FIR vs IIR

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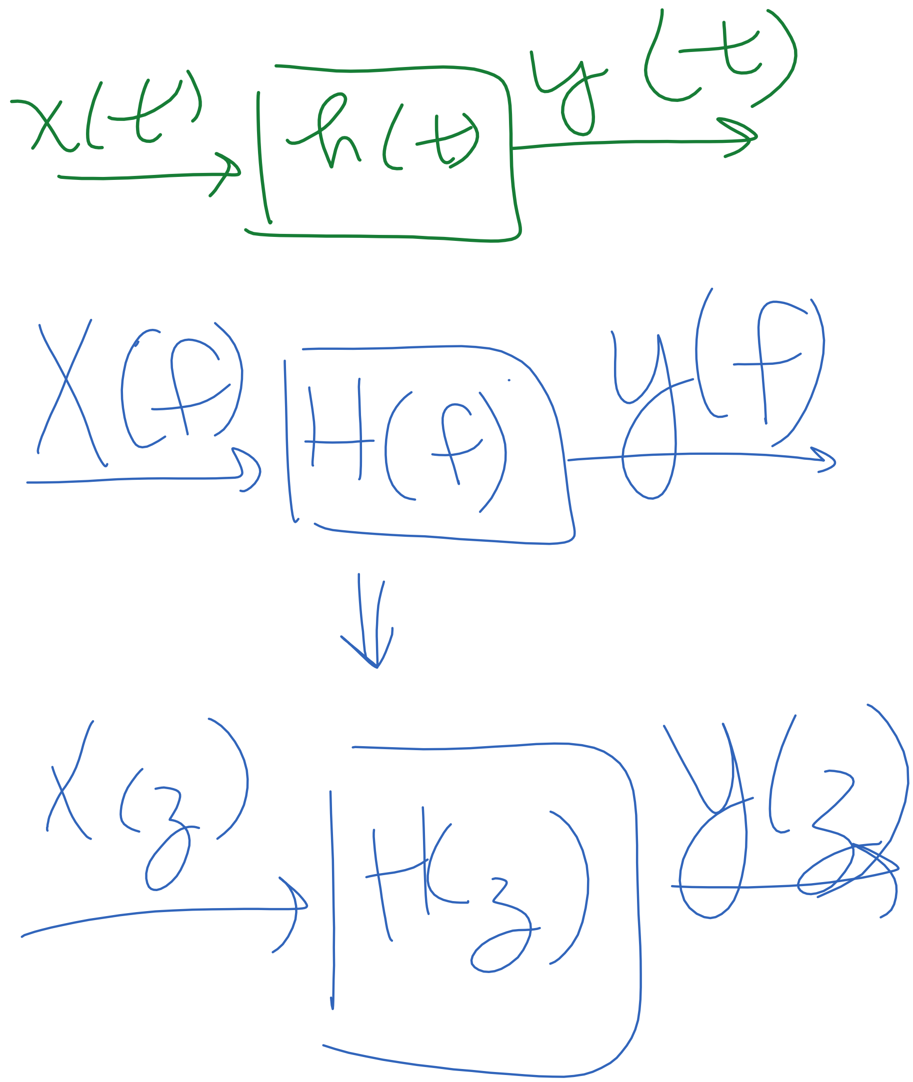
FIR = finite impulse response

Vs

IIR = infinite impulse response



Z-transform Pairs and Properties

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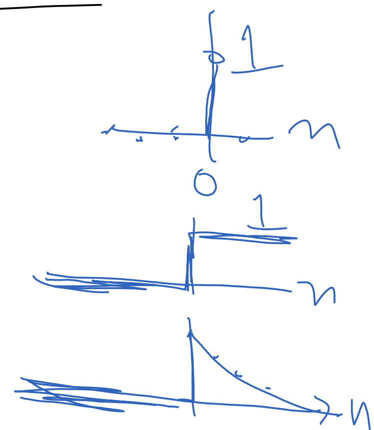
$x[n]$ $\xrightarrow{\mathcal{Z}\{\}}$	$X(z)$
$\delta[n] \longleftrightarrow$	1
$u[n] \longleftrightarrow$	$\dfrac{1}{1-z^{-1}}$
$a^n u[n] \longleftrightarrow$	$\dfrac{1}{1-az^{-1}}$



