

Inductive reasoning

- How do we know that our recursive code will work?
- iterMul terminates because b is initially positive, and decrease by 1 each time around loop; thus must eventually become less than 1
- recurMul called with $b = 1$ has no recursive call and stops
- recurMul called with $b > 1$ makes a recursive call with a smaller version of b ; must eventually reach call with $b = 1$

Mathematical induction

- To prove a statement indexed on integers is true for all values of n :
 - Prove it is true when n is smallest value (e.g. $n = 0$ or $n = 1$)
 - Then prove that if it is true for an arbitrary value of n , one can show that it must be true for $n+1$

Example

- $0 + 1 + 2 + 3 + \dots + n = (n(n+1))/2$
- Proof
 - If $n = 0$, then LHS is 0 and RHS is $0*1/2 = 0$, so true
 - Assume true for some k , then need to show that
 - $0 + 1 + 2 + \dots + k + (k+1) = ((k+1)(k+2))/2$
 - LHS is $k(k+1)/2 + (k+1)$ by assumption that property holds for problem of size k
 - This becomes, by algebra, $((k+1)(k+2))/2$
 - Hence expression holds for all $n \geq 0$

What does this have to do with code?

- Same logic applies

```
def recurMul(a, b):  
    if b == 1:  
        return a  
    else:  
        return a + recurMul(a, b-1)
```

- Base case, we can show that recurMul must return correct answer
- For recursive case, we can assume that recurMul correctly returns an answer for problems of size smaller than b, then by the addition step, it must also return a correct answer for problem of size b
- Thus by induction, code correctly returns answer