Individual Homework #3

Problem 1

Find inverse matrix of

$$\begin{pmatrix} -6 & 4 & 3 & 3 \\ 2 & -2 & 0 & -1 \\ -2 & -5 & -2 & 3 \\ -1 & -1 & 0 & 1 \end{pmatrix}$$

Per Gauss:

$$\left(\begin{array}{cccc|cccc} -6 & 4 & 3 & 3 & 1 & 0 & 0 & 0 \\ 2 & -2 & 0 & -1 & 0 & 1 & 0 & 0 \\ -2 & -5 & -2 & 3 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ -6 & 4 & 3 & 3 & 1 & 0 & 0 & 0 \\ 2 & -2 & 0 & -1 & 0 & 1 & 0 & 0 \\ -2 & -5 & -2 & 3 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 10 & 3 & -3 & 1 & 0 & 0 & -6 \\ 0 & -4 & 0 & 1 & 0 & 1 & 0 & 2 \\ 0 & -3 & -2 & 1 & 0 & 0 & 1 & -2 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 10 & 3 & -3 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1.2 & -0.2 & 0.4 & 1 & 0 & -0.4 \\ 0 & 0 & -1.1 & 0.1 & 0.3 & 0 & 1 & -3.8 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0.3 & -0.3 & 0.1 & 0 & 0 & -0.6 \\ 0 & 0 & 1.2 & -0.2 & 0.4 & 1 & 0 & -0.4 \\ 0 & 0 & -1.1 & 0.1 & 0.3 & 0 & 1 & -3.8 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0.3 & -0.3 & 0.1 & 0 & 0 & -0.6 \\ 0 & 0 & 0.1 & -0.1 & 0.7 & 1 & 1 & -4.2 \\ 0 & 0 & -1.1 & 0.1 & 0.3 & 0 & 1 & -3.8 \end{array}\right)$$

Answer:

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0.3 & -0.3 & 0.1 & 0 & 0 & -0.6 \\ 0 & 0 & 0.1 & -0.1 & 0.7 & 1 & 1 & -4.2 \\ 0 & 0 & -1 & 0 & 1 & 1 & 2 & -8 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0.3 & -0.3 & 0.1 & 0 & 0 & -0.6 \\ 0 & 0 & 1 & -1 & 7 & 10 & 10 & -42 \\ 0 & 0 & -1 & 0 & 1 & 1 & 2 & -8 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0.3 & -0.3 & 0.1 & 0 & 0 & -0.6 \\ 0 & 0 & 1 & -1 & 7 & 10 & 10 & -42 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 10 & 3 & -3 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & -1 & -1 & -2 & 8 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 10 & 0 & -3 & 4 & 3 & 6 & -30 \\ 0 & 0 & 1 & 0 & -1 & -1 & -2 & 8 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 10 & 0 & 0 & -20 & -30 & -30 & 120 \\ 0 & 0 & 1 & 0 & -1 & -1 & -2 & 8 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & -2 & -3 & -3 & 12 \\ 0 & 0 & 1 & 0 & -1 & -1 & -2 & 8 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 2 & 3 & 3 & -13 \\ 0 & 1 & 0 & 0 & -2 & -3 & -3 & 12 \\ 0 & 0 & 1 & 0 & -1 & -1 & -2 & 8 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

$$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -6 & -8 & -9 & 37 \\ 0 & 1 & 0 & 0 & -2 & -3 & -3 & 12 \\ 0 & 0 & 1 & 0 & -1 & -1 & -2 & 8 \\ 0 & 0 & 0 & 1 & -8 & -11 & -12 & 50 \end{array}\right)$$

Answer:

$$\begin{pmatrix} -6 & -8 & -9 & 37 \\ -2 & -3 & -3 & 12 \\ -1 & -1 & -2 & 8 \\ -8 & -11 & -12 & 50 \end{pmatrix}$$

Problem 2

Solve equation for X:

$$\left(\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 8 & 5 & 1 & 3 & 7 & 2 \end{pmatrix}^{11} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 5 & 6 & 2 & 7 & 3 & 1 \end{pmatrix}^{-1}\right)^{149} \cdot X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 1 & 5 & 4 & 7 & 8 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 8 & 5 & 6 & 2 & 7 & 3 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 7 & 1 & 3 & 4 & 6 & 2 \end{pmatrix}$$

