FIR vs IIR

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FIR = finite impulse response

Vs

IIR = infinite impulse response

$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

$$X(f) \rightarrow \boxed{H(f)} \rightarrow Y(f)$$

$$\downarrow$$

$$X(z) \rightarrow \boxed{H(z)} \rightarrow Y(z)$$

Z-transform Pairs and Properties

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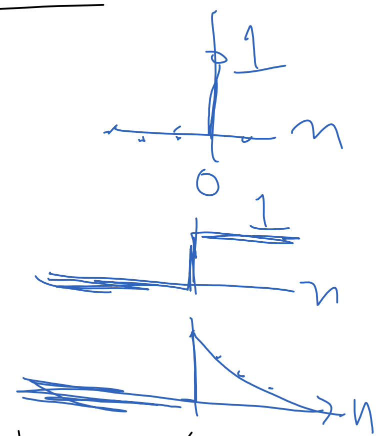
$$x[n] \xrightarrow{\mathcal{Z}\{\}} X(z)$$

$$\delta[n] \longleftrightarrow 1$$

$$u[n] \longleftrightarrow \frac{1}{1-z^{-1}}$$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}}$$



Properties

$$c_1 x_1[n] + c_2 x_2[n] \longleftrightarrow c_1 X_1(z) + c_2 X_2(z)$$

$$x[n-n_0] \longleftrightarrow z^{-n_0} X(z)$$

$$x_1[n] * x_2[n] \longleftrightarrow X_1(z) \cdot X_2(z)$$

Example $x[n] = 0.7^n u[n]$

$$y[n] = x[n-3] \longrightarrow Y(z) = ?$$

$$Y(z) = z^{-3} X(z)$$

$$X(z) = \frac{1}{1-0.7z^{-1}}$$

$$Y(z) = \frac{z^{-3}}{1-0.7z^{-1}}$$

Relationship between transfer function and impulse response

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$$y_I[n] = x_I[n] * \delta[n]$$

$$Y_I(z) = X_I(z) \cdot \underbrace{Z\{\delta[n]\}}_{1}$$

$$Y_I(z) = X_I(z)$$

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$\delta[n] \longrightarrow \boxed{x_I[n]} \longrightarrow \cancel{y}_I[n]$$

h[n] = the response of the system when the input is an impulse function

= i.e. "impulse response"

$$\delta[n] \longrightarrow \boxed{h[n]} \longrightarrow h[n]$$

H(z) is the Z-transform of the impulse response

i.e., H(f) the transfer function of a system is the Fourier transform of the impulse response

$$H(f) = \mathcal{F}\{h[n]\}$$

Basic approach

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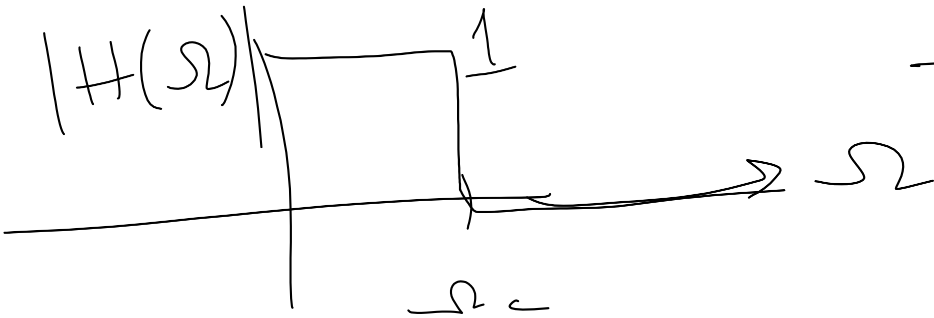

$$\begin{tikzcd} \arrow[r, "\mathcal{Z}^{-1}\{\}", "\mathcal{F}^{-1}\{\}"'] & \boxed{h[n]} \arrow[d] \\ x[n] \arrow[r] & \circledast \arrow[d] \\ & y[n] \end{tikzcd}$$

Limitations of filtering

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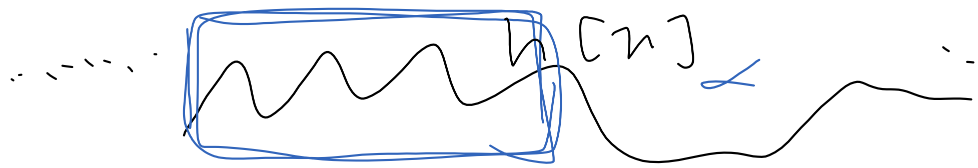

