Stochastic Adaptive Control

Exercise part 18

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The focus of this excercise is modelling and identification of dynamic stochastic systems. For generating data set, you can either use the work bench (*wb* and *sysinit*) or use the matlab procedure *sim* from the identification toolbox (check eg. *ident/sim*).

Excercise 1

(The effect of model structure). In this exercise we consider a system given by:

$$y_t = \frac{B}{A} u_{t-1} + e_t$$

where $e_{\in} \mathbf{N}_{iid}\left(0, \sigma^2\right)$ and

$$A(q^{-1}) = 1 - 1.5\ q^{-1} + 0.7\ q^{-2}$$
$$B(q^{-1}) = 1 + 0.5\ q^{-1}$$

Assume the input signal is a PRBS signal.

Question 1.1 Determine the variance, σ^2, such that the variance contribution (from the noise) to the output is 1/100 of the contribution from the the (determistic) input signal. Use e.g. *trfvar* or simply *var* . □

Produce 2 data sets (2 simulations) with the input being a PRBS signal.

Question 1.2 Estimate the parameters (correct order of the polynomials) using *arx* or *estarx* . Check the estimate and compare with correct values. Also check the correlation of the residuals (use both data sets). □

Question 1.3 Estimate the parameters using an IV4 method and perform the check as in the previous question. □

Question 1.4 Do the previous question, but with an OE method. □

Now, assume e_t is no longer white, but given as:

$$e_t = \frac{1}{1 - 0.9\ q^{-1}}\xi_t$$

where $\xi_t \in \mathbf{N}_{iid}\left(0, \sigma_\xi^2\right)$.

Question 1.5 Determine σ_ξ^2 such that the variance of e_t is as found in first question. Hint: use *trfvar* . □

Question 1.6 Check the perfomance (in terms of bias, size of confidence interval) of ARX, OE, IV4 and PEM method. □

Excercise 2

(Estimating in the ARMAX structure.)

Consider a dynamic system (ARMAX structure) given by

$$A(q^{-1})y_t = B(q^{-1})u_{t-1} + C(q^{-1})e_t$$

where $e_{\in}\mathbf{N}_{iid}\left(0, \sigma^2\right)$ and

$$A(q^{-1}) = 1 - 1.5\ q^{-1} + 0.7\ q^{-2}$$
$$B(q^{-1}) = 1 + 0.1\ q^{-1}$$
$$C(q^{-1}) = 1 - 0.8(q^{-1})$$

Let the input signal be a PRBS signal and let $\sigma^2 = 0.1$.

Question 2.1 Estimate the parameters in a ARX model. Check the estimate and their correct values and their uncertainty. Use e.g. *estpres* . Also check the covariance function of the residuals. □

Question 2.2 Estimate the parameters in the ARMAX model. Check the estimate and their correct values and their uncertainty. Use e.g. *estpres* . Also check the covariance function of the residuals. □

Now assume σ^2 is reduced (try eg. $1e-3,\ 1e-9$) and produce a data set with this property.

Question 2.3 Again, estimate the parameters in the ARMAX model and check the estimate, their uncertainty and their correct values. □

Question 2.4 Answer the previous question with a OE and a IV estimate. □

Excercise 3

(Estimating in the BJ structure.)

Consider a dynamic system given by

$$y_t = q^{-1}\frac{1 - 1.5\ q^{-1} + 0.7\ q^{-2}}{1 + 0.1\ q^{-1}}u_t + \frac{1 + 0.2q^{-1}}{1 - 0.8q^{-1}}e_t$$

where $e_{\in}\mathbf{N}_{iid}\left(0, \sigma^2\right)$. Let the input signal be a PRBS signal and let σ^2 be such that the varaince contribution from noise and input signal is the same.

Question 3.1 Determine σ^2. □

Produce two data set with the discribed model.

Question 3.2 Estimate the parameters in a BJ model. Check the estimate and their correct values and their uncertainty. Use e.g. *estpres* . Also check the covariance function of the residuals. □

Question 3.3 Find a suitable ARMAX structure equivalent with the system. Estimate the parameters in this model . Check the estimate and their correct values and their uncertainty. check the covariance function of the residuals. □

Question 3.4 Answer the previous question with a OE and a IV estimate. □

Excercise 4

(Estimating in the L-structure.)

Consider a dynamic system given by

$$(1 - 0.8q^{-1})y_t = q^{-1}\frac{1 - 1.5\ q^{-1} + 0.7\ q^{-2}}{1 + 0.1\ q^{-1}}u_t + \frac{1 + 0.2q^{-1}}{1 - 0.8q^{-1}}e_t$$

where $e_{\in}\mathbf{N}_{iid}\left(0, \sigma^2\right)$. Let the input signal be a PRBS signal and let σ^2 be such that the varaince contribution from noise and input signal is the same.

Question 4.1 Determine σ^2. □

Produce two data set with the discribed model.

Question 4.2 Estimate the parameters in the L-structure. Check the estimate and their correct values and their uncertainty. Use e.g. *estpres* . Also check the covariance function of the residuals. □

Question 4.3 Find a suitable ARMAX structure equivalent with the system. Estimate the parameters in this model . Check the estimate and their correct values and their uncertainty. check the covariance function of the residuals. □

Question 4.4 Answer the previous question with a OE and a IV estimate. □
