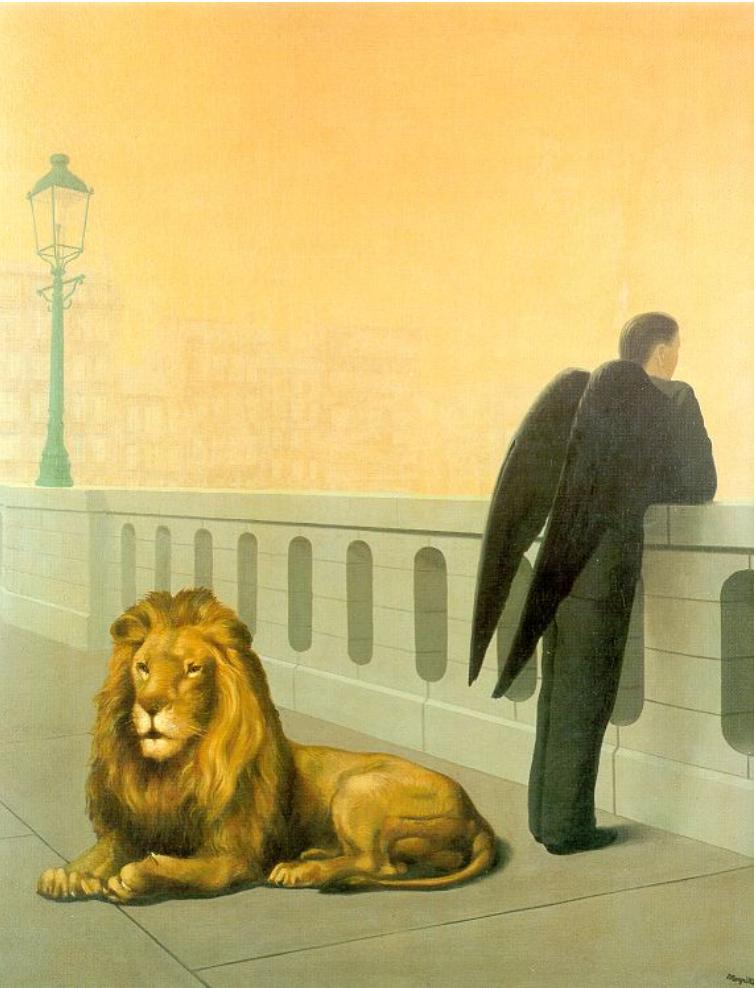


# Midterm Review



Magritte, *Homesickness*

Computational Photography

Derek Hoiem, University of Illinois

# Major Topics

- Linear Filtering
  - How it works
  - Template and Frequency interpretations
  - Image pyramids and their applications
  - Sampling (Nyquist theorem, application of low-pass filtering)
- Light and color
  - Lambertian shading, shadows, specularities
  - Color spaces (RGB, HSV, LAB)
  - Image-based lighting
- Techniques
  - Finding boundaries: intelligent scissors, graph cuts, where to cut and why
  - Texture synthesis: idea of sampling patches to synthesize, filling order
  - Compositing and blending: alpha compositing, Laplacian blending, Poisson editing
- Warping
  - Transformation matrices, homogeneous coordinates, solving for parameters via system of linear equations
- Modeling shape
  - Averaging and interpolating sets of points

# Major Topics

- Camera models and Geometry
  - Pinhole model: diagram, intrinsic/extrinsic matrices, camera center (or center of projection), image plane
  - Focal length, depth of field, field of view, aperture size
  - Vanishing points and vanishing lines (what they are, how to find them)
  - Measuring relative lengths based on vanishing points and horizon
- Interest points
  - Trade-offs between repeatability and distinctiveness for detectors and descriptors
  - Harris (corner) detectors and Difference of Gaussian (blob) detectors
  - SIFT representation: what transformations is it robust to or not robust to
- Image stitching
  - Solving for homography
  - RANSAC for robust detection of inliers
- Object recognition and search
  - Use of “visual words” to speed search
  - Idea of geometric verification to check that points have consistent geometry
- Other camera systems
  - Kinect, lightfield camera, synthetic aperture --- how do they work?

# Preparing for the Exam

Topics include:

- **Filtering** (inc. apply, design, interpret simple filters, frequency interpretation, properties of filters)
  - **Lighting** (inc. lighting and material models)
  - **Camera models** (inc. pinhole model and its applications, effects of lens and camera controls)
  - **Single view geometry** (inc. relative measurements within an image)
  - **Interest points and correspondence** (inc. interest point detectors, matching, retrieval based on interest points)
  - **Alignment/transformations** (inc. global transformations, setting up matrix to solve for a transformation, RANSAC)
  - **Color spaces, pyramids, and sampling**
  - "Topics of Interest" and other
- 
- Each bullet has 10-20 points allocated (out of 100)
  - There are 10 questions.
  - Questions about special topics typically require that you understand the key concepts that are presented.
- 
- **Bring a photo ID**
  - **You cannot use notes or calculator**

# Today's review

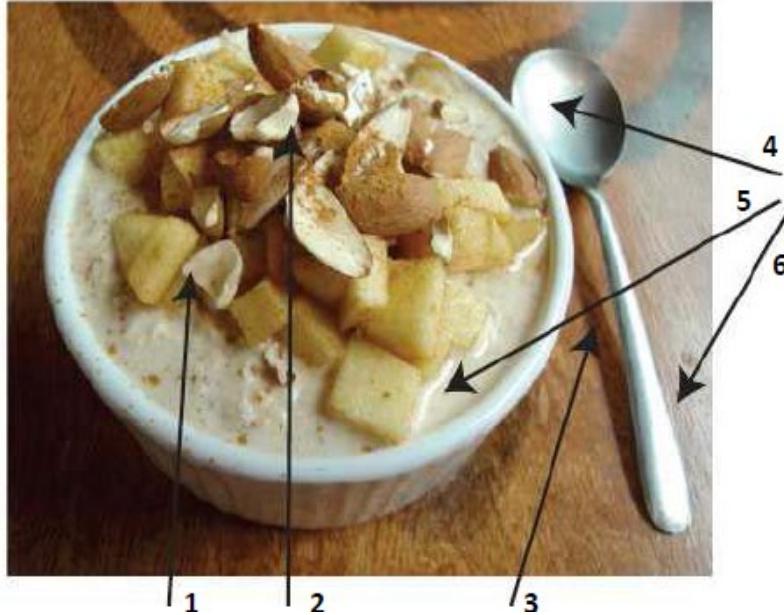
1. Light
2. Camera capture and geometry
3. Image filtering
4. Region selection and compositing
5. Solving for transformations

## Purposes

- Remind you of key concepts
- Chance for you to ask questions

# 1. Light and color

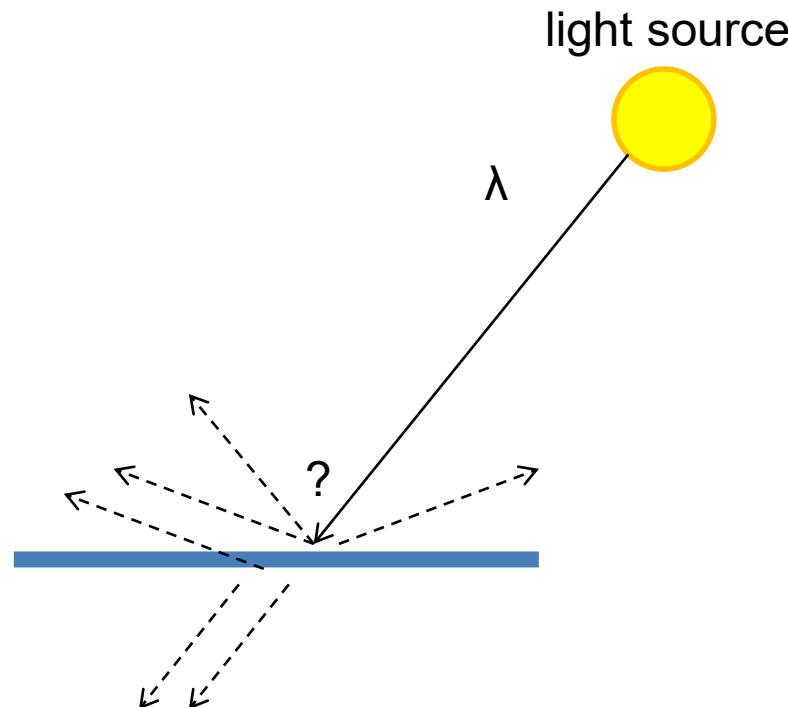
- Lighting
  - Lambertian shading, shadows, specularities
  - Color spaces (RGB, HSV, LAB)



# How is light reflected from a surface?

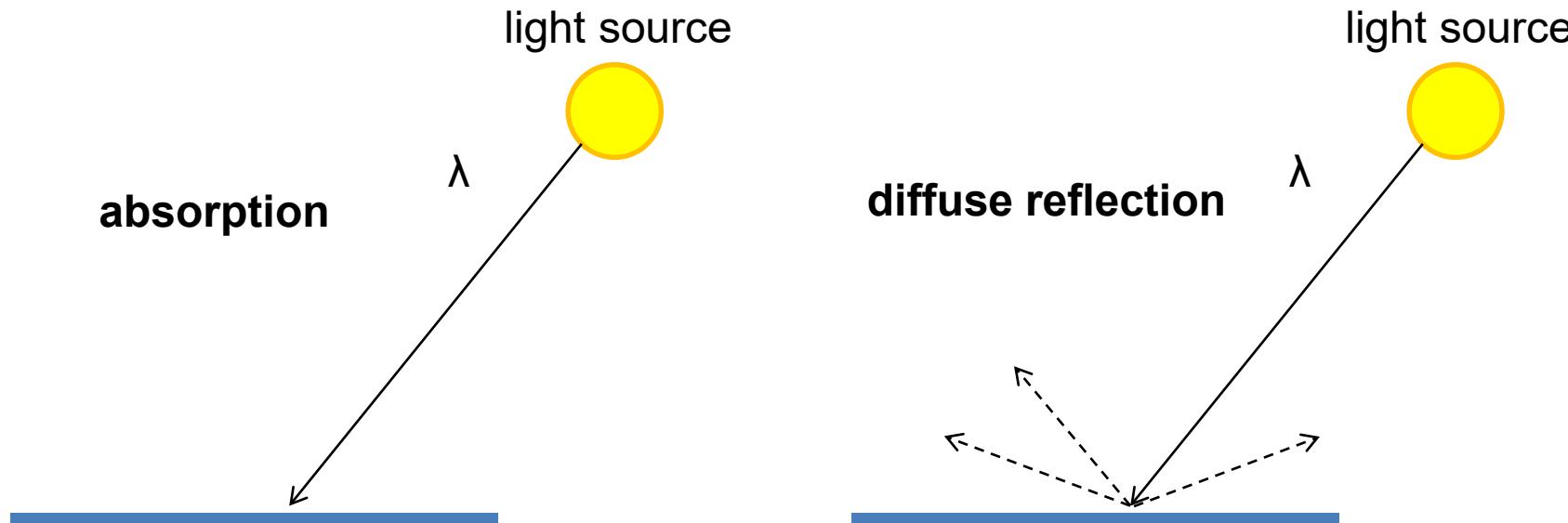
Depends on

- Illumination properties: wavelength, orientation, intensity
- Surface properties: material, surface orientation, roughness, etc.



# Lambertian surface

- Some light is absorbed (function of albedo)
- Remaining light is reflected equally in all directions (diffuse reflection)
- Examples: soft cloth, concrete, matte paints



# Diffuse reflection

*Intensity does depend on illumination angle*  
because less light comes in at oblique angles.

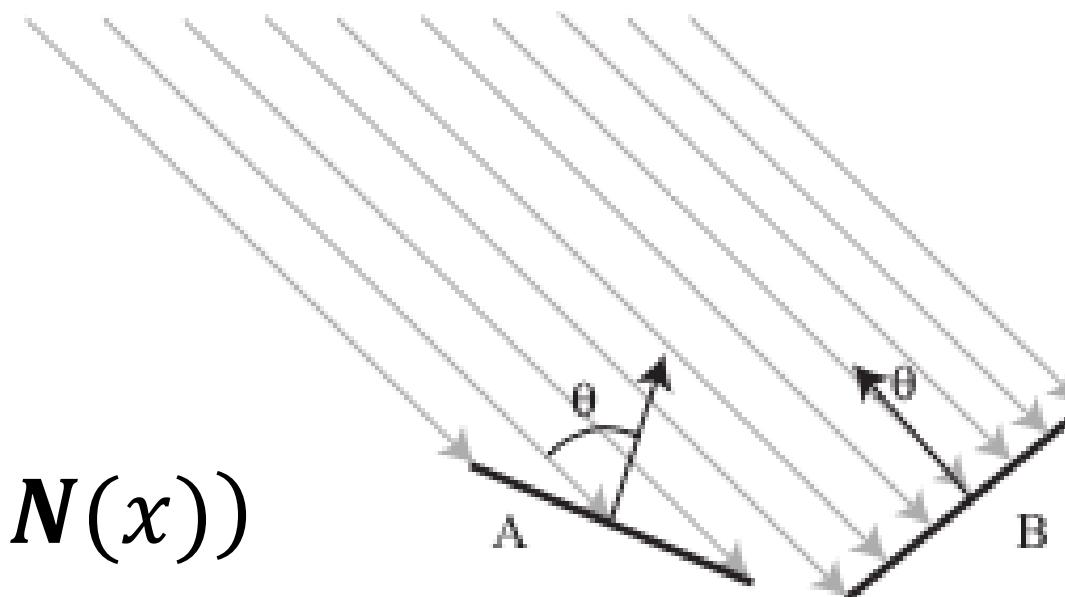
$\rho$  = albedo

$S$  = directional source

$N$  = surface normal

$I$  = image intensity

$$I(x) = \rho(x)(S \cdot N(x))$$

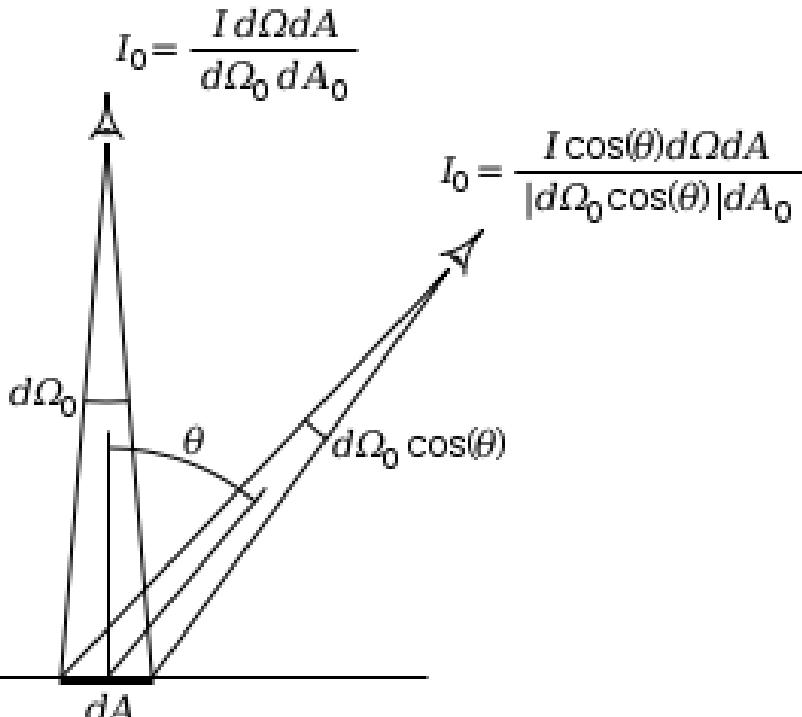
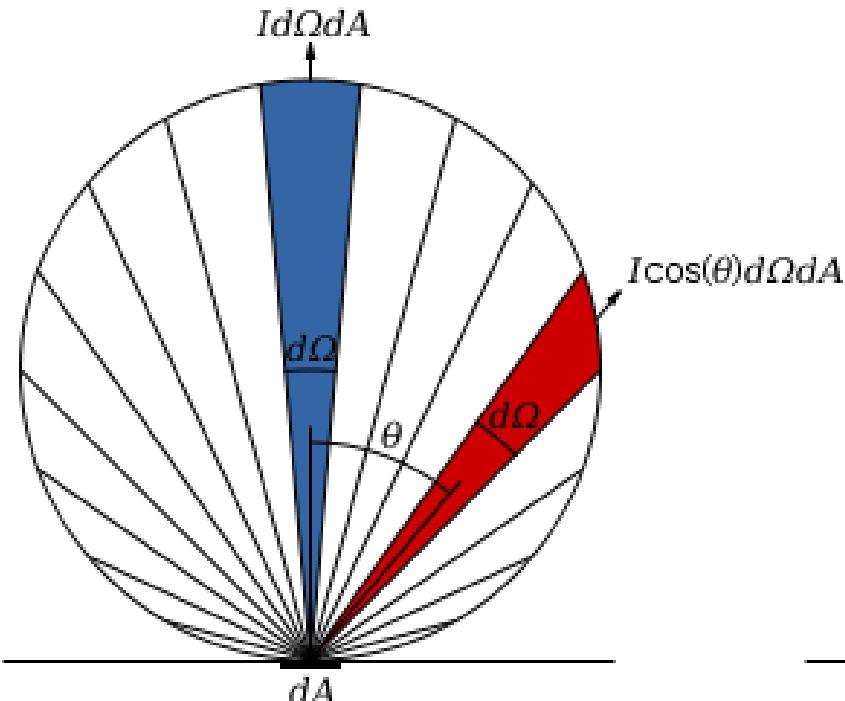




# Diffuse reflection

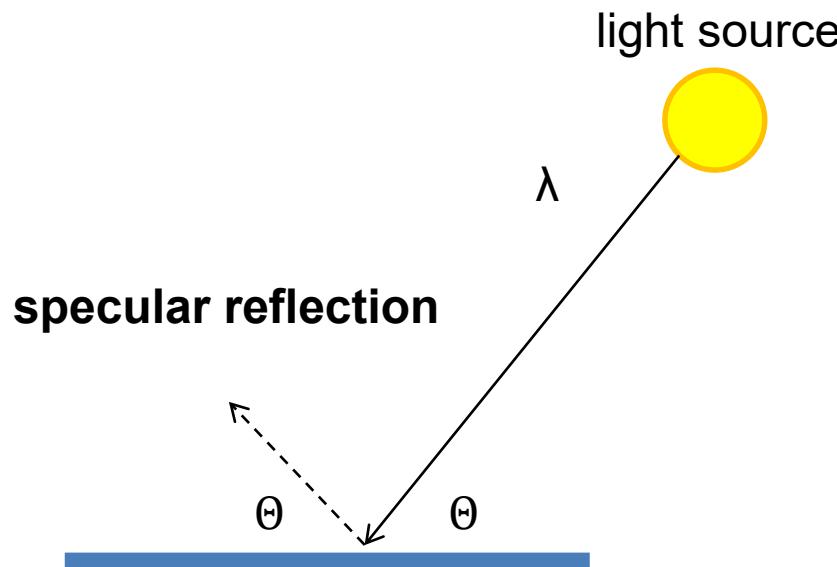
*Intensity does not depend on viewer angle.*

