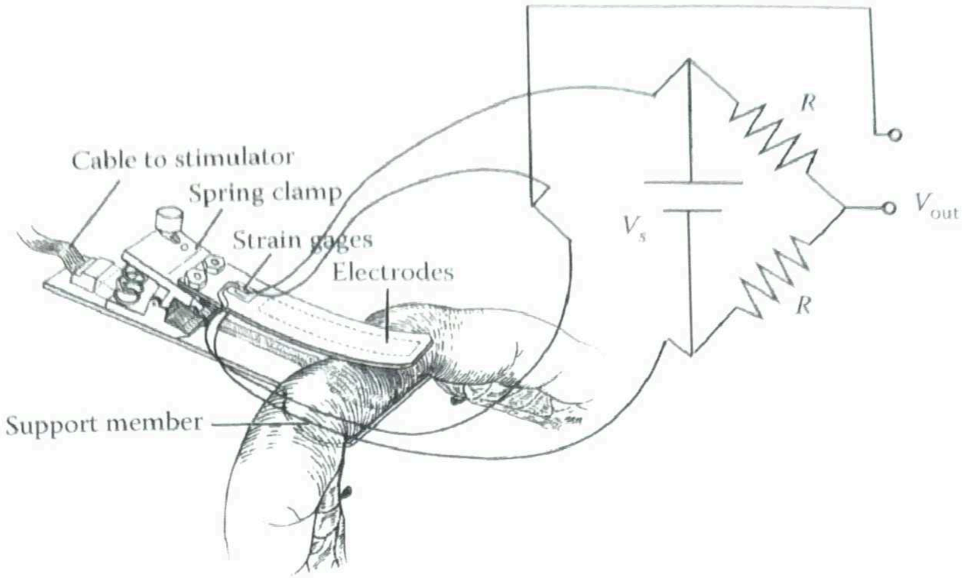

Figure 1.6 A strain gage probe used to measure motility of the intestine. The bridge circuit is used to convert the differential chance in resistance from a pair of strain gages into a change in voltage.

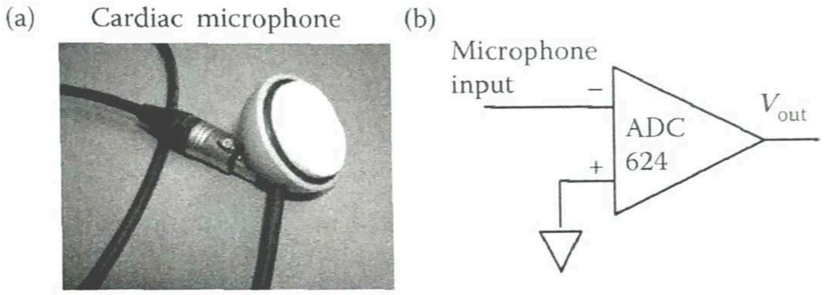

Figure 1.7 (a) A cardiac microphone that uses a piezoelectric element to generate a small voltage when deformed by sound pressure. (b) The detector circuit for this transducer is just a low-noise amplifier.

A bridge circuit detector is used in conjunction with a pair of differentially configured strain gages: when the intestine contracts, the end of the cantilever beam moves downward and the upper strain gage (visible) is stretched and increases in resistance whereas the lower strain gage (not visible) compresses and decreases in resistance. The output of the bridge circuit can be found from simple circuit analysis to be $V_{out} = V_S R/2$, where V_S is the value of the source voltage. If the transducer operates based on a change in inductance or capacitance, the above techniques are still useful except a sinusoidal voltage source must be used.

If the transducer element is a voltage generator, the first stage is usually an amplifier. Figure 1.7a shows a cardiac microphone used to monitor the sounds of the heart that is based on a piezoelectric element. The piezoelectric element generates a small voltage when sound pressure deforms this element; so, the detector stage is just a low-noise amplifier (Figure 1.7b). If the transducer produces a current output, as is the case in many electromagnetic detectors, then a current-to-voltage amplifier (also termed a transconductance amplifier) is used to produce a voltage output. In some circumstances, additional amplification beyond the first stage may be required.

