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Directional Surveying: Rotating and Sliding Operations Give Different Wellbore Position Accuracy

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Abstract

We present a novel method for analysing the position uncertainty of directional surveys. By this method, the uncertainty can be accurately calculated by conceptually simple analytical formulae. The theory covers any correlation in measurement error and any correlation in toolface, for both stationary and continuous surveys.

The results demonstrate the importance of handling the toolface dependence properly. A direct benefit is that too optimistic or overly pessimistic accuracy estimates are avoided. Furthermore, the difference between rotating and sliding operation is significant for many error terms. For example, a misalignment error typically causes 10 times higher position uncertainty in sliding mode than in rotating mode, for a 3000 m long survey.

These findings should have implications for the future specification of survey instrument operation procedures, and thus the design of directional survey programs.

Introduction

The toolface is defined as the angle between one axis of the instrument-based co-ordinate system, normal to the tool axis, and the instrument's high-side direction. In borehole surveying accuracy analysis, the propagation of error is in many cases complicated by the toolface dependence of the relation between sensor (measurement) error and position error.

In traditional MWD magnetic directional surveys the instrument package rotates as part of the drillstring, thus randomizing the angular orientation between consecutive measurements. In contrast, many gyro survey operations are carried out in sliding mode, i. e., the instrument does not rotate or rotates very slowly during the survey. Many instruments may be operated in either mode, depending on the survey program.

These different modes of operation have considerable effect on the estimation of the wellbore position accuracy. In rotating mode the instrument's toolface angle will be uncorrelated between the measurements, whereas in sliding mode the toolface angle is highly correlated. In either case, the actual value is often unknown for real surveys, and always unknown for planned surveys.

We have developed a theory that takes these statistical properties of the toolface angle into account. As a consequence, we are able to analyse the position accuracy for either operational mode by simple, effective, and purely analytical algorithms. The theory also covers any combination of stationary and continuous determinations of azimuth and inclination.

This paper presents the theory and shows its applicability to position accuracy analysis. To our knowledge, this is the first rigorous treatment of the subject.

Theory

We shall deal with toolface dependent measurements of azimuth and inclination. Such measurements are done either stationary (typical for MWD and stationary gyro surveys) or continuously (by integrating the sensor readings; typical for continuous gyro surveys). Consequently, we address four situations:

- stationary Azimuth -stationary Inclination
- continuous Azimuth -continuous Inclination
- stationary Azimuth -continuous Inclination
- continuous Azimuth -stationary Inclination

which are treated together in a common methodology and general formulas.

Furthermore, we deal with two stochastic variables: sensor error and toolface. The main subject of this paper is to show how the correlation between consecutive measurements for each of these variables affects the position uncertainty, represented by the covariance between coordinates.

By definition, the correlation coefficient ρ may be any number between -1 and 1. We derive general formulae valid for any ρ. To demonstrate the significance of this factor, however, we shall frequently compare the cases of full correlation (ρ = *1*, systematic) and no correlation (ρ = *0*, random) between the stations.

A sensor error is described as random or systematic depending on the nature of the error source (Ref. 1).

The toolface is characterised as uncorrelated (random) when the instrument rotates considerably between measurements. This is typical for traditional MWD surveys. If the instrument does not rotate, or rotates very slowly during the survey (sliding mode), the toolface will be correlated (systematic; non-varying) between the measurements. This is typical for many gyro surveys. In either case, the actual value of the toolface is assumed unknown.

Thus, we consider the following situations:

- random Sensor error -random Toolface
- random Sensor error -systematic Toolface
- systematic Sensor error -random Toolface
- systematic Sensor error -systematic Toolface

We have organized the material such that Appendices A through E, when read in sequential order, give a logical presentation of the theory.

Appendices A and B give the necessary background on error propagation through linearised equations for this particular problem. Furthermore, a general expression for position covariances is derived in Appendix C.

The main method is derived in Appendix D. It is shown how the general covariance expression simplifies for various toolface and sensor error properties. The method covers both stationary and continuous measurements and any combination of them. A simple method to calculate the expectation values with respect to toolface is also described in Appendix D.

Appendix E presents an alternative method. The result is a new set of equations that do not depend upon toolface.

Assumptions.

- A basic assumption for the analysis is that the relation between sensor and position error can be linearised. This approximation is generally good, since the error of each measurement will be small. The assumption of linearised relations is used throughout this paper.
- The developed method is valid for any error distribution for the sensor errors. However, there are one exception; the expection value for the sensor error is zero.
- All toolface angles are statistically independent of all sensor errors.
- When using correlation coefficients between different sensors, they are all assumed to have the same value. Accordingly for correlation coefficients between different toolface angles.
- Magnetic and geographical azimuth (referenced to magnetic and geographical north, respectively) should be used in the weighting functions, whereas grid direction (referenced to map projection north) should be used in the wellbore trajectory description. For simplicity, we assume that all these are equal, denoted by the term "azimuth".
- The term "sliding" is used to denote situations where the toolface angle does not change during a survey. However, in a real sliding drilling operation, the toolface may change slowly over a curved wellbore section. In such cases, the formulae for non-varying toolface will be approximately correct.

Method.

