PHYS 4421: Introduction to Continuum Physics
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Project Name?

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1 Introduction

Our goal is to study the Kuramoto-Sivashinsky equation:

$$u_t = (u^2)_x - u_{xx} - \nu u_{xxxx} \tag{1}$$

where $u(x,t)$ is considered a function of one spatial coordinate x and the time t. The subscripts denote partial differentiation and ν is a damping parameter, which we will call the viscosity.

We assume periodic boundary conditions on the $x \in [0, L]$ interval:

$$u(x+L,t) = u(x,t) \tag{2}$$

which allows a Fourier series expansion:

$$u(x,t) = \sum_{-\infty}^{+\infty} b_k(t) e^{ik2\pi x/L} \tag{3}$$

Since $u(x,t)$ is real, it holds:

$$b_k = b^*_{-k} \tag{4}$$

Substituting eq. (3) into eq. (1) we get:

$$\dot{b}_k = (2\pi/L)^2 \left(k^2 - (2\pi/L)^2 \nu k^4\right) b_k + (2\pi/L) ik \sum_{m=-\infty}^{+\infty} b_m b_{k-m} \tag{5}$$

In general we will work with L as a controlling parameter and set $\nu = 1$. Yet, for the first numerical exploration of the K-S equation we will work with ν as a control parameter and set $L = 2\pi$ in order to compare our results with the previous ones given in [1].

From eq. (5) we notice that $\dot{b}_0 = 0$ and thus b_0 is an integral of the equations, or from (3), the average of the solution $\int dx u(x,t)$ is a constant, which (following [1]) we set to zero. This leads to $b_0 = 0$.

To simplify the system of equations further we choose the b_k's purely imaginary:

$$b_k = i a_k \tag{6}$$

with a_k real. Then eq. (5) reads:

$$\dot{a}_k = (2\pi/L)^2 \left(k^2 - (2\pi/L)\,\nu k^4\right) a_k - (2\pi/L) k \sum_{m=-\infty}^{+\infty} a_m a_{k-m} \tag{7}$$

Inserting eq. (6) into eq. (4) we get:

$$a_{-k} = -a_k \tag{8}$$

To integrate this system of equations numerically we will truncate the system of equations to a finite length by setting $a_k = 0$ for $k > N$. Then eq. (7) with the help of eq. (8) can be cast into the form:

$$\begin{aligned} \dot{a}_k &= \left(\frac{2\pi}{L}\right)^2 \left(k^2 - (2\pi/L)\,\nu k^4\right) a_k \\ &\quad - \frac{2\pi}{L} k \left(\sum_{m=1}^{k} a_m a_{k-m} - \sum_{m=k+1}^{N} a_m a_{m-k} - \sum_{m=1}^{N-k} a_m a_{k+m} \right) \end{aligned} \tag{9}$$

which only contains the a_k's for which $1 \leq k \leq N$. Thus we only need N instead of $2N$ equations ($a_0 = 0$, since we demanded that the average of the solution vanishes). Following the arguments in [1] we choose N=16 and $\nu = 0.029910$, ($L = 2\pi$). For the integration we use a fourth order Runge-Kutta scheme with constant step size.

Two projections of a typical trajectory in the a_k space onto different three-dimensional subspaces are shown in Figures 1 and 2.

2 Bibliography

1. F. Christiansen, P. Cvitanović, V. Putkaradze *Hopf's last hope: spatiotemporal chaos in terms of unstable recurrent patterns*, Nonlinearity **10**, 50 (1997)
2. P. Holmes, J.L. Lumley and G. Berkooz, *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*, Cambridge U. Press, Cambridge 1996.

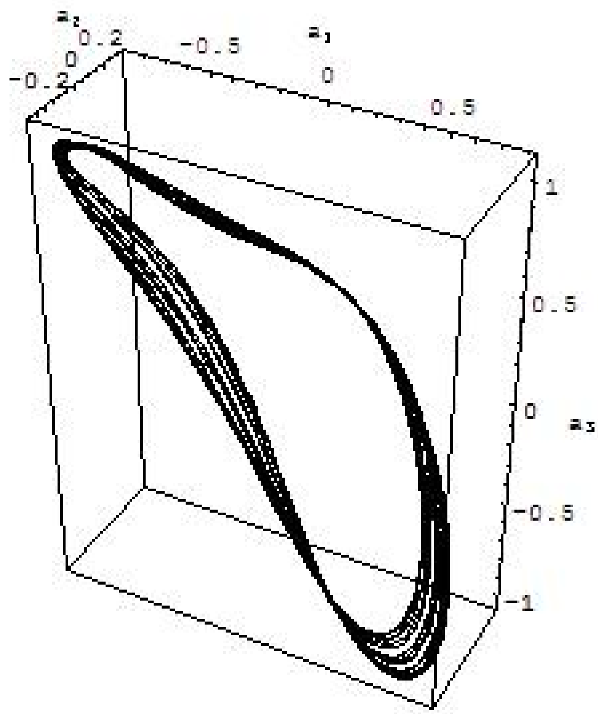

Figure 1: Projection of the trajectory onto the a_1, a_2, a_3 subspace. N=16 and $\nu = 0.029910,\ (L = 2\pi)$.

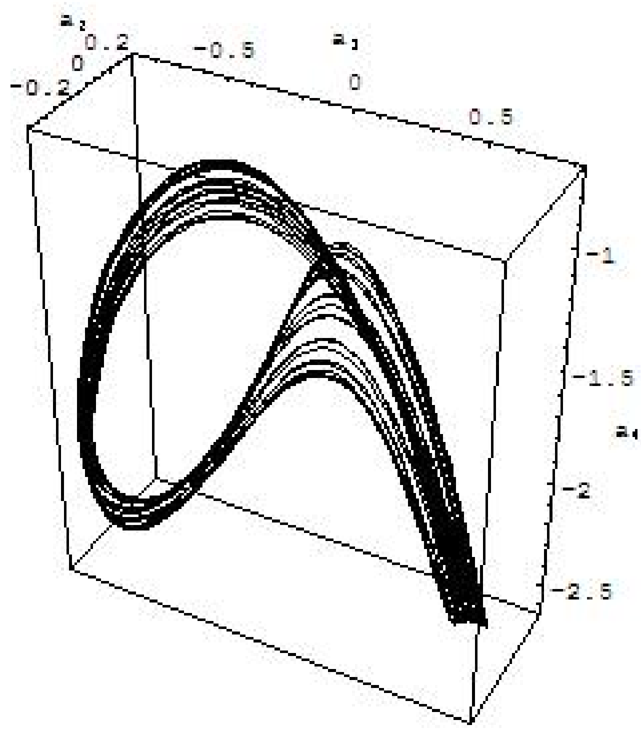

Figure 2: Projection of the trajectory onto the a_1, a_2, a_4 subspace. N=16 and $\nu = 0.029910,\ (L = 2\pi)$.