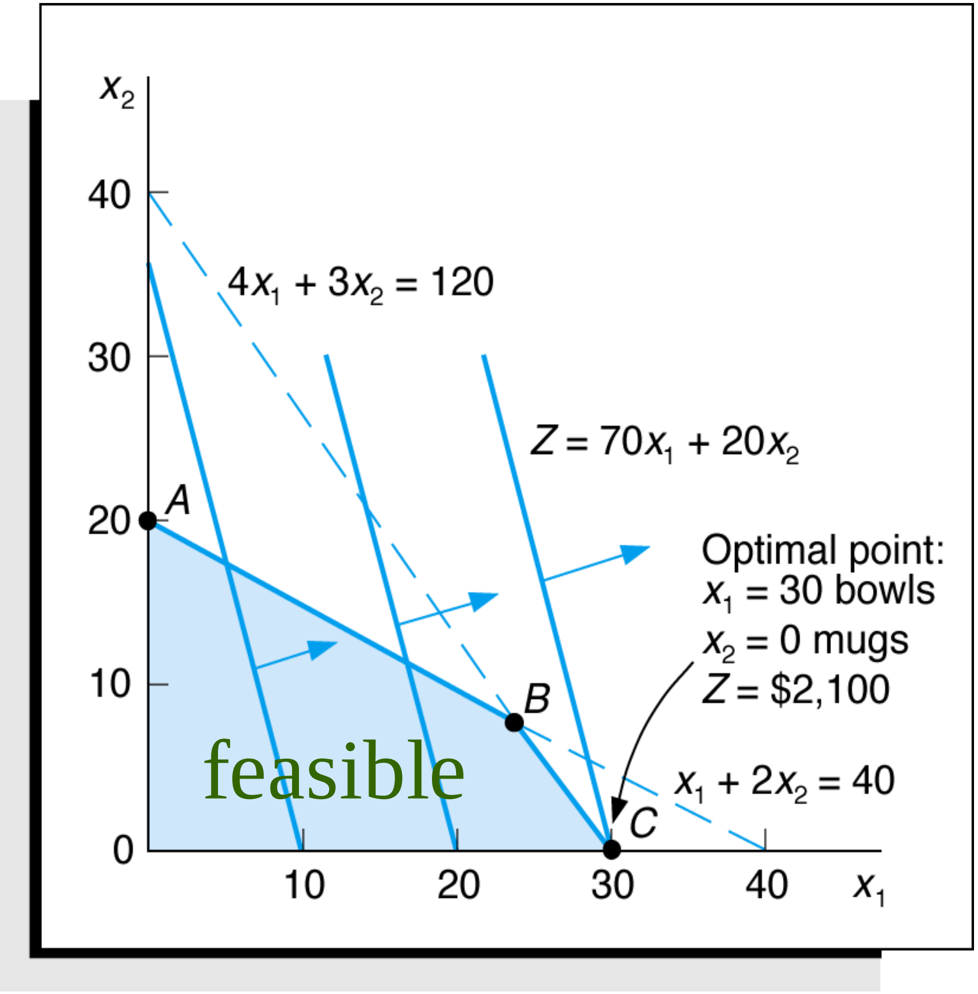
$x_1, x_2 \geq 0$



Solutions at All Corner Points

Optimal Solution for New Objective Function
Graphical Solution of Maximization Model

Maximize $Z = \$70x_1 + \$20x_2$
subject to: $1x_1 + 2x_2 \leq 40$
$4x_1 + 3x_2 \leq 120$
$x_1, x_2 \geq 0$



Optimal Solution with $Z = 70x_1 + 20x_2$

Linear Programming Model: Standard Form

$$\text{Max } Z = 40x_1 + 50x_2 + 0s_1 + 0s_2$$

subject to: $1x_1 + 2x_2 + s_1 = 40$

$$4x_1 + 3x_2 + s_2 = 120$$

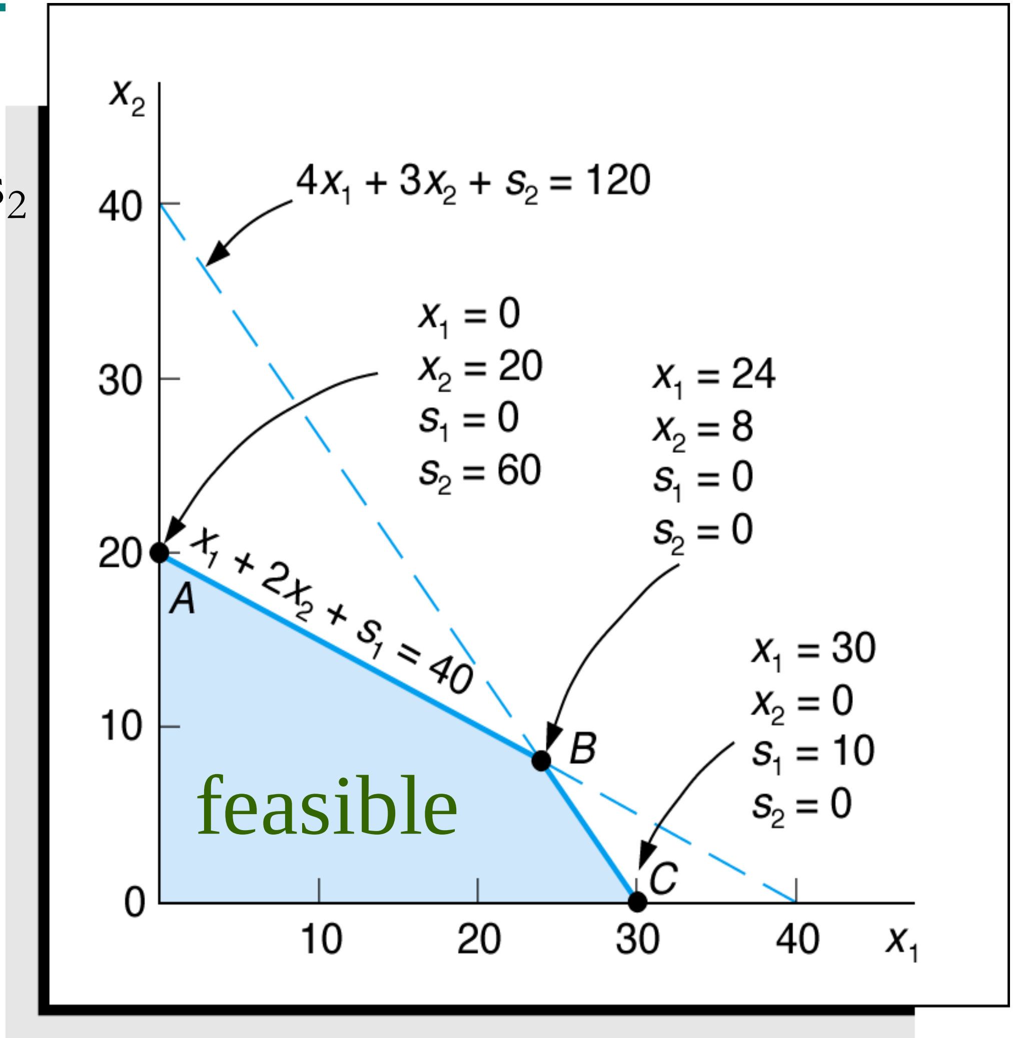
$$x_1, x_2, s_1, s_2 \geq 0$$

Where:

x_1 = number of bowls

x_2 = number of mugs

s_1, s_2 are slack variables



Solution Points A, B, and C with Slack

What if we add a plate and 5 hours of labor?

Profit \$30
Labor = 0.5 hours, clay = 2 lbs

Maximize $Z = \$40x_1 + \$50x_2 + \$30x_3$

subject to: $1x_1 + 2x_2 + 0.5x_3 \leq 45$
$4x_1 + 3x_2 + 2x_3 \leq 120$
$x_1, x_2 \geq 0$

max 40x1 + 50x2 + 30x3

ST

x1 + 2x2 + 0.5x3 <= 45

4x1 + 3x2 + 2x3 <= 120

x1 >= 0

x2 >= 0

x3 >= 0

OBJECTIVE FUNCTION VALUE

1) 1860.000

VARIABLE	VALUE	REDUCED COST
X1	0.000000	**20.000000**
X2	12.000000	0.000000
X3	42.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	4.000000
3)	0.000000	14.000000
4)	0.000000	0.000000
5)	12.000000	0.000000
6)	42.000000	0.000000

Reduced Cost

is the amount by which an objective function coefficient would have to improve (so increase for maximization problem, decrease for minimization problem) before it would be possible for a corresponding variable to assume a positive value in the optimal solution.

Only positive for decision variables with a value of 0.

What if mugs had a profit of $60?

OBJECTIVE FUNCTION VALUE **max 40x1 + 50x2 + 30x3**

1) 1860.000

VARIABLE	VALUE	REDUCED COST
X1	**0.000000**	**20.000000**
X2	12.000000	0.000000
X3	42.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	4.000000
3)	0.000000	14.000000
4)	0.000000	0.000000
5)	12.000000	0.000000
6)	42.000000	0.000000

OBJECTIVE FUNCTION VALUE **max 60x1 + 50x2 + 30x3**

1) 1860.000

VARIABLE	VALUE	REDUCED COST
X1	21.000000	0.000000
X2	12.000000	0.000000
X3	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	4.000000
3)	0.000000	14.000000
4)	21.000000	0.000000
5)	12.000000	0.000000
6)	0.000000	0.000000

Dual Price / Shadow Price

is the instantaneous change, per unit of the constraint, in the objective value of the optimal solution obtained by relaxing the constraint.

In other words, it is the marginal utility of relaxing the constraint, or, equivalently, the marginal cost of strengthening the constraint.

What if we add 5 hours of labor? The objective increases by 5*4 = $20.

