Calculus, Homework 9

> Find derivatives

Problem A

$$f(x) = \frac{2 + x^2}{\sqrt{1 + x^4}}$$

$$f(x)' = \frac{(2 + x^2)'\sqrt{1 + x^4} - (2 + x^2)\sqrt{1 + x^4}'}{\sqrt{1 + x^4}^2} = \frac{2x\sqrt{1 + x^4} - (2 + x^2)(x^4)'\frac{1}{2\sqrt{1+x^4}}}{\sqrt{1 + x^4}^2} =$$

$$\frac{2x(1 + x^4) - \frac{1}{2}4x^3(2 + x^2)}{\sqrt{1 + x^4}^3} = \frac{2x(1 + x^4 - 2x^2 - x^4)}{\sqrt{1 + x^4}^3} = \frac{2x(1 - 2x^2)}{\sqrt{1 + x^4}^3}$$

Problem B

$$f(x) = e^{3x}(x + 3)$$

$$f'(x) = e^{3x}(x + 3)' + (e^{3x})'(x + 3) = e^{3x} + e^{3x}(3x)'(x + 3) = e^{3x}(1 + 3(x + 3)) = e^{3x}(3x + 10)$$

Problem C

$$f(x) = x^2 2^x + x^3 3^x$$

> $a^x = e^{\ln(a^x)} = e^{x \ln a} \Rightarrow (a^x)' = (x \ln a)' e^{x \ln a} = a^x \ln a$

$$f'(x) = (x^2)(2^x)' + (x^2)'(2^x) + (x^3)(3^x)' + (x^3)'(3^x) = x^2 2^x \ln 2 + 2x(2^x) + x^3 3^x \ln 3 + 3x^2 3^x =$$

$$2^x(x^2 \ln 2 + 2x) + 3^x(x^3 \ln 3 + 3x^2)$$

Problem D

$$f(x) = \sin x \cdot \cos^2 3x$$

$$f'(x) = (\sin x)' \cos^2 3x + \sin x(\cos^2 3x)' = (\cos x)\cos^2 3x + \sin x(2\cos 3x)(\cos 3x)' =$$

$$(\cos x)\cos^2 3x + \sin x(2\cos 3x)(-\sin 3x)(3x)' = \cos x \cdot \cos^2 3x - 6 \sin x \cdot \cos 3x \cdot \sin 3x =$$

$$\cos x \cdot \cos^2 3x - 3 \sin x \cdot \sin 6x$$

Problem E

$$f(x) = e^{2x}(3\cos 3x - 2\sin 3x)$$

$$f'(x) = e^{2x}(3\cos 3x - 2\sin 3x)' + (e^{2x})'(3\cos 3x - 2\sin 3x) =$$

$$e^{2x}(3(\cos 3x)' - 2(\sin 3x)') + 2e^{2x}(3\cos 3x - 2\sin 3x) =$$

$$e^{2x}(-9\sin 3x - 6\cos 3x) + 2e^{2x}(3\cos 3x - 2\sin 3x) =$$

$$e^{2x}(-9\sin 3x - 6\cos 3x + 6\cos 3x - 4\sin 3x) = -13e^{2x}\sin 3x$$

Problem F

$$f(x) = x^{a^a} + a^{x^a} + a^{a^x} \quad (a > 0)$$

$$f'(x) = (x^{a^a})' + (a^{x^a})' + (a^{a^x})' = a^a x^{a^a - 1} + (x^a)' a^{x^a} \ln a + (a^x)' a^{a^x} \ln a =$$

$$a^a x^{a^a-1} + a x^{a-1} a^{x^a} \ln a + a^x \ln a \cdot a^{a^x} \ln a = a^a x^{a^a-1} + x^{a-1} a^{x^a+1} \ln a + a^{a^x+x} \ln^2 a$$

Problem G

$$f(x) = \arccos \frac{1+x^3}{1-x^3}$$

> $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$$f'(x) = \left(\frac{1+x^3}{1-x^3}\right)' \left(-\frac{1}{\sqrt{1-\left(\frac{1+x^3}{1-x^3}\right)^2}}\right) =$$

$$\frac{(1+x^3)'(1-x^3) - (1+x^3)(1-x^3)'}{(1-x^3)^2} \left(-\frac{1}{\sqrt{1-\left(\frac{1+x^3}{1-x^3}\right)^2}}\right) =$$

$$\frac{3x^2(1-x^3) + 3x^2(1+x^3)}{(1-x^3)^2} \left(-\frac{1}{\sqrt{1-\left(\frac{1+x^3}{1-x^3}\right)^2}}\right) =$$

$$-\frac{1}{\sqrt{1-\left(\frac{1+x^3}{1-x^3}\right)^2}}\frac{6x^2}{(1-x^3)^2} = -\frac{6x^2}{\sqrt{1-\frac{(1+x^3)^2}{(1-x^3)^2}}(1-x^3)^2} =$$

$$-\frac{6x^2}{\frac{1}{1-x^3}\sqrt{(1-x^3)^2-(1+x^3)^2}(1-x^3)^2} = -\frac{6x^2}{\sqrt{(1-x^3-1-x^3)(1-x^3+1+x^3)}(1-x^3)} =$$

$$-\frac{6x^2}{\sqrt{-4x^3}(1-x^3)} = -\frac{6x^2}{2x\sqrt{-x}(1-x^3)} = -\frac{3x}{\sqrt{-x}(1-x^3)}$$