Properties

$$c_1 x_1[n] + c_2 x_2[n] \longleftrightarrow c_1 X_1(z) + c_2 X_2(z)$$

$$x[n-n_0] \longleftrightarrow z^{-n_0} X(z)$$

$$x_1[n] * x_2[n] \longleftrightarrow X_1(z) \cdot X_2(z)$$

Example $x[n] = 0.7^n u[n]$

$$y[n] = x[n-3] \rightarrow Y(z) = ?$$

$$Y(z) = z^{-3} X(z)$$

$$X(z) = \frac{1}{1-0.7z^{-1}}$$

$$Y(z) = \frac{z^{-3}}{1-0.7z^{-1}}$$

Relationship between transfer function and impulse response

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$$y[n] = x[n] * \delta[n]$$

$$Y(z) = X(z) \cdot \underbrace{\mathcal{Z}\{\delta[n]\}}_{1}$$

$$Y(z) = X(z)$$

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$$

$$\delta[n] \rightarrow \boxed{x[n]} \rightarrow \cancel{y}[n]$$

h[n] = the response of the system when the input is an impulse function

= i.e. "impulse response"

$$\delta[n] \rightarrow \boxed{h[n]} \rightarrow h[n]$$

H(z) is the Z-transform of the impulse response

i.e., H(f) the transfer function of a system is the Fourier transform of the impulse response

$$H(f) = \mathcal{F}\{h[n]\}$$

Basic approach

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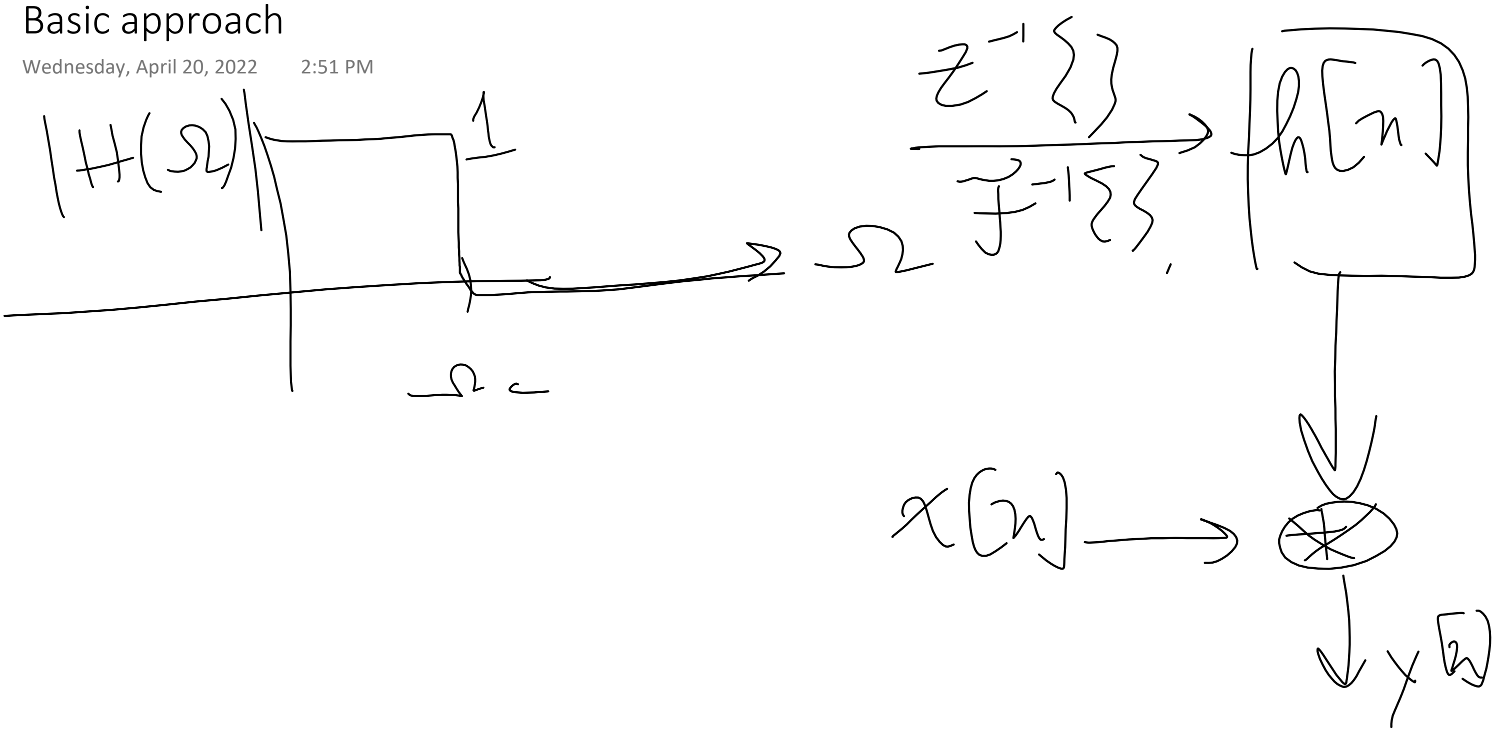