In the following permutation, there are 3 cycles:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 8 & 5 & 1 & 3 & 7 & 2 \end{pmatrix}$$

- mod 3: $\xrightarrow{0} 1 \xrightarrow{1} 4 \xrightarrow{2} 5$
- mod 4: $\xrightarrow{0} 2 \xrightarrow{1} 6 \xrightarrow{2} 3 \xrightarrow{3} 8$
- mod 1: $\xrightarrow{0} 7$

This permutation is composed with the permutation 11 times, a composition of 12 permutations in total. Since $12 \bmod 3 = 0, 12 \bmod 4 = 0, 12 \bmod 1 = 0$, then since

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 8 & 5 & 1 & 3 & 7 & 2 \end{pmatrix}^{11} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 7 & 1 & 3 & 4 & 6 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 7 & 1 & 3 & 4 & 6 & 2 \end{pmatrix}$$

is fhe first mapping, then after 11 compositions it would bring us back to square one.

The resulting permutation consists of a single cycle of length 8:

- $1 \to 8 \to 2 \to 5 \to 3 \to 7 \to 6 \to 4 \to 1$

Composing this permutation with itself $149 - 1 = 148$ times would be identical to composing it $148 \bmod 8 = 4$ times, thus we need to shift each position in the permutation above 4 spaces along the cycle:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 7 & 1 & 3 & 4 & 6 & 2 \end{pmatrix}^{149} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 5 & 7 & 1 & 3 & 4 & 6 & 2 \end{pmatrix}^{5} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 8 & 3 & 1 & 5 & 2 & 6 \end{pmatrix}$$

Sweet, now we only need to solve the following equation:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 8 & 3 & 1 & 5 & 2 & 6 \end{pmatrix} \cdot X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 3 & 1 & 5 & 4 & 7 & 8 & 2 \end{pmatrix}$$

Write up as a tri-permutation (custom notation, oopsie):

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ ? & ? & ? & ? & ? & ? & ? & ? \\ 6 & 3 & 1 & 5 & 4 & 7 & 8 & 2 \end{pmatrix}$$

Use mappings from

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 7 & 4 & 8 & 3 & 1 & 5 & 2 & 6 \end{pmatrix}$$

to fill in the gaps and get

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 5 & 6 & 2 & 1 & 3 & 7 \\ 6 & 3 & 1 & 5 & 4 & 7 & 8 & 2 \end{pmatrix} \Rightarrow X = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 5 & 6 & 2 & 1 & 3 & 7 \end{pmatrix}$$

Answer:

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 4 & 5 & 6 & 2 & 1 & 3 & 7 \end{pmatrix}$$

Problem 3

Find whether a permutation is odd or even:

$$\sigma = \begin{pmatrix} 1 & 2 & \dots & 97 & 98 & \dots & 131 & 132 & \dots & 256 \\ 160 & 161 & \dots & 256 & 126 & \dots & 159 & 1 & \dots & 125 \end{pmatrix}$$

Calculate the number of inversions:

- There are $256 - 159 = 97$ elements in the first group, $159 - 125 = 34$ elements in the second group, and 125 elements in the third group.
- Both groups 2 and 3 are wholly inverted in relation to the first group.
- The third group is inverted in relation to the second group.

Therefore, the total number of inversions:

$$97 \times (34 + 125) + 34 \times 125 = 15423 + 4250 = 19673$$

The number of inversions is odd $\Rightarrow$ the permutation is odd.

> It is also fair to mention that this entire permutation turns out to be a single cycle of even (256) length; therefore, since cycles of even lengths are odd, the entire permutation is odd as well.

