Initialization of tensor algebra

Definitions

Functions

Elasticity

Isotropy (2 ctes)

Cubic (3 ctes)

Transverse isotropy (5 ctes)

Fabric-based tetragonal (2-3 ctes)

Fabric-based orthotropy (3 ctes)

Fabric-based orthotropy new (3 ctes, 4 exposants)

Stiffness

In[809]:=

```
ssortzc2[{λ0_, λs0_, μ0_, l_}, rho_, {m1_, m2_, m3_}] :=
 (λ0 + 2 μ0) * rho^k1 * m1^(2 l) * tcro[mm1, mm1] + (λ0 + 2 μ0) * rho^k2 *
   m2^(2 l) * tcro[mm2, mm2] + (λ0 + 2 μ0) * rho^k3 * m3^(2 l) * tcro[mm3, mm3] +
 2 * μ0 * rho^((k2 + k3) / 2) * m2^l * m3^l * (tdou[mm2, mm3] + tdou[mm3, mm2]) +
 2 * μ0 * rho^((k3 + k1) / 2) * m3^l * m1^l * (tdou[mm3, mm1] + tdou[mm1, mm3]) +
 2 * μ0 * rho^((k1 + k2) / 2) * m1^l * m2^l * (tdou[mm1, mm2] + tdou[mm2, mm1]) +
 λs0 * (rho^(k2 / 2) * rho^(k3 / 2) * m2^l * m3^l * (tcro[mm2, mm3] + tcro[mm3, mm2])) +
 λs0 * (rho^(k1 / 2) * rho^(k3 / 2) * m3^l * m1^l * (tcro[mm1, mm3] + tcro[mm3, mm1])) +
 λs0 * (rho^(k1 / 2) * rho^(k2 / 2) * m1^l * m2^l * (tcro[mm1, mm2] + tcro[mm2, mm1]))
```

In[810]:=
```
MatrixForm[Simplify[proj4[ssortzc2[{λ0, λs, μ0, l}, rho, {m1, m2, m3}]]]]
```
Out[810]//MatrixForm=

$$\begin{pmatrix} m1^{2\,l}\,rho^{k1}\,(\lambda 0+2\,\mu 0) & m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\lambda s & m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\lambda s & 0 & \\ m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\lambda s & m2^{2\,l}\,rho^{k2}\,(\lambda 0+2\,\mu 0) & m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\lambda s & 0 & \\ m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\lambda s & m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\lambda s & m3^{2\,l}\,rho^{k3}\,(\lambda 0+2\,\mu 0) & 0 & \\ 0 & 0 & 0 & 2\,m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\mu 0 & \\ 0 & 0 & 0 & 0 & 2\,m \\ 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[811]:=
```
MatrixForm[Simplify[proj4[ssortzc2[{λ0, λs, μ0, l}, 1, {1, 1, 1}]]]]
```
Out[811]//MatrixForm=

$$\begin{pmatrix} \lambda 0+2\,\mu 0 & \lambda s & \lambda s & 0 & 0 & 0 \\ \lambda s & \lambda 0+2\,\mu 0 & \lambda s & 0 & 0 & 0 \\ \lambda s & \lambda s & \lambda 0+2\,\mu 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,\mu 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\,\mu 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\,\mu 0 \end{pmatrix}$$

In[812]:=
```
Simplify[orthosticonst[ssortzc2[{λ0, λs0, μ0, l}, rho, {1, 1, 1}]]]
```
Out[812]=

$$\left\{-2\,rho^{\frac{k1+k2}{2}}\,\mu 0-2\,rho^{\frac{k1+k3}{2}}\,\mu 0+2\,rho^{\frac{k2+k3}{2}}\,\mu 0+rho^{k1}\,(\lambda 0+2\,\mu 0),\right.$$
$$-2\,rho^{\frac{k1+k2}{2}}\,\mu 0+2\,rho^{\frac{k1+k3}{2}}\,\mu 0-2\,rho^{\frac{k2+k3}{2}}\,\mu 0+rho^{k2}\,(\lambda 0+2\,\mu 0),$$
$$2\,rho^{\frac{k1+k2}{2}}\,\mu 0-2\,rho^{\frac{k1+k3}{2}}\,\mu 0-2\,rho^{\frac{k2+k3}{2}}\,\mu 0+rho^{k3}\,(\lambda 0+2\,\mu 0),\ rho^{\frac{k2+k3}{2}}\,\lambda s0,$$
$$\left.rho^{\frac{k1+k3}{2}}\,\lambda s0,\ rho^{\frac{k1+k2}{2}}\,\lambda s0,\ rho^{\frac{k2+k3}{2}}\,\mu 0,\ rho^{\frac{k1+k3}{2}}\,\mu 0,\ rho^{\frac{k1+k2}{2}}\,\mu 0\right\}$$

