Assignment 5

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Abstract

Your abstract.

1 Question 1: State space formulation

In this problem, the trajectory of a satellite is observed, taking 50 measurements of the radius and the angle. The measurement are poisoned with withe noise, and it can be described as follow:

$$r_t^m = r_t^p + \epsilon_{rt}$$
$$\theta_t^m = \theta_t^p + \epsilon_{\theta t}$$

where $\epsilon_r t \sim N(0, 2000^2)$, $\epsilon_{\theta t} \sim N(0, 0.03^2)$ and $\epsilon_r t$, $\epsilon_{\theta t}$ are independent.

The model of the trajectory, can be described with the following stochastic, equations:

$$r_t^p = r_{t-1}^p + \varepsilon_{rt}^p$$
$$\theta_t^p = \theta_{t-1}^p + v_{t-1}^p + \varepsilon_{\theta t}^p$$
$$v_t^p = v_{t-1}^p + \varepsilon_{v_\theta t}^p$$

where r_t^p is the real radius, θ_t^p is the angle, and $v_{\theta t}^p$ is the angular velocity. The model describe a circular orbit with some perturbations, which are modeled by the following distributions:

$$\varepsilon_{rt}^p \sim N(0, 500^2), \quad \varepsilon_{\theta t}^p \sim N(0, 0.005^2), \quad \varepsilon_{v_\theta t}^p \sim N(0, 0.005^2)$$

With the given information we can construct the stochastic space model as

$$X_t = AX_{t-1} + e_1$$

$$Y_t = CX_t + e_2$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$X_t = \begin{pmatrix} X_{1,t} \\ X_{2,t} \\ X_{3,t} \end{pmatrix} = \begin{pmatrix} r_t^p \\ \theta_t^p \\ v_{\theta t}^p \end{pmatrix}, \quad Y_t = \begin{pmatrix} r_t^m \\ \theta_t^m \end{pmatrix} = \begin{pmatrix} Y_{1,t} \\ Y_{2,t} \end{pmatrix}$$

$$e_1 \sim N(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 500^2 & 0 & 0 \\ 0 & 0.005^2 & 0 \\ 0 & 0 & 0.005^2 \end{bmatrix}), \quad e_2 \sim N(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2000^2 & 0 \\ 0 & 0.03^2 \end{bmatrix}))$$

The third state, the satellite velocity, is not directly measured. However, thanks to the correlation between the velocity and the angle this variable is possible to estimate. Precisely, the Kalman filter implemented in the next section address, among others, this problem.

2 Question 2: Kalman filter implementation

Below it can found the implementation of the kalaman filter. It is based in the formulas provides in the slides from lecture 11.

```
############## Kalaman filter initalization
X_estimated <- matrix(0,nrow = 3, ncol = 60)
stored_K <- matrix(0,nrow = 3, ncol = 60)
stored_klist <- list()
stored_SigmaXX <- list()

Xhat <- matrix(c(38000,0,0.0269356),nrow=3)
SigmaXXhat<- matrix(c(1000,0,0,0,0,0,0,0,0),nrow = 3)
SigmaYYhat <- matrix(c(1,0,0,1),nrow = 2)
SigmaXYhat <- SigmaXXhat%*%t(C)

for(n in 1:50){
  #estimation
  #K = SigmaXYhat%*%solve(SigmaYYhat)
  K = SigmaXXhat%*%t(C)%*%solve(SigmaYYhat)
  Xestim = Xhat + K%*%(Y[,n]-C%*%Xhat)
  SigmaXX <- SigmaXXhat - K%*%SigmaYYhat%*%t(K)

  #Storing data
  X_estimated[,n] <-Xestim
  stored_SigmaXX[[n]] <- SigmaXX

  #prediction
  Xhat <- A%*%Xestim
  SigmaXXhat <- A%*%SigmaXX%*%t(A) + Sigma1
  SigmaYYhat <- C%*%SigmaXXhat%*%t(C) + Sigma2
  SigmaXYhat <- SigmaXXhat%*%t(C)
}
```

An important issue to point is the initialization of the filter. The chosen starting point is:

$$X_t = \begin{pmatrix} X_{1,t=0} \\ X_{2,t=0} \\ X_{3,t=0} \end{pmatrix} = \begin{pmatrix} 38000 \\ 0 \\ 0.026 \end{pmatrix}$$

The starting radius and angle are extracted from the first measurement. On the other hand, the angular velocity is calculated for the given radius. It is the necessary velocity for the satellite to be on that specific orbit.

