1. Sistemes de numeració.

Com a regla general, si no ens diuen el contrari, en aquelles conversions de part decimal d'un nombre en base 10 a binari ens quedarem amb tres *bits* per cada xifra del nombre original. De tota manera als exercicis resolts aquí es calcularan totes les xifres.

1. (a) $100110_2 = 2^5 + 2^2 + 2 = 38_{10}$
   (b) $110011_2 = 2^5 + 2^4 + 2 + 1 = 51_{10}$
   (c) $110111_2 = 2^5 + 2^4 + 2^2 + 2 + 1 = 55_{10}$
   (d) $1001,10_2 = 2^3 + 1 + 2^{-1} = 9,5_{10}$
   (e) $101010110,001_2 = 2^8 + 2^6 + 2^4 + 2^2 + 2 + 2^{-3} = 342,125_{10}$

2. (a) $93_{10} \longrightarrow$

$$\begin{array}{r|l} 93 & 2 \\ \cline{2-2} 13 & 46 \\ 1 & \end{array} \quad \begin{array}{r|l} 46 & 2 \\ \cline{2-2} 06 & 23 \\ 0 & \end{array} \quad \begin{array}{r|l} 23 & 2 \\ \cline{2-2} 03 & 11 \\ 1 & \end{array} \quad \begin{array}{r|l} 11 & 2 \\ \cline{2-2} 1 & 5 \end{array} \quad \begin{array}{r|l} 5 & 2 \\ \cline{2-2} 1 & 2 \end{array} \quad \begin{array}{r|l} 2 & 2 \\ \cline{2-2} 0 & 1 \end{array}$$

$$93_{10} = 1011101_2$$

(b) $647_{10} \longrightarrow$

$$\begin{array}{r|l} 647 & 2 \\ \cline{2-2} 04\ & 323 \\ 07 & \\ 1 & \end{array} \quad \begin{array}{r|l} 323 & 2 \\ \cline{2-2} 12\ & 161 \\ 03 & \\ 1 & \end{array} \quad \begin{array}{r|l} 161 & 2 \\ \cline{2-2} 1 & 80 \end{array} \quad \begin{array}{r|l} 80 & 2 \\ \cline{2-2} 0 & 40 \end{array} \quad \begin{array}{r|l} 40 & 2 \\ \cline{2-2} 0 & 20 \end{array}$$

$$\begin{array}{r|l} 20 & 2 \\ \cline{2-2} 0 & 10 \end{array} \quad \begin{array}{r|l} 10 & 2 \\ \cline{2-2} 0 & 5 \end{array} \quad \begin{array}{r|l} 5 & 2 \\ \cline{2-2} 1 & 2 \end{array} \quad \begin{array}{r|l} 2 & 2 \\ \cline{2-2} 0 & 1 \end{array}$$

$$647_{10} = 1010000111_2$$

(c) $310_{10} \longrightarrow$

$$\begin{array}{r|l} 310 & 2 \\ \cline{2-2} 11\ & 155 \\ 10 & \\ 0 & \end{array} \quad \begin{array}{r|l} 155 & 2 \\ \cline{2-2} 15 & 77 \\ 1 & \end{array} \quad \begin{array}{r|l} 77 & 2 \\ \cline{2-2} 17 & 38 \\ 1 & \end{array} \quad \begin{array}{r|l} 38 & 2 \\ \cline{2-2} 18 & 19 \\ 0 & \end{array} \quad \begin{array}{r|l} 19 & 2 \\ \cline{2-2} 1 & 9 \end{array}$$

$$\begin{array}{r|l} 9 & \underline{2} \\ 1 & 4 \end{array} \quad \begin{array}{r|l} 4 & \underline{2} \\ 0 & 2 \end{array} \quad \begin{array}{r|l} 2 & \underline{2} \\ 0 & 1 \end{array}$$

$$310_{10} = 100110110_2$$

(d) $131_{10} \longrightarrow$

$$\begin{array}{r|l} 131 & \underline{2\ \ \ } \\ 11 & 65 \\ 1 & \end{array} \quad \begin{array}{r|l} 65 & \underline{2\ \ \ } \\ 05 & 32 \\ 1 & \end{array} \quad \begin{array}{r|l} 32 & \underline{2\ \ \ } \\ 12 & 16 \\ 0 & \end{array} \quad \begin{array}{r|l} 16 & \underline{2} \\ 0 & 8 \end{array} \quad \begin{array}{r|l} 8 & \underline{2} \\ 0 & 4 \end{array}$$