1.5 Analog Signal Processing and Filters

While the most extensive signal processing is performed on digitized data using algorithms implemented in software, some analog signal processing is usually necessary. The most common analog signal processing restricts the frequency range or *bandwidth* of the signal using *analog filters*. It is this filtering that usually sets the bandwidth of the overall measurement system. Since signal bandwidth has a direct impact on the process of converting an analog signal into an equivalent (or nearly equivalent) digital signal, it is often an essential element in any biomedical measurement system. Filters are defined by several properties: filter type, bandwidth, and attenuation characteristics. The latter can be divided into initial and final characteristics. Each of these properties is described and discussed in the next section.

1.5.1 Filter Types

Analog filters are electronic devices that remove selected frequencies. Filters are usually termed according to the range of frequencies they *do not* suppress. Thus, *lowpass* filters allow low frequencies to pass with minimum attenuation while higher frequencies are attenuated. Conversely, *highpass* filters pass high frequencies, but attenuate low frequencies. *Bandpass* filters reject frequencies above and below a *passband* region. An exception to this terminology is *bandstop* filters that pass frequencies on either side of a range of attenuated frequencies.

These filter types can be illustrated by a plot of the filter's *spectrum*, that is, a plot showing how the filter treats the signal at each frequency over the frequency range of interest. Figure 1.8 shows stereotypical frequency spectra or frequency plots of the four different filter types described above. The filter gain is the ratio of output signal amplitude to input signal amplitude as a function of frequency:

$$\text{Gain}(f) = \frac{\text{Output values}(f)}{\text{Input values}(f)} \tag{1.2}$$

The lowpass filter has a filter gain of 1.0 for the lower frequencies (Figure 1.8). This means that the output equals the input at those frequencies. However, as frequency increases, the gain drops, indicating that the output signal also drops for a given input signal. The highpass filter has exactly the opposite behavior with respect to frequency (Figure 1.8). As frequency increases, the gain and output signal increase so that at higher frequency, the gain is 1.0 and the output

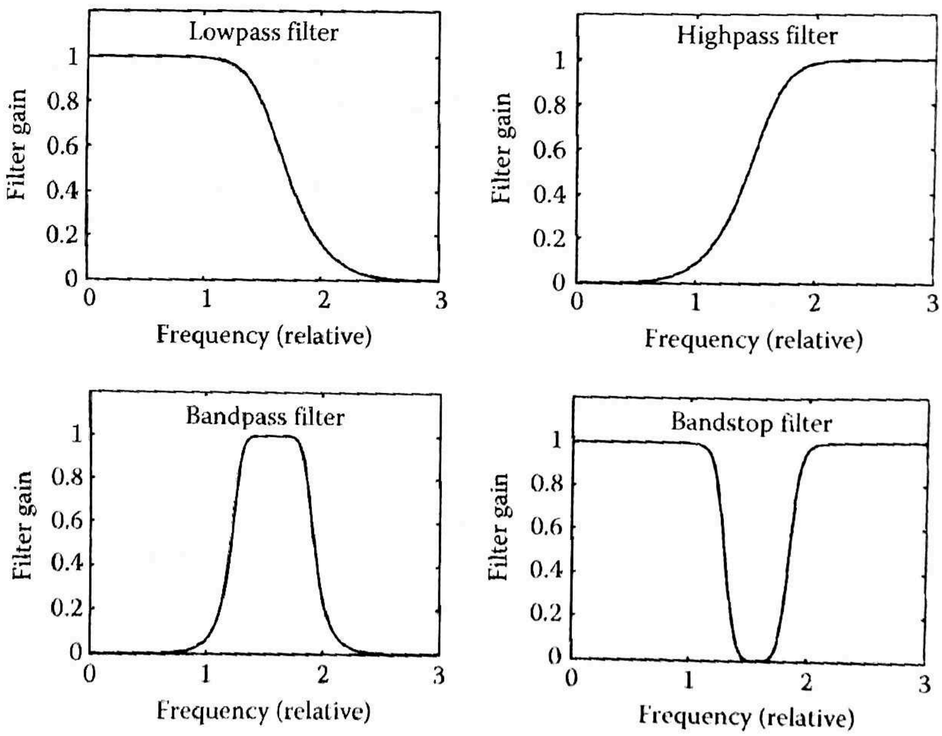

Figure 1.8 Influence on signal frequency of the four basic filter types.

equals the input. The bandpass filter is a combination of these two filters; so, the gain and output increase with frequency up to a certain frequency range where the gain is constant at 1.0, then gain decreases with further increases of frequency (Figure 1.8). The spectrum of the band-stop filter is the inverse of the bandpass filter (Figure 1.8).

