Final Review Problems

1. Show that the Hamiltonian Cycle problem for directed graphs is in NP-Complete.

2. Macrosoft has a 24-hour-a-day, 7-days-a-week toll free hotline that is being set up to answer questions regarding a new product. The following table summarizes the number of full-time equivalent employees (FTEs) that must be on duty in each time block. Macrosoft may hire both full-time and part-time employees.

- Full-timers work 8-hour shifts with hourly wages of $15.20
- Part-timers work 4-hour shifts with hourly wages of $12.95.
- Employees may start work only at the beginning of 1 of the 6 intervals.
- Part-time employees can only answer 5 calls in the time a full-time employee can answer 6 calls. (i.e., a part-time employee is only 5/6 of a full-time employee.)
- At least two-thirds of the employees working at any one time must be full-time employees.

Formulate an LP to determine how to staff the hotline at minimum cost.

Interval	Time	FTEs
1	0-4	15
2	4-8	10
3	8-12	40
4	12-16	70
5	16-20	40
6	20-0	35

3. Suppose you have four production plants for making cars. Each works a little differently in terms of labor needed, materials, and energy used per car:

Location	labor hrs	materials	energy
plant1	2	3	15
plant2	3	4	10
plant3	4	5	9
plant4	5	6	7

Suppose we need to produce at least 400 cars at plant 3 according to a labor agreement. We have 3300 hours of labor and 4000 units of material available. We are allowed to use at most 12000 units of energy, and we want to maximize the number of cars produced. Formulate this as a linear programming problem: (1) what are the variables, (2) what is the objective in terms of these variables, and (3) what are the constraints.

4. Consider the problem

4-SAT: Given a CNF formula Φ with four literals per clause, is there a satisfying assignment?

Prove that 4-SAT is NP-complete.

Final Review Problems

5. Let G = (V, E) be a DAG, where each edge is annotated with some positive length. Let s be a source vertex in G. Suppose we run Dijkstra's algorithm to compute the distance from s to each vertex $v \in V$, and then order the vertices in increasing order of their distance from s. Are we guaranteed that this is a valid topological sort of G?

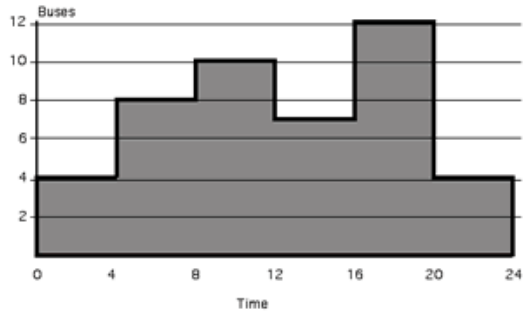
6. Consider a connected weighted directed graph G = (V, E, w). Define the *fatness* of a path P to be the maximum weight of any edge in P. Give an efficient algorithm that, given such a graph and two vertices $u,v \in V$, finds the minimum possible fatness of a path from u to v in G.

7. Let T be a complete binary tree with n vertices. Finding a path from the root of T to a given vertex v in T using breadth-first search takes

a) O(n)
b) O(lgn)
c) O(nlgn)
d) O(logn)
e) $O(n^2)$

8. A Hamiltonian path in a graph G=(V,E) is a simple that includes every vertex in V. Design an algorithm to determine if a directed acyclic graph (DAG) G has a Hamiltonian path. Your algorithm should run in O(V+E). Provide a written description of your algorithm including why it works, pseudocode and an explanation of the running time.

9. Consider a bus company scheduling drivers for its buses. The requirement for buses varies from hour to hour because of customer demand as shown in below.



Time 0 represents midnight, and times are shown with a 24 hour clock starting at midnight. For example, four buses must run from midnight to 4 a.m., while eight buses must run from 4 a.m. until 8 a.m. We assume that the bus requirements are the same every day.

Determine how many drivers to schedule at each starting time to cover the requirements for buses. Drivers work eight hour shifts that start at times: 0, 4, 8, 12, 16 or 20. For example, a driver starting at time 0 can drive a bus from time 0 to 8. A driver scheduled to start at time 20 works for the final four hours of the day and the first four hours of the next day. The goal is to minimize the number of drivers used. Note that although a driver can be hired for an eight hour period, there is no requirement that he drive a bus for the entire period. He might be idle for a four hour interval within the period. Formulate the problem as a linear program with an objective function and all constraints.

Final Review Problems

10. Consider a variant of BIN-PACKING Problem where the bin size is b (not necessarily an integer) and each item has size less than b/3. This version is still NP-complete, though we won't prove this here. Your problem is to give an algorithm that *approximates* the optimal packing into bins, and prove a bound on the quality of your approximation. In particular, prove that if your algorithm uses 3a+1 bins for some integer a, then the optimal algorithm uses at least 2a+1 bins.