Stochastic Adaptive Control

Excercise part 15

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The focus of this excercise is analyses and control of stochatic systems given in external form ie. described by transfer functions. At your disposal for solving this exercise is the following procedures

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wb                    work bench

sfak                  Spectral factorization

spec2sd               Spectrum to spectral density
sd2spec               Spectral density to spectrum

dsn*                  Design of various controllers.

trfvar                Determine the variance amplification (AC gain) of
                      a transfer function.
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Excercise 1

(MV_0-control.) Consider the system

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + C(q^{-1})e_t$$

where

$$\begin{aligned} A(q^{-1}) &= 1 - 1.5q^{-1} + 0.7q^{-2} \\ B(q^{-1}) &= 1 - 0.5q^{-1} \qquad k = 1 \\ C(q^{-1}) &= 1 - 0.9q^{-1} + 0.18q^{-2} \qquad \sigma^2 = 0.1 \end{aligned}$$

Introduces this description into *sysinit.m* .

Question 1.1 Design (i.e. determine the Q, R and S polynominals) a MV_0 controller. Do it by hand i.e. use e.g. *truncimp* to solve this simple version of the diophantine equation. Check with the steps in *dsnmv0* □

Question 1.2 Find a state space realization of the the QRS controller. Check e.g. with the solution (which is just one solution) in *wb* . □

Question 1.3 Determine the closed loop transfer function from reference signal to output and control signal (as text book transfer functions). □

Question 1.4 Perform a deterministic simulation (set *dets* in *wb* to 1 which gives a deterministic simulation with a step as reference signal) and compare the result with expected from the closed loop transfer functions. □

Also notice, the work bench (in the end) is set up to determine the closed loop transfer functions from noise and reference to the output and control action (ie. 4 transfer functions). In the stochastic situation it also determine the variance of the output and control signal.

Question 1.5 Compare the resulting closed loop transfer functions (determined in the end of the work bench) with the expected ones from the text book. □

Question 1.6 Perform a stochastic simulation (set *dets* in *wb* to zero which also results in a zero reference). Compare the obtained emperical (J_c and J_u) variance with theoretical values (determined in the end of *wb*). □

Excercise 2

(Different control strategies.) The Nomoto model of a ship can be described by the following model. The yaw rate r is the derivative of the heading ψ, ie.

$$\dot{\psi} = r$$

and is a first order dynamic function of the rudder angle u, ie.

$$\dot{r} + \tau\ r = \kappa u$$

For simplicity, let $\kappa = 100$ and $\tau = 1$.

Question 2.1 Find the transfer function from rudder angle (control input) to the the heading. □

Question 2.2 Find the discrete time version when the sampling frequency is 10 Hz. Check the location of the zeros and poles of the description. □

This description can also be brought into the form

$$A(q^{-1})y_t = q^{-k}B(q^{-1})u_t + v_t$$

where v_t is the resulting disturbance. Assume, that the properties of the disturbances has been analysed and the analysis have resulted in the following spectral density

$$\phi(\omega) = 0.124 - 0.0176cos(\omega) - 0.024cos(2\omega)$$

where ω is the normalized angular frequency. Notice, *sfak* produce a nonmonic polynomial. Let m be a specktrum. Then the following matlab commands will give the C polynomial (monic) and the variance of the innovation, e_t.

```
Cn=sfak(m); vare=Cn(1)^2; C=Cn/Cn(1);
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Question 2.3 Find the ARMAX description of the system, ie. A, B, k, C and σ^2. □

Introduces this description into *sysinit.m* . Notice, the work bench (in the end) is set up to determine the closed loop transfer functions from noise and reference to the output and control action (ie. 4 transfer functions). In the stochastic situation it also determine the variance of the output and control signal.

The performace of a controller can be judged from its ability to track a refence (servo problem) and to reduce the effect of the noise or disturbance (regulation problem). The tracking ability is best evaluated in connection a step change and in a deterministic situation. On the other hand it is best to see the effect of the controller in a stochastic situation if the reference is constant (and zero). The work best is set up to produce a deterministic simulation (ie. the variance of the noise ($vare = 0$) is zero and the reference signal is a step change) if the variable *dets* is 1. If *dets* is 0 the reference is zeros and the variance of the innovation is as determined in the previuous questions.

Question 2.4 Design a MV_0 controller and check the closed loops deterministic step response. □

Question 2.5 Also check the stochastic response, ie. run the work bench with *dets=1* . Determine the variance of the output and control and compare with the simulations. □

A reference model can be of the type:

$$H_w = \frac{1-\beta}{1-\beta q^{-1}} \tag{1}$$

where β is chosen properly. Proper means such that the output response and the control action is reasonable both in a deterministic and a stochastic situation.

Question 2.6 Design a PZ controller and check the closed loops deterministic step response (find a suitable value for β). Also check the stochastic response and compare with theoretical values (determine the variance of the output and control). □

The oscilations in the control is due to the system zero which is rather close to the stability limit. If we use a GSP controller we might avoid canceling the zero(s).

Question 2.7 Design a GSP controller (in which we do not cancel any zeros) and check the closed loops deterministic step response (find a suitable value for β). Also check the stochastic response and compare with theoretical values (determine the variance of the output and control). □

The tuning parameter in the MV_1 controllers is ρ. In the procedure *dsnmv1a* the controller which minimize the cost

$$J_1 = \mathbf{E}\left\{(y_{t+k} - w_t)^2 + \rho u_t^2\right\}$$

is minimized.

Question 2.8 Design a MV_1 controller and check the closed loops stochastic response (find a suitable value for ρ). Compare theoretical values for error and control variance and compare with values obtained in simulations. Also check the deterministic step response. □

The problem when designing a controller minimizing the cost mentioned above is the steady state error for a non zero reference. This is avoided when minizing

$$J_2 = \mathbf{E}\left\{(y_{t+k} - w_t)^2 + \rho(u_t - u_{t-1})^2\right\}$$

The design is performed in *dsnmv1* .

Question 2.9 Design a MV_1 controller (in which we are minimizing J_2) and check the closed loops deterministic step response (find a suitable value for ρ). In your search for a proper values for β you might let it increasing from a very small number by the recursion

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beta=beta*5; wb
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Also check the stochastic response and compare with theoretical values (determine the variance of the output and control). □

The reference model with respect to the reference has to have a DC amplification equal to 1. This is not the case for the stochastic reference. A simple reference model is

$$y_m^e = \frac{1}{1 - \gamma q^{-1}} e_t \tag{2}$$

Question 2.10 Design a MV_2 controller and check the closed loops stochastic response (find a suitable value for β and γ. Try the combination of $\beta = [0.1 \quad 0.9]$ and $\gamma = [0.1 \quad 0.9]$). Compare theoretical values for error and control variance and compare with values obtained in simulations. Also check the deterministic step response. □

The LQG controller is (also for the external model structure) based on an infinite horizon strategy.

Question 2.11 Design a LQG controller and check the closed loops stochastic response (find a suitable value for ρ) Compare theoretical values for error and control variance and compare with values obtained in simulations. Also check the deterministic step response. □