Need to convolve with infinitely long impulse response
But we can only use an impulse response of finite duration = >
Truncate h[n]

Theoretically: $x[n] \circledast h[n] \longrightarrow y[n]$

Reality: $x[n] \circledast \left(h[n] \times w[n]\right) \longrightarrow y[n]$

Choose this window to reduce the same effects we've seen before
That result from windowing (or truncating) our signal.
i..e, use a window that has lower side lobes in the freq domain,

Basic FIR filters

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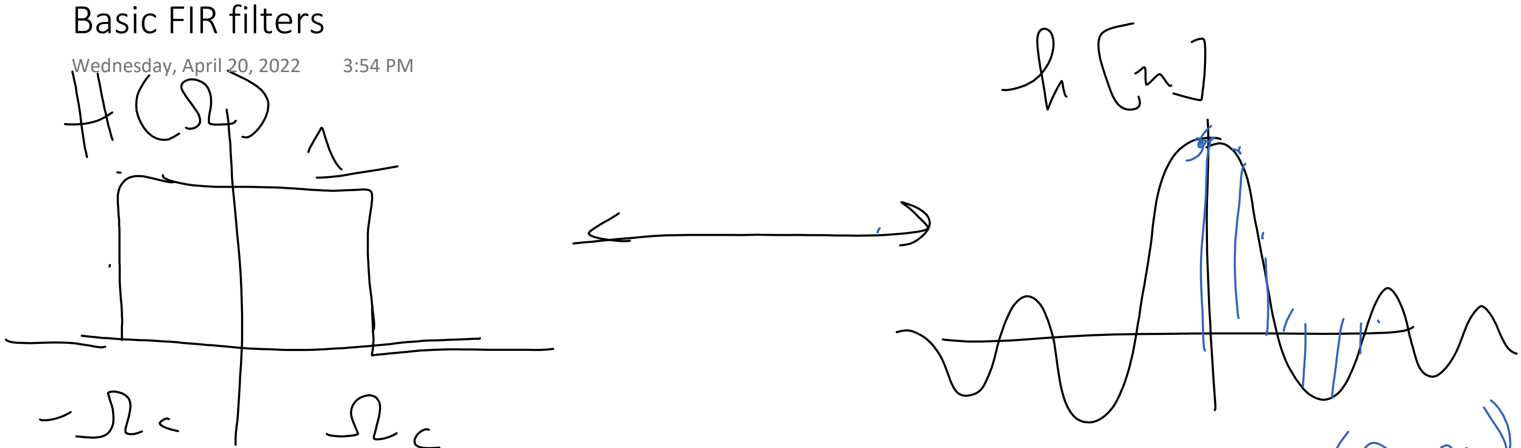
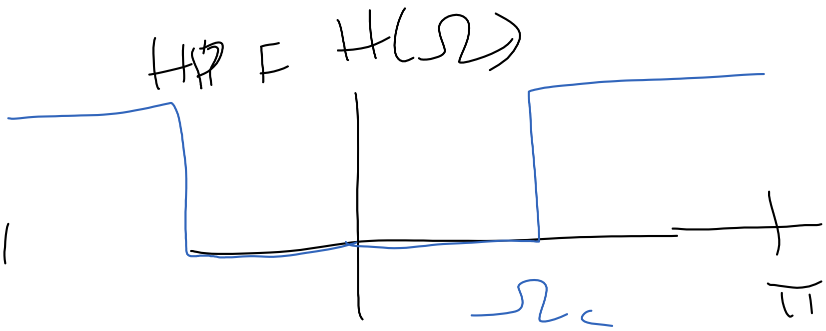


$$H(\Omega) = \begin{cases} 1, & |\Omega| < \Omega_c \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \frac{\sin(\Omega_c n)}{\pi n}$$



$\underline{h[n]}$

$$H_{HP}(\Omega) = f(H_{LP}(\Omega))$$

$$H_{HP}(\Omega) = 1 - H_1$$

$$H(\Omega) = \begin{cases} 1, & |\Omega| > \Omega_c \\ 0, & \text{otherwise} \end{cases}$$