What is a filter?

N.   Mao 2016 in *Advances in Technical Nonwovens*

raw data

motion artifact.
electric noise.
neural action potentials.

DSP filter.

neural action potentials
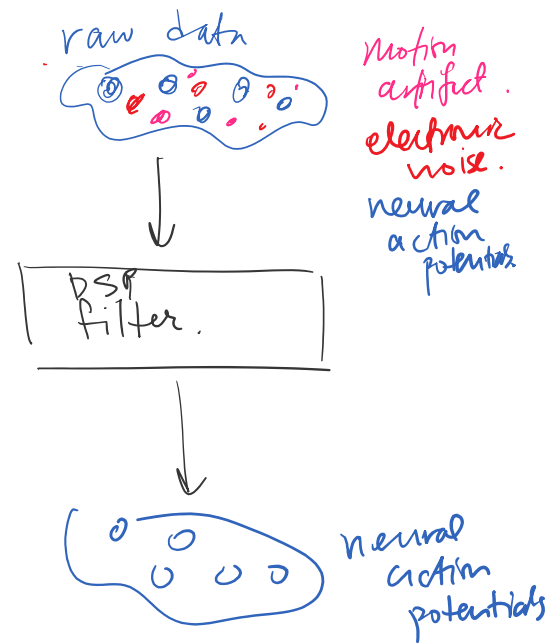
Transfer function review
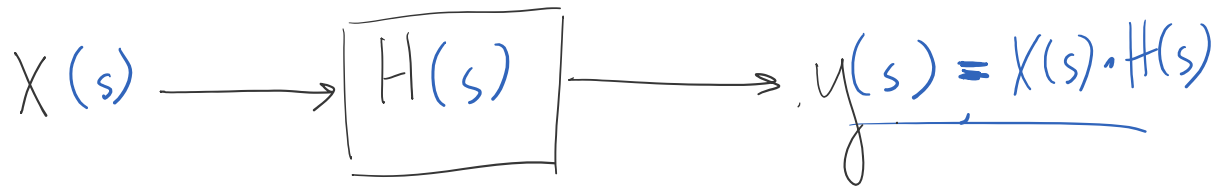
$$X(s) \longrightarrow \boxed{H(s)} \longrightarrow Y(s) = X(s) \cdot H(s)$$

or $\quad H(s) = \dfrac{Y(s)}{X(s)} \qquad$ @ $H(s)$ describes the I/O relationship

Recall: $\quad s = \sigma + j\omega \quad$ frequency

evaluate @ $s = j\omega$

$$x(t) \quad\downarrow \mathcal{F}\{\} \qquad\qquad y(t) \quad\uparrow \mathcal{F}^{-1}\{\}$$

$$X(j\omega) \longrightarrow \boxed{H(j\omega)} \longrightarrow Y(j\omega)$$
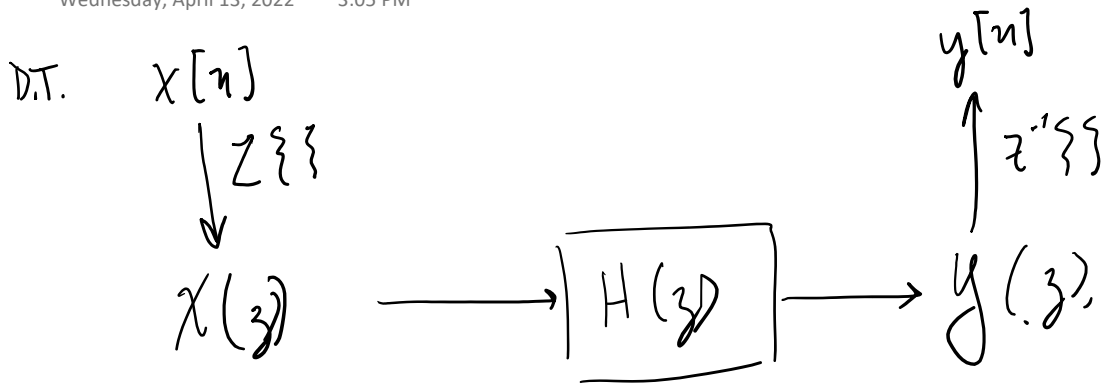
(or $X(f)$)
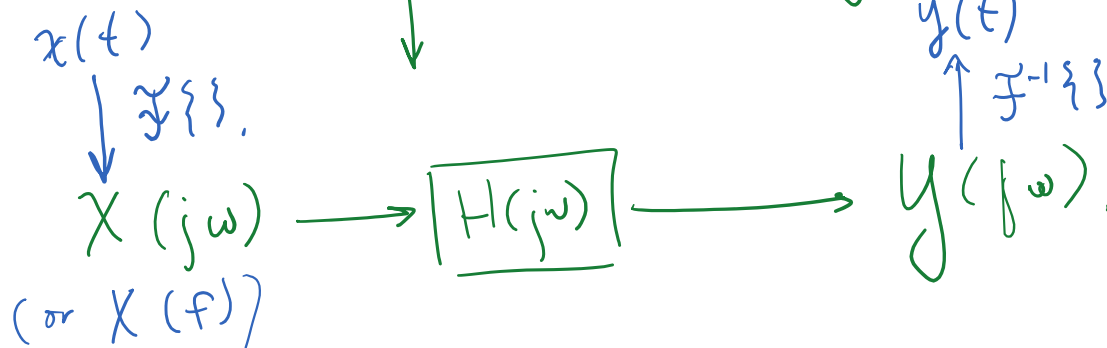
X(f) is the frequency spectrum of x(t)

frequency.