3 Question 3: Trajectory reconstruction

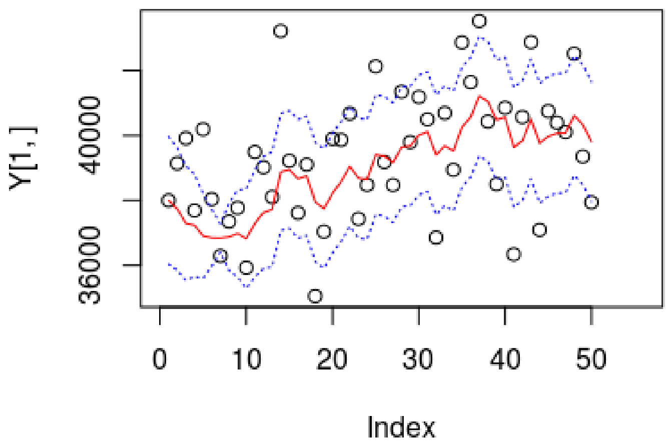

Figure 1: Radius reconstruction

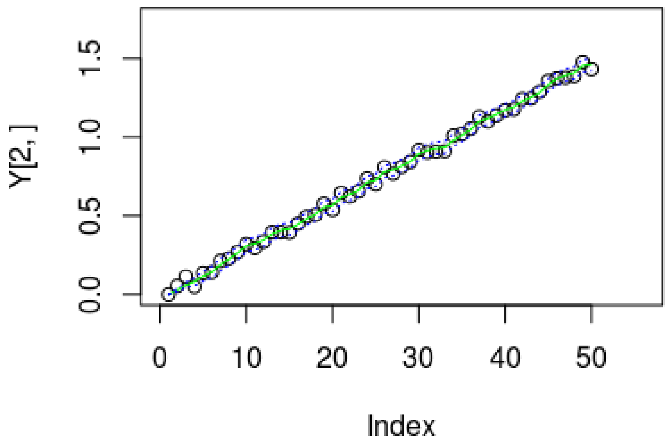

Figure 2: Angle reconstruction

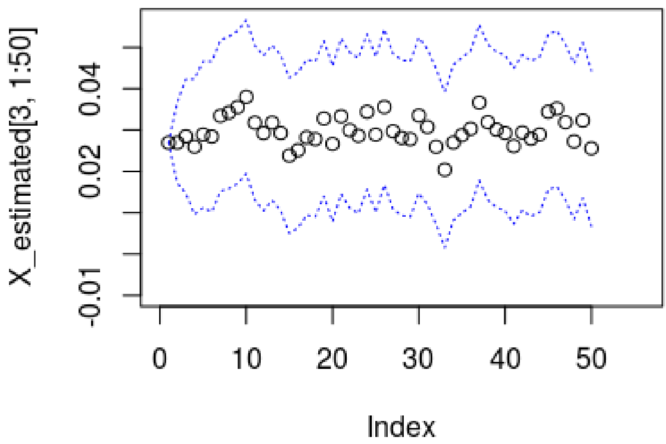

