Non-deterministic signals

Monday, November 23, 2020 9:25 AM

Deterministic vs. stochastic

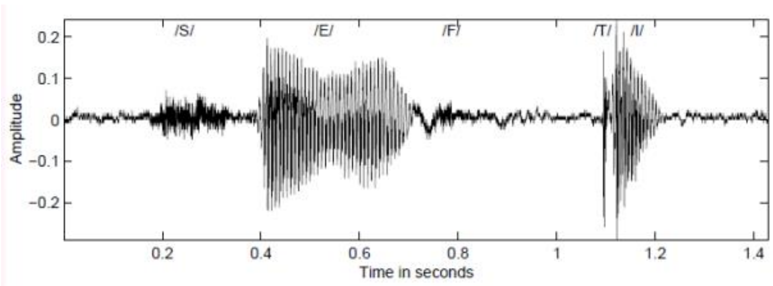
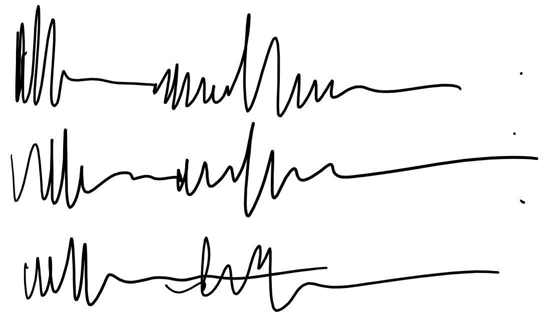


Figure 3.1: Top: Speech signal of the word "safety" uttered by a male speaker.
Rangayyan, 2nd Ed.



- Random noise
- What would the signal look like on the next trial? The next 100 trials?

stimuli → $\boxed{\text{stochastic process}}$ → response

EMI

Sunday, January 3, 2021 1:42 AM

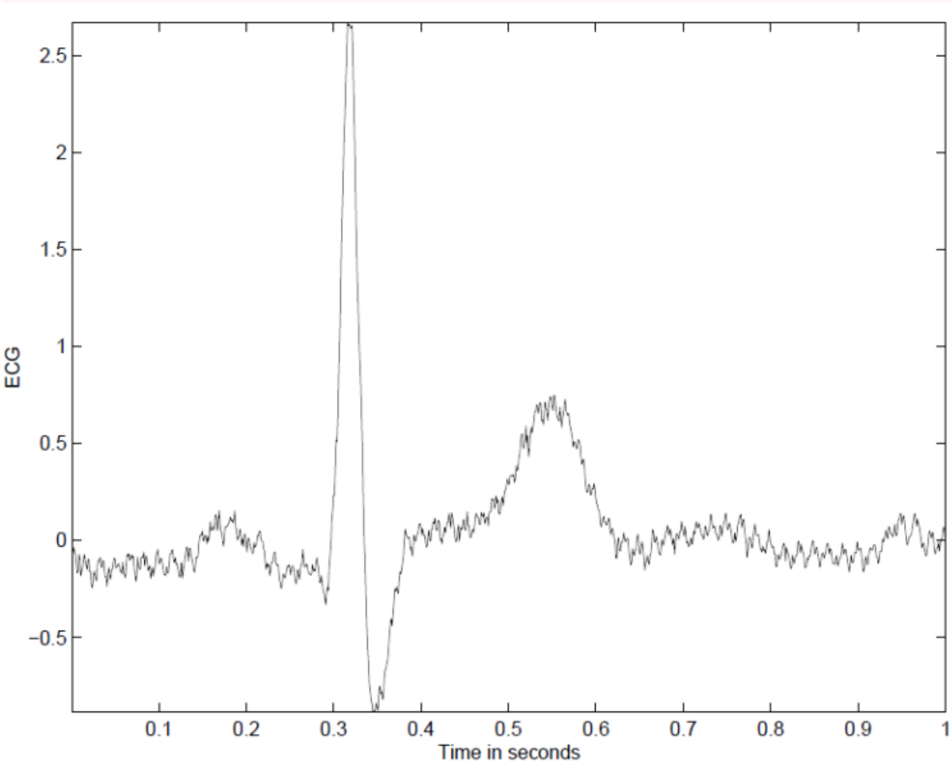
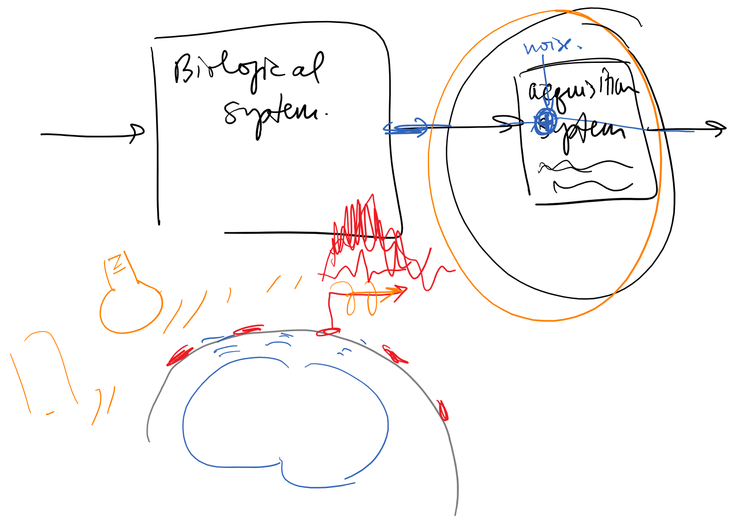


Figure 3.7: ECG signal with power-line (60 Hz) interference.

