Dynamical Systems 1
Course 01617
E16

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Time and location

Mondays 13:00 - 17:00, typically with lectures from 13:00-14:30 and exercises afterwards. We will be located in

Building 3??, room ??

Laptop and Maple

Bring your laptop to classes. Occasionally we will use Maple for illustrations so it will be a good idea to have this installed on your laptop. You will not need Maple for algebraic manipulations - these will typically be easier to do by hand.

Course assessment

3 assignments and 1 multiple choice. Details:

- Assignment $\#1$. Hand out: Week 2. Hand in: Week 4.
- Multiple choice. 25% of full mark. 1hr in course week 7.
- Assignment $\#2$. Hand out: Week 8. Hand in: Week 10.
- Assignment $\#3$. Hand out: Week 12. Hand in: Week 14.

Assignments $\#1$, $\#2$ and $\#3$ count for 75% of the total mark.

Course material

We will primarily follow [Per00]. It is available at the campus book store. But occasionally we will also use [Str94] and lecture notes on campusnet.

Course responsible

- Poul G. Hjorth (PGH). Contact: pghj@dtu.dk
- Kristian Uldall Kristiansen (KUK). Contact: krkri@dtu.dk

Tentative course plan

- **Basics**: Weeks 1-3 (All KUK)
- **Local theory**: Weeks 4-9 (KUK: 4,6,7. PGH: 5,8-12).
- **Global theory (for planar systems)**: Weeks 10-12 (PGH)
- **Repetition**: Week 13. (PGH and KUK).

Basics

- Week 1: Introduction by [Str94] and lecture notes.
  - Phase space: Solution approach vs. geometric approach
  - Motivating examples
  - 1D systems
- Week 2: Linear systems [Sections 1.4-1.7 and 1.9 in [Per00]]
  - Linear solution theory: Matrix exponential. (Example of a flow).
  - 2D linear systems: saddles, sinks and foci.
  - Stability theory. Invariant subspaces E^s, E^u and E^c.
- Week 3: Nonlinear flow. [Section 2.5 in [Per00]]
  - Flow and flow defined by nonlinear differential equations.
  - Invariant sets. Equilibria as an example of invariant set.
  - Stable and unstable sets.

Local theory

- Week 4: Equilibria and hyperbolicity. [Sections 2.6 and 2.7 in [Per00]]
  - Equilibria. Linearization. Hyperbolicity.
  - Stable manifold theorem. No need to introduce manifolds, really. Just use graphs.
- Week 5: Hartman-Grobman [Section 2.8 in [Per00]]
  - Topological conjugacy.
  - Hartman-Grobman
  - Consequences.
- Week 6: Stability [Section 2.9 in [Per00]]
  - Definitions.
  - Theorem: Stability of linearization vs stability of nonlinear system.
- Week 7: Summary+Mid-term!
- Week 8: Center manifolds [Section 2.12 in [Per00]]
  - Recall linear invariant spaces.
  - Theorem.
  - Applications. There are many.
- Week 9: Local bifurcations I [Section 4.2 in [Per00]]
  - Structural stability.
  - Implicit function theorem.
  - Focus on normal forms: saddle-node, transcritical, pitchfork.
  - Theorem on saddle-node.
  - Applications.
- Week 10: Local bifurcations II [Sections 3.3 and 4.4 in [Per00]]
  - Definition of periodic orbits and their stability (here we need something which is not local to describe a bifurcation which is local...)

  - Hopf normal form.
  - Theorem.
  - Applications.

Global theory (for planar systems)

- Week 11: Concepts from phase plane analysis [Lecture notes]
  - Use of null-clines.
  - Use of "trapping regions".
  - Use of "invariant manifolds".
  - Applications.
- Week 12: The Poincare-Bendixson Theory [Section 3.7 in [Per00]]
  - Limit sets.
  - Theorem.
  - Applications.
- Week 13: Repetition.

References

[Str94] Strogatz, S. H., Nonlinear Dynamics and Chaos - with Applications to Physics, Biology, Chemistry, and Engineering, Studies in Nonlinearity, 1994.

[Per00] Perko, L., Differential Equations and Dynamical Systems, Texts in Applied Mathematics 7, Springer-Verlag, New York, 2000.