Announcements

Wednesday, March 16, 2022 3:13 PM

PS5 due Wed. 3/23

- STFT and Spectrogram earlier on syllabus
- Exam 2 during Week 15
- Project presentations during final exam period

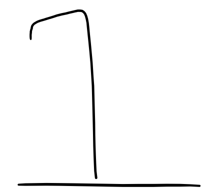
Power spectrum - definitions

Monday, March 14, 2022 10:12 AM

PARSEVAL'S THEOREM

$$\int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-\infty}^{\infty} |X(f)|^2\,df$$

Avg. power $$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\,dt = \frac{1}{T}\int_{-\infty}^{\infty}\boxed{|X(f)|^2}\,df$$



power spectral density. [Watts/Hz]

"P.S.D."

DFT - approximation

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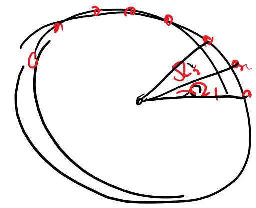
$$x[n] \longrightarrow \boxed{\mathcal{F}\{\ \}} \longrightarrow \underset{\text{DTFT.}}{X(f)} \longrightarrow \boxed{\text{Computation}} \longrightarrow \underset{\text{DFT.}}{\tilde{X}(f)}$$

1. While f is continuous, we can only compute the DFT at discrete frequencies.

$\Rightarrow \uparrow N_{fft} \Rightarrow \uparrow$ resolution in freq.



2. $$X(f) = \sum_{n=0}^{\infty} x[n]\, e^{j2\pi f n T_s}$$

$$\tilde{X}(f) = \sum_{n=0}^{L-1} x[n]\, e^{j2\pi f n T_s}.$$

While the true DTFT sums an infinite number of points, when we compute the DFT, we only have a finite number of time points

Frequency resolution

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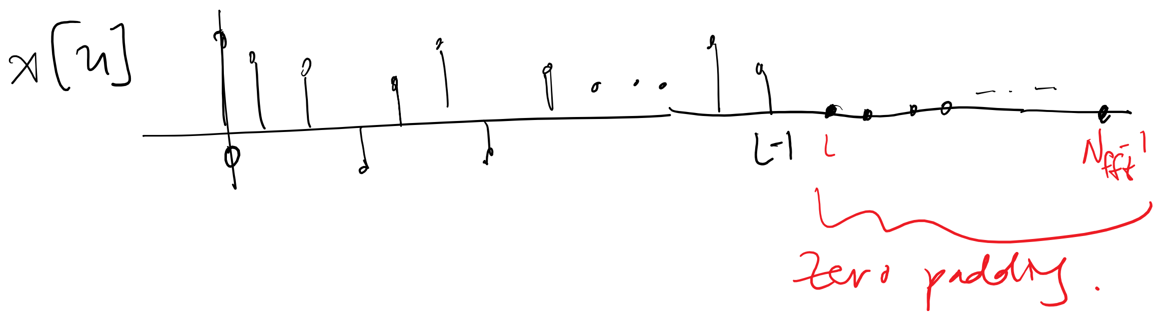
Zero-padding demo

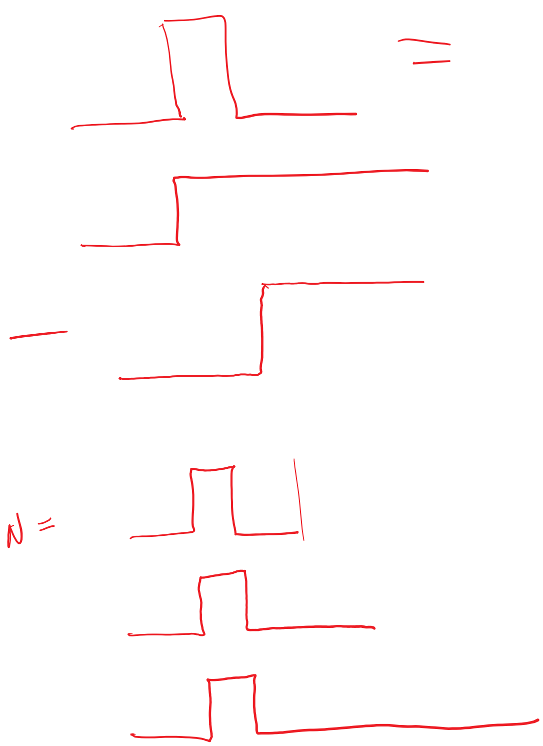
$$X(f) = \sum_{n=0}^{\infty} x[n] e^{j2\pi f n T_s}$$

$$\tilde{X}(f) = \sum_{n=0}^{L-1} x[n] e^{j2\pi f n T_s}.$$

zero padding:



Example



Windowing effect

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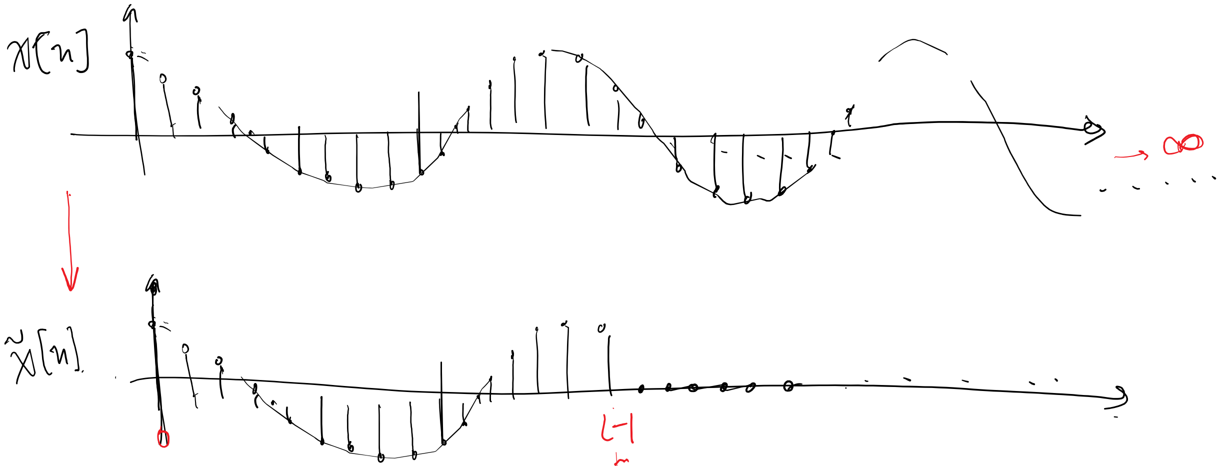
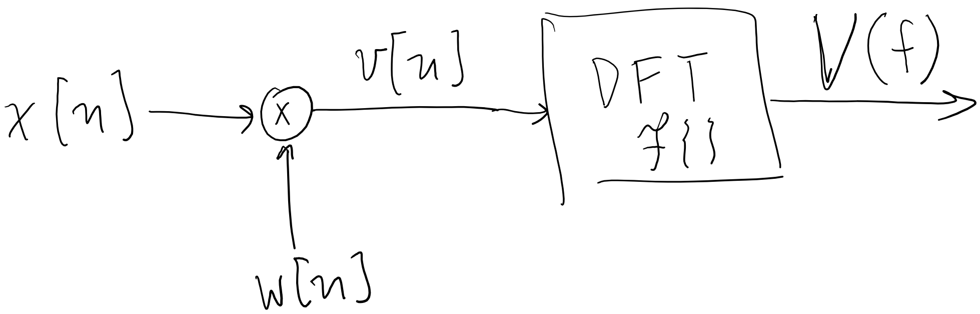


