# B | Numerical examples' details

This appendix contains details on the numerical examples of Section 5.1. More specifically, the exact equations of all benchmark functions used, a graphical representation of the respective solution spaces, as well as a table with the initial points selected and the optima obtained for all optimization runs can be found herein.

Rosenbrock's function

$$\mathcal{P}_{\mathrm{NLP}} = \begin{cases} \underset{\mathbf{X}}{\text{minimize}} & g_0(\mathbf{X}) = (2.5 - x_1)^2 + 100 \cdot ((x_2 - 0.5) - (x_1 - 1.5)^2)^2 \\ \text{s.t.} & g_1(\mathbf{X}) = (x_1 - 2.5)^3 - x_2 + 1.5 \le 0 \\ & g_2(\mathbf{X}) = x_1 + x_2 - 4 \le 0 \end{cases}$$
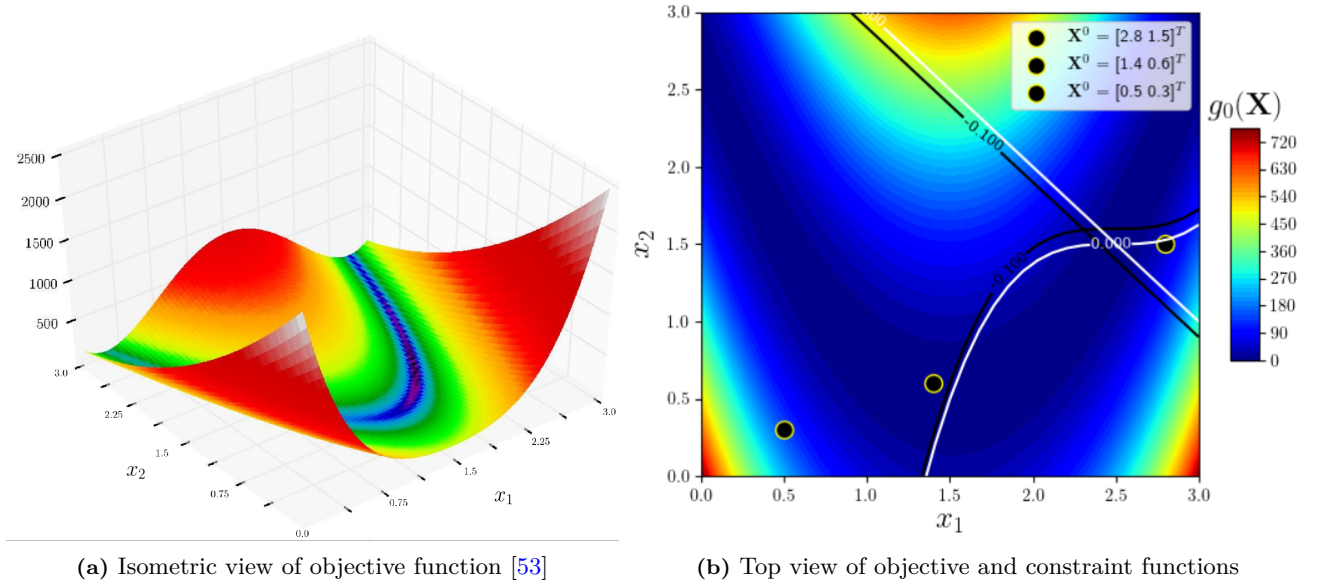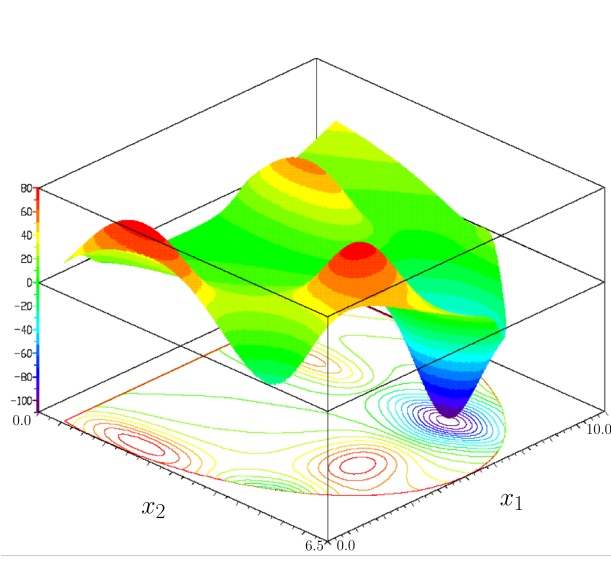