The contribution from one particular sensor error to the uncertainty in station *m+1* is given by (A-14):

$$dX_{m+1} = \sum_{i=1}^{m} h_{X_i} d\varepsilon_i \qquad \ldots(1)$$

where $d\varepsilon_i$ is the sensor error in station *i*. *X* can be any of the *N* (north), *E* (east), or *V* (vertical) co-ordinates. The h_X expression depends upon

- The weighting function in station *i* (inclusive toolface angle)
- The wellbore geometry from station i to the station *m+1* where covariance shall be calculated.

It should be emphasised that h_X changes dependent of the station *m+1* to be analysed.

The general expression for the covariance is derived in Appendix C (C-7):

$$Cov(X,Y) = \sum_{i=1}^{m} \sum_{j=1}^{m} E\{h_{X_i} h_{Y_j}\} \rho_{ij} \sigma^2 \qquad \ldots(2)$$

where the statistical characteristics of $d\varepsilon_i$ are described through the standard deviation σ^2 and the correlation coefficient ρ_{ij} between stations *i* and *j*.

In Appendix D we derive the simplified expressions for

- Random error - Sliding mode
- Random error - Rotating mode
- Systematic error - Sliding mode
- Systematic error - Rotating mode

These expressions are given in the equations (D-2,3,4).

The expectation value of h_X, h_Y, and the product $h_X h_Y$ is the key issue for calculating the covariances. Appendix D shows how these values can be found by evaluating h_X etc. for discrete toolface values and averaging the results according to certain rules. The number of toolface angles is eight or four, depending on the form of the weighting function. The toolfaces must be evenly distributed around the circle.

Alternative method.

This metod is described in Appendix E. Here we utilize that the expectation values for trigonometric functions that occur in the weighting functions, can be calculated analytically. The actual values are shown in Table 1. Inserting these values, we see that each sensor error term can be replaced by up to five fictive error terms that are mutually independent and have the same variance as the original error term. In contrast to the original weighting function, the coefficients for these fictive terms are independent of the toolface angle and can thus be handled like all other error terms that are not influenced by the toolface.

The operational mode and original error term characteristics determine whether the fictive error terms should be treated as random or systematic.

This alternative method results in less computational work and fewer data variables than the method described above. However, weighting functions including the fictive terms have to be derived from the original weighting function.

Results and Examples

The mode of operating the survey tool, rotating or sliding, has significant impact on the estimated position variances and covariances. This can be seen from the formulae alone. The results are also dependent of wellbore geometry.

To discuss toolface dependence and illustrate the effects of the operational mode, we choose an example wellbore with constant inclination (75°) and constant azimuth (60°) located at 60°N. The survey separation increment is 30m. The survey speed is assumed to be 0.8m/s for continuous gyro and inertial surveys.

We have left out uncertainties from initialisation for continuous measurements in this summary.

Random sensor errors

Stationary measurements. There is no difference between the results for sliding and rotating mode for any error term. The standard deviations (SD) for the wellbore position co-ordinates grow proportionally with the square root of measured depth (MD). This situation is not illustrated.

Continuous measurements. There is no difference between the results for sliding and rotating mode for any error term. The SD values along the wellbore grow proportionally with ($MD^{1.5}$). A typical error term in this class is gyro random walk.

Figure 1 shows the standard deviation for the North co-ordinate when the standard deviation for the gyro bias error term is 0.15°/sqrt(hour).

Systematic sensor errors

Stationary measurements. For this situation we need to distingish between error types.

Tool misalignments, accelerometer biases, magnetometer biases and gyro biases are examples that give SD values that grow linearly with MD in sliding mode, and proportionally with the square root of MD in rotating mode. The effects are illustrated for a tool misalignment error with SD 0.1° in Figure 2. The weighting function is given in (B-3,4). From this figure it is seen that the SD at MD = 3000m is ten times higher in sliding mode, compared to the SD value for rotating mode.

The other type is represented by the scale errors for the sensors mentioned above. The SD values grow linearly with MD for both sliding and rotating tool. However the SD values are normally slightly less in the rotating mode. This situation is not illustrated.

Continuous measurements. Also here we have to distinguish between error types.

Gyro biases (also called gyro drift) represent the first one. This type is illustrated by (B-7,8), and the effect is shown in Figure 3. The gyro bias value is 0.15°/hour. In sliding mode the SD values grow proportionally with the square of MD, while the relationship is linear in rotating mode.

The second type is gyro scale errors, for which the SD values grow proportionally with the square of MD in both sliding and rotating mode. However, the SD values are normally slightly less in the rotating mode. The effect of this type is illustrated in Figure 4 for a gyro scale error of 0.01.

Combined measurement types

A combination of continuous azimuth and stationary inclination, or eventually vice versa, will give results between those reported above.

Conclusions

We have developed a novel and general method for calculation of the position uncertainty in directional surveys. The method is applicable to the error models for both stationary and continuous survey operations, and for both random and systematic error sources. A major achievement is that the dependence upon toolface, including the toolface's statistical properties, is analysed by purely analytical means. This enables us to evaluate and compare the effects of sliding and rotating survey operations.

We have shown the mathematical derivation of the method, along with two alternatives for efficient incorporation of the statistical properties. The theory is thus well suited for fast and efficient computer analysis.

