Stochastic Adaptive Control

Project part 12

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This project (and project 24) has to be handed in on (or before) the last day in the examination period (which in 2016 is 1.6.2016).

The focus of this project is analysis of stochastic systems, state estimation and control. The application is the coupled tank system known from exercise 1 and other earlier exercises.

In a part of the project you are ask to do some simulations. You can use the workbench (wb.m from dist12) or you can program it yourself. In the project there are given some suggestions for using some matlab procedure. You are welcome to use other procedures or functions.

Excercise 1

The two tank system is seen in Figure 1. The two levels h_1 and h_2 are measured and available for

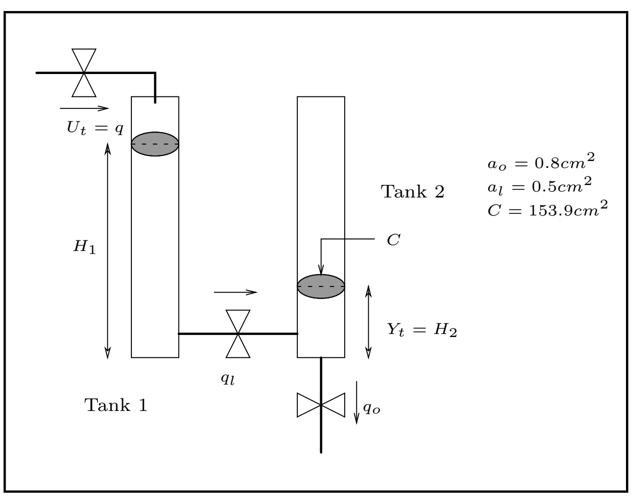

Figure 1. The coupled tanks system

digital control as illustrated in Figure 2. The sampled measurements are used (in the computer controller) to determine the control action, which through a D/A converter affects the pump and the flow into the first tank. We will assume that the continuous nonlinear system model has been linearized and sampled ($T_s = 5$ sec.) such that the control object is a discrete time system (We did this in Exercise 1).

The discrete time linear system is described by the process equation

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_{t+1} = A_s \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_t + B_s u_t + v_t \tag{1}$$

where v_t is a random vectors.

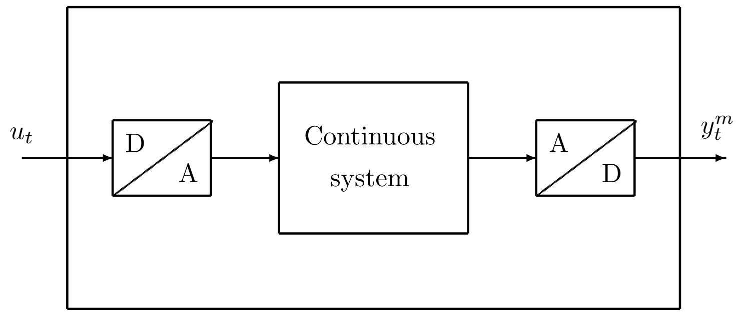

Figure 2. The sampled data control system

Assume that the uncertainty in the process equation is due to an non ideal pump. Let η_t be a perturbation of the control signal. Then

$$v_t = B_s \eta_t,$$

The uncertainty is assumed to follow $\eta_t \in N_{iid}(0, R_{1a})$ (i.e. η_t is a sequence of independent identically distributed Gaussian variable).

The objective is to control the system in such a manner that the (controlled) output

$$y_t = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_t = C x_t \tag{2}$$

is as close as possible to a set point (or reference signal).

Furthermore, the measurements are

$$y_t^m = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}_t = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_t + e_t = C_m x_t + e_t \tag{3}$$

where $e_t \in N_{iid}(0, R_2)$. Notice the difference between the measurements and the output.

The system matrices (including variance matrices) are accessible through the procedure *tsystem* (Use *help tsystem* for details).

