Discrete Maths, Homework 19

Problem 1

The probability space: sequences (x_1, x_2) of length 2 that consist of whole number in the $[0, 9]$ range. All outcomes are equally probable. Find the probability of the event "$x_1 \neq x_2$".

Given some x_1, the chance for the event $x_1 = x_2$ (the next digit being the very same digit out of 10) occurring is $\frac{1}{10}$.

Thus, the chance of $x_1 \neq x_2$ is $1 - \frac{1}{10} = \frac{9}{10}$.

> Alternatively, there are only 10 sequences that have same numbers out of 100 possible sequences, so the chance could be deduced from this information as well.

Problem 2

Probability space: sequences (x_1, x_2, x_3, x_4) of length 4 that consist of whole numbers from 0 to 2. All outcomes are equally probable. Find the probability of the event "there are 0, 1, and 2 in the sequence".

Total number of possible sequences: $3^4 = 81$.

Take all 3-digit words of length 3, there are 27 in total. 3 of them cannot be extended to a valid word of length 3: $000, 111, 222$, leaving us with 24 possible valid beginnings.

Words that contain all three valid digits result in three possible valid sequences. There are $3! = 6$ such words, resulting in $6 \times 3 = 18$ possible 4-letter words.

3-letter words that are with 2 digits which are the same and some other single digit can be extended to a valid word a single possible way, thus there are, $24 - 6 = 18$ such words, giving us the final number of valid sequences, $18 + 18 = 36$ valid words.

Thus, the chance of the required event is $\frac{36}{81} = \frac{4}{9}$.

Problem 3

Probability space is all whole numbers from 1000 to 9999. All outcomes are equally probable. Find the probability of the event "the sum of digits is equal to 9".

Using the stars and bars formula, we need to place 3 bars between 8 ones:

$$(1)111|1|11|11 \mapsto 4122$$

Thus, the number of such numbers for $n = 4, k = 8$ would be

$$\left(\binom{n}{k}\right) = \binom{n+k-1}{k} = \binom{4+8-1}{8} = \binom{11}{8} =$$

$$= \frac{11!}{8!3!} = \frac{11 \times 10 \times 9}{3 \times 2 \times 1} = 11 \times 5 \times 3 = 165$$

Thus, the probability is $\dfrac{165}{9000} = \dfrac{11}{600}$.

Problem 4

Probability space: binary words of length 23. All outcomes are equally probable. Find the probability of an event "on first 11 positions there are less ones than on the last 12".

Take any word like the following and reverse its bits:

$$\underbrace{00010101101}_{5}|\underbrace{001010100111}_{6} \mapsto \underbrace{11101010010}_{11-5=6}|\underbrace{110101011000}_{12-6=6}$$

This way, we may establish a bijection between all the words that have less ones on the first 11 positions and those that a more or equal number of ones on the first 11 positions, compared to the last 12. Since we obviously can do that for every single number, splitting the entire probability space into 2 equal parts, and the probability in the answer would be $\dfrac{1}{2}$.