Numerical examples show that sliding and rotating operations may give significantly different position uncertainties. However, this depends upon the specific error terms considered. We also observe that some error terms give uncertainty effects that should not be neglected even in rotating mode.

The results of this work should have considerable consequences for instrument design, operational procedures, and survey programs.

Nomenclature

D , MD	alonghole depth
I	wellbore inclination
A	wellbore azimuth
N	north co-ordinate
E	east co-ordinate
V	vertical co-ordinate (downward)
τ	toolface angle
$d\varepsilon$	sensor error
ρ	correlation coefficient
σ , SD	standard deviation
$E\{\}$	expectation operator

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Appendix A. Propagation of uncertainty

Wellbore trajectory equation

It is assumed throughout this paper that the wellbore's position is derived from a number of discrete measurements. This is obvious for tools measuring at predefined locations (MWD and gyro in stationary mode), but also for continuous instruments (continuous gyro and inertial tools) the measurements are discretised by sampling an integrated function.

We therefore start by considering an arbitrary measurement station with index i. We shall consider the common case that the position is derived from measurements of depth D, azimuth A, and inclination I; however, this assumption is not crucial to the further analysis, and other measured quantities may be handled similarly. The increment in one position coordinate (here: X) due to measurements in station i is:

$$\Delta X_i = f_X\left(\Delta D_i, A_i, I_i\right) \quad \text{...(A-1)}$$

Notice that the function f_X (wellbore trajectory model) may depend also on measurements in the neighbour station i-1, but this is neglected here. It can be shown that the uncertainty analysis results are more or less insensitive to the actual wellbore trajectory model (Ref. 3).

The X co-ordinate in station $(m+1)$ is expressed as the sum of the start co-ordinate and all m previous increments:

$$X_{m+1} = X_0 + \sum_{i=1}^{m} \Delta X_i \quad \text{...(A-2)}$$

Accordingly, (A-3) is the basis for the uncertainty analysis

$$dX_{m+1} = \sum_{i=1}^{m} d\Delta X_i \quad \text{...(A-3)}$$

assuming that the start co-ordinate uncertainty equals zero.

Linearised error propagation equation. The uncertainty in the position increment is found by a linearisation (Taylor expansion to first order). The following analysis is restricted to the azimuth and inclination terms, as depth measurements are not toolface dependent.

$$d\Delta X_i = f_{X_i}^{A} dA_i + f_{X_i}^{I} dI_i \quad \text{...(A-4)}$$

where the functions

$$f_{X_i}^{A} = \frac{\delta f_X(i)}{\delta A_i} \qquad f_{X_i}^{I} = \frac{\delta f_X(i)}{\delta I_i} \quad \text{...(A-5,6)}$$

denote the partial derivatives evaluated in station i. These quantities are not dependent of the toolface angle. However, dA_i and dI_i are toolface dependent.

Weighting functions

Stationary azimuth and inclination

$$dA_i = \frac{\delta A(i)}{\delta \varepsilon_i} d\varepsilon_i = A_i^{\varepsilon} d\varepsilon_i \quad \text{...(A-7)}$$

$$dI_i = \frac{\delta I(i)}{\delta \varepsilon_i} d\varepsilon_i = I_i^{\varepsilon} d\varepsilon_i \quad \text{...(A-8)}$$

where $d\varepsilon_i$ is the error source contribution in station i, and the notation A_i^{ε} is introduced for simplisity.

Continuous azimuth and inclination. For continuous gyro and inertial tools some of the error terms are a part of the integration process for dA and/or dI. The azimuth conditions become

$$\begin{aligned} dA_i &= dA_{i-1} + \Delta dA_i = dA_{i-1} + \frac{\delta A(i)}{\delta \varepsilon_i} d\varepsilon_i \\ &= dA_{i-1} + A_i^{\varepsilon} d\varepsilon_i = \sum_{k=1}^{i} A_k^{\varepsilon} d\varepsilon_k \end{aligned} \quad \text{...(A-9)}$$

Similar for inclination:

$$dI_i = dI_{i-1} + I_i^{\varepsilon} d\varepsilon_i = \sum_{k=1}^{i} I_k^{\varepsilon} d\varepsilon_k \quad \text{...(A-10)}$$

General expression for weighting functions. The relations (A-7..10) are called the weighting functions (Ref. 1, 2). The dependence on toolface angle arises in these relations. We now introduce general expressions covering all possible combinations of azimuth and inclination mode and insert into (A-4):

$$d\Delta X_i = f_{X_i}^{A} \sum_{k=\beta}^{i} A_k^{\varepsilon} d\varepsilon_k + f_{X_i}^{I} \sum_{k=\gamma}^{i} I_k^{\varepsilon} d\varepsilon_k \qquad \text{...(A-10)}$$

where $\beta = 1$ means continuous azimuth
$\beta = i$ means stationary azimuth
$\gamma = 1$ means continuous inclination
$\gamma = i$ means stationary inclination

The total uncertainty in station ($m+1$):

$$dX_{m+1} = \sum_{i=1}^{m} \left(f_{X_i}^{A} \sum_{k=\beta}^{i} A_k^{\varepsilon} d\varepsilon_k + f_{X_i}^{I} \sum_{k=\gamma}^{i} I_k^{\varepsilon} d\varepsilon_k \right) \qquad \text{(A-11)}$$