Within each class, filters are also defined by the frequency ranges that they pass, termed the filter bandwidth, and the sharpness with which they increase (or decrease) attenuation as frequency varies. (Again, band-stop filters are an exception as their bandwidth is defined by the frequencies they reject.) Spectral sharpness is specified in two ways: as an initial sharpness in the region where attenuation first begins, and as a slope further along the attenuation curve. These various filter properties are best described graphically in the form of a frequency plot (sometimes referred to as a *Bode* plot), a plot of filter gain against frequency. Filter gain is simply the ratio of the output voltage divided by the input voltage, V_{out}/V_{in}, often taken in dB. (The dB operation is defined in Section 2.1.4, but is simply a scaled log operation.) Technically, this ratio should be defined for all frequencies for which it is nonzero, but practically, it is usually stated only for the frequency range of interest. To simplify the shape of the resultant curves, frequency plots sometimes plot gain in dB against the log of frequency.* When the output/input ratio is given analytically as a function of frequency, it is termed the *transfer function*. Hence, the frequency plot of a filter's output/input relationship can be viewed as a graphical representation of its transfer function (Figure 1.8).

1.5.2 Filter Bandwidth

The bandwidth of a filter is defined by the range of frequencies that are not attenuated. These unattenuated frequencies are also referred to as *passband* frequencies. Figure 1.9a shows the frequency plot of an ideal filter, a filter that has a perfectly flat passband region and an infinite attenuation slope. Real filters may indeed be quite flat in the passband region, but will attenuate with a gentler slope, as shown in Figure 1.9b. In the case of an ideal filter (Figure 1.9a), the bandwidth (the region of unattenuated frequencies) is easy to determine: specifically, the bandwidth ranges between 0.0 and the sharp attenuation at f_c Hz. When the attenuation begins gradually, as in Figure 1.9b, defining the passband region is problematic. To specify the bandwidth in this filter, we must identify a frequency that defines the boundary between the attenuated and unattenuated portions of the frequency curve. This boundary has been somewhat arbitrarily defined as the frequency when the attenuation is 3 dB.† In Figure 1.9b, the filter would have a bandwidth of 0.0 to f_c Hz, or simply f_c Hz. The filter whose frequency characteristics are shown in Figure 1.9c has a sharper attenuation characteristic, but still has the same bandwidth (f_c Hz). The bandpass filter whose frequency characteristics are shown in Figure 1.9d has a bandwidth of $f_h - f_l$ in Hz.

1.5.3 Filter Order

The slope of a filter's attenuation curve is related to the complexity of the filter: more complex filters have a steeper slope, approaching the ideal filter as shown in Figure 1.9a. In analog filters, complexity is proportional to the number of energy storage elements in the circuit. These could be either inductors or capacitors, but are generally capacitors for practical reasons. Using standard circuit analysis, it can be shown that each independent energy storage device leads to an additional order of a polynomial in the denominator of the transfer function equation (Equation 1.2) that describes the filter. (The denominator of the transfer function is also

* When gain is plotted in dB, it is in logarithmic form, since the dB operation involves taking the log (see Section 2.1.4). Plotting gain in dB against log frequency puts the two variables in similar metrics and results in more straight-line plots.

† This defining point is not entirely arbitrary because when the signal is attenuated at 3 dB, its amplitude is 0.707 ($10^{-3/20}$) of what it was in the passband region and it has half the power of the unattenuated signal since $0.707^2 = 1/2$. Accordingly, this point is also known as the *half-power point*.

