Linear Algebra, Individual Homework 8, Variant 18

Problem 1

Complement vector $v = \frac{1}{10}(-5, 1, 7, 5)$ to an orthonormal basis in $\mathbb{R}^4$.

Add three vectors to the existing one:

$$\langle v, e_2, e_3, e_4 \rangle$$

Since we have added the standard vectors, it's obvious that the span above would be linearly independent. Thus, apply the Gram-Schmidt orthogonalization approach:

$$f_{k+1} = e_{k+1} - \sum_{i=1}^{k} \frac{(f_i, e_i)}{(f_i, f_i)} f_i$$

$$f_1, = \frac{1}{10}(-5, 1, 7, 5)$$

$$\begin{aligned}
f_2 &= (0,1,0,0) - \frac{0.1}{1}\frac{1}{10}(-5,1,7,5) \\
&= \frac{1}{100}(5, 99, -7, -5) \\
f_3 &= (0,0,1,0) - \frac{0.7}{1}\frac{1}{10}(-5,1,7,5) + \frac{0.07}{0.99}\frac{1}{100}(5,99,-7,-5) \\
&= (0,0,1,0) - \frac{7}{100}(-5,1,7,5) - \frac{7}{9900}(-5,-99,7,5) \\
&= \frac{1}{99}(35, 0, 50, -35) \\
f_4 &= (0,0,0,1) - \frac{0.5}{1}\frac{1}{10}(-5,1,7,5) + \frac{0.05}{0.99}\frac{1}{100}(5,99,-7,-5) + \frac{35}{99}\frac{33^2}{25 \cdot 22}\frac{1}{99}(35,0,50,-35 \\
&= (0,0,0,1) - \frac{1}{20}(-5,1,7,5) + \frac{5}{9900}(5,99,-7,-5) + \frac{7}{990}(35,0,50,-35) \\
&= \frac{1}{2}(1,0,0,1)
\end{aligned}$$

Now normalize all the vectors:

$$
\begin{aligned}
\frac{1}{10}(-5,1,7,5) &\rightsquigarrow \frac{1}{10}(-5,1,7,5) \\
\frac{1}{100}(5,99,-7,-5) &\rightsquigarrow \frac{1}{\frac{1}{100}\sqrt{5^2+99^2+7^2+5^2}}\frac{1}{100}(5,99,-7,-5) \\
&\rightsquigarrow \frac{1}{30\sqrt{11}}(5,99,-7,-5) \\
\frac{1}{99}(35,0,50,-35) &\rightsquigarrow \frac{1}{\frac{5}{99}\sqrt{7^2+10^2+7^2}}\frac{5}{99}(7,0,10,-7) \\
&\rightsquigarrow \frac{1}{3\sqrt{22}}(7,0,10,-7) \\
\frac{1}{2}(1,0,0,1) &\rightsquigarrow \frac{1}{\sqrt{2}}(1,0,0,1)
\end{aligned}
$$

Problem 2

Subspace U of a euclidean space $\mathbb{R}^4$ is defined by a system of equations

$$
\begin{cases}
-7x_1 - 2x_2 + x_3 - 30x_4 = 0 \\
6x_2 - 24x_3 + 6x_4 = 0
\end{cases}
$$

For vector $v = (-1,1,-5,-3)$ find its projection to U, its orthogonal part in relation to U and the distance from it to U.

First, find the basis that is generated by the matrix

$$
\begin{pmatrix} -7 & -2 & 1 & -30 \\ 0 & 6 & -24 & 6 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 1 & -4 \\ 0 & 1 & -4 & 1 \end{pmatrix}
$$

Fundamental system of solutions would be:

$$
\begin{cases}
x_1 = -x_3 + 4x_4 \\
x_2 = 4x_3 - x_4 \\
x_3 = x_3 \\
x_4 = x_4
\end{cases}
$$

$$
A = \begin{pmatrix} -1 & 4 \\ 4 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \implies \mathbf{e} = \langle (-1,4,1,0), (4,-1,0,1) \rangle
$$