- Amount of reflected light proportional to  $\cos(\theta)$
- Visible solid angle also proportional to  $\cos(\theta)$



# Specular Reflection

- Reflected direction depends on light orientation and surface normal
- E.g., mirrors are mostly specular



Flickr, by piratejohnny

# Many surfaces have both specular and diffuse components

- Specularity = spot where specular reflection dominates (typically reflects light source)

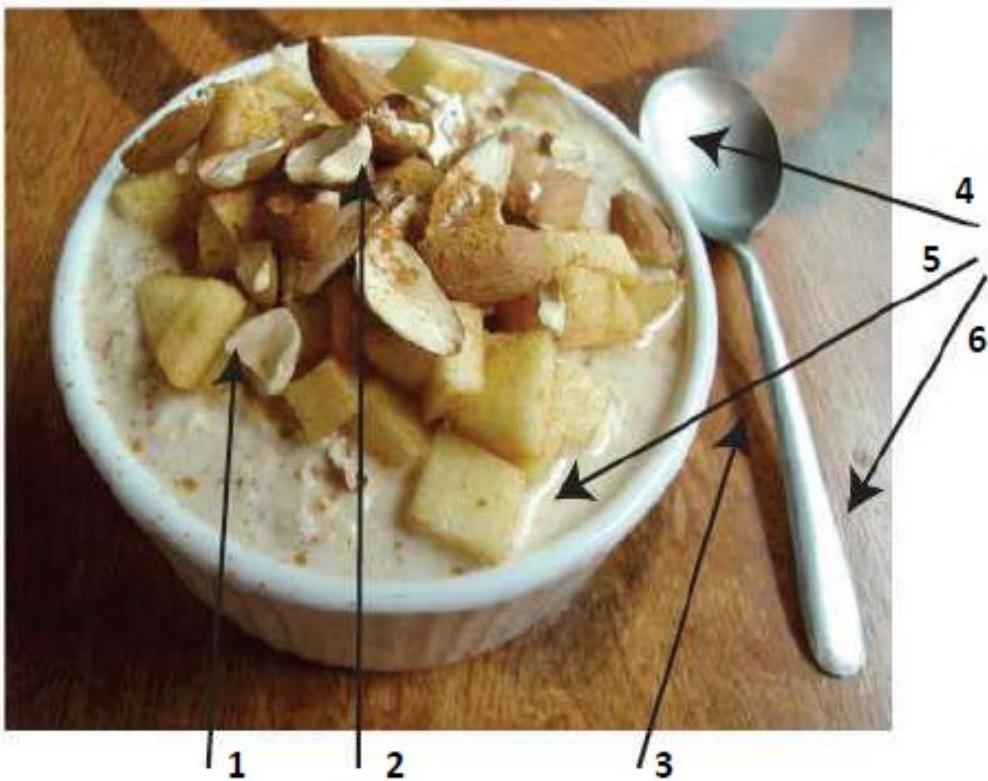


Photo: northcountryhardwoodfloors.com



# Questions

1.

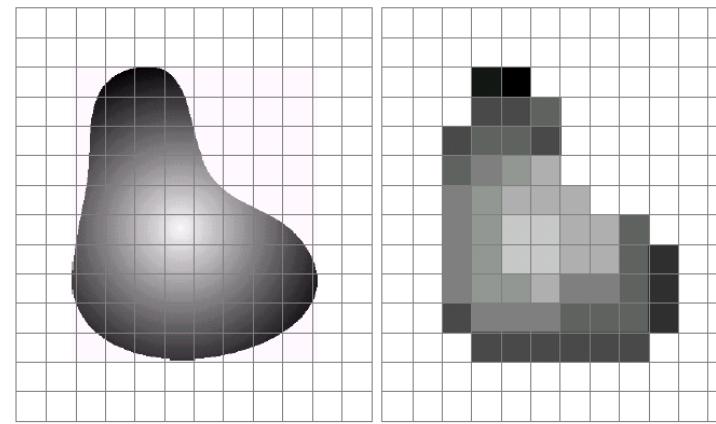
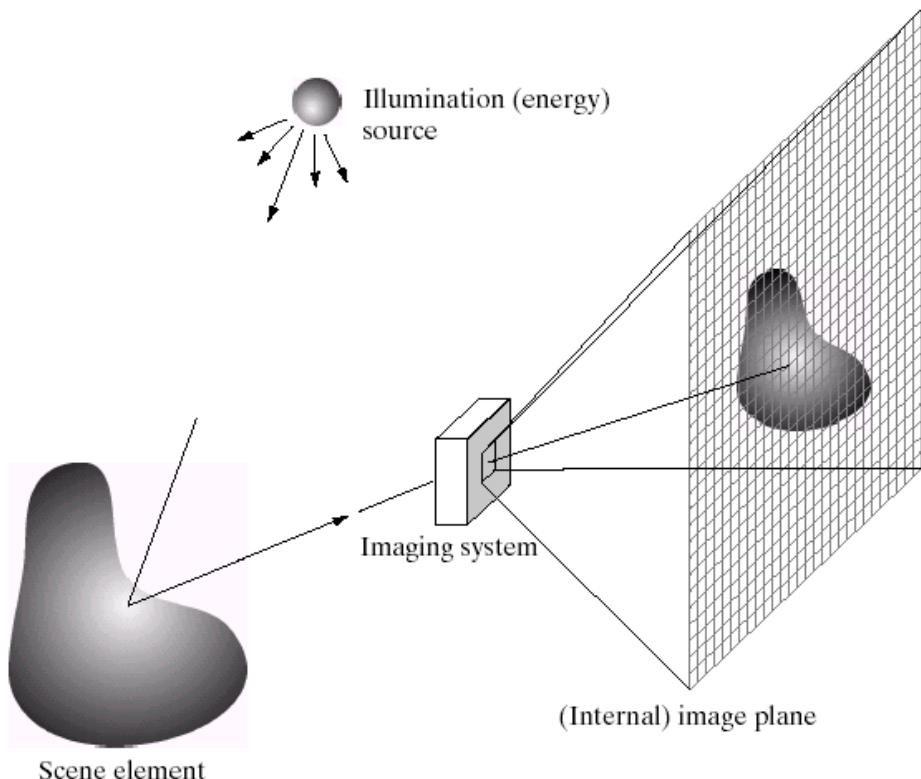


- A. For each of the arrows in the above image, name the reasons the pixel near the end of the arrow has its brightness value and explain very briefly. The arrow pointing to milk is pointing to the thin bright line at the edge of the piece of apple; the arrow pointing to the spoon handle is pointing to the bright area on the handle.

Possible factors: albedo, shadows, texture, specularities, curvature, lighting direction

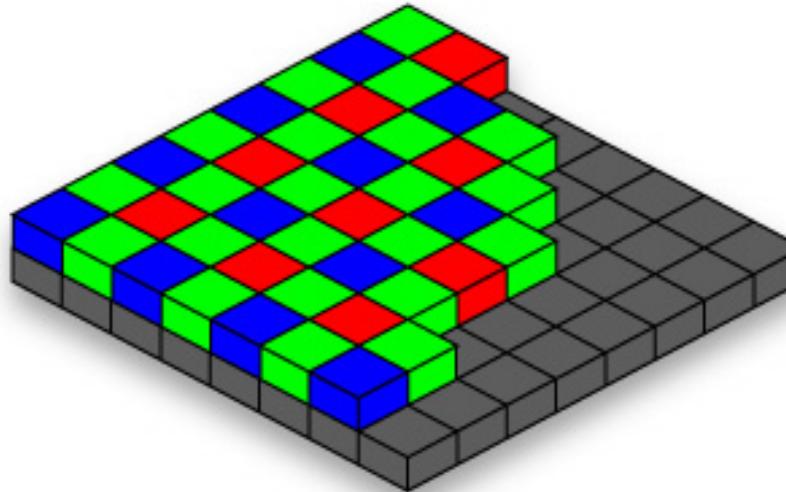
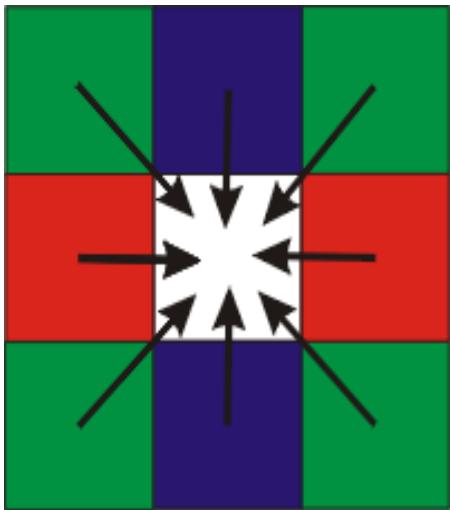
# Discretization

- Because pixel grid is discrete, pixel intensities are determined by a range of scene points

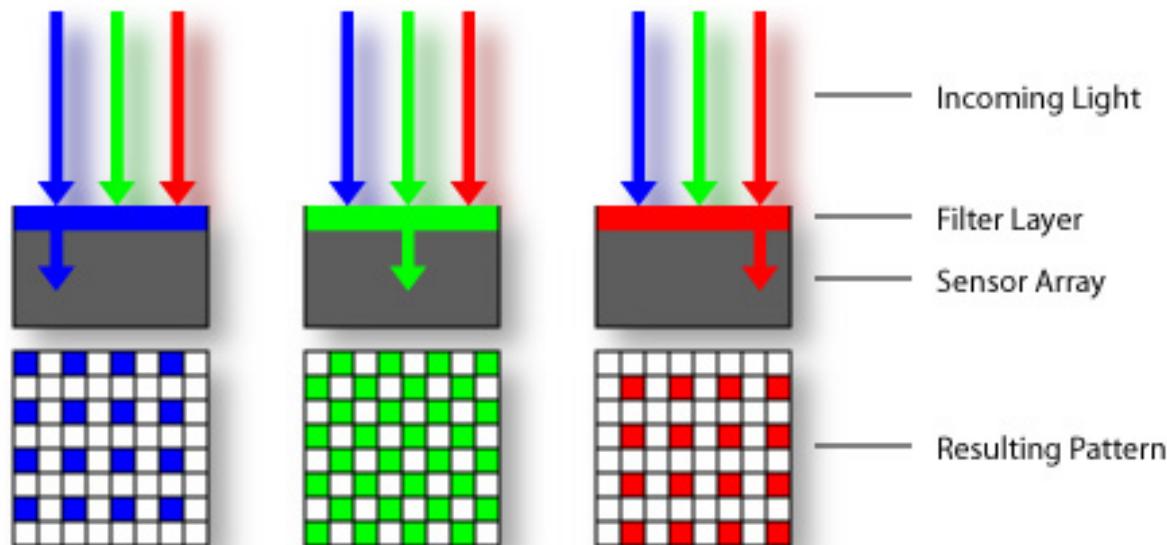


**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

# Color Sensing: Bayer Grid

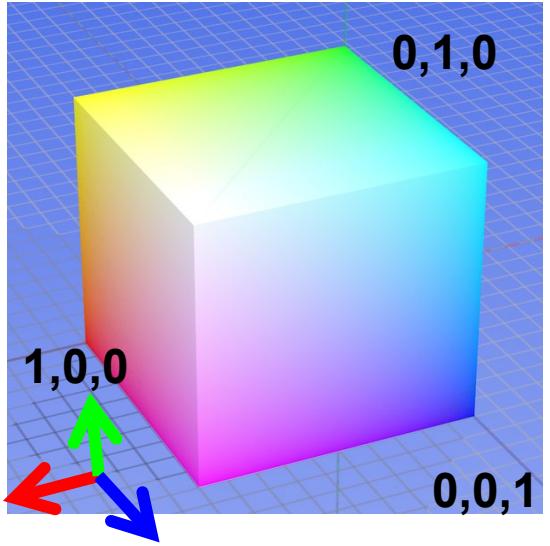


Estimate RGB at each cell  
from neighboring values

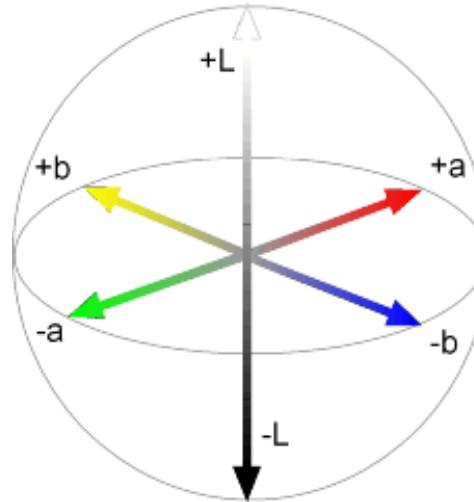


# Color spaces

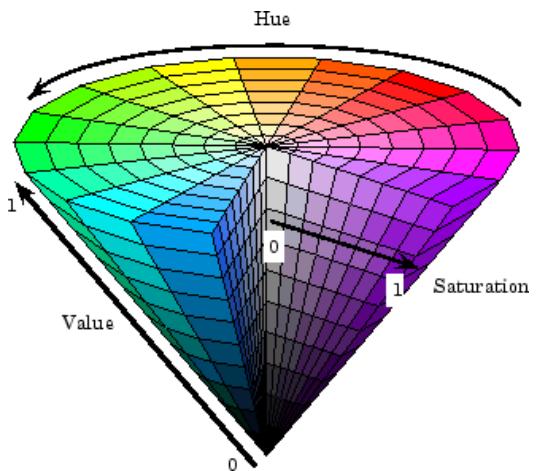
RGB



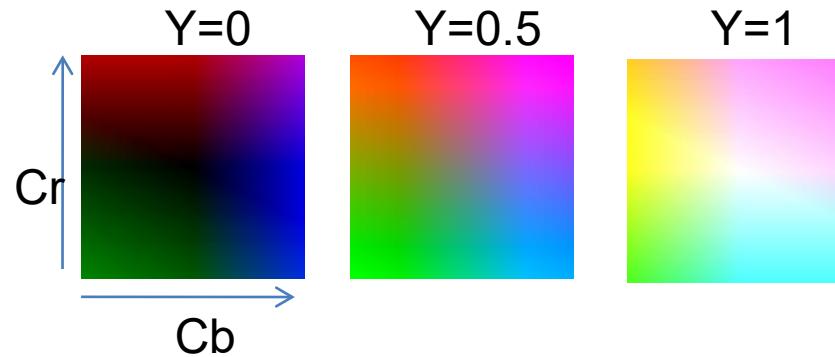
LAB



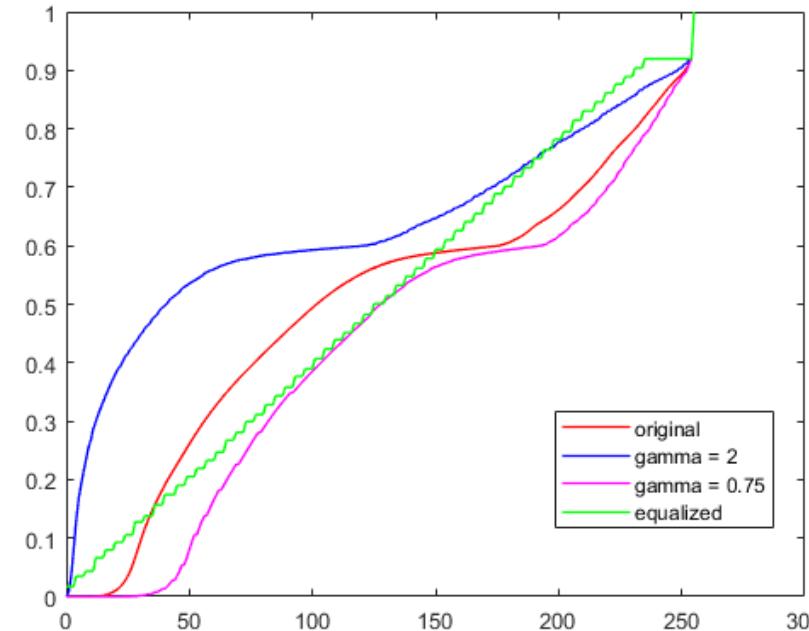
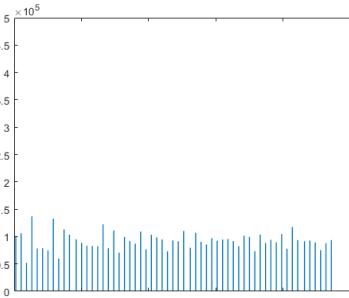
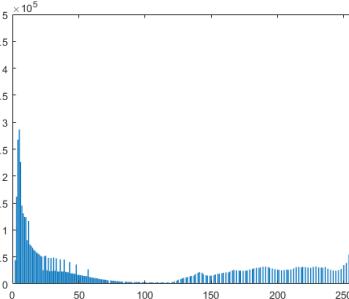
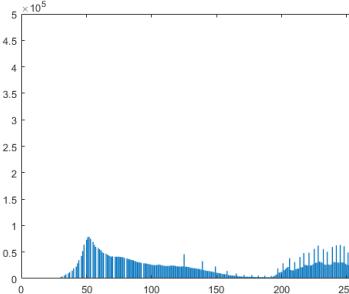
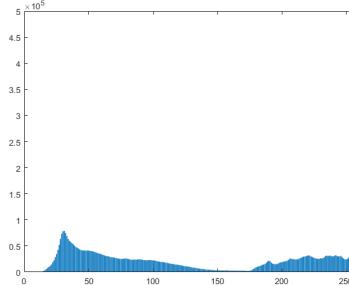
HSV