Problem H

$$f(x) = 2^{\operatorname{arctg}\sqrt{1+x^2}}$$

> $(\operatorname{arctg} x)' = \frac{1}{x^2+1}$

$$f'(x) = (\operatorname{arctg}\sqrt{1+x^2})' 2^{\operatorname{arctg}\sqrt{1+x^2}} \ln 2 = \frac{\sqrt{1+x^2}'}{\sqrt{1+x^2}^2+1} 2^{\operatorname{arctg}\sqrt{1+x^2}} \ln 2 =$$

$$\frac{(x^2)'}{2\sqrt{1+x^2}(2+x^2)} 2^{\operatorname{arctg}\sqrt{1+x^2}} \ln 2 = \frac{x\ln 2 \cdot 2^{\operatorname{arctg}\sqrt{1+x^2}}}{\sqrt{1+x^2}(2+x^2)}$$

Problem I

$$f(x) = (1+x)^{\frac{1}{x}}$$

$$f'(x) = (e^{\ln((1+x)^{\frac{1}{x}})})' = e^{\ln((1+x)^{\frac{1}{x}})}(\ln((1+x)^{\frac{1}{x}}))' = (1+x)^{\frac{1}{x}}\left(\frac{1}{x}\ln(1+x)\right)' =$$

$$(1+x)^{\frac{1}{x}}\left(\left(\frac{1}{x}\right)'\ln(1+x) + \frac{1}{x}(\ln(1+x))'\right) = (1+x)^{\frac{1}{x}}\left(-\frac{\ln(1+x)}{x^2} + \frac{1}{x}\cdot\frac{1}{x}\right) =$$

$$(1+x)^{\frac{1}{x}}\left(\frac{1-\ln(1+x)}{x^2}\right) = \frac{(1-\ln(1+x))(1+x)^{\frac{1}{x}}}{x^2}$$

Problem J

$$f(x) = (\arccos x)^2\left[\ln^2(\arccos x) - \ln\arccos x + \frac{1}{2}\right]$$

> $(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$

$$f'(x) = \overbrace{((\arccos x)^2)' \left[\ln^2(\arccos x) - \ln\arccos x + \frac{1}{2}\right]}^{A} + \overbrace{(\arccos x)^2 \left[\ln^2(\arccos x) - \ln\arccos x + \frac{1}{2}\right]'}^{B} =$$

$$A = \frac{-2\arccos x}{\sqrt{1-x^2}} \left[\ln^2(\arccos x) - \ln\arccos x + \frac{1}{2}\right] = \frac{\arccos x}{\sqrt{1-x^2}} \left[2\ln\arccos x - 2\ln^2(\arccos x) - 1\right]$$

$$B = \arccos^2 x \left[(\ln^2(\arccos x))' - (\ln\arccos x)' + \left(\frac{1}{2}\right)'\right] =$$

$$\arccos^2 x(\ln(\arccos x)(\ln\arccos x - 1))' =$$

$$\arccos^2 x((\ln(\arccos x))'(\ln\arccos x - 1) + (\ln(\arccos x))(\ln\arccos x - 1)') =$$

$$\arccos^2 x \left(\frac{(\arccos x)'(\ln\arccos x - 1)}{\arccos x} + \frac{(\arccos x)' \ln\arccos x}{\arccos x}\right) =$$

$$\arccos^2 x \left(-\frac{1}{\sqrt{1-x^2}} \frac{2\ln\arccos x - 1}{\arccos x}\right) = \frac{\arccos x(1 - 2\ln\arccos x)}{\sqrt{1-x^2}}$$

$$f'(x) = A + B = \frac{\arccos x}{\sqrt{1-x^2}} \left[2\ln\arccos x - 2\ln^2(\arccos x) - 1\right] + \frac{\arccos x(1 - 2\ln\arccos x)}{\sqrt{1-x^2}} =$$

$$\frac{\arccos x}{\sqrt{1-x^2}} \left[2\ln\arccos x - 2\ln^2(\arccos x) - 1 + 1 - 2\ln\arccos x\right] = -\frac{2\ln^2\arccos x \cdot \arccos x}{\sqrt{1-x^2}}$$