$\tilde{x}[n] = x[n] \times w[n]$



$$V(f) = X(f) * W(f) \longleftrightarrow v[n] = x[n] \cdot w[n]$$

Review of convolution

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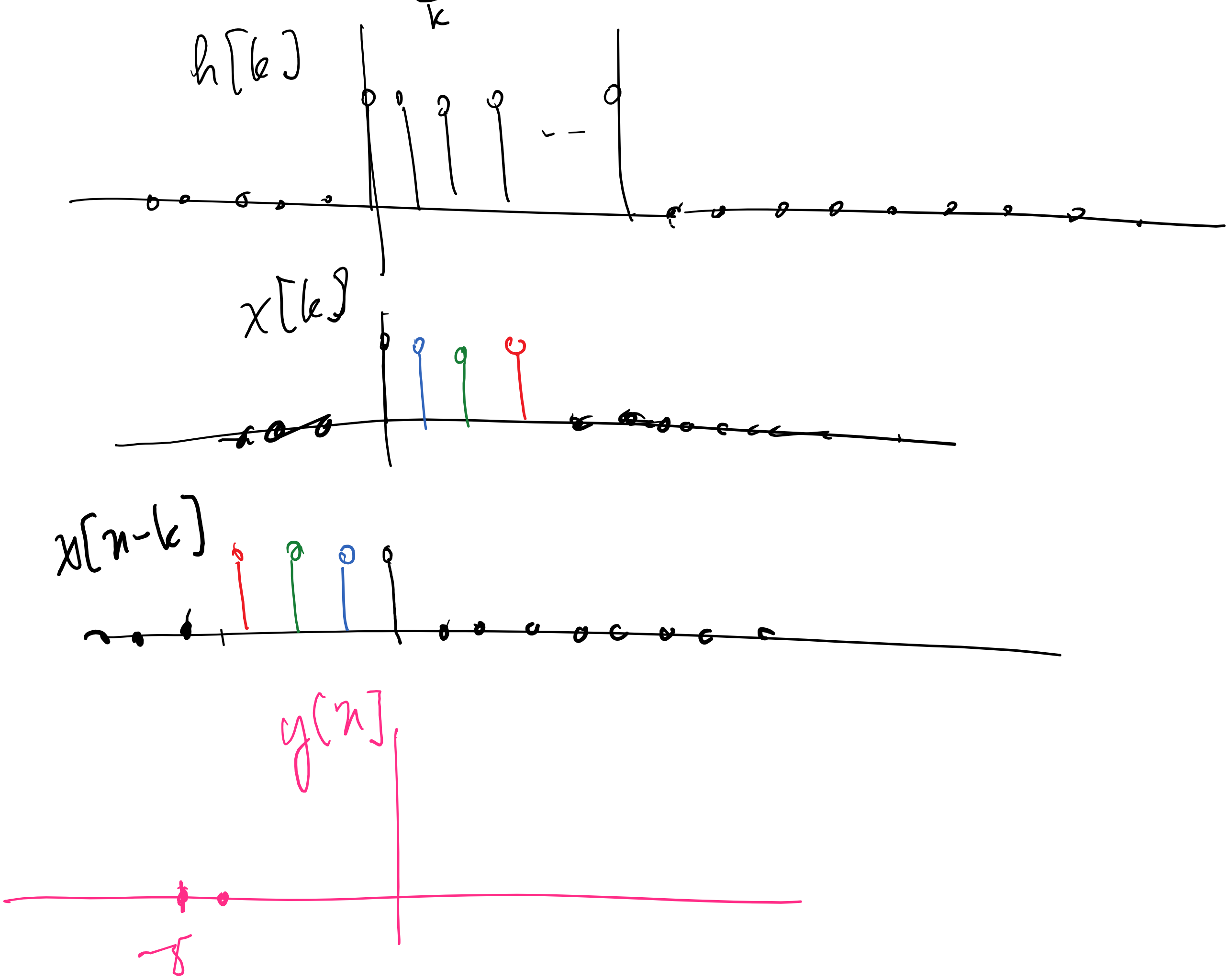
$$y[n] = x[n] * h[n]$$

$$= \sum_{k=0}^{\infty} x[k]\, h[n-k]$$

$$= \sum_{k} h[k]\, x[n-k]$$



Windowing effect

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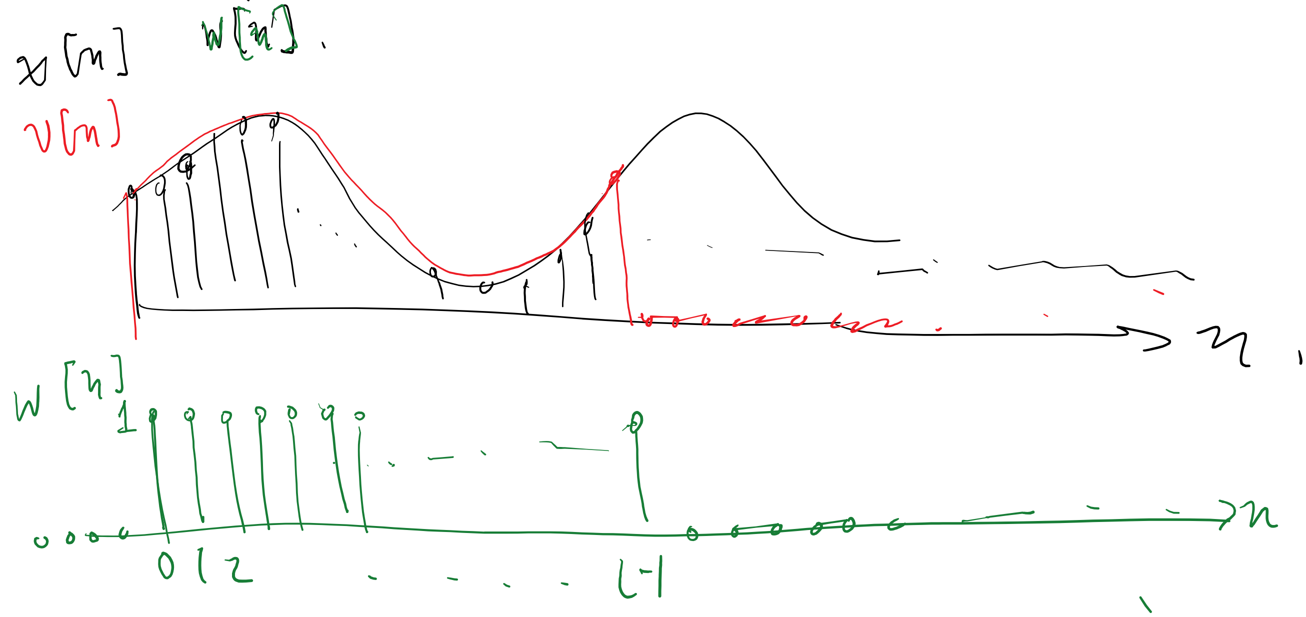
$$x[n] \longrightarrow \otimes \xrightarrow{v[n]} \boxed{\mathcal{F}\{\ \}} \xrightarrow{\tilde{V}(f)} \boxed{|\cdot|^2} \longrightarrow P_v(f)$$

