



POLITECNICO
MILANO 1863

Three-Dimensional Bin Packing with Vertical Support

Jacopo Libè
952914

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Case study

- Large warehouses
- Mixed-case palletization
- No control over items' shape (strongly heterogeneous)
- Pallets wrapped during loading procedure



Figure: Example of pit palletization (Schäfer Case Picking — SSI SCHÄFER)

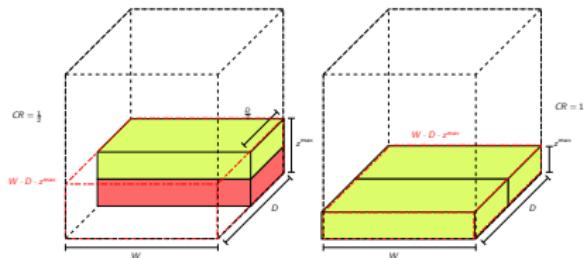


Figure: Cage ratio of two different bin configurations

Definition

An item has vertical support if one of the following conditions hold:

- **Condition 1:** at least a percentage α_s of its base area is resting on other items
- **Condition 2:** at least 3 of its vertices are resting over other items and **Condition 1** holds with a lower percentage

Introduction

Vertical Support

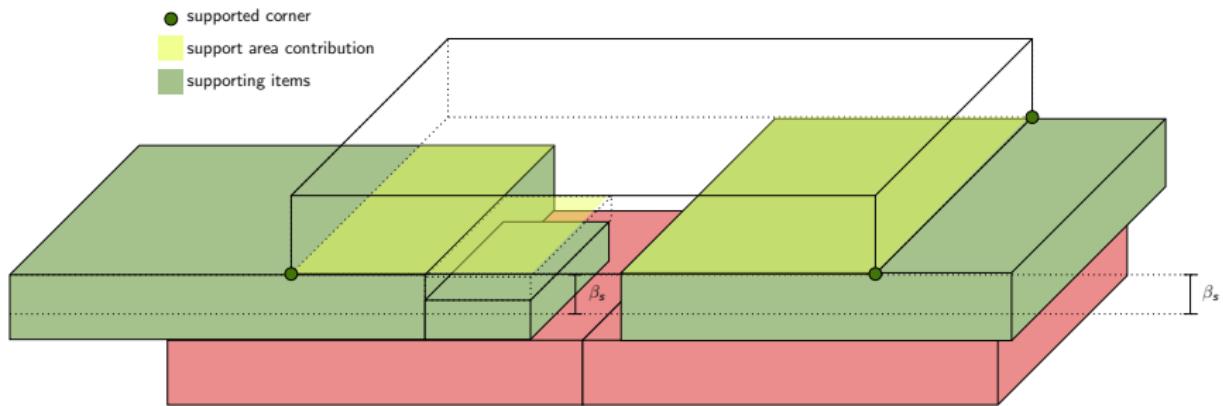


Figure: Representation of an item with conditions 1 and 2 of vertical support given
 $\alpha_s = 0.5, \beta_s$

- The problem is NP-Hard
- Exact methods only for small instances
- Existing 3D-BPP heuristics don't consider practical constraints
- Solutions for container loading and pallet loading problems are layer based

minimize number of used bins

then, maximize average cage ratio of the used bins

subject to all items are assigned to one and only one bin

all items are inside the bin's bounds

no overlaps between items in the same bin

all items have vertical support

MILP Proxy Model - Objective Function

minimize	$\sum_{b \in B} (Hv_b + z_b^{\max})$	
then, maximize		
subject to		
number of used bins		
average cage ratio of the used bins		
all items are assigned to one and only one bin	$\sum_{b \in B} u_{ib} = 1$	$\forall i \in I, \forall b \in B$
all items are inside the bin's bounds	$u_{ib} \leq v_b$	$\forall i \in I, \forall b \in B$
no overlaps between items in the same bin	$v_b \geq v_c$	$\forall (b, c) \in B : b < c$
all items have vertical support	$x_i + w_i \leq W$	$\forall i \in I$
	$y_i + d_i \leq D$	$\forall i \in I$
	$z_i + h_i \leq H$	$\forall i \in I$
	$(x_i + w_i) - x_j \leq W(1 - x_{ij}^P)$	$\forall i, j \in I$
	$x_j - (x_i + w_i) + 1 \leq Wx_{ij}^P$	$\forall i, j \in I$
	$(y_i + d_i) - y_j \leq D(1 - y_{ij}^P)$	$\forall i, j \in I$
	$y_j - (y_i + d_i) + 1 \leq Dy_{ij}^P$	$\forall i, j \in I$
	$(z_i + h_i) - z_j \leq H(1 - z_{ij}^P)$	$\forall i, j \in I$
	$z_j - (z_i + h_i) + 1 \leq Hz_{ij}^P$	$\forall i, j \in I$
	$x_{ij}^P + x_{ji}^P + y_{ij}^P + y_{ji}^P + z_{ij}^P + z_{ji}^P \geq u_{ib} + u_{jb} - 1$	$\forall i, j \in I, \forall b \in B$
	$z_b^{\max} \geq (z_i + h_i) - H(1 - u_{ib})$	$\forall i \in I, \forall b \in B$

