POLITECNICO MILANO 1863

SCUOLA DI INGEGNERIA INDUSTRIALE E DELL'INFORMAZIONE

Executive Summary of the Thesis

Three-dimensional bin packing with vertical support

Laurea Magistrale in Computer Science and Engineering - Ingegneria Informatica

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1. Introduction

Recent progress in the digitalization of industrial processes led to a rise in studies on the Three-Dimensional Bin Packing Problem (3D-BPP). The problem consists in packing a set of items in the minimum number of bins without any overlap. When considering real-world settings, the addition of new practical constraints is required. Previous studies in other fields related to container loading and pallet loading have shown that static stability of the bins is a crucial aspect to consider (Bortfeldt and Wäscher [2013]). In this thesis, we address a version of the bin packing problem stemming from a real case study of mixed-case palletization: the Three-Dimensional Bin Packing Problem with Vertical Support (3D-BPPVS). We extend the standard formulation of the bin packing problem by ensuring that all items that are packed inside a bin will not fall, and we refer to this property as the vertical support. Vertical stability is usually ensured between horizontal or vertical slices of items as a constraint on the minimum amount of area which rests on other items (e.g., Gzara et al. [2020]; Paquay et al. [2016]). Each item can also be rotated along its vertical axis by 90 degrees.

Our research stems from the case study of a logistics company in northern Italy. The company manages large warehouses where automated lines bring boxes to different packing stations, and then they are loaded onto pallets of standard size. Since the company is dealing directly with customers' orders, boxes have very different sizes and are usually packed in smaller quantities. Moreover, the assortment of items to pack is strongly heterogeneous which makes the use of layered approaches have sub-optimal results. During the palletization, levels of already packed items are wrapped to ensure better overall stability of the pallet. This wrapping procedure requires that the amount of unused space between items is minimal. The company measures this property with a metric called cage ratio. Cage ratio is the ratio between the volume of the packed items inside a bin and the volume of the cuboid which surrounds them, the cage. The cage has the same base as the bin and height equal to the highest packed item inside the bin. Current commercial solutions employed by the company have solutions with around 60% cage ratio, and a target of 70% was set as a benchmark for our work. A visual representation of the cage ratio is shown in fig. 1.

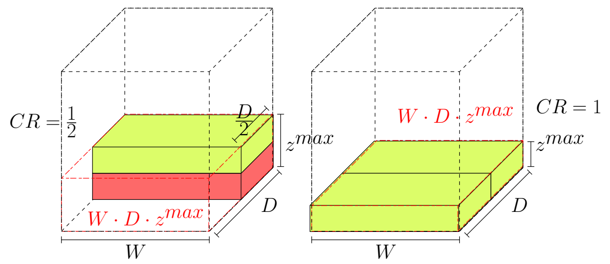

Figure 1: Cage ratio of two different bin configurations

2. Literature Review

The 3D-BPP is the generalization of the one-dimensional bin packing problem which is NP-Hard (Martello et al. [2000]). Exact methods can only solve small instances of the problem which means that most solutions proposed in the literature are heuristics. Heuristics for the 3D-BPP are designed to solve the standard problem and don't take into account practical constraints. Martello et al. [2000] provided a set of benchmark instances for 3D-BPP heuristics, and an exact method based on a two-level branch-and-bound algorithm. Their method used a staircase placement strategy, where a series of corner points were identified as possible placement points to evaluate. The method was later extended in Crainic et al. [2008] to find new niches previously ignored by the introduction of Extreme Points. Gonçalves and Resende [2013] introduced a biased random-key genetic algorithm (BRKGA) which is one of the best performing heuristics on the benchmark instances of Martello et al. [2000]. The algorithm was later modified with variable neighborhood descent variation presented in Zudio et al. [2018] which improved the number of generations needed to find high-quality results.

