Dynamical Systems 1 Kristian Uldall Kristiansen and Frank Schilder
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Week 3

On today

In this course, we will primarily focus on dynamical systems that arise from initial value problems of differential equations:

$$\dot{x} = f(x), \quad x(0) = x_0. \tag{1}$$

Here $x = x(t)$ but we often suppress this, in line with the phase space approach of dynamical systems theory. Today we will focus on well-posedness of such initial value problems, that is: Existence, uniqueness and smooth dependency on initial data, properties that make (1) amendable to dynamical systems theory. We will define a nonlinear flow and introduce the concept of invariance.

Read

Section 2.5 in [Per00]. Background reading: Sections 2.1-2.3. Might be useful to skip the proofs initially. Focus on definitions, the theorems and the examples!

NB!

The lecture will be short 45 min. We will have 3 hrs tutorials instead. So bring the exercise sheets from last if you have not finished it.

Key words

Well-posedness, flow, orbits, invariant sets.

Exercises

Exercise 1 Problem 1 in problem set 5 on p. 100.

Exercise 2 Problem 3 in problem set 5 on pp. 100-101. Can you write a Maple code that animates the flow of $N_\epsilon(x_0)$?

Exercise 3 Problem 5 in problem set 5 on p. 101. Show that S is invariant (a) directly using the computed flow and (b) using the equations of motion. Sketch the flow.

Exercise 4 Consider $\dot{x} = \sqrt{|x|}$. Show that

$$x = \frac{t^2}{4}$$

solves the equation for $t \geq 0$. Is it true that any solution of $\dot{x} = \sqrt{|x|}$ is monotonically increasing/decreasing or constant as we learned in week 1? Can you explain the apparant paradox? Compare with the Theorem on p. 74 in [Per00].

Exercise 5* Problem 6 in problem set 5 on p. 101.

Exercise 6** Problems 6,8, and then 5 on pp. 78-79. The purpose of this set of exercises is to prove the local existence and uniqueness of solutions to the initial value problem $\dot{x} = f(x)$, $x(0) = x_0$. Hint: Lipschitz functions are defined in Definition 2 on p. 71.

Next week

Next week we will begin our study of nonlinear systems locally near an equilibrium. We start of by presenting and analysing the stable manifold theorem.

References

[Per00] Perko, L., Differential Equations and Dynamical Systems, Texts in Applied Mathematics 7, Springer-Verlag, New York, 2000.