OBJECTIVE FUNCTION VALUE **max 40x1 + 50x2 + 30x3**

1) 1860.000

VARIABLE	VALUE	REDUCED COST
X1	0.000000	20.000000
X2	12.000000	0.000000
X3	42.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	**4.000000**
3)	0.000000	14.000000
4)	0.000000	0.000000
5)	12.000000	0.000000
6)	42.000000	0.000000

OBJECTIVE FUNCTION VALUE **max 40x1 + 50x2 + 30x3**

1) 1880.000

VARIABLE	VALUE	REDUCED COST
X1	0.000000	20.000000
X2	16.000000	0.000000
X3	36.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	4.000000
3)	0.000000	14.000000
4)	0.000000	0.000000
5)	16.000000	0.000000
6)	36.000000	0.000000

LP Model Formulation – Minimization

Decision Variables:

x_1 = bags of Super-gro
x_2 = bags of Crop-quick

The Objective Function:

Minimize $Z = \$6x_1 + 3x_2$
Where: $\$6x_1$ = cost of bags of Super-Gro
$\$3x_2$ = cost of bags of Crop-Quick

Model Constraints:

$2x_1 + 4x_2 \geq 16$ lb (nitrogen constraint)
$4x_1 + 3x_2 \geq 24$ lb (phosphate constraint)
$x_1, x_2 \geq 0$ (non-negativity constraint)

