Stochastic Adaptive Control

Exercise part 1

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5. februar 2016

The purpose of this first exercise is to get acquainted with a simulation model and the work bench (matlab code). In the actual case the simulation model (ie. the plant model) is a coupled tank system, which is a slightly nonlinear second order system. The focus is on stationary points, linearization, offset, eigenvalues, sampling and control.

The work bench can be found on the course web page:

`http://www2.imm.dtu.dk/courses/02421/dist1.zip`

Get (you can see how on the web page) the files and read the Read.Me file. It is recommended to use a discrete time linear model (sflag=1) in the development phase and only use the nonlinear model (sflag=3) i the final run. The solution can be downloaded (in a separate directory) and examined with the command solution.

For solving the exercises you can use a toolbox available ont he course homepage and on the G-bar (under ˜nkpo/toolbox).

Excercise 1

We consider the problem of controlling the levels in a two coupled tank system (see figure 1). The control signal (U_t) is the in-flow to the first tank and the objective is to control the level (y_t) in the second tank. The two tank are connected through a pipe. The set point, ie. the desired value of the level in the second tank, is denoted as w_t.

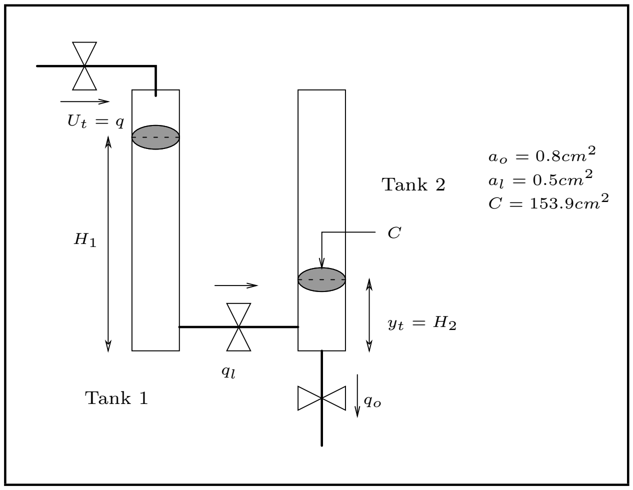
The state of the system is chosen to be

$$\begin{aligned} X_1 &= H_2 && \text{ie. the level in the second tank} \\ X_2 &= H_1 - H_2 && \text{ie. the difference in levels} \end{aligned}$$

then the dynamics of the system can be described as

$$C(\dot{X}_1 + \dot{X}_2) = U - q_l \tag{1}$$

$$C\dot{X}_1 = q_l - q_o \tag{2}$$



Figur 1. The coupled tanks system

where C denotes the area of the cross section of the tank (the two tank have the same cross section). The flows, q_l and q_o, are assumed to obey the Bernoulli equation, ie.

$$q_l = \sigma_l a_l \sqrt{2gX_2} \tag{3}$$

$$q_o = \sigma_o a_o \sqrt{2gX_1} \tag{4}$$

Note it assumed that the tank system is operated in its normal mode where $X_1 \geq 0$ and $X_2 \geq 0$. The constants of the system are

$$C = 153.938\ cm^2 \qquad g = 981\ cm/s^2$$

$$a_l = 0.5\ cm \qquad a_o = 0.8\ cm$$

$$\sigma_l = 0.44 \qquad \sigma_o = 0.31$$

(These parameters are available in the matlab file tank_par.m).

Question 1.1 Determine the steady state values of state vector and control signal X_0 and U_0 which results in an output, corresponding to a desired value w_0. (Don't let you be confused by the fact, that X_0 is the steady state vector, whereas X_1 and X_2 are the elements in the state vector X.) Ie. determine the mapping from w_0 to X_0 and U_0. Implement this as a matlab function which obey the syntax

```
[U0,X0]=offset(w0)
```

What is the values for X_0 and U_0 if $w_0 = 19\ cm$. □

In the following we will use this point (related to $w_0 = 19\ cm$) as our operation point.

The state equations, (1) and (2), can be written as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -a_1 & a_2 \\ a_1 & -2a_2 \end{bmatrix} \begin{bmatrix} \sqrt{X_1} \\ \sqrt{X_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} U \tag{5}$$

where

$$a_1 = \sigma_0 a_0 \frac{\sqrt{2g}}{C} \qquad a_2 = \sigma_l a_l \frac{\sqrt{2g}}{C}$$

Now, let x denote the deviation in X from X_0 and let u denote the deviation in U from U_0. Then the linearized description of the tank system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Bu \tag{6}$$

where

$$A = \begin{bmatrix} -a_1 & a_2 \\ a_1 & -2a_2 \end{bmatrix} \begin{bmatrix} \frac{1}{2\sqrt{X_{10}}} & 0 \\ 0 & \frac{1}{2\sqrt{X_{20}}} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix}$$

Question 1.2 Verify (5) and (6) and determine the eigenvalues of the system. Also determine the time constants of the system. □

In the following we will use a sampling period equal $T_s = 5$ *sec*.

Question 1.3 Determine the discrete time (linear) model and its eigen values. (Hint: use e.g. *c2d, eig*). □

In the distributed version of the work bench, the set point (*wt*) change after a short period from w_0 to $w_0 + 5$ *cm*.

Question 1.4 Use the work bench (ie. edit the wb.m file) to produce a step response and determine (approx.) the rise time of the system. □

The matlab command `dlqr` can be applied to determine a controller ie. L in

$$u_t = -Lx_t$$

such that the cost function

$$J_t = \sum_{i=1}^{\infty} x_i^T Q x_i + u_i^T R u_i$$

is minimized. (Write `help dlqr` for checking the syntax). In the distributed work bench a discrete time model (A,B,C) is determined by *sysinit* .

Question 1.5 Use `dlqr` to determine a controller (ie. L) that minimize the cost for $Q = I$ and $R = 0.01$. □

In order to cope with set point deviations (from w_0) the following control law

$$u_t = Mw_t - Lx_t \tag{7}$$

is used instead.

Question 1.6 Determine M such that the steady state error is zero (for the discrete time linear model). Use the work bench to produce a response (for the closed loop) to the given variation in the set point. □