Det & Eigenvalues

In[813]:=
```
Simplify[Det[proj4[ssortzc2[{λ0, λs, μ0, l}, rho, {m1, m2, m3}]]]]
```
Out[813]=

$$8\,m1^{4\,l}\,m2^{4\,l}\,m3^{4\,l}\,rho^{2\,(k1+k2+k3)}\,\mu 0^{3}\,(\lambda 0-\lambda s+2\,\mu 0)^{2}\,(\lambda 0+2\,(\lambda s+\mu 0))$$

In[814]:=
```
Simplify[Eigenvalues[proj4[ssortzc2[{λ0, λs, μ0, l}, 1, {1, 1, 1}]]]]
```
Out[814]=

$$\{2\,\mu 0,\ 2\,\mu 0,\ 2\,\mu 0,\ \lambda 0-\lambda s+2\,\mu 0,\ \lambda 0-\lambda s+2\,\mu 0,\ \lambda 0+2\,(\lambda s+\mu 0)\}$$

Compliance

In[815]:=

```
eeortzc2[{ϵ0_, ν0_, μ0_, l_}, rho_, {m1_, m2_, m3_}] :=
 (1 / ϵ0 / rho^k1 / m1^(2 l) * tcro[mm1, mm1] + 1 / ϵ0 / rho^k2 / m2^(2 l) *
    tcro[mm2, mm2] + 1 / ϵ0 / rho^k3 / m3^(2 l) * tcro[mm3, mm3] -
   ν0 / ϵ0 / rho^((k2 + k3) / 2) / m2^l / m3^l * (tcro[mm2, mm3] + tcro[mm3, mm2]) -
   ν0 / ϵ0 / rho^((k3 + k1) / 2) / m3^l / m1^l * (tcro[mm1, mm3] + tcro[mm3, mm1]) -
   ν0 / ϵ0 / rho^((k1 + k2) / 2) / m1^l / m2^l * (tcro[mm1, mm2] + tcro[mm2, mm1]) +
   1 / 2 / μ0 / rho^((k1 + k2) / 2) / m1^l / m2^l * (tdou[mm1, mm2] + tdou[mm2, mm1]) +
   1 / 2 / μ0 / rho^((k3 + k1) / 2) / m3^l / m1^l * (tdou[mm1, mm3] + tdou[mm3, mm1]) +
   1 / 2 / μ0 / rho^((k2 + k3) / 2) / m2^l / m3^l * (tdou[mm3, mm2] + tdou[mm2, mm3]))
```

In[816]:=

```
MatrixForm[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]]
```

Out[816]//MatrixForm=

$$\begin{pmatrix} \frac{m1^{-2\,l}\,rho^{-k1}}{\epsilon 0} & -\frac{m1^{-l}\,m2^{-l}\,rho^{\frac{1}{2}(-k1-k2)}\,\nu 0}{\epsilon 0} & -\frac{m1^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k1-k3)}\,\nu 0}{\epsilon 0} & 0 & 0 \\ -\frac{m1^{-l}\,m2^{-l}\,rho^{\frac{1}{2}(-k1-k2)}\,\nu 0}{\epsilon 0} & \frac{m2^{-2\,l}\,rho^{-k2}}{\epsilon 0} & -\frac{m2^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k2-k3)}\,\nu 0}{\epsilon 0} & 0 & 0 \\ -\frac{m1^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k1-k3)}\,\nu 0}{\epsilon 0} & -\frac{m2^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k2-k3)}\,\nu 0}{\epsilon 0} & \frac{m3^{-2\,l}\,rho^{-k3}}{\epsilon 0} & 0 & 0 \\ 0 & 0 & 0 & \frac{m2^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k2-k3)}}{2\,\mu 0} & 0 \\ 0 & 0 & 0 & 0 & \frac{m1^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k1-k} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[817]:=

```
FullSimplify[
 MatrixForm[Inverse[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]]]]
```