S P E C T R U M.

Discrete-time domain transfer function
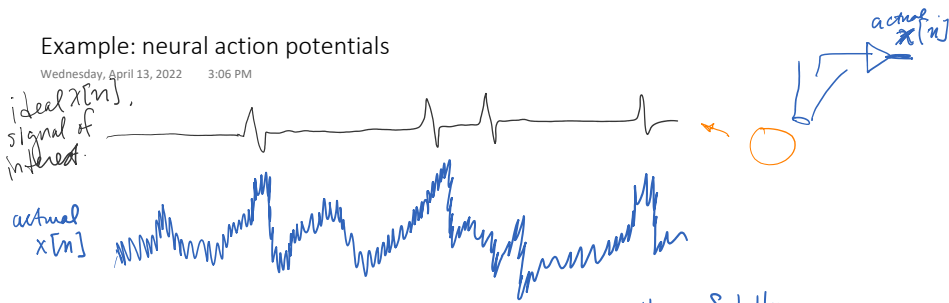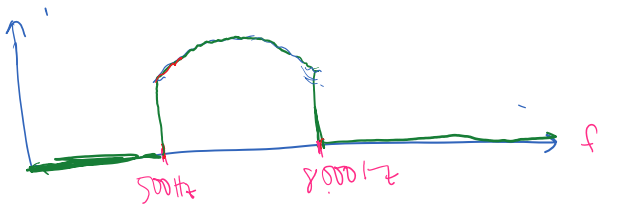
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D.T. $x[n]$

$$\downarrow Z\{\ \}$$

$$X(z) \longrightarrow \boxed{H(z)} \longrightarrow Y(z)$$

$y[n]$

$$\uparrow Z^{-1}\{\ \}$$

z-transform:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n} \quad \xrightarrow{\; z = re^{j\Omega}\big|_{e^{j\Omega}}\;} \quad X(j\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$
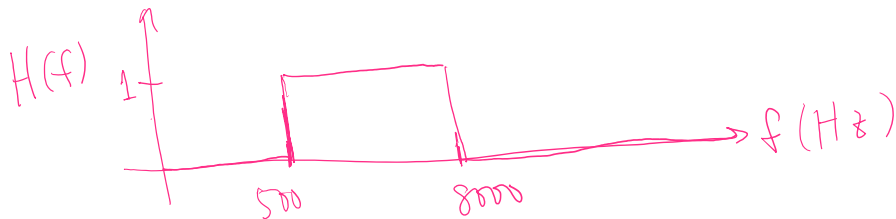
Example: neural action potentials

actual $x[n]$

ideal $x[n]$,
signal of
interest:

actual
$x[n]$

neural action potentials signal bandwidth $\sim 500Hz \sim 8\,kHz$.

$|X(f)|$

f

500Hz   8000Hz

$$x[n] \longrightarrow \boxed{filter} \longrightarrow y[n].$$

Desired
$|y(f)|$

$$X(f) \longrightarrow \boxed{H(f)} \longrightarrow y(f).$$

f

500Hz   8000Hz
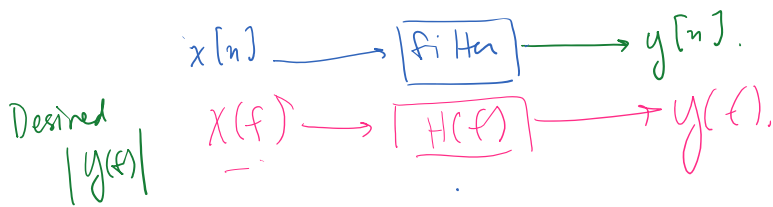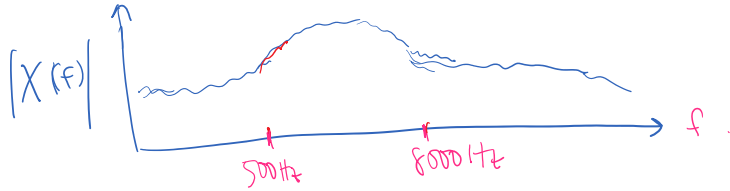
What ideal $H(f)$ will give me this ideal $y(f)$?

$$b/c \quad y(f) = X(f) \cdot H(f).$$

$H(f)$ 1

$f(Hz)$

500   8000

Types of filters - *Ideal*

Band pass filter.

$|H(f)|$ vs f, $f_{c,\ell}$, $f_{c,h}$

Low pass filter.

$|H(f)|$, $f_{c,\ell}$

High pass filter.

