

## CS 325 - Homework 7

1. (6 pts) Let  $X$  and  $Y$  be two decision problems. Suppose we know that  $X$  reduces to  $Y$  in polynomial time. Which of the following can we infer? Explain.

- a. If  $Y$  is NP-complete then so is  $X$ .
- b. If  $X$  is NP-complete then so is  $Y$ .
- c. If  $Y$  is NP-complete and  $X$  is in NP then  $X$  is NP-complete.
- d. If  $X$  is NP-complete and  $Y$  is in NP then  $Y$  is NP-complete.
- e. If  $X$  is in P, then  $Y$  is in P.
- f. If  $Y$  is in P, then  $X$  is in P.

2. (4 pts) Consider the problem COMPOSITE: given an integer  $y$ , does  $y$  have any factors other than one and itself? For this exercise, you may assume that COMPOSITE is in NP, and you will be comparing it to the well-known NP-complete problem SUBSET-SUM: given a set  $S$  of  $n$  integers and an integer target  $t$ , is there a subset of  $S$  whose sum is exactly  $t$ ? Clearly explain whether or not each of the following statements follows from that fact that COMPOSITE is in NP and SUBSET-SUM is NP-complete:

- a. SUBSET-SUM  $\leq_p$  COMPOSITE.
- b. If there is an  $O(n^3)$  algorithm for SUBSET-SUM, then there is a polynomial time algorithm for COMPOSITE.
- c. If there is a polynomial algorithm for COMPOSITE, then  $P = NP$ .
- d. If  $P \neq NP$ , then **no** problem in NP can be solved in polynomial time.

3. (8 pts) A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. P that HAM-PATH =  $\{ (G, u, v) : \text{there is a Hamiltonian path from } u \text{ to } v \text{ in } G \}$  is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

4. (12 pts) K-COLOR. Given a graph  $G = (V, E)$ , a  $k$ -coloring is a function  $c: V \rightarrow \{1, 2, \dots, k\}$  such that  $c(u) \neq c(v)$  for every edge  $(u, v) \in E$ . In other words the number 1, 2, ...,  $k$  represent the  $k$  colors and adjacent vertices must have different colors. The decision problem K-COLOR asks if a graph can be colored with at most  $K$  colors.

- a. The 2-COLOR decision problem is in P. Describe an efficient algorithm to determine if a graph has a 2-coloring. What is the running time of your algorithm?
- b. It is known that the 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.