Out[817]//MatrixForm=

$$\begin{pmatrix} \frac{m1^{2\,l}\,rho^{k1}\,\epsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2} & -\frac{m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & -\frac{m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & 0 & 0 \\ -\frac{m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & \frac{m2^{2\,l}\,rho^{k2}\,\epsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2} & -\frac{m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & 0 & 0 \\ -\frac{m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & -\frac{m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & \frac{m3^{2\,l}\,rho^{k3}\,\epsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2} & 0 & 0 \\ 0 & 0 & 0 & 2\,m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\mu 0 & 0 \\ 0 & 0 & 0 & 0 & 2\,m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\mu \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[818]:=

```
orthoelaconst[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]
```

Out[818]=

$$\left\{m1^{2\,l}\,rho^{k1}\,\epsilon 0,\ m2^{2\,l}\,rho^{k2}\,\epsilon 0,\ m3^{2\,l}\,rho^{k3}\,\epsilon 0,\ m2^{l}\,m3^{-l}\,rho^{k2+\frac{1}{2}(-k2-k3)}\,\nu 0,\right.$$
$$m1^{-l}\,m3^{l}\,rho^{\frac{1}{2}(-k1-k3)+k3}\,\nu 0,\ m1^{l}\,m2^{-l}\,rho^{k1+\frac{1}{2}(-k1-k2)}\,\nu 0,$$
$$\left.m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\mu 0,\ m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\mu 0,\ m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\mu 0\right\}$$

Det & Eigenvalues

In[819]:=
```
Simplify[Det[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]]]
```

Out[819]=
$$-\frac{m1^{-4\,l}\,m2^{-4\,l}\,m3^{-4\,l}\,rho^{-2\,(k1+k2+k3)}\,(1+\nu 0)^2\,(-1+2\,\nu 0)}{8\,\epsilon 0^3\,\mu 0^3}$$

In[820]:=
```
Simplify[Eigenvalues[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, 1, {1, 1, 1}]]]]
```

Out[820]=
$$\left\{\frac{1}{2\,\mu 0},\ \frac{1}{2\,\mu 0},\ \frac{1}{2\,\mu 0},\ \frac{1+\nu 0}{\epsilon 0},\ \frac{1+\nu 0}{\epsilon 0},\ \frac{1-2\,\nu 0}{\epsilon 0}\right\}$$

Relationships

In[821]:=
```
FullSimplify[Solve[proj4[ssortzc2[{λ0, λs0, μ0, l}, rho, {m1, m2, m3}]] ==
    Inverse[proj4[eeortzc2[{ε0, ν0, μ0, l}, rho, {m1, m2, m3}]]], {λ0, λs0}]]
```

Out[821]=
$$\left\{\left\{\lambda 0 \to -2\,\mu 0+\frac{\varepsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2},\ \lambda s0 \to -\frac{\varepsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2}\right\}\right\}$$

In[822]:=
```
FullSimplify[Solve[proj4[ssortzc2[{λ0, λs0, μ0, l}, rho, {m1, m2, m3}]] ==
    Inverse[proj4[eeortzc2[{ε0, ν0, μ0, l}, rho, {m1, m2, m3}]]], {ε0, ν0}]]
```

Out[822]=
$$\left\{\left\{\varepsilon 0 \to \lambda 0+2\,\mu 0-\frac{2\,\lambda s0^2}{\lambda 0+\lambda s0+2\,\mu 0},\ \nu 0 \to \frac{\lambda s0}{\lambda 0+\lambda s0+2\,\mu 0}\right\}\right\}$$

Fabric-based orthotropy new 2 (3 ctes, 7 exponents)