Re-formulation of weighting functions

We now rearrange this expression such that each $d\varepsilon$ term appears just one time in the summation:

$$dX_{m+1} = \sum_{i=1}^{m} \left(A_i^{\varepsilon} \sum_{k=i}^{p} f_{X_k}^{A} + I_i^{\varepsilon} \sum_{k=i}^{q} f_{X_k}^{I} \right) d\varepsilon_i \qquad \text{...(A-12)}$$

where $p = m$ means continuous azimuth
$p = i$ means stationary azimuth
$q = m$ means continuous inclination
$q = i$ means stationary inclination

The rearrangement is done by interchanging the summations in (A-11) with the proper changes in summation limits. A notational change of summation indicies then yields (A-12).

We now introduce

$$h_{X_i} = A_i^{\varepsilon} \sum_{k=i}^{p} f_{X_k}^{A} + I_i^{\varepsilon} \sum_{k=i}^{q} f_{X_k}^{I} \qquad \text{...(A-13)}$$

Notice that in the continuous case ($p = m$ and/or $q = m$), h_X depends on all intervals between station i and ($m+1$). Equation (A-12) simplifies to

$$dX_{m+1} = \sum_{i=1}^{m} h_{X_i} d\varepsilon_i \qquad \text{...(A-14)}$$

Appendix B. Examples of weighting functions

General

The position increment is shown in (A-1) and the error propagation in (A-4). As an example these quantities are derived for the North co-ordinate:

$$\Delta N_i = f_N\left(\Delta D_i, A_i, I_i\right) = \Delta D_i \sin(I_i)\cos(A_i) \qquad \text{...(B-1)}$$

$$\begin{aligned} d\Delta N_i &= f_{N_i}^{A} dA_i + f_{N_i}^{I} dI_i \\ &= -\Delta D_i \sin(I_i)\sin(A_i) dA_i \\ &\quad + \Delta D_i \cos(I_i)\cos(A_i) dI_i \end{aligned} \qquad \text{...(B-2)}$$

where dA_i and dI_i are the weighting functions.

Staionary term - Tool misalignment

In Ref. 1, eq. {5.1.3-1,3}, the weighting functions for the tool's x axis misalignment are shown to be

$$dA_i = -(\cos(\tau_i)/\sin(I_i)) \cdot d\varepsilon_i \qquad \text{...(B-3)}$$

$$dI_i = \sin(\tau_i) \cdot d\varepsilon_i \qquad \text{...(B-4)}$$

Inserting (B-3,4) into (B-2) gives

$$d\Delta N_i = h_{N_i} d\varepsilon_i \qquad \text{...(B-5)}$$

where

$$\begin{aligned} h_{N_i} &= \Delta D_i \sin(A_i)\cos(\tau_i) \\ &\quad + \Delta D_i \cos(I_i)\cos(A_i)\sin(\tau_i) \end{aligned} \qquad \text{...(B-6)}$$

Continuous term - Gyro bias

An example of a continuous weighting function (azimuth) is shown in Ref 1, eq. {8.1.1-13}

$$dA_i = dA_{i-1} - \Delta t_i \sin(I_i)\sin(\tau_i) d\varepsilon_i \qquad \text{...(B-7)}$$

where Δt_i is the time interval since the previous update of azimuth. The h_N expression now becomes

$$h_{N_i} = \Delta t_i \sin(I_i)\sin(\tau_i) \sum_{k=i}^{m} \Delta D_k \sin(I_k)\sin(A_k) \qquad \text{...(B-8)}$$

Appendix C. Covariance definition and expressions

The covariance between two random variables X and Y is by definition

$$Cov(X,Y) = E\{XY\} - E\{X\}E\{Y\} \qquad \text{...(C-1)}$$

where E is the expectation value operator. It is easily shown that the covariance arises only from the deviations dX and dY, thus it can be written:

$$Cov(X,Y) = E\{dXdY\} - E\{dX\}E\{dY\} \qquad \text{...(C-2)}$$

Throughout this paper it is assumed that the expectation values for dX and dY is zero. The expression thus simplifies to

$$Cov(X,Y) = E\{dXdY\} \qquad \text{...(C-3)}$$

X and Y represent any of the position co-ordinates N(north), E(east) and V(vertical), thus (C-4) is the general expression for the variances and covariances in the *NEV* system when inserting (A-14) in (C-3).

$$Cov(X,Y) = E\{\sum_{i=1}^{m}\sum_{j=1}^{m} h_{X_i} h_{Y_j} d\varepsilon_i d\varepsilon_j\} \qquad \text{...(C-4)}$$

The h functions are dependent on the toolface. As the toolfaces are statistically independent of the sensor errors, (C-4) can be reformulated to

$$Cov(X,Y) = \sum_{i=1}^{m}\sum_{j=1}^{m} E\{h_{X_i} h_{Y_j}\} E\{d\varepsilon_i d\varepsilon_j\} \qquad \text{...(C-5)}$$