This is our basis, so we may apply the formula for an orthogonal projection:

$$\mathrm{pr}_U v = A(A^T A)^{-1} A^T v$$

$$\mathrm{pr}_U v = \begin{pmatrix} -1 & 4 \\ 4 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \left(\begin{pmatrix} -1 & 4 & 1 & 0 \\ 4 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 4 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \right)^{-1} \begin{pmatrix} -1 & 4 & 1 & 0 \\ 4 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -4 \\ -3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 4 \\ 4 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 18 & -8 \\ -8 & 18 \end{pmatrix}^{-1} \begin{pmatrix} -1 & 4 & 1 & 0 \\ 4 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -4 \\ -3 \end{pmatrix} =$$

$$= \frac{1}{130} \begin{pmatrix} -1 & 4 \\ 4 & -1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 9 & 4 \\ 4 & 9 \end{pmatrix} \begin{pmatrix} -1 & 4 & 1 & 0 \\ 4 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -4 \\ -3 \end{pmatrix} =$$

$$= \frac{1}{130} \begin{pmatrix} 7 & 32 \\ 32 & 7 \\ 9 & 4 \\ 4 & 9 \end{pmatrix} \begin{pmatrix} -1 & 4 & 1 & 0 \\ 4 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -4 \\ -3 \end{pmatrix} =$$

$$= \frac{1}{130} \begin{pmatrix} 121 & -4 & 7 & 32 \\ -4 & 121 & 32 & 7 \\ 7 & 32 & 9 & 4 \\ 32 & 7 & 4 & 9 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ -4 \\ -3 \end{pmatrix} =$$

$$= \frac{8}{130} \begin{pmatrix} -32 \\ -7 \\ -4 \\ -9 \end{pmatrix}$$

The orthogonal projection would be just the original vector minus its projection:

$$\mathrm{ort}_U v = v - \mathrm{pr}_U v = \begin{pmatrix} -1 \\ 1 \\ -5 \\ -3 \end{pmatrix} - \frac{8}{130} \begin{pmatrix} -32 \\ -7 \\ -4 \\ -9 \end{pmatrix} = \frac{3}{65} \begin{pmatrix} 21 \\ 31 \\ -103 \\ -53 \end{pmatrix}$$

Distance would be just the length of the orthogonal projection vector:

$$\mathrm{dist}_U v = \frac{3}{65} \sqrt{21^2 + 31^2 + 103^2 + 53^2} = \frac{6}{65} \sqrt{3705}$$

Problem 3

In euclidean space $\mathbb{R}^4$, two subspaces $U = \langle u_1, u_2 \rangle$ and $W = \langle w_1, w_2 \rangle$ are given, where $u_1 = (-1, -2, -2, 1), u_2 = (-1, 3, 1, -3), w_1 = (-2, -2, 1, -1), w_2 = (3, 3, 1, -1)$. Find the vector $v \in \mathbb{R}^4$, for which $\mathrm{pr}_U v = (14, 3, 13, 6)$ and $\mathrm{ort}_W v = (-5, 5, 20, 20)$.

Rewrite the subspaces as matrices:

$$A_U = \begin{pmatrix} -1 & -1 \\ -2 & 3 \\ -2 & 1 \\ 1 & -3 \end{pmatrix}, \quad A_W = \begin{pmatrix} -2 & 3 \\ -2 & 3 \\ 1 & 1 \\ -1 & -1 \end{pmatrix}$$

We know that

$$\mathrm{pr}_U v = A_U (A_U^T A_U)^{-1} A_U^T v$$

and

$$\mathrm{ort}_W v = v - A_W (A_W^T A_W)^{-1} A_W^T v$$

Calculate the coefficient matrices:

