Discrete Maths, Homework 12

Your Result:

Тикки

Share Your Result

Problem 12.1

How many components are there is a forest with 6 vertices and 4 edges? Give an example of such forest.

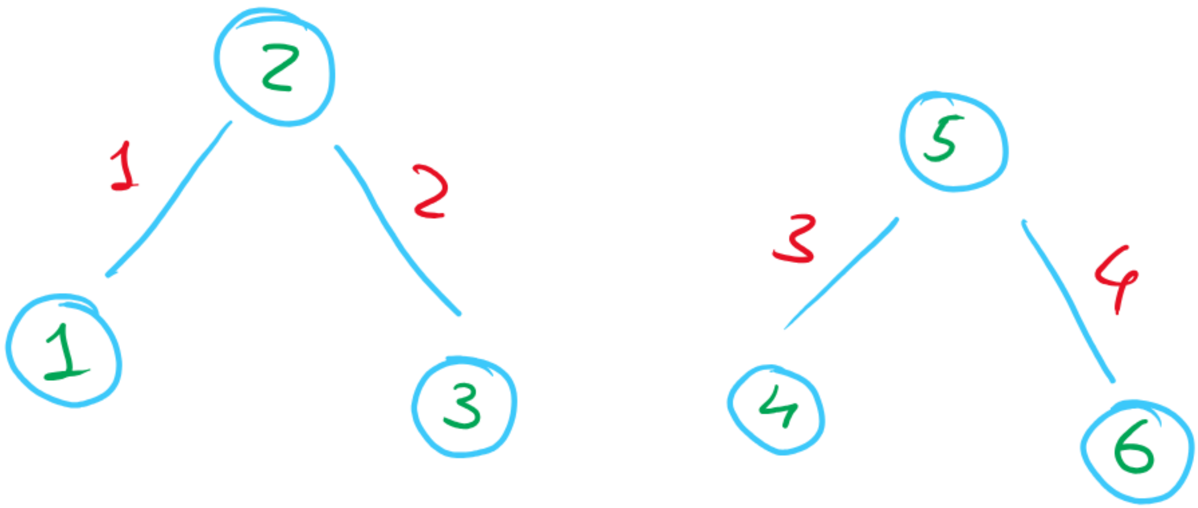
Cyclomatric number of a forest is 0. Therefore,

$$r(G) = m - n + c$$

$$0 = 4 - 6 + c \Rightarrow c = 2$$

Answer: 2

Example:



Problem 12.2

How many simple paths could there be in a tree on n vertices?

Take any vertex x_n out of n options and then take any vertex y_{n-1} out of $n-1$ options. Since any two vertices in a tree are connected by a single path, the total number of paths is the number of all vertex pairs, aka $n(n-1)$.

Answer: $n(n-1)$

Problem 12.3

Find the maximum number of vertices in a connected graph, the sum of vertex degrees of which is equal to 20.

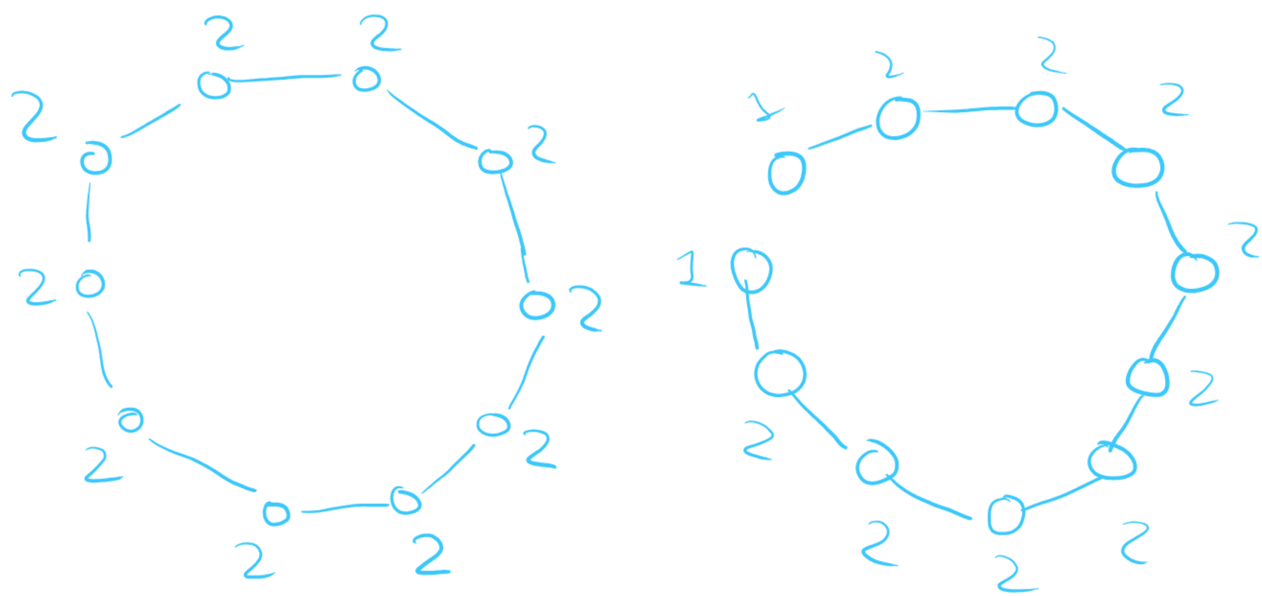
For the number of vertices to be maximum, we need to minimize connections between vertices while still maintaining connectiveness (the number of components is equal to 1). For this reason, the cyclomatic number should be equal to 0. There are $\frac{20}{2} = 10$ vertices in total.