YCbCr



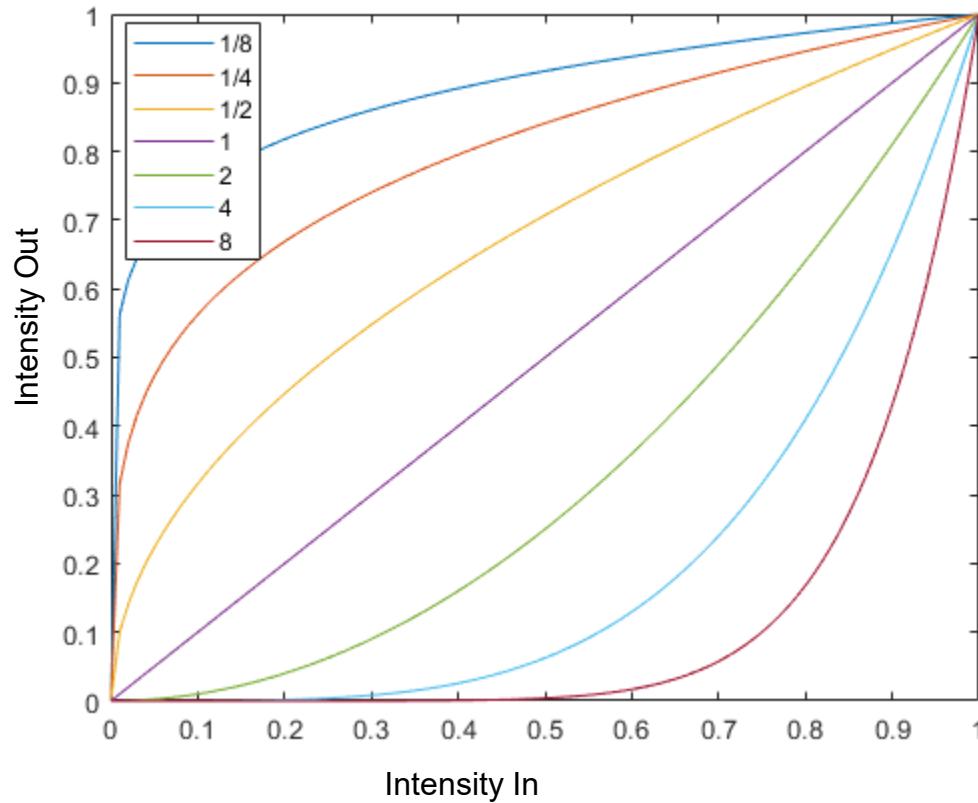
# Image Histograms



Cumulative Histograms

# Gamma adjustment

$$i_{out} = i_{in}^{\gamma}$$



$$\gamma = 0.5$$



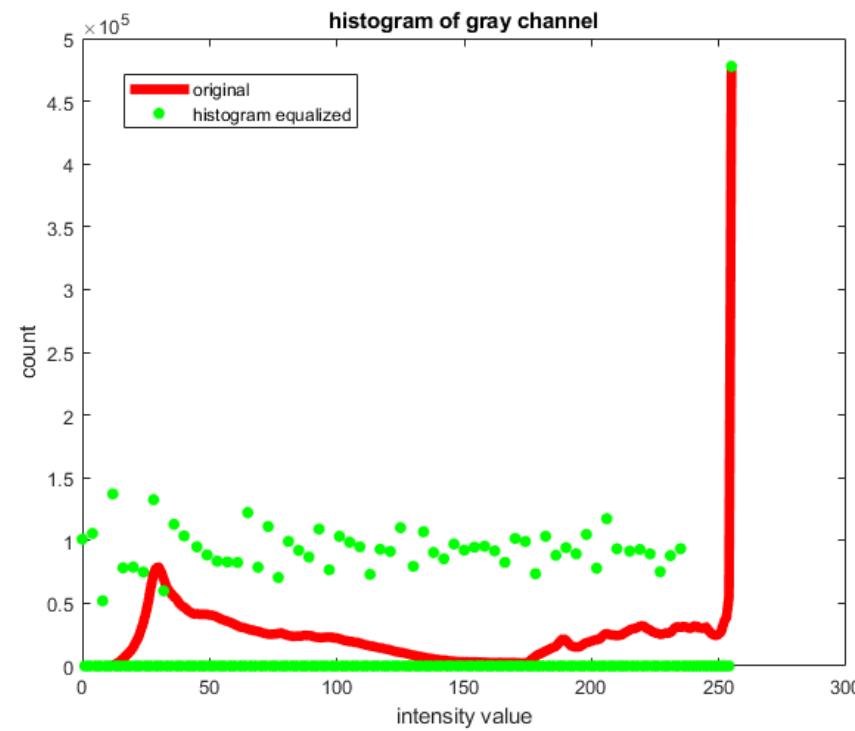
$$\gamma = 1$$



$$\gamma = 2$$

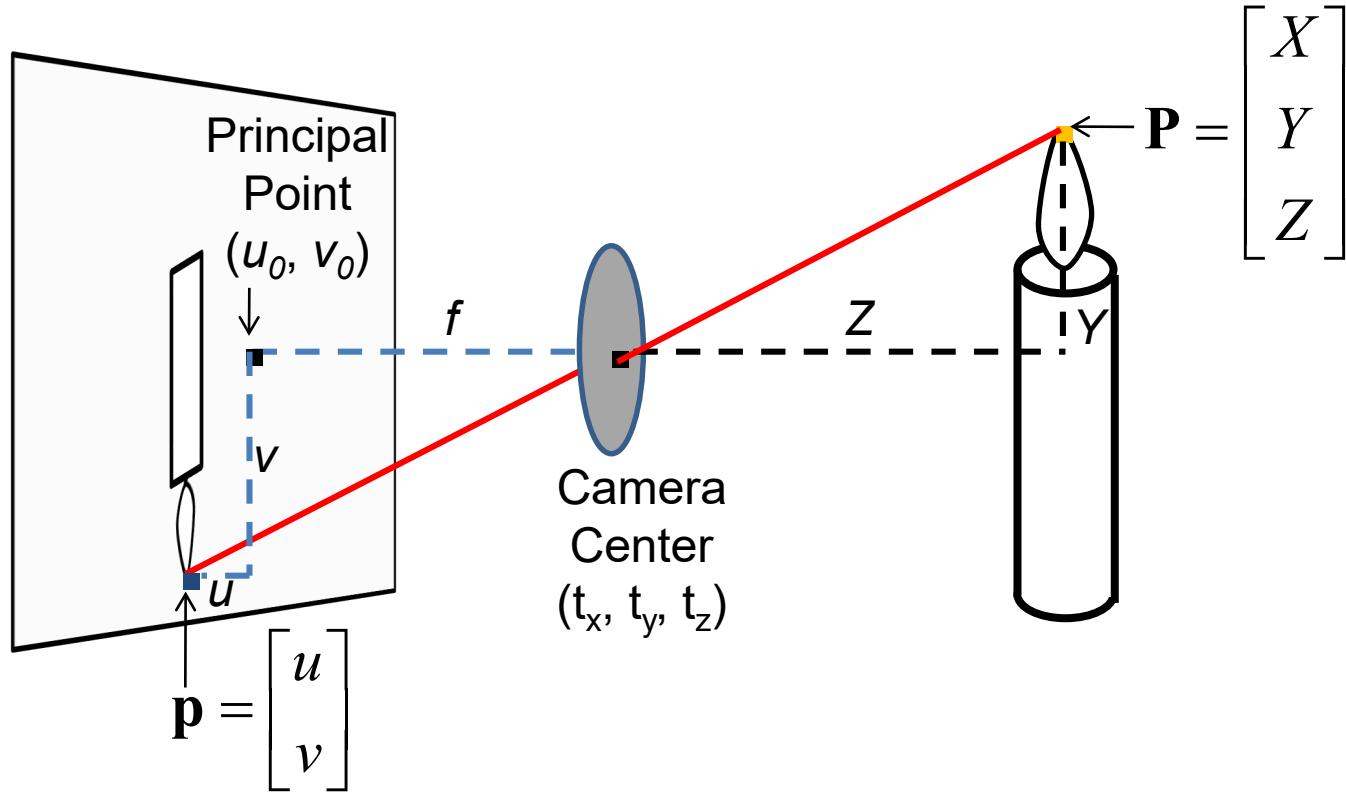


# Histogram Equalization



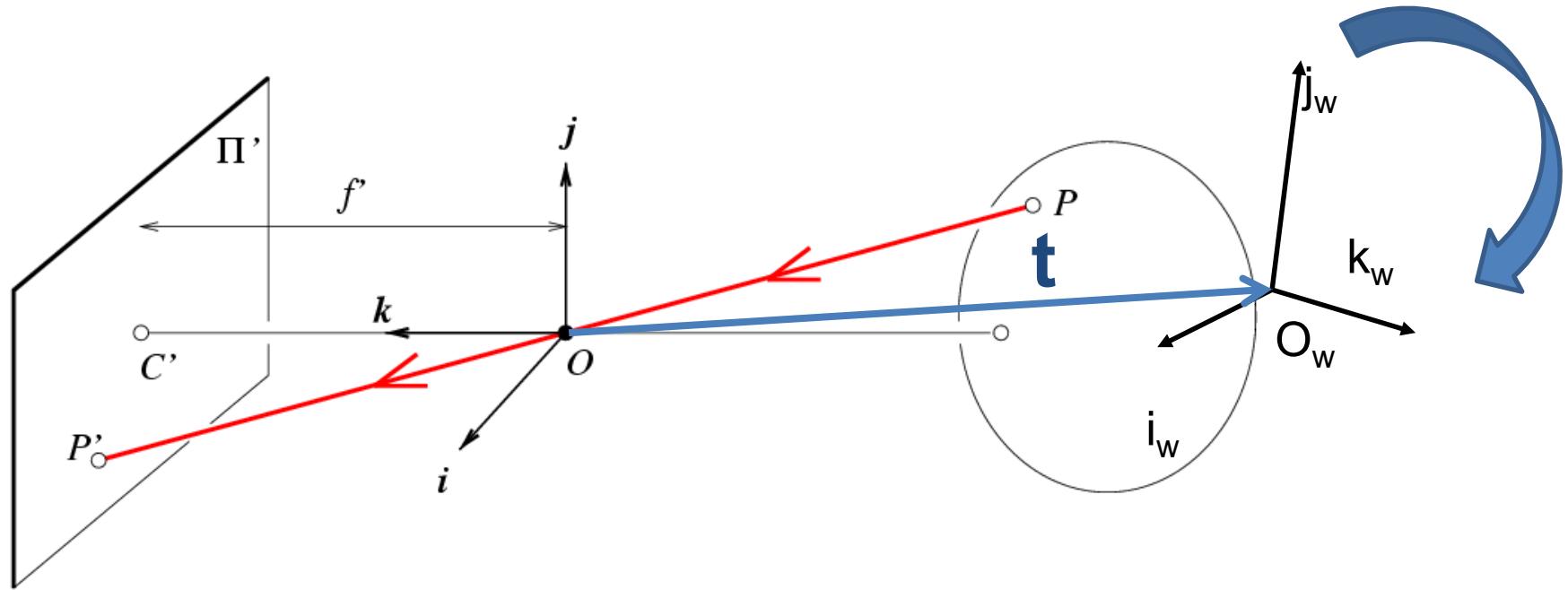
## 2. Camera Capture and Geometry

# Pinhole Camera



Useful figure to remember

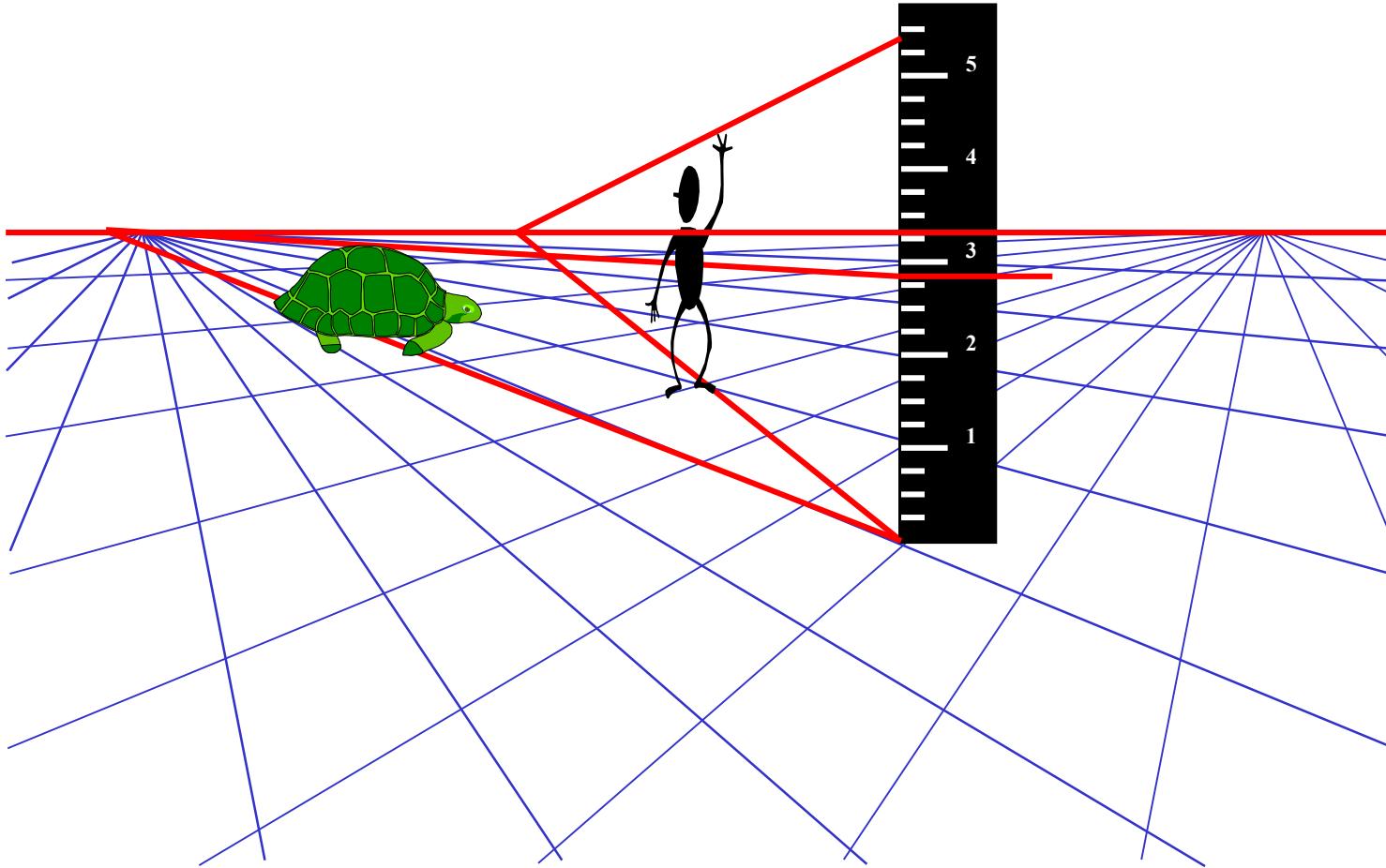
# Projection Matrix



**R**

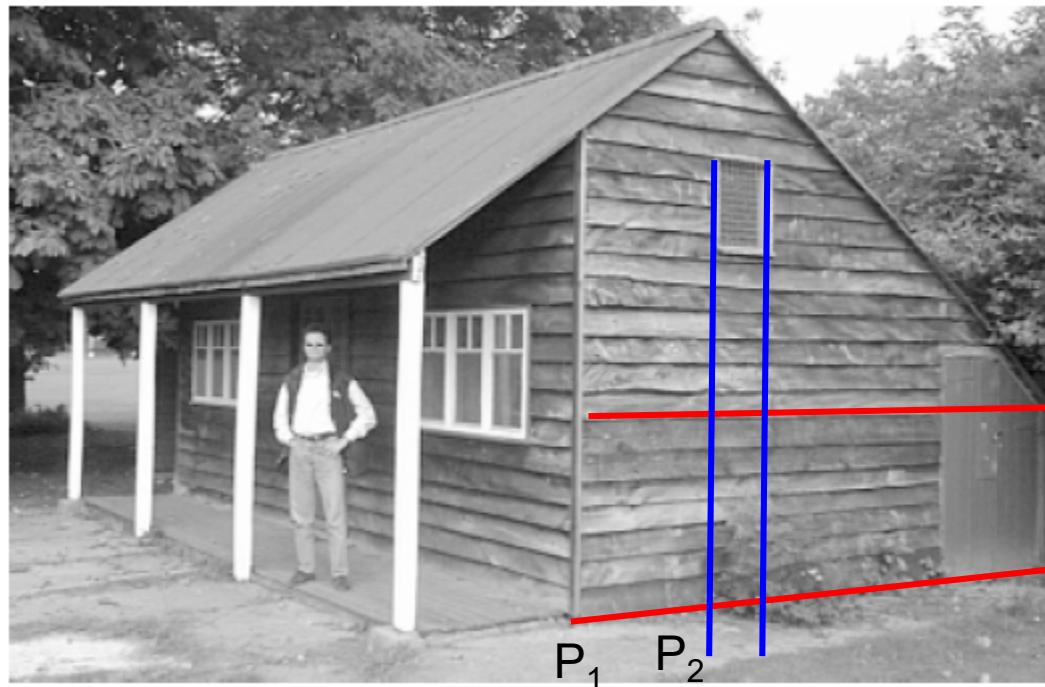
$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X} \xrightarrow{w} \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

# Single-view metrology



Assume the man is 6 ft tall.

- What is the height of the building?
- How long is the right side of the building compared to the small window on the right side of the building?



cross-ratio

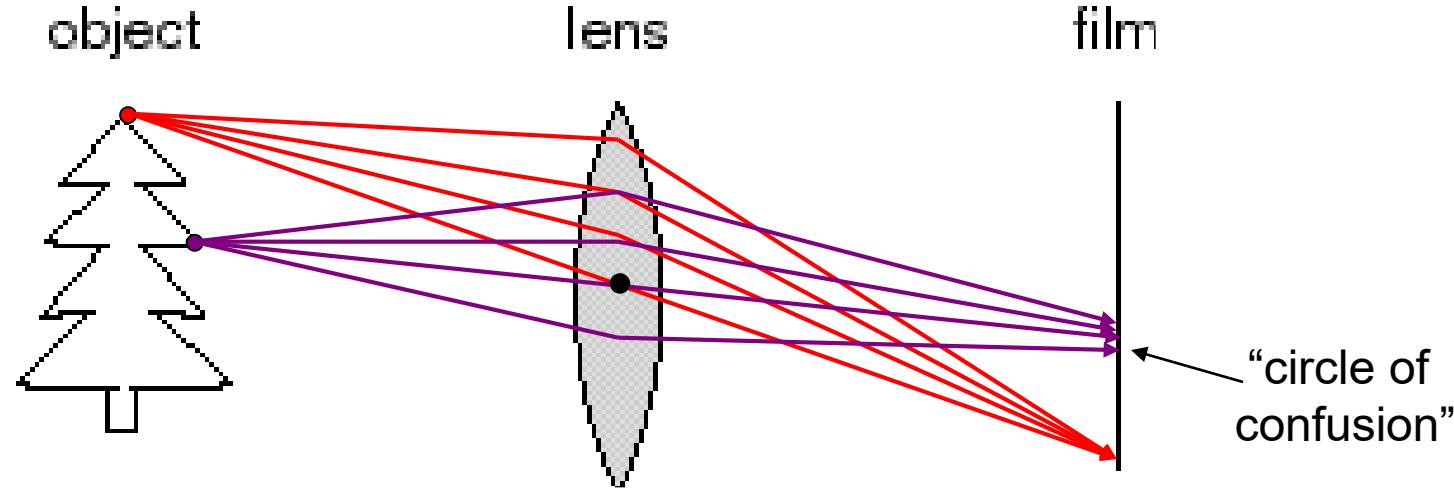
$$\frac{\|P_3 - P_1\| \|P_4 - P_2\|}{\|P_3 - P_2\| \|P_4 - P_1\|}$$