Further

$$E\{d\varepsilon_i d\varepsilon_j\} = \rho_{ij}\sigma^2 \qquad \text{...(C-6)}$$

where ρ_{ij} is the correlation coefficient between the measurements indexed i and j, and σ^2 is assumed to be constant for all stations. (C-5) now becomes

$$Cov(X,Y) = \sum_{i=1}^{m}\sum_{j=1}^{m} E\{h_{X_i} h_{Y_j}\}\rho_{ij}\sigma^2 \qquad \text{...(C-7)}$$

Appendix D. Calculation of covariances

Covariance expressions

Before calculating the covariances we split the right-hand side of (C-7) into two parts, the first one containing non-diagonal elements and the second one the diagonal elements. This is convenient for understanding the difference between sliding and rotating operations.

$$Cov(X,Y) = \sum_{i=1}^{m}\sum_{j\neq i}^{m} E\{h_{X_i} h_{Y_j}\}\rho_{ij}\sigma^2 + \sum_{i=1}^{m} E\{h_{X_i} h_{Y_i}\}\rho_{ii}\sigma^2 \qquad \text{...(D-1)}$$

Random error source

Sliding and rotating tool. In this situation the first part of (D-1) equals zero because $\rho_{ij} = 0$ when $i \neq j$ .The covariance will be the same for sliding and rotating operations.

$$Cov(X,Y) = \sum_{i=1}^{m} E\{h_{X_i} h_{Y_i}\}\sigma^2 \qquad \text{...(D-2)}$$

Systematic error source

Sliding tool. All stations have identical toolfaces in this situation, i.e.; $\rho_{ij} = 1$ for all i and j. This means that the original expression (C-7) can not be reduced. However, it can be written as

$$Cov(X,Y) = E\{\sum_{i=1}^{m} h_{X_i} \sum_{j=1}^{m} h_{Y_j}\}\sigma^2 \qquad \text{...(D-3)}$$

Rotating tool. In this situation, h_X and h_Y are independent when $i \neq j$; hence $E\{h_X h_Y\} = E\{h_X\}E\{h_Y\}$ in the double sum of (D-1). This double sum can be expressed as a sum over all i and j, minus the diagonal terms $i = j$. The covariance thus becomes

$$Cov(X,Y) = E\{\sum_{i=1}^{m} h_{X_i}\}E\{\sum_{j=1}^{m} h_{Y_j}\}\sigma^2 - \sum_{i=1}^{m} E\{h_{X_i}\}E\{h_{Y_i}\}\sigma^2 + \sum_{i=1}^{m} E\{h_{X_i} h_{Y_i}\}\sigma^2 \qquad \text{...(D-4)}$$

Expectation values for toolface dependent expressions

For all known error models (Refs. 1,2) all weighting functions can be written on the form

$$A_i^\varepsilon = r_i^A \sin^2(\tau_i) + s_i^A \sin(\tau_i)\cos(\tau_i) + t_i^A \cos^2(\tau_i) + u_i^A \sin(\tau_i) + v_i^A \cos(\tau_i) + w_i^A \quad \text{...(D-5)}$$

$$I_i^\varepsilon = r_i^I \sin^2(\tau_i) + s_i^I \sin(\tau_i)\cos(\tau_i) + t_i^I \cos^2(\tau_i) + u_i^I \sin(\tau_i) + v_i^I \cos(\tau_i) + w_i^I \quad \text{...(D-6)}$$

with toolface-dependent coefficients *r…w*. Insertion of these expressions in (A-13) yields

$$\begin{aligned} h_{X_i} = & \left[r_i^A \sin^2(\tau_i) + s_i^A \sin(\tau_i)\cos(\tau_i) + t_i^A \cos^2(\tau_i) + u_i^A \sin(\tau_i) + v_i^A \cos(\tau_i) + w_i^A\right] \cdot \sum_{k=i}^{p} f_{X_k}^A \\ & + \left[r_i^I \sin^2(\tau_i) + s_i^I \sin(\tau_i)\cos(\tau_i) + t_i^I \cos^2(\tau_i) + u_i^I \sin(\tau_i) + v_i^I \cos(\tau_i) + w_i^I\right] \cdot \sum_{k=i}^{q} f_{X_k}^I \end{aligned} \quad \text{...(D-7)}$$

By introducing

$$a_{X_i} = r_i^A \cdot \sum_{k=i}^{p} f_{X_k}^A + r_i^I \cdot \sum_{k=i}^{q} f_{X_k}^I \quad \text{...(D-8)}$$

$$b_{X_i} = s_i^A \cdot \sum_{k=i}^{p} f_{X_k}^A + s_i^I \cdot \sum_{k=i}^{q} f_{X_k}^I \quad \text{...(D-9)}$$

etc., (D-7) can be written

$$h_{X_i} = a_{X_i} \sin^2(\tau_i) + b_{X_i} \sin(\tau_i)\cos(\tau_i) + c_{X_i} \cos^2(\tau_i) + d_{X_i} \sin(\tau_i) + e_{X_i} \cos(\tau_i) + f_{X_i} \quad \text{...(D-10)}$$

and similarly for h_Y.