$|H(f)|$, $f_{c,h}$
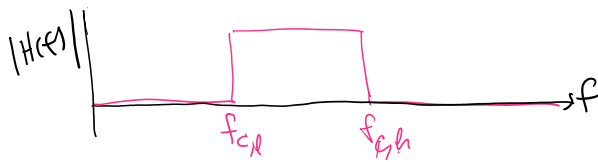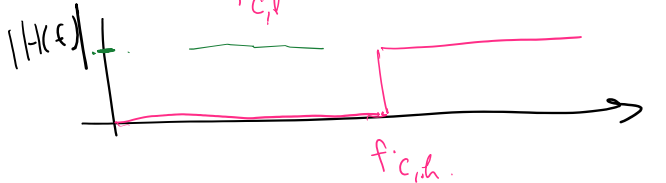
$f_c =$ "cutoff" frequency
a.k.a. corner frequency   a.k.a. 3db frequency.

$G_{PB} =$ pass band gain.

More realistic.

$|H(f)|_{db}$ G_{PB} f
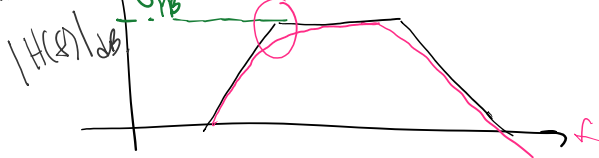
$$|H(f)| = \text{"Gain"} \quad b/c. \quad \frac{|Y(f)|}{|X(f)|} = |H(f)| = \text{Gain},$$

or how much input gets scaled by (at each frequency).

$X(f) =$ frequency spectrum

$|X(f)| =$ magnitude spectrum.

$H(f) =$ frequency response.
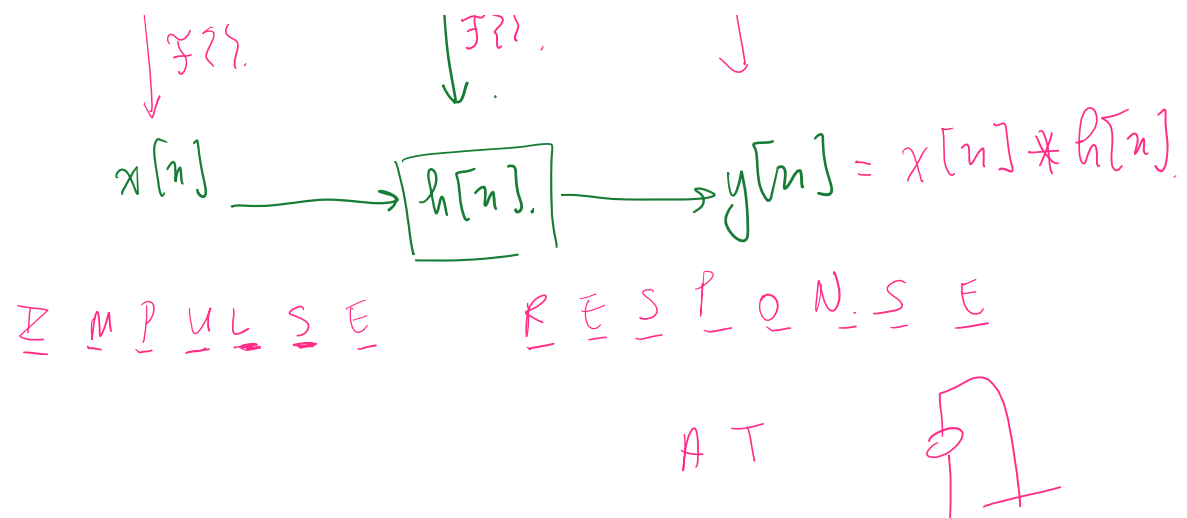
$|H(f)| =$ magnitude response.

$$X(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f).$$

$$Y(f) = X(f) \cdot H(f)$$

$\mathcal{F}\{\}$. $\mathcal{F}\{\}$. $\mathcal{F}\{\}$.

$$\downarrow \mathcal{F}\{\} \qquad \downarrow \mathcal{F}\{\} \qquad \downarrow$$

$$x[n] \longrightarrow \boxed{h[n].} \longrightarrow y[n] = x[n] * h[n]$$

I M P U L S E   R E S P O N S E

A T

FIR filter process

Finite Impulse Response
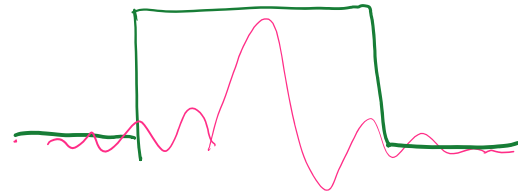
Objective: to find the impulse response of the desired filter frequency response.

→ Design $H_{ideal}(f)$,

→ $H_{ideal}(f) \rightarrow \boxed{\mathcal{F}^{-1}\{\}} \longrightarrow h_{ideal}[n]$

Actually, $h_{ideal} \cdot W[n] = \underline{h[n]}$

→ $y[n] = x[n] \circledast h_{ideal}[n]$.

Example: eeg bandpass filter

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