P<sub>4</sub>

P<sub>3</sub>

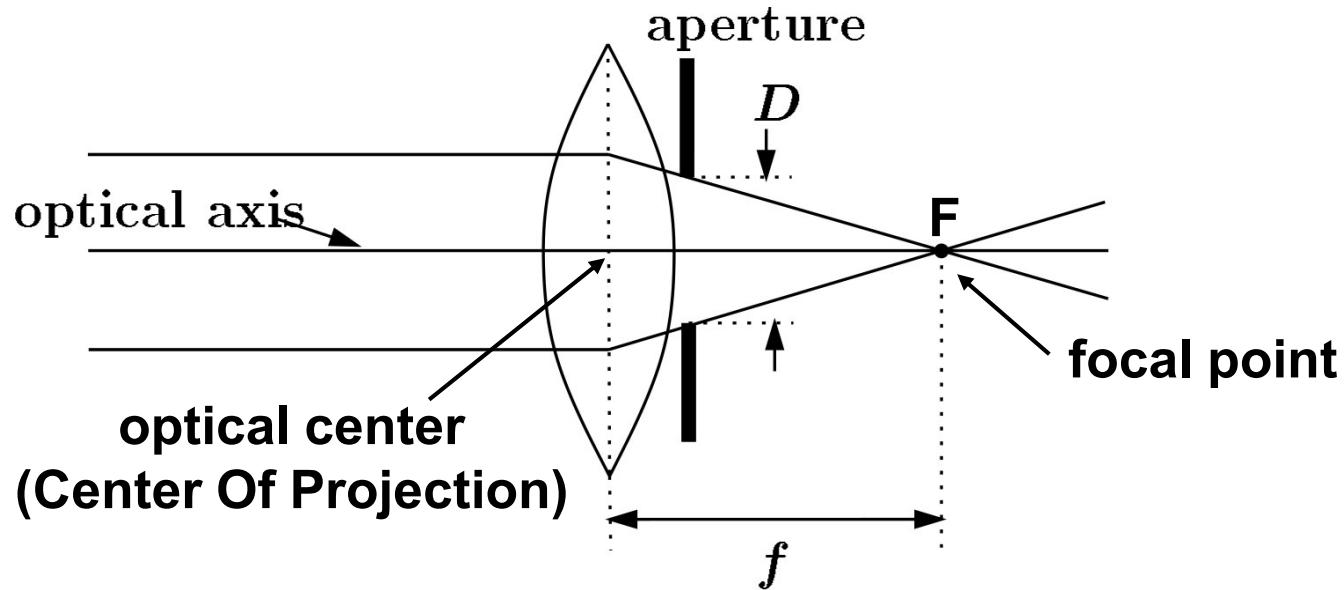
P<sub>1</sub> P<sub>2</sub>

# Adding a lens



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
    - other points project to a “circle of confusion” in the image
  - Changing the shape of the lens changes this distance

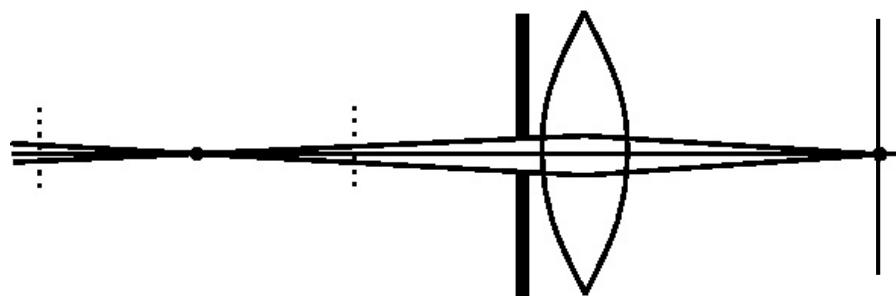
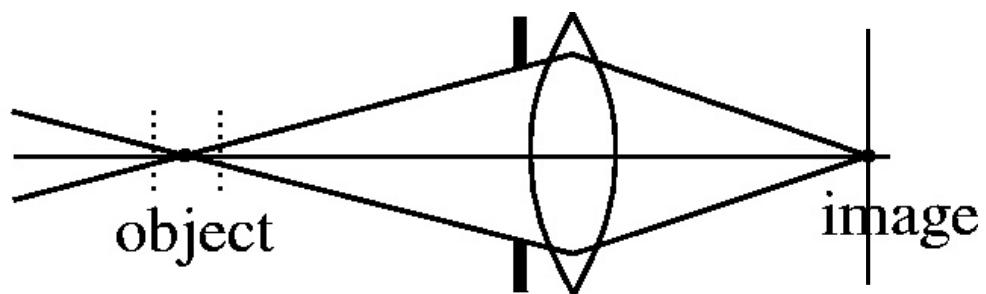
# Focal length, aperture, depth of field



A lens focuses parallel rays onto a single focal point

- focal point at a distance  $f$  beyond the plane of the lens
- Aperture of diameter  $D$  restricts the range of rays

# The aperture and depth of field



Main way to increase depth of field: Decrease aperture size

F-number (f/#) = focal\_length / aperture\_diameter

- E.g., f/16 means that the focal length is 16 times the diameter
- When you set the f-number of a camera, you are setting the aperture

# The Photographer's Great Compromise

What we want	How we get it	Cost
More spatial resolution	Increase focal length	Light, FOV
	Decrease focal length	DOF
Broader field of view	Decrease aperture	Light
	Increase aperture	DOF
More depth of field	Shorten exposure	Light
	Lengthen exposure	Temporal Res
More light		

### 3. Linear filtering

- Can think of filtering as
  - A function in the spatial domain (e.g., compute average of each 3x3 window)
  - Template matching
  - Modifying the frequency of the image

# Filtering in spatial domain

Slide filter over image and take dot product at each position

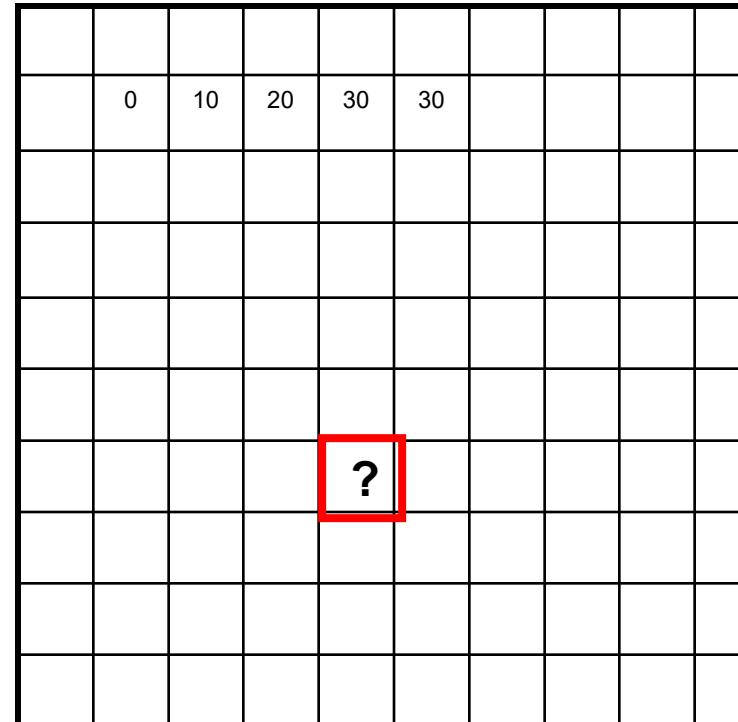
$f[.,.]$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$h[.,.]$

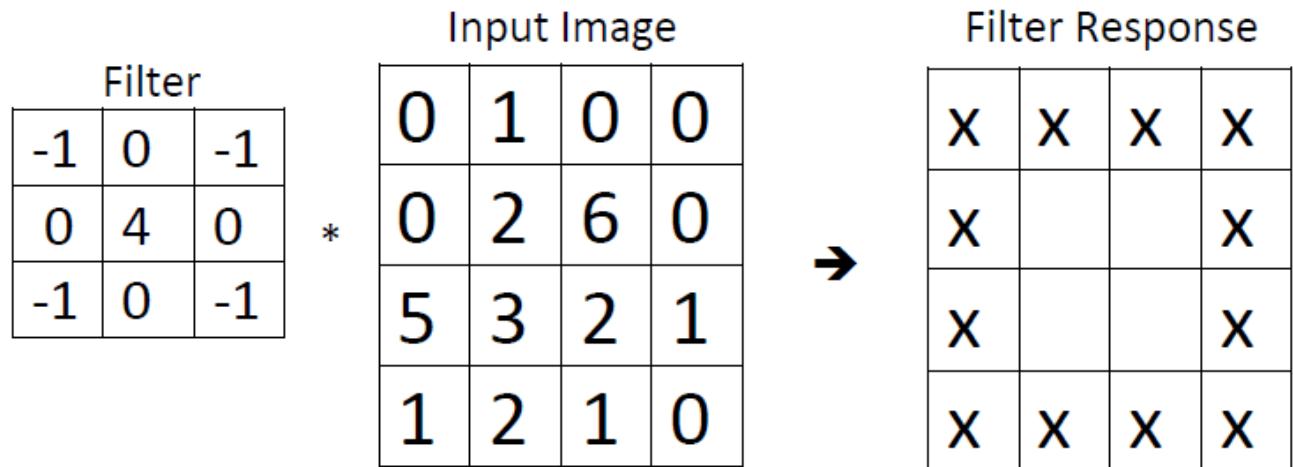
$$g[\cdot, \cdot] \frac{1}{9}$$

A 3x3 grid of orange squares, each containing the number 1. The grid is enclosed in a black border. Above the grid, the formula  $g[\cdot, \cdot] \frac{1}{9}$  is written.



## 1.Filters

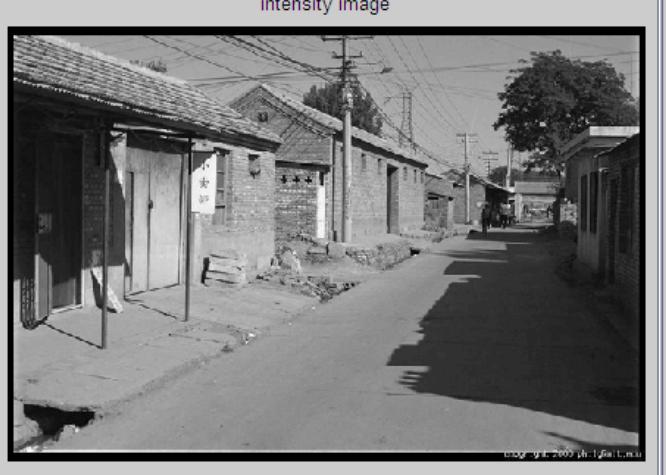
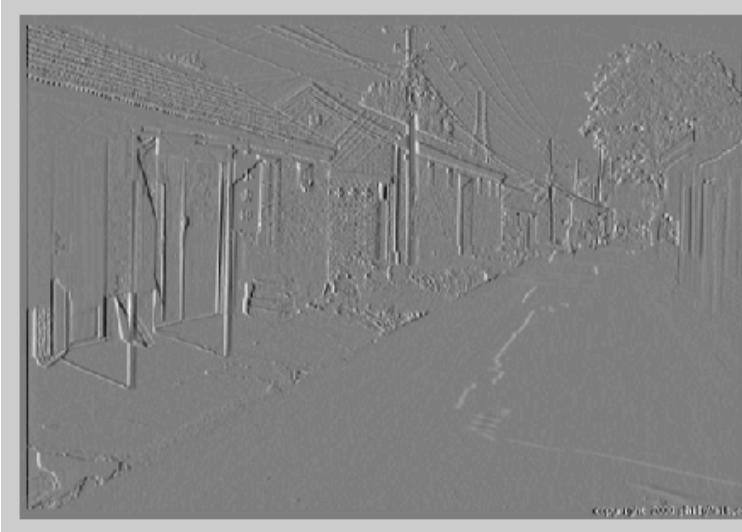
- a) Compute the center four values of the filter response, using the filter and image given below.



# Filtering in spatial domain

1	0	-1
2	0	-2
1	0	-1

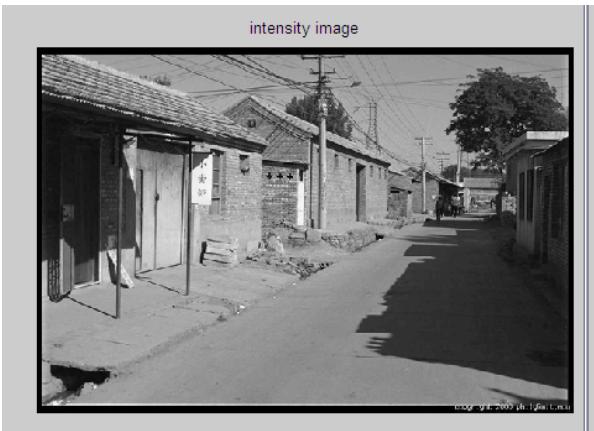
intensity image


$$\text{intensity image} \quad * \quad \begin{matrix} \text{filter} \\ \begin{matrix} \text{---} & \text{---} \\ \text{---} & \text{---} \end{matrix} \end{matrix} = \quad \text{filtered image}$$


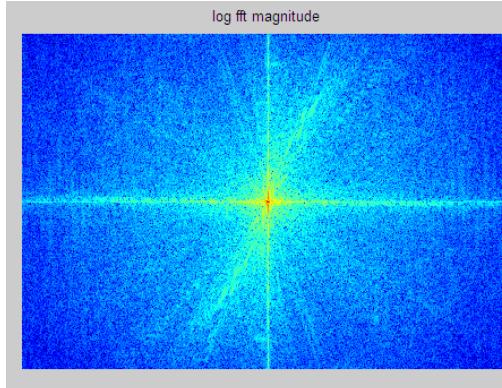
# Filtering in frequency domain

- Can be faster than filtering in spatial domain  
(for large filters)
- Can help understand effect of filter
- Algorithm:
  1. Convert image and filter to fft (fft2 in matlab)
  2. Pointwise-multiply ffts
  3. Convert result to spatial domain with ifft2

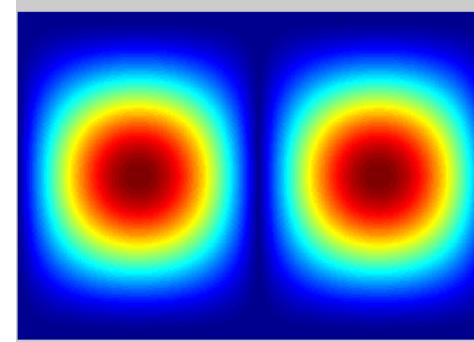
# Filtering in frequency domain



FFT

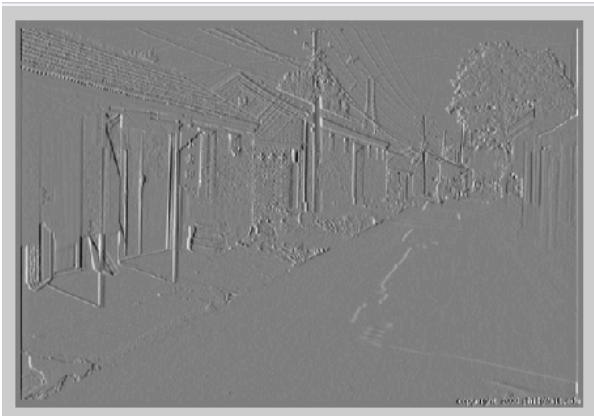


$\times$



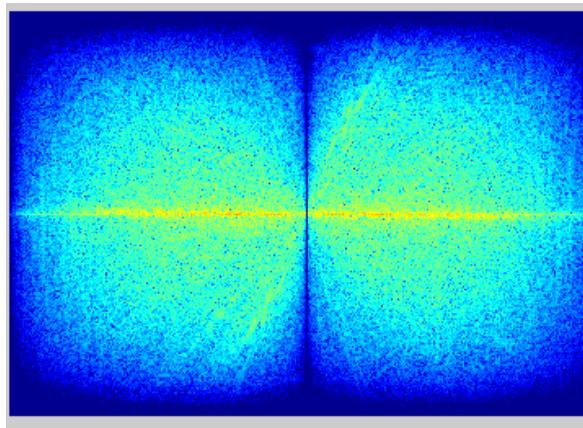
FFT

||



Inverse FFT

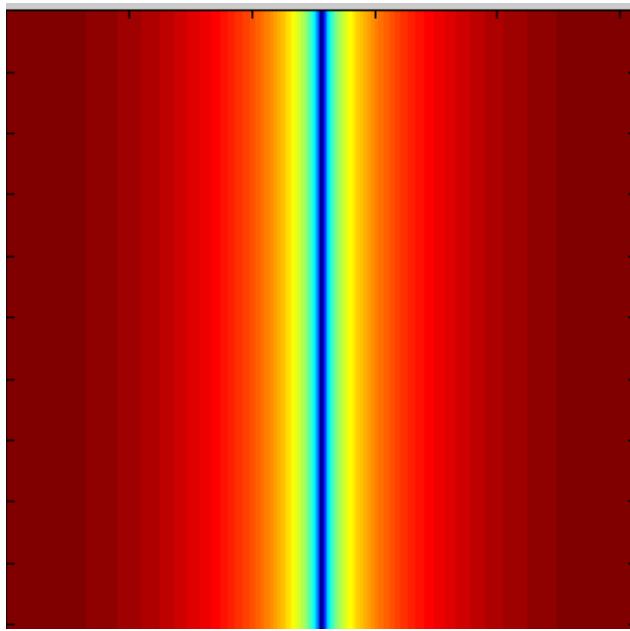
←



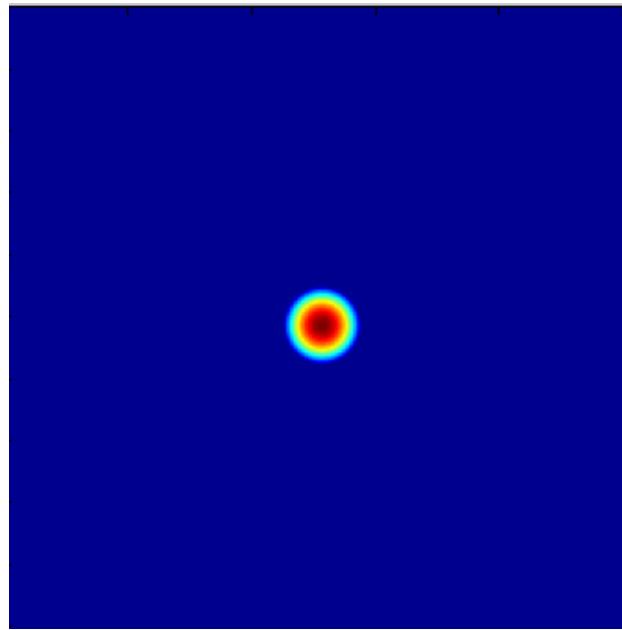
# Filtering in frequency domain

- Linear filters for basic processing
  - Edge filter (high-pass)
  - Gaussian filter (low-pass)

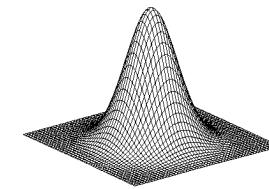
$$[-1 \ 1]$$



FFT of Gradient Filter



FFT of Gaussian

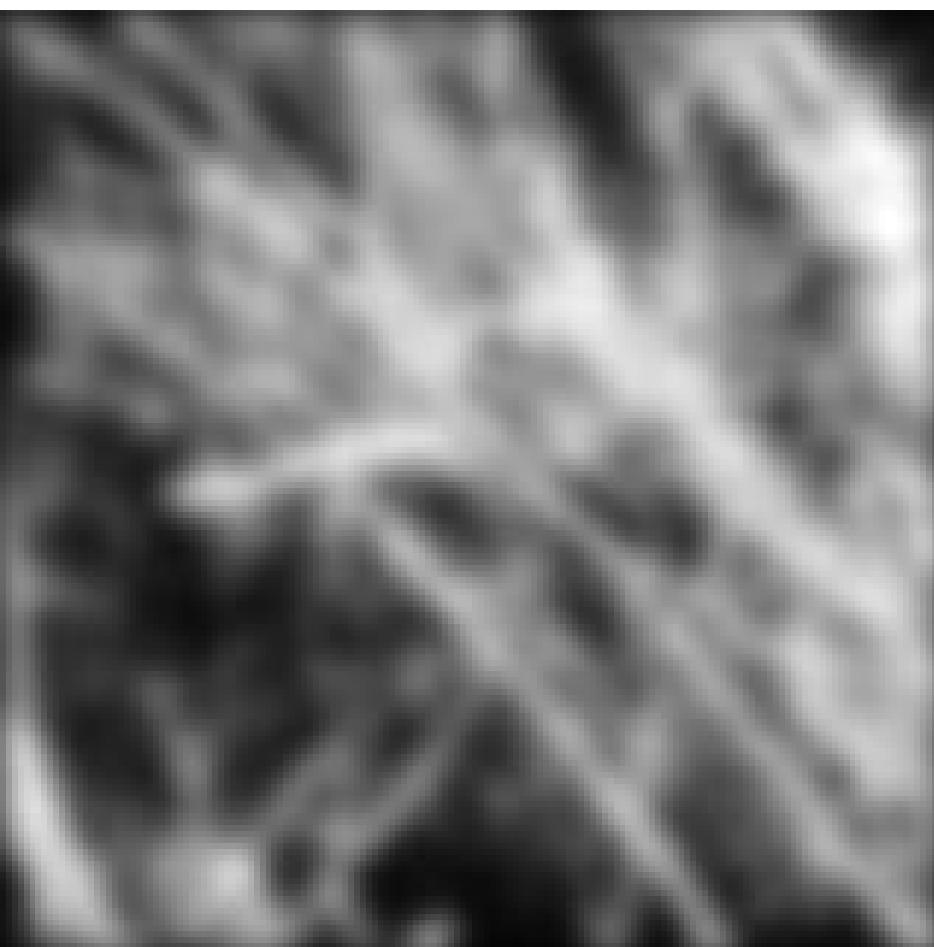


Gaussian

# Filtering

Why does the Gaussian give a nice smooth image, but the square filter give edgy artifacts?