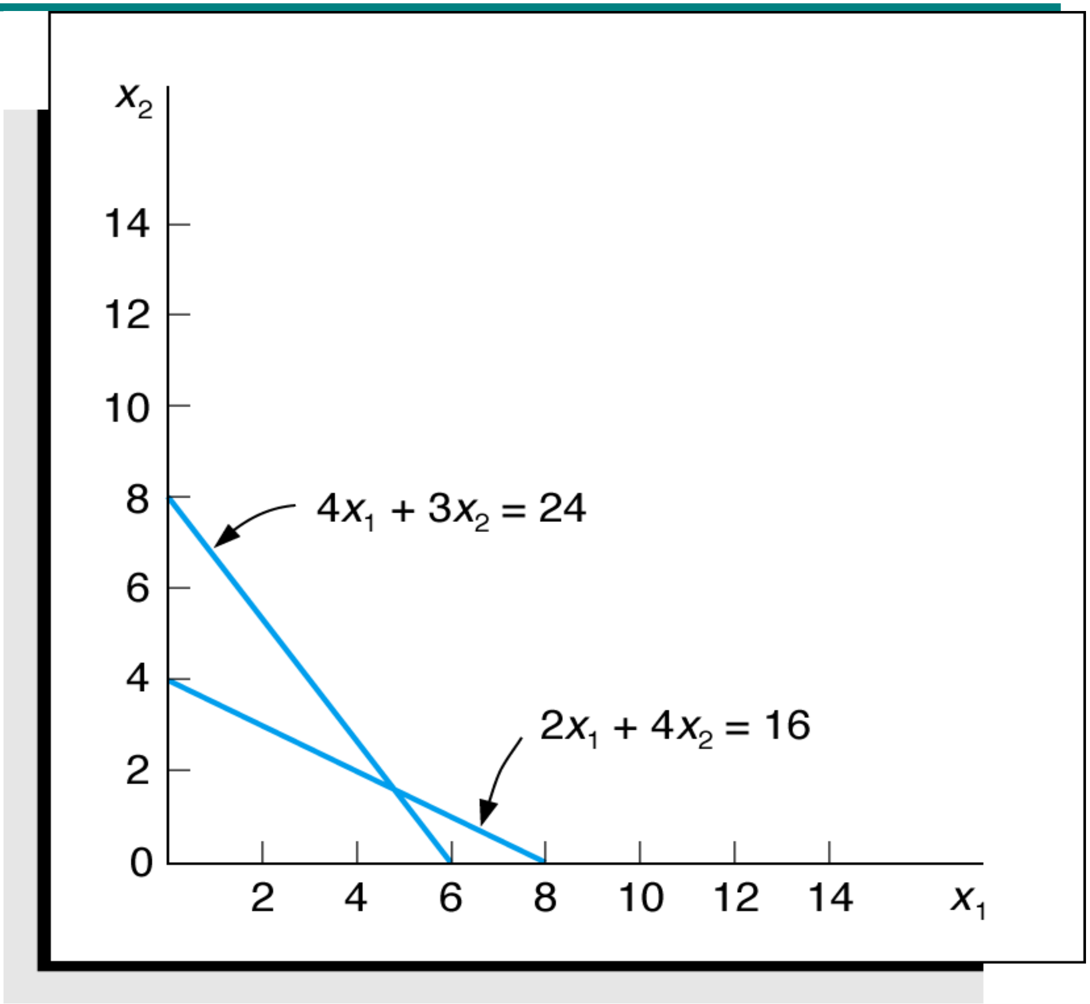
Constraint Graph – Minimization

Minimize $Z = \$6x_1 + \$3x_2$

subject to: $2x_1 + 4x_2 \geq 16$

$4x_1 + 3x_2 \geq 24$

$x_1, x_2 \geq 0$



Graph of Both Model Constraints

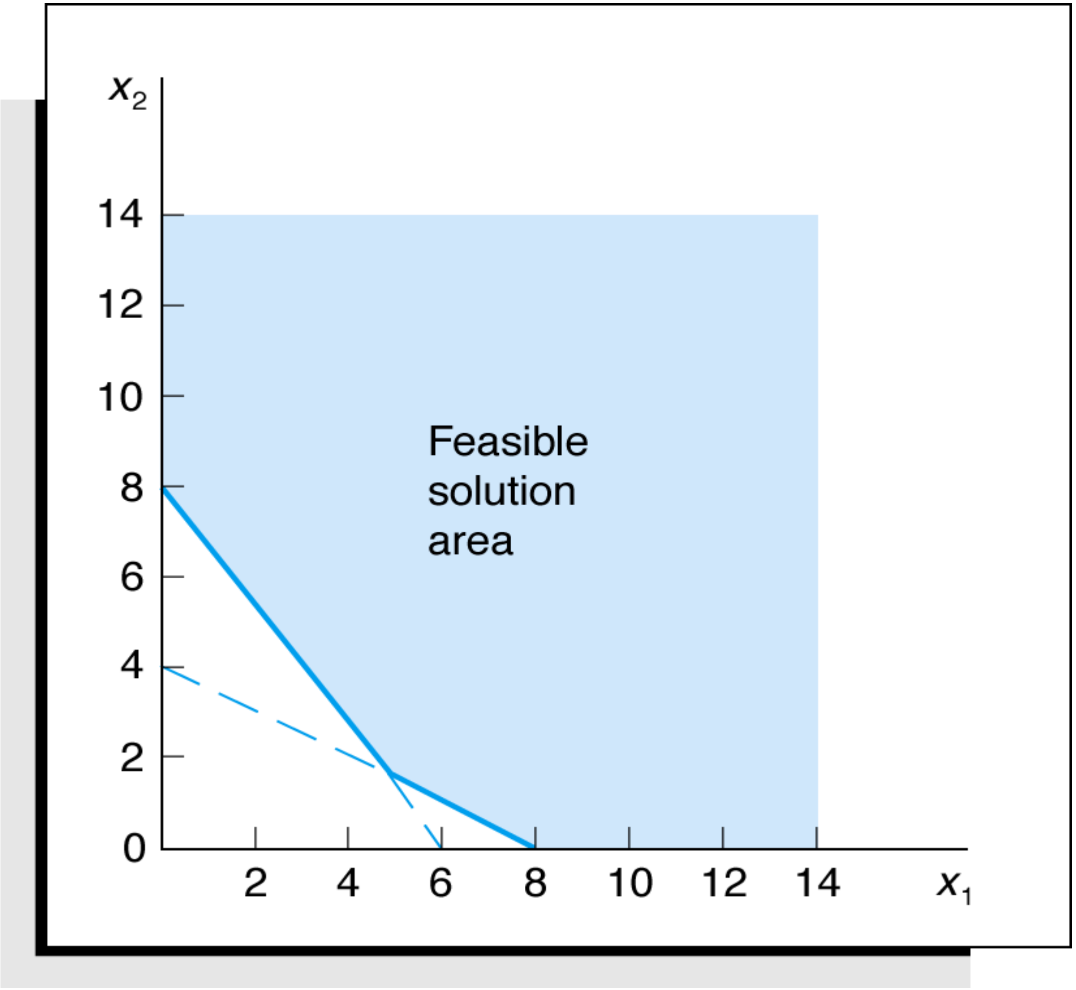
Feasible Region– Minimization

Minimize $Z = \$6x_1 + \$3x_2$

subject to: $2x_1 + 4x_2 \geq 16$

$4x_1 + 3x_2 \geq 24$

$x_1, x_2 \geq 0$



Feasible Solution Area

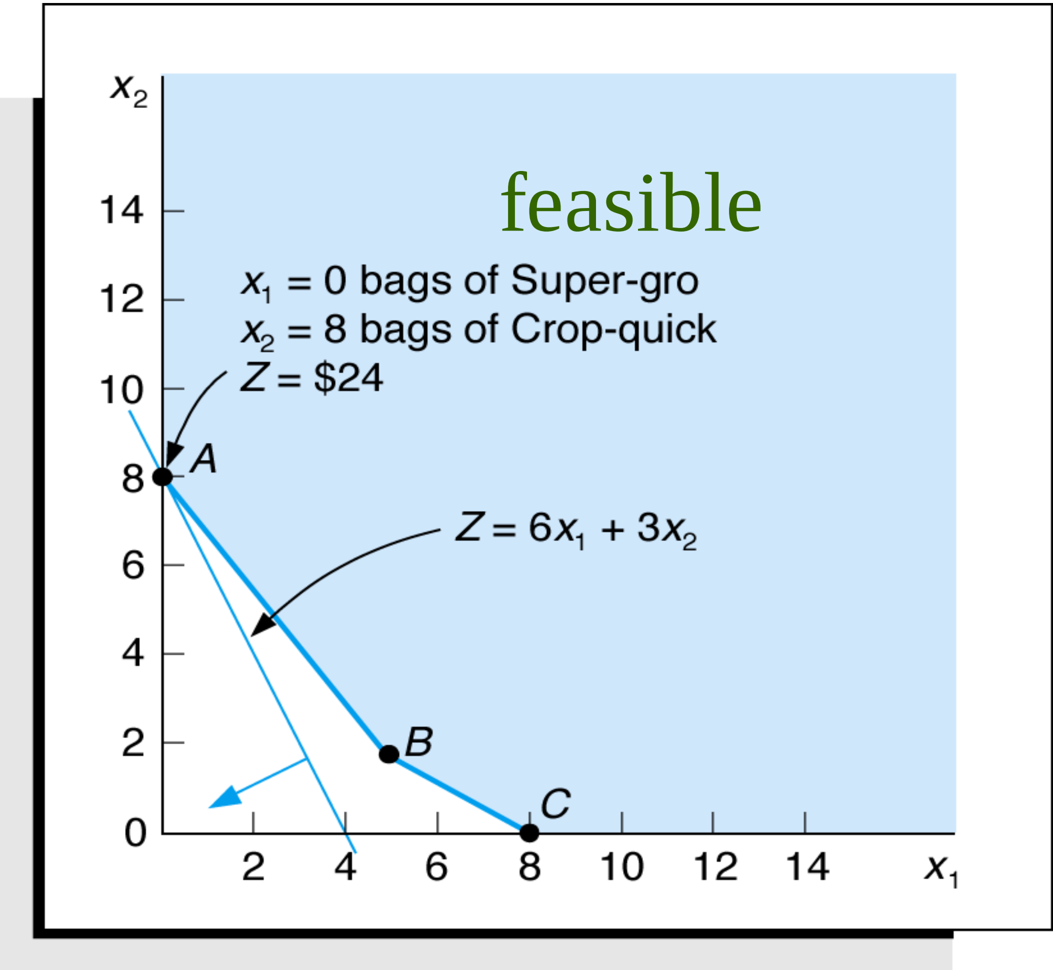
Optimal Solution Point – Minimization

Minimize $Z = \$6x_1 + \$3x_2$

subject to: $2x_1 + 4x_2 \geq 16$

$4x_1 + 3x_2 \geq 24$

$x_1, x_2 \geq 0$



Optimum Solution Point

Surplus Variables – Minimization

- A surplus variable is ***subtracted from a ≥ constraint*** to convert it to an equation (=).
- A surplus variable ***represents an excess*** above a constraint requirement level.
- A surplus variable ***contributes nothing*** to the calculated value of the objective function.
- Subtracting surplus variables in the farmer problem constraints:

$$2x_1 + 4x_2 - s_1 = 16 \text{ (nitrogen)}$$
$$4x_1 + 3x_2 - s_2 = 24 \text{ (phosphate)}$$

Graphical Solutions – Minimization

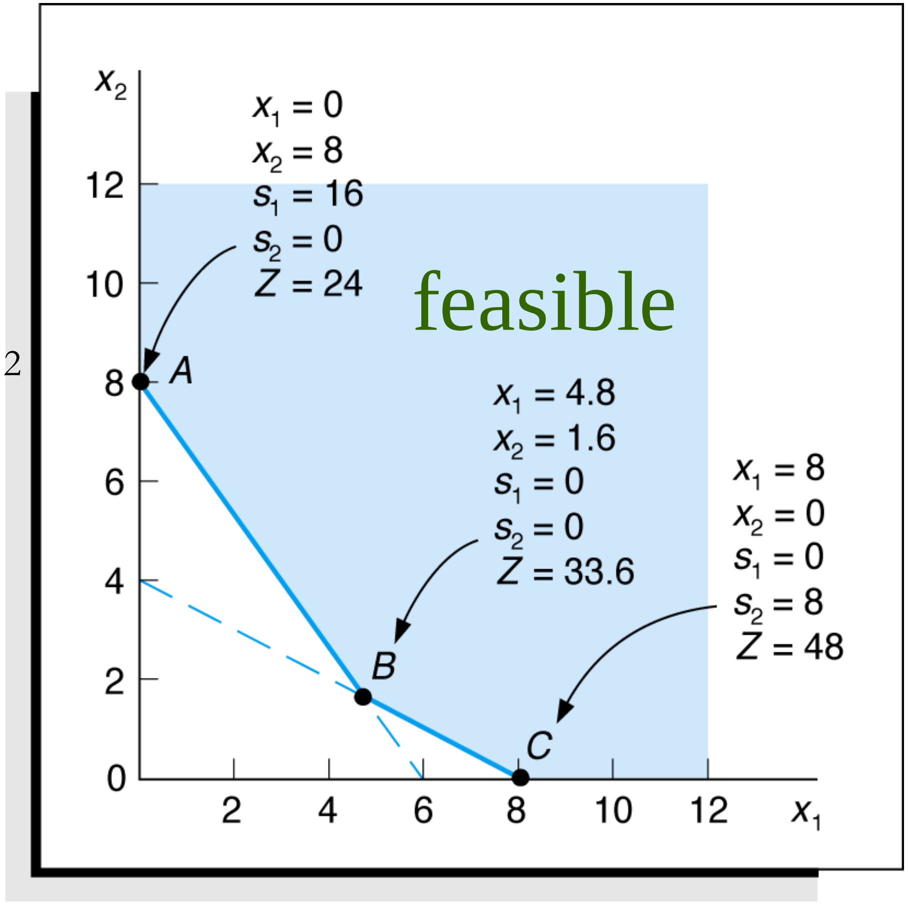
$$\text{Minimize } Z = \$6x_1 + \$3x_2 + 0s_1 + 0s_2$$

subject to:

$$2x_1 + 4x_2 - s_1 = 16$$

$$4x_1 + 3x_2 - s_2 = 24$$

$$x_1, x_2, s_1, s_2 \geq 0$$



Graph of Fertilizer Example

Irregular Types of Linear Programming Problems

For some linear programming models, the general rules do not apply.

Special types of problems include those with:

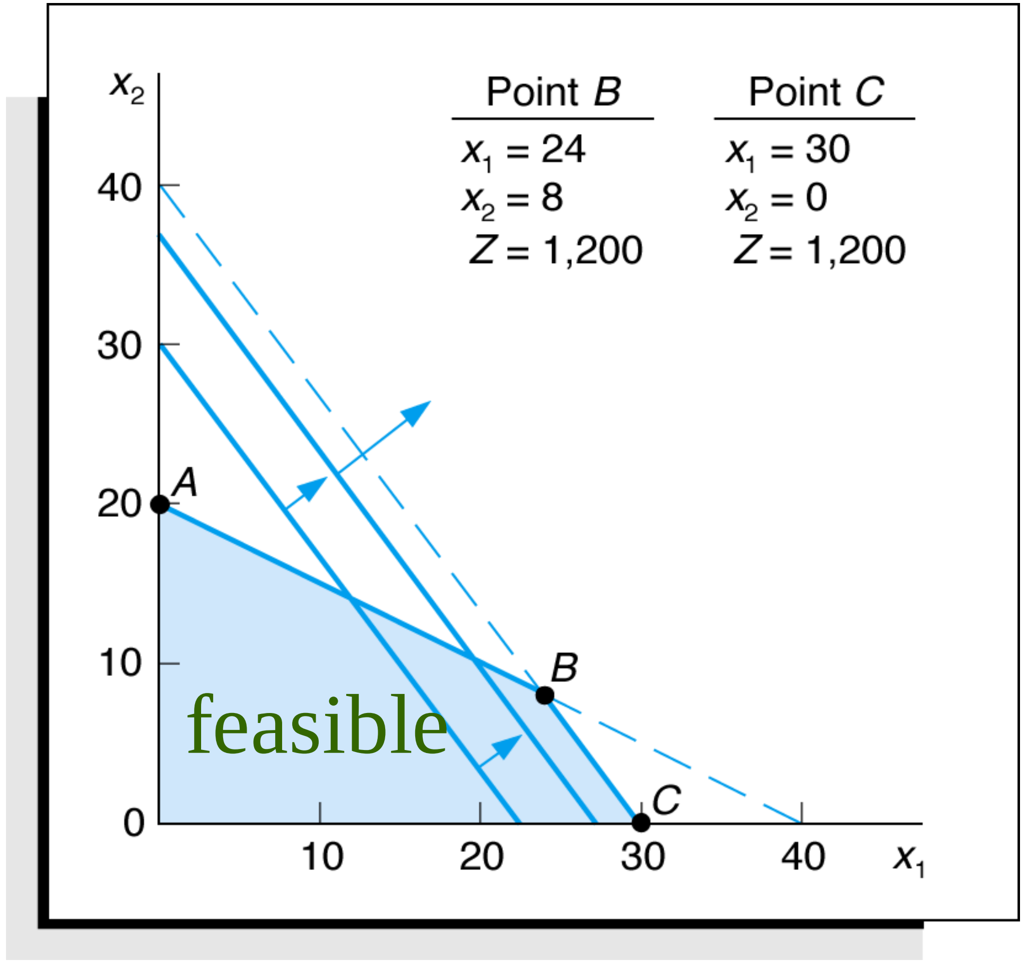
- Multiple optimal solutions
- Infeasible solutions
- Unbounded solutions

Multiple Optimal Solutions Beaver Creek Pottery

The objective function is **parallel** to a constraint line.