Answer:

$$\operatorname{sgn} \sigma = -1$$

Problem 4

Calculate:

$$\det \begin{vmatrix} 0 & 0 & x & 0 & 0 & 2 \\ x & x & 9 & x & 0 & 1 \\ 0 & 7 & 0 & 8 & 4 & 4 \\ 0 & 2 & 0 & 0 & 0 & 7 \\ 5 & 4 & x & x & 4 & 5 \\ 0 & 4 & 2 & 2 & 5 & 5 \end{vmatrix}$$

$$x \det \begin{vmatrix} x & x & x & 0 & 1 \\ 0 & 7 & 8 & 4 & 4 \\ 0 & 2 & 0 & 0 & 7 \\ 5 & 4 & x & 4 & 5 \\ 0 & 4 & 2 & 5 & 5 \end{vmatrix} + 2 \det \begin{vmatrix} x & x & 9 & x & 0 \\ 0 & 7 & 0 & 8 & 4 \\ 0 & 2 & 0 & 0 & 0 \\ 5 & 4 & x & x & 4 \\ 0 & 4 & 2 & 2 & 5 \end{vmatrix}$$

$$2x \det \begin{vmatrix} x & x & 0 & 1 \\ 0 & 8 & 4 & 4 \\ 5 & x & 4 & 5 \\ 0 & 2 & 5 & 5 \end{vmatrix} + 7x \det \begin{vmatrix} x & x & x & 0 \\ 0 & 7 & 8 & 4 \\ 5 & 4 & x & 4 \\ 0 & 4 & 2 & 5 \end{vmatrix} + 4 \det \begin{vmatrix} x & 9 & x & 0 \\ 0 & 0 & 8 & 4 \\ 5 & x & x & 4 \\ 0 & 2 & 2 & 5 \end{vmatrix}$$

$$2x \det \begin{vmatrix} x & x & 0 & 1 \\ 0 & 8 & 4 & 4 \\ 5 & x & 4 & 5 \\ 0 & 2 & 5 & 5 \end{vmatrix} + 7x \det \begin{vmatrix} x & x & x & 0 \\ 0 & 7 & 8 & 4 \\ 5 & 4 & x & 4 \\ 0 & 4 & 2 & 5 \end{vmatrix} + 32 \det \begin{vmatrix} x & 9 & 0 \\ 5 & x & 4 \\ 0 & 2 & 5 \end{vmatrix} + 16 \det \begin{vmatrix} x & 9 & x \\ 5 & x & x \\ 0 & 2 & 2 \end{vmatrix}$$

$$2x\det\begin{vmatrix} x & x & 0 & 1 \\ 0 & 8 & 4 & 4 \\ 5 & x & 4 & 5 \\ 0 & 2 & 5 & 5 \end{vmatrix} + 7x\det\begin{vmatrix} x & x & x & 0 \\ 0 & 7 & 8 & 4 \\ 5 & 4 & x & 4 \\ 0 & 4 & 2 & 5 \end{vmatrix} + 64\det\begin{vmatrix} x & 0 \\ 5 & 4 \end{vmatrix} + 160\det\begin{vmatrix} x & 9 \\ 5 & x \end{vmatrix} + 32\det\begin{vmatrix} x & x \\ 5 & x \end{vmatrix} + 32\det\begin{vmatrix} x & 9 \\ 5 & x \end{vmatrix}$$

$$2x\det\begin{vmatrix} x & x & 0 & 1 \\ 0 & 8 & 4 & 4 \\ 5 & x & 4 & 5 \\ 0 & 2 & 5 & 5 \end{vmatrix} + 7x\det\begin{vmatrix} x & x & x & 0 \\ 0 & 7 & 8 & 4 \\ 5 & 4 & x & 4 \\ 0 & 4 & 2 & 5 \end{vmatrix} + 256x + 160(x^2-45) + 32(x^2-5x) + 32(x^2-45)$$

$$2x\det\begin{vmatrix} x & x & 0 & 1 \\ 0 & 8 & 4 & 4 \\ 5 & x & 4 & 5 \\ 0 & 2 & 5 & 5 \end{vmatrix} + 7x\det\begin{vmatrix} x & x & x & 0 \\ 0 & 7 & 8 & 4 \\ 5 & 4 & x & 4 \\ 0 & 4 & 2 & 5 \end{vmatrix} + 224x^2 + 96x - 8640$$

