ANL 9

(2) a) $B_i^n(x) \geq 0$ dla $x \in [0,1]$ i osiąga dokł. jedno maksimum

$B_k^n(x) = \binom{n}{k} x^k (1-x)^{n-k}$

I) Najpierw $x=1$ i $x=0$:

$k=0$:

$B_0^n(x) = (1-x)^n \geq 0$

max w 0, Mz w 1

$k=n$:

$B_n^n(x) = x^n \geq 0$

max w 1, Mz w 0

$k \in (0,n)$:

$B_k^n(0) = 0 \quad B_k^n(1) = 0$

II) Sprawdźmy ekstrema dla $x \in (0,1)$

$[B_i^n(x)]' = 0$

reg. Łańcuchowa

$\binom{n}{k}[x^k(1-x)^{n-k}]' = \binom{n}{k}[kx^{k-1}(1-x)^{n-k} - x^k(1-x)^{n-k-1}(n-k)] = 0$

stąd $\binom{n}{k} \neq 0$, $1-x \neq 0$, $x \neq 0$, $(1-x)^{n-k} \neq 0$, $x^k \neq 0$

Dzielimy przez $\binom{n}{k}x^{k-1}(1-x)^{n-k-1}$

$k(1-x) - (n-k)x = 0$

$k - kx - nx + kx = 0$

dokł. jeden ekstremum (maksimum) $\leftarrow$ $\boxed{x = \frac{k}{n}}$

b) $\sum_{i=0}^{n} B_i^n(t) = 1$

$\sum_{i=0}^{n} B_i^n(t) = \sum_{i=0}^{n}\binom{n}{i}x^i(1-x)^{n-i} = [x+(1-x)]^n = 1^n = 1$

$y = 1-x$

$\sum_{i=0}^{n}\binom{n}{i}x^i y^{n-i} = (x+y)^n$

ANL - 3

② c) $B_i^n(u) = (1-u)B_i^{n-1}(u) + uB_{i-1}^{n-1}(u),\ 0 \le i \le n$

$$P \overset{def}{=} (1-u)\left[\binom{n-1}{i}u^i(1-u)^{n-1-i}\right] + u\left[\binom{n-1}{i-1}u^{i-1}(1-u)^{n-1-i+1}\right] =$$

$$= \left[\binom{n-1}{i}u^i(1-u)^{n-i}\right] + \left[\binom{n-1}{i-1}u^i(1-u)^{n-i}\right]$$

$$= \left[\binom{n-1}{i} + \binom{n-1}{i-1}\right]u^i(1-u)^{n-i}$$

$\downarrow$ tr. Pascale

$$= \binom{n}{i}u^i(1-u)^{n-i} = B_i^n(u) \quad \blacksquare$$

d) $B_i^n(u) = \frac{n+1-i}{n+1}B_i^{n+1}(u) + \frac{i+1}{n+1}B_{i+1}^{n+1}(u),\ 0 \le i \le n$

$$\frac{(n+1-i)}{(n+1)} \cdot \underbrace{\frac{(n+1)!}{i!(n+1-i)!}}_{\binom{n+1}{i}} u^i(1-u)^{n+1-i} + \frac{(i+1)}{(n+1)} \cdot \frac{(n+1)! \cdot u^{i+1}(1-u)^{n-i}}{(i+1)!(n+1-i-1)!}$$

$$\frac{n!}{i!(n-i)!}u^i(1-u)^{n+1-i} + \frac{n!}{i!(n-i)!}u^{i+1}(1-u)^{n-i}$$

$$\binom{n}{i}u^i(1-u)^{n-i}\left[\underbrace{(1-u)+u}_{=1}\right] = \binom{n}{i}u^i(1-u)^{n-i} = B_i^n(u)$$