$$\begin{aligned}
A_U (A_U^T A_U)^{-1} A_U^T &= \begin{pmatrix} -1 & -1 \\ -2 & 3 \\ -2 & 1 \\ 1 & -3 \end{pmatrix} \left(\begin{pmatrix} -1 & -2 & -2 & 1 \\ -1 & 3 & 1 & -3 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -2 & 3 \\ -2 & 1 \\ 1 & -3 \end{pmatrix} \right)^{-1} \begin{pmatrix} -1 & -2 & -2 & 1 \\ -1 & 3 & 1 & -3 \end{pmatrix} \\
&= \begin{pmatrix} -1 & -1 \\ -2 & 3 \\ -2 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 10 & -10 \\ -10 & 20 \end{pmatrix}^{-1} \begin{pmatrix} -1 & -2 & -2 & 1 \\ -1 & 3 & 1 & -3 \end{pmatrix} \\
&= \frac{1}{10} \begin{pmatrix} -1 & -1 \\ -2 & 3 \\ -2 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & -2 & -2 & 1 \\ -1 & 3 & 1 & -3 \end{pmatrix} \\
&= \frac{1}{10} \begin{pmatrix} -3 & -2 \\ -1 & 1 \\ -3 & -1 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} -1 & -2 & -2 & 1 \\ -1 & 3 & 1 & -3 \end{pmatrix} \\
&= \frac{1}{10} \begin{pmatrix} 5 & 0 & 4 & 3 \\ 0 & 5 & 3 & -4 \\ 4 & 3 & 5 & 0 \\ 3 & -4 & 0 & 5 \end{pmatrix}
\end{aligned}$$

$$A_W(A_W^TA_W)^{-1}A_W^T = \begin{pmatrix}-2 & 3\\ -2 & 3\\ 1 & 1\\ -1 & -1\end{pmatrix}\left(\begin{pmatrix}-2 & -2 & 1 & -1\\ 3 & 3 & 1 & -1\end{pmatrix}\begin{pmatrix}-2 & 3\\ -2 & 3\\ 1 & 1\\ -1 & -1\end{pmatrix}\right)^{-1}\begin{pmatrix}-2 & -2 & 1 & -1\\ 3 & 3 & 1 & -1\end{pmatrix}$$

$$= \begin{pmatrix}-2 & 3\\ -2 & 3\\ 1 & 1\\ -1 & -1\end{pmatrix}\begin{pmatrix}10 & -10\\ -10 & 20\end{pmatrix}^{-1}\begin{pmatrix}-2 & -2 & 1 & -1\\ 3 & 3 & 1 & -1\end{pmatrix}$$

$$= \frac{1}{10}\begin{pmatrix}-2 & 3\\ -2 & 3\\ 1 & 1\\ -1 & -1\end{pmatrix}\begin{pmatrix}2 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}-2 & -2 & 1 & -1\\ 3 & 3 & 1 & -1\end{pmatrix}$$

$$= \frac{1}{10}\begin{pmatrix}-1 & 1\\ -1 & 1\\ 3 & 2\\ -3 & -2\end{pmatrix}\begin{pmatrix}-2 & -2 & 1 & -1\\ 3 & 3 & 1 & -1\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 0 & -1 & 1\end{pmatrix}$$

Now we have the following:

$$\frac{1}{10}\begin{pmatrix}5 & 0 & 4 & 3\\ 0 & 5 & 3 & -4\\ 4 & 3 & 5 & 0\\ 3 & -4 & 0 & 5\end{pmatrix} v = (14,3,13,6)^T$$

$$\begin{pmatrix}5 & 0 & 4 & 3\\ 0 & 5 & 3 & -4\\ 4 & 3 & 5 & 0\\ 3 & -4 & 0 & 5\end{pmatrix} v = \begin{pmatrix}140\\ 30\\ 130\\ 60\end{pmatrix} \rightsquigarrow \left(\begin{array}{cccc|c}1 & 0 & \frac{4}{5} & \frac{3}{5} & 28\\ 0 & 1 & \frac{3}{5} & -\frac{4}{5} & 6\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right)$$

$$v - \frac{1}{2}\begin{pmatrix}1 & 1 & 0 & 0\\ 1 & 1 & 0 & 0\\ 0 & 0 & 1 & -1\\ 0 & 0 & -1 & 1\end{pmatrix} v = (-5,5,20,20)^T \rightsquigarrow \frac{1}{2}\begin{pmatrix}1 & -1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1\end{pmatrix} v = (-5,5,20,20)^T$$

$$\begin{pmatrix}1 & -1 & 0 & 0\\ -1 & 1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & 1\end{pmatrix} v = \begin{pmatrix}-10\\ 10\\ 40\\ 40\end{pmatrix} \rightsquigarrow \left(\begin{array}{cccc|c}1 & -1 & 0 & 0 & -10\\ 0 & 0 & 1 & 1 & 40\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{array}\right)$$