The concept of vertical support received most of its contribution from the literature of Container Loading Problems (CLP) and Pallet Loading Problems (PLP). As noted in Bortfeldt and Wäscher [2013], static stability is usually implicitly enforced as a consequence of load compactness, or explicitly guaranteed by using filler material in a post-processing step. Most heuristics for CLPS and PLPs try to build dense layers composed of similar items that they stack, reducing the problem to a one-dimensional bin packing problem. Layers are filtered based on the fill-rate and when they are below a certain threshold they are discarded (e.g., Alonso et al. [2020]; Elhedhli et al. [2019]). This means that when no new layer can be built, new bins are opened, simpler placement methods are used to pack the remaining items or filler material is used to complete the layers. Other more recent methods in the PLPs literature ensure support explicitly as a post-processing step when spacing layers relative to eachother (e.g., Gzara et al. [2020]).

Our solution to the problem fills the gap in the research by finding solutions to the 3D-BPPVS without explicitly building layers, and without the use of filler material.

3. Proposed Solution

Based on other publications from the literature and our case study partner's insights we define the property of vertical support. Given a support area threshold α_s and a maximum vertical gap below which an item can be considered as effectively supporting another one β_s. We define an item as supported if one of the following conditions holds

Condition 1. the sum of the overlap area over the XY-plane with every other item on which it is resting is greater than α_s times its base area. (area support)

Condition 2. the number of its corners resting on another item is greater or equal to 3, and condition 1 holds with a threshold α'_s where $\alpha'_s < \alpha_s$. (vertex support)

A visual representation of the conditions 1 and 2 of support is illustrated in fig. 2.

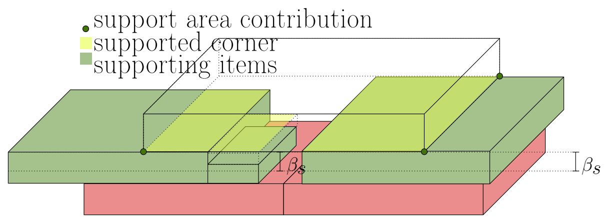

Figure 2: Representation of an item with conditions 1 and 2 of vertical support given $\alpha_s = 0.5, \beta_s$

We propose an heuristic that combines a constructive heuristic with a beam-search algorithm. The main idea of the heuristic is to build solutions to the 3D-BPPVS without explicitly building layered solutions. The constructive heuristic is designed to solve a single-bin bin packing problem with vertical support, while the beam-search expands the heuristic's solutions by exploring different sequences of item placements. Since our heuristic is constructive in nature, we start from an empty solution and build it iteratively by packing new items or opening new bins. To represent a partial solution, we defined a structure containing all the history of the placements of items and opening of bins.

4. Conclusions

In this thesis, we presented an heuristic for the Three-Dimensional Bin Packing Problem with Vertical Support. We modified the two-dimensional Extreme Points algorithm of Crainic et al. [2008] to consider vertical support. We then used this modified algorithm in a constructive heuristic which builds solutions to the single bin three-dimensional bin packing problem by filling planes, generated based on the previously inserted items. Finally, we introduced a beam-search algorithm which evaluates different sequences of item placements by using our proposed constructive heuristic, and removes duplicate solutions at each iteration.
Our heuristic achieved an average gap of 5.37% against the best solutions provided by other heuristics, however we were able to solve the same problem in a fraction of their computational time. We consider this as a great result since it states that our algorithm is competitive also in the realm of 3D-BPP without support. We generated a data set of problem instances based on real-world products from our case study, and we used them to evaluate our heuristic. In most configurations, our solutions exceeded the target metric of 70% cage ratio, with some configurations having a negligible execution time. In table 1 we report the average execution time (TT), bin used (B), and cage ratio (CR) across all our 100 case study instances for each configuration of our heuristic. Different placement modes are marked as PS (single placement) and PM (multiple items per placements), and each row represent a different value of k states considered in the beam-search. In table 2 we analyze the tradeoff between running times and cage ratio obtained by each configuration of the heuristic. We list the difference between the average running time of each configuration (TT) and the best average running time ($TT*$) together with the difference between best average cage ratio ($CR*$) and the average cage ratio obtained by each configuration (CR). We identified that for our partner needs a good configuration consists in using the PM placement mode with $k \in 20, 50$.
Further research could introduce new practical constraints considered in the literature like family groupings, load-bearing, and compatibilities between items. Improvement heuristics could also be adapted to account for the support constraint like, for example, space defragmentation techniques introduced by Zhu et al. [2012].