Limitations of filtering

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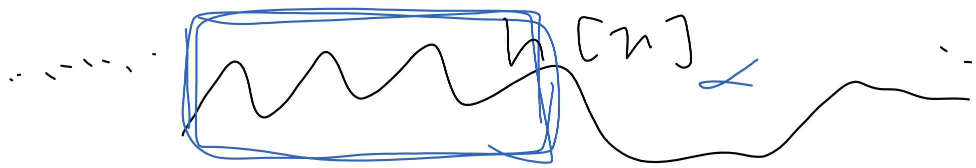

Need to convolve with infinitely long impulse response
But we can only use an impulse response of finite duration = >
Truncate h[n]

Theoretically: $x[n] * h[n] \rightarrow y[n]$

Reality: $x[n] * \left(h[n] \times w[n] \right) \rightarrow y[n]$

Choose this window to reduce the same effects we've seen before
That result from windowing (or truncating) our signal.
i..e, use a window that has lower side lobes in the freq domain,

Basic FIR filters

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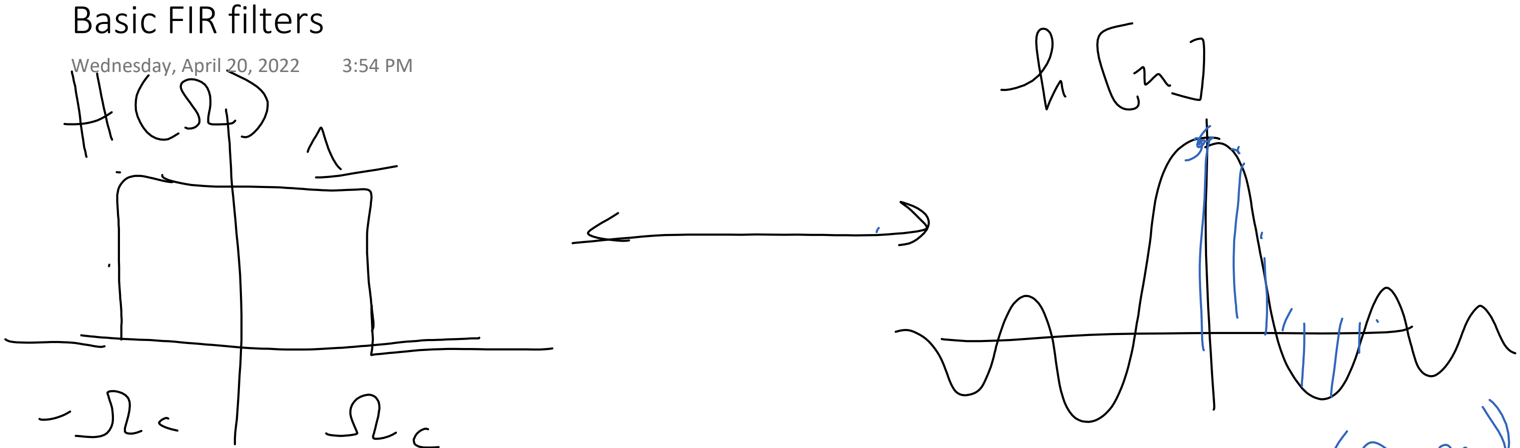
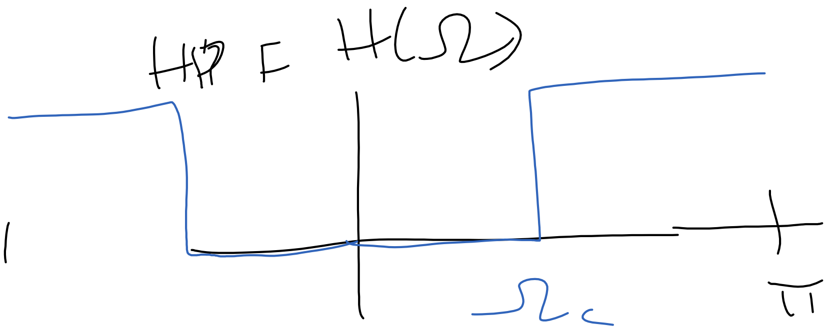