Figure 3: Angular velocity estimation

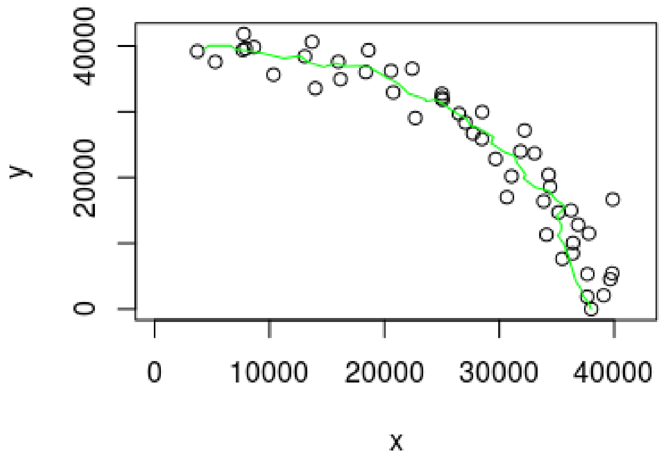

Figure 4: Estimated trajectory

4 Question 4: Predictions

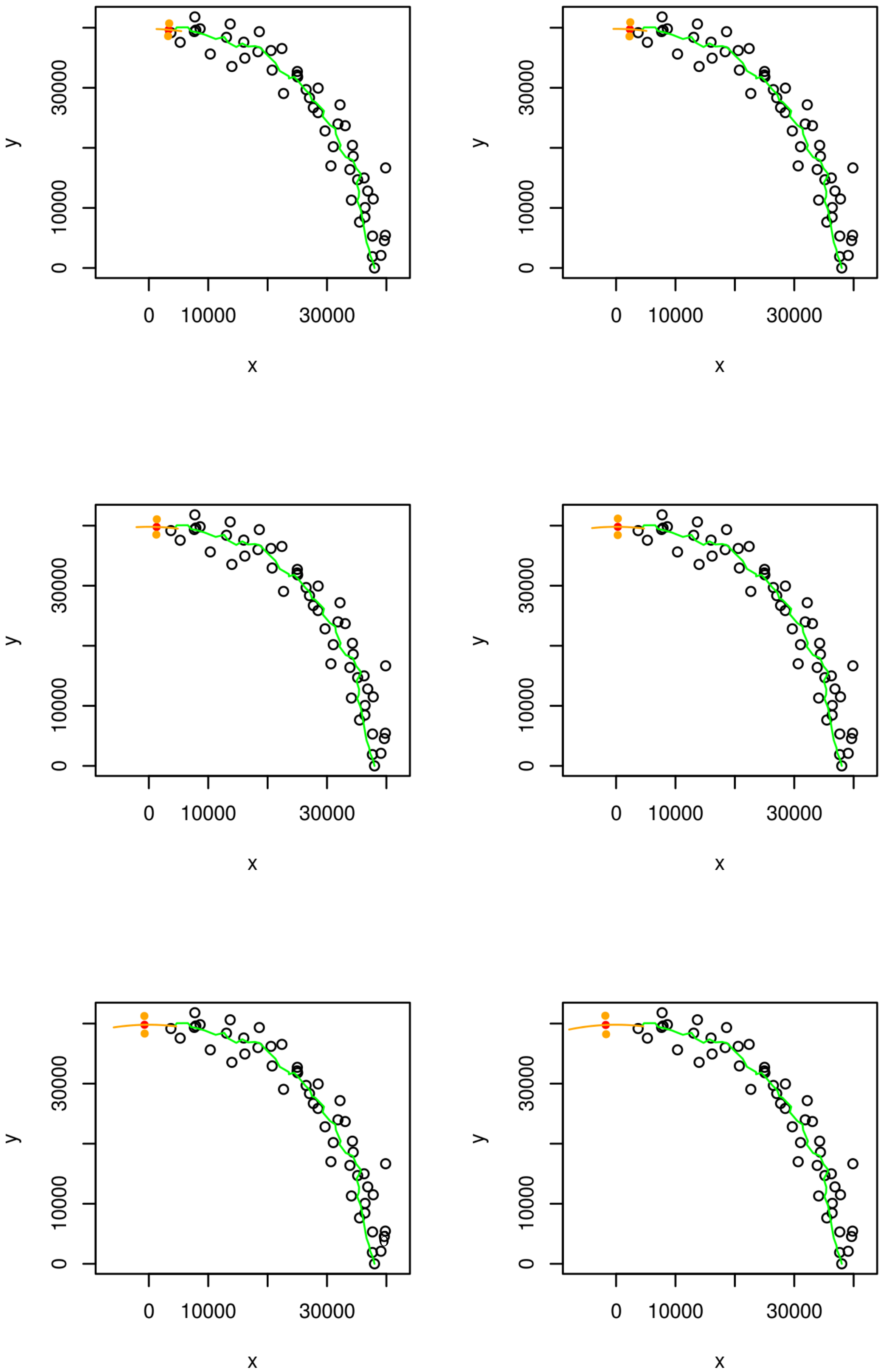

Figure 5: Estimated trajectory