Microeconomics-2 (advanced course)
FES HSE

Viner Daniil @danya_vin

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Contents

1 Classic monopoly

1.1 Definition

A firm is **a monopoly** if no other firms produces the same good or a close substitute.

Comment. We differentiate among the products referring to the cross-price elasticity: if its value is small, then the potential substitutes are not close enough to be a substitute.

A monopolist has market power in the sense that the amount of output that it is able to sell responds continuously on the price it charges.

Obviously, that the monopolist suggests consumers a good at a price of p.

1.2 Examples of a monopoly

The main reason why a monopoly exists is the fact that other firms find it impossible or unprofitable to enter the market (or to continue the competition with more bigger firms).

The greatest examples of a monopoly are:

- Natural monopolies, when a single company can produce and offer to sell a product or service at a lower cost than its competitors can

- Legal protection of a patent holder, when the government prohibits using some technologies, ideas and producing methods except by those who created them

1.3 Model of a monopoly

Let's suppose there are many consumers of a good and they are price-takers. We'll assume that a good which we're considering is normal, which means that the function of demand $-$ $D(p)$ is decreasing.

$y = D(p)$ — the output if the monopoly, $c(y)$ — costs function of the firm-monopolist.

So, the monopolist chooses the price p^m which is the solution of the maximization problem:

$$\Pi(p) = pD(s) - c(D(p)) \longrightarrow \max_{p \geqslant 0} \qquad (1)$$

Logically, $y^m = D(p^m)$ is the output of the monopolist.

Sometimes it's easier to use an inverse function $p = p(y) = D^{-1}(y)$.

Then, output y^m is the solution of this problem:

$$\Pi(y) = p(y)y - c(y) \longrightarrow \max_{y \geqslant 0} \qquad (2)$$

1.4 First-order condition

$$\begin{cases} \Pi'(y^m) \leqslant 0 \\ y^m \cdot \Pi(y^m) = 0 \end{cases} \qquad (3)$$

Condition that $y^m > 0$ is guaranteed by:

$$p(0) > c'(0) \qquad (4)$$

Proof. If $y^m = 0$, next condition must be done:

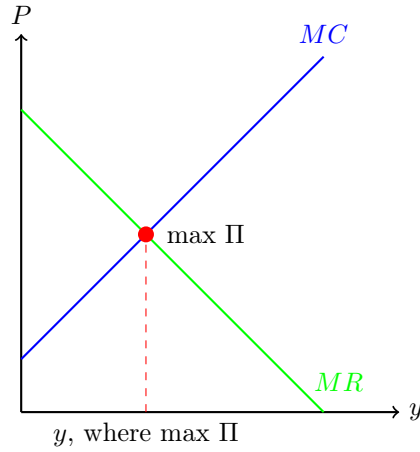$$\Pi'(0) = p(0) - c'(0) \leqslant 0$$

which contradicts $p(0) > c'(0)$ $\qquad\qquad\qquad \square$

$$\Pi(y) = p(y)y - c(y)$$
$$\Pi'(y) = (p(y)y - c(y))' = 0$$
$$= p(y) \cdot p'(y)y = c'(y), \quad \text{at } y = y^m > 0 \tag{5}$$

Then, $p^m = p(y^m)$

Let's $TC = D(p)p$, $TC = c(D(p))$, then as we maximize Π

$$\Pi' = 0$$
$$TR' - TC' = 0$$
$$MR - MC = 0$$
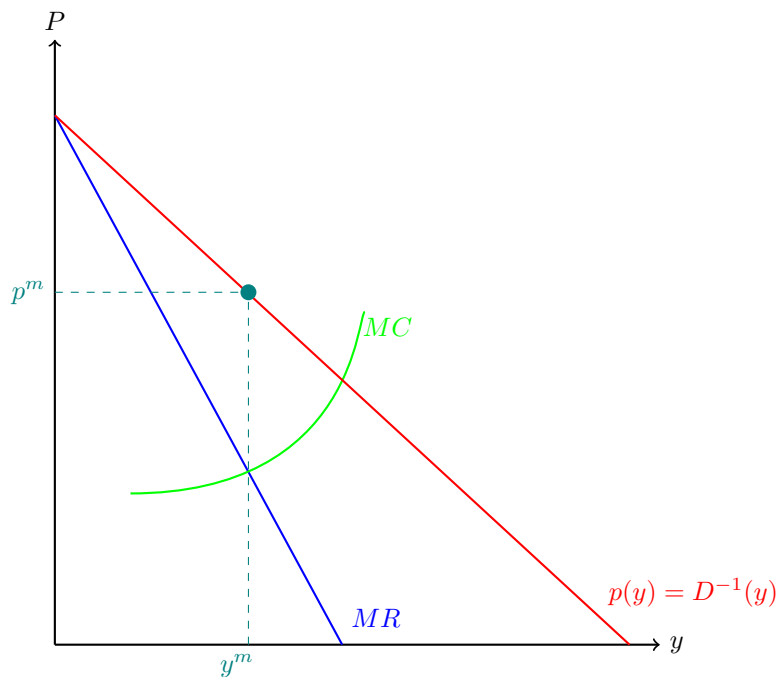