$$H(\Omega) = \begin{cases} 1, & |\Omega| < \Omega_c \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \frac{\sin(\Omega_c n)}{\pi n}$$



$$H(\Omega) = \begin{cases} 1, & |\Omega| > \Omega_c \\ 0, & \text{otherwise} \end{cases}$$

h[n]

$$H_{HP}(\Omega) = f(H_{LP}(\Omega))$$

$$H_{HP}(\Omega) = 1 - H_1$$

Example FIR filter for respiratory signal

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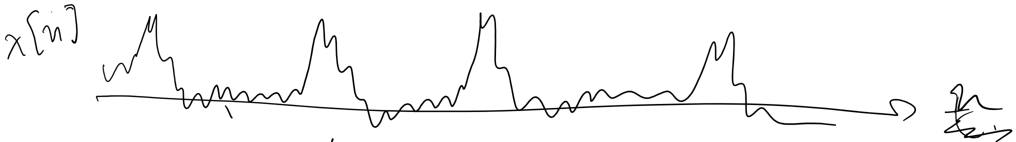
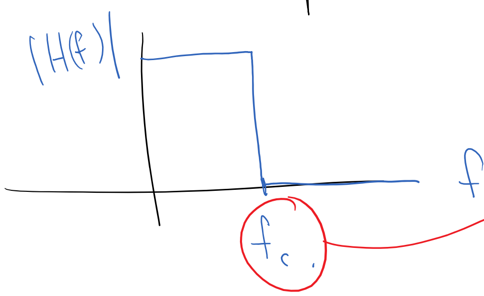


↓ high freq. noise.

⇓

Low pass filter.

Design H(f)



→ look @ spectrum X(f). (fft).

$f_c \approx 1$ Hz.

↓ $\mathcal{F}^{-1}\{\ \}$

$$h[n] = \frac{\mathrm{sinc}(\Omega_c n)}{\pi n}$$

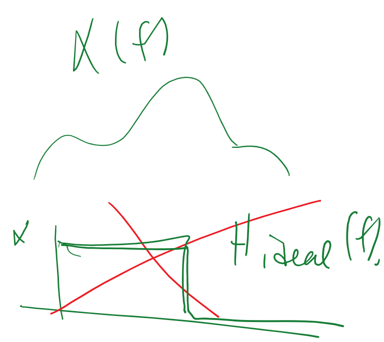
$w[n] \rightarrow \otimes \rightarrow h_w[n]$.