Noise sources

Monday, February 1, 2021 1:43 PM

Intrinsic noise
- Shot noise
- Thermal noise
- 1/f noise (pink)



Extrinsic noise
- Motion artifact
- Muscle artifact
- EMI = electromagnetic interference

Artifacts

Sunday, January 3, 2021 1:35 AM

Fetal ECG artifact

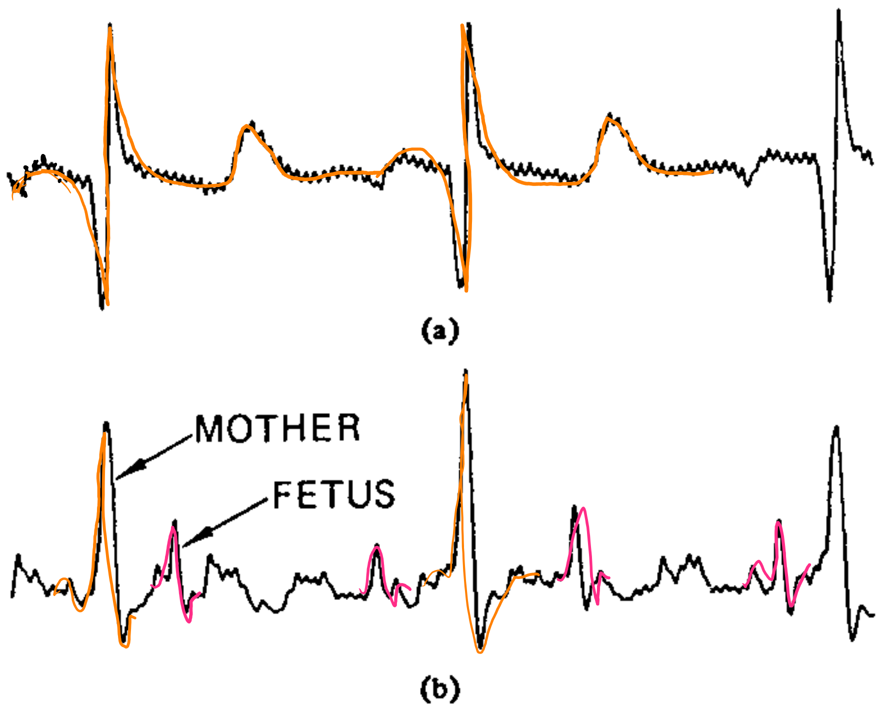
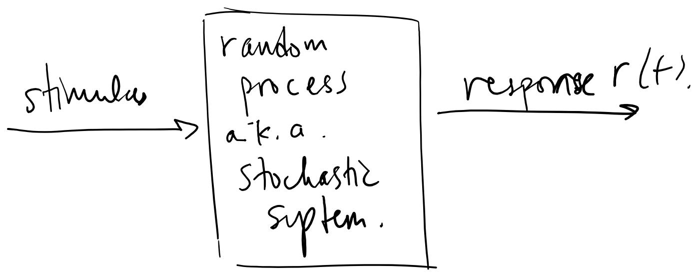


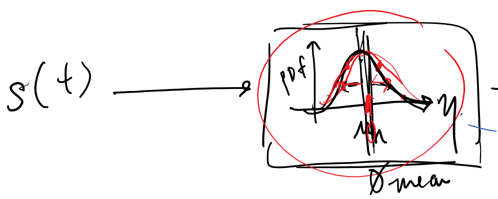
Figure 3.9: ECG signals of a pregnant woman from abdominal and chest leads: (a) chest-lead ECG, and (b) abdominal-lead ECG; the former presents the maternal ECG whereas the latter is a combination of the maternal and fetal ECG signals. (See also Figure 3.101.) Reproduced with permission from B. Widrow, J.R. Glover, Jr., J.M. McCool, J. Kaunitz, C.S. Williams, R.H. Hearn, J.R. Zeidler, E. Dong, Jr., R.C. Goodlin, Adaptive noise cancelling: Principles and applications, Proceedings of the IEEE, 63(12):1692–1716, 1975. c
IEEE