Maximize $Z = \$40x_1 + 30x_2$
subject to: $1x_1 + 2x_2 \leq 40$
$4x_1 + 3x_2 \leq 120$
$x_1, x_2 \geq 0$

Where:
x_1 = number of bowls
x_2 = number of mugs



Example with Multiple Optimal Solutions

An Infeasible Problem

Every possible solution **violates** at least one constraint:

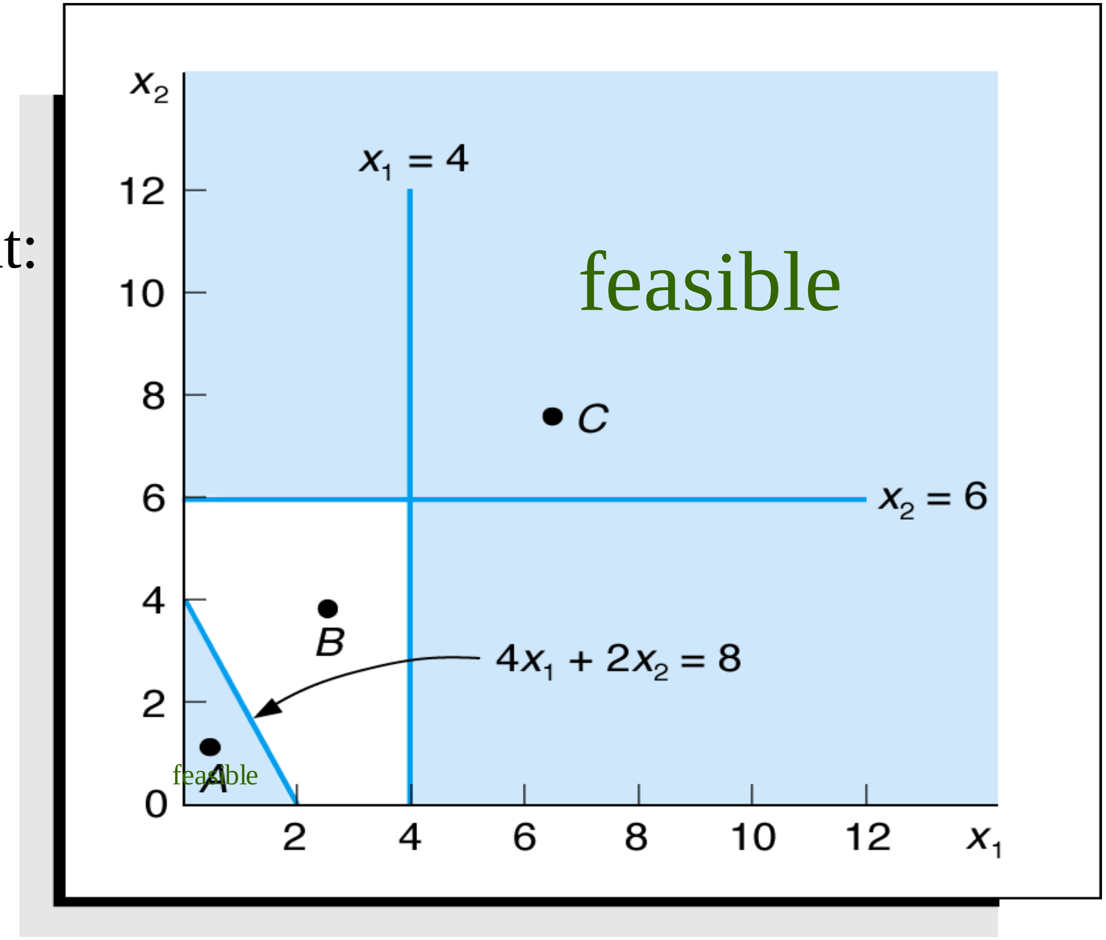
Maximize $Z = 5x_1 + 3x_2$

subject to: $4x_1 + 2x_2 \le 8$

$x_1 \ge 4$

$x_2 \ge 6$

$x_1, x_2 \ge 0$



Graph of an Infeasible Problem

An Unbounded Problem

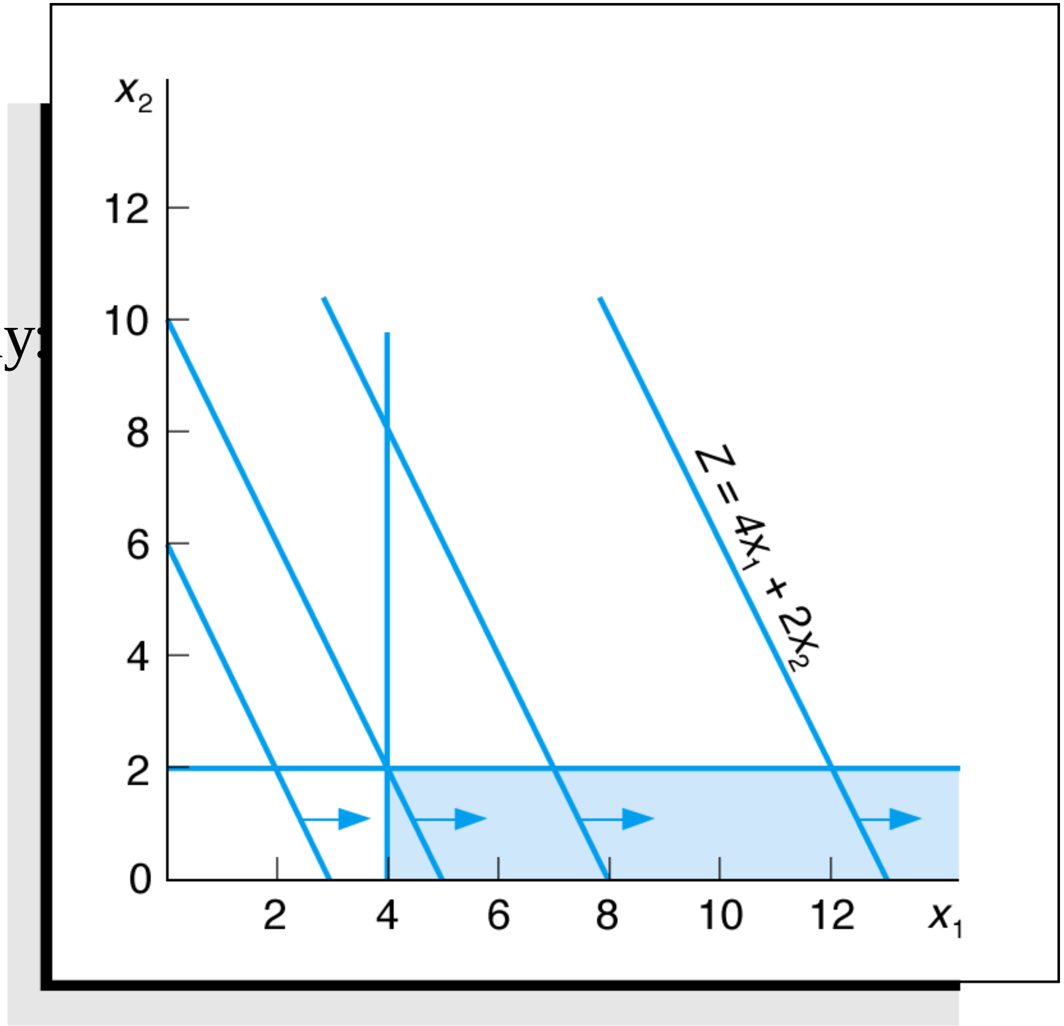
Value of the objective function increases indefinitely:

Maximize $Z = 4x_1 + 2x_2$

subject to: $x_1 \geq 4$

$x_2 \leq 2$

$x_1, x_2 \geq 0$



Graph of an Unbounded Problem

Simplex Method

A large variety of Simplex-based algorithms exist to solve LP problems.

Other (polynomial time) algorithms have been developed for solving LP problems:

- Khachian algorithm (1979)
- Kamarkar algorithm (AT&T Bell Labs, mid 80s)
- See Section 4.10

BUT,

none of these algorithms have been able to beat Simplex in actual practical applications.

HENCE,

Simplex (in its various forms) is and will most likely remain the most dominant LP algorithm for at least the near future

Extreme point theorem:

If the maximum or minimum value of a linear function defined over a polygonal convex region exists, then it is to be found at the boundary of the region.

Convex set:

A set (or region) is convex if, for any two points (say, x_1 and x_2) in that set, the line segment joining these points lies entirely within the set.

A point is by definition convex.

What does the extreme point theorem imply?

- A finite number of extreme points implies a finite number of solutions!
- Hence, search is reduced to a finite set of points
- However, a finite set can still be too large for practical purposes
- Simplex method provides an efficient systematic search guaranteed to converge in a finite number of steps.

Formulations

There are many ways to formulate linear programs:

- **objective (or cost) function**
  maximize $c^T x$, or
  minimize $c^T x$, or
  find any feasible solution
- **(in)equalities**
  $Ax \leq b$, or
  $Ax \geq b$, or
  $Ax = b$, or any combination
- **nonnegative variables**
  $x \geq 0$, or not

Fortunately it is pretty easy to convert among forms

Formulations

The two **most common** formulations:

Canonical form		Standard form
minimize $c^T x$ subject to $Ax \geq b$ $x \geq 0$	**slack** variables $\longrightarrow$	minimize $c^T x$ subject to $Ax = b$ $x \geq 0$

e.g.

$7x_1 + 5x_2 \geq 7$ $x_1, x_2 \geq 0$	$\xrightarrow{y_1}$	$7x_1 + 5x_2 - y_1 = 7$ $x_1, x_2, y_1 \geq 0$

Geometric View of Canonical Form

A **polytope** in n-dimensional space

- Each inequality corresponds to a half-space.
- The "feasible set" is the intersection of the half-spaces
- This corresponds to a polytope
- Polytopes are **convex**: if x,y is in the polytope, so is the line segment joining them.
- The optimal solution is at a vertex (i.e., a corner).