Question 1.1 Determine the stationary distribution (type and parameters) of the output in the uncontrolled situation. Include an argumentation. □

The workbench (wb2.m) has been set up such that the structure *Dtsys* contains the tank system.

Question 1.2 Produce experimental results by simulation (with your own code or by using wb2). Compare the experimental results with theoretical results related to the previous question. Discuss whatever the experimental results match the theoretical results. Perform a number (100) of simulations and compare the results (with theoretical results). □

Let us now focus on control of the system and let us first assume perfect state information (ie. we know the states of the system). The aim is to control the output, but with a limited amount of control action. Consequently let

$$Q_1 = C^T C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \qquad\qquad Q_2 = 0.01$$

Question 1.3 Design a stationary ($N \to \infty$) LQ controller based on the assumptions mentioned above. (Use eg. *dlqr* or *sslq* .) Additionally find the closed loop description of the system with the controller and determine the theoretical stationary ($t_0 \to -\infty$) distribution of the output and the control signal. □

Question 1.4 Produce experimental results by simulations. Compare the experimental results with theoretical results related to the previous question. Include a discussion. □

The controller design above is a state feedback controller. However, since we have not got perfect state information a state estimator is needed.

Question 1.5 Design an ordinary stationary Kalman filter (ie. write down the recursions and find the gain). Find the stationary distribution (type and parameters) of the estimation error in the Kalman filter. □

Question 1.6 Verify experimentally the results found above. □

Question 1.7 Which statistical properties (on which signal) from the Kalman filter can be used to check that the model assumptions are correct? □

Question 1.8 Perform such a check experimentally (by simulation). □

Question 1.9 Find the closed loop description for the system (ie. write down the recursion). Notice, the closed loop system has $2n$ states, where n is the number of states in the open loop system. Also find the stationary distribution of the states, output and control. □

Question 1.10 Check experimentally (by simulation) the distribution of the states, output and control. □

It turns out that the assumption that η_t is white does not hold. Analysis shows that the state noise is more reasonably approximated by the following process model:

$$\begin{aligned} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}_{t+1} &= \begin{bmatrix} 1.8 & 1 \\ -0.95 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}_t + \begin{bmatrix} 1.5 \\ 0 \end{bmatrix} \zeta_t \\ \eta_t &= \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}_t \end{aligned}$$

where $\zeta_t \in N_{iid}(0, R_{1n})$. The system matrices are accessible through the procedure *nsystem* . (Use *help nsystem* for details).

Question 1.11 Determine R_{1n} such that the stationary distribution of η_t is unchanged (compared to previous). Use *e.q. dlyap* to determine the stationary variance gain from ζ_t to η_t. □

Question 1.12 Augment the tank system description with the noise process model mentioned above. □

The workbench (wb.m) has been set up such that the structure *Dtsys* contains the tank system and *Dnsys* contains the process noise system.

Question 1.13 Verify experimentally the distribution of η_t. □

Question 1.14 Design a stationary LQ controller based on the augmented state space description. Assume perfect state information. The control objective is unchanged. Find the closed loop description and find the (theoretical) stationary distributions of the output and control signal. □

Question 1.15 Verify experimentally the distribution of the output and control signal. □

Question 1.16 The variance of the output and control is different from that obtained in Question 1.3 (Although the variance and mean of v_t (and η_t) is unchanged). Give a qualitative explanation. □

Due to hardware limitations it turns out that it is problematic to implement the ordinary Kalman filter.

Question 1.17 What limitations in the hardware (sensors and digital computer) could make implementation of an ordinary Kalman filter problematic? □

Question 1.18 Design a stationary *predictive* Kalman filter instead (ie. state the recursions and find the gain). □

Question 1.19 Determine the total closed loop description (with tanksys, noise and estimator states). Furthermore determine the stationary distribution of the closed loop system states, output and control. □

Question 1.20 Verify experimentally the stationary distribution of the closed loop system states, output and control. □