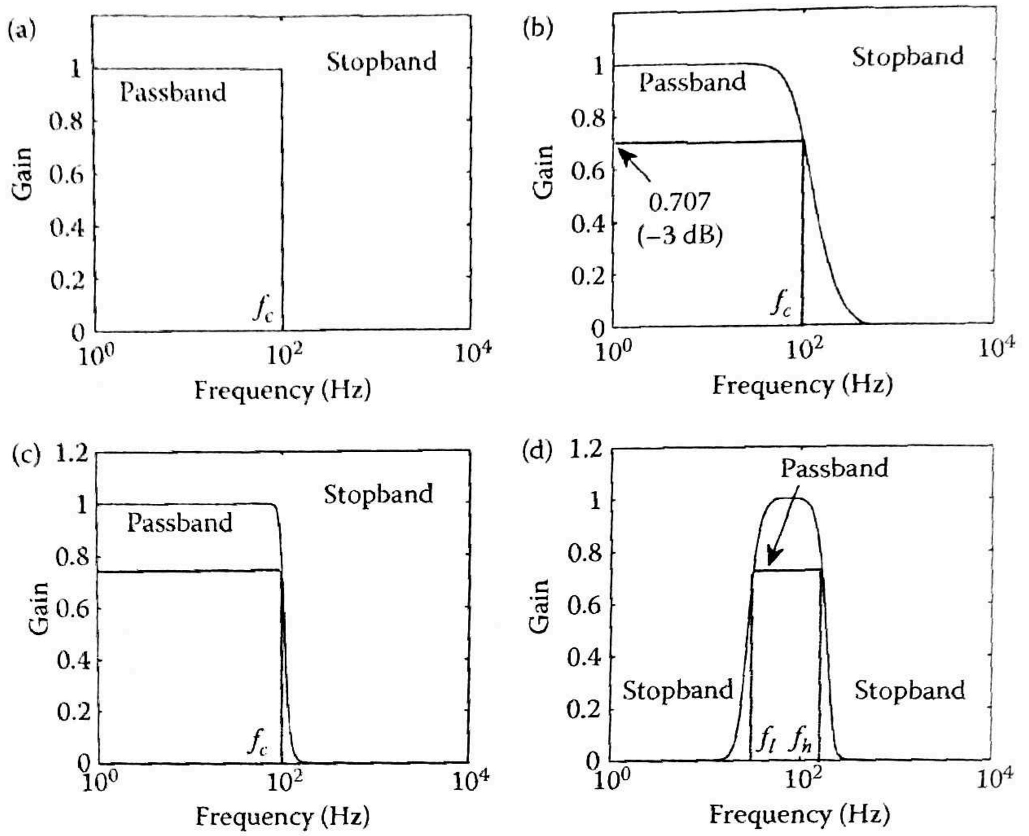

Figure 1.9 Frequency plots of ideal and realistic filters. Each of the frequency plots shown here has a linear vertical axis, but often, the vertical axis is plotted in dB. The horizontal axis is in log frequency. (a) Ideal lowpass filter. (b) Realistic lowpass filter with a gentle attenuation characteristic. (c) Realistic lowpass filter with a sharp attenuation characteristic. (d) Bandpass filter.

referred to as the *characteristic equation* because it defines the basic characteristics of the related system.) As with any polynomial equation, the number of roots of this equation will depend on the order of the equation; hence, filter complexity (i.e., the number of energy storage devices) is equivalent to the number of roots in the denominator of the transfer function. In electrical engineering, it has long been common to call the roots of the denominator equation *poles*. Thus, the complexity of a filter is also equivalent to the number of poles in the transfer function. For example, a *second-order* or *two-pole* filter has a transfer function with a second-order polynomial in the denominator and would contain two independent energy storage elements (very likely two capacitors).

Applying an asymptote analysis to the transfer function, it can be shown that for frequencies much greater than the cutoff frequency, f_c, the slope of most real-world filters is linear if it is plotted on a log-versus-log plot. Figure 1.10 shows the frequency characteristics of the transfer function of a first-order (single-pole) filter with a cutoff frequency, f_c, of 5 Hz plotted in both linear (Figure 1.10a) and dB versus log frequency (Figure 1.10b) format. Converting the vertical axis to dB involves taking the log (see Section 2.1.4); so Figure 1.10b is a log–log plot. At the cutoff frequency of 5 Hz, the frequency characteristic is curved, but at higher frequencies, above 10 Hz, the curve straightens out to become a downward slope that decreases 20 dB for each order of magnitude, or decade, increase in frequency. For example, at 50 Hz, the frequency characteristic has a value of −20 dB and at 500 Hz the value would be ⊠40 dB although this is not shown in the graph. Plotting dB versus log frequency leads to the unusual units for the slope of dB/decade. Nonetheless, this type of plot is often used because it generates straight-line segments for the frequency characteristics of real-world filters and because taking logs extends the range of values presented on both axes. Both linear and dB versus