Random processes

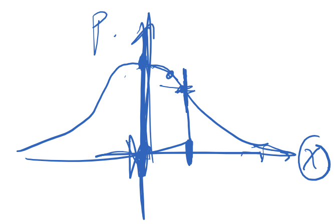
Wednesday, February 3, 2021 3:09 PM

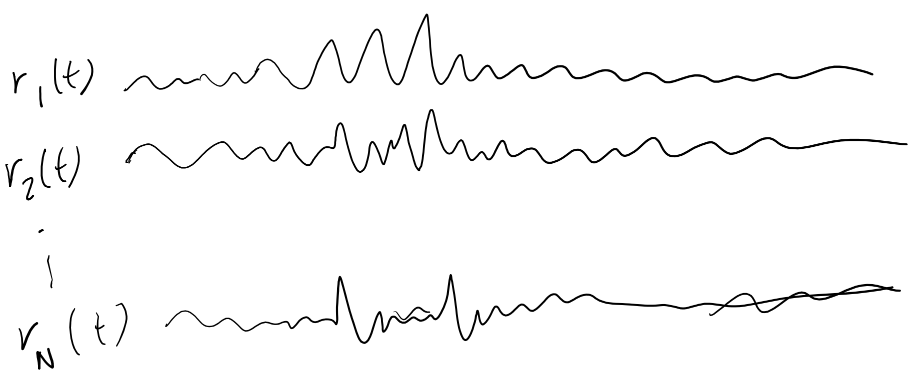
Random processes generate stochastic signals. This means that the output is NOT deterministic. There is some uncertainty, or noise, in the output





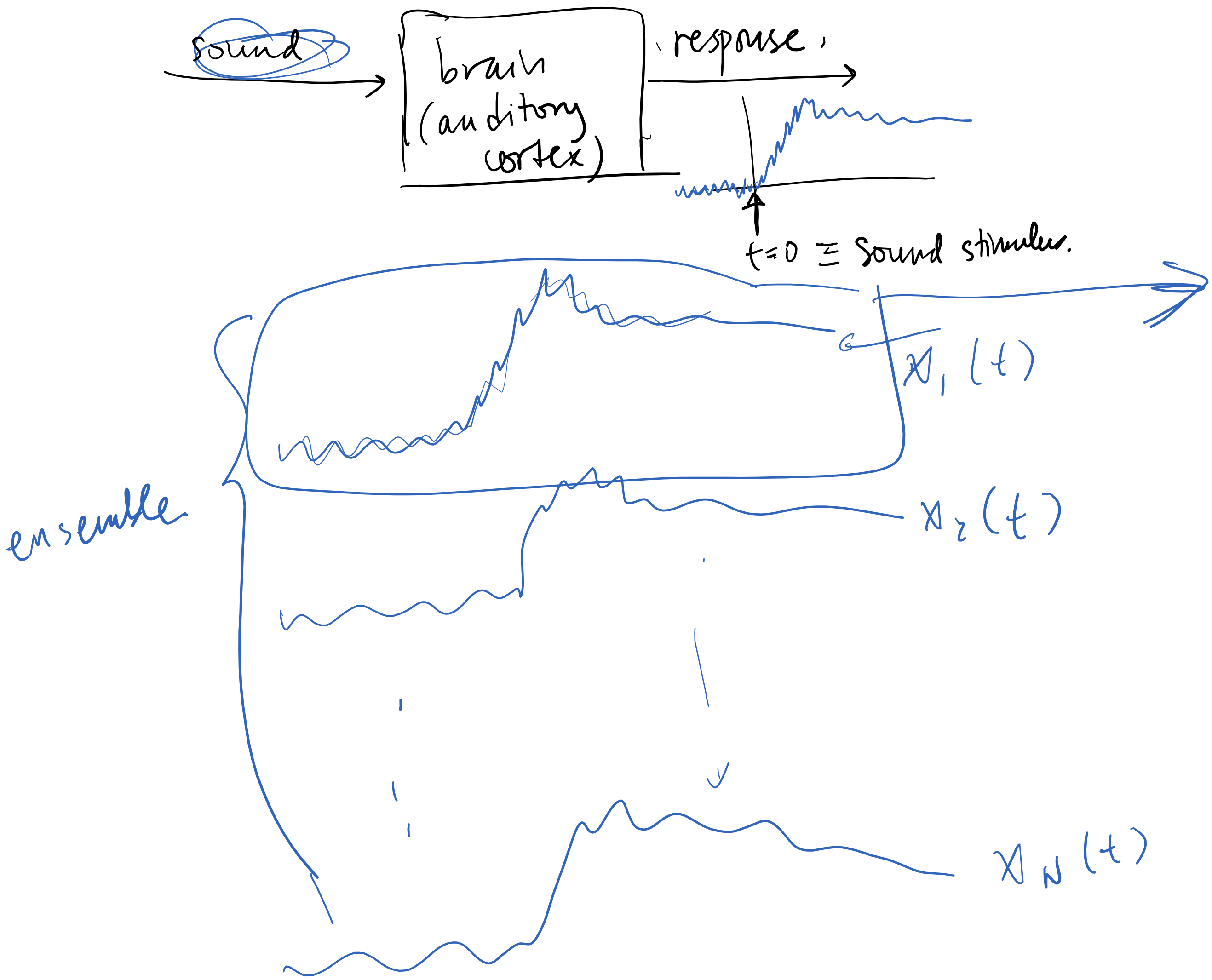
i represents which observation or trial.