$$w[n] \uparrow$$



$$v[n] = x[n]\cdot w[n]$$

$$\updownarrow \mathcal{F}$$

$$V(f) = X(f) * \underline{W(f)}$$

Window types

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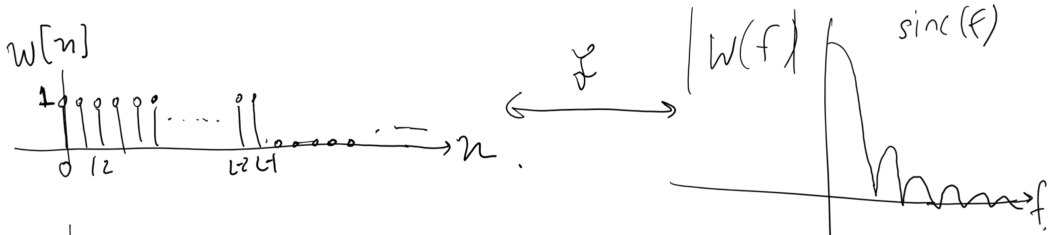
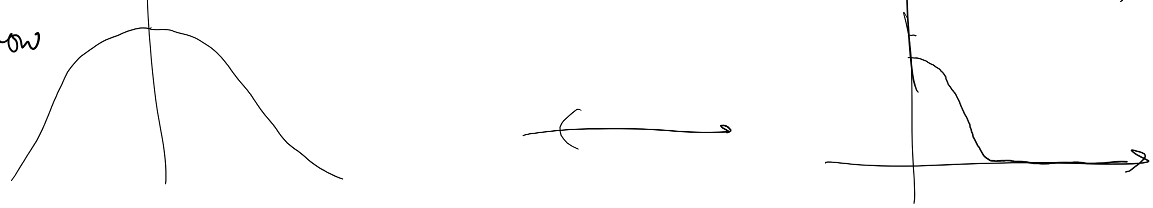
Rectangular window =



Hamming window



Bartlett

There's a tradeoff between the width of the main lobe and the amplitude of the side lobes (slightly more spectral smearing but less spectral leakage)

Hanning .

Effect of convolving W(f) with X(f)

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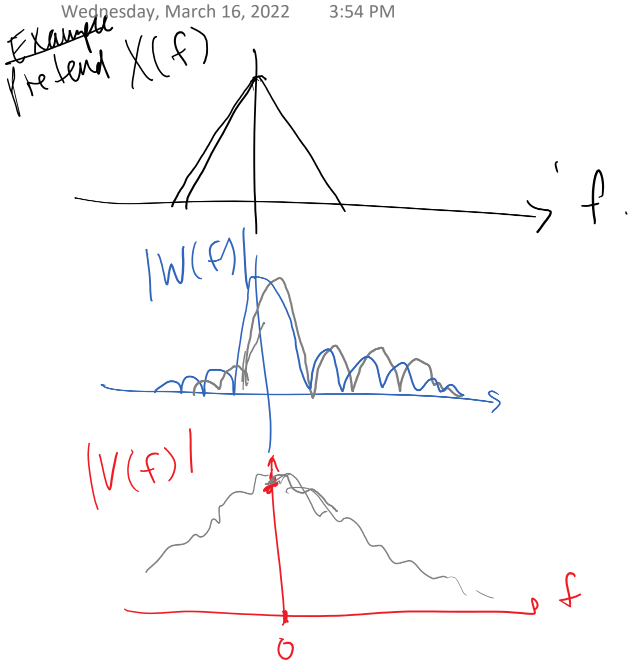