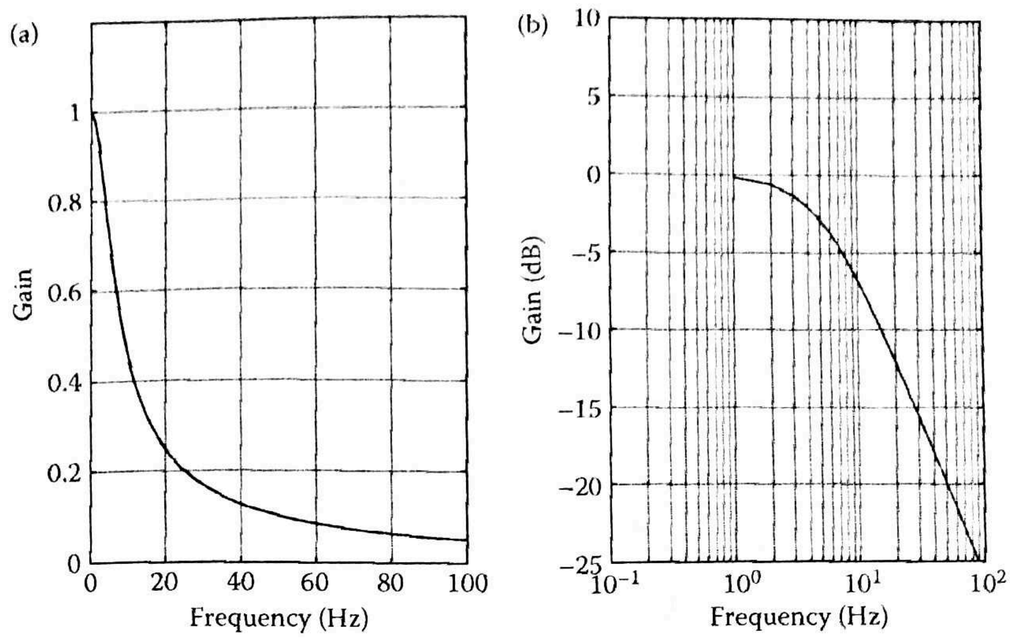

Figure 1.10 Two representations of the gain characteristics (i.e., transfer function) for a first-order filter. (a) A linear plot of gain against frequency. (b) The same curve is plotted with gain in dB, a log function, against log frequency. The attenuation slope above the cutoff frequency becomes a straight line with a slope of 20 dB/decade.

log frequency plotting is used in this book; the axes will describe which type of plot is being presented.

The downward slope of 20 dB/decade seen for the first-order filter shown in Figure 1.10b generalizes, in that for each additional filter pole added (i.e., each increase in filter order), the slope is increased by 20 dB/decade. (In a lowpass filter, the downward slope is sometimes referred to as the filter's *roll-off*.) Figure 1.11 shows the frequency plot of a second-order, two-pole filter

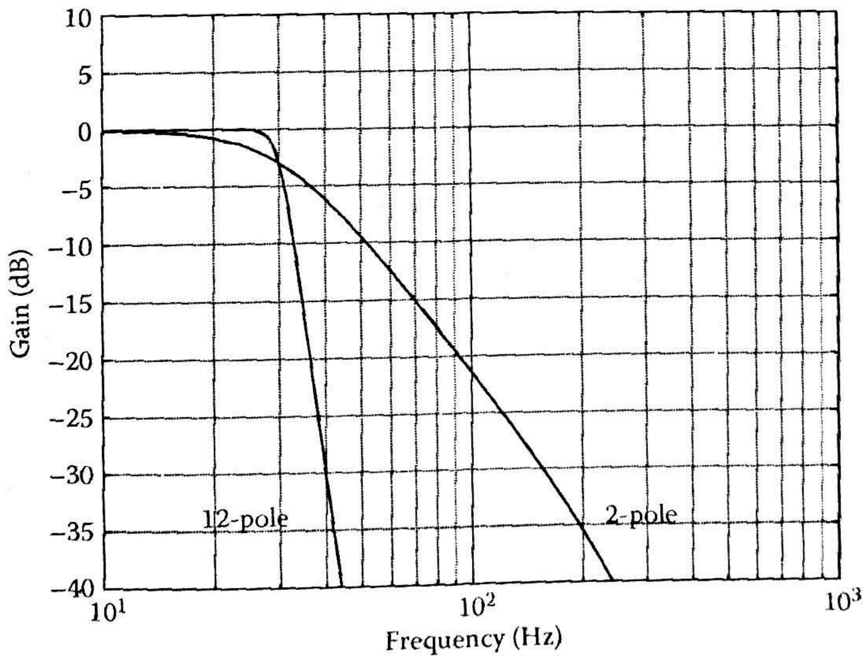

Figure 1.11 A dB versus log frequency plot of a second-order (two-pole) and a 12th-order (12-pole) lowpass filter having the same cutoff frequency. The higher-order filter more closely approaches the sharpness of an ideal filter.

