– CHEMICAL OSCILLATORS

YOUTUBE

MODEL FOR CHLORINE DIOXIDE–IODINE–MALONIC ACID REACTION

$$\dot{x} = a - x - \frac{4xy}{1+x^2},$$

$$\dot{y} = bx\left(1 - \frac{y}{1+x^2}\right).$$

PARAMETERS: $a, b > 0$

VARIABLES: $x, y \geq 0$.

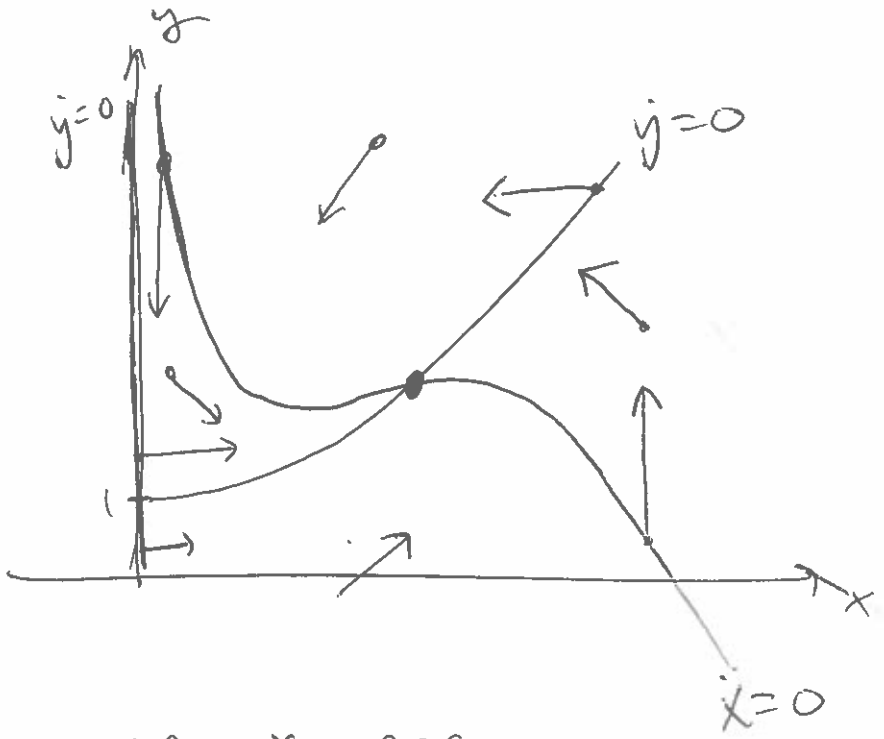
NULLCLINES:

$$N_x = \{(x,y) \mid \dot{x} = 0\}$$
$$N_y = \{(x,y) \mid \dot{y} = 0\}$$

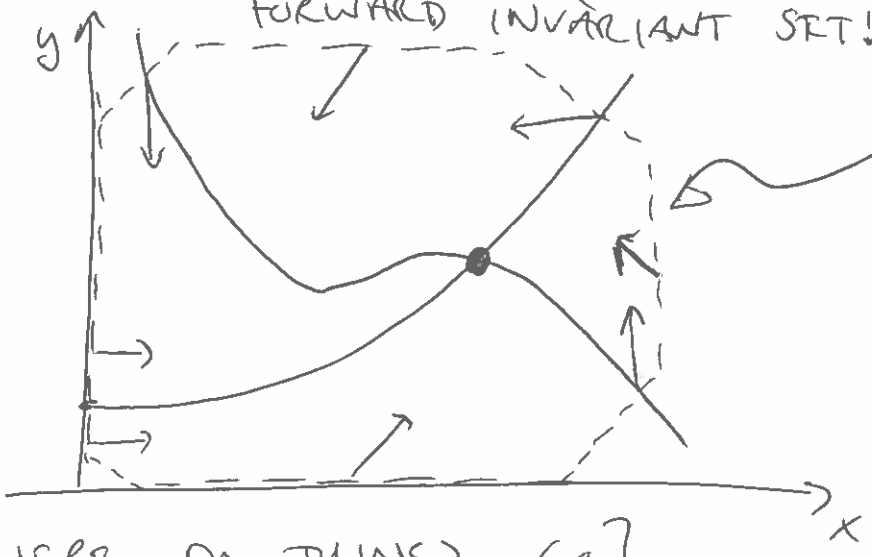
~~HERTER~~ HERE

N_x: $y = \dfrac{(a-x)(1+x^2)}{4x}$

N_y: $x = 0$ ✓ $y = 1 + x^2$ //



TRAPPING REGION (WEEK 11)
FORWARD INVARIANT SET!



T: $\forall x \in T \Rightarrow \varphi_t(x) \in T$ $\forall t \geq 0$ //

WHERE DO THINGS GO?

EQUILIBRIUM:

$$1+x^2 = \frac{(a-x)(1+x^2)}{4x}$$

$$4x = a - x, \quad 5x = a, \quad \boxed{x_* = \frac{a}{5}}$$

$$\boxed{y_* = 1 + \frac{a^2}{25}}$$

LINEARIZATION?