Find

$$\det\begin{vmatrix} x & x & 0 & 1 \\ 0 & 8 & 4 & 4 \\ 5 & x & 4 & 5 \\ 0 & 2 & 5 & 5 \end{vmatrix} = -\det\begin{vmatrix} 5 & x & 4 & 5 \\ 0 & 8 & 4 & 4 \\ x & x & 0 & 1 \\ 0 & 2 & 5 & 5 \end{vmatrix} = -\det\begin{vmatrix} 5 & x & 4 & 5 \\ 0 & 8 & 4 & 4 \\ 0 & 0.2(-5+x)x & -0.8x & 1-x \\ 0 & 2 & 5 & 5 \end{vmatrix} =$$

$$-\det\begin{vmatrix} 5 & x & 4 & 5 \\ 0 & 8 & 4 & 4 \\ 0 & 0 & -0.1(-13+x)x & 0.1(10-15x+x^2) \\ 0 & 0 & 4 & 4 \end{vmatrix} = \det\begin{vmatrix} 5 & x & 4 & 5 \\ 0 & 8 & 4 & 4 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & -0.1(-13+x)x & 0.1(10-15x+x^2) \end{vmatrix} =$$

$$\det\begin{vmatrix} 5 & x & 4 & 5 \\ 0 & 8 & 4 & 4 \\ 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 1-0.2x \end{vmatrix} = 5\times 8\times 4\times\left(1-\frac{x}{5}\right) = -32x+160$$

Find

$$\det\begin{vmatrix} x & x & x & 0 \\ 0 & 7 & 8 & 4 \\ 5 & 4 & x & 4 \\ 0 & 4 & 2 & 5 \end{vmatrix} = -\det\begin{vmatrix} 5 & 4 & x & 4 \\ 0 & 7 & 8 & 4 \\ x & x & x & 0 \\ 0 & 4 & 2 & 5 \end{vmatrix} = -\det\begin{vmatrix} 5 & 4 & x & 4 \\ 0 & 7 & 8 & 4 \\ 0 & 0.2x & -0.2(-5+x)x & -0.8x \\ 0 & 4 & 2 & 5 \end{vmatrix} =$$

$$-\det\begin{vmatrix} 5 & 4 & x & 4 \\ 0 & 7 & 8 & 4 \\ 0 & 0 & \frac{1}{35}(27-7x)x & -\frac{32}{25}x \\ 0 & 0 & -\frac{18}{7} & \frac{19}{7} \end{vmatrix} = \det\begin{vmatrix} 5 & 4 & x & 4 \\ 0 & 7 & 8 & 4 \\ 0 & 0 & -\frac{18}{7} & \frac{19}{7} \\ 0 & 0 & \frac{1}{35}(27-7x)x & -\frac{32}{25}x \end{vmatrix} = \det\begin{vmatrix} 5 & 4 & x & 4 \\ 0 & 7 & 8 & 4 \\ 0 & 0 & -\frac{18}{7} & \frac{19}{7} \\ 0 & 0 & 0 & -\frac{1}{90}x(9+19x) \end{vmatrix} =$$

$$\frac{5\times 7\times -18\times -x(9+19x)}{7\times 90} = 19x^2+9x$$

Finally,

$$2x(-32x+160)+7x(19x^2+9x)+256x+160(x^2-45)+32(x^2-5x)+32(x^2-45)=$$

$$133x^3+(63+32+192-64)x^2+(-160+320+256)x-8640=133x^3+223x^2+416x-8640$$

Answer:

> I beg for this to be correct otherwise my sanity will literally implode and cause me to die (also I didn't continue with the algebraic expansion because I kept messing up for some reason and the answer went to hell)

$$133x^3+223x^2+416x-8640$$

Problem 5

Find the coefficient before x^5 in the determinant expression:

$$\begin{vmatrix} 2 & 5 & 10 & 9 & x & 8 & 10 \\ 5 & 8 & 8 & x & 5 & 6 & 8 \\ 10 & 8 & x & 8 & 7 & 8 & 9 \\ 9 & x & 8 & 8 & 1 & 6 & 8 \\ x & 5 & 7 & 1 & 7 & 9 & x \\ 8 & 6 & 8 & 6 & 9 & x & 4 \\ 10 & 8 & 9 & 8 & x & 4 & 6 \end{vmatrix}$$