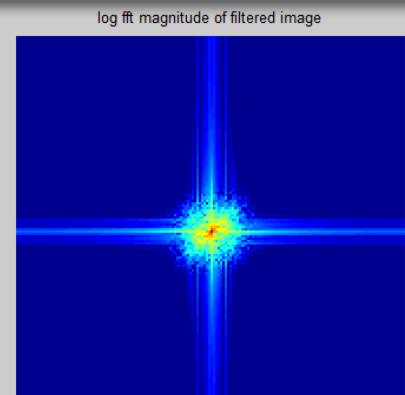
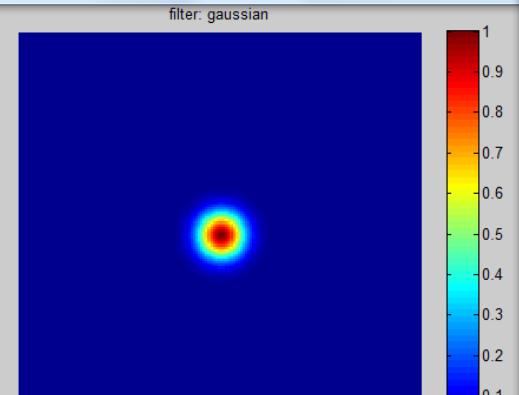
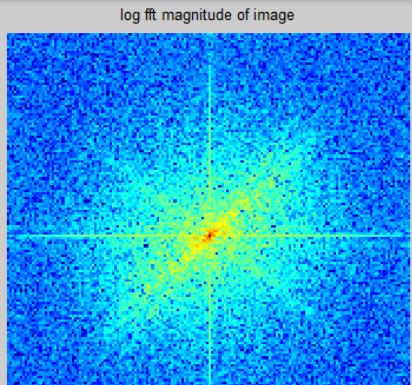
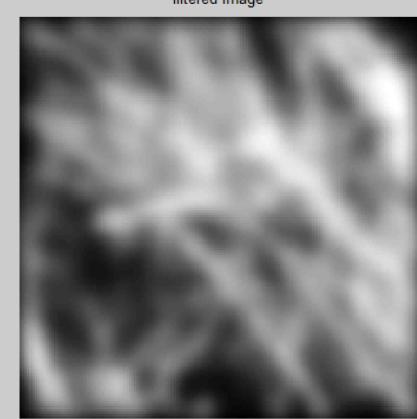
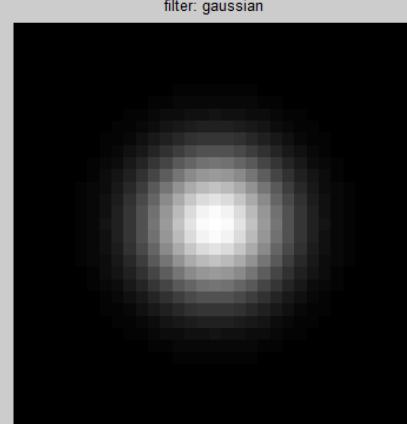
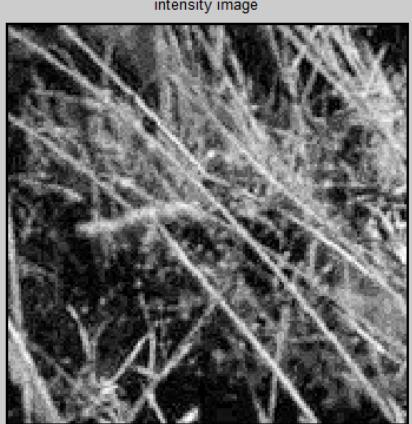
Gaussian



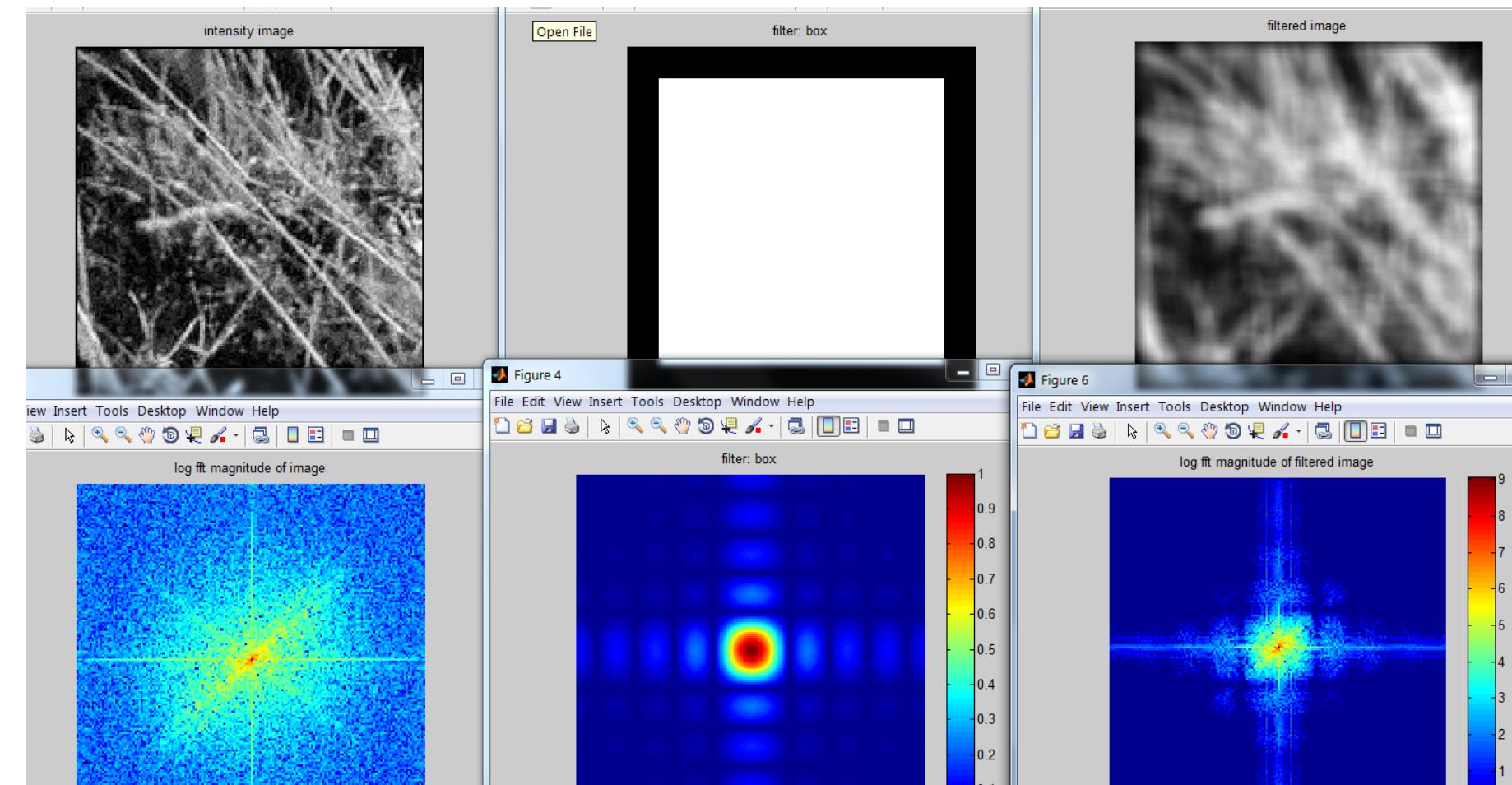
Box filter



# Gaussian



# Box Filter



# Question

1. Use filtering to find pixels that have at least three white pixels among the 8 surrounding pixels (assume a binary image)
2. Write down a filter that will compute the following function

$$\text{fil}(y, x) = -0.5 * \text{im}(y+1, x) + \text{im}(y, x) - 0.5 * \text{im}(y-1, x)$$

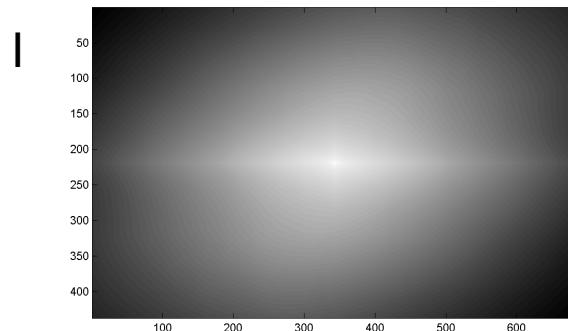
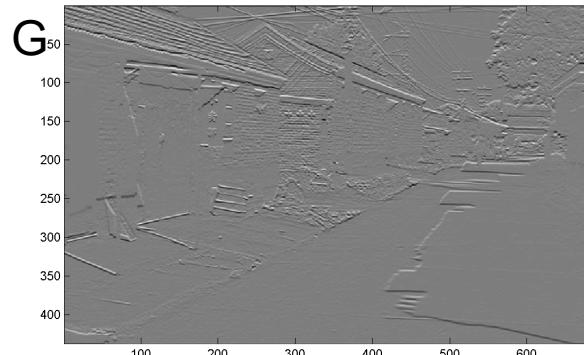
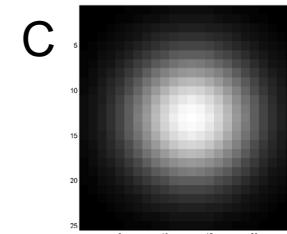
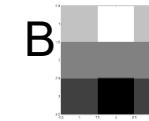
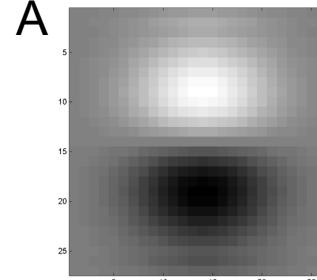
for each  $x, y$

# Question

3. Fill in the blanks:

$$\begin{array}{l} \text{a) } \underline{\quad} = D * B \\ \text{b) } \overline{A} = \underline{\quad} * \underline{\quad} \\ \text{c) } F = \overline{D} * \underline{\quad} \\ \text{d) } \underline{\quad} = D * C \end{array}$$

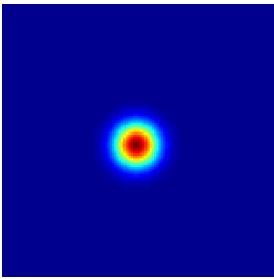
Filtering Operator



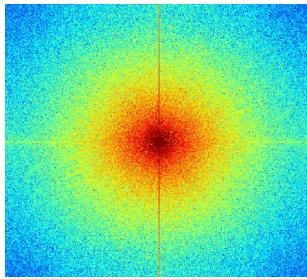
# Question

Match the spatial domain image to the Fourier magnitude image

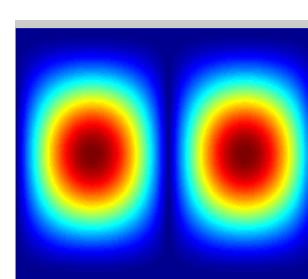
1



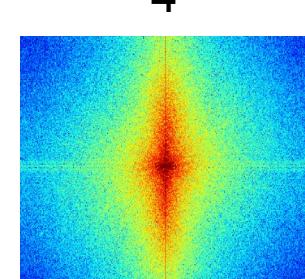
2



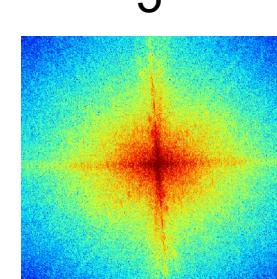
3



4



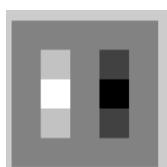
5



B



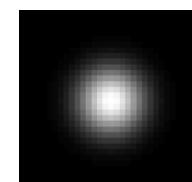
A



C



D



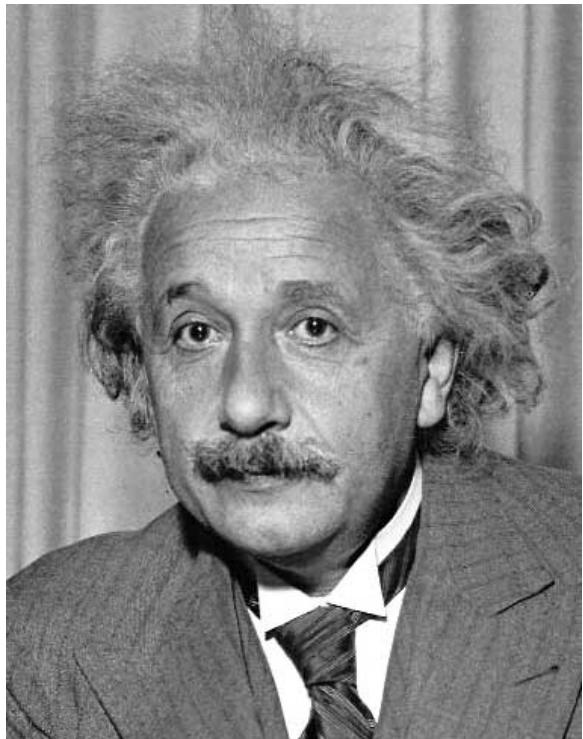
E



# Matching with filters

- Goal: find  in image
- Method 2: SSD

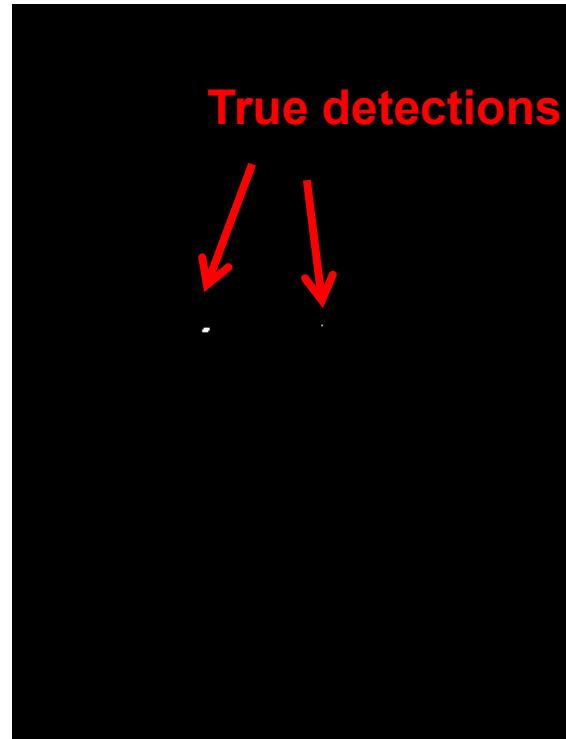
$$h[m,n] = \sum_{k,l} (g[k,l] - f[m+k, n+l])^2$$



Input



1 -  $\sqrt{\text{SSD}}$



Thresholded Image

# Matching with filters

- Goal: find  in image
- Method 3: Normalized cross-correlation

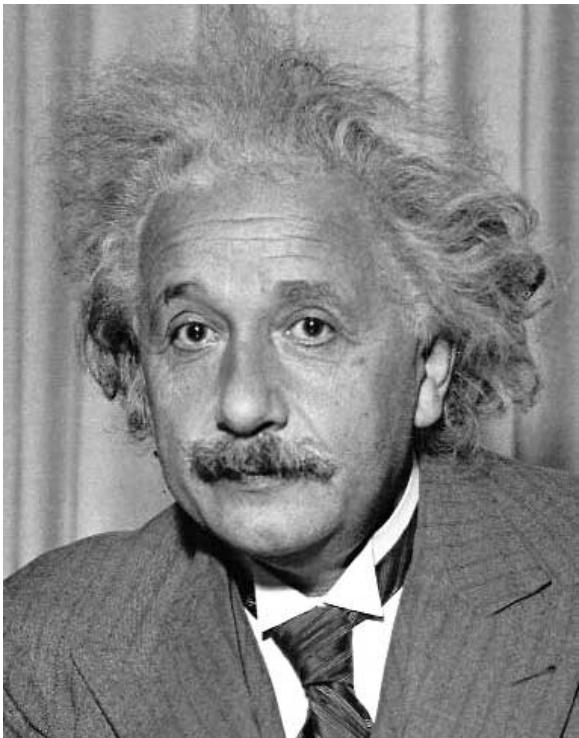
$$h[m,n] = \frac{\sum_{k,l} (g[k,l] - \bar{g})(f[m-k, n-l] - \bar{f}_{m,n})}{\left( \sum_{k,l} (g[k,l] - \bar{g})^2 \sum_{k,l} (f[m-k, n-l] - \bar{f}_{m,n})^2 \right)^{0.5}}$$

mean template                                    mean image patch

Matlab: `normxcorr2(template, im)`

# Matching with filters

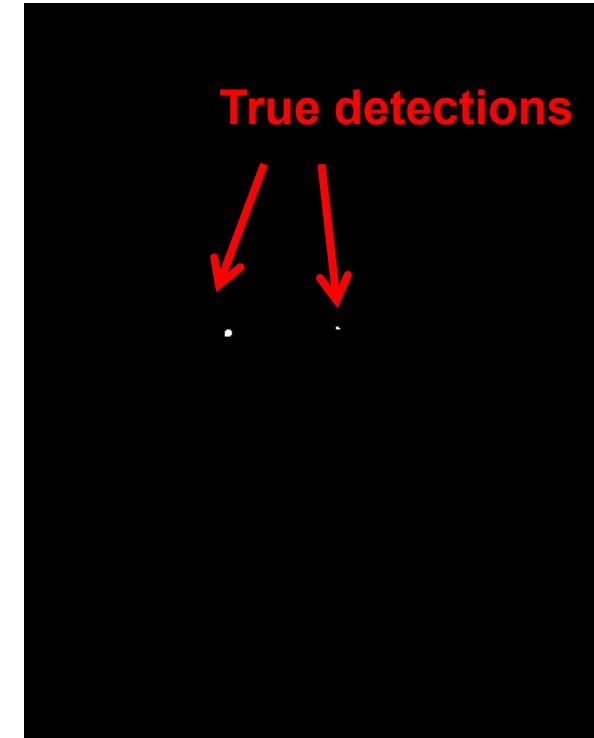
- Goal: find  in image
- Method 3: Normalized cross-correlation