Geometric View of Canonical Form

minimize:

$$z = -2x_1 - 3x_2$$

subject to:

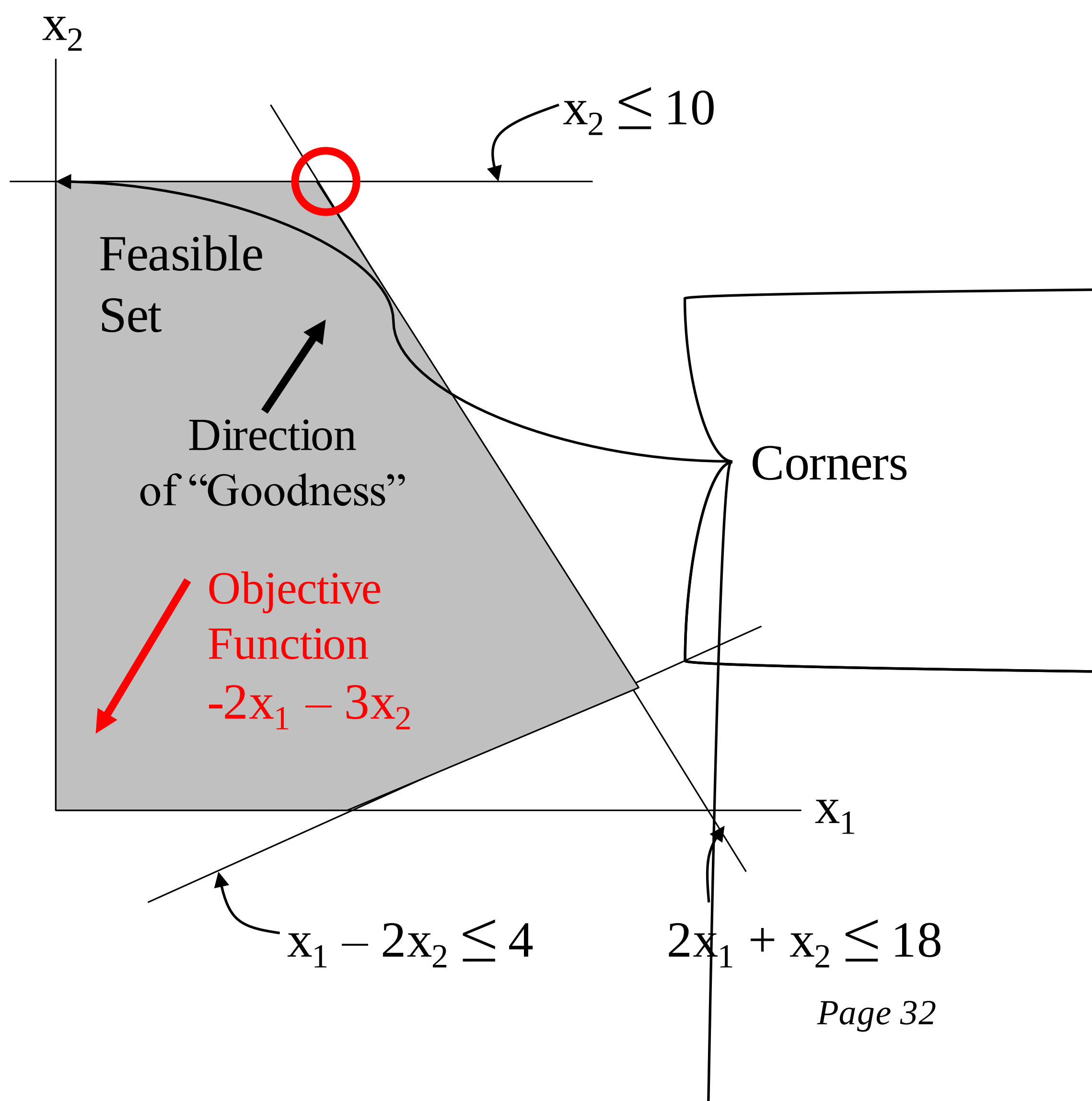
$$x_1 - 2x_2 \leq 4$$

$$2x_1 + x_2 \leq 18$$

$$x_2 \leq 10$$

$$x_1, x_2 \geq 0$$

An intersection of 5 halfspaces



Geometric View of Canonical Form

A **polytope** in n-dimensional space

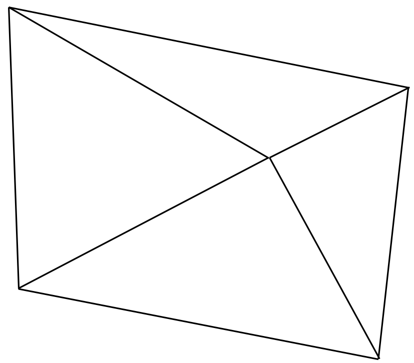
- Each inequality corresponds to a half-space.
- The "feasible set" is the intersection of the half-spaces.
- This corresponds to a polytope
- The optimal solution is at a corner.

Simplex moves around on the surface of the polytope

Interior-Point methods move within the polytope

The Simple Essense of Simplex

Polytope P



Input: min $f(x) = cx$

s.t. x in $P = \{x: Ax \leq b, x \geq 0\}$

Consider Polytope P from canonical form as a graph $G = (V,E)$ with
V = polytope vertices,
E = polytope edges.

1) Find *any* vertex v of P.
2) While there exists a neighbor u of v in G with $f(u) < f(v)$, update v to u.
3) Output v.

Choice of neighbor if several u have $f(u) < f(v)$?

Basic Steps of Simplex

1. Begin the search at an extreme point (i.e., a basic feasible solution).
2. Determine if the movement to an adjacent extreme can improve on the optimization of the objective function. If not, the current solution is optimal. If, however, improvement is possible, then proceed to the next step.
3. Move to the adjacent extreme point which offers (or, perhaps, *appears* to offer) the most improvement in the objective function.
4. Continue steps 2 and 3 until the optimal solution is found or it can be shown that the problem is either unbounded or infeasible.

Simplex Running Time

For dense matrices, takes O(n(n+m)) time per iteration

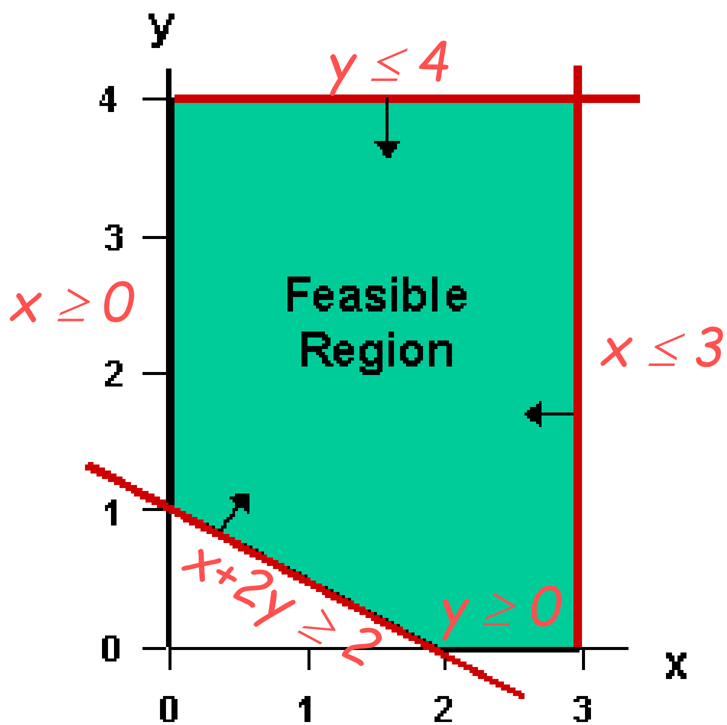
Can take an **exponential** number of iterations.

In practice, sparse methods are used for the iterations.

Linear Programming in 2 dimensions

Name	Vars	Constraints	Objective
linear programming (LP)	real	linear inequalities	linear function

2 variables:
feasible region is a convex polygon

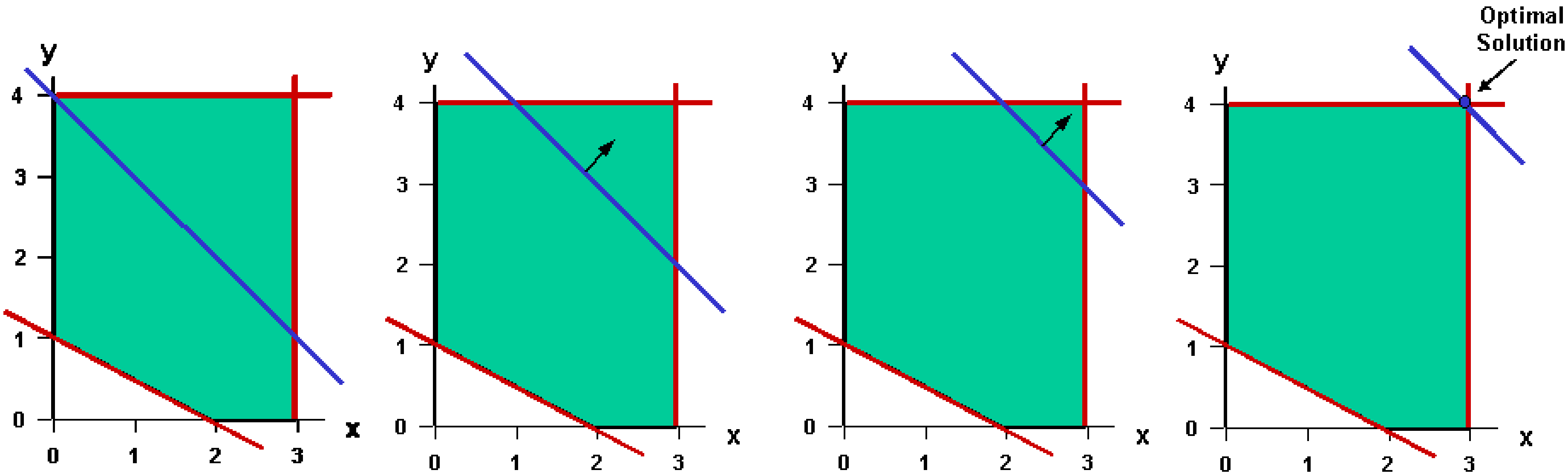


boundary of feasible region comes from the constraints

Linear Programming in 2 dimensions

Name	Vars	Constraints	Objective
linear programming (LP)	real	linear inequalities	linear function

"level sets" of the objective x+y (sets where it takes a certain value)

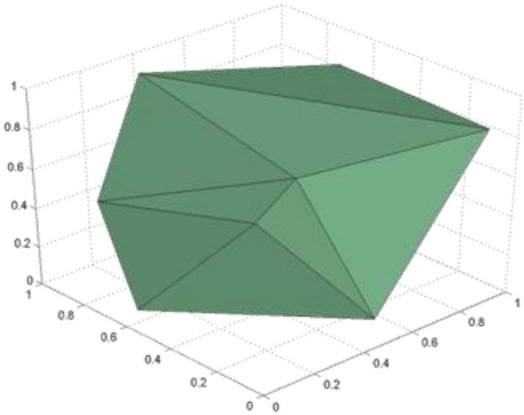


x+y = 4   *x+y = 5*   *x+y = 6*   *x+y = 7*

Linear Programming in *n* dimensions

Name	Vars	Constraints	Objective
linear programming (LP)	real	linear inequalities	linear function

