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CS325

HW 6

1. Shortest Paths using LP: (7 points)
Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables. • We can compute the shortest path from s to t in a weighted directed graph by solving. max dt subject to ds = 0 dv – du ≤ w(u,v) for all (u,v)∈E • We can compute the single-source by changing the objective function to max $\sum v \in V\ dv$ Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.
Objectives Function/Constraints
max dt
Ds = 0
Dv – du =< w(u, v) for all (u, v)
See Excel

a) Find the distance of the shortest path from G to C in the graph below.
Distance: 16
b) Find the distances of the shortest paths from G to all other vertices.
G-A: 7 (GHA)
G-B: 11 (GHB)
G-C: 16 (GHBC)
G-D: 2 (GD)
G-E: 21 (GHBE)
G-F: 17 (GHAF)
G-H: 3 (GH)

2. Product Mix: (7 points) Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is $0.75 per tie for all four types of ties.

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

Answer: See Excel file for the formulation of a linear program
<u>Optimal Number of Ties</u>
Silk = 7000
Poly = 12625
Blend 1 = 13100
Blend 2 = 8500
Total: 119686

3. Transshipment Model (10 points) This is an extension of the transportation model. There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant (pi) must be shipped to a Warehouse (wj) before being shipped to the Retailer (rk). Each Plant will have an associated supply (si) and each Retailer will have a demand (dk). The number of plants is n, number of warehouses is q and the number of retailers is m. The edges (i,j) from plant (pi)to warehouse (wj) have costs associated denoted cp(i,j). The edges (j,k) from a warehouse (wj)to a retailer (rk) have costs associated denoted cw(j,k).

Answer: See Excel Sheet 2
Part A: Determine the number of refrigerators to be shipped from the plants to the warehouses and then warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost?
Minimum Cost: $17,100
Optimal Shipping Routes, where Key = Plant1-Warehouse1 (P1-W1)

P1-W1	150
P1-W2	0
P2-W1	200
P2-W2	250
P3-W1	0
P3-W2	150
P3-W3	100
P4-W2	0
P4-W3	150
W1-R1	100
W1-R2	150
W1-R3	100
W1-R4	0
W2-R3	0
W2-R4	200
W2-R5	200
W2-R6	0
W3-R4	0
W3-R5	0
W3-R6	150
W3-R7	100

Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?

There is no optimal solution. There is no feasible solution to meet all the demand for all the warehouses that are available.

Part C: Instead of closing Warehouse 2 management has decide to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?
Yes, it is feasible.

P1-W1	150
P1-W2	0
P2-W1	350
P2-W2	100
P3-W1	0
P3-W2	0
P3-W3	250
P4-W2	0
P4-W3	150
W1-R1	100
W1-R2	150
W1-R3	100
W1-R4	150
W2-R3	0
W2-R4	50
W2-R5	50
W2-R6	0
W3-R4	0
W3-R5	150
W3-R6	150
W3-R7	100

4. Making Change (6 points)
Given coins of denominations (value) $1 = v1 < v2 < \ldots < vn$, we wish to make change for an amount A using as few coins as possible. Assume that vi's and A are integers. Since v1= 1 there will always be a solution. Solve the coin change using integer programming. For each the following denomination sets and amounts formulate the problem as an integer program with an objective function and constraints, determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used?

Answer: Used Python to implement the algorithm for this/standard CoinChange coding challenge

a) V = [1, 5, 10, 25] and A = 202.

10 Coins.
8x of 25, 2x of 1

b) V = [1, 3, 7, 12, 27] and A = 293
14 Coins.
10x of 27, 1x of 1, 3, 7, and 12