Input

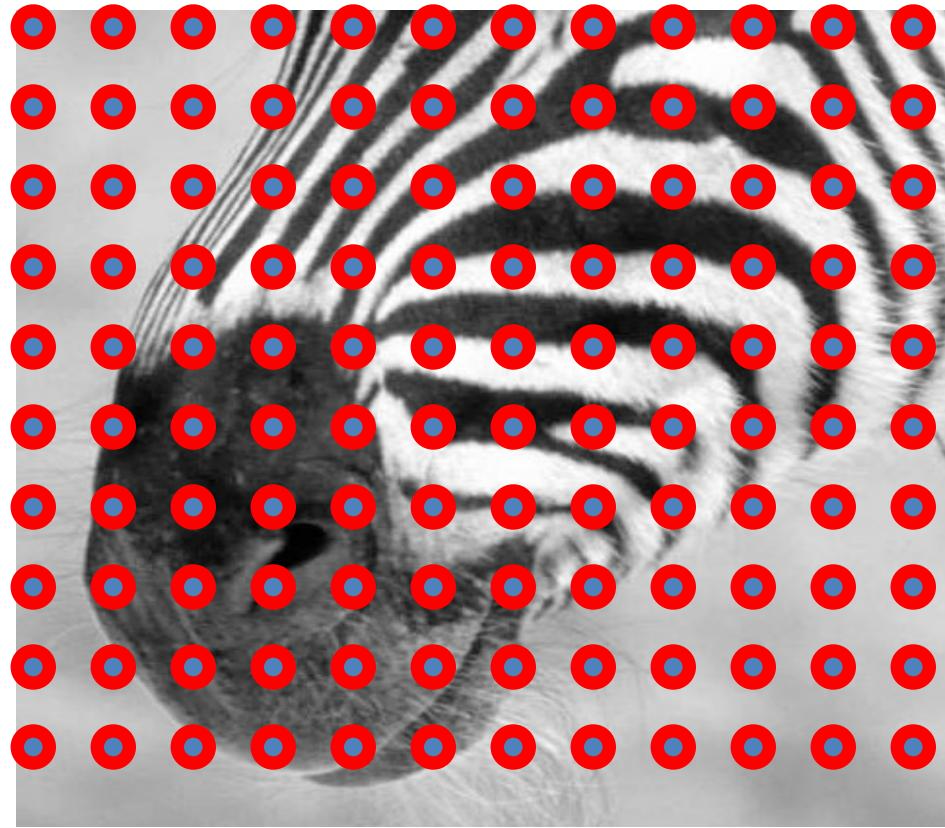


Normalized X-Correlation



True detections  
↓  
↓  
Thresholded Image

# Subsampling by a factor of 2

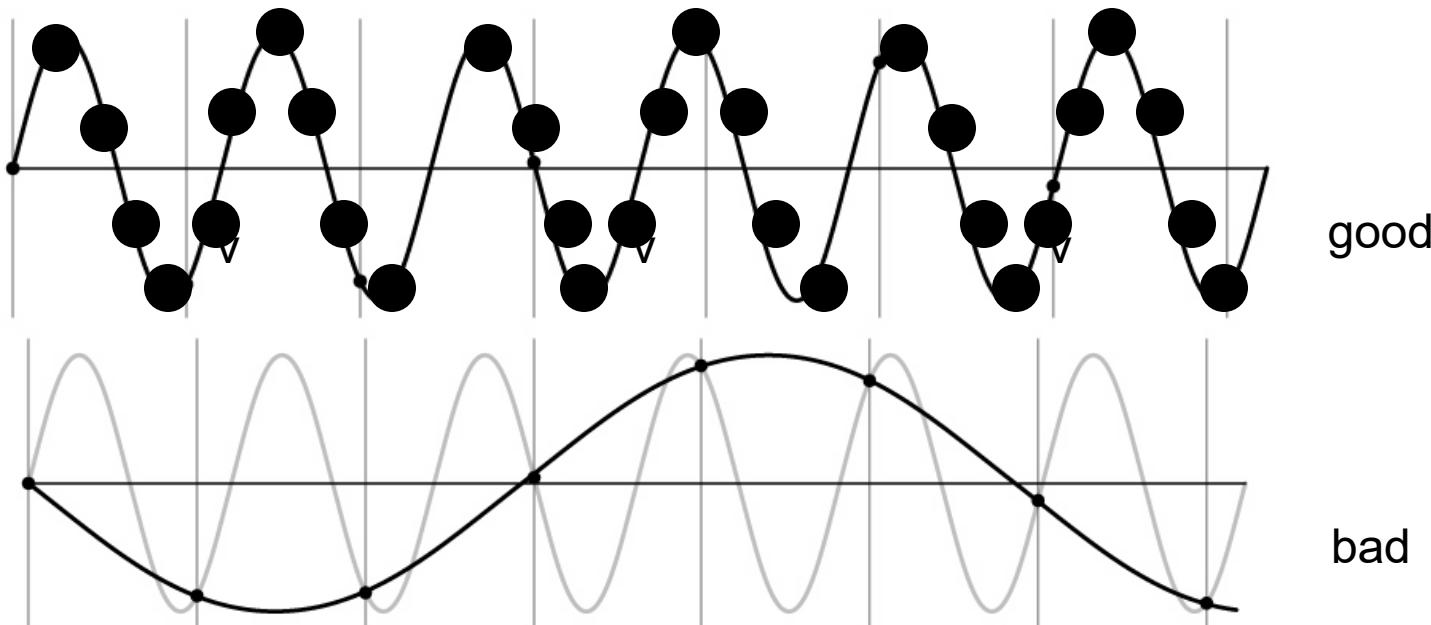


Problem: This approach causes “aliasing”

Throw away every other row and column to create a 1/2 size image

# Nyquist-Shannon Sampling Theorem

- When sampling a signal at discrete intervals, the sampling frequency must be  $\geq 2 \times f_{\max}$
- $f_{\max}$  = max frequency of the input signal
- This will allow to reconstruct the original perfectly from the sampled version



# Algorithm for downsampling by factor of 2

1. Start with  $\text{image}(h, w)$

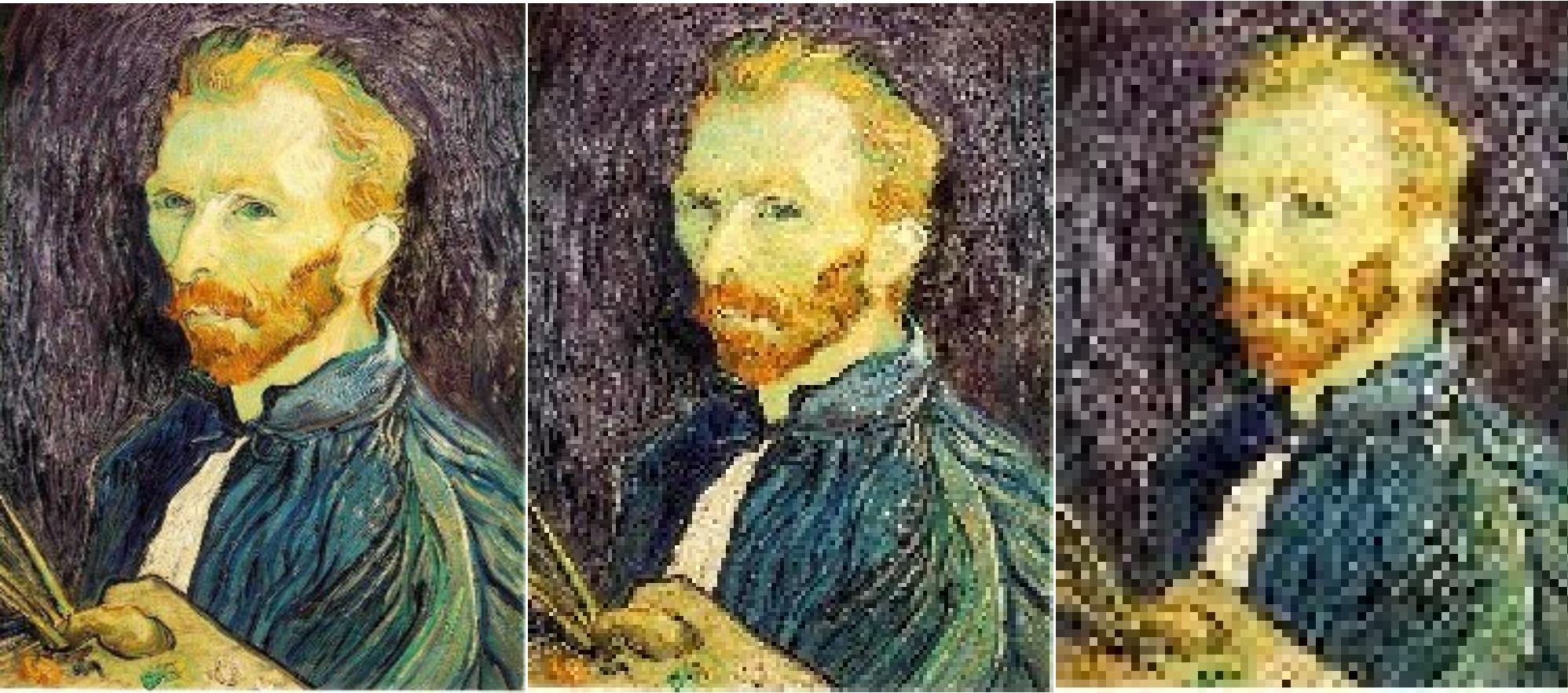
2. Apply low-pass filter

```
im.blur = imfilter(image, fspecial('gaussian', 13, 2))
```

3. Sample every other pixel

```
im.small = im.blur(1:2:end, 1:2:end);
```

# Subsampling without pre-filtering

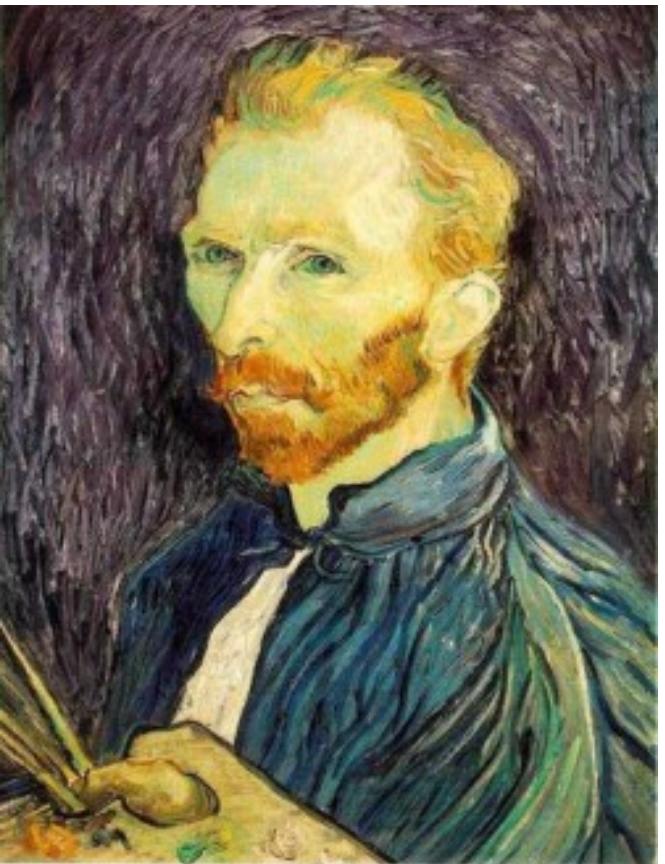


1/2

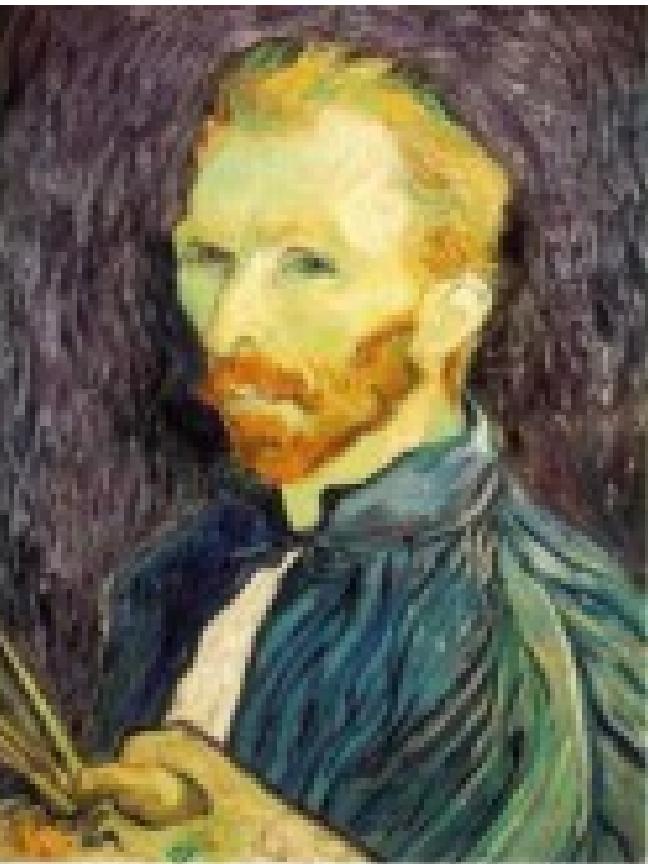
1/4 (2x zoom)

1/8 (4x zoom)

# Subsampling with Gaussian pre-filtering



Gaussian 1/2



G 1/4



G 1/8

# Sampling

**Why does a lower resolution image still make sense to us? What do we lose?**

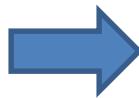
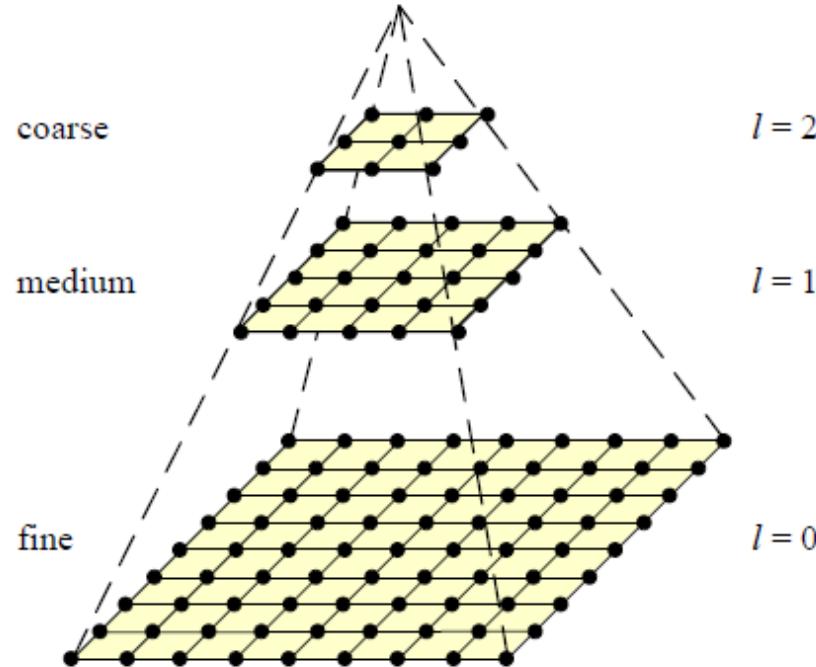


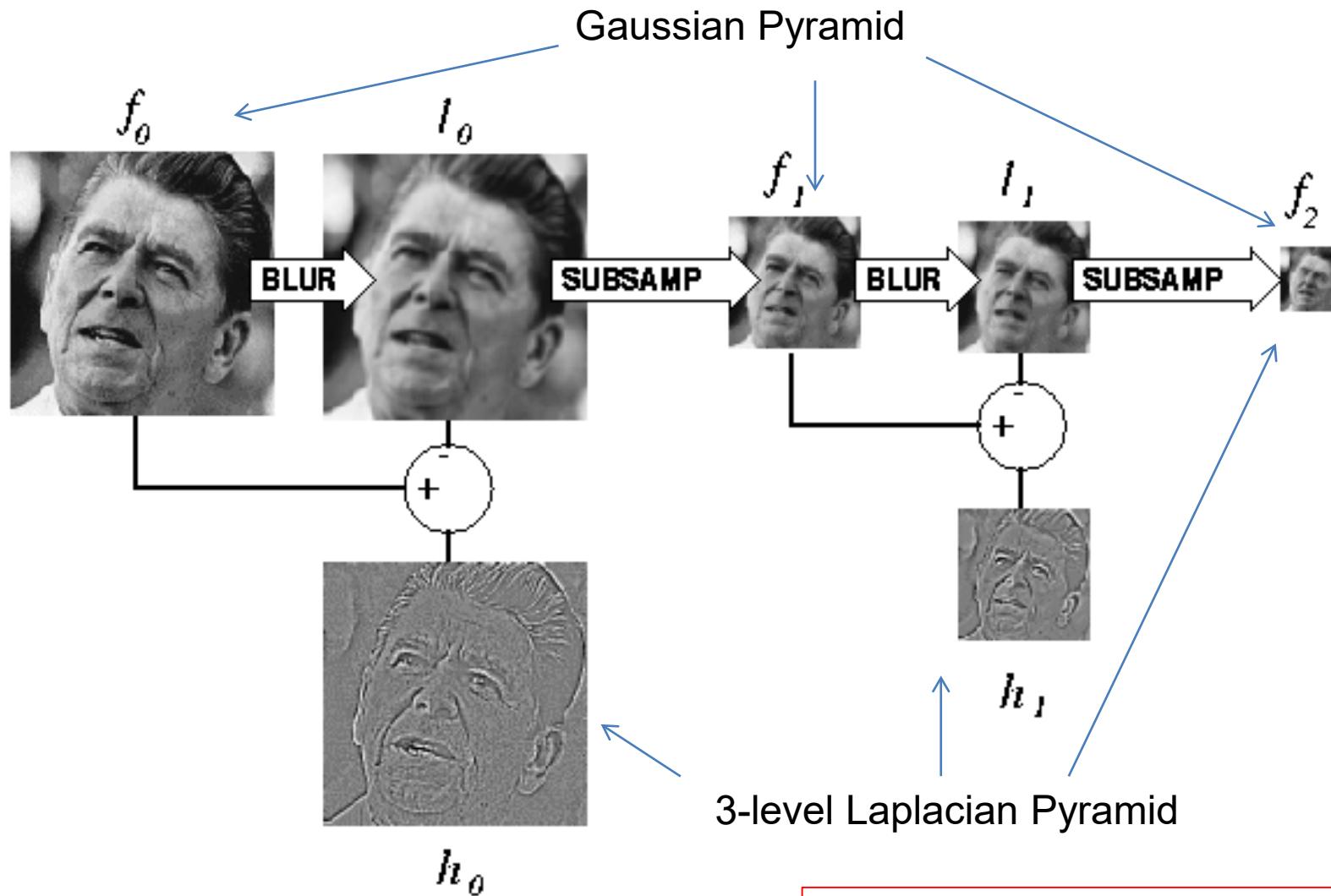
Image: <http://www.flickr.com/photos/igorms/136916757/>