[illegible]

LINKARIZATION

$$A = Df(x_*, y_*) = \frac{1}{a^2+25}\begin{bmatrix} 3a^2-125 & -20a \\ 2ba^2 & -5ba \end{bmatrix}$$

EIGENVALUES ? !!

$$\lambda_1\lambda_2 = \det(A) = \frac{25ba}{a^2+25} > 0$$

NOT A SADDLE! FOCUS / NODE!

$$\lambda_1+\lambda_2 = \operatorname{tr}(A) = \frac{3a^2-5ab-125}{a^2+25}$$

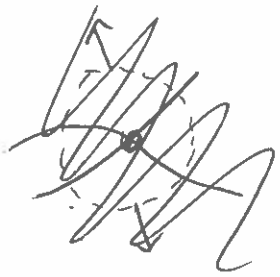
$$\operatorname{Re}(\lambda_{1,2}) < 0 \iff \operatorname{tr}(A) < 0$$

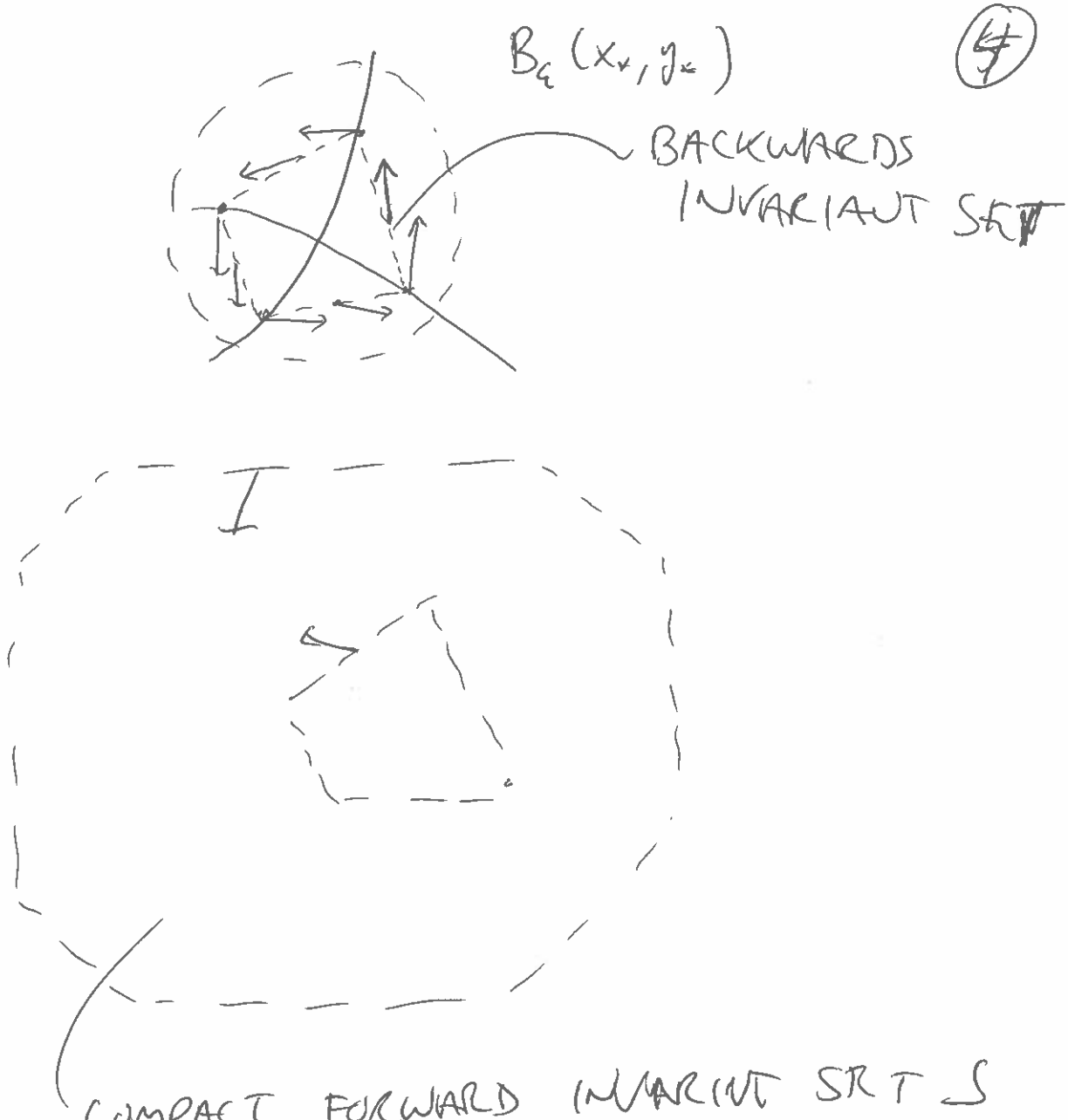
$$b < \frac{3a^2-125}{a^2+25}$$

HENCE:

$\operatorname{Re}(\lambda_{1,2}) > 0$ IF $\operatorname{tr}(A) \geq 0$ i.e. $b \geq \dfrac{3a^2-125}{a^2+25}$

IF EQ (x_*, y_*) UNSTABLE, WHAT IS INSIDE ?





COMPACT FORWARD INVARINT SET S
WHICH DOES NOT CONTAIN EQ.
POINCARE-BENDIXSON: S CONTAINS
~~LIMIT CYCLE~~ PERIODIC ORBIT!

WEEK 12.

BUT NOTICE:

$tr(A) = 0$ WHEN $b = b_{cr}(a)$!

CHANGES STABILITY! EQ. NON-HYPERBOLIC

HOPF THEOREM: SUPPOSE (x_*, y_*) WEEK 10

WITH $\lambda_{1,2}(b)$ COMPLEX:

$$Re\,\lambda_{1,2}(b_{cr}) = 0, \quad tr\,\lambda_{1,2}(b_{cr}) \neq 0$$

$$\left(Re'\,\lambda_{1,2}\right)(b_{cr}) \neq 0$$

$\Rightarrow$ THEN A FAMILY OF PERIODIC ORBITS EMERGES FROM THE EQ AT $b = b_{cr}$