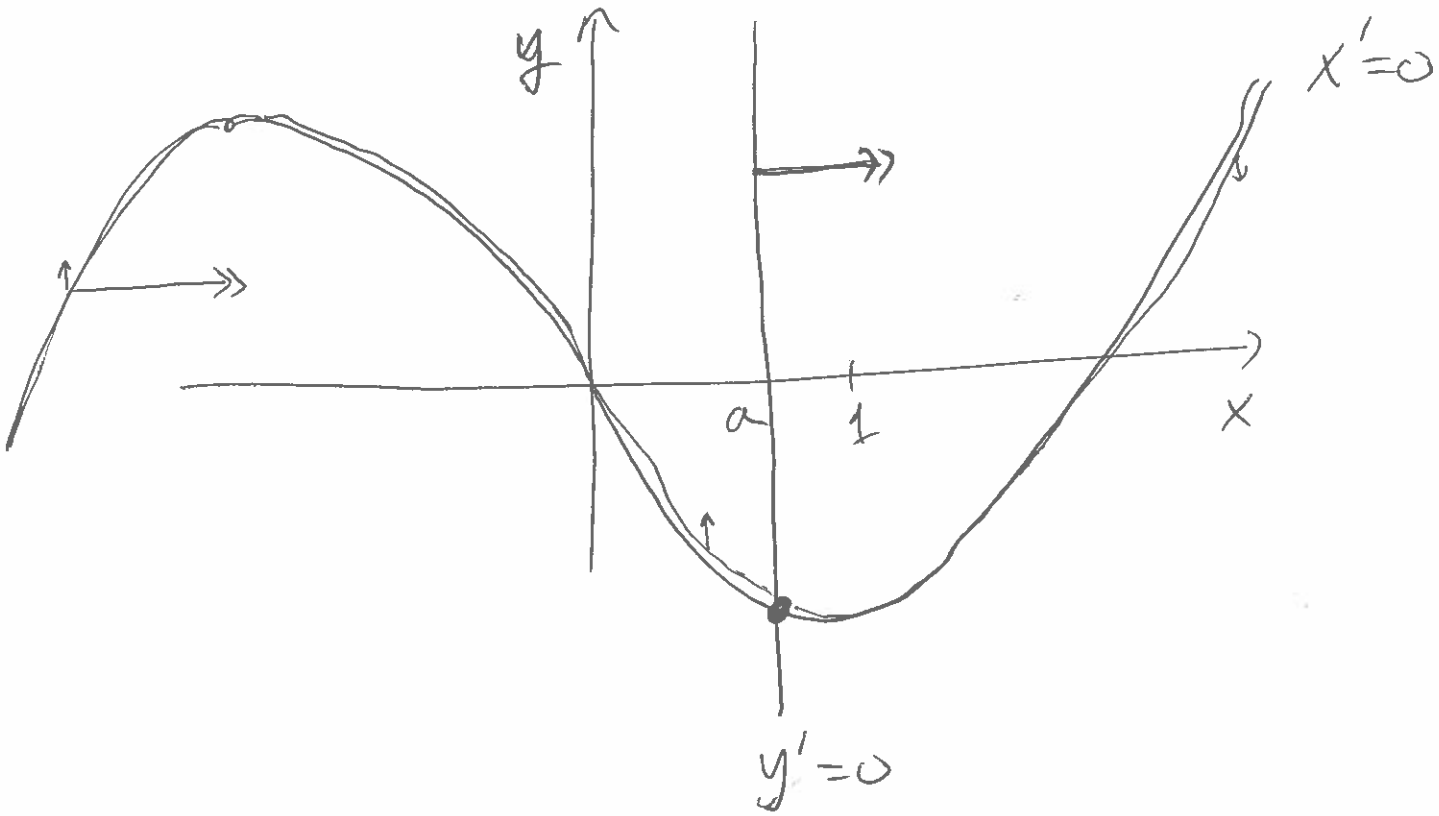
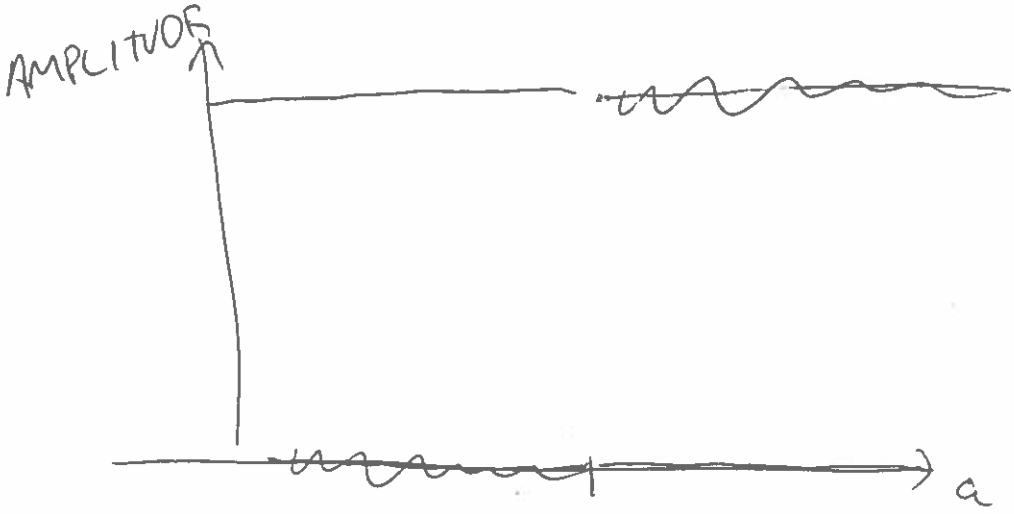
Van der Pol :