3 variables:
feasible region is a convex polyhedron

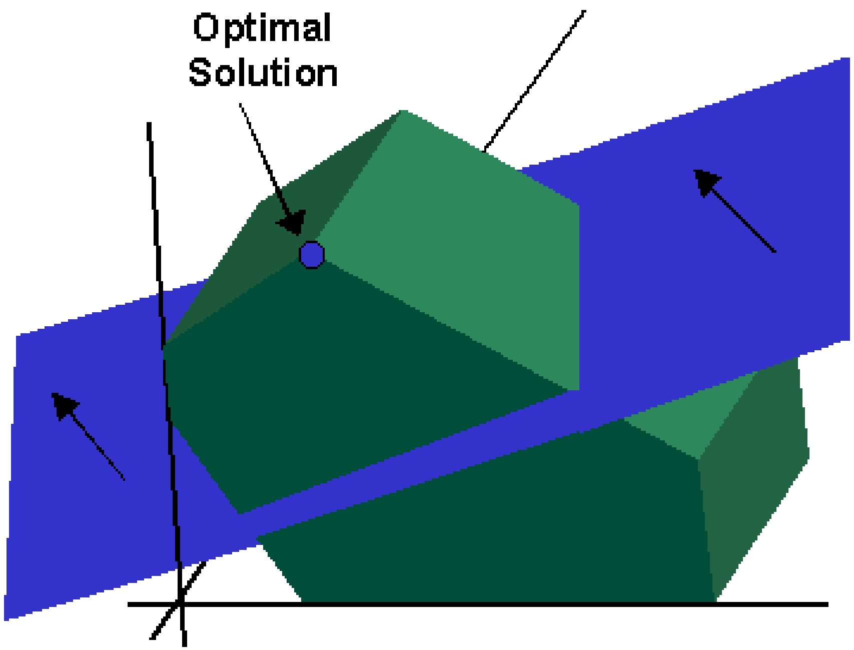


Linear Programming in *n* dimensions

Name	Vars	Constraints	Objective
linear programming (LP)	real	linear inequalities	linear function

here level set is a plane
(in general, a hyperplane)

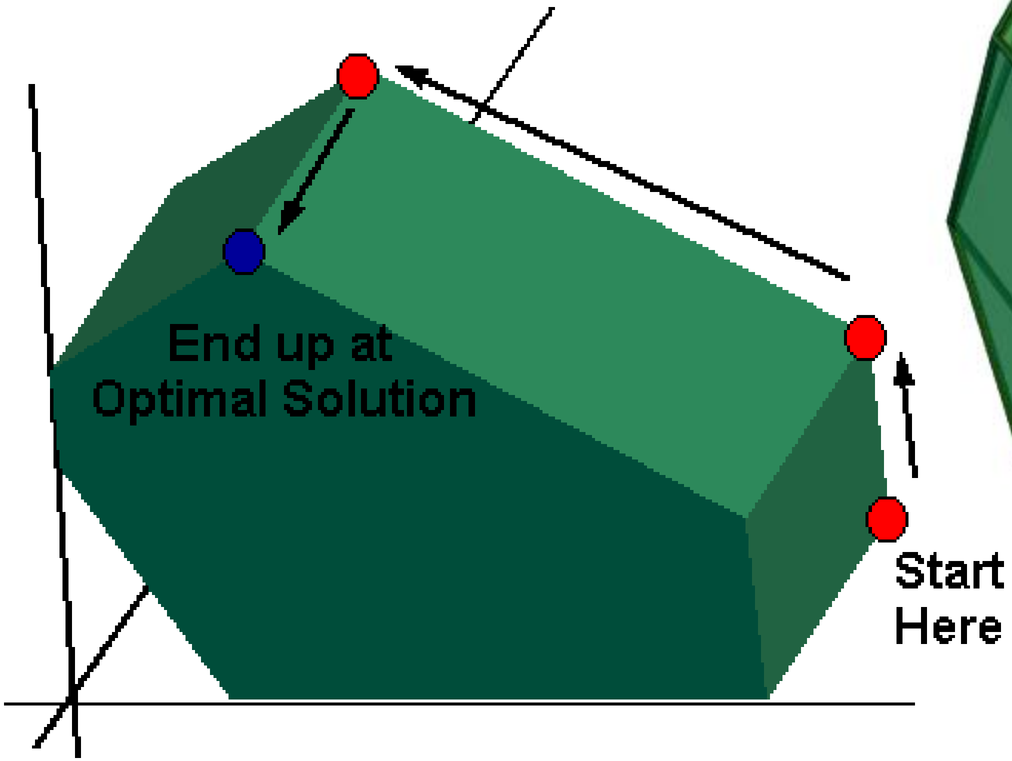
If an LP optimum is finite, it can always be achieved at a corner ("vertex") of the feasible region.

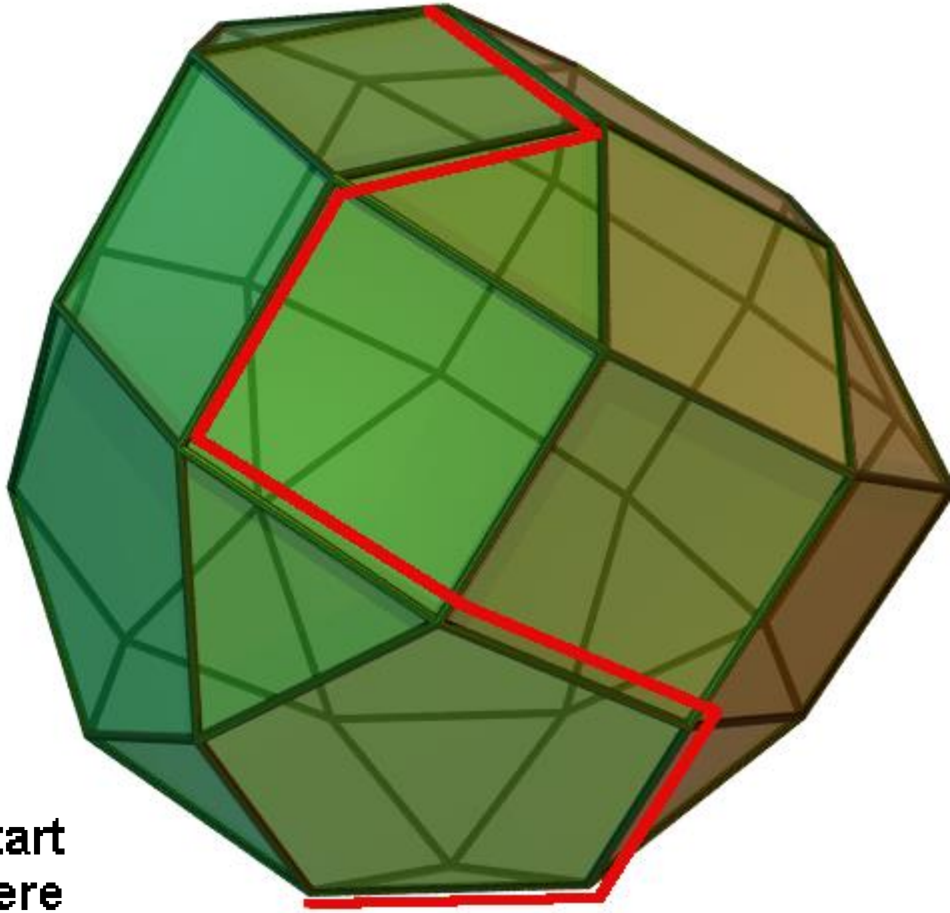


(Can there be infinite solutions? Multiple solutions?)

Simplex Method for Solving an LP

At every step, move to an adjacent vertex that improves the objective.





Trail Mix Problem

- Acme Snacks produces three different trail mix blends by mixing the following raw ingredients: peanuts, raisins, soynuts, and pretzels.

- These ingredients contain the following nutrients: vitamins, protein, calcium, and fat in the following quantities:

	Nutrient, k			
Ingredient, i	Vitamins	Protein	Calcium	Crude Fat
peanuts	8	10	6	8
raisins	6	5	10	6
soynuts	10	12	6	6
pretzels	4	18	6	9

Let a_{ik} = quantity of nutrient k per kg of ingredient i

Constraints

- *The company contracts for the following demands.*

Demand (kg)	Mix 1	Mix 2	Mix 3
d_j	10,000	6,000	8,000

- There are limited availabilities of the raw ingredients.

Supply (kg)	peanuts	raisins	soynuts	pretzels
s_i	6,000	10,000	4,000	5,000

- The different mixes have "quality" bounds per kilogram.

	Vitamins		*Protein*		*Calcium*		*Crude fat*	
	min	*max*	*min*	*max*	*min*	*max*	*min*	*max*
Mix 1	*6*	--	*6*	--	*7*	--	*4*	*8*
Mix 2	*6*	--	*6*	--	*6*	--	*4*	*8*
Mix 3	*4*	*6*	*6*	--	*6*	--	*4*	*8*

The above values represent bounds: l_{jk} *and* u_{jk}

Costs and Notation

Cost per kg of the raw ingredients is as follows:

	peanuts	*raisins*	*soynuts*	*pretzels*
cost/kg, c_i	*20¢*	*12¢*	*24¢*	*12¢*

Formulate problem as a linear program whose solution yields desired trail mix production levels at minimum cost.