Ensemble data - oddball response

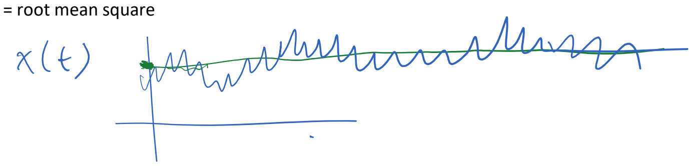
Monday, February 1, 2021 2:50 PM

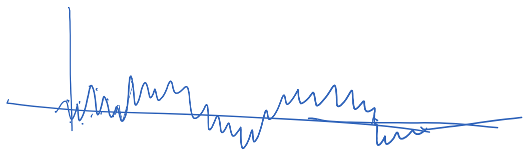


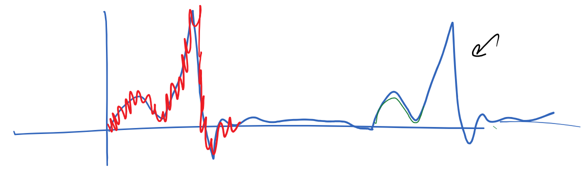
RMS

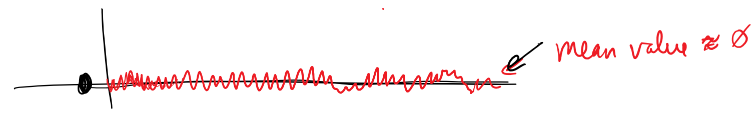
Monday, February 1, 2021 2:49 PM

RMS = root mean square









$$x_{RMS} = \sqrt{\frac{1}{T}\int_0^T [x(t)]^2 \, dt}. \qquad (C.T.)$$

$$x_{RMS} = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2} \qquad (D.T.),$$

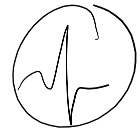
$$\sigma_x \equiv x_{RMS}.$$

SNR of ensemble average

Wednesday, February 3, 2021 3:14 PM

SNR = signal-to-noise ratio.

$$SNR \triangleq \frac{\text{RMS value } \cancel{\text{of signal}}}{\text{RMS value of noise}}.$$



$$x(t) = s(t) + \eta(t).$$

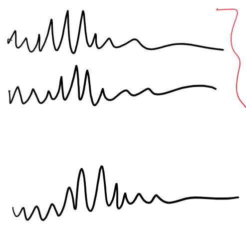
stochastic process $\xrightarrow{x(t)}$

$x_1(t)$

$x_2(t)$

$\vdots$

$x_N(t)$



Ensemble Average.

$$\text{SNR of } x_i(t) \overset{?}{\gtreqless} \text{SNR of } z(t) \triangleq \boxed{\frac{1}{N}\sum_{i=1}^{N} x_i(t)}$$

$$\text{SNR of } x_i(t) = \frac{\sigma_{x_s}}{\sigma_{x_\eta}}$$

$$\text{SNR of } z(t) = \boxed{\frac{\sigma_{z_s}}{\sigma_{z_\eta}}} = \frac{\sigma_{x_s}}{} \Rightarrow$$

$$z = \frac{1}{N}\left(\sum_{k=1}^{N} x_k\right) = \frac{1}{N}\left(x_1 + x_2 + \dots x_N\right),$$

$$\sigma_{z_\eta} = ? \qquad \sigma_{z_\eta}^2 = \left(E\{z_\eta^2\}\right) - \left(E\{z_\eta\}\right)^2.$$

$$y = x_1 + x_2 + \dots x_N.$$

$$N^2 \cdot \sigma_{z_\eta}^2 = \sigma_y^2 = E\{y^2\} - \left(E\{y\}\right)^2.$$

$$E\{y_\eta\} = N \cdot E\{\eta\}.$$

$$E\{y_\eta^2\} = \iint \dots \int \underbrace{(\eta_1 + \eta_2 + \dots \eta_N)^2}_{\eta_1^2 + \eta_2^2 + \dots \eta_N^2 \; + \int \cancel{\eta_1 \cdot \eta_2} \, f \dots} f_{\eta_1}(\eta_1) \cdot f_{\eta_2}(\eta_2) \dots f_{\eta_N}(\eta_N)$$

$$= \underbrace{\int \eta_1^2 f_{\eta_1}(\eta_1)} + \underbrace{\int \eta_2^2 f_{\eta_2}(\eta_2)} + \dots \underbrace{\int \eta_N^2 f_N(\eta_N)}.$$

$$= N \cdot \underbrace{\int \eta^2 f_\eta(\eta)}_{\sigma_\eta^2} \Rightarrow \sigma_{y_\eta}^2 = N \cdot \sigma_\eta^2.$$

$$\sigma_{z_\eta}^2 = \frac{1}{N^2}\sigma_{y_\eta}^2 = \frac{1}{N}\sigma_\eta^2.$$

$$\sigma_{z_\eta}^2 = \frac{1}{N^2}\sigma_{y_\eta}^2 = \frac{1}{N}\sigma_\eta$$

$$\underline{\sigma_{z_\eta}} = \frac{1}{\sqrt{N}}\underline{\sigma_\eta}$$

$$\underline{SNR_z} = \frac{\sigma_{x_s}}{\frac{1}{\sqrt{N}}\sigma_\eta} = \sqrt{N}\boxed{\frac{\sigma_{x_s}}{\sigma_\eta}}$$