$$\begin{array}{r|l} 4 & \underline{2} \\ 0 & 2 \end{array} \quad \begin{array}{r|l} 2 & \underline{2} \\ 0 & 1 \end{array}$$

$$131_{10} = 10000011_2$$

(e) $258,75_{10} \longrightarrow$

$$\begin{array}{r|l} 258 & \underline{2\ \ \ \ \ } \\ 05\ \ & 129 \\ 18 & \\ 0 & \end{array} \quad \begin{array}{r|l} 129 & \underline{2\ \ \ } \\ 09 & 64 \\ 1 & \end{array} \quad \begin{array}{r|l} 64 & \underline{2\ \ \ } \\ 04 & 32 \\ 0 & \end{array} \quad \begin{array}{r|l} 32 & \underline{2\ \ \ } \\ 12 & 16 \\ 0 & \end{array} \quad \begin{array}{r|l} 16 & \underline{2} \\ 0 & 8 \end{array}$$

$$\begin{array}{r|l} 8 & \underline{2} \\ 0 & 4 \end{array} \quad \begin{array}{r|l} 4 & \underline{2} \\ 0 & 2 \end{array} \quad \begin{array}{r|l} 2 & \underline{2} \\ 0 & 1 \end{array}$$

$$258_{10} = 100000010_2$$

$$0,75 \times 2 = 1,5 \geq 1 \Rightarrow 1$$
$$0,5 \times 2 = 1 \geq 1 \Rightarrow 1$$

$$0,75_{10} = 0,11_2 \rightarrow 258,75_{10} = 10000010,11_2$$

(f) $1,625_{10} \longrightarrow$

$$0,625 \times 2 = 1,25 \geq 1 \Rightarrow 1$$
$$0,25 \times 2 = 0,5 < 1 \Rightarrow 0$$
$$0,5 \times 2 = 1 \geq 1 \Rightarrow 1$$

$$1,625_{10} = 1,101_2$$

(g) $19,3125_{10} \longrightarrow$

$$\begin{array}{cc|c} 1 & 9 & 2 \\ \cline{3-3} & 1 & 9 \end{array} \quad \begin{array}{c|c} 9 & 2 \\ \cline{2-2} 1 & 4 \end{array} \quad \begin{array}{c|c} 4 & 2 \\ \cline{2-2} 0 & 2 \end{array} \quad \begin{array}{c|c} 2 & 2 \\ \cline{2-2} 0 & 1 \end{array}$$

$$19_{10} = 10011_2$$

$$0,3125 \times 2 = 0,625 < 1 \Rightarrow 0$$
$$0,625 \times 2 = 1,25 \geq 1 \Rightarrow 1$$
$$0,25 \times 2 = 0,5 < 1 \Rightarrow 0$$
$$0,5 \times 2 = 1 \geq 1 \rightarrow 1$$
$$19,3125_{10} = 10011,0101_2$$

3. (a) $13_{16} = 1 \cdot 16^1 + 3 \cdot 16^0 = 19_{10}$

   (b) $65_{16} = 6 \cdot 16^1 + 5 \cdot 16^0 = 101_{10}$

   (c) $3F0_{16} = 3 \cdot 16^2 + F \cdot 16^1 + 0 \cdot 16^0 = 3 \cdot 16^2 + 15 \cdot 16^1 + 0 \cdot 16^0 = 1008_{10}$

   (d) $D0CE_{16} = D \cdot 16^3 + 0 \cdot 16^2 + C \cdot 16^1 + E \cdot 16^0 = 13 \cdot 16^3 + 0 \cdot 16^2 +$