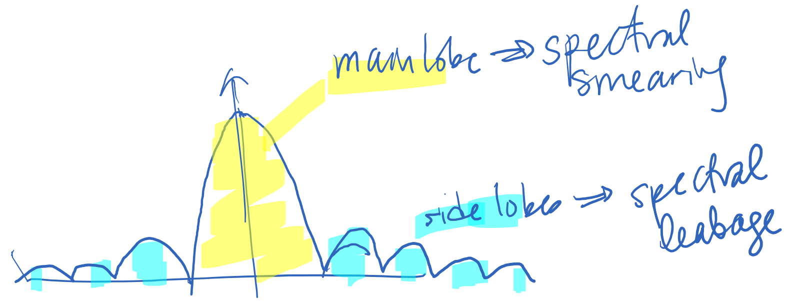

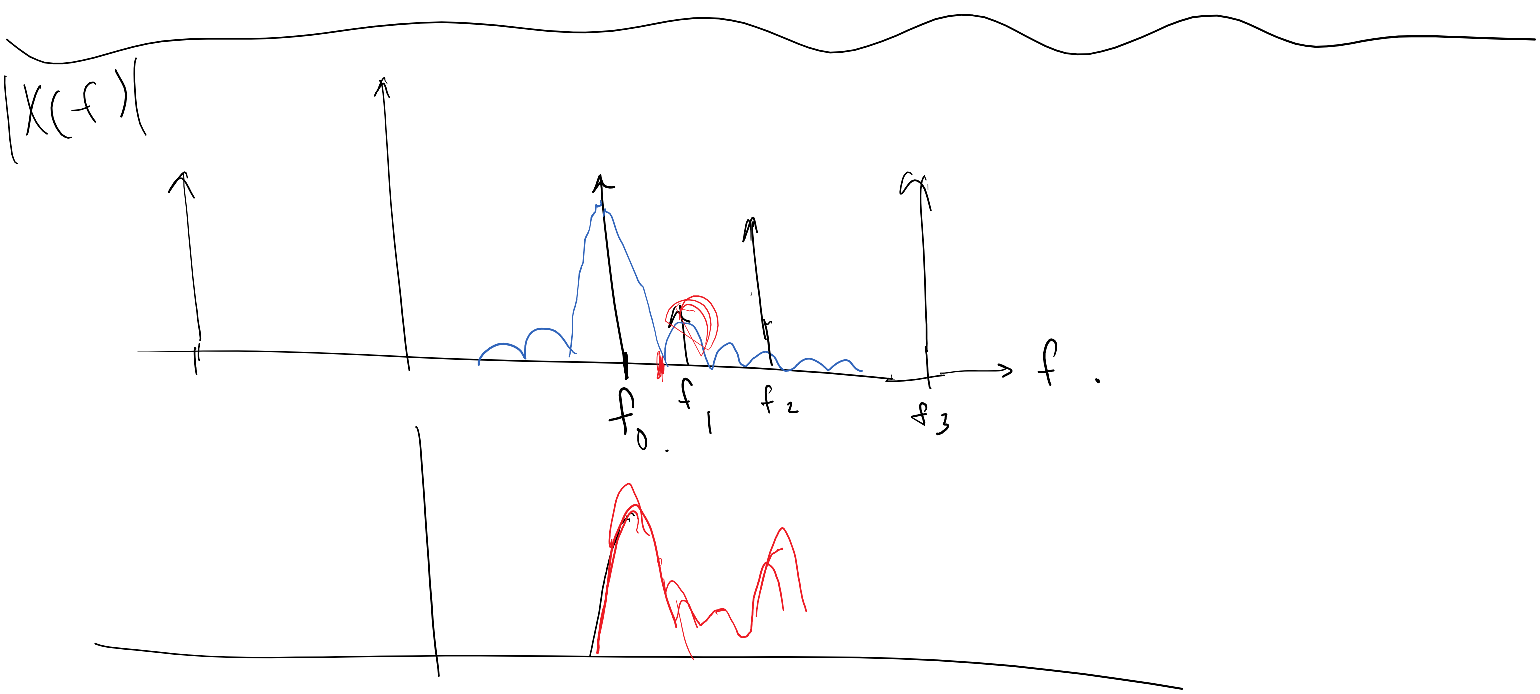

Variability in estimate

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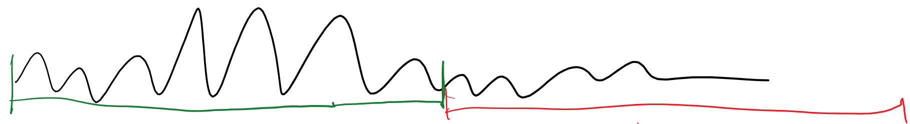
stochastic process $\xrightarrow{x(t)}$ is from a random process.