Expectation values by averaging

When evaluating the covariances (D-2,3,4) we need to calculate expectation values like $E\{h_X\}$, $E\{h_Y\}$ and $E\{h_X h_Y\}$. With h_X and h_Y on the general form (D-10), the expectation values can be found by evaluating the expressions with a discrete set of toolface angles, and averaging the results. The set of toolface angles depends on the complexity of h_X and h_Y as follows:

If one or both of h_X and h_Y has one or more of the coefficients *a*, *b* and *c* different from zero, the expectation values will be obtained by averaging over eight toolface angles equally distributed around the circle, for example 0°, 45°, 90°, °135°, 180°, 225°, 270°, 315°.

If $a = b = c = 0$, but one or both of h_X and h_Y has one or more of the coefficients *d* and *e* different from zero, the expectation values will be obtained by averaging over four toolface angles equally distributed around the circle, for example 0°, 90°, 180°, 270°.

If h_X and h_Y are independent of toolface (only *f* different from zero), we just use the function value without any averaging process.

Returning for a moment to the general formula (C-7), and inserting the *h* functions, we find 36 different product terms. Several of these terms will have expectation values identical to zero. Table 1 lists the non-zero expectation values. This table of course also applies to (D-2,3,4).

In equations (D-2,3,4), expectation values are calculated at two levels. Terms on the form

$$E\{\sum_{i=1}^{m} h_{X_i} \sum_{j=1}^{m} h_{Y_j}\}$$

$$E\{\sum_{i=1}^{m} h_{X_i}\} \qquad E\{\sum_{j=1}^{m} h_{Y_j}\}$$

are conveniently evaluated by first summing h_X and h_Y throughout the wellbore for one toolface value, then repeating for the next toolface and so on, and finally averaging the results.

Terms on the form

$$\sum_{i=1}^{m} E\{h_{X_i} h_{Y_i}\} \qquad \sum_{i=1}^{m} E\{h_{X_i}\} E\{h_{Y_i}\}$$

must be calculated by averaging at each single station. The procedure is thus: Evaluate h_X (or h_Y or the product) with all four or eight toolface values in station *i*; average the results; repeat this in the next station; and finally, sum the averages throughout the well.

Appendix E. Alternative method for calculation of covariances

The method that was derived in Appendix D implies averaging over several toolface angles. This can be omitted by introducing some fictive error sources instead of the real one. The alternative approach is shown in this appendix.

We consider the expression for h_X and the similar one for h_Y :

$$\begin{aligned} h_{X_i} &= a_{X_i}\sin^2(\tau_i) + b_{X_i}\sin(\tau_i)\cos(\tau_i) \\ &+ c_{X_i}\cos^2(\tau_i) + d_{X_i}\sin(\tau_i) \\ &+ e_{X_i}\cos(\tau_i) + f_{X_i} \end{aligned} \quad \ldots\text{(E-1)}$$

$$\begin{aligned} h_{Y_j} &= a_{Y_j}\sin^2(\tau_j) + b_{Y_j}\sin(\tau_j)\cos(\tau_j) \\ &+ c_{Y_j}\cos^2(\tau_j) + d_{Y_j}\sin(\tau_j) \\ &+ e_{Y_j}\cos(\tau_j) + f_{Y_j} \end{aligned} \quad \ldots\text{(E-2)}$$

The contribution to the covariance, *Cov(X,Y),* from the observation pair *ij* is now expressed in (E-3). The expectation value will be dependent on the correlation between the toolface angles. The correlation coefficient, $\rho(\tau_i\tau_j)$, expresses the correlation between the toolface angles in the stations *i* and *j*. By inserting the values from Table 1, the general expression for the expectation value can therefore be written:

$$\begin{aligned} E\{h_{Xi}h_{Yj}\}E\{d\varepsilon_i d\varepsilon_j\} &= \Big[\tfrac{1}{4}a_{X_i}a_{Y_j} + \tfrac{1}{4}a_{X_i}c_{Y_j} \\ &+ \tfrac{1}{2}a_{X_i}f_{Y_j} + \tfrac{1}{4}c_{X_i}a_{Y_j} + \tfrac{1}{4}c_{X_i}c_{Y_j} + \tfrac{1}{2}c_{X_i}f_{Y_j} \\ &+ \tfrac{1}{2}f_{X_i}a_{Y_j} + \tfrac{1}{2}f_{X_i}c_{Y_j} + f_{X_i}f_{Y_j}\Big]\rho_{ij}\sigma^2 \\ &+ \Big[\tfrac{1}{8}a_{X_i}a_{Y_j} - \tfrac{1}{8}a_{X_i}c_{Y_j} + \tfrac{1}{8}b_{X_i}b_{Y_j} - \tfrac{1}{8}c_{X_i}a_{Y_j} \\ &+ \tfrac{1}{8}c_{X_i}c_{Y_j} + \tfrac{1}{2}d_{X_i}d_{Y_j} + \tfrac{1}{2}e_{X_i}e_{Y_j}\Big]\rho(\tau_i\tau_j)\rho_{ij}\sigma^2 \end{aligned} \quad .\text{(E-3)}$$

(E-3) is equivalent to the "quadratic" form shown in (E-4).

