



**POLITECNICO  
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**SCUOLA DI INGEGNERIA INDUSTRIALE  
E DELL'INFORMAZIONE**

EXECUTIVE SUMMARY OF THE THESIS

## Three-dimensional bin packing with vertical support

LAUREA MAGISTRALE IN COMPUTER SCIENCE AND ENGINEERING - INGEGNERIA INFORMATICA

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### 1. Introduction

Recent progress in the digitalization of industrial processes led to a rise in studies on the Three-Dimensional Bin Packing Problem (3D-BPP). The problem consists in packing a set of items in the minimum number of bins without any overlap. When considering real-world settings, the addition of new practical constraints is required. Previous studies in other fields related to container loading and pallet loading have shown that static stability of the bins is a crucial aspect to consider (Bortfeldt and Wäscher [2013]). In this thesis, we address a version of the bin packing problem stemming from a real case study of mixed-case palletization: the Three-Dimensional Bin Packing Problem with Vertical Support (3D-BPPVS). We extend the standard formulation of the bin packing problem by ensuring that all items that are packed inside a bin will not fall, and we refer to this property as the vertical support.

Our research stems from the case study of a logistics company in northern Italy. The company manages large warehouses where automated lines bring boxes to different packing stations, and then they are loaded onto pallets of standard size. Since the company is dealing directly with customers' orders, boxes have very

different sizes and are usually packed in smaller quantities. Moreover, the assortment of items to pack is strongly heterogeneous which makes the use of layered approaches to have sub-optimal results. During the palletization, the lower levels of already packed items are wrapped to ensure better overall stability of the pallet. This wrapping procedure requires that the amount of unused space between items is minimal. The company measures this property with a metric called cage ratio. Cage ratio is the ratio between the volume of the packed items inside a bin and the volume of the cuboid which surrounds them, the cage. The cage has the same base as the bin and height equal to the highest packed item inside the bin. Current commercial solutions employed by the company have solutions with around 60% cage ratio, and a target of 70% was set as a benchmark for our work.

### 2. Gap Identification

The 3D-BPP is the generalization of the one-dimensional bin packing problem which is NP-Hard (Martello et al. [2000]). Exact methods can only solve small instances of the problem which means that most solutions proposed in the literature are heuristics. The concept of vertical support received most of its contribution from

the literature of Container Loading Problems (CLP) and Pallet Loading Problems (PLP). As noted in Bortfeldt and Wäscher [2013], static stability is usually implicitly enforced as a consequence of load compactness, or explicitly guaranteed by using filler material in a postprocessing step. Most heuristics for CLPS and PLPs try to build dense layers composed of similar items that they then stack, reducing the problem to a one-dimensional bin packing problem. Layers are filtered based on the fill-rate and when they are below a certain threshold they are discarded (e.g., Alonso et al. [2020]; Elhedhli et al. [2019]). This means that when no new layer can be built, new bins are opened, simpler placement methods are used to pack the remaining items or filler material is used to complete the layers.

Our solution to the problem fills the gap in the research by finding solutions to the 3D-BPPVS without explicitly building layers, and without the use of filler material.

### 3. Proposed Solution

### 4. Results

Table 1: Average execution time of literature results with bin gap

Heuristic		Execution Time (s)				Bin Gap (%)
		$n = 50$	$n = 100$	$n = 150$	$n = 200$	
PM	$k = 1$	0.05	0.11	0.28	0.55	4.57
	$k = 5$	0.08	0.39	1.02	2.16	4.32
	$k = 10$	0.15	0.74	1.98	4.12	4.29
	$k = 20$	0.29	1.45	3.89	8.07	4.05
	$k = 50$	0.72	3.63	9.72	20.47	3.95
PS	$k = 1$	0.04	0.18	0.51	1.08	4.35
	$k = 5$	0.12	0.74	2.19	4.79	4.01
	$k = 10$	0.23	1.43	4.19	9.39	3.94
	$k = 20$	0.47	2.81	8.48	18.93	3.74
	$k = 50$	1.15	6.74	21.03	45.78	3.52
BRKGA-VD		17.13	80.63	190.50	369.75	0.00

### 5. Conclusions

A final section containing the main conclusions of your research/study have to be inserted here.

### 6. Acknowledgements

Here you might want to acknowledge someone.

### References

[1] Alonso, M. T., Alvarez-Valdes, R., and Parreño, F. (2020). A grasp algorithm for

multi container loading problems with practical constraints. *JOR*, 18(1):49–72.

- [2] Bortfeldt, A. and Wäscher, G. (2013). Constraints in container loading – a state-of-the-art review. *European Journal of Operational Research*, 229(1):1–20.
- [3] Elhedhli, S., Gzara, F., and Yildiz, B. (2019). Three-dimensional bin packing and mixed-case palletization. *INFORMS Journal on Optimization*, 1(4):323–352.
- [4] Martello, S., Pisinger, D., and Vigo, D. (2000). The three-dimensional bin packing problem. *Operations research*, 48(2):256–267.