Chao's Rules

a blog

GaTech graduate research project, spring 2011

`RepoFile`, rev. : last edit by , //

Chao Shi (adviser: Predrag Cvitanović)

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Contents

Chapter 1

Research blog

[1]

2011-03-16 PC to Ruslan, Stefan and Evangelos Chao Shi, shichao116@gmail.com is starting (rather late) to work on slicing and dicing Kuramoto Sivashinsky as a 1st year graduate student project. Hopefully we can get him to speed with your and Stefan's help within the next six months. At the moment he is reading parts of ChaosBook and our articles, but it might be wise that you teach him immediately how to use your KS programs and data, so we do not waste time on that. Chao can compute.

2011-03-16 PC Chao, please keep track of what goes on in siminos/blog/ and siminos/lyapunov/ - you will be getting emails about updates. There is some current excitement there concerning the 'physical' dimensionality of the Kuramoto-Sivashinsky strange attractors.

2011-03-17 CS to Ruslan, Stefan and Evangelos Hi! Nice to meet you all. It is the first time I type something here. I am still reading Chapter 4. (and [1]) I will catch up with you as soon as I can. Maybe I will have a lot of questions to consult you. Hope you won't bother:)

2011-03-16 PC Why reference to Ellis, Gay-Balmaz, Holm and Ratiu [1]?

2011-03-19 CS to PC Yesterday I finished Chapter 4 but skipped the detail of the some examples. We discussed the first half of Chapter 4. Since there're pretty much mathematical detail and examples, we decide to break it into two parts. By the way, the notation of this chapter at the beginning is a little bit confusing and took me a while to get the concept of what those formulas really mean. I have some other questions about Chapter 4 but but now I am not used to typing math symbols here, I might ask you later directly or write it here if I get familiar with it.

[1] `RepoFile`, rev. : last edit by , //

2011-03-21 PC You have the dasbuch/book/chapter/*.tex source code, you can just clip and paste formulas to here.

2011-03-19 CS to PC Also I have a question about Example 3.2 in Chapter 3. It states that the choice of the coordinates of the pinball game are smart because they conserve the phase space volume. I don't understand this, would you mind explain it more specifically?

2011-03-25 PC Glad you asked the question about choice of billiard coordinates. Please write up the solution to exercise 2.1; it's worth doing it in class as well, a concrete example of how symplectic invariance preserves area for each (q, p) dual coordinate pair.

2011-03-19 CS to PC Going from here, I am also wondering how to choose phase space coordinates? Does phase/state space coordinates have any requirement and whether conservation of space volume is such a requirement? What's the meaning of conservation of phase space requirement? In my understanding in Hamiltonian flows, conservation of phase space volume means conservation of energy, am I right?

2011-03-25 PC Now, it is more subtle than that; time dependent flow can be symplectic, but energy is not conserved; I think Percival and D. Richards [2] (I have it in the CNS library) discuss that well. Symplectic invariance is *much stronger* requirement than either either energy conservation or phase-space volume conservation, see Section 7.4 Poincaré invariants and Appendix D.4 Stability of Hamiltonian flows: symplectic transformations preserve area for each (q, p) dual coordinate pair.

2011-03-22 PC to Chao This might be of interest to Adam, let him know about it: arXiv:1103.3981, *Chains of rotational tori and filamentary structures close to high multiplicity periodic orbits in a 3D galactic potential*, by Katsanikas, Patsis and Pinotsis.

They write " This paper discusses phase space structures encountered in the neighborhood of periodic orbits with high order multiplicity in a 3D autonomous Hamiltonian system with a potential of galactic type. We consider 4D spaces of section and we use the method of color and rotation [Patsis and Zachilas 1994] in order to visualize them. As examples we use the case of two orbits, one 2-periodic and one 7-periodic. We investigate the structure of multiple tori around them in the 4D surface of section and in addition we study the orbital behavior in the neighborhood of the corresponding simple unstable periodic orbits. By considering initially a few consequents in the neighborhood of the orbits in both cases we find a structure in the space of section, which is in direct correspondence with what is observed in a resonance zone of a 2D autonomous Hamiltonian system. However, in our 3D case we have instead of stability islands rotational tori, while the chaotic zone connecting the points of the unstable periodic orbit is replaced by filaments extending in 4D following a smooth color variation. For more intersections, the consequents of the orbit which started in the neighborhood of the unstable periodic orbit, diffuse in phase space and form a cloud that occupies a large volume surrounding the region containing the rotational tori. In this cloud the colors of the points are mixed. The same structures have been observed in the neighborhood of all m-periodic orbits we have examined in the system. This indicates a generic behavior. ”

2011-03-19 CS to PC What do you mean by "take the Hall out of library"?:) I actually don’t understand your last email.