Indices/ sets

$i \in I$ *ingredients { peanuts, raisins, soynuts, pretzels }*

$j \in J$ *products { mix 1, mix 2, mix 3}*

$k \in K$ *nutrients { vitamins, protein, calcium, crude fat }*

Data

d_j	*demand for product j (kg)*
s_i	*supply of ingredient i (kg)*
l_{jk}	*lower bound on number of nutrients of type k per kg of product j*
u_{jk}	*upper bound on number of nutrients of type k per kg of product j*
c_i	*cost per kg of ingredient i*
a_{ik}	*number of nutrients k per kg of ingredient i*

Decision Variables

x_{ij}	*amount (kg) of ingredient i used in producing product j*

LP Formulation of Mix Problem

$$\min \quad \sum_{i\in I}\sum_{j\in J} c_i x_{ij}$$

$$s.t. \quad \sum_{i\in I} x_{ij} = d_j \qquad \forall\, j \in J$$

$$\sum_{j\in J} x_{ij} \le s_i \qquad \forall\, i \in I$$

$$\sum_{i\in I} a_{ik} x_{ij} \ge l_{jk}\, d_j \qquad \forall\, j \in J,\ k \in K$$

$$\sum_{i\in I} a_{ik} x_{ij} \le u_{jk} d_j \qquad \forall\, j \in J,\ k \in K$$

$$x_{ij} \ge 0 \qquad \forall\, i \in I,\ j \in J$$

LP:Labor Planning

Addresses staffing needs over a specific time period.

Hong Kong Bank of Commerce:

- 12 Full time workers available, but may fire some.
- Use part time workers who has to work for 4 consequtive hours in a day.
- Luch time is one hour between 11a.m. and 1p.m. shared by full time workers.
- Total part time hours is less than 50% of the day’s total requirement.
- Part-timers earn $4/hr (=$16/day) and full timers earn $50/day.

LP:Labor Planning (Cont'd.)

Time Period	Minimum labor required
9a.m.-10a.m.	10
10a.m.-11a.m.	12
11a.m.-noon	14
Noon-1p.m.	16
1p.m.-2p.m.	18
2p.m.-3p.m.	17
3p.m.-4p.m.	15
4p.m.-5p.m.	10

LP:Labor Planning (cont'd.)

F : # Full time tellers per day

P_i : # Part time tellers who start work at time slot $i, i = 1,2\ldots,5.$

$$Min\ Daily\ Personnel\ Cost = \$50F + \$16\sum P_i$$

$$\begin{aligned}
F + P_1 \quad & \geq 10 \\
F + P_1 + P_2 \quad & \geq 12 \\
0.5F + P_1 + P_2 + P_3 \quad & \geq 14 \\
0.5F + P_1 + P_2 + P_3 + P_4 \quad & \geq 16 \\
F + P_2 + P_3 + P_4 + P_5 & \geq 18 \\
F + P_3 + P_4 + P_5 & \geq 17 \\
F + P_4 + P_5 & \geq 15 \\
F + P_5 & \geq 10
\end{aligned}$$

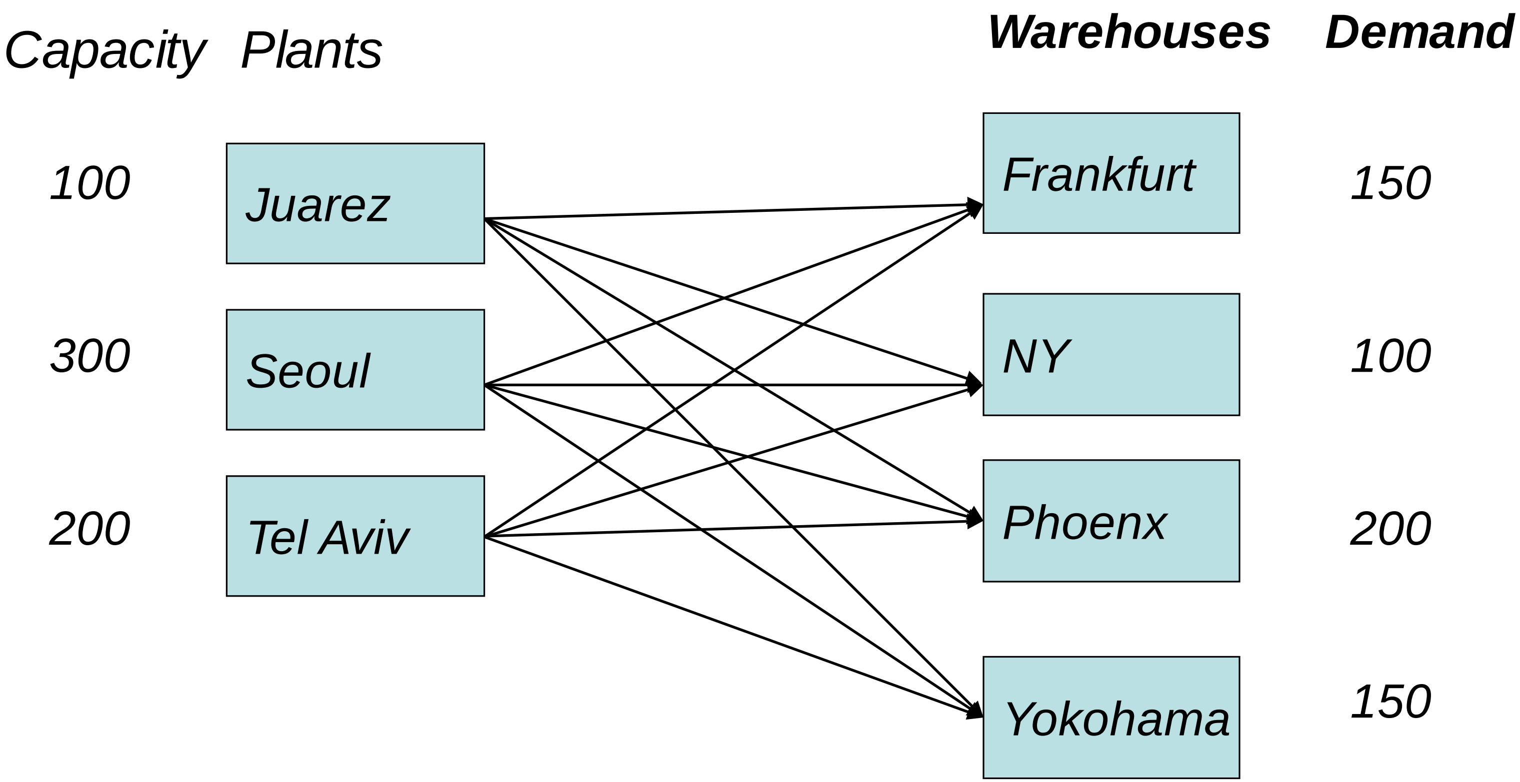
$$F \leq 12$$

$$4\sum P_i \leq 0.5(10 + 12 + 14 + \ldots + 10)$$

$$F, P_i \geq 0$$

Applications of LP:Transportation Models

Sporting goods company



LP:Transportation Models (cont'd.)

What are the optimal shipping quantities from the plants to the warehouses, if the demand has to be met by limited capacities while the shipping cost is minimized?

Shipping Costs per pair of skis

	Destination			
From Plant	Frankfurt	NY	Phoenix	Yokohama
Juarez	$19	$7	$3	$21
Seoul	15	21	18	6
Tel Aviv	11	14	15	22

LP:Transportation Models (cont'd.)

X_{ij}: Number of units shipped from plant i to warehouse j. i=1,2,3 and j=1,2,3,4.

Minimize shipping costs=$19X_{11}+7X_{12}+3X_{13}+21X_{14}$
$+15X_{21}+21X_{22}+18X_{23}+6X_{24}$
$+11X_{31}+14X_{32}+15X_{33}+22X_{34}$

From Plant	Destination: Frankfurt	NY	Phoenix	Yokohama	Capacity
Juarez	X11	X12	X13	X14	100
Seoul	X21	X22	X23	X24	300
Tel Aviv	X31	X32	X33	X34	200
Demand	150	100	200	150	600

LP:Transportation Models (cont'd.)

subject to

#shipped from a plant can not exceed the capacity:

$$X_{11}+X_{12}+X_{13}+X_{14}\le 100 \text{ (Juarez Plant)}$$

$$X_{21}+X_{22}+X_{23}+X_{24}\le 300 \text{ (Seoul Plant)}$$

$$X_{31}+X_{32}+X_{33}+X_{34}\le 200 \text{ (Tel Aviv Plant)}$$

#shipped to a warehouse can not be less than the demand:

$$X_{11}+X_{21}+X_{31}\ge 150 \text{ (Frankfurt)}$$

$$X_{12}+X_{22}+X_{32}\ge 100 \text{ (NY)}$$

$$X_{13}+X_{23}+X_{33}\ge 200 \text{ (Phoenix)}$$

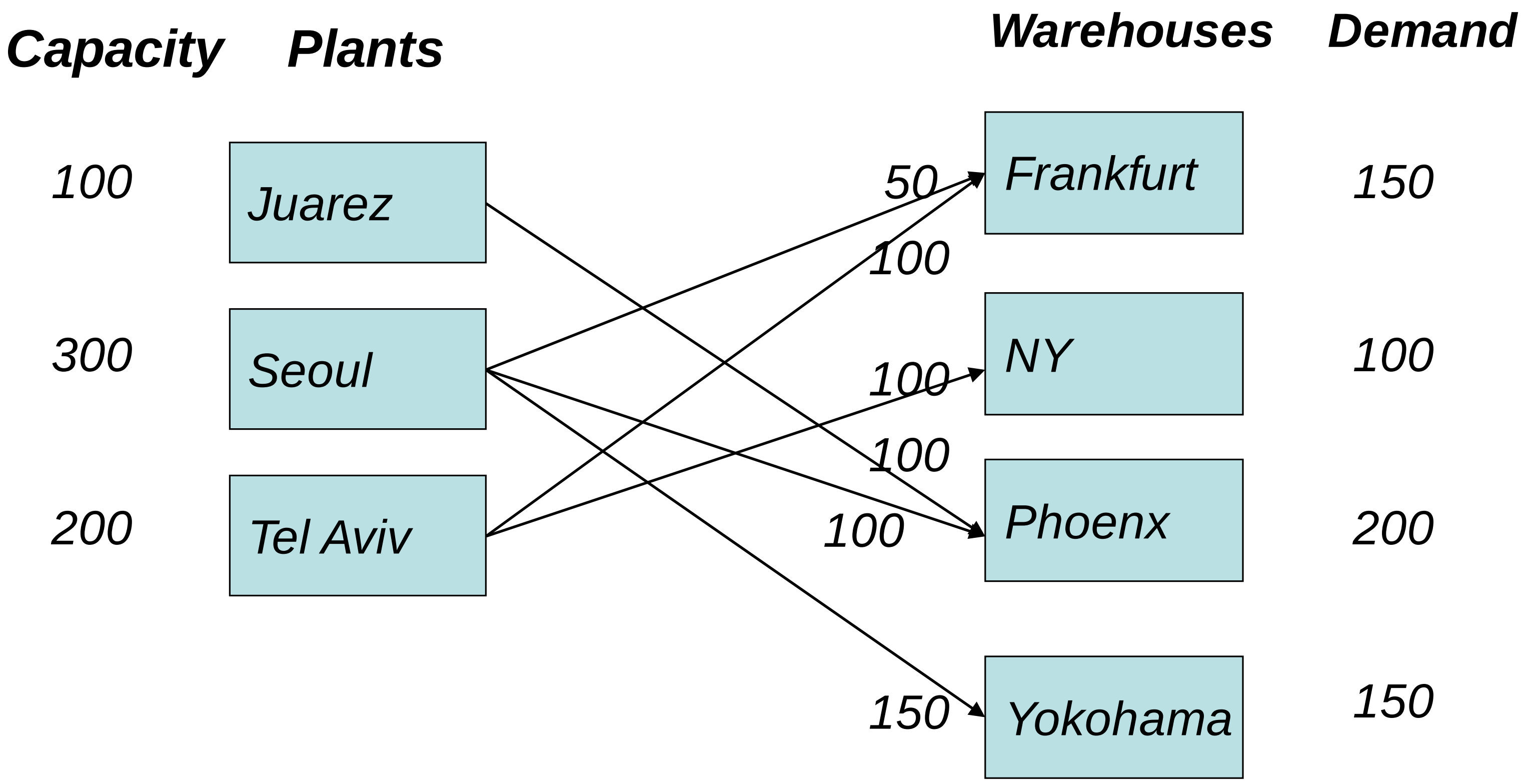
$$X_{14}+X_{24}+X_{34}\ge 150 \text{ (Yokohama)}$$

Nonnegativity

$$X_{ij}\ge 0 \text{ for all i,j.}$$

LP:Transportation Models (cont'd.)

Optimal Solution: Optimal cost=$6,250



Decision Problem

min(max (x1, x2, x3))

ST 3x1 + 2x2 – 5x3 <= 8

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min(max (x1, x2, x3))

ST 3x1 + 2x2 – 5x3 <= 8

min = t

ST $3x_1+2x_2- 5x_3 <=8$

$x_1<=t$ x1 – t <= 0

$x_2<=t$ x2 – t <= 0

$x_3<=t$ x3 – t <= 0

Decision Problem

min = t

ST $3x_1+2x_2- 5x_3$ <=8

x_1<=t   x1 – t <= 0

x_2<=t   x2 – t <= 0

x_3<=t   x3 – t <= 0

Unbounded: x's can keep getting smaller

min (max(x1, x2, x3))

			max	$3x1+2x2-\ 5x3$ <= 8	
x1	**x2**	**x3**	**0**	**constraint**	**met**
2	1	5	5	-17	TRUE
-2	-3	8	8	-52	TRUE
3	-2	2	3	-5	TRUE
-10	-20	-7	-7	-35	TRUE
-10	-20	-8	-8	-30	TRUE
-10	-20	-9	-9	-25	TRUE
-12	-22	-10	-10	-30	TRUE
-14	-24	-11	-11	-35	TRUE
-16	-26	-12	-12	-40	TRUE
-18	-28	-13	-13	-45	TRUE
-20	-30	-14	-14	-50	TRUE
-22	-32	-15	-15	-55	TRUE
-24	-34	-16	-16	-60	TRUE
-26	-36	-17	-17	-65	TRUE
-28	-38	-18	-18	-70	TRUE
-30	-40	-19	-19	-75	TRUE
-32	-42	-20	-20	-80	TRUE
-34	-44	-21	-21	-85	TRUE
-36	-46	-22	-22	-90	TRUE
-38	-48	-23	-23	-95	TRUE
-40	-50	-24	-24	-100	TRUE
-42	-52	-25	-25	-105	TRUE
		min	-25		

Shortest Paths

We can compute the length of the shortest path from s to t in a weighted directed graph by solving the following very simple linear programming problem.

$$\begin{array}{lll} \text{maximize} & d_t & \\ \text{subject to} & d_s = 0 & \\ & d_v - d_u \leq \ell_{u\to v} & \text{for every edge } u\to v \end{array}$$

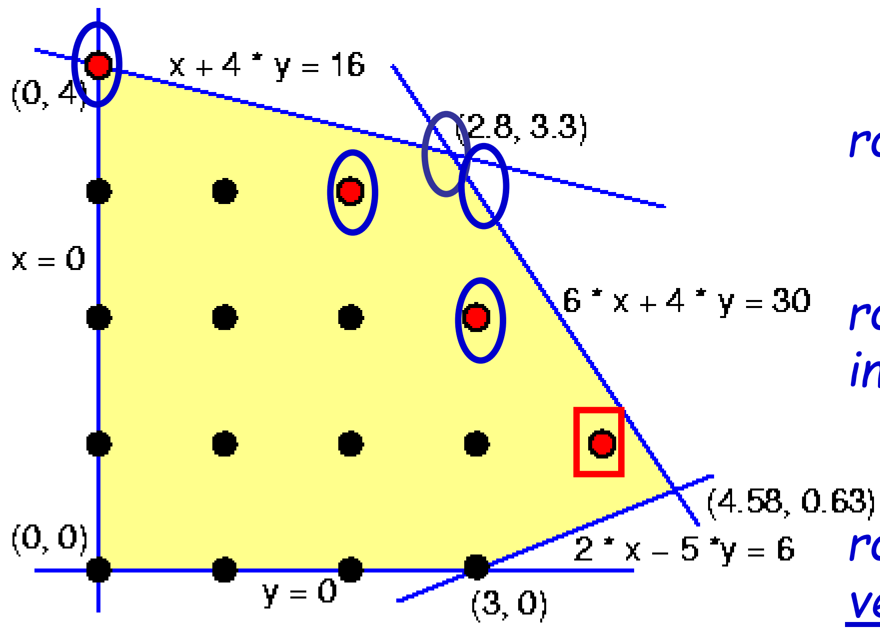
Here, $\ell_{u\to v}$ is the length of the edge $u\to v$. Each variable d_v represents a tentative shortest-path distance from s to v. The constraints mirror the requirement that every edge in the graph must be relaxed. These relaxation constraints imply that in any feasible solution, d_v is *at most* the shortest path distance from s to v. Thus, somewhat counterintuitively, we are correctly *maximizing* the objective function to compute the *shortest* path! In the optimal solution, the objective function d_t is the actual shortest-path distance from s to t, but for any vertex v that is not on the shortest path from s to t, d_v may be an underestimate of the true distance from s to v. However, we can obtain the true distances from s to every other vertex by modifying the objective function:

Variations

- Integer Programming
- Mixed Integer Programming

Integer Linear Programming (ILP)

Name	Vars	Constraints	Objective
integer linear prog. (ILP)	integer	linear inequalities	linear function



round to nearest int (3,3)?
No, infeasible.

round to nearest feasible int (2,3) or (3,2)?
No, suboptimal.

round to nearest integer <u>vertex</u> (0,4)?
No, suboptimal.

Function to maximize: $f(x, y) = 6 * x + 5 * y$

Optimum LP solution $(x, y) = (2.8, 3.3)$

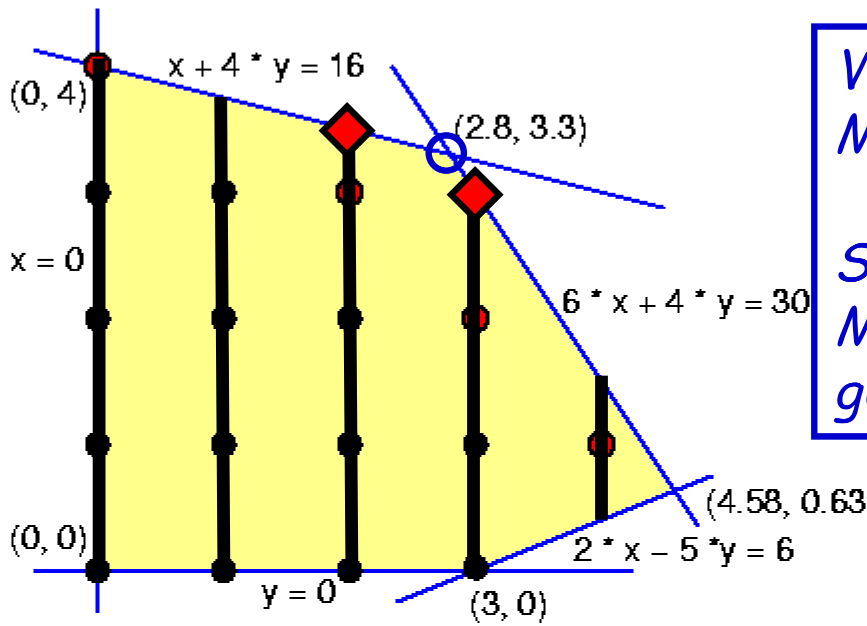
Pareto optima: (0, 4), (2, 3), (3, 2), (4, 1)

Optimum ILP solution $(x, y) = (4, 1)$

Mixed Integer Programming (MIP)

Name	Vars	Constraints	Objective
linear programming (LP)	real	linear inequalities	linear function
integer linear prog. (ILP)	integer	linear inequalities	linear function
mixed integer prog. (MIP)	int&real	linear inequalities	linear function

x still integer but y is now real



We'll be studying MIP solvers.

SCIP mainly does MIP though it goes a bit farther.

Figure 7.2 The feasible polyhedron for a three-variable linear program.