Stiffness

In[823]:=
```
ssortzc2[{λ0_, λs0_, μ0_, l_}, rho_, {m1_, m2_, m3_}] :=
  (λ0 + 2 μ0) * rho^k1 * m1^(2 l) * tcro[mm1, mm1] + (λ0 + 2 μ0) * rho^k2 *
    m2^(2 l) * tcro[mm2, mm2] + (λ0 + 2 μ0) * rho^k3 * m3^(2 l) * tcro[mm3, mm3] +
   2 * μ0 * rho^k4 * m2^l * m3^l * (tdou[mm2, mm3] + tdou[mm3, mm2]) +
   2 * μ0 * rho^k5 * m3^l * m1^l * (tdou[mm3, mm1] + tdou[mm1, mm3]) +
   2 * μ0 * rho^k6 * m1^l * m2^l * (tdou[mm1, mm2] + tdou[mm2, mm1]) +
   λs0 * (rho^(k2 / 2) * rho^(k3 / 2) * m2^l * m3^l * (tcro[mm2, mm3] + tcro[mm3, mm2])) +
   λs0 * (rho^(k1 / 2) * rho^(k3 / 2) * m3^l * m1^l * (tcro[mm1, mm3] + tcro[mm3, mm1])) +
   λs0 * (rho^(k1 / 2) * rho^(k2 / 2) * m1^l * m2^l * (tcro[mm1, mm2] + tcro[mm2, mm1]))
```

In[824]:=
```
MatrixForm[Simplify[proj4[ssortzc2[{λ0, λs, μ0, l}, rho, {m1, m2, m3}]]]]
```
Out[824]//MatrixForm=

$$\begin{pmatrix} m1^{2\,l}\,rho^{k1}\,(\lambda 0+2\,\mu 0) & m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\lambda s & m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\lambda s & 0 & \\ m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\lambda s & m2^{2\,l}\,rho^{k2}\,(\lambda 0+2\,\mu 0) & m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\lambda s & 0 & \\ m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\lambda s & m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\lambda s & m3^{2\,l}\,rho^{k3}\,(\lambda 0+2\,\mu 0) & 0 & \\ 0 & 0 & 0 & 2\,m2^{l}\,m3^{l}\,rho^{k4}\,\mu 0 & \\ 0 & 0 & 0 & 0 & 2\,m1^{l} \\ 0 & 0 & 0 & 0 & \end{pmatrix}$$

In[825]:=
```
MatrixForm[Simplify[proj4[ssortzc2[{λ0, λs, μ0, l}, 1, {1, 1, 1}]]]]
```
Out[825]//MatrixForm=

$$\begin{pmatrix} \lambda 0+2\,\mu 0 & \lambda s & \lambda s & 0 & 0 & 0 \\ \lambda s & \lambda 0+2\,\mu 0 & \lambda s & 0 & 0 & 0 \\ \lambda s & \lambda s & \lambda 0+2\,\mu 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\,\mu 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\,\mu 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\,\mu 0 \end{pmatrix}$$

In[826]:=
```
Simplify[orthosticonst[ssortzc2[{λ0, λs0, μ0, l}, rho, {1, 1, 1}]]]
```
Out[826]=

$$\{2\,rho^{k4}\,\mu 0-2\,rho^{k5}\,\mu 0-2\,rho^{k6}\,\mu 0+rho^{k1}\,(\lambda 0+2\,\mu 0),\ -2\,rho^{k4}\,\mu 0+2\,rho^{k5}\,\mu 0-2\,rho^{k6}\,\mu 0+rho^{k2}\,(\lambda 0+2\,\mu 0),\ -2\,rho^{k4}\,\mu 0-2\,rho^{k5}\,\mu 0+2\,rho^{k6}\,\mu 0+rho^{k3}\,(\lambda 0+2\,\mu 0),\ rho^{\frac{k2+k3}{2}}\,\lambda s0,\ rho^{\frac{k1+k3}{2}}\,\lambda s0,\ rho^{\frac{k1+k2}{2}}\,\lambda s0,\ rho^{k4}\,\mu 0,\ rho^{k5}\,\mu 0,\ rho^{k6}\,\mu 0\}$$