# Gaussian pyramid

- Useful for coarse-to-fine matching
- Applications include multi-scale object detection, image alignment, optical flow, point tracking

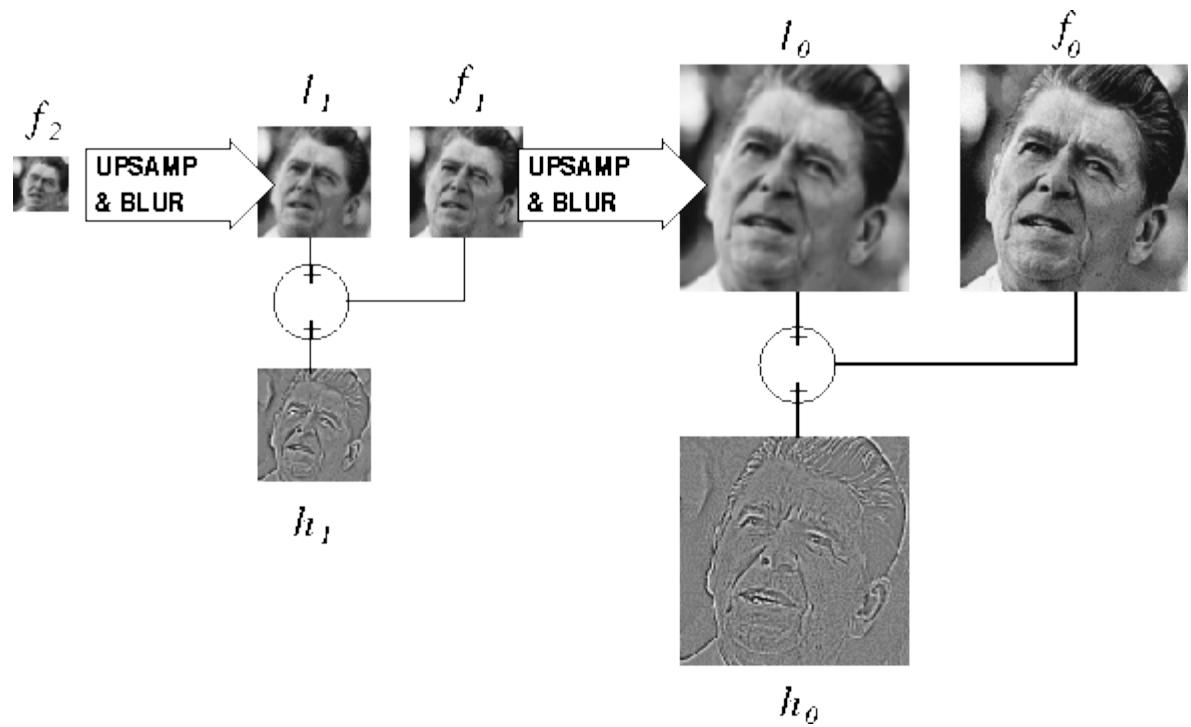


# Computing Gaussian/Laplacian Pyramid



Useful figure to remember

# Image reconstruction from Laplacian pyramid

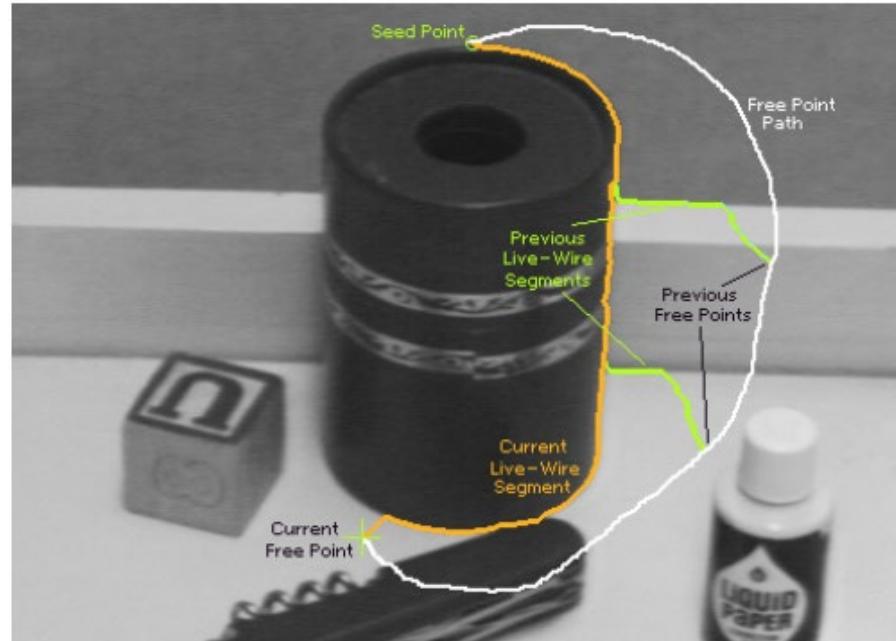


# 4. Region selection and compositing

- Selecting image regions
  - Intelligent scissors
  - Graph cuts
- Compositing
  - Alpha masks
  - Feathering
  - Laplacian pyramid blending
  - Poisson blending

# Intelligent Scissors

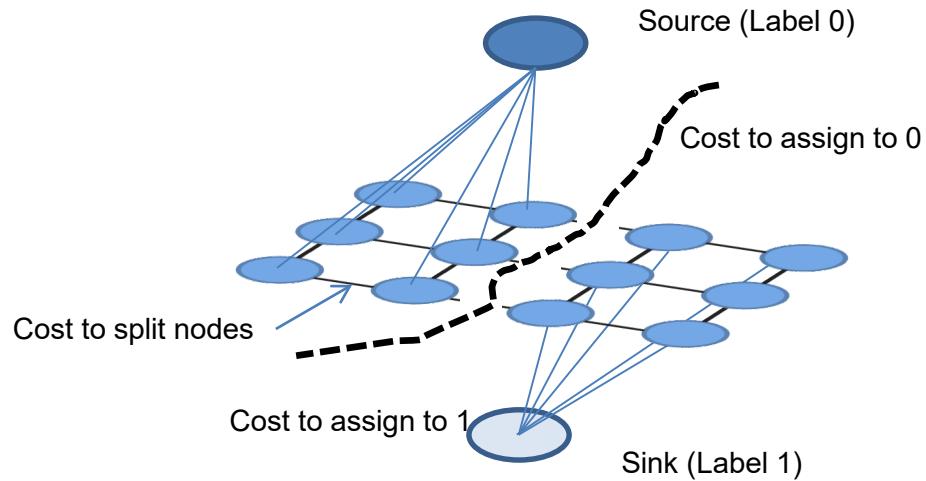
- You can treat the image as a graph
  - Nodes = pixels, edges connect neighboring pixels



Intelligent Scissors: Good boundaries are a short (high gradient) path through the graph

# Graph Cuts

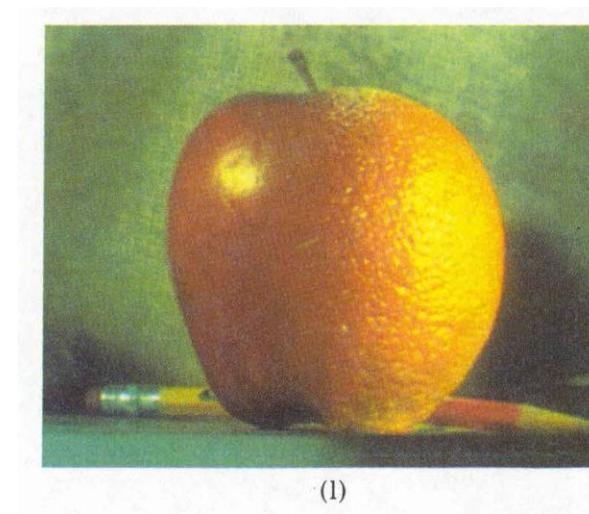
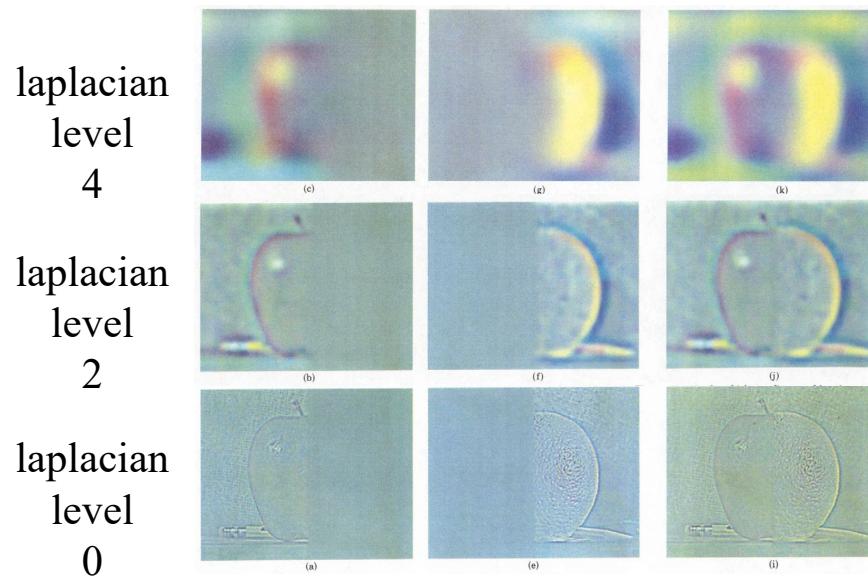
- You can treat the image as a graph
  - Nodes = pixels, edges connect neighboring pixels



Graph cut: Good boundaries are a cheap cut, where some pixels want to be foreground, and some to be background

# Compositing and Blending

- Feathering: blur mask around its edges
- Laplacian blending: blend low-frequency slowly, high frequency quickly
  - Blend with alpha mask values ranging from 0 to 1



# Question

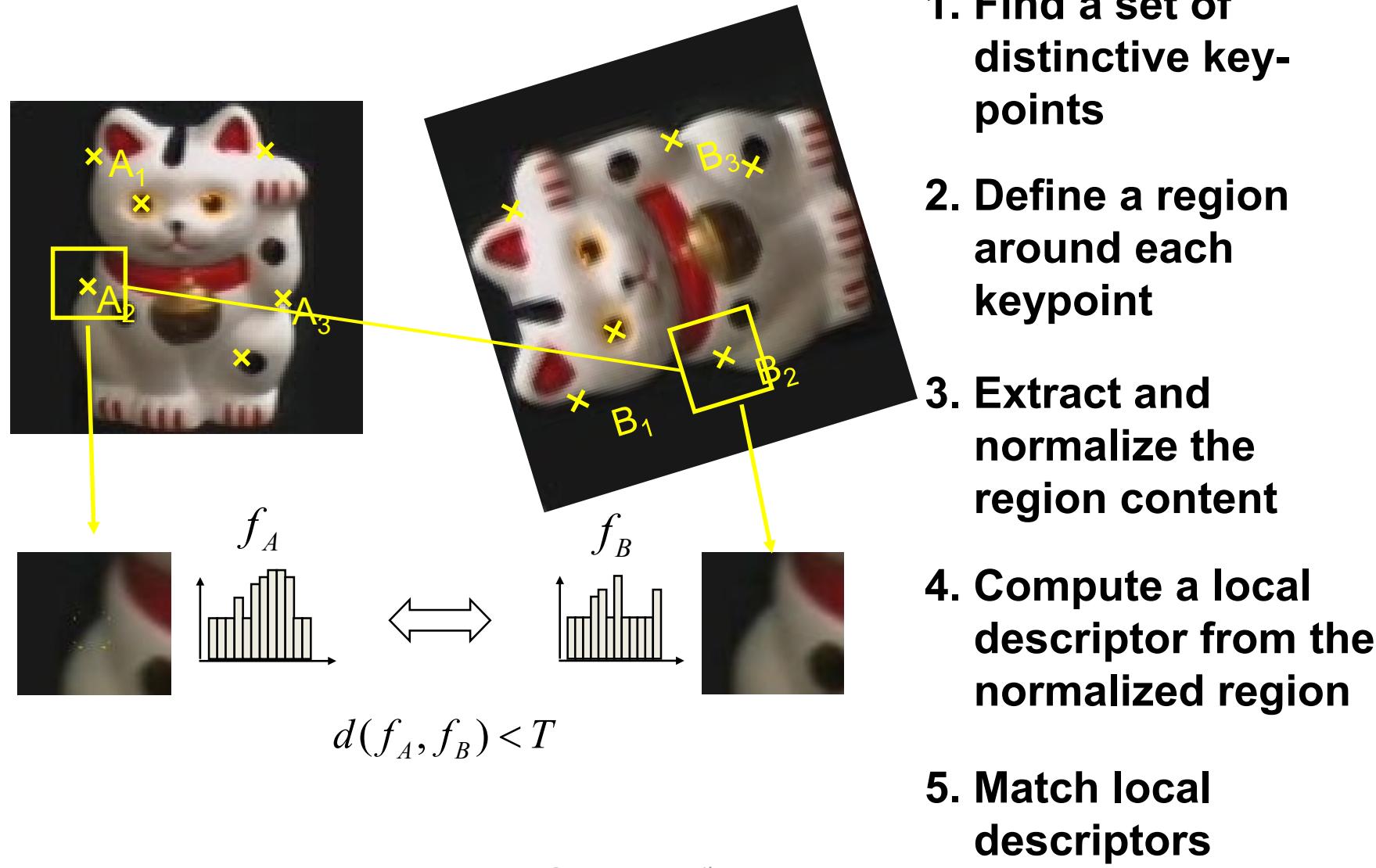
1) I am trying to blend this bear into this pool.

What problems will I have if I use:

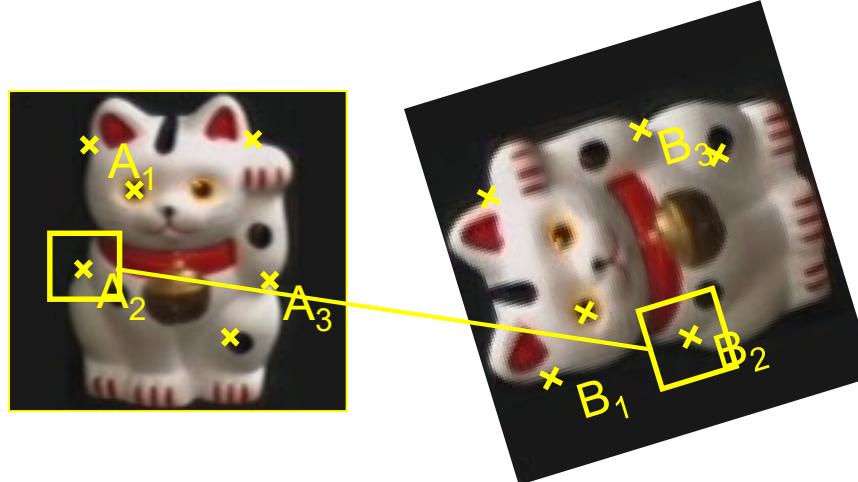
- a) Alpha compositing with feathering
- b) Laplacian pyramid blending
- c) Poisson editing?



