Discrete Maths, Last Homework

Problem 1

Find the last two digits of 99^{1000}.

$$99^{1000} \equiv (-1)^{1000} \equiv 1 \mod 100$$

This implies that the last two digits are 01.

Problem 2

Prove that numbers a^2 and b^2 give the same remainders when dividing by $a - b$ if a, b are positive integers and $a > b$.

$$a^2 - b^2 = (a+b)(a-b) \equiv 0 \mod (a-b)$$

Since the number is divisible by $(a-b)$, its remainder $\mod (a-b)$ is 0, which implies that the same remainder is given.

Problem 3

Let x, y be integers. Prove that $x + 10y$ is divisible by $13 \iff u + 4x$ is divisible by 13.

$$\begin{aligned} x + 10y \equiv 0 \mod 13 \iff \\ 4x + 40y \equiv 0 \mod 13 \iff \\ 4x + 39y + y \equiv 0 \mod 13 \iff \\ 4x + y \equiv 0 \mod 13 \end{aligned}$$

Problem 4

Solve a comparsion $53x \equiv 1 \mod 42$ using extended Euclidean algorithm.

$$53x \equiv 1 \mod 42$$

We need to find the inverse of $53 \mod 42$. For this, solve

$$53x + 42y \equiv 1 \mod 42$$

Write some code to execute extended euclidean algorithm:

```
def extended_euclidean_algorithm(a, b):
    s, t = 1, 0
    s_prev, t_prev = 0, 1

    r, r_prev = a, b

    steps = []

    while r != 0:
        quotient = r_prev // r
        r_prev, r = r, r_prev - quotient * r
        s_prev, s = s, s_prev - quotient * s
        t_prev, t = t, t_prev - quotient * t

        steps.append((r_prev, s_prev, t_prev))

    return steps

a = 53
b = 42
steps = extended_euclidean_algorithm(a, b)

for step in steps:
    print(f"r={step[0]}, x={step[1]}, y={step[2]}")
```

As an answer, we get

```
r=53, x=1, y=0
r=42, x=0, y=1
r=11, x=1, y=-1
r=9, x=-3, y=4
r=2, x=4, y=-5
r=1, x=-19, y=24
```

$-19 \equiv 23 \mod 42 \implies 23$ is the solution. Check it:

$$53 \times 23 \equiv 1219 \equiv 1 \mod 42$$

Answer: 23.