y, where max Π

1.5 Second-order condition

Let $TR(y) = p(y)y$ and $MR(y) = TR'(y)$, then

$$(MR(y^m))' < (MC(y^m))' \tag{6}$$

1.6 Illustration of the monopolist problem

Here p^m and y^m are the price and output at which customers will buy the volume of good at which $MR = MC$.

1.7   Inverse elasticity pricing rule

Definition.

$$\frac{p^m - MC(y^m)}{p^m} = \frac{1}{|\varepsilon_d|}, \quad \text{for } |\varepsilon_d| > 1 \tag{7}$$

The right part of the formula (7) is called **Lerner's Index**. It shows the degree of the monopoly power. At the same time it shows how the mark-up price depends on the elasticity of demand.

Comment. If the elasticity of demand is infinite, then the firm is a price taker and has no market power.

Comment. The greater the elasticity, the smaller the markup the manufacturer can make because in that case customers will refuse to consume.

Let's find the formula (7) by mathematical transformation. We know that:

- $p'(y^m)y^m + p(y^m) = c'(y^m)$

- $p(y^m) = c'(y^m)$

- $\varepsilon_d = \dfrac{\mathrm{d}y}{\mathrm{d}p} \cdot \dfrac{p}{y} < 0$

So, transform by using that equations we get

$$p(y^m)\left[1 + \frac{p'(y^m)y^m}{p(y^m)}\right] = c'(y^m)$$

$$p^m\left[1 - \frac{1}{|\varepsilon_d|}\right] = c'(y^m)$$

$$p^m - \frac{p^m}{|\varepsilon_d|} = c'(y^m)$$

$$\frac{p^m - c'(y^m)}{p^m} = \frac{1}{|\varepsilon_d|}$$

$$\frac{p^m - MC(y^m)}{p^m} = \frac{1}{|\varepsilon_d|}$$

1.8   Monopoly's output in comparison with the perfectly competitive industry

Consider a monopoly with the $c(y) \in C^1$ costs function and a perfectly competitive industry with the same costs $c(y)$. Let $p(y) \in C^1$ be an inverse demand function. Then given $p' > 0$, $c' > 0$, $c'' > 0$ and $p(0) > c'(0)$

Proposition. The monopoly output y^m will be less than the output $\overline{y}$ of the perfectly competitive industry

$$y^m < \overline{y} \tag{8}$$

Proof. It's simple. On the one hand, y^m maximizes Π of the monopoly, then

$$p(y^m)y^m - c(y^m) \geqslant p(\overline{y})\overline{y} - c(\overline{y})$$

On the other hand, $\overline{y}$ provides maximum profit to the price-taking firm at a constant price $p(\overline{y})$, then

$$p(\overline{y})\overline{y} - c(\overline{y}) \geqslant p(\overline{y})y^m - c(y^m)$$

Adding these equations we get $p(y^m)y^m \geqslant p(\overline{y})y^m \implies$ $\boxed{y^m < \overline{y}}^a$

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[a]Remember, that $p(y)$ is an inverse function of demand and $y^m \neq \overline{y}$ because of $p(\overline{y}) - c'(\overline{y}) \leqslant 0$
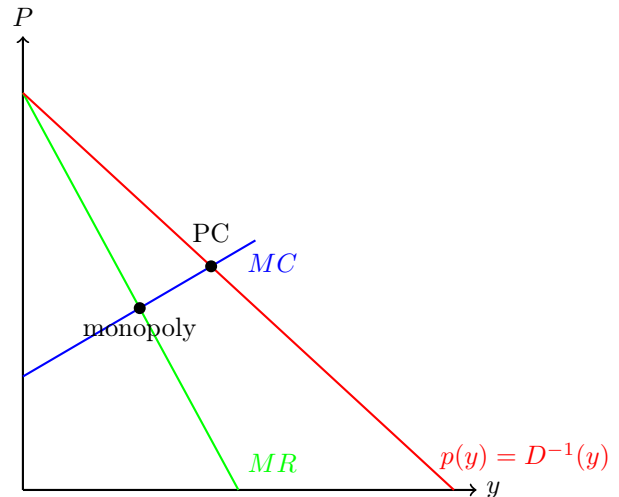
1.9 Comparative Statics