Det & Eigenvalues

In[827]:=
```
Simplify[Det[proj4[ssortzc2[{λ0, λs, μ0, l}, rho, {m1, m2, m3}]]]]
```
Out[827]=

$$8\,m1^{4\,l}\,m2^{4\,l}\,m3^{4\,l}\,rho^{k1+k2+k3+k4+k5+k6}\,\mu 0^{3}\,(\lambda 0-\lambda s+2\,\mu 0)^{2}\,(\lambda 0+2\,(\lambda s+\mu 0))$$

In[828]:=
```
Simplify[Eigenvalues[proj4[ssortzc2[{λ0, λs, μ0, l}, 1, {1, 1, 1}]]]]
```
Out[828]=

$$\{2\,\mu 0,\ 2\,\mu 0,\ 2\,\mu 0,\ \lambda 0-\lambda s+2\,\mu 0,\ \lambda 0-\lambda s+2\,\mu 0,\ \lambda 0+2\,(\lambda s+\mu 0)\}$$

Compliance

In[829]:=

```
eeortzc2[{ϵ0_, ν0_, μ0_, l_}, rho_, {m1_, m2_, m3_}] :=
 (1 / ϵ0 / rho^k1 / m1^(2 l) * tcro[mm1, mm1] + 1 / ϵ0 / rho^k2 / m2^(2 l) *
    tcro[mm2, mm2] + 1 / ϵ0 / rho^k3 / m3^(2 l) * tcro[mm3, mm3] -
   ν0 / ϵ0 / rho^((k2 + k3) / 2) / m2^l / m3^l * (tcro[mm2, mm3] + tcro[mm3, mm2]) -
   ν0 / ϵ0 / rho^((k3 + k1) / 2) / m3^l / m1^l * (tcro[mm1, mm3] + tcro[mm3, mm1]) -
   ν0 / ϵ0 / rho^((k1 + k2) / 2) / m1^l / m2^l * (tcro[mm1, mm2] + tcro[mm2, mm1]) +
   1 / 2 / μ0 / rho^k6 / m1^l / m2^l * (tdou[mm1, mm2] + tdou[mm2, mm1]) +
   1 / 2 / μ0 / rho^k5 / m3^l / m1^l * (tdou[mm1, mm3] + tdou[mm3, mm1]) +
   1 / 2 / μ0 / rho^k4 / m2^l / m3^l * (tdou[mm3, mm2] + tdou[mm2, mm3]))
```

In[830]:=

```
MatrixForm[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]]
```

Out[830]//MatrixForm=

$$\begin{pmatrix}
\frac{m1^{-2\,l}\,rho^{-k1}}{\epsilon 0} & -\frac{m1^{-l}\,m2^{-l}\,rho^{\frac{1}{2}(-k1-k2)}\,\nu 0}{\epsilon 0} & -\frac{m1^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k1-k3)}\,\nu 0}{\epsilon 0} & 0 & 0 \\
-\frac{m1^{-l}\,m2^{-l}\,rho^{\frac{1}{2}(-k1-k2)}\,\nu 0}{\epsilon 0} & \frac{m2^{-2\,l}\,rho^{-k2}}{\epsilon 0} & -\frac{m2^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k2-k3)}\,\nu 0}{\epsilon 0} & 0 & 0 \\
-\frac{m1^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k1-k3)}\,\nu 0}{\epsilon 0} & -\frac{m2^{-l}\,m3^{-l}\,rho^{\frac{1}{2}(-k2-k3)}\,\nu 0}{\epsilon 0} & \frac{m3^{-2\,l}\,rho^{-k3}}{\epsilon 0} & 0 & 0 \\
0 & 0 & 0 & \frac{m2^{-l}\,m3^{-l}\,rho^{-k4}}{2\,\mu 0} & 0 \\
0 & 0 & 0 & 0 & \frac{m1^{-l}\,m3^{-l}\,rho^{-k5}}{2\,\mu 0} \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\qquad
\frac{m1^{-l}\,m2}{2}
$$

In[831]:=

```
FullSimplify[
 MatrixForm[Inverse[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]]]]
```