SNR of x_i

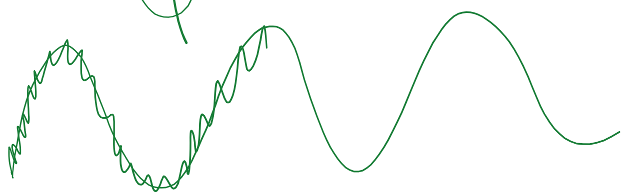
dB

Wednesday, February 2, 2022 3:27 PM

dB = decibels

$$\boxed{SNR_{dB} = 20 \cdot \log_{10}(SNR)}$$

$$\frac{\sigma_s}{\sigma_\eta}$$



$$\frac{\sigma_s^2}{\sigma_\eta^2}$$

$$\log\left(\frac{\sigma_s}{\sigma_\eta}\right)^2 = 2 \cdot \log\left(\frac{\sigma_s}{\sigma_\eta}\right) \times 10$$

$$\sigma_s = 10 \cdot \sigma_\eta \Rightarrow \frac{\sigma_s}{\sigma_\eta} = 10$$

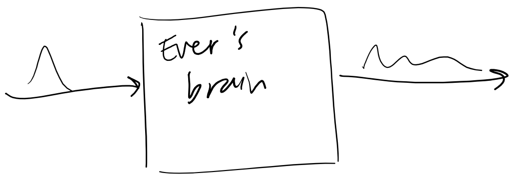
$$SNR_{dB} = 20 \cdot \log_{10}\left(\frac{\sigma_s}{\sigma_\eta}\right)$$

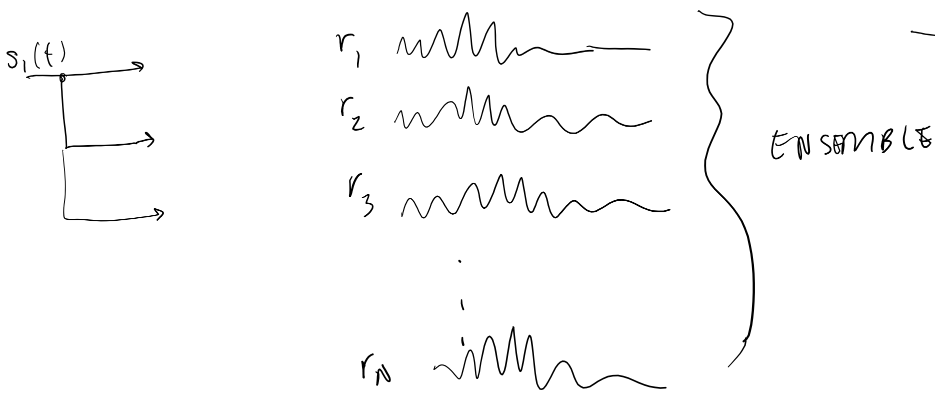
$$= 20 \cdot \underbrace{\log_{10}(10)}$$