$\blacksquare$

ANL-3

(3) $B_k^n(t) = \binom{n}{k} t^k (1-t)^{n-k}$

$$\begin{cases} B_n^n(t) = \binom{n}{n} t^n (1-t)^0 = t^n \\ B_{n-1}^n(t) = \binom{n}{n-1} t^{n-1}(1-t) \\ \vdots \\ B_1^n(t) = \binom{n}{1} t^1 (1-t)^{n-1} \\ B_0^n(t) = \binom{n}{0} \cancel{t^0}(1-t)^n = (1-t)^n \end{cases}$$

jest ich $n+1$

$\alpha_n B_n^n + \alpha_{n-1} B_{n-1}^n + \ldots + \alpha_0 B_0^n = 0$

Współczynniki przy t^n => Wielomian jako wektor - $\begin{array}{c} \begin{bmatrix} y_n \\ y_{n-1} \\ \vdots \\ y_0 \end{bmatrix} \end{array} \begin{array}{l} t^n \\ t^{n-1} \\ \vdots \\ t^0 \end{array}$

$$\begin{array}{c} t^n \\ t^{n-1} \\ \vdots \\ t^1 \\ 1 = t^0 \end{array} \begin{bmatrix} \neq 0 & \neq 0 & \neq 0 & \cdots & \neq 0 \\ & \neq 0 & \neq 0 & \cdots & \neq 0 \\ & & \neq 0 & \cdots & \vdots \\ & 0 & & \ddots & \neq 0 \\ & & & & \neq 0 \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$\alpha_n \begin{bmatrix} \neq 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \alpha_{n-1} \begin{bmatrix} \neq 0 \\ \neq 0 \\ \vdots \\ 0 \end{bmatrix} + \ldots + \alpha_0 \begin{bmatrix} \neq 0 \\ \neq 0 \\ \vdots \\ \neq 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

el. Gausse

Współczynniki przy:

1) $\alpha_0 \cdot \overset{\neq 0}{X_1} = 0 \Rightarrow \alpha_0 = 0$

x) $\overset{=0}{\alpha_0} \overset{\neq 0}{X_2} + \alpha_1 \overset{\neq 0}{y_2} \Rightarrow \alpha_1 = 0$

x^2) $\underbrace{\alpha_0 \overset{\neq 0}{X_3}}_{0} + \underbrace{\alpha_1 \overset{\neq 0}{y_3}}_{0} + \alpha_2 \overset{\neq 0}{Z_3} = 0 \Rightarrow \alpha_2 = 0$ itd

$\alpha_0 = \alpha_1 = \ldots = \alpha_n = 0$

Czyli wielomiany B_k^n są liniowo niezależne i jest ich $n+1$ więc tworzą bazę Π_n

ANL - 9

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(5) $P_n(t) = \sum_{i=0}^{n} B_i^n(t) W_i \qquad 0 \le t \le 1$

$\uparrow$ pkt kontrolne

$P_n(t) = \binom{n}{0}^{=1} t^{0 =1} (1-t)^n W_0 + \binom{n}{1} t^1 (1-t)^{n-1} W_1 + \dots + \binom{n}{n}^{=1} t^n (1-t)^{0 =1} W_n$

Wyciągamy $(1-t)$ kiedy się da:

$P_n(t) = W_n \binom{n}{n} t^n + (1-t)\Big[W_{n-1}\binom{n}{n-1} t^{n-1} + (1-t)\big[W_{n-2}\binom{n}{n-2} t^{n-2} + \dots$

$+ \dots + (1-t)\big[W_1 \binom{n}{1} t + (1-t) W_0 \binom{n}{0}\big]\big]\Big] \dots$

Czyli $w_0 = W_0$

$w_{i+1} = W_{i+1}\binom{n}{i+1} t^{i+1} + w_i (1-t)$ wtedy $w_n = P_n(t)$

Musimy jeszcze umieć policzyć t^i liniowo (prosto przemnożyć w każdej iteracji $\cdot t$)

oraz mając $\binom{n}{i}$ uzyskać $\binom{n}{i+1}$

$\binom{n}{0} = 1, \binom{n}{1} = n, \binom{n}{2} = \frac{n(n-1)}{2!}, \binom{n}{3} = \binom{n}{2} \cdot \frac{(n-2)}{3}$ czyli

$\binom{n}{k} = \binom{n}{k-1} \cdot \left(\frac{n-k}{k}\right)$

Algorytm: (n, t, W[ ])

```
w := W0          // W0 (n 0) t^0
bin := n         // do liczenia (n i)
pow := t         // do liczenia t^i
e := 1-t
for (i = 1; i ≤ n; i += 1)
    w := Wi · bin · pow + w · e
    pow := pow · t