Out[831]//MatrixForm=

$$\begin{pmatrix}
\frac{m1^{2\,l}\,rho^{k1}\,\epsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2} & -\frac{m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & -\frac{m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & 0 & 0 \\
-\frac{m1^{l}\,m2^{l}\,rho^{\frac{k1+k2}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & \frac{m2^{2\,l}\,rho^{k2}\,\epsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2} & -\frac{m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & 0 & 0 \\
-\frac{m1^{l}\,m3^{l}\,rho^{\frac{k1+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & -\frac{m2^{l}\,m3^{l}\,rho^{\frac{k2+k3}{2}}\,\epsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2} & \frac{m3^{2\,l}\,rho^{k3}\,\epsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2} & 0 & 0 \\
0 & 0 & 0 & 2\,m2^{l}\,m3^{l}\,rho^{k4}\,\mu 0 & 0 \\
0 & 0 & 0 & 0 & 2\,m1^{l}\,m3^{l}\,rho^{k5}\,\mu 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}
\qquad
2
$$

In[832]:=

```
orthoelaconst[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]
```

Out[832]=

$$\Big\{m1^{2\,l}\,rho^{k1}\,\epsilon 0,\ m2^{2\,l}\,rho^{k2}\,\epsilon 0,\ m3^{2\,l}\,rho^{k3}\,\epsilon 0,$$
$$m2^{l}\,m3^{-l}\,rho^{k2+\frac{1}{2}(-k2-k3)}\,\nu 0,\ m1^{-l}\,m3^{l}\,rho^{\frac{1}{2}(-k1-k3)+k3}\,\nu 0,$$
$$m1^{l}\,m2^{-l}\,rho^{k1+\frac{1}{2}(-k1-k2)}\,\nu 0,\ m2^{l}\,m3^{l}\,rho^{k4}\,\mu 0,\ m1^{l}\,m3^{l}\,rho^{k5}\,\mu 0,\ m1^{l}\,m2^{l}\,rho^{k6}\,\mu 0\Big\}$$

Det & Eigenvalues

In[833]:=
```
Simplify[Det[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, rho, {m1, m2, m3}]]]]
```

Out[833]=
$$-\frac{m1^{-4\,l}\,m2^{-4\,l}\,m3^{-4\,l}\,rho^{-k1-k2-k3-k4-k5-k6}\,(1+\nu 0)^2\,(-1+2\,\nu 0)}{8\,\epsilon 0^3\,\mu 0^3}$$

In[834]:=
```
Simplify[Eigenvalues[proj4[eeortzc2[{ϵ0, ν0, μ0, l}, 1, {1, 1, 1}]]]]
```

Out[834]=
$$\left\{\frac{1}{2\,\mu 0},\ \frac{1}{2\,\mu 0},\ \frac{1}{2\,\mu 0},\ \frac{1+\nu 0}{\epsilon 0},\ \frac{1+\nu 0}{\epsilon 0},\ \frac{1-2\,\nu 0}{\epsilon 0}\right\}$$

Relationships

In[835]:=
```
FullSimplify[Solve[proj4[ssortzc2[{λ0, λs0, μ0, l}, rho, {m1, m2, m3}]] ==
   Inverse[proj4[eeortzc2[{ε0, ν0, μ0, l}, rho, {m1, m2, m3}]]], {λ0, λs0}]]
```

Out[835]=
$$\left\{\left\{\lambda 0 \to -2\,\mu 0+\frac{\varepsilon 0\,(-1+\nu 0)}{-1+\nu 0+2\,\nu 0^2},\ \lambda s0 \to -\frac{\varepsilon 0\,\nu 0}{-1+\nu 0+2\,\nu 0^2}\right\}\right\}$$

In[836]:=
```
FullSimplify[Solve[proj4[ssortzc2[{λ0, λs0, μ0, l}, rho, {m1, m2, m3}]] ==
   Inverse[proj4[eeortzc2[{ε0, ν0, μ0, l}, rho, {m1, m2, m3}]]], {ε0, ν0}]]
```

Out[836]=
$$\left\{\left\{\varepsilon 0 \to \lambda 0+2\,\mu 0-\frac{2\,\lambda s0^2}{\lambda 0+\lambda s0+2\,\mu 0},\ \nu 0 \to \frac{\lambda s0}{\lambda 0+\lambda s0+2\,\mu 0}\right\}\right\}$$

Fabric-based cortical bone orthotropy (3 ctes)