Simplify:

$$\begin{vmatrix} x & 5 & 7 & 1 & 7 & 9 & x \\ 9 & x & 8 & 8 & 1 & 6 & 8 \\ 10 & 8 & x & 8 & 7 & 8 & 9 \\ 5 & 8 & 8 & x & 5 & 6 & 8 \\ 2 & 5 & 10 & 9 & x & 8 & 10 \\ 8 & 6 & 8 & 6 & 9 & x & 4 \\ 10 & 8 & 9 & 8 & x & 4 & 6 \end{vmatrix}$$

$$\begin{vmatrix} x & 5 & 7 & 1 & 7 & 9 & x \\ 9 & x & 8 & 8 & 1 & 6 & 8 \\ 10 & 8 & x & 8 & 7 & 8 & 9 \\ 5 & 8 & 8 & x & 5 & 6 & 8 \\ 2 & 5 & 10 & 9 & x & 8 & 10 \\ 8 & 6 & 8 & 6 & 9 & x & 4 \\ 8 & 3 & -1 & -1 & 0 & -4 & -4 \end{vmatrix}$$

$$\begin{vmatrix} x & 5 & 7 & 1 & 7 & 9 & 0 \\ 9 & x & 8 & 8 & 1 & 6 & -1 \\ 10 & 8 & x & 8 & 7 & 8 & -1 \\ 5 & 8 & 8 & x & 5 & 6 & 3 \\ 2 & 5 & 10 & 9 & x & 8 & 8 \\ 8 & 6 & 8 & 6 & 9 & x & -4 \\ 8 & 3 & -1 & -1 & 0 & -4 & -12 \end{vmatrix}$$

We need to consider $\binom{6}{5} = 6$ options of choosing 5 lines out of 6 lines that have x after simplifications and calculate determinants of those minors:

$$\begin{vmatrix} x & 5 & 7 & 1 & 7 & 9 & \overset{M_{17}}{\boxed{0}} \\ 9 & x & 8 & 8 & 1 & 6 & \overset{M_{27}}{\boxed{-1}} \\ 10 & 8 & x & 8 & 7 & 8 & \overset{M_{37}}{\boxed{-1}} \\ 5 & 8 & 8 & x & 5 & 6 & \overset{M_{47}}{\boxed{3}} \\ 2 & 5 & 10 & 9 & x & 8 & \overset{M_{57}}{\boxed{8}} \\ 8 & 6 & 8 & 6 & 9 & x & \overset{M_{67}}{\boxed{-4}} \\ \overset{M_{17}}{\boxed{8}} & \overset{M_{27}}{\boxed{3}} & \overset{M_{37}}{\boxed{-1}} & \overset{M_{47}}{\boxed{-1}} & \overset{M_{57}}{\boxed{0}} & \overset{M_{67}}{\boxed{-4}} & -12 \end{vmatrix}$$

We only take the auxiliary diagonal of each of the minors, since we don't need the x-s (multiply respective values highlighted above), thus for $kx^5 \in \det A$ (funny notation, I know). M_{ij} denotes which columns/lines have not been taken as a part of the algebraic expansion, and now to evaluate all the minors/algebraic complements after the expansion over x has taken place:

$$x^5 M_{17} = x^5 \begin{vmatrix} x & 0 \\ 8 & -12 \end{vmatrix}$$

$$x^5 M_{27} = x^5 \begin{vmatrix} x & -1 \\ 3 & -12 \end{vmatrix}$$

$$x^5 M_{37} = x^5 \begin{vmatrix} x & -1 \\ -1 & -12 \end{vmatrix}$$

$$x^5 M_{47} = x^5 \begin{vmatrix} x & 3 \\ -1 & -12 \end{vmatrix}$$

$$x^5 M_{57} = x^5 \begin{vmatrix} x & 8 \\ 0 & -12 \end{vmatrix}$$

$$x^5 M_{57} = x^5 \begin{vmatrix} x & -4 \\ -4 & -12 \end{vmatrix}$$

Now, only take the auxiliary diagoal since we don't care about x^6:

$k = -(8 \times 0 + 3 \times -1 + -1 \times -1 + -1 \times 3 + 0 \times 8 + -4 \times -4) = -(0 - 3 + 1 - 3 + 0 + 16) = -11$

Answer:

$$-11$$