(a) Isometric view of objective function [53]

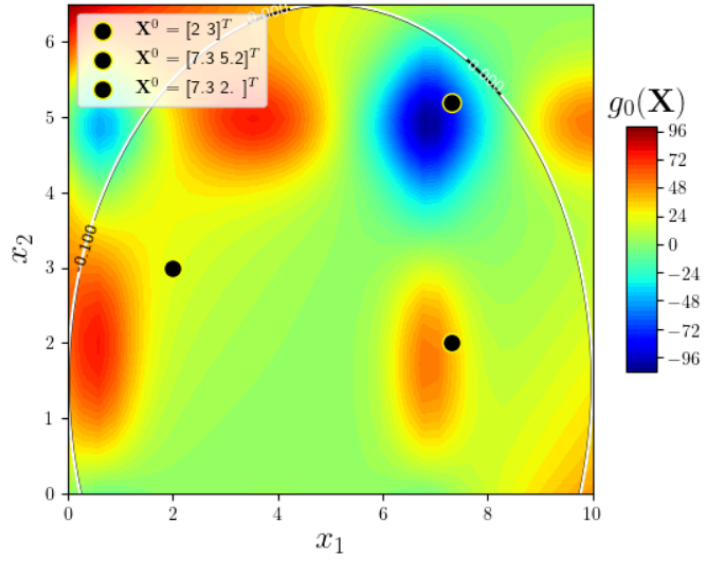**(b)** Top view of objective and constraint functions

Figure B.1: Rosenbrock's function with cubic and linear constraints [18]

Mishra's bird function

$$\mathcal{P}_{\mathrm{NLP}} = \begin{cases} \underset{\mathbf{X}}{\text{minimize}} & g_0(\mathbf{X}) = \sin(x_2 - 6.5) \cdot e^{[1-\cos(x_1-10)]^2} + \cos(x_1 - 10) \cdot e^{[1-\sin(x_2-6.5)]^2} + (x_1 - x_2 - 3.5)^2 \\ \text{s.t.} & g_1(\mathbf{X}) = (x_1 - 5.)^2 + (x_2 - 1.5)^2 - 25 \le 0 \end{cases}$$

(a) Isometric view of objective function [19]

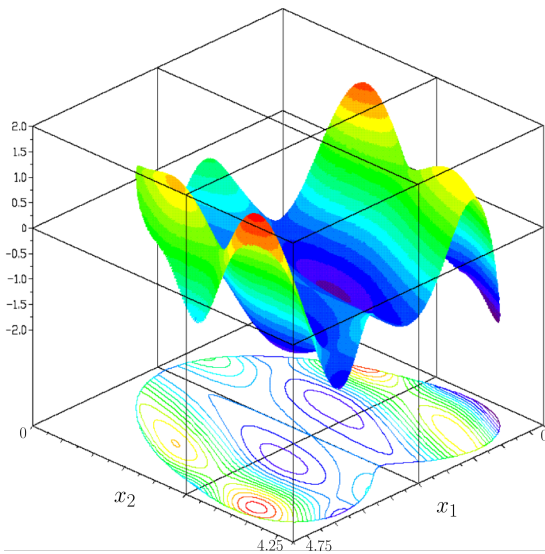

(b) Top view of objective and constraint functions

Figure B.2: Mishra's bird function with circular constraint [19]
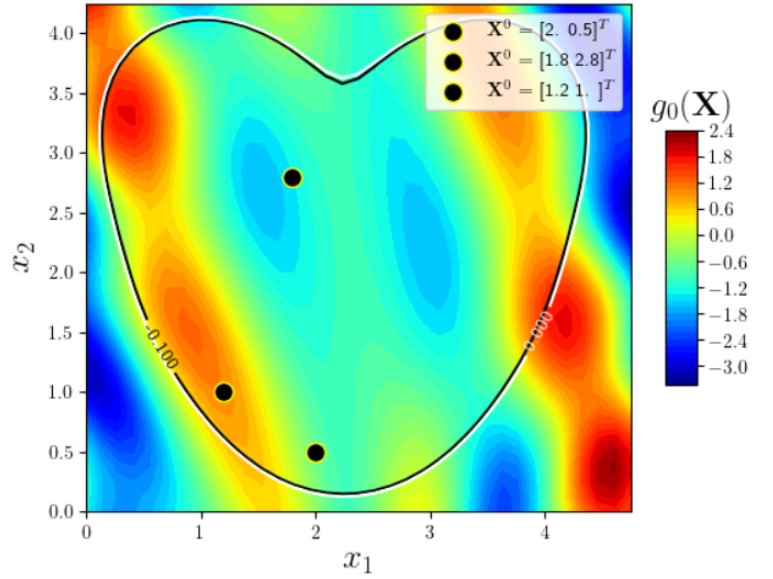
Townsend's function

$$\mathcal{P}_{\text{NLP}} = \begin{cases} \underset{\mathbf{X}}{\text{minimize}} & g_0(\mathbf{X}) = -(\cos((x_1 - 2.35) \cdot (x_2 - 2.5)))^2 - (x_1 - 2.25) \cdot \sin(3 \cdot (x_1 - 2.25) + x_2 - 2.5) \\ \text{s.t.} & g_1(\mathbf{X}) = (x_1 - 2.25)^2 + (x_2 - 2.5)^2 - [2\cos(t) - 0.5\cos(2t) - 0.25\cos(3t) - \dfrac{1}{8}\cos(4t)]^2 \\ & \quad - [2\sin(t)]^2 \le 0 \end{cases}$$

where,

$$t = \arctan2(x_1 - 2.25, \ x_2 - 2.5)$$

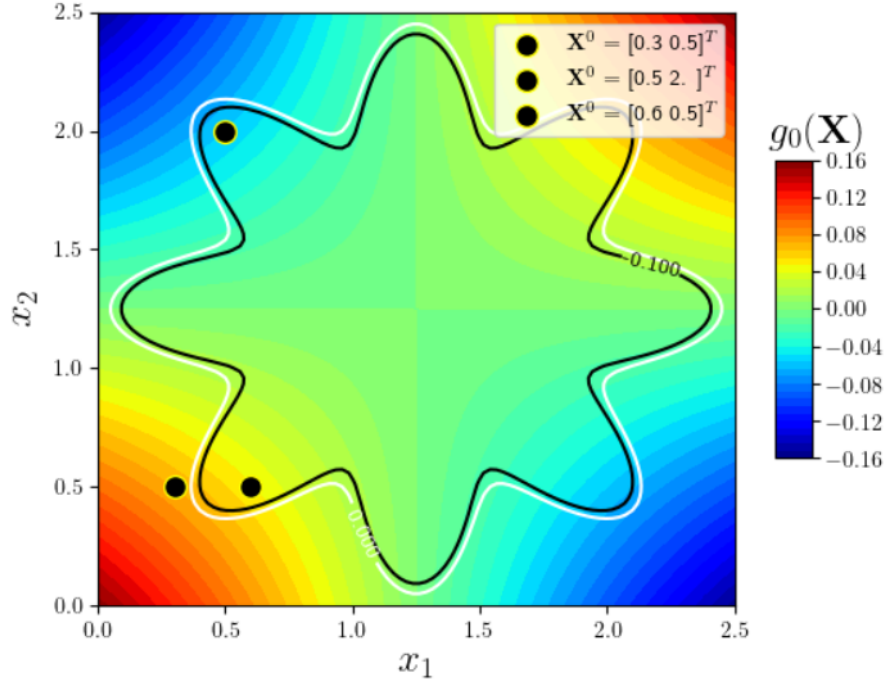**(a)** Isometric view of objective function [20]

(b) Top view of objective and constraint functions

Figure B.3: Townsend's function [20]

Simionescu's function

$$\mathcal{P}_{\text{NLP}} = \begin{cases} \underset{\mathbf{X}}{\text{minimize}} & g_0(\mathbf{X}) = 0.1 \cdot (x_1 - 1.25) \cdot (x_2 - 1.25) \\[2mm] \text{s.t.} & g_1(\mathbf{X}) = (x_1 - 1.25)^2 + (x_2 - 1.25)^2 - \left[1 + 0.2\cos\left(8\arctan\dfrac{(x_1 - 1.25)}{(x_2 - 1.25)}\right)\right]^2 \end{cases}$$

Figure B.4: Simionescu's function [21]