# Keypoint Matching



# Key trade-offs



## Localization



More Points

Robust to occlusion

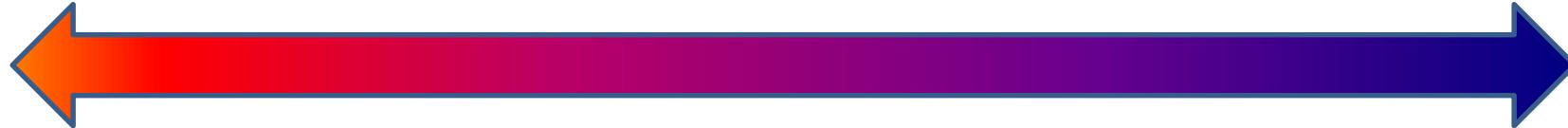
Works with less texture

More Repeatable

Robust detection

Precise localization

## Description



More Robust

Deal with expected variations

Maximize correct matches

More Selective

Minimize wrong matches

# Harris Detector [Harris88]

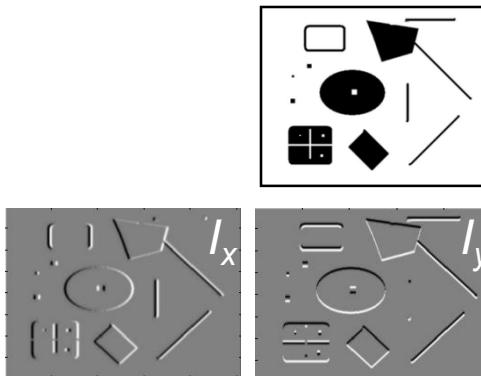
- Second moment matrix

$$\mu(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

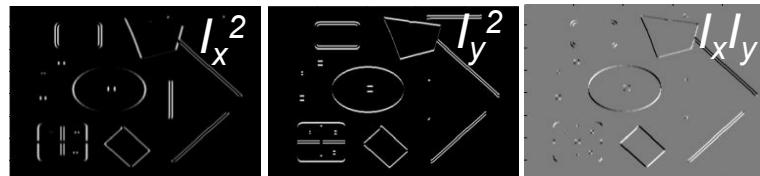
$$\det M = \lambda_1 \lambda_2$$

$$\text{trace } M = \lambda_1 + \lambda_2$$

1. Image derivatives



2. Square of derivatives



3. Gaussian filter  $g(\sigma_I)$



4. Cornerness function – both eigenvalues are strong

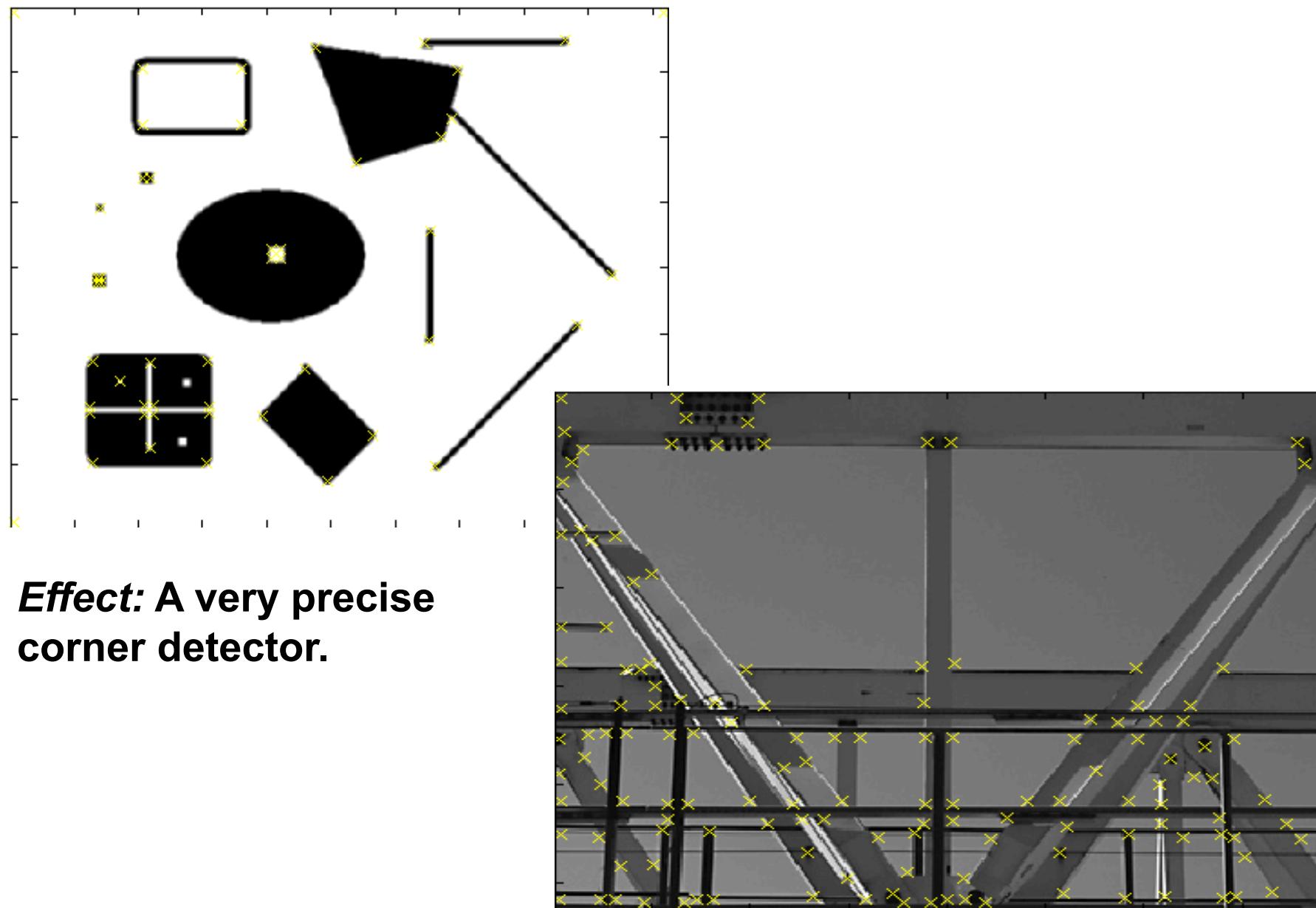
$$har = \det[\mu(\sigma_I, \sigma_D)] - \alpha [\text{trace}(\mu(\sigma_I, \sigma_D))^2] =$$

$$g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha [g(I_x^2) + g(I_y^2)]^2$$

5. Non-maxima suppression



# Harris Detector – Responses [Harris88]



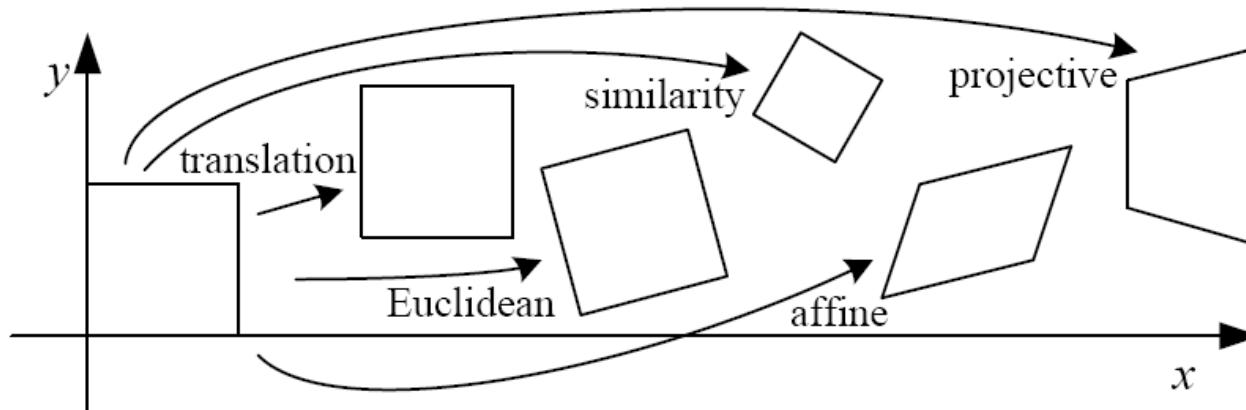
# Applications

- Image stitching
  - Matching keypoints
  - Solving for homography
  - RANSAC
- Object recognition
  - Clustering keypoints and creating tf-idf tables for fast retrieval
  - Geometric verification

# 5. Solving for transformations

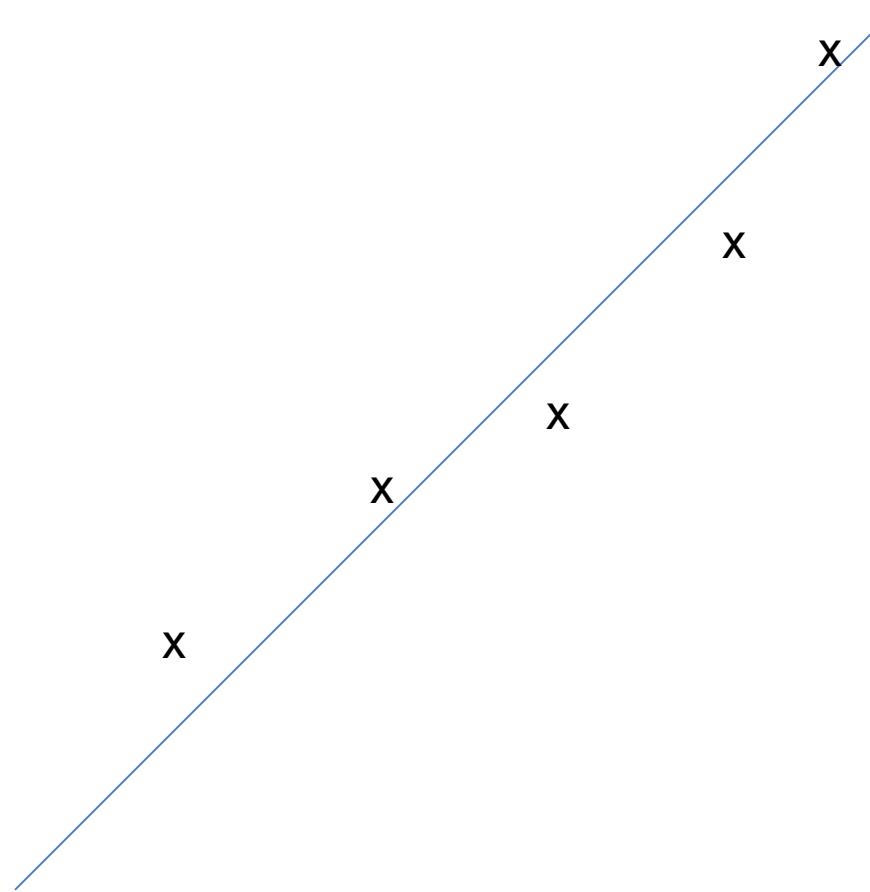
- Map between 2D coordinates using linear projection

Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$[ \mathbf{I}   \mathbf{t} ]_{2 \times 3}$	2	orientation + ...	
rigid (Euclidean)	$[ \mathbf{R}   \mathbf{t} ]_{2 \times 3}$	3	lengths + ...	
similarity	$[ s\mathbf{R}   \mathbf{t} ]_{2 \times 3}$	4	angles + ...	
affine	$[ \mathbf{A} ]_{2 \times 3}$	6	parallelism + ...	
projective	$[ \tilde{\mathbf{H}} ]_{3 \times 3}$	8	straight lines	



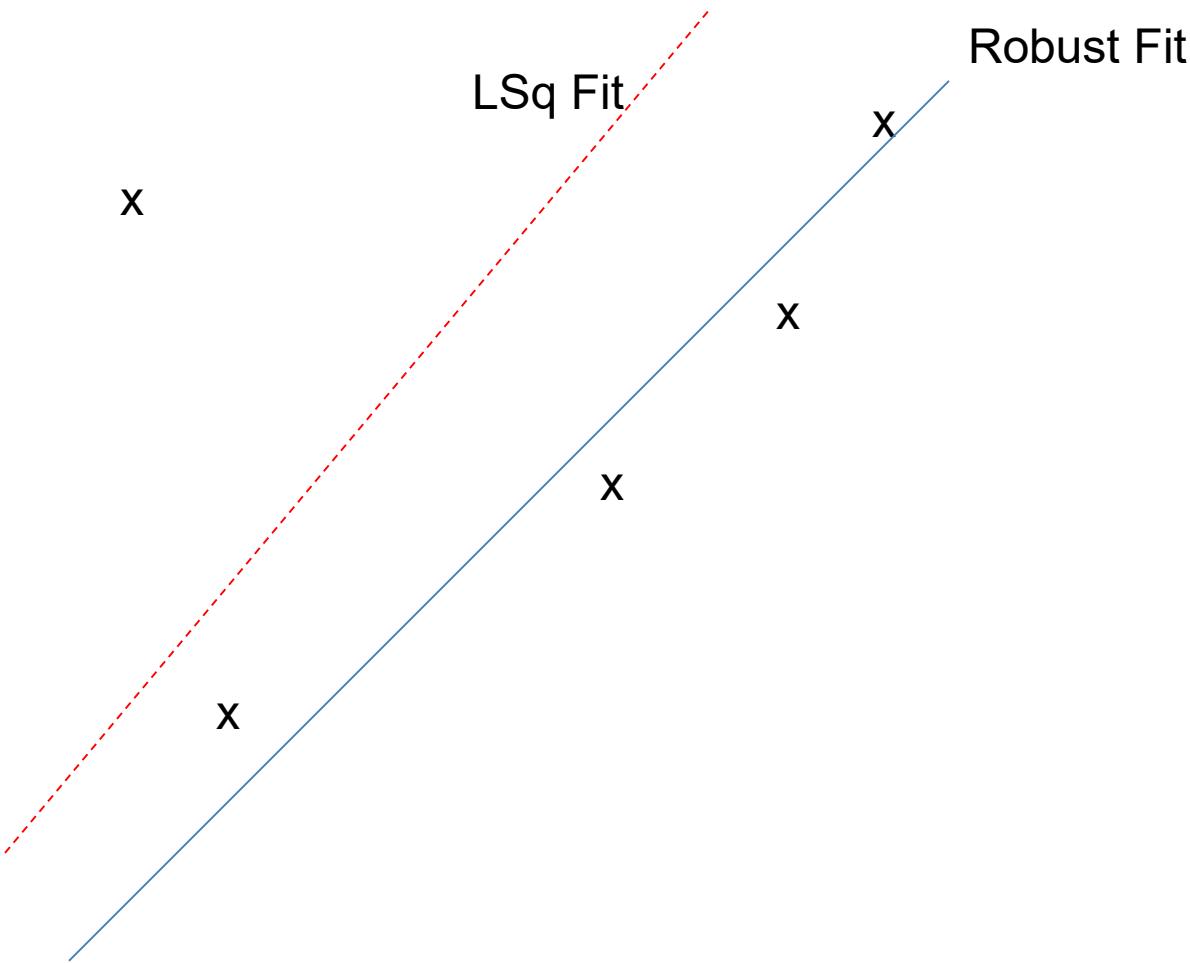
# Least-squares Solving

If all points (or correspondences) fit the model with some noise, better to use all for least squares estimate



# Least-squares Solving

If some points are outliers, robust approach such as RANSAC is needed



# RANSAC: RANdom SAmple Consensus

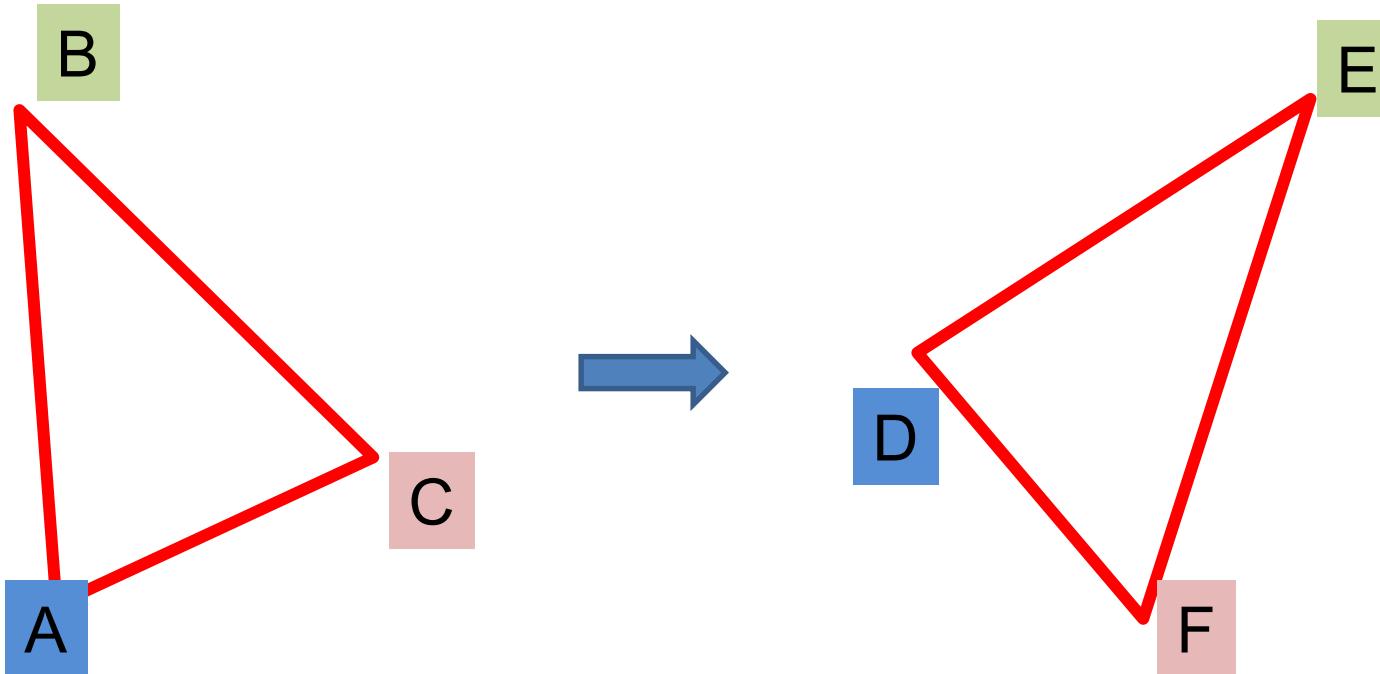
## RANSAC Algorithm

- Repeat N times
  1. Randomly select a sample
    - Select just enough points to recover the parameters
  2. Fit the model with random sample
  3. See how many other points agree
- Best estimate is one with most agreement
  - can use agreeing points to refine estimate

Example of 2D line fitting

# Question

Suppose we have two triangles, ABC and DEF, related by a general affine transformation. Solve for the transformation.



# Good luck!

- Questions?

# Take-home questions

- 2) How would you make a sharpening filter using gradient domain processing? What are the constraints on the gradients and the intensities?

# Take-home question

Suppose you have estimated three vanishing points corresponding to orthogonal directions. How can you recover the rotation matrix that is aligned with the 3D axes defined by these points?

- Assume that intrinsic matrix  $K$  has three parameters
- Remember, in homogeneous coordinates, we can write a 3d point at infinity as  $(X, Y, Z, 0)$

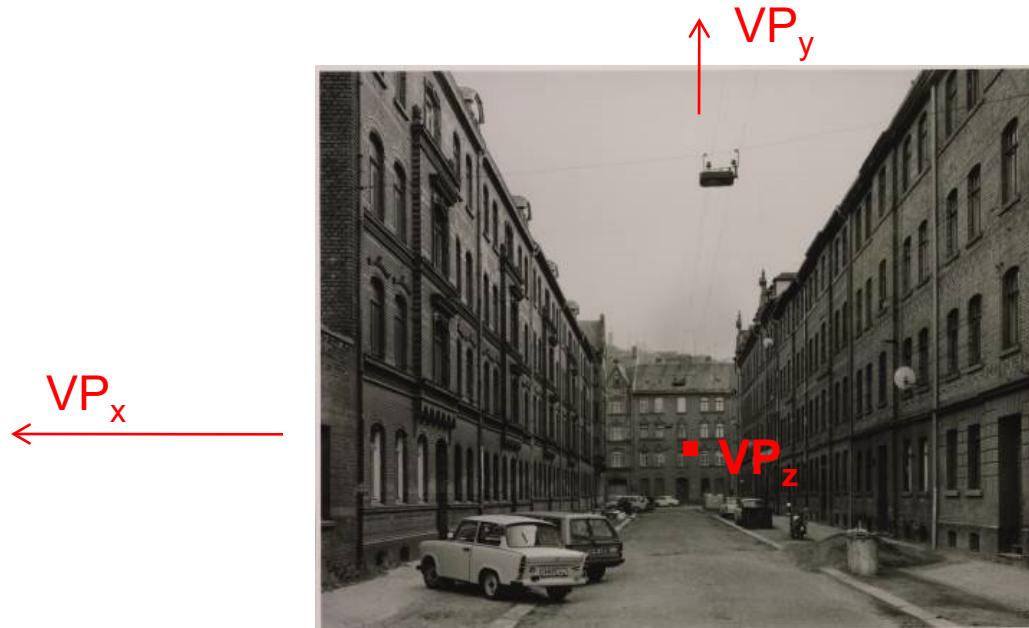


Photo from online Tate collection