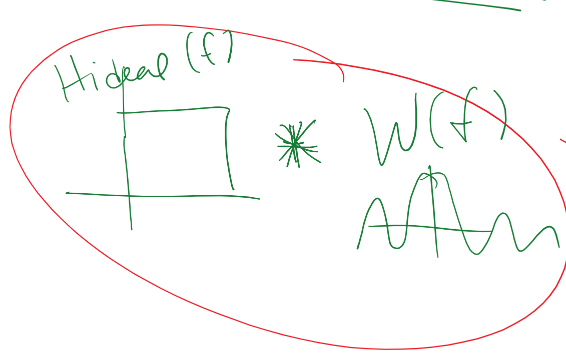
eg hamming, kaiser,

$x[n] \rightarrow (*) \rightarrow y[n]$

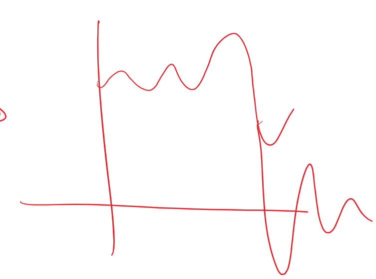
H(f)

$X(f) \rightarrow \otimes \longrightarrow Y(f) =$





→



Review of Ztransform

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$$H(z) = \frac{b_n z^n + b_{n-1} z^{n-1} + b_{n-2} z^{n-2} + \dots b_1 z + b_0}{a_m z^m + a_{m-1} z^{m-1} + \dots + a_1 z + a_0} = b_n + b_{n-1} z^{-1} + b_{n-2} z^{-2} \dots + b_0 z^{-n}$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$(a_m z^m + a_{m-1} z^{m-1} + \dots a_1 z + a_0) Y(z) = (b_n z^n + b_{n-1} z^{n-1} + b_{n-2} z^{n-2} + \dots b_1 z + b) X(z)$$

z^{-1}

$$a_m y[n] + a_{m-1} y[n-1] + a_{m-2} y[n-2] + \dots$$
$$= b_n x[n] + b_{n-1} x[n-1] + \dots b_0$$

FIR1

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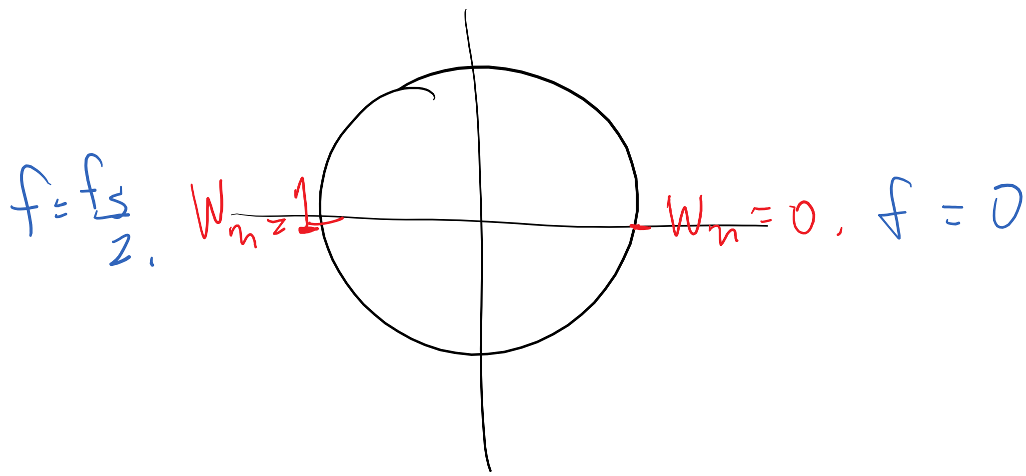
%% Alternate way using fir1
Wn = normalized frequency
B = fir1(N,Wn);



$$W_n = \frac{f_c}{(f_s/2)}$$

FIR filter summary

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Create filter either by

1. create appropriate sinc fn.
2. fir1 command

Filter signal either by

1. conv
2. filt filt

Check filter

1) Look at filter's frequency response H(f)
2) Look at signal spectra before and after filtering i.e., X(f) and Y(f)
3) Look at signals in time before and after i.e., x(t) and y(t)