   $12 \cdot 16^1 + 14 \cdot 16^0 = 53454_{10}$

   (e) $0,2_{16} = 0 \cdot 16^0 + 2 \cdot 16^{-1} = 0,125_{10}$

   (f) $12,9_{16} = 1 \cdot 16^1 + 2 \cdot 16^0 + 9 \cdot 16^{-1} = 18,5625_{10}$

   (g) $F1,A_{16} = F \cdot 16^1 + 1 \cdot 16^0 + A \cdot 16^{-1} = 15 \cdot 16^1 + 1 \cdot 16^0 + 10 \cdot 16^{-1} = 241,625_{10}$

   (h) $C8,D_{16} = C \cdot 16^1 + 8 \cdot 16^0 + D \cdot 16^{-1} = 12 \cdot 16^1 + 8 \cdot 16^0 + 13 \cdot 16^{-1} = 200,8125_{10}$

4. (a)

$$3,A2_{16} \rightarrow 0011,1010\ 0010_2 \rightarrow 011,101\ 000\ 100_2 \rightarrow 3,504_8 \rightarrow$$

$$\rightarrow 3,6328125_{10}$$

(b)

$$1B1,9 \rightarrow 0001\ 1011\ 0001, 1001_2 \rightarrow 110\ 110\ 001, 100\ 100_2 \rightarrow$$

$$\rightarrow 661,44_8 \rightarrow 433,5625_{10}$$

(c)

$$6416213A,17B_{16} \rightarrow$$

$$\rightarrow 0110\ 0100\ 0001\ 0110\ 0010\ 0001\ 0011\ 1010, 0001\ 0111\ 1011_2 \rightarrow$$

$$\rightarrow 001\ 100\ 100\ 000\ 101\ 100\ 010\ 000\ 100\ 111\ 010, 000\ 101\ 111\ 011_2 \rightarrow$$

$$\rightarrow 14405420472,0573_8 \rightarrow 1679171898,0092529296_{10}$$

5. (a) Podem passar el nombre a base 10, després a binari i d'allà és trivial obtenir el nombre en hexadecimal

$$204231,134_5 =$$

$$2{\cdot}5^5+0{\cdot}5^4+4{\cdot}5^3+2{\cdot}5^2+3{\cdot}5^1+1{\cdot}5^0+1{\cdot}5^{-1}+3{\cdot}5^{-2}+4{\cdot}5^{-3} = 6816,352_{10}$$

Part entera

$$\begin{array}{r|l} 6816 & 2 \\ \cline{2-2} 08\ \ \ & 3408 \\ 016 & \\ 0 & \end{array} \quad \begin{array}{r|l} 3408 & 2 \\ \cline{2-2} 14\ \ \ & 1704 \\ 008 & \\ 0 & \end{array} \quad \begin{array}{r|l} 1704 & 2 \\ \cline{2-2} 10\ \ & 852 \\ 04 & \\ 0 & \end{array}$$

$$\begin{array}{r|l} 852 & 2 \\ \cline{2-2} 05\ & 426 \\ 12 & \\ 0 & \end{array} \quad \begin{array}{r|l} 426 & 2 \\ \cline{2-2} 02\ & 213 \\ 06 & \\ 0 & \end{array} \quad \begin{array}{r|l} 213 & 2 \\ \cline{2-2} 013 & 106 \\ 1 & \end{array} \quad \begin{array}{r|l} 106 & 2 \\ \cline{2-2} 06 & 53 \\ 0 & \end{array}$$

$$\begin{array}{r|l} 53 & 2 \\ \cline{2-2} 13 & 26 \\ 1 & \end{array} \quad \begin{array}{r|l} 26 & 2 \\ \cline{2-2} 06 & 13 \\ 0 & \end{array} \quad \begin{array}{r|l} 13 & 2 \\ \cline{2-2} 1 & 6 \end{array} \quad \begin{array}{r|l} 6 & 2 \\ \cline{2-2} 0 & 3 \end{array} \quad \begin{array}{r|l} 3 & 2 \\ \cline{2-2} 1 & 1 \end{array}$$

$$= 0001\ 1010\ 1010\ 0000_2 = 1AA0_{16}$$

Part decimal

$$\begin{aligned} 0,352_{10} = 0,&0101\ 1010\ 0001\ 1100\ 1010\ 1100\ 0000 \\ &1000\ 0011\ 0001\ 0010\ 0110\ 1110\ 1000 \end{aligned}$$

$$= 0,5A1CAC083126E8_{16}$$

$$204231,1345_5 = 1A9F,5C28F_{16}$$

Alternativament, passem a base 10 i després amb mètodes vistos en exercicis anteriors, a hexadecimal