Let $p(y) \in C^2$, $p' < 0$, $MC = c$, $p(0) > c$, $(MR)' < 0$. Then

$$\frac{dy^m(c)}{dc} < 0 \quad \text{and} \quad \frac{dp^m(c)}{dc} > 0$$

2 Quasilinear Economy

Consider an economy where there are m consumers and n firms. All of consumers have quasilinear preferences, for i-th consumer his utility is $U_i(x_i, z_i) = v_i(x_i) + z_i$, where x_i is the only good to be consumed and z_i is the money owned by this consumer.

Properties of $v_i(x_i)$ functions:

- $v_i'(x_i) > 0$
- $v_i''(x_i) < 0$
- $v_i(0) = 0$

2.1 Main theorem of the quasilinear economy

Let $\bar{x} = (x_1, x_2, \ldots, x_m)$, $\bar{y} = (y_1, y_2, \ldots, y_n)$, $\bar{r} = (r_1, r_2, \ldots, r_n)$ and $\bar{z} = (z_1, z_2, \ldots, z_m)$

Allocation $(\widehat{\bar{x}}, \widehat{\bar{y}}, \widehat{\bar{r}}, \widehat{\bar{z}})$ is Pareto-optimal if and only if it is a solution of the problem

$$\begin{cases} \sum_i \nu_i(x_i) - \sum_j c_j(y_j) \longrightarrow \max \\ \sum_i (x_i) \leq \sum_i y_j \\ x_i \geq 0, \ y_j \geq 0 \\ \text{for all } i \in I \text{ and } j \in J \end{cases} \tag{9}$$

Moreover $\widehat{r_j} = c_j(\widehat{y_j})$ and $\sum_i \widehat{z_i} = \sum_i w_i - \sum_j \widehat{r_j}$.

2.2 Welfare analysis under monopoly

The first line in (9) is the main goal of consumers, i.e. to maximize the welfare. That function is called a **welfare indicator**. Then we have 2 sets of indices, $I = \{1, 2, \ldots, m\}$ and $J = \{1, 2, \ldots, n\}$.

At *Pareto-optimal allocation* it takes the maximum value W_{max}. If an allocation *is not* Pareto-optimal, then $W < W_{max}$ and this difference $W_{max} - W$ is called **deadweight loss** (DWL).

We give the following definitions for Consumers' (CS) and producers' (PS) surplus:

$$\begin{aligned} CS &= \sum_{i \in I} \nu_i(x_i) - p \sum_{i \in I} x_i \\ PS &= p \sum_{j \in J} y_j - \sum_{j \in J} c_j(y_j) \end{aligned} \tag{10}$$

Welfare indicator at (x, y) allocation equals $W(x, y) = CS + PS$. Moreover, we can identify