2011-03-25 PC Reference to Hall is in the *Lie police* section of Siminos blog. It is good idea to keep reading updates of the three blogs - yours, siminos/blog and siminos/lyapunov, they are all related to your work.

2011-03-29 Ruslan Hi Chao. If you want to use Matlab for your explorations of the Kuramoto-Sivashinsky, then all my Matlab files can be found in /siminos/chao/matlab/ruslan. File ksdupo.m is the primary file. I’m using the cells feature of Matlab, so this file contains many sections that can be run independently. There is not much in the way of comments, so you’ll have to work it out for yourself. I’ll be happy to guide you through it if you ask specific questions.

2011-03-30 CS to PC Hi Professor Cvitanović, I am now reading Chapter 9. I have a question from the paragraph following the definition of free action: The splitting of a group G into a stabilizer G_p and *m-1* coset cG_p relates to an orbit M_p to *m-1* other distinct orbits cM_p. All of them have equivalent stabilizers, or more precisely, the points on the same group orbit have *conjugate stabilizers*: $G_{cp} = cG_pc^{-1}$. For the last sentence, does it mean that if G_p is a stabilizer of M_p, then cG_pc^{-1} is a stabilizer of cM_p?

2011-03-31 PC Yes, you are right. I have now incorporated “if G_p is a stabilizer of M_p, then cG_pc^{-1} is a stabilizer of cM_p” into discrete.tex, thanks.

I intend to excise the dreaded word ‘stabilizer’ from the text, just have forgotten to do it [click] here. Suggestion - print out the chapter, replace by hand word ‘stabilizer’ everywhere by ’symmetry’ and let’s sit together and see whether the chapter is easier to read.

2011-03-31 PC Just curious (it’s your blog, you do what you want with it): why did you undo the `svn-multi`? The reason why I installed it is that when you print paper copies of the blog it keeps track of the svn version, date of last edit of a given file. This is very useful when you have bunch of handwritten edits of earlier versions that you would like to keep track of. Evangelos has problems with system managers in France, so he introduced the switch `\svnmultifalse`, which you can also comment out in blog.tex. But I think there should be no problem, so I have reverted to the version prior to your commenting out `svn-multi` lines.

2011-03-31 CS Sorry about that. I undo the `svn-multi` because otherwise I can not compile the .tex file and generate pdf document. I have not yet find another way to reconcile the `svn-multi`. Before that I can make two versions, one with `svn-multi` commented kept for my own use and one the same with this for your convenience.

2011-04-06 PC Here is my configuration of `WinEdt` (but that is a matter of taste)

```
[x] MikTeX 4.9 full distribution installation
[x] WinEdt, see www.cns.gatech.edu/CNS-only/WinEdt.txt
configuration wizard, wrapping:
        [ ] disable wrapping
[ ] remove TeX; from Conventional (Soft) Wrapping
[ ] fixed right margin 74
[ ] disable indent soft wrapping
in preferences, disable
[ ] Wrap Mode, and [ ] Line Wrapping Enabled
```

In order to read `siminos/blog` you need to `LaTeX`, then `dvi -> ps` and then either read the `ps` file, or convert `ps -> pdf`. This requires that you install these:

```
[ ] www.ghostscript.com
    [ ] gs901w32 -> C:\Program Files\gs\gs9.01\bin\gswin32.exe
[ ] ghostview pages.cs.wisc.edu/~ghost/gsview/
    manually changed 'execution modes' to
    [ ] C:\Program Files\Ghostgum\gsview\gsview32.exe
```

2011-04-05 CS Worked out exercise 2.1. Finished Chapter 9 with all the details in the examples, have had exemplified pictures of symmetry and group actions.

2011-04-06 PC Not so fast - exercise 2.1 is not finished until you write down your solution.

2011-04-05 CS A question about the last sentence in the first paragraph of Section 5.4 discussed in the group study last week: Why does the neighborhood size scale as $1/|\Lambda_p|$? Wouldn't it scale as $|\Lambda_p|$?