$$\frac{dx}{dt} = x' = y - x\left(\frac{x^2}{3} - 1\right), \quad \text{FAST}$$

$$\frac{dy}{dt} \quad y' = \varepsilon(a - x). \quad \text{SLOW}$$

WITH $\varepsilon << 1$.





≈ 1

DISCONTINUOUS?

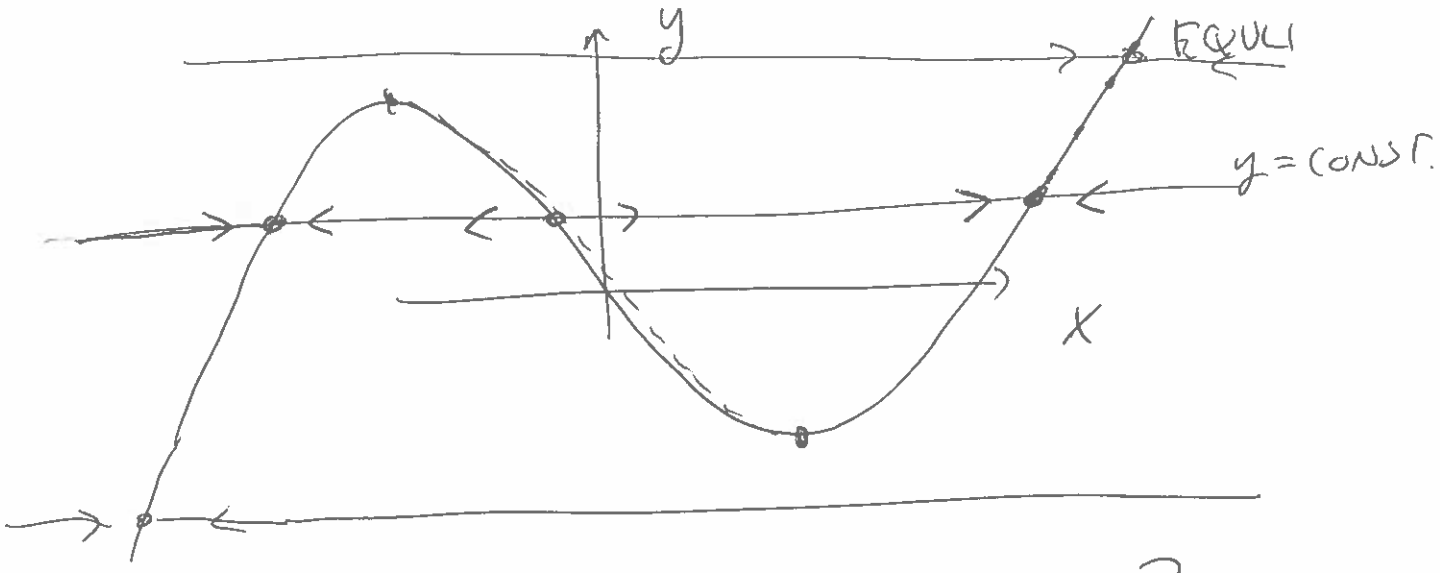
EQ: $x_* = a, \quad y_* = a\left(\frac{a^2}{3} - 1\right)$

$$A = Df(x_*, y_*) = \begin{bmatrix} -a^2+1 & 1 \\ -\varepsilon & 0 \end{bmatrix}. \quad \begin{array}{l} tr(A) = 1-a^2 \\ det(A) = \varepsilon > 0. \end{array}$$

HOPF AT $a = \pm 1$! FOR $\varepsilon > 0$.