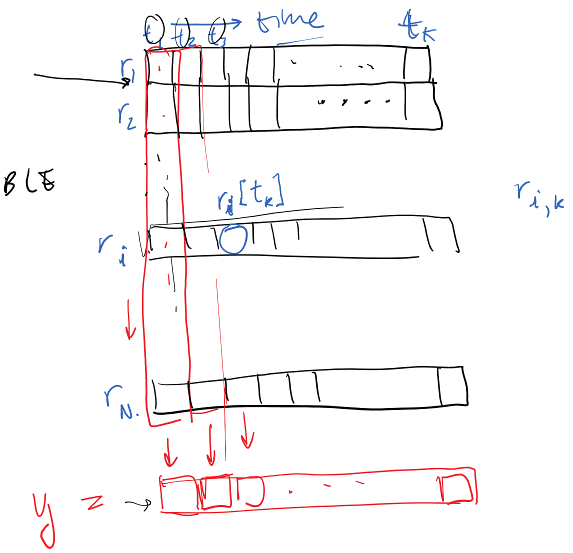
$$20 \cdot 1 = 20\,dB$$

Ensemble Average

Wednesday, February 2, 2022 3:12 PM







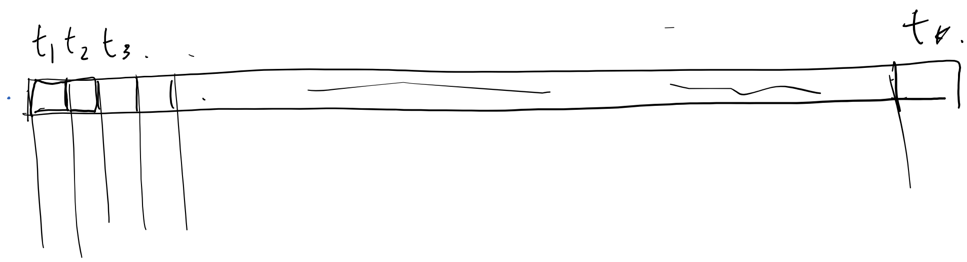
ENS. AVG

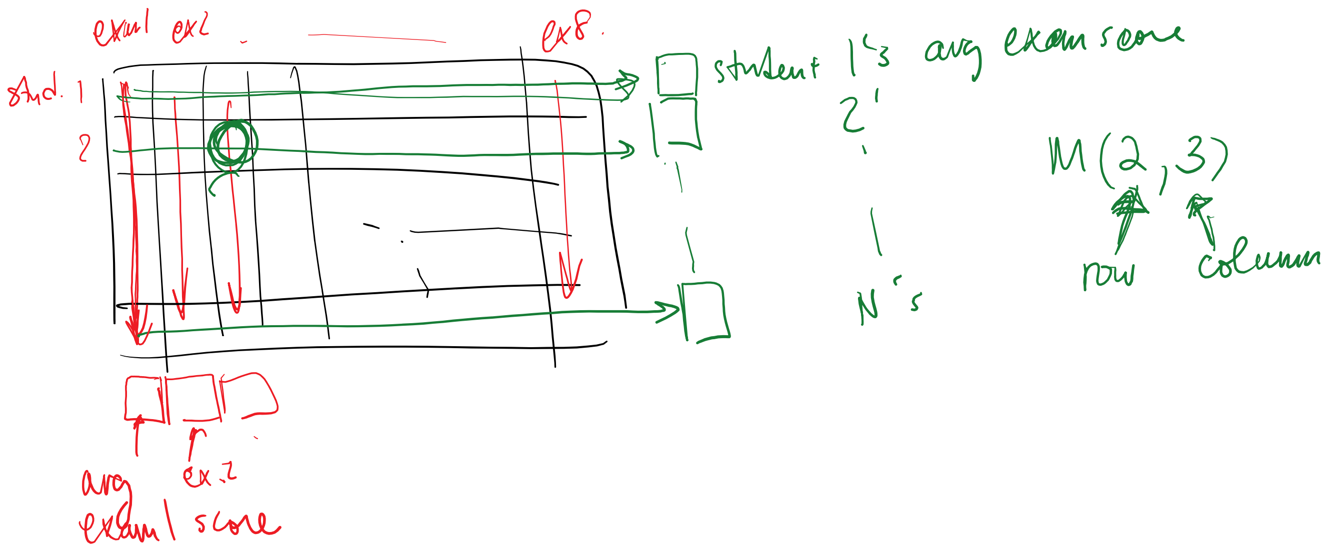
$$\boxed{y[t_k] = \frac{1}{N}\sum_{i=1}^{N} r_i[t_k].}$$

$$SNR_y = \sqrt{N}\, SNR_r$$

Matlab - creating time vector; and mean

Wednesday, February 3, 2021 4:06 PM





Dimension 1 = rows
Dimension 2 = column

Std dev of ensemble response

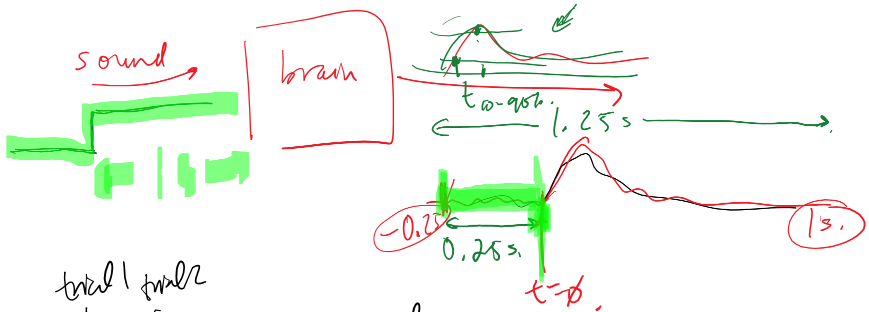
Monday, February 8, 2021 3:27 PM

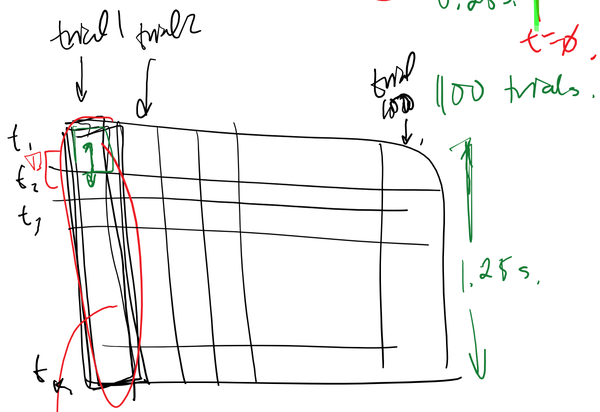
Step response, response time - saccade, visual response