Fabric-based cortical bone (2 ctes, 4 exponents)

Fabric-based cortical bone (2 ctes, 4 alternative exponents)

Fabric-based transverse isotropy (3 ctes)

Fabric-based transverse isotropy bis (3 ctes)

Fabric-based transverse isotropy bis (5 ctes)

Orthotropy (9 ctes)

Cortical bone

Elasticity, Transverse isotropy, Cai et al., 2019

Elasticity, Mirzaali et al. 2016

ECM

Mineralisation

Trabecula basic transverse isotropic elasticity

Lamella basic elasticity MCF axes

Lamella basic elasticity bone axis

Lamella basic elasticity bone axis B

Lamella basic elasticity bone axis C

Lamella basic elasticity bone axis D

Lamella basic elasticity bone axis E

Lamella basic elasticity bone axis F

Lamella basic elasticity bone axis G (Final for publication)

Reduction factor of indentation modulus after wetting (Wolfram et al., 2010, Bone)

In[1149]:=
```
wetfactor = 0.778;
```

Osteonal versus interstitial bone areal density (Pazzaglia et al., 2013, J Anat)

In[1150]:=
```
ostfrac = 0.482; intfrac = 0.518;
```

Indentation moduli for osteonal and interstitial cortical bone (Lo, 2023, MSc thesis, unpublished)

In[1151]:=
```
ampli = (19 * ostfrac + 22.4 * intfrac) / 19
```
Out[1151]=
```
1.09269
```

Fabric from vertebral cortical shell (Dall'Ara et al., 2013, JMBBM)

In[1152]:=
```
modaxi = (14.6 + 14.91 + 14.16) / 3
```
Out[1152]=
```
14.5567
```

In[1153]:=
```
modcir = (11.63 + 12.45 + 13.07) / 3
```
Out[1153]=
```
12.3833
```

In[1154]:=
```
modrad = (8.21 + 9.21 + 7.59) / 3
```
Out[1154]=
```
8.33667
```

In[1155]:=
```
{msv1, msv2, msv3} = {fsv1, fsv2, fsv3} /.
  NSolve[{fsv2 / fsv3 == (Sqrt[modcir / modaxi]), fsv1 / fsv2 == Sqrt[modrad / modcir],
      fsv1 + fsv2 + fsv3 == 3, fsv1 > 0, fsv2 > 0, fsv3 > 0}, {fsv1, fsv2, fsv3}][[1]]
```
Out[1155]=
```
{0.847416, 1.03281, 1.11978}
```

Isotropic modulus from (Franzoso et al., 2009, J Biomech Eng)

In[1156]:=
```
elamiso = 24.66 * wetfactor * ampli / msv3^2
```
Out[1156]=
```
16.7189
```

Correction with respect to isotropic modulus from F in % (negligible, no

need to update)

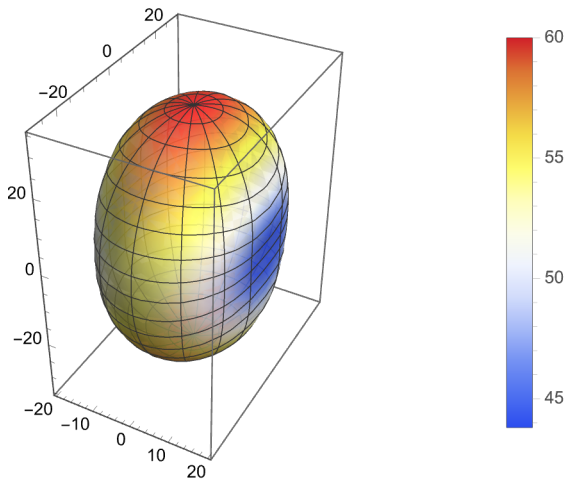
In[1157]:=
```
(1 - elamiso / 16.796513031759368`) * 100
```
Out[1157]=
```
0.461864
```