PUT $\varepsilon = 0$:

$$\left.\begin{array}{l} x' = y - x\left(\frac{x^2}{3} - 1\right) \\ y' = 0 \quad \text{PARAMETER} \end{array}\right\} \text{LAYER PROBLEM}$$



BUT y NOT PARAMETER?

τ: FAST TIME

$t = \varepsilon\tau$ SLOW TIME

$$\boxed{\varepsilon \frac{dx}{dt} = \varepsilon \frac{d\tau}{dt} \frac{dx}{d\tau} = x' = y - x\left(\frac{x^2}{3} - 1\right)}$$

PUT $\varepsilon = 0$:

$$0 = y - x\left(\frac{x^2}{3} - 1\right)$$

$$\dot{y} = a - x$$

} REDUCED PROBLEM

RESTRICTED TO: ↖ ATTRACTING!

$$y \overset{*}{=} x\left(\frac{x^2}{3} - 1\right) \Rightarrow x = x(y) \; !!$$

$$\dot{y} \overset{\text{by } *}{=} (x^2 - 1)\dot{x} = a - x$$

$$\dot{x} \overset{\text{D}}{=} \frac{a - x}{x^2 - 1} \quad \text{FOR } x \neq \pm 1.$$

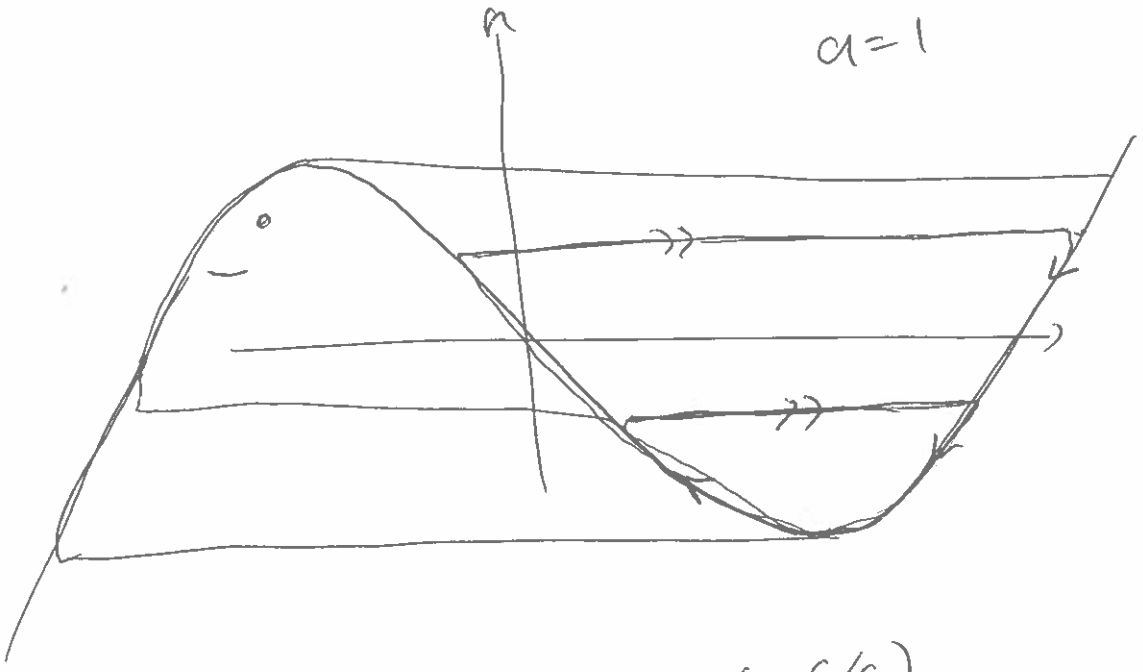
NOTICE FOR $a = 1$ THEN RHS

$$\lim_{x \to 1} \frac{1 - x}{x^2 - 1} = \lim_{x \to 1} \frac{1 - x}{(x-1)(x+1)}$$

$$= \ell - \frac{1}{2}$$

~~FOR (D) IT~~

$\Rightarrow$ THERE EXISTS A SOL OF (D) THROUGH $x = 1$!



WIDTH $\simeq$ $\mathcal{O}(e^{-c/a})$ $c>0$!!