minimize then, maximize subject to	number of used bins average cage ratio of the used bins all items are assigned to one and only one bin all items are inside the bin's bounds no overlaps between items in the same bin all items have vertical support	$\min \quad \sum_{b \in B} (Hv_b + z_b^{max})$ $\text{s.t.} \quad \begin{aligned} & \sum_{b \in B} u_{ib} + \sum_{b \in B} u_{jb} = 1 && \forall (i, j) \in I^{OR} \\ & u_{ib} \leq v_b && \forall i \in I, \forall b \in B \\ & v_b \geq v_c && \forall (b, c) \in B : b < c \\ \\ & x_i + w_i \leq W && \forall i \in I \\ & y_i + d_i \leq D && \forall i \in I \\ & z_i + h_i \leq H && \forall i \in I \\ & (x_i + w_i) - x_j \leq W(1 - x_{ij}^P) && \forall i, j \in I \\ & x_j - (x_i + w_i) + 1 \leq Wx_{ij}^P && \forall i, j \in I \\ & (y_i + d_i) - y_j \leq D(1 - y_{ij}^P) && \forall i, j \in I \\ & y_j - (y_i + d_i) + 1 \leq Dy_{ij}^P && \forall i, j \in I \\ & (z_i + h_i) - z_j \leq H(1 - z_{ij}^P) && \forall i, j \in I \\ & z_j - (z_i + h_i) + 1 \leq Hz_{ij}^P && \forall i, j \in I \\ & x_{ij}^P + x_{ji}^P + y_{ij}^P + y_{ji}^P + z_{ij}^P + z_{ji}^P \geq u_{ib} + u_{jb} - 1 && \forall i, j \in I, \forall b \in B \\ & z_b^{max} \geq (z_i + h_i) - H(1 - u_{ib}) && \forall i \in I, \forall b \in B \end{aligned}$
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minimize then, maximize subject to	$\sum_{b \in B} (Hv_b + z_b^{max})$ $\sum_{b \in B} u_{ib} + \sum_{b \in B} u_{jb} = 1$ $u_{ib} \leq v_b$ $v_b \geq v_c$ $x_i + w_i \leq W$ $y_i + d_i \leq D$ $z_i + h_i \leq H$ $(x_i + w_i) - x_j \leq W(1 - x_{ij}^P)$ $x_j - (x_i + w_i) + 1 \leq Wx_{ij}^P$ $(y_i + d_i) - y_j \leq D(1 - y_{ij}^P)$ $y_j - (y_i + d_i) + 1 \leq Dy_{ij}^P$ $(z_i + h_i) - z_j \leq H(1 - z_{ij}^P)$ $z_j - (z_i + h_i) + 1 \leq Hz_{ij}^P$ $x_{ij}^P + x_{ji}^P + y_{ij}^P + y_{ji}^P + z_{ij}^P + z_{ji}^P \geq u_{ib} + u_{jb} - 1$ $z_b^{max} \geq (z_i + h_i) - H(1 - u_{ib})$	number of used bins average cage ratio of the used bins all items are assigned to one and only one bin all items are inside the bin's bounds no overlaps between items in the same bin all items have vertical support	$\forall (i, j) \in I^{OR}$ $\forall i \in I, \forall b \in B$ $\forall (b, c) \in B : b < c$ $\forall i \in I$ $\forall i \in I$ $\forall i \in I$ $\forall i, j \in I$
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minimize number of used bins
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 all items are inside the bin's bounds
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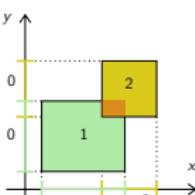
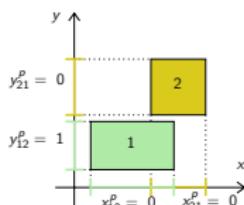


Figure: Precedences variables (2D case)

$$\begin{aligned}
 & \text{min} && \sum_{b \in B} (Hv_b + z_b^{max}) \\
 \text{s.t.} & & \sum_{b \in B} u_{ib} + \sum_{b \in B} u_{jb} = 1 & \forall (i, j) \in I^{OR} \\
 & & u_{ib} \leq v_b & \forall i \in I, \forall b \in B \\
 & & v_b \geq v_c & \forall (b, c) \in B : b < c \\
 & & x_i + w_i \leq W & \forall i \in I \\
 & & y_i + d_i \leq D & \forall i \in I \\
 & & z_i + h_i \leq H & \forall i \in I \\
 & & (x_i + w_i) - x_j \leq W(1 - x_{ij}^P) & \forall i, j \in I \\
 & & x_j - (x_i + w_i) + 1 \leq Wx_{ij}^P & \forall i, j \in I \\
 & & (y_i + d_i) - y_j \leq D(1 - y_{ij}^P) & \forall i, j \in I \\
 & & y_j - (y_i + d_i) + 1 \leq Dy_{ij}^P & \forall i, j \in I \\
 & & (z_i + h_i) - z_j \leq H(1 - z_{ij}^P) & \forall i, j \in I \\
 & & z_j - (z_i + h_i) + 1 \leq Hz_{ij}^P & \forall i, j \in I \\
 & & x_{ij}^P + x_{ji}^P + y_{ij}^P + y_{ji}^P + z_{ij}^P + z_{ji}^P \geq u_{ib} + u_{jb} - 1 & \forall i, j \in I, \forall b \in B \\
 & & z_b^{max} \geq (z_i + h_i) - H(1 - u_{ib}) & \forall i \in I, \forall b \in B
 \end{aligned}$$

minimize number of used bins

then, maximize average cage ratio of the used bins

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MILP Proxy Model - Discretized Vertical Support