2011-04-06 PC Mhm, clearly not written clearly enough, but perhaps the most important property of an unstable flow that one has to understand. The product of expanding multipliers $|\Lambda_p|$ blows up exponentially with time, but the *neighborhood shrinks* exponentially with time, Detroit-like. Does looking at Figure 5.1 help? Does reading Sect. 1.5.1 help? If you
2CB understand it, can you rewrite

Nearby points aligned along the stable (contracting) directions remain in the neighborhood of the trajectory $x(t) = f^t(x_0)$; the ones to keep an eye on are the points which leave the neighborhood along the unstable directions because all nonlinear phenomena comes from these directions. The sub-volume $|\mathcal{M}_{x_0}| = \prod_i^e \Delta x_i$ of the set of points which get no further away from $f^t(x_0)$ than L, the typical size of the system, is fixed by the condition that $\Delta x_i \Lambda_i = O(L)$ in each expanding direction i. Hence the neighborhood size scales as $|\mathcal{M}_{x_0}| \propto O(L^{d_e})/|\Lambda_p| \propto 1/|\Lambda_p|$ where Λ_p is the product of expanding Floquet multipliers (5.7) only; contracting ones play a secondary role. "

so it makes sense to you. If you and Adam do not understand it, then bring it up for discussion in the study group.

2011-04-07 CS to PC Got it.

2011-04-10 ES to CS Chao, my Fortran 90 code to integrate KSe, find periodic orbits etc is in the svn repository `vaggelis`. The file 00README.txt explains what you'll find in each directory. The basic routines in the directory `libraries` are in general well documented. The driver routines in `testing` and `production` are less well documented. In most low-level directories there are three subdirectories (branches, trunk and tags). The current code is always in trunk, ignore the rest. I think that, for what you will have to do, you will find Ruslan's matlab code easier to use (KSe is not that expensive to integrate in terms of CPU time, at least for small system size). In any case I'd be glad to answer your questions about where to find what and how to use the code.

2011-04-11 CS rewrote Paragraph 1 of Section 5.4 as follows:

Nearby points aligned along the stable (contracting) directions remain in the neighborhood of the trajectory $x(t) = f^t(x_0)$; the ones to keep an eye on are the points which leave the neighborhood along the unstable directions because almost all nonlinear and chaotic phenomena comes from these directions. The sub-volume $|\mathcal{M}_{x_0}| = \prod_i^e \Delta x_i$ of the set of points which get no further away from $f^t(x_0)$ than L, the typical size of the system, is fixed by the condition that $\Delta x_i \Lambda_i = O(L)$ in each expanding direction i. Hence the neighborhood size scales as $|\mathcal{M}_{x_0}| \propto O(L^{d_e})/|\Lambda_p| \propto 1/|\Lambda_p|$ where Λ_p is the product of expanding Floquet multipliers (5.7) only(see section 1.5.1 and figure 1.9 for example); contracting ones play a secondary role

2011-04-12 PC Thanks, I have now rewritten introduction of Chapter 5 as well as the section 5.4 is the spirit you suggest, emphasizing the key role the concept of 'neighborhood' will play.

2011-04-11 CS Finished Chapter 6. Generally be able to understand it but feel like there's whole lot more content underneath that as in the KS transformation for example. There should be a lot tricks and methods to construct such regularization. And I wonder what kind of singularity could be regularized. But I guess that this is not an easy question and should not be the emphasis to my project.

2011-04-12 PC

2011-04-11 CS A trivial error: eqn.(6.13) should be "$\sqrt{x}dx = 2dt$", rather than "$\sqrt{x}dx = \sqrt{2}dt$"

2011-04-12 PC No error is 'trivial.' Thanks.

2011-04-12 PC Remember to write up your solution to exercise 2.1 before you forget it. If it is good, we might rewrite the Chapter 8 *Billiards* before the study group takes it up.

2011-04-12 CS I'll present Chap.8 next Friday as planned. Since a test on Friday is waiting for me, I'll write the solution on weekends.

Chapter 2

Exercises

Exercise 2.1 ***Birkhoff coordinates.*** *Prove that the Birkhoff coordinates are phase space volume preserving.*

Solution 2.1 - Birkhoff coordinates.

Assume the radius of two circles is $R = 1$, the distance between the centers of two circles is L, the magnitude of the ball's momentum is 1. Thus, $p \sin\phi = \sin\phi$.