$\therefore$ $|X(f)|$ depends on when I record my signal

$x[n]$



calculate PSD $\rightarrow$ $P_{x_A}(f)$.

calculate PSD $\rightarrow$ $P_{x_B}(f)$ . . . $P_{xx}(f)$

average

$\downarrow$

averaged periodogram estimate.

Estimate variability, averaging across windows

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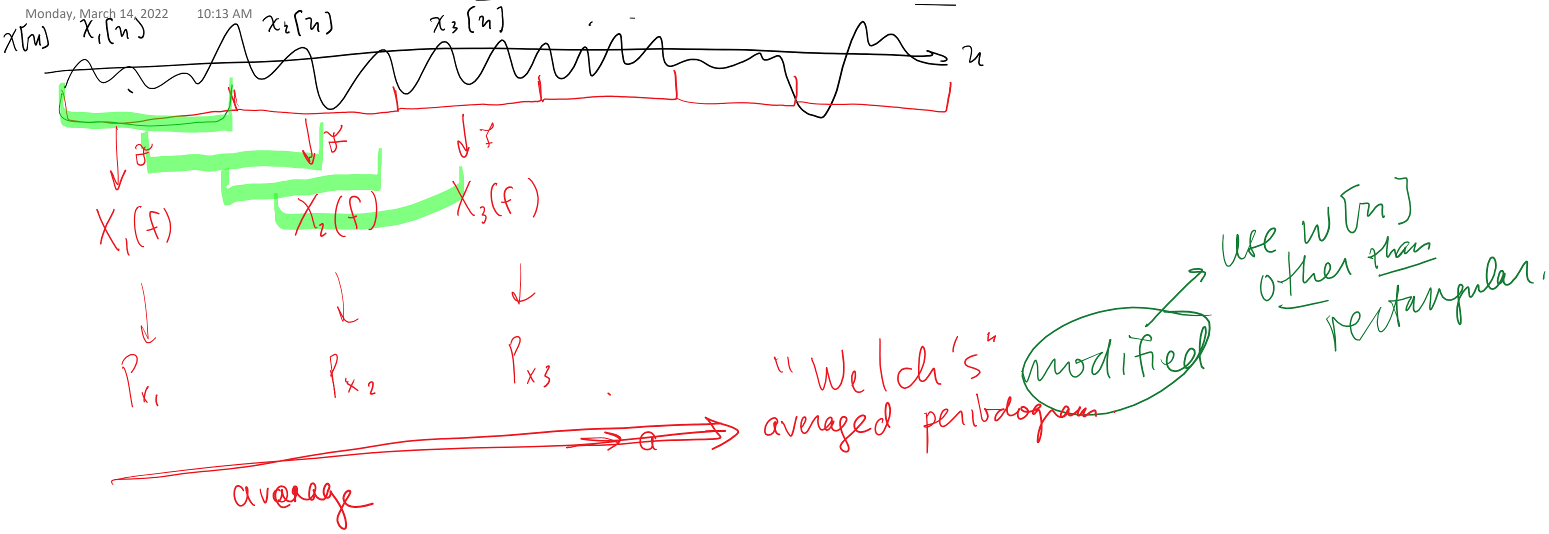