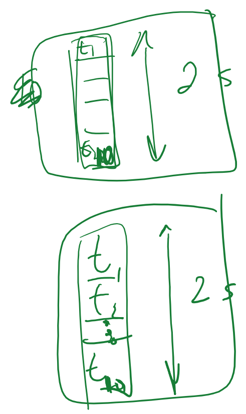
Monday, February 1, 2021 2:50 PM

Ensemble avg example in matlab

Wednesday, February 2, 2022 3:03 PM

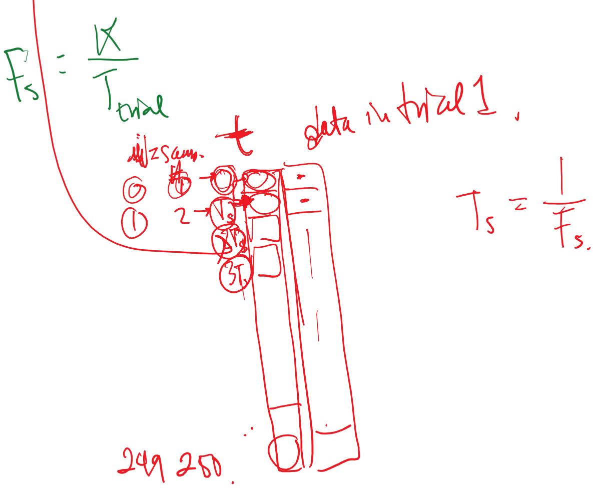






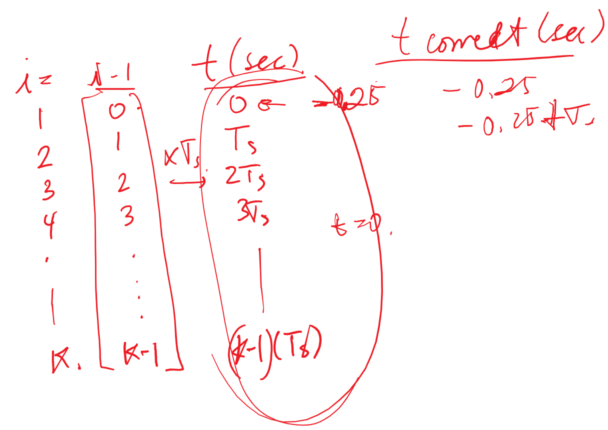
10 samples/2sec

=5 samp/sec

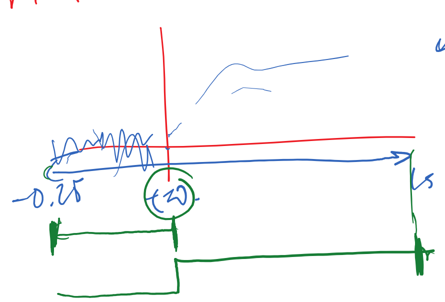


$F_s = \frac{K}{T_{trial}}$

$T_s = \frac{1}{F_s}$



plot Trial 1 data vs. time.