$$\begin{aligned} &E\{h_{Xi}h_{Yj}\}E\{d\varepsilon_i d\varepsilon_j\} = \\ &\Big[\tfrac{1}{2}a_{X_i} + \tfrac{1}{2}c_{X_i} + f_{X_i}\Big]\cdot\Big[\tfrac{1}{2}a_{Y_j} + \tfrac{1}{2}c_{Y_j} + f_{Y_j}\Big]\rho_{ij}\sigma^2 \\ &+ \Big[\sqrt{\tfrac{1}{8}}a_{X_i} - \sqrt{\tfrac{1}{8}}c_{X_i}\Big]\cdot\Big[\sqrt{\tfrac{1}{8}}a_{Y_j} - \sqrt{\tfrac{1}{8}}c_{Y_j}\Big]\rho(\tau_i\tau_j)\rho_{ij}\sigma^2 \\ &+ \Big[\sqrt{\tfrac{1}{8}}b_{X_i}\Big]\cdot\Big[\sqrt{\tfrac{1}{8}}b_{Y_j}\Big]\rho(\tau_i\tau_j)\rho_{ij}\sigma^2 \\ &+ \Big[\sqrt{\tfrac{1}{2}}d_{X_i}\Big]\cdot\Big[\sqrt{\tfrac{1}{2}}d_{Y_j}\Big]\rho(\tau_i\tau_j)\rho_{ij}\sigma^2 \\ &+ \Big[\sqrt{\tfrac{1}{2}}e_{X_i}\Big]\cdot\Big[\sqrt{\tfrac{1}{2}}e_{Y_j}\Big]\rho(\tau_i\tau_j)\rho_{ij}\sigma^2 \end{aligned} \quad \text{(E-4)}$$

Fictive error sources

(E-4) shows that the expression can be realised by substituting the original error source, *dε*, by five statistically independent fictive error sources $d\varepsilon_1$, $d\varepsilon_2$, $d\varepsilon_3$, $d\varepsilon_4$, and $d\varepsilon_5$, which all have the same expectation value and variance as the original one. In addition, they have station-to-station correlation properties that reflect the correlation properties of the rotating or sliding operation.

The coefficients for the fictive error sources are :

Error term no.	Coefficients
1	$(\tfrac{1}{2}a_{X_i} + \tfrac{1}{2}c_{X_i} + f_{X_i})$
2	$(\sqrt{\tfrac{1}{8}}a_{X_i} - \sqrt{\tfrac{1}{8}}c_{X_i})$
3	$(\sqrt{\tfrac{1}{8}}b_{X_i})$
4	$(\sqrt{\tfrac{1}{2}}d_{X_i})$
5	$(\sqrt{\tfrac{1}{2}}e_{X_i})$

Example. The *h* expression for a gyro bias term was shown in (B-7). This is on the form (E-1) with $a = b = c = e = f = 0$ and $d \neq 0$. Hence, the alternative realisation implies only fictive term no. 4, and the coefficient becomes from (E-8):

$$\sqrt{\tfrac{1}{2}}\Delta t_i \sin(I_i)\sum_{k=i}^{m}\Delta D_k \sin(I_k)\sin(A_k)$$

The correlation properties (random or systematic) of each fictive error term are described through the operational mode and the characteristics of the original error source. (E-4) is helpful to get the results listed below. Remember that the five fictive terms always are statistically independent.

Systematic error source

Sliding tool

$d\varepsilon_1$	systematic
$d\varepsilon_2$, $d\varepsilon_3$, $d\varepsilon_4$, $d\varepsilon_5$	systematic

Rotating tool

$d\varepsilon_1$	systematic
$d\varepsilon_2$, $d\varepsilon_3$, $d\varepsilon_4$, $d\varepsilon_5$	random

Random error source

Sliding and rotating tool

$d\varepsilon_1$	random
$d\varepsilon_2$, $d\varepsilon_3$, $d\varepsilon_4$, $d\varepsilon_5$	random

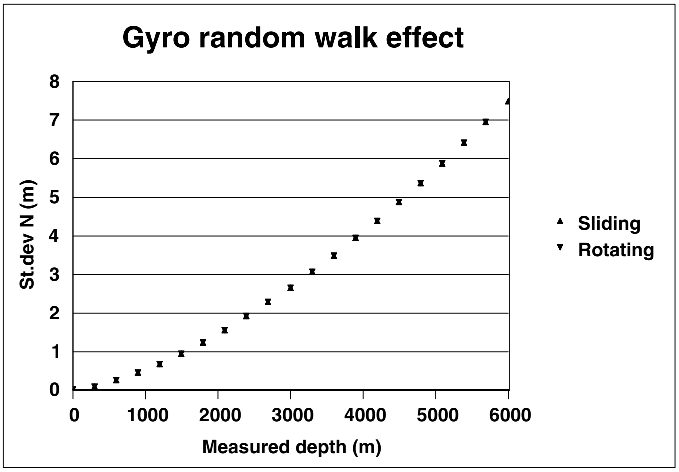

Figure 1 — Effect of Gyro random walk (0.15°/sqrt(h)): Standard deviation for the North co-ordinate along a straight line wellbore (I=75°, A=60°) at 60°N.