Combining these two systems together, we get:

$$\left(\begin{array}{cccc|c} 1 & 0 & \frac{4}{5} & \frac{3}{5} & 28 \\ 0 & 1 & \frac{3}{5} & -\frac{4}{5} & 6 \\ 1 & -1 & 0 & 0 & -10 \\ 0 & 0 & 1 & 1 & 40 \end{array}\right) \rightsquigarrow \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 & 20 \end{array}\right)$$

which implies that the required vector is

$$v = \begin{pmatrix} 0 \\ 10 \\ 20 \\ 20 \end{pmatrix}$$

Problem 4

In space $W = \mathbb{R}[x]_{\leq 2}$ equipped with a structure of a euclidean space in relation to some (unknown) dot product, the volume of a parallelipiped generated by vectors $-2 + 4x + 5x^2, 6 - 10x - 18x^2, 4 - 6x - 12x^2$ is equal to 4. Find the volume of a parallelipiped generated by vectors $2 - 2x + 4x^2, -4 + 5x - 11x^2, -4 + 6x - 10x^2$.

Map the vectors to two matrices, V, U:

$$V = \begin{pmatrix} -2 & 6 & 4 \\ 4 & -10 & -6 \\ 5 & -18 & -12 \end{pmatrix}, \quad U = \begin{pmatrix} 2 & -4 & -4 \\ -2 & 5 & 6 \\ 4 & -11 & -10 \end{pmatrix}$$

Now find such matrix C such that

$$VC = U$$

We know that $\operatorname{Vol} P(\langle v_1, v_2, v_3 \rangle) = \sqrt{\det G(\langle v_1, v_2, v_3 \rangle)}$, where $G(\dots)$ is a Gram matrix.

We also know that $G(\langle u_1, u_2, u_3 \rangle) = C^T G(\langle v_1, v_2, v_3 \rangle) C$. Therefore, $\operatorname{Vol} P(u_1, u_2, u_3) = \sqrt{\det C^T G(v_1, v_2, v_3) C} = |\det C| \sqrt{\det G(v_1, v_2, v_3)} = |\det C| \operatorname{Vol} P(\langle v_1, v_2, v_3 \rangle)$

Thus, we may simply find C and calculate the answer:

$$C = V^{-1} U$$

$$V^{-1} = \begin{pmatrix} -3 & 0 & -1 \\ -\frac{9}{2} & -1 & -1 \\ \frac{11}{2} & \frac{3}{2} & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} -3 & 0 & -1 \\ -\frac{9}{2} & -1 & -1 \\ \frac{11}{2} & \frac{3}{2} & 1 \end{pmatrix} \begin{pmatrix} 2 & -4 & -4 \\ -2 & 5 & 6 \\ 4 & -11 & -10 \end{pmatrix} = \begin{pmatrix} -10 & 23 & 22 \\ -11 & 24 & 22 \\ 12 & -\frac{51}{2} & -23 \end{pmatrix} \rightsquigarrow$$

$$\begin{pmatrix} 1 & -1 & 0 \\ -11 & 24 & 22 \\ 1 & -\frac{3}{2} & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & -1 & 0 \\ -11 & 24 & 22 \\ 0 & -\frac{1}{2} & -1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 & 0 & 0 \\ -11 & 13 & 22 \\ 0 & -\frac{1}{2} & -1 \end{pmatrix}$$

$$\operatorname{Vol} P(u_1, u_2, u_3) = |\det C| \operatorname{Vol} P(v_1, v_2, v_3) = |-2| \operatorname{Vol} P(v_1, v_2, v_3) = 2 \times 4 = 8$$

Problem 5

Let L be a three-dimensional surface in $\mathbb{R}^5$ passing through points $v_0 = (-6, -6, 1, -9, -7), v_1 = (-2, 1, 10, -14, -4), v_2 = (-1, -6, -2, -5, 2), v_3 = (-15, -6, 5, -6, -2)$.

Find a point in L closest to point $v = (5, 0, 0, 10, 1)$ and the distance from it to v.