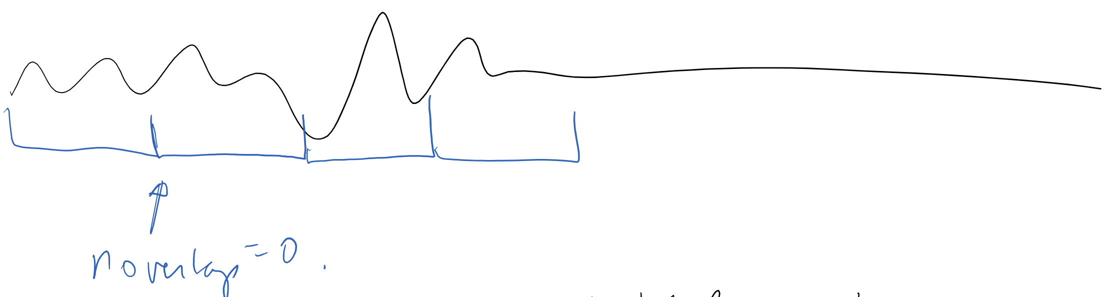

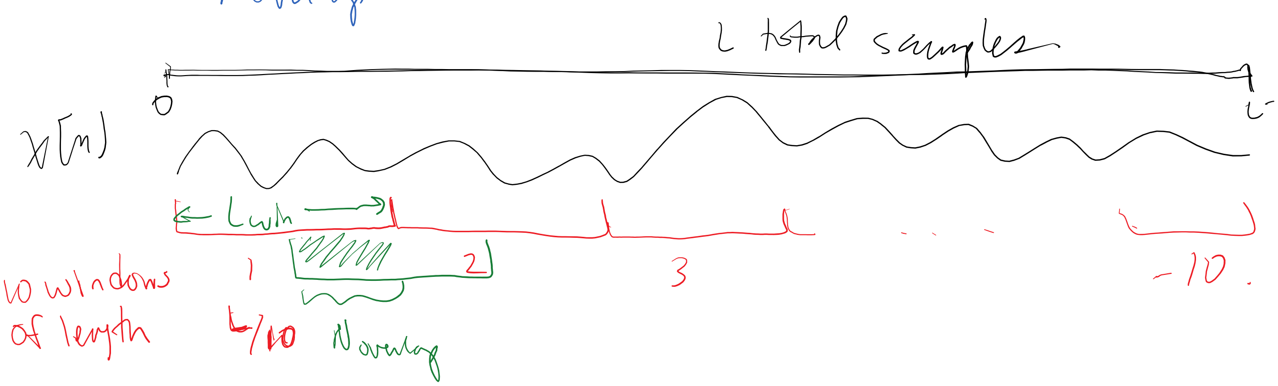

Periodogram

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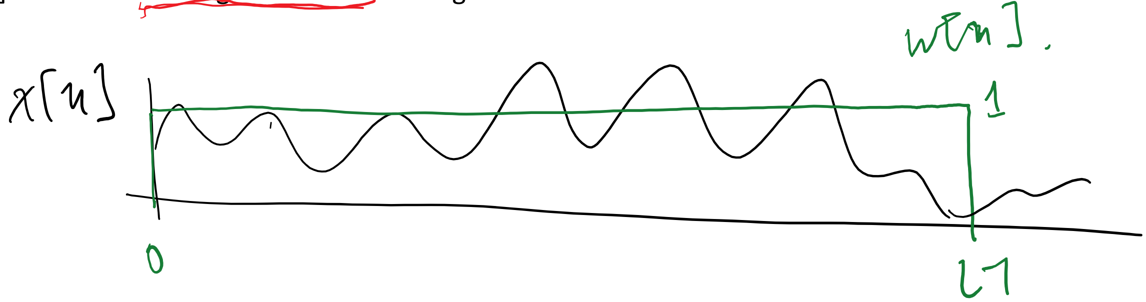
w[n] = Just one rectangular window of length L



$$\Big\downarrow \mathcal{F}$$

$$|X(f)|^2 = \hat{P}_x$$

Modified PSD estimate

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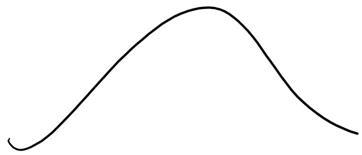
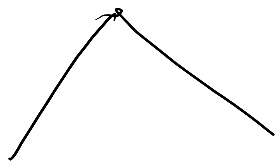
$\rightarrow w[n] =$ not rectangular.

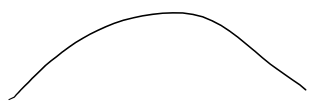
eg



Hamming



Triangular.



Bartlett.