First, we have to find the map:$(\theta_1, \sin(\phi_1)) \mapsto (\theta_2, \sin(\phi_2))$

Look at ΔABD, by Sine Theorem, we have $\frac{\sin(\pi-\phi_i)}{\overline{BD}} = \frac{\sin(\beta)}{\overline{AB}}$, where $\overline{BD} = L^{'}$, $\overline{AB} = R$, $\sin(\pi - \phi_1) = \sin(\phi_1)$

$$\Rightarrow \frac{\sin(\phi_1)}{L'} = \frac{\sin(\beta)}{R} \tag{2.1}$$

Look at ΔECD, still by Sine Theorem, we have $\frac{\sin(\beta)}{\overline{EC}} = \frac{\sin(\phi_2)}{\overline{CD}}$, where $\overline{EC} = R$, $\overline{CD} = L^{'} - L$

$$\Rightarrow \frac{\sin(\phi_2)}{L\prime - L} = \frac{\sin(\beta)}{R} \tag{2.2}$$

$$\beta = \phi_1 - \theta_1 \tag{2.3}$$

From the first equation, we have

$$L\prime = \frac{\sin(\phi_1)}{\sin(\beta)} R = \frac{\sin(\phi_1)}{\sin(\phi_1 - \theta_1)} \tag{2.4}$$

exc2_1.jpg

Substitute this into the second equation, we have

$$\frac{\sin(\phi_2)}{\frac{\sin(\phi_1)}{\sin(\phi_1-\theta 1)}R-L}=\frac{\sin(\phi_1-\theta_1)}{R} \tag{2.5}$$

$$\Rightarrow \sin(\phi_2)=\sin(\phi_1)-\frac{L}{R}\sin(\phi_1-\theta_1) \tag{2.6}$$

Also we have $\theta_2=\pi-\phi_2-\beta=\pi-\phi_2-(\phi_1-\theta_1)$
Now we have got the map. Next step is to show it's phase space volume preserving. rewrite the map by substituting $p_i=\sin(\phi_i),\phi_i=\arcsin(p_i)$*:*

$$p_2=p_1-\frac{L}{R}p_1\cos(\theta_1)+\frac{L}{R}\sqrt{1-p_1^2}\sin(\theta_1) \tag{2.7}$$

$$\theta_2=\pi+\theta_1-\arcsin(p_1)-\arcsin(p_2) \tag{2.8}$$

$$\begin{aligned}
dp_2 &= dp_1-\frac{L}{R}\cos(\theta_1)dp_1+\frac{L}{R}\sin(\theta_1)p_1d\theta_1-\frac{Lp_1\sin(\theta 1)}{R\sqrt{1-p_1^2}}dp_1+ \qquad (2.9)\\
&\quad \frac{L}{R}\sqrt{1-p_1^2}\cos\theta 1d\theta_1\\
&= (1-\frac{L}{R}\cos\theta_1-\frac{Lp_1\sin(\theta_1)}{R\sqrt{1-p_1^2}})dp_1+(\frac{L}{R}\sin(\theta_1)p_1+\frac{L}{R}\sqrt{1-p_1^2}\cos\theta 1)d\theta_1\\
d\theta_2 &= d\theta_1-\frac{1}{\sqrt{1-p_1^2}}dp_1-\frac{1}{\sqrt{1-p_2^2}}dp_2\\
dp_2\wedge d\theta_2 &= (1-\frac{L}{R}\cos(\theta_1)-\frac{Lp_1\sin(\theta_1)}{R\sqrt{1-p_1^2}})dp_1\wedge d\theta_1+\\
&\quad \frac{1}{\sqrt{1-p_1^2}}(\frac{L}{R}\sin(\theta_1)p_1+\frac{L}{R}\sqrt{1-p_1^2\cos(\theta_1)})dp_1\wedge d\theta_1 \qquad (2.10)\\
&= dp_1\wedge d\theta_1
\end{aligned}$$

Bibliography

[1] D. C. P. Ellis, F. Gay-Balmaz, D. D. Holm, and T. S. Ratiu, Lagrange-Poincaré field equations, Submitted to J. Geometry and Physics, arXiv:0910.0874, 2009.

[2] I. Percival and D. Richards, *Introduction to Dynamics* (Cambridge Univ. Press, Cambridge, 1996).