

Following an apparently highly arbitrary formal language, though later we will show how it represents an implmenetation of a generic theorem prover.

Let  $\Sigma = \{0, 1\}^*$  be a set bit vectors of varying length, called symbols.

Define a word  $w \in \mathcal{W} \subset \Sigma^*$  to be a set<sup>1</sup> of symbols such that every two symbols in the word do not share a common prefix. For example, a word may contain the symbols 0010 and 1010 as they do not share a common prefix, but a string containing the symbols 0010 and 0011 is not considered a word on our language.

Define the set of sentences  $\mathcal{S} = \mathcal{W}^*$  to be the set of all sequences of words. We are given a set of rules  $\mathcal{R} \subset \mathcal{S} \times \mathcal{S}$  denoted by  $s_1 \implies s_2$ , with semantics such that if all words on  $s_1$  can be proved, then all words on  $s_2$  are proved. We are also given a set  $\mathcal{F} \subset \mathcal{W}$  of facts. We construct the set of ground words  $\mathcal{G}$  by:

$$\frac{w \in \mathcal{F}}{w \in \mathcal{G}} \quad (1)$$

means that every fact word is a ground word. The next rule

$$\frac{\begin{array}{l} w_1 \in \mathcal{G} \\ w_2 \in \mathcal{W} \\ w_1 \cup w_2 \in \mathcal{W} \end{array}}{w \in \mathcal{G}} \quad (2)$$

states that if a word is ground, then if its union (as a word is a set) with another word (valid word but not necessarily ground) yields another valid word, then this latter word is ground too. The last one is modus ponens:

$$\frac{\begin{array}{l} \{s_1, s_2\} \subset \mathcal{S} \\ s_1 \implies s_2 \\ \forall \omega \in s_1, \omega \in \mathcal{G} \\ w \in s_2 \end{array}}{w \in \mathcal{G}} \quad (3)$$

reads that if  $s_1, s_2$  are sentences and we're given that  $s_1 \implies s_2$ , and all the words  $\omega$  in  $s_1$  are ground, then all words on  $s_2$  are ground.

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<sup>1</sup>Set as unordered indeed, unlike words in common languages where order of letters matters.