Stochastic Adaptive Control

Excercise part 14

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The focus of this excercise is analyses and control of stochatic systems given in external form ie. described by transfer functions.

- Dioph
- prediction
- regulator implementation
- MV0

Excercise 1

(Diophantine equation) Consider an ARMA process given by

$$A(q^{-1})y_t = C(q^{-1})e_t$$

where

$$\begin{aligned} A(q^{-1}) &= 1 - 1.5q^{-1} + 0.7q^{-2} \\ C(q^{-1}) &= 1 - 0.2q^{-1} + 0.5q^{-2} \qquad \sigma^2 = 0.1 \end{aligned}$$

where e_t obey the standard assumptions.

Let us first consider a simple version of the Diophantine equation:

$$C(q^{-1}) = A(q^{-1})G(q^{-1}) + q^{-k}S(q^{-1}) \tag{1}$$

where $ord(G) = k - 1$.

Question 1.1 Determine the 5 first coefficient of the impulse response for the transfer operator:

$$H_n(q) = \frac{C}{A}$$

Hint: Use *dimpulse* and notice Matlab uses z notation and not z^{-1}. You might have to pad with zeros in order to obtain a correct result. Notice, the convention in which both $A(q^{-1})$ and $C(q^{-1})$ are monic implies that the impulse response start with 1. □

The unknowns in Diophantine equation in (1) is the polynomials $G(q^{-1})$ and $S(q^{-1})$ (or equivalent the coefficients in these two polynomials). The Diophantine equation is in fact a set of linear equations with an equation for each $q^{-i} \quad i = 0,\ 1,\ \ldots$. From (1) it is clear $S(q^{-1})$ will only contribute to coefficients to q^{-i} where $i \geq k$. It is also clear that $G(q^{-1})$ (in general) will contribute to coefficients to q^{-i} where $i \geq 0$. Consequently, $G(q^{-1})$, can be found as the truncated impulse response for $\frac{C}{A}$.

Question 1.2 Find $G(q^{-1})$ for $k = 2$. □

Question 1.3 Check the error polynomial from

$$E(q^{-1}) = C(q^{-1}) - A(q^{-1})G(q^{-1})$$

where $G(q^{-1})$ is found as just described. **Hint:** use eg. *polmul* and *polsum* . Compare with the Diophantine equation in (1) and $S(q^{-1})$. Find $S(q^{-1})$. Also check the command *poldiopk.m* in the distribution. □

Question 1.4 Solve the problem (i.e. find $G(q^{-1})$ and $S(q^{-1})$) for $k = 3$ and compare with the previous solution (for $k = 2$). Look especially on $G(q^{-1})$. □

Let us now focus on the more general version of the Diophantine equation:

$$C(q^{-1}) = A(q^{-1})G(q^{-1}) + \bar{B}(q^{-1})S(q^{-1}) \tag{2}$$

where $\bar{B}(q^{-1}) = q^{-k}B(q^{-1})$. The procedure *dioph.m* is developed for solving this problem. In the following let

$$B(q^{-1}) \quad = \quad 1 - 0.5q^{-1} \qquad\qquad k = 2$$

Question 1.5 The Diophantine equation in (2) consists of a number of linear equations in the parameters in the two unknown polynomials, $R(q^{-1})$ and $S(q^{-1})$. Investigate the set of equations by writing (2) and the coefficients to q^{-i}. Also compare with the printout from *dioph* (with show= 1). □

Excercise 2

(Prediction). Consider a process given by:

$$y_t = \frac{1 + 0.6q^{-1}}{1 - 1.4q^{-1} + 0.6q^{-2}} e_t$$

where $e_t \in \mathbf{N}_{iid}(0, 0.7)$ (white noise).

Question 2.1 Produce a simulation (with length 500 steps) with this process. **Hint:** use *dlsim* . □

Question 2.2 Design a 3-step ahead predictor for this process. **Hint:** use *dsnprd* . □

Question 2.3 Use the previous found simulation of y_t as input to the predictor and find the predictions. Plot the simulation and predicted values in same figure (eventually pad with zeros in order to obtain signal related to same instant of time). Also determine the varianse of the prediction error. □

Question 2.4 Now increase the time delay k to 5 and repeat the previous questions. □

Excercise 3

(MV$_0$ controller) Consider a system given by

$$\begin{array}{rcll} A(q^{-1}) & = & 1 - 1.5q^{-1} + 0.7q^{-2} & \\ B(q^{-1}) & = & 1 - 0.5q^{-1} & k = 2 \\ C(q^{-1}) & = & 1 - 0.2q^{-1} + 0.5q^{-2} & \sigma^2 = 0.1 \end{array}$$

where e_t obey the standard assumptions.

Question 3.1 Determine the uncontrolled variance of the output. **Hint:** the procedure *trfvar* determines the variance of the output from a system with a given transfer function and unit input variance. □

Question 3.2 Design a MV$_0$ controller using *dioph* for solving the Diophantine equation. Compare your steps and results with similar obtained with *dsnmv0* . □

Question 3.3 Determine the theoretical variance of the error due to the stochastic disturbances. □

The procedure *dsnmv0* determine the polynomials in the MV_0 controller

$$R(q^{-1})u_t = Q(q^{-1})w_t - S(q^{-1})y_t \tag{3}$$

and the *armax2ss* -procedure can be used to determine a state space realization of the controller, ie. A_r, B_r, C_r and D_r in

$$\begin{array}{rcl} x_{t+1}^{(r)} & = & A_r x_t^{(r)} + B_r \begin{bmatrix} w_t \\ -y_t \end{bmatrix} \\ u_t & = & C_r x_t^{(r)} + D_r \begin{bmatrix} w_t \\ -y_t \end{bmatrix} \end{array}$$

Question 3.4 Find the matrices in the realization of the controller e.g. by means of *armax2ss* . □

Question 3.5 Verify (use e.g. *ss2trf*), that the transfer function from w_t and y_t is as given by (3). □

The work bench (with *wb.m* as main file) is set up to simulate a ARMAX system controlled by an external controller (such as the MV$_0$ controller). The system definition is to be found in *sysinit.m* . The implementation of the controller is also (already) programmed in *wb.m* .

Question 3.6 Perform a deterministic simulation (set s2 in *sysinit.m*) and check the performance (check out the produced plot by running *wb.m*). Especially, check the tracking ability in the output and the (wild) control actions. □

Question 3.7 Change the workbench to simulate a stochastic run (set s2 in *sysinit*) with a constant reference (set refsig to 1). Run a simulation and determine (the empirical variance of the output). Compare this with the theoretical. □