Firstly, let's shift the surface to be a subspace of the entire space. To do this, we simply shift $v_0 \mapsto (0, 0, 0, 0, 0)$ and for all other vectors, $v_i \mapsto v_i - v_0$:

$$y_1 = v_1 - v_0 = \begin{pmatrix} -2 \\ 1 \\ 10 \\ -14 \\ -4 \end{pmatrix} - \begin{pmatrix} -6 \\ -6 \\ 1 \\ -9 \\ -7 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 9 \\ -5 \\ 3 \end{pmatrix}$$

$$y_2 = v_2 - v_0 = \begin{pmatrix} -1 \\ -6 \\ -2 \\ -5 \\ -2 \end{pmatrix} - \begin{pmatrix} -6 \\ -6 \\ 1 \\ -9 \\ -7 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -3 \\ 4 \\ 5 \end{pmatrix}$$

$$y_3 = v_3 - v_0 = \begin{pmatrix} -15 \\ -6 \\ 5 \\ -6 \\ -2 \end{pmatrix} - \begin{pmatrix} -6 \\ -6 \\ 1 \\ -9 \\ -7 \end{pmatrix} = \begin{pmatrix} -9 \\ 0 \\ 4 \\ 3 \\ 5 \end{pmatrix}$$

Thus, we may write a parametric vector equation of the plane Ξ:

$$\Xi = v_0 + \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \begin{pmatrix} t_1 & t_2 & t_3 \end{pmatrix}$$

If we also shift the given point v by v_0, then our task collapses to finding the distance between this point y and the following subspace Ω:

$$\Omega = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \begin{pmatrix} t_1 & t_2 & t_3 \end{pmatrix}, \quad y = v - v_0 = \begin{pmatrix} 5 \\ 0 \\ 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -6 \\ -6 \\ 1 \\ -9 \\ -7 \end{pmatrix} = \begin{pmatrix} 11 \\ 6 \\ -1 \\ 19 \\ 8 \end{pmatrix}$$

Now we may simply calculate the distance by finding the absolute value of the orthogonal projection vector and the closest point by finding the projection of the vector y to our subspace. Then, to find the original point, we would shift it back by v_0.

> Ah shit, here we go again:

We know that

$$\mathrm{pr}_Y y = A_Y(A_Y^T A_Y)^{-1} A_Y^T y$$

and

$$\mathrm{ort}_Y y = y - A_Y(A_Y^T A_Y)^{-1} A_Y^T y$$

Calculate the coefficient matrix:

$$A_Y = \begin{pmatrix} 4 & 5 & -9 \\ 7 & 0 & 0 \\ 9 & -3 & 4 \\ -5 & 4 & 3 \\ 3 & 5 & 5 \end{pmatrix}$$

$$A_Y(A_Y^T A_Y)^{-1}A_Y^T = \begin{pmatrix} 4 & 5 & -9 \\ 7 & 0 & 0 \\ 9 & -3 & 4 \\ -5 & 4 & 3 \\ 3 & 5 & 5 \end{pmatrix} \left(\begin{pmatrix} 4 & 7 & 9 & -5 & 3 \\ 5 & 0 & -3 & 4 & 5 \\ -9 & 0 & 4 & 3 & 5 \end{pmatrix} \begin{pmatrix} 4 & 5 & -9 \\ 7 & 0 & 0 \\ 9 & -3 & 4 \\ -5 & 4 & 3 \\ 3 & 5 & 5 \end{pmatrix} \right)^{-1} \begin{pmatrix} 4 & 7 & 9 & -5 \\ 5 & 0 & -3 & 4 \\ -9 & 0 & 4 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 5 & -9 \\ 7 & 0 & 0 \\ 9 & -3 & 4 \\ -5 & 4 & 3 \\ 3 & 5 & 5 \end{pmatrix} \begin{pmatrix} 180 & -12 & 72 \\ -12 & 75 & 70 \\ 0 & -20 & -31 \end{pmatrix}^{-1} \begin{pmatrix} 4 & 7 & 9 & -5 & 3 \\ 5 & 0 & -3 & 4 & 5 \\ -9 & 0 & 4 & 3 & 5 \end{pmatrix}$$