Fabric-based orthotropic elasticity tensor for cortical bone tissue

In[1158]:=
```
stilam = blow4[Inverse[
    proj4[eeortzc[{elamiso, 0.34, elamiso / 2 / (1 + 0.34), 1}, {msv1, msv2, msv3}]]]]
```
Out[1158]=
```
{{{{18.4795, 0., 0.}, {0., 11.6024, 0.}, {0., 0., 12.5794}},
   {{0., 5.45996, 0.}, {5.45996, 0., 0.}, {0., 0., 0.}},
   {{0., 0., 5.91973}, {0., 0., 0.}, {5.91973, 0., 0.}}},
  {{{0., 5.45996, 0.}, {5.45996, 0., 0.}, {0., 0., 0.}},
   {{11.6024, 0., 0.}, {0., 27.4496, 0.}, {0., 0., 15.3315}},
   {{0., 0., 0.}, {0., 0., 7.2148}, {0., 7.2148, 0.}}},
  {{{0., 0., 5.91973}, {0., 0., 0.}, {5.91973, 0., 0.}},
   {{0., 0., 0.}, {0., 0., 7.2148}, {0., 7.2148, 0.}},
   {{12.5794, 0., 0.}, {0., 15.3315, 0.}, {0., 0., 32.2672}}}}
```

In[1159]:=
```
orthoelaconst[
 eeortzc[{elamiso, 0.34, elamiso / 2 / (1 + 0.34), 1}, {msv1, msv2, msv3}]]
```
Out[1159]=
```
{12.0061, 17.8339, 20.9639, 0.313593, 0.449276, 0.278969, 7.2148, 5.91973, 5.45996}
```

In[1161]:=
```
elaplot[eeortzc[{elamiso, 0.34, elamiso / 2 / (1 + 0.34), 1}, {msv1, msv2, msv3}]]
```
Out[1161]=



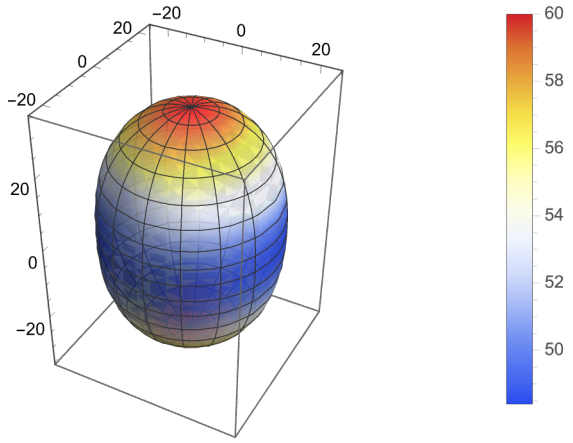
Fabric-based transverse isotropy elasticity tensor for bone tissue used in the homogenisation model

In[1162]:=
```
msv12 = (msv1 + msv2) / 2
```
Out[1162]=
```
0.940112
```

In[1163]:=
```
orthoelaconst[
 eeortzc[{elamiso, 0.34, elamiso / 2 / (1 + 0.34), 1}, {msv12, msv12, msv3}]]
```
Out[1163]=
```
{14.7764, 14.7764, 20.9639, 0.285448, 0.404978, 0.34, 6.56727, 6.56727, 5.51357}
```

In[1164]:=
```
%[[3]] / %[[1]]
```
Out[1164]=
```
1.41874
```

In[1165]:=
```
elaplot[eeortzc[{elamiso, 0.34, elamiso / 2 / (1 + 0.34), 1}, {msv12, msv12, msv3}]]
```
Out[1165]=



Comparison with the experimental elasticity tensor for density=1 (Cai et al. 2019, Acta Biomater)

In[1171]:=
```
MatrixForm[
 {orthoelaconst[eeortzc[{1000 * elamiso, 0.34, 1000 * elamiso / 2 / (1 + 0.34), 1},
    {msv12, msv12, msv3}]], orthoelaconst[blow4[comcai]]}]
```
Out[1171]//MatrixForm=

$$\begin{pmatrix} 14\,776.4 & 14\,776.4 & 20\,963.9 & 0.285448 & 0.404978 & 0.34 & 6567.27 & 6567.27 & 5513.57 \\ 13\,399.2 & 13\,399.2 & 21\,221.2 & 0.245206 & 0.38835 & 0.408439 & 6324.32 & 6324.32 & 4756.76 \end{pmatrix}$$