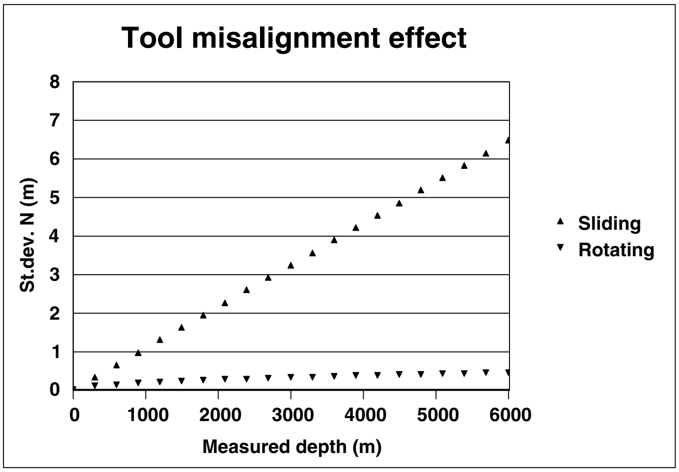

Figure 2 — Effect of Tool misalignment (0.1°): Standard deviation for the North co-ordinate along a straight line wellbore (I=75°, A=60°) at 60°N.

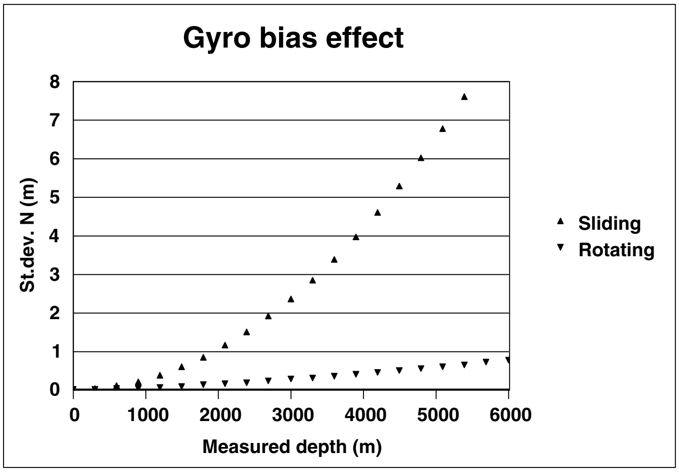

Figure 3 — Effect of Gyro bias (0.15°/h): Standard deviation for the North co-ordinate along a straight line wellbore (I=75°, A=60°) at 60°N.

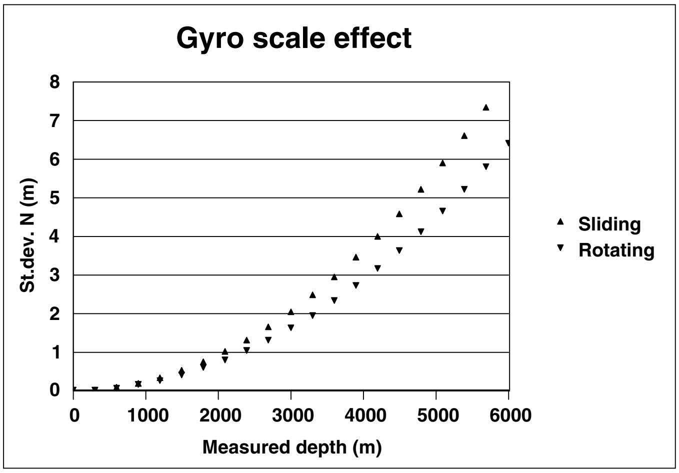

Figure 4 — Effect of Gyro scale (0.01): Standard deviation for the North co-ordinate along a straight line wellbore (I=75°, A=60°) at 60°N.

TABLE 1 Expectation values for trigonometric functions

Coefficient	**Cefficients dependent of toolface**	$E[\ldots]$ for $\rho(\tau_i \tau_j) = 1$	$E[\ldots]$ for $\rho(\tau_i \tau_j) = 0$
$a_{X_i} a_{Y_j}$	$\sin^2(\tau_i) \cdot \sin^2(\tau_j)$	3/8	1/4
$a_{X_i} c_{Y_j}$	$\sin^2(\tau_i) \cdot \cos^2(\tau_j)$	1/8	1/4
$a_{X_i} f_{Y_j}$	$\sin^2(\tau_i)$	1/2	1/2
$b_{X_i} b_{Y_j}$	$\sin(\tau_i) \cdot \cos(\tau_i) \cdot \sin(\tau_j) \cdot \cos(\tau_j)$	1/8	0
$c_{X_i} a_{Y_j}$	$\cos^2(\tau_i) \cdot \sin^2(\tau_j)$	1/8	1/4
$c_{X_i} c_{Y_j}$	$\cos^2(\tau_i) \cdot \cos^2(\tau_j)$	3/8	1/4
$c_{X_i} f_{Y_j}$	$\cos^2(\tau_i)$	1/2	1/2
$d_{X_i} d_{Y_j}$	$\sin(\tau_i) \cdot \sin(\tau_j)$	1/2	0
$e_{X_i} e_{Y_j}$	$\cos(\tau_i) \cdot \cos(\tau_j)$	1/2	0
$f_{X_i} a_{Y_j}$	$\sin^2(\tau_j)$	1/2	1/2
$f_{X_i} c_{Y_j}$	$\cos^2(\tau_j)$	1/2	1/2
$f_{X_i} f_{Y_j}$	1	1	1