$$= \frac{1}{144756} \begin{pmatrix} 4 & 5 & -9 \\ 7 & 0 & 0 \\ 9 & -3 & 4 \\ -5 & 4 & 3 \\ 3 & 5 & 5 \end{pmatrix} \begin{pmatrix} 925 & 1812 & 6240 \\ 372 & 5580 & 13464 \\ -240 & -3600 & -13356 \end{pmatrix} \begin{pmatrix} 4 & 7 & 9 & -5 & 3 \\ 5 & 0 & -3 & 4 & 5 \\ -9 & 0 & 4 & 3 & 5 \end{pmatrix}$$

$$= \frac{1}{144756} \begin{pmatrix} 3400 & 2748 & -27924 \\ 6475 & 12684 & 43680 \\ 6249 & -14832 & -37656 \\ -3857 & 2460 & -17412 \\ 3435 & 15336 & 19260 \end{pmatrix} \begin{pmatrix} 4 & 7 & 9 & -5 & 3 \\ 5 & 0 & -3 & 4 & 5 \\ -9 & 0 & 4 & 3 & 5 \end{pmatrix}$$

$$= \frac{1}{144756} \begin{pmatrix} 278656 & 23800 & -89340 & -89780 & -115680 \\ -303800 & 45325 & 194943 & 149401 & 301245 \\ 289740 & 43743 & -49887 & -203541 & -243693 \\ 153520 & -26999 & -111741 & -23111 & 86331 \\ -82920 & 24045 & 61947 & 101949 & 183285 \end{pmatrix}$$

Finally get the resulting vector:

$$\frac{1}{144756} \begin{pmatrix} 278656 & 23800 & -89340 & -89780 & -115680 \\ -303800 & 45325 & 194943 & 149401 & 301245 \\ 289740 & 43743 & -49887 & -203541 & -243693 \\ 153520 & -26999 & -111741 & -23111 & 86331 \\ -82920 & 24045 & 61947 & 101949 & 183285 \end{pmatrix} \begin{pmatrix} 11 \\ 6 \\ -1 \\ 19 \\ 8 \end{pmatrix} = \frac{1}{144756} \begin{pmatrix} 666096 \\ 1983786 \\ -2317338 \\ 1890006 \\ 2573514 \end{pmatrix}$$

And to get the coordinates of the closest point, shift it back by v_0:

$$\frac{1}{144756} \begin{pmatrix} 666096 \\ 1983786 \\ -2317338 \\ 1890006 \\ 2573514 \end{pmatrix} + \begin{pmatrix} -6 \\ -6 \\ 1 \\ -9 \\ -7 \end{pmatrix} = \frac{1}{144756} \begin{pmatrix} -202440 \\ 1115250 \\ -2172582 \\ 587202 \\ 1560222 \end{pmatrix} = \frac{1}{24126} \begin{pmatrix} -33740 \\ 185875 \\ -362097 \\ 97867 \\ 260037 \end{pmatrix}$$

Now get the orthogonal projection:

$$\begin{pmatrix} 5 \\ 0 \\ 0 \\ 10 \\ 1 \end{pmatrix} - \frac{1}{24126} \begin{pmatrix} -33740 \\ 185875 \\ -362097 \\ 97867 \\ 260037 \end{pmatrix} = \frac{1}{24126} \begin{pmatrix} 154370 \\ -185875 \\ 362097 \\ 143393 \\ -235911 \end{pmatrix}$$

and the length of this vector would be

$$\frac{\sqrt{154370^2 + 185875^2 + 362097^2 + 143393^2 + 235911^2}}{24126} = \frac{4}{12063}\sqrt{4151709411}$$

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excuse the fuck
\frac{1}{144756}
    {{278656 , 23800 , -89340 , -89780 , -115680},
    {-303800 , 45325 , 194943 , 149401 , 301245},
    {289740 , 43743 , -49887 , -203541 , -243693},
    {153520 , -26999 , -111741 , -23111 , 86331},
    {-82920 , 24045 , 61947 , 101949 , 183285}}
    {{925 , 1812 , 6240},
    {372 , 5580 , 13464},
    {-240 , -3600 , -13356}}
{{4,7,9,-5,3},{5,0,-3,4,5},{-9,0,4,3,5}}
{{4,5,9},{7,0,0},{9,-3,4},{-5,4,3},{3,5,5}}
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