Welch's modified periodogram

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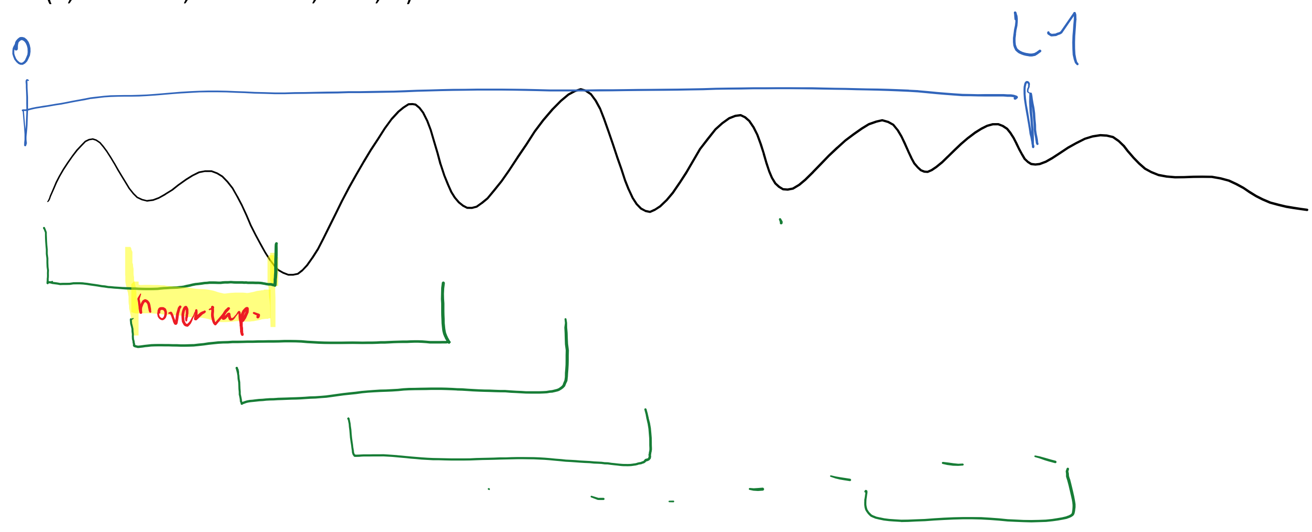
[Pxx,F] = pwelch(X,WINDOW,NOVERLAP,NFFT,Fs)



N windows, each of length $\frac{L}{N}$

In class example

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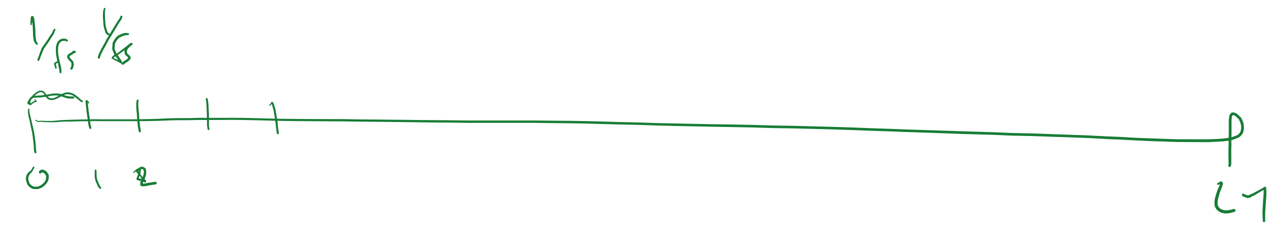
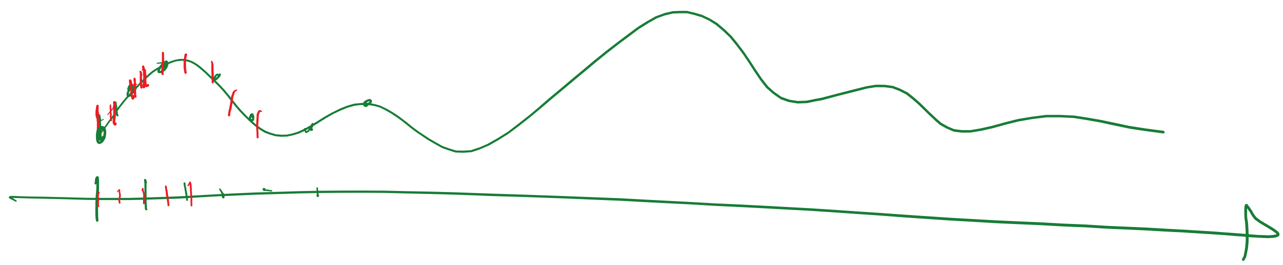
Determine and plot the power spectra of heart rate variability during both normal and meditative sets.





Sleep EEG

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Brain rhythms

Stage	< 4Hz.	4-7 Hz.	7-13 Hz
0	some		Strong.
1		strong.	
2		strong	
3	strong		peak.
4	strong		
5	diffuse	diffuse	diffuse

To get the data from the epoch structure:

dat16 = Epoch(16).Data(:,4);

Would give you 4th channel from epoch 16