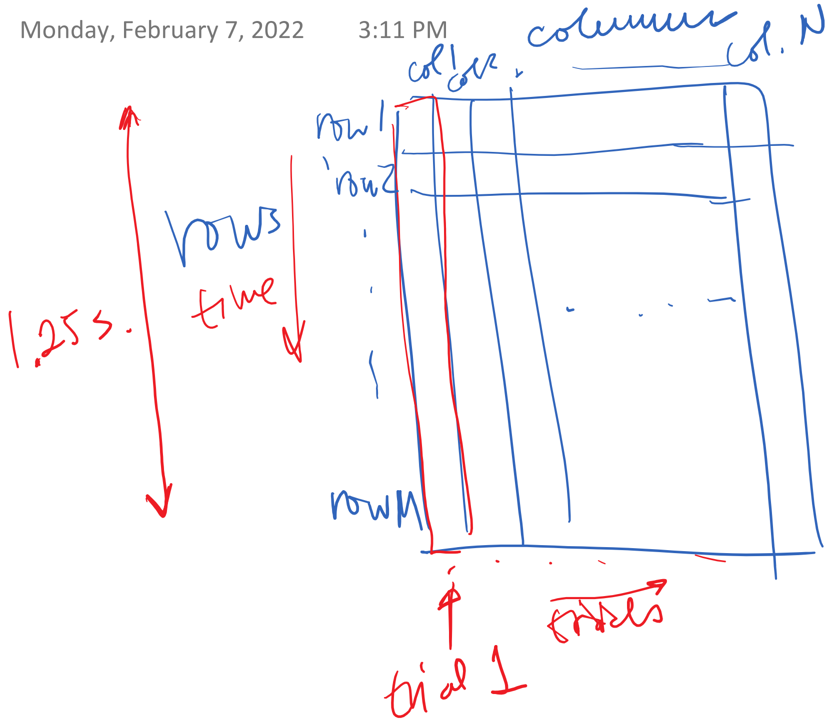


PS 2 ensemble data - experimental paradigm

Wednesday, February 3, 2021 3:58 PM

Matrices

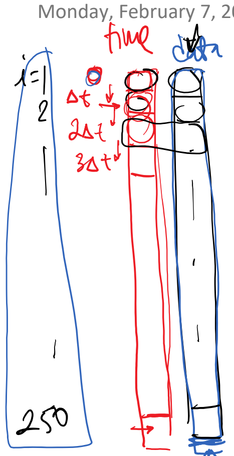
Monday, February 7, 2022 3:11 PM

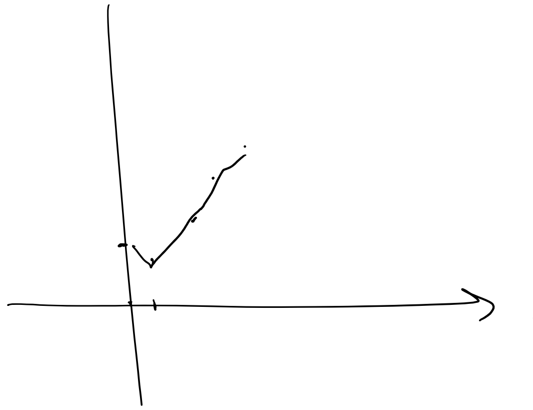


M x N matrix,
# #
1st dimension 2nd dim.

Creating time vector

Monday, February 7, 2022 3:19 PM





$$\Delta t = \frac{1.25\,s}{250} = 0.005\,s$$

$(= 5\,ms)$

$y_1 \triangleq$ response from trial 1
@ the 1st column,
or all elements in column 1

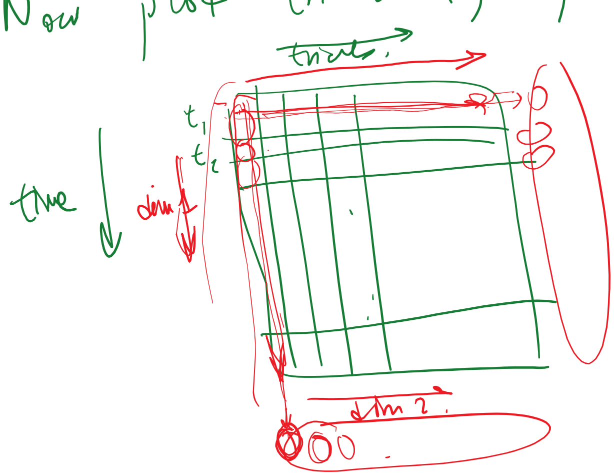
k	t	t_{act}
1	0 s	-0.25
2	Δt	-0.2495
3	$\boxed{2\Delta t}$	-0.2490
4		
⋮	⋮	0 s
L	$\boxed{(L-1)\Delta t}$	~1 s;

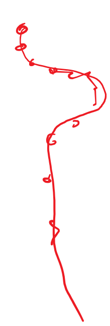
$k = 1:1:250.$

$t = (k-1) * T_s;$

$t_{act} = t - 0.25;$

Now plot trials 1, 11, & 21 all on same plot.

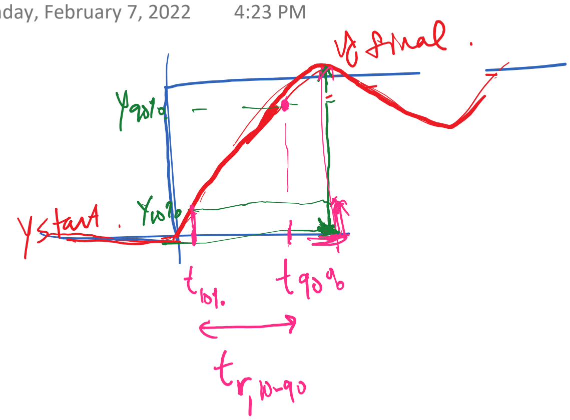




ens. avg = average across observations (or trials).

10 - 90% rise time

Monday, February 7, 2022 4:23 PM



To find ystart:
Eyeball what you think the average value is

To find yfinal:
Just use the max of the signal

Y10% = (yfinal - ystart)*0.10 + ystart
Y90% = (yfinal - ystart)*0.90 + ystart

Now you want to know at what time did the signal reach the 10% value (y10%)?
Eyeball at what time did y reach y10%? That's t10%

At what time did the signal reach the 90% value (y90%)?
Eyeball at what time did y reach y90%? That's t90%

The 10 to 90% rise time is the difference between t90% and t10%
t_r = t90% - t10%

In-class code will be posted on canvas