Table 1: Summary of case study tests

	PS			PM		
k	*TT (ms)*	*B*	*CR (%)*	*TT (ms)*	*B*	*CR (%)*
1	423.87	1.37	65.87	65.18	1.31	**70.70**
5	1,597.54	1.34	69.19	185.22	1.29	**73.08**
10	2,627.52	1.32	70.35	344.90	1.27	**73.56**
20	5,373.79	1.34	70.78	620.95	1.27	**74.57**
50	14,203.10	1.31	72.11	1,279.96	1.29	**74.61**
100	26,934.21	1.31	73.23	2,340.37	1.26	**75.36**
200	48,944.90	1.30	73.89	4,465.78	1.25	**76.39**

Table 2: Case study experiments trade off between average execution times and average cage ratio

k	*PS*		*PM*	
	CR – CR (%)*	*TT – TT* (ms)*	*CR* – CR (%)*	*TT – TT* (ms)*
1	10.56	358.69	5.73	0.00
5	7.24	1,532.36	3.35	120.04
10	6.08	2,562.34	2.87	279.72
20	5.65	5,308.61	**1.85**	**555.77**
50	4.32	14,137.92	**1.82**	**1,214.78**
100	3.20	26,869.03	1.07	2,275.19
200	2.54	48,879.72	0.04	4,400.60

5. Acknowledgements

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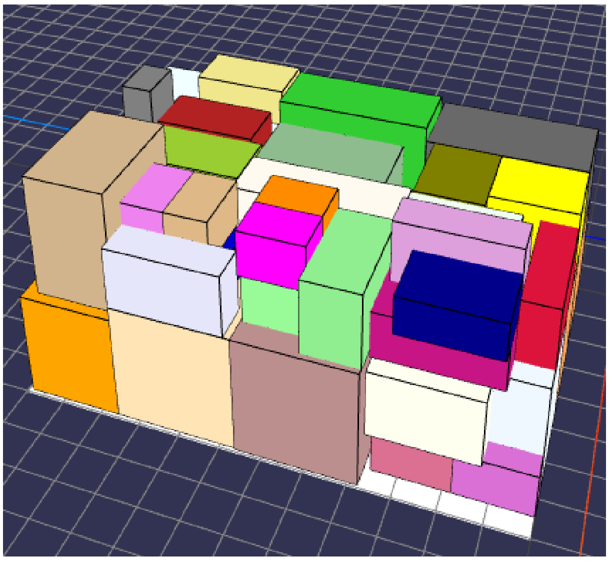

Figure 3: Solution to Instance 82 from case study tests

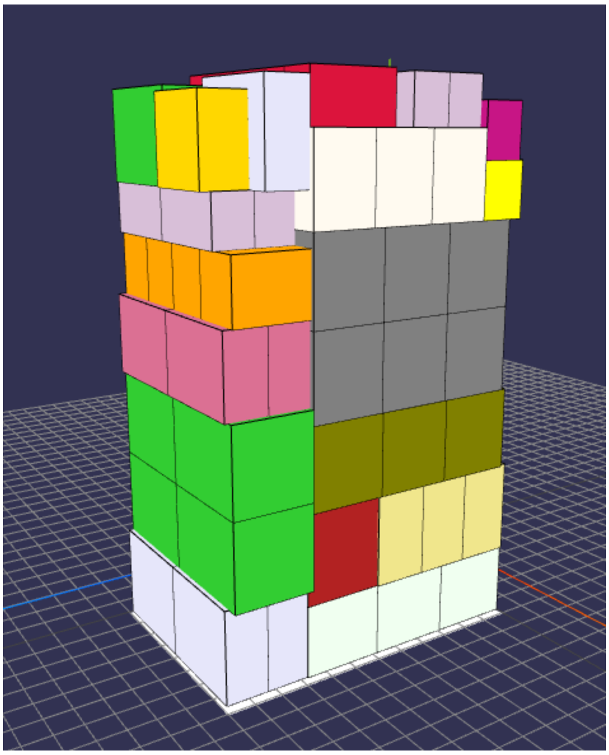

Figure 4: Solution to Instance 66, Bin 1 from case study tests

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