02685 Assignment 01: Ordinary Differential Equations

John Bagterp Jørgensen

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1 The Test Problem and DOPRI54

Consider the initial value problem (test equation)

$$\dot{x}(t) = \lambda x(t) \quad x(0) = 1 \qquad \lambda = -1$$

and (harmonic oscillator)

$$\ddot{x}(t) = -x(t) \qquad x(0) = 1; \qquad \dot{x}(0) = 0$$

1. Implement the following methods for these problems: 1) the explicit Euler method, 2) the Implicit Euler method, 3) the Trapezoidal Method, 4) the Classical Runge-Kutta method, 5) the two methods in DOPRI54
2. Derive the analytical solutions for these problems
3. Plot the global error of the numerical solution at time $t = 10$.
4. Plot the local error at $t = t_0 + h$ as function of step size h.
5. Plot the estimated local error by the method.
6. Discuss the results and the quality of the error estimator for DOPRI54
7. Can you come up with other (simple) methods for estimation of the local error

2 The Van der Pol System

For different choices of the step size h do:

1. Test your methods (1-5 from Problem 1) on the Van der Pol problem ($\mu = 3$). Plot the estimates of the errors as well as the solution.
2. Test your methods on the Van der Pol problem with $\mu = 100$. Describe what happens.

3 Design your own Explicit Runge-Kutta Method

Design your own explicit Runge-Kutta method and apply it to the test equation and the Van der Pol equation.

1. Write up the order conditions for an embedded Runge-Kutta method with 3 stages. The solution you advance must have order 3 and the embedded method used for error estimation must have order 2.
2. Derive the coefficients for the error estimator.
3. Write up the Butcher tableau for your method and the corresponding equations defining the method.
4. Test your solver on the test equation with $\lambda = -1$.
5. Verify the order of your method by plotting the local error as function of step size (for the test equation).
6. Compute $R(h\lambda)$ for your method and make a stability plot of your method.
7. Test your solver on the van der Pol equation with $\mu = 3$. Compare the solution you get with the solution you get when using `ode15s`.

4 Step Size Controller

Implement the five methods considered in problem 1 with adaptive time steps using

1. an asymptotic step size controller
2. a PI step size controller

Test and compare your methods on the Van der Pol problem with $\mu = 3$. You must test your methods for different absolute and relative tolerances.

5 ESDIRK23

1. Implement ESDIRK23 with fixed step size.
2. Test your implementation on the Van der Pol equation with $\mu = 3$ and $\mu = 100$. Compare the solution and the number of function evaluations with your own Explicit Runge-Kutta method.
3. Plot the stability region of the ESDIRK23 method. Is it A-stable? Is it L-stable? Discuss the practical implications of the stability region of ESDIRK23.
4. Implement ESDIRK23 with variable step size. Test it on the Van der Pol problem.

Hand in a short report describing your findings to Anne-Mette Larsen, Building 303B, Office 105. We must receive the report no later than: March 16 2017, no later than 13:00 (just at the beginning of the lecture). You must also upload an electronic version to CampusNet (a pdf file, and a zip file with all matlab code, latex code etc used to answer the questions).

On the report you should clear write name, study number and group number.

REMEMBER: Your report should include an answer of the questions and Matlab code documenting how you found the solution. In CampusNet you should upload the pdf as well as zip file with all Matlab code.