**Question 1:** Consider the following problem

$$\max(x_1 + x_2)$$
$$s.t. |x_1 - x_2| \leq 10$$

Can I solve this problem with a linear program? If so, how?

**Answer:**

$$\begin{aligned} \max \quad & x_1 + x_2 \\ s.t. \quad & x_1 - x_2 \leq 10 \\ & x_1 - x_2 \geq -10 \end{aligned}$$

**This finds an unbounded solution.**

**Question 2:** Consider the following problem

$$\min(\max\{x_1, x_2, x_3\})$$
$$s.t. \quad 3x_1 + 2x_2 - 5x_3 \leq 8$$

Can I find a solution for this problem with a linear program? If so, how?

**Answer:**

$$\begin{aligned} \min \quad & t \\ s.t. \quad & 3x_1 + 2x_2 - 5x_3 \leq 8 \\ & x_1 \leq t \\ & x_2 \leq t \\ & x_3 \leq t \end{aligned}$$

**This finds an unbounded solution**

**Question 3**

7.2. Duckwheat is produced in Kansas and Mexico and consumed in New York and California. Kansas produces 15 shnupells of duckwheat and Mexico 8. Meanwhile, New York consumes 10 shnupells and California 13. The transportation costs per shnupell are \$4 from Mexico to New York, \$1 from Mexico to California, \$2 from Kansas to New York, and \$3 and from Kansas to California.

Write a linear program that decides the amounts of duckwheat (in shnupells and fractions of a shnupell) to be transported from each producer to each consumer, so as to minimize the overall transportation cost.

**Answer:** Let $x_{i,j}$ be the number of shnupells of duckwheat produced in city i and consumed in city j.

$$
\begin{array}{ll}
\min & 4 \cdot x_{M,N} + 1 \cdot x_{M,C} + 2 \cdot x_{K,N} + 3x_{K,C} \\
s.t. & x_{K,N} + x_{K,C} \le 15 \\
 & x_{M,N} + x_{M,C} \le 8 \\
 & x_{K,N} + x_{M,N} \ge 10 \\
 & x_{K,C} + x_{M,C} \ge 13 \\
 & x_{K,N}, x_{M,N}, x_{M,N}, x_{M,C} \ge 0
\end{array}
$$

Here I've represented the supplies as less-than-or-equal inequalities and the demands as greater-than-or-equal inequalities, because if you represent the demands as less-than-or-equal inequalities you get the trivial solution of zero shipments being sent.

**This finds an optimal solution**

## Question 4

7.3. A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, upto the maximum available limits given below.

- Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, and revenue \$1,000 per cubic meter.
- Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, and revenue \$1,200 per cubic meter.
- Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, and revenue \$12,000 per cubic meter.

Write a linear program that optimizes revenue within the constraints.

**Solution**

- Let $q_i$ denote the quantity (in cubic meters) of material $i$
- The linear program will be the following:

$$\text{maximize } 1000q1 + 1200q2 + 12000q3$$
$$2q1 + q2 + 3q3 \leq 100$$
$$q1 + q2 + q3 \leq 60$$
$$q1 \leq 40$$
$$q2 \leq 30$$
$$q3 \leq 20$$
$$q1, q2, q3 \geq 0$$