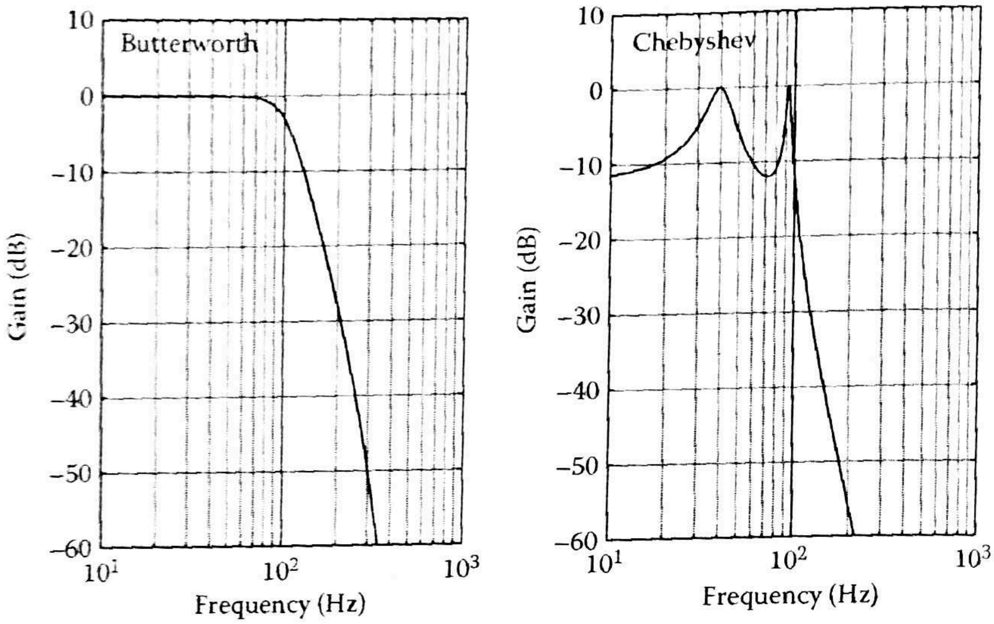

Figure 1.12 Two filters that have the same cutoff frequency (100 Hz) and the same order (four-pole), but differing in the sharpness of the initial slope. The filter labeled Chebyshev has a steeper initial slope, but contains ripples in the passband region.

with a slope of 40 dB/decade and a 12th-order lowpass filter. Both filters have the same cutoff frequency, f_c, hence the same bandwidth. The steeper slope or roll-off of the 12-pole filter is apparent. In principle, a 12-pole lowpass filter would have a slope of 240 dB/decade (12 × 20 dB/decade). In fact, this frequency characteristic is theoretical because in real analog filters, parasitic components and inaccuracies in the circuit elements limit the actual attenuation that can be obtained. The same rationale applies to highpass filters, except that the frequency plot decreases with decreasing frequency at a rate of 20 dB/decade for each highpass filter pole.

1.5.4 Filter Initial Sharpness

As shown above, both the slope and the initial sharpness increase with filter order (number of poles), but increasing filter order also increases the complexity and hence the cost of the filter. It is possible to increase the initial sharpness of the filter's attenuation characteristics without increasing the order of the filter, if you are willing to accept some unevenness or *ripple* in the passband. Figure 1.12 shows two lowpass, fourth-order filters having the same cutoff frequency, but differing in the initial sharpness of the attenuation. The one-marked Butterworth has a smooth passband, but the initial attenuation is not as sharp as the one marked Chebyshev, which has a passband that contains ripples. This property of analog filters is also seen in digital filters and is discussed in detail in Chapter 4.

EXAMPLE 1.1

An ECG signal of 1 V peak to peak has a bandwidth from 0.01 to 100 Hz. (Note that this frequency range has been established by an official standard and is meant to be conservative.) It is desired to reduce any noise in the signal by at least 80 dB for frequencies above 1000 Hz. What is the order of analog filter required to achieve this goal?

Solution

Since the signal bandwidth must be at least 100 Hz, the filter's cutoff frequency, f_c, must be not less than 100 Hz, but the filter must reduce the signal by 80 dB within 1 decade. Since typical