Problem №1

The cost function of a non-discriminating monopolist is $c(y) = y^2 + 12$, and the inverse demand function is $p(y) = 24 - y$.

Subproblem №1

From Micro—1 we remember, that $MR = TR'$ and $TR = p(y) \cdot y$. So, let's calculate

$$MR = TR' = (p(y)y)' = (24y - y^2)' = 24 - 2y$$

As we know, $MR = MC$. Moreover, $MC = c'(y) = 2y$. Logically:

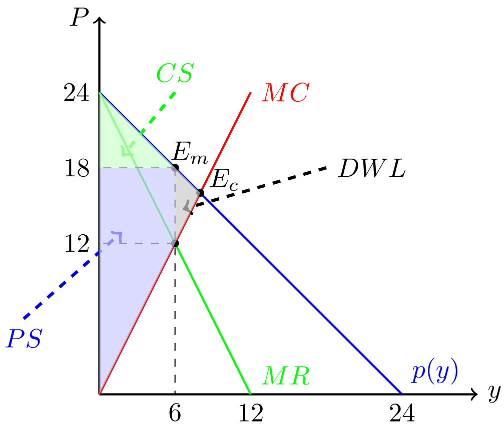
$$\begin{aligned} 24 - 2y &= 2y \\ y^* &= 6 \text{ — optimal } y\text{, then} \\ p^* &= p(6) = 18 \text{ — optimal price} \end{aligned}$$

Let's put that equilibrium price and output into profit function:

$$\Pi = \underbrace{24 \cdot 6 - 36}_{TR} - \underbrace{36 - 12}_{c} = 60$$

Subproblem №2

CS, PS and DWL can be computed as squares of figures shown below



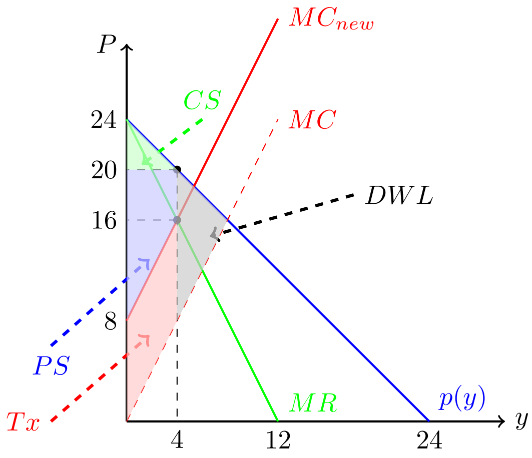
Moreover, in the picture we illustrated answers on **Subproblem №1**

$CS = \dfrac{(24 - 18) \cdot 6}{2} = 18$

$PS = \dfrac{18 + 6}{2} \cdot 6 = 72$

$DWL = \dfrac{6 \cdot 2}{2} = 6$

Subproblem №3



$MC_{new} = MR \implies 2y + 8 = 24 - 2y \implies y^* = 4,\ p^* = 20$

$CS_{new} = 4 \cdot 4 \cdot \frac{1}{2} = 8$

$PS_{new} = \frac{1}{2} \cdot (12 + 4) \cdot 4 = 32$

$DWL_{new} = \frac{1}{2} \cdot 4 \cdot 12 = 24$

$SW = CS_{new} + PS_{new} + Tx + DWL_{new} = 8 + 32 + 8 + 24 = 72$

$Tx = \text{quantity} \cdot \text{tax} = 4 \cdot 8 = 32$

Subproblem №4

Basic profit formula: $\Pi = p(y) \cdot y - c(y)$

In our constraints it'll be like

$$\Pi = 24y - y^2 - y^2 - 12$$

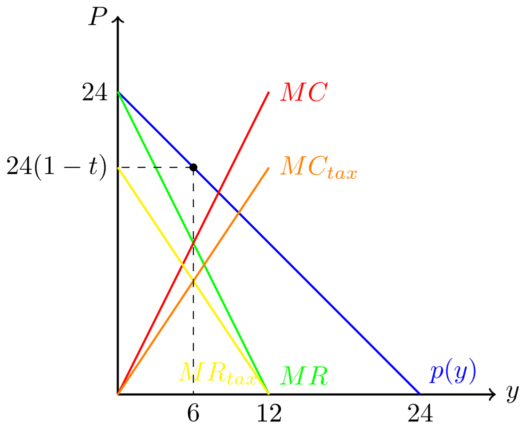
Imagine that government created some income tax t, our progit formula will transform into:

$$\Pi = \underbrace{(1-t)(24y - y^2)}_{TR_{\text{new}}} - \underbrace{(1-t)(y^2 + 12)}_{TC_{\text{new}}}$$

Let's calculate MR_4 and MC_4

$$\begin{cases} MR_4 = ((1-t)((24-y)y))' = (1-t)(24-2y) \\ MC_4 = (1-t)2y \end{cases} \implies \begin{cases} y^* = 6 \\ p^* = p(y^*) = 18 \end{cases}$$

It means, that $\Pi = 18 \cdot 6 \cdot (1-t) \implies$ profit is less than without taxes



Significantly, that in monopoly with income taxes optimal keeps the same as in previous subproblems

$MR'_{tax} = 0 = (1-t)(24-2y) \Longrightarrow y^* = 12$

$Tx = 60 \cdot t$

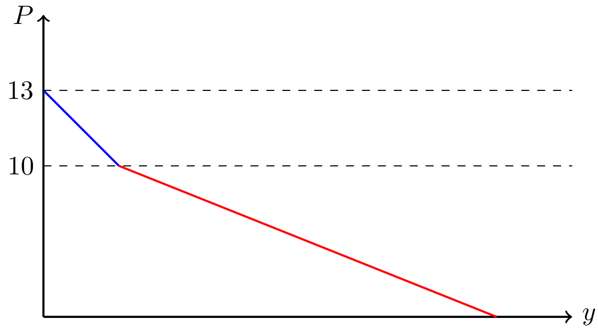
$PS_4 = TR - VC = \Pi + FC = 60(1-t) + 12(1-t)$

So, $SW = 90 + 72t$

Problem №2

A monopolist uses technology with a cost function $c(y) = \frac{5}{3}y^2$ and can sell his product in two regions, price discrimination between which is prohibited. The inverse function of demand for monopoly goods in the first region is $p_1 = 10 - y_1$, and in the second region $p_2 = 13 - 0.5y_2$

Subproblem №1



$y_1 = \begin{cases} 10 - p_1, \ p_1 \leqslant 10 \\ 0, \ p > 10 \end{cases}$, $y_2 = \begin{cases} 26 - 2p_2, \ p_1 \leqslant 13 \\ 0, \ p > 13 \end{cases}$ $\Longrightarrow y_1 + y_2 = \begin{cases} 36 - 3p, \ p \leqslant 10 \\ 26 - 2p, \ p \in (10, 13) \\ 0, \ p > 13 \end{cases}$

From the system of $(y_1 + y_2)$ we'll find p^*:

$$p^* = \begin{cases} 12 - \frac{y}{3}, \ y \in [0, 6] \\ 13 - \frac{y}{2}, \ y \in (6, 36] \end{cases}$$

$$\Pi = \begin{cases} 13y - 0.5y^2 - \dfrac{5}{3}y^2, \ y \in [0, 6] \\ 12y - \dfrac{y^2}{3} - \dfrac{5}{3}y^2, \ y \in (6, 36] \end{cases}$$

If we analyze this profit, we'll realize that after $y = 6$ the graphic will be under horizontal line that's why we don't need to draw it and take into consideration

So, we analyze the first equation. It's a parabola, branches down, so the maximum will be in the top. Skip simple calculations and we get optimal price and quantity:

$$\begin{cases} y^* = 3 \\ p^* = p(y^*) = 12 \end{cases}$$

Then, $\Pi = 39 - \frac{13}{2} \cdot 3 = 19.5$

Another subproblems will be solved on the next seminar