$$W = \sum_{i=1}^m v_i(x_i) - c(y)$$

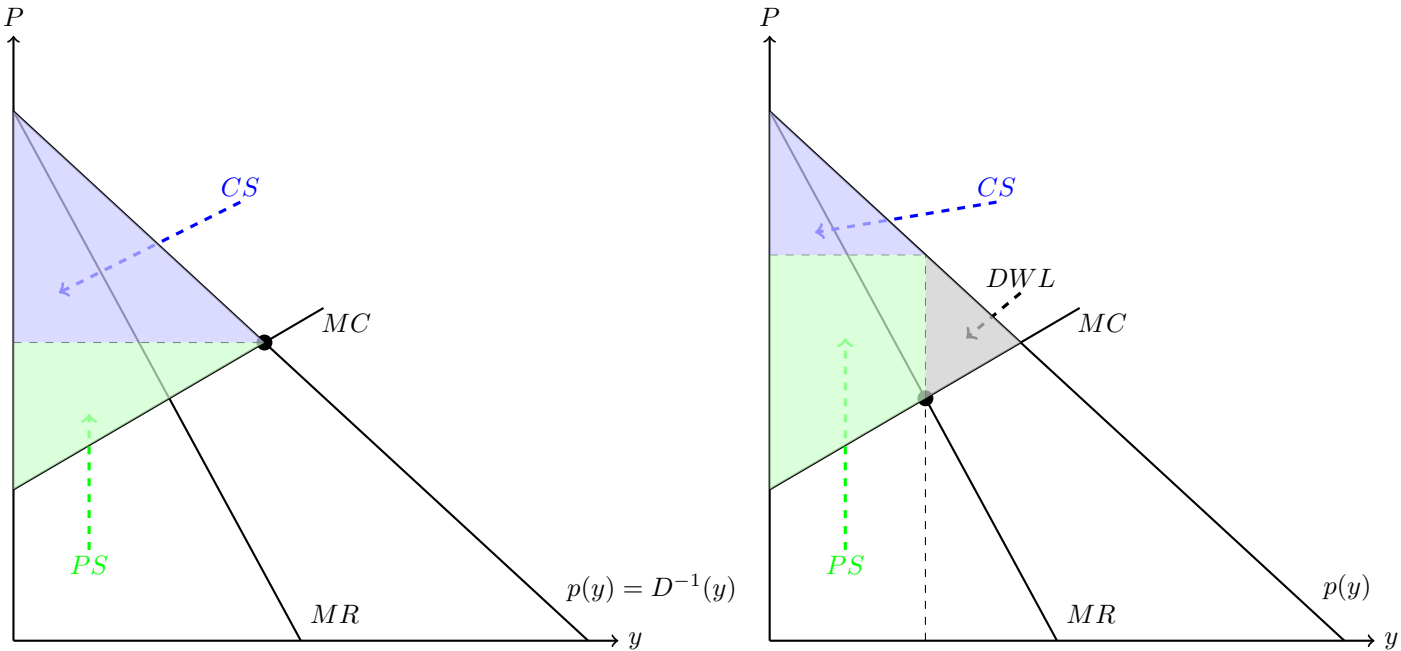Let $y = \sum_{i \in I} x_i = x -$ output of a monopoly

1. $W(y) = v(y) - c(y) \longrightarrow \max\limits_{y>0} \implies v'(y) = c'(y)$
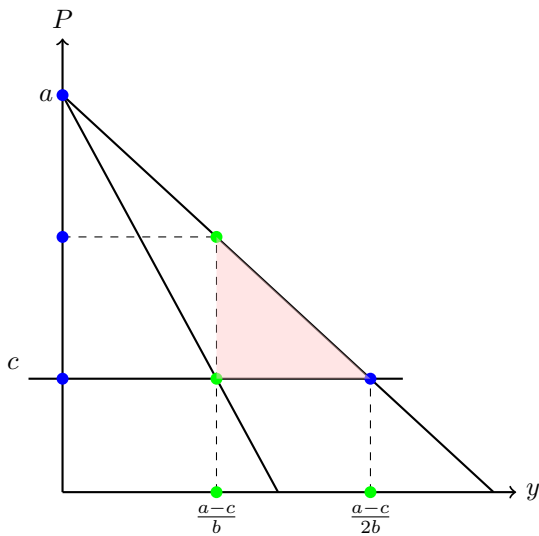
2. First-order condition:
$$(p(y)y)' - c'(y) = 0$$
$$p'(y)y + p(y) - c'(y) = 0$$

3. If we differentiate the formula $W(y) = v(y) - c(y)$:
$$W'(y^m) = v'(y^m) - c'(y^m) > 0$$
$$= p(y^m) - c'(y^m)$$
$$= -p'(y^m)y^m > 0$$

2.3 DWL formula

For instance, we have $p(y) = a - by$ and $c'(y) = c$ then
$$TR = ay - by^2$$
$$MR = a - 2by$$

Now, how try to find DWL? Optimal output will be $\hat{y} = \dfrac{a-c}{b}$, while monopoly's:
$$y^m = \frac{a-c}{2b}$$

So,
$$DWL = \int_{y^m}^{\hat{y}} [(a - bt) - c]\, dt = \frac{(a-c)^2}{8b}$$

2.4 The concept of a representative consumer

If an aggregate demand on good can be represented by solving a utility maximization problem of a sole consumer, then we say that in such economy a representative consumer exists. This is exactly the case in a quasilinear economy.

In other words there exists a function $\nu(x)$ such that by solving the problem

$$\begin{cases} \nu(x) + \sum_{i \in I} z_i \to \max \\ p \cdot \sum_{i \in I} x_i + \sum_{i \in I} z_i \leq \sum_{i \in I} w_i \end{cases} \tag{11}$$

we can find the aggregate demand function $D(p) = \sum_{i \in I} x_i(p)$