Stochastic Adaptive Control

Exercise part 19

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The purpose of this exercise is to get acquainted with system identification in connection to

- Multivariable discrete time models
- State space discrete time models
- Continuous time models

The application is the coupled tank apparatus from excercise 1.

Excercise 1

We consider the problem of controlling the levels in a two coupled tank system (see figure 1). The control signal (U_t) is the in-flow to the first tank and the objective is to control the level (y_t) in the second tank. The two tank are connected through a pipe. The set point, ie. the desired value of the level in the second tank, is denoted as w_t.

The state of the system is chosen to be

$$\begin{array}{rcll} X_1 & = & H_2 & \text{ie. the level in the second tank} \\ X_2 & = & H_1 - H_2 & \text{ie. the difference in levels} \end{array}$$

then the dynamics of the system can be described as

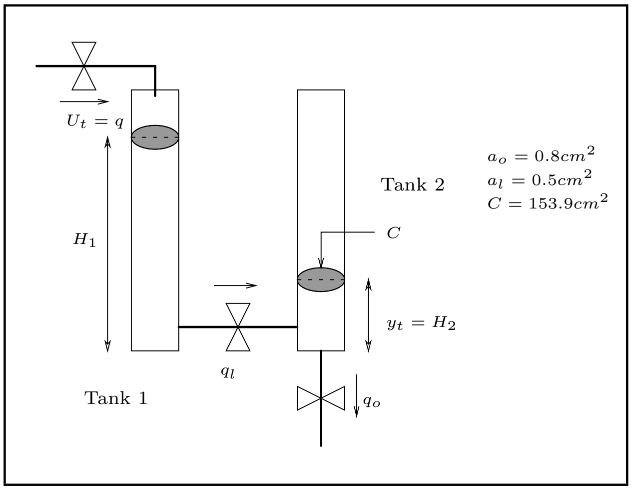
$$C(\dot{X}_1 + \dot{X}_2) = U - q_l \qquad (1)$$

$$C\dot{X}_1 = q_l - q_o \qquad (2)$$

where C denotes the area of the cross section of the tank (the two tank have the same cross section). The flows, q_l and q_o, are assumed to obey the Bernoulli equation, ie.

$$q_l = \sigma_l a_l \sqrt{2gX_2} \qquad (3)$$

$$q_o = \sigma_o a_o \sqrt{2gX_1} \qquad (4)$$



Figur 1. The coupled tanks system

Note it assumed that the tank system is operated in its normal mode where $X_1 \geq 0$ and $X_2 \geq 0$. The constants of the system are

$$C = 153.938\ cm^2 \qquad g = 981\ cm/s^2$$

$$a_l = 0.5\ cm \qquad a_o = 0.8\ cm$$

$$\sigma_l = 0.44 \qquad \sigma_o = 0.31$$

(These parameters are available in the matlab file tank_par.m).

In offset.m the steady state values of the state vector and the control signal X_0 and U_0 which results in an output, corresponding to a desired value w_0. (Don't let you be confused by the fact, that X_0 is the steady state vector, whereas X_1 and X_2 are the elements in the state vector X.)

In the following we will use this point (related to $w_0 = 19\ cm$) as our operation point.

The state equations, (1) and (7), can be written as

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} -a_1 & a_2 \\ a_1 & -2a_2 \end{bmatrix} \begin{bmatrix} \sqrt{X_1} \\ \sqrt{X_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} U \tag{5}$$

where

$$a_1 = \sigma_0 a_0 \frac{\sqrt{2g}}{C} \qquad a_2 = \sigma_l a_l \frac{\sqrt{2g}}{C}$$

Now, let x denote the deviation in X from X_0 and let u denote the deviation in U from U_0. Then the linearized description of the tank system is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + Bu \tag{6}$$

where

$$A = \begin{bmatrix} -a_1 & a_2 \\ a_1 & -2a_2 \end{bmatrix} \begin{bmatrix} \frac{1}{2\sqrt{X_{10}}} & 0 \\ 0 & \frac{1}{2\sqrt{X_{20}}} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix}$$

In the following we will use a sampling period equal $T_s = 5\ sec.$

In the distributed version of the work bench, the set point (wt) change after a short period from w_0 to $w_0 + 5\ cm$.

Furthermore, the measurements (or the measured outputs) are

$$y_t^m = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}_t = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_t + e_t = C_m x_t + e_t \tag{7}$$

where $e_t \in N_{iid}(0, R_2)$. Notice the difference between the measurements and the output.

Question 1.1 Perform a deterministic experiment and estimate the parameters in a MARX model. □

Question 1.2 Perform a stochastic experiment and estimate the parameters in a MARX model. □

Question 1.3 Perform a stochastic experiment and estimate the parameters in a continuous time MARX model. □

Question 1.4 Estimate the parameters in a linear state space model. □

Question 1.5 Assume X_0 and U_0 are known. Estimate the parameters, a_1 and a_2 in a linear (continuous time) grey box model. □