Pre-Computed Parameter

$$O(i, j, k, h)$$

$$O(i, j, 1, 2) = 1$$

Variables

$$s_{ijb}^{12} = 1$$

$$(x_j - x_i, y_j - y_i)$$

s.t.

$$\begin{aligned} z_j - (z_i + h_i) &\leq \beta_s + H(1 - z_{ij}^c) && \forall (i, j) \in I : i \neq j \\ z_j - (z_i + h_i) &\geq -\beta_s - H(1 - z_{ij}^c) && \forall (i, j) \in I : i \neq j \\ s_{ij} &\leq z_{ij}^p && \forall (i, j) \in I \\ s_{ij} &\leq z_{ij}^c && \forall (i, j) \in I \\ s_{ij} &\geq z_{ij}^p + z_{ij}^c - 2 && \forall (i, j) \in I : i \neq j \\ \sum_{j \in I} s_{ij} &\leq \sum_{b \in B} u_{ib} && \forall i \in I \\ z_i &\leq H(1 - g_i) && \forall i \in I \\ \sum_{(k,h) \in \Delta, b \in B : O(i,j,k,h) \neq 0} s_{ijb}^{kh} &\leq s_{ij} && \forall (i, j) \in I \\ \sum_{(k,h) \in \Delta : O(i,j,k,h) \neq 0} s_{ijb}^{kh} &\leq u_{ib} && \forall (i, j, b) \in I^B \\ \sum_{(k,h) \in \Delta : O(i,j,k,h) \neq 0} s_{ijb}^{kh} &\leq u_{jb} && \forall (i, j, b) \in I^B \\ x_j - x_i &\geq \gamma k - 2W(1 - s_{ijb}^{kh}) && \forall (k, h) \in \Delta, \forall (i, j, b) \in I^B : O(i, j, k, h) \neq 0 \\ x_j - x_i &\leq \gamma(k+1) + 2W(1 - s_{ijb}^{kh}) && \forall (k, h) \in \Delta, \forall (i, j, b) \in I^B : O(i, j, k, h) \neq 0 \\ y_j - y_i &\geq \gamma h - 2D(1 - s_{ijb}^{kh}) && \forall (k, h) \in \Delta, \forall (i, j, b) \in I^B : O(i, j, k, h) \neq 0 \\ y_j - y_i &\leq \gamma(h+1) + 2D(1 - s_{ijb}^{kh}) && \forall (k, h) \in \Delta, \forall (i, j, b) \in I^B : O(i, j, k, h) \neq 0 \\ \sum_{(k,h) \in \Delta, b \in B : j \in I : i \neq j \wedge O(i,j,k,h) \neq 0} O(i,j,k,h) s_{ijb}^{kh} &\geq \alpha_s w_i d_i - w_i d_i g_i && \forall i \in I \end{aligned}$$

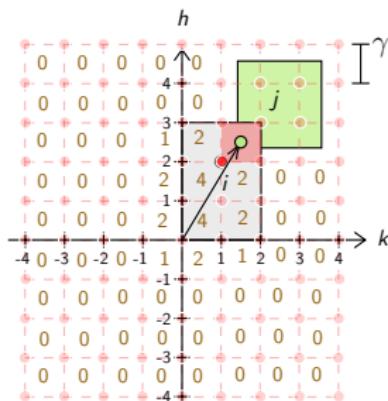


Figure: Space discretization

Proposed Heuristic

Overview

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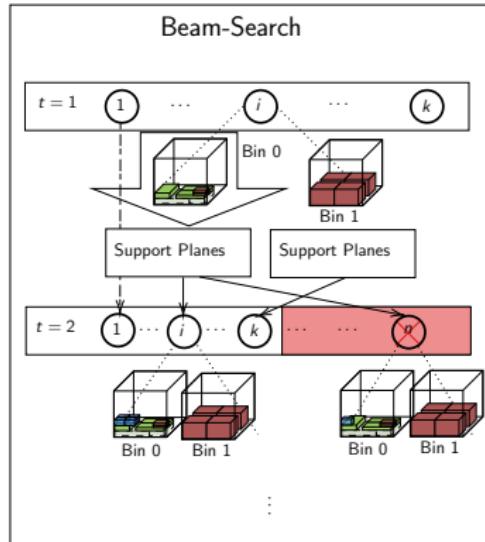


Figure: Conceptual representation of the proposed heuristic

Proposed Heuristic Optimizations

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Computational Experiments

Conclusions

Results & Future Developments

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