$$r(G) = m - n + c$$

$$0 = 10 - n + 1$$

$$n = 11$$

Take a graph that would be a cycle on 10 vertices: C_{10}. Take one vertex, and duplicate it, creating a break in the cycle to get our required example (P_{11}):

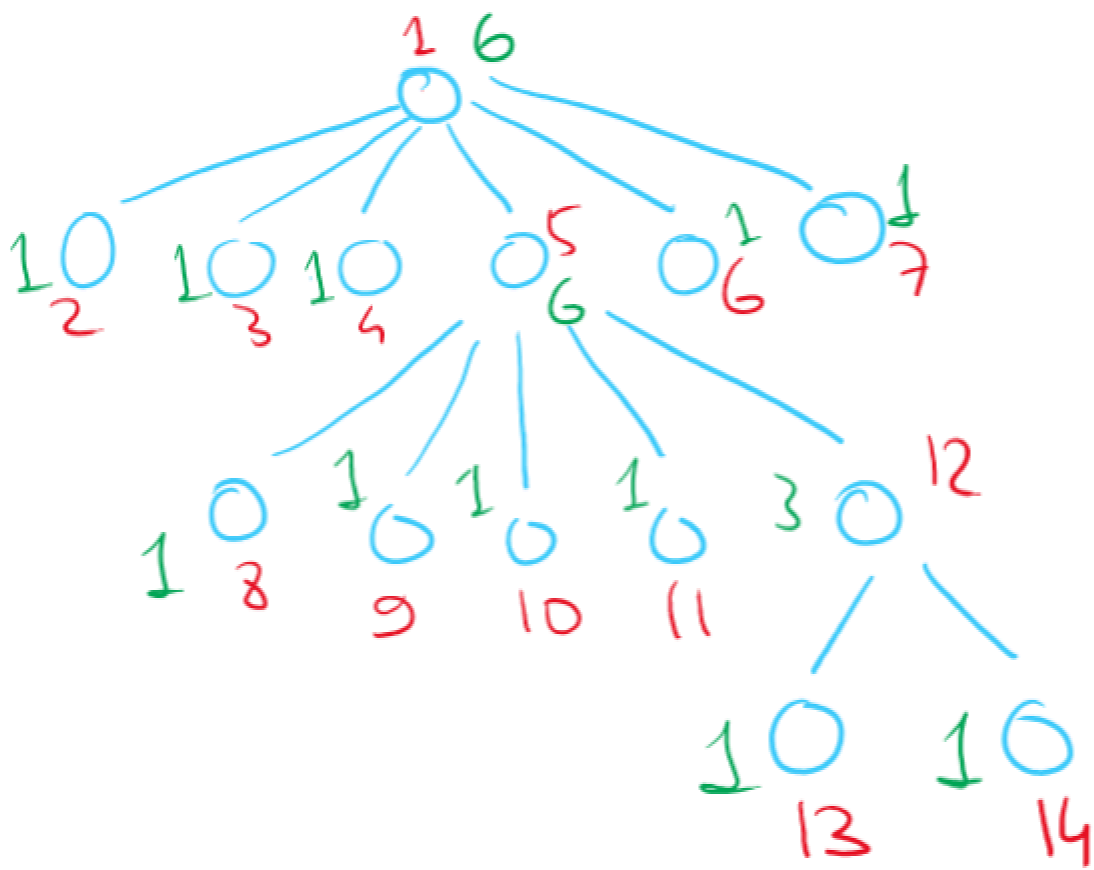


Answer: 11

Problem 12.4

Subproblem A

Give an example of a tree on 14 vertices, which has exactly 2 vertices of degree 6 and has no vertices of degree 2.



Subproblem B

In a tree on 13 vertices there are exactly 2 vertices of degree 6. Does it follow that there is a vertex of degree 2 in this tree?

In a tree, the number of edges is one less than the number of vertices. Sum of degrees of all n vertices is equal to $(n-1)\times 2 = (13-1)\times 2 = 24$

Two vertices should have their sum of degrees equal to 6, which leaves $24 - 6 \times 2 = 12$ for 11 vertices.

Each vertex has its degree equal to at least 1 (otherwise the vertex is isolated, which is impossible). After assigning degree of 1 to each vertex, per **Dirichlet** principle, the degree of at least one of the vertices (which are not equal to 6) should be at least 2, which implies that there is a vertex of degree 2 in the tree.