Linear Taylor Expansion

$$\tilde{f}_L(x) = f(x^{(k)}) + \frac{\partial f}{\partial x}\bigg|_{x^{(k)}} \cdot (x - x^{(k)})$$

$$y = T^{-1}(x) = x$$
$$\downarrow$$
$$x = T(y) = y$$

Reciprocal Taylor Expansion

$$\tilde{f}_R(x) = f(x^{(k)}) + \frac{\partial f}{\partial x}\bigg|_{x^{(k)}} \cdot (-x^{(k)})^2 \cdot \left(\frac{1}{x} - \frac{1}{x^{(k)}}\right)$$

$$y = T^{-1}(x) = \frac{1}{x}$$
$$\downarrow$$
$$x = T(y) = \frac{1}{y}$$

CONLIN Expansion

$$\tilde{f}_{CONLIN}(x) = f(x^{(k)}) + \underbrace{\frac{\partial f}{\partial x}^{+}\bigg|_{x^{(k)}}}_{\max\left\{0,\ \frac{\partial f}{\partial x}\big|_{x^{(k)}}\right\}} (x - x^{(k)}) + \underbrace{\frac{\partial f}{\partial x}^{-}\bigg|_{x^{(k)}}}_{\max\left\{0,\ -\frac{\partial f}{\partial x}\big|_{x^{(k)}}\right\}} \cdot (x^{(k)})^2 \left(\frac{1}{x} - \frac{1}{x^{(k)}}\right)$$

$$y = T^{-1}(x) = \begin{cases} x, & \text{if } \frac{\partial f}{\partial x} \geq 0 \\ \frac{1}{x}, & \text{if } \frac{\partial f}{\partial x} < 0 \end{cases}$$
$$\downarrow$$
$$x = T(y) = \begin{cases} y, & \text{if } \frac{\partial f}{\partial x} \geq 0 \\ \frac{1}{y}, & \text{if } \frac{\partial f}{\partial x} < 0 \end{cases}$$

MMA Expansion

$$\tilde{f}_{MMA}(x) = f(x^{(k)}) + \underbrace{\frac{\partial f}{\partial x}^{+}\bigg|_{x^{(k)}} \cdot (U - x^{(k)})^2}_{p_{ij}^{(k)}} \cdot \left(\frac{1}{U - x} - \frac{1}{U - x^{(k)}}\right)$$
$$+ \underbrace{\frac{\partial f}{\partial x}^{-}\bigg|_{x^{(k)}} \cdot (x^{(k)} - L)^2}_{q_{ij}^{(k)}} \cdot \left(\frac{1}{x - L} - \frac{1}{x^{(k)} - L}\right)$$

$$y = T^{-1}(x) = \begin{cases} \frac{1}{U - x}, & \text{if } \frac{\partial f}{\partial x} \geq 0 \\ \frac{1}{x - L}, & \text{if } \frac{\partial f}{\partial x} < 0 \end{cases}$$
$$\downarrow$$
$$x = T(y) = \begin{cases} U - \frac{1}{y}, & \text{if } \frac{\partial f}{\partial x} \geq 0 \\ \frac{1}{y} - L, & \text{if } \frac{\partial f}{\partial x} < 0 \end{cases}$$

Generalized Expansion

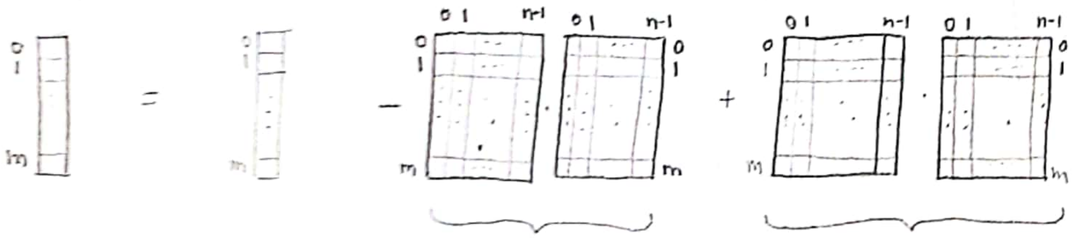
$$y = T^{-1}(x)$$
$$x = T(y)$$

$$\tilde{f}_{gen}(x) = \tilde{f}(T(y)) = \underbrace{f(T(y^{(k)}))}_{f(x^{(k)})} + \underbrace{\frac{\partial f}{\partial T}\bigg|_{T(y^{(k)})}}_{\frac{\partial f}{\partial x}\big|_{x^{(k)}}} \cdot \frac{\partial T}{\partial y}\bigg|_{y^{(k)}} \cdot (y - y^{(k)}) \Rightarrow$$

$$\tilde{f}_{gen}(x) = f(x^{(k)}) + \underbrace{\frac{\partial f}{\partial x}\bigg|_{x^{(k)}} \cdot \frac{\partial T}{\partial y}\bigg|_{y^{(k)}}}_{P^{(k)}} \cdot (y - y^{(k)}) = \underbrace{f(x^{(k)}) - P^{(k)} \cdot y^{(k)}}_{\text{zero-order term}} + \underbrace{P^{(k)} \cdot y}_{\text{first-order term}}$$

Multiple Functions (g_j) and Variables (x_i)

$$\underbrace{\tilde{g}_j(\underset{\sim}{x})} = \underbrace{g_j(\underset{\sim}{x}^{(k)})} - \underbrace{P_{ji}^{(k)} \cdot y_{ij}^{(k)}} + \underbrace{P_{ji}^{(k)} \cdot y_{ij}}$$



Dot product of j^{th} row of $P_{ji}^{(k)}$ with j^{th} column of $y_{ij}^{(k)}$

Dot product of j^{th} row of $P_{ji}^{(k)}$ with j^{th} column of y_{ij}

Adding 2^{nd}-order diagonal Hessian

$$\tilde{g}_{DQA}(\underset{\sim}{x}) = g(\underset{\sim}{x}^{(k)}) + \sum_{i=1}^{n} \underbrace{\frac{\partial g}{\partial x_i}\Big|_{\underset{\sim}{x}^{(k)}} \cdot \frac{\partial T}{\partial y_i}\Big|_{\underset{\sim}{y}^{(k)}}}_{P^{(k)}} \cdot (y_i - y_i^{(k)}) + \frac{1}{2}\sum_{i=1}^{n} \underbrace{\frac{\partial^2 g}{\partial x_i^2}\Big|_{\underset{\sim}{x}^{(k)}} \cdot \frac{\partial^2 T}{\partial y_i^2}\Big|_{\underset{\sim}{y}^{(k)}}}_{Q^{(k)}} \cdot (y_i - y_i^{(k)})^2 \Rightarrow$$

$$\Rightarrow \tilde{g}_{DQA}(\underset{\sim}{x}) = g(\underset{\sim}{x}^{(k)}) + \sum_{i=1}^{n} P_i^{(k)} \cdot (y_i - y_i^{(k)}) + \frac{1}{2}\sum_{i=1}^{n} Q_i^{(k)} \cdot (y_i - y_i^{(k)})^2 \Rightarrow$$

$$\Rightarrow \tilde{g}_{DQA}(\underset{\sim}{x}) = \underbrace{g(\underset{\sim}{x}^{(k)}) - \sum_{i=1}^{n} P_i^{(k)} \cdot y_i^{(k)} + \frac{1}{2}\sum_{i=1}^{n} Q_i^{(k)} (y_i^{(k)})^2}_{\text{zero-order term}} + \underbrace{\sum_{i=1}^{n} P_i^{(k)} \cdot y_i - \sum_{i=1}^{n} Q_i^{(k)} \cdot y_i^{(k)} \cdot y_i}_{\text{first-order term}} + \underbrace{\frac{1}{2}\sum_{i=1}^{n} Q_i^{(k)} \cdot y_i^2}_{\text{second-order term}}$$