Part entera

$$\begin{array}{r|l} 204231 & 16 \\ \cline{2-2} 44 & 12764 \\ 122 & \\ 103 & \\ 71 & \\ 7 & \end{array} \qquad \begin{array}{r|l} 12764 & 16 \\ \cline{2-2} 156 & 797 \\ 124 & \\ 12 & \end{array} \qquad \begin{array}{r|l} 797 & 16 \\ \cline{2-2} 157 & 49 \\ 13 & \end{array} \qquad \begin{array}{r|l} 49 & 16 \\ \cline{2-2} 1 & 3 \end{array}$$

$$204231_{10} = 31DC7_{16}$$

Part decimal

$$\begin{aligned}
&0,36 \times 16 = 5,76 \geq 1 \Rightarrow 5 \\
&0,76 \times 16 = 12,16 \geq 1 \Rightarrow 12 \equiv C \\
&0,16 \times 16 = 2,56 \geq 1 \Rightarrow 2 \\
&0,56 \times 16 = 8,96 \geq 1 \Rightarrow 8 \\
&0,96 \times 16 = 15,36 \geq 1 \Rightarrow 15 \equiv F \\
&-------------- \\
&0,36 \times 16 = 5,76 \geq 1 \Rightarrow 5
\end{aligned}$$

$$0,36_{10} = \overline{5C28F}...16$$

(b) $165433_7 = 1 \cdot 7^5 + 6 \cdot 7^4 + 5 \cdot 7^3 + 4 \cdot 7^2 + 3 \cdot 7^1 + 3 \cdot 7^0 = 33148_{10}$

$$\begin{array}{r|l} 33148 & 16 \\ \cline{2-2} 114 & 2071 \\ 28 & \\ 12 & \end{array} \qquad \begin{array}{r|l} 2071 & 16 \\ \cline{2-2} 47 & 129 \\ 151 & \\ 7 & \end{array} \qquad \begin{array}{r|l} 129 & 16 \\ \cline{2-2} 1 & 8 \end{array}$$

$$33148_{10} = 817C_{16}$$

6. (a) $62 \rightarrow 0110\ 0010$
   (b) $25 \rightarrow 0010\ 0101$
   (c) $274 \rightarrow 0010\ 0111\ 0100$
   (d) $284 \rightarrow 0010\ 1000\ 0100$
   (e) $42,91 \rightarrow 0100\ 0010,\ 1001\ 0001$
   (f) $5,014 \rightarrow 0101,\ 0000\ 0001\ 0100$

7. (a) $1001 \rightarrow 9$
   (b) $0101 \rightarrow 5$
   (c) $0110\ 0001 \rightarrow 61$
   (d) $0100\ 0111 \rightarrow 47$
   (e) $0011\ 0110,\ 1000 \rightarrow 36,8$
   (f) $0011\ 1000,\ 1000\ 1000 \rightarrow 38,88$

2. Introducció als circuits lògics.

1. (a) $f(a,b) = ab + a$

a	b	$ab + a$
0	0	0
0	1	0
1	0	1
1	1	1

   (b) $f(a,b) = (a \oplus b)\overline{b}$

a	b	$(a \oplus b)\overline{b}$
0	0	0
0	1	1
1	0	0
1	1	0

(c) $f(a,b) = \overline{(\overline{a} + b)} \oplus (a \cdot \overline{b})$

a	b	$\overline{(\overline{a} + b)} \oplus (a \cdot \overline{b})$
0	0	0
0	1	0
1	0	0
1	1	0

(d) $f(a,b,c) = (a \cdot b) + c$

a	b	c	$ab + c$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(e) $f(a,b,c) = \overline{(a \cdot b) \oplus c}$

a	b	c	$\overline{(a \cdot b) \oplus c}$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

(f) $f(a, b, c, d) = \overline{\overline{(\overline{a} + b)} \oplus (c \cdot \overline{d})}$

a	b	c	d	$\overline{\overline{(\overline{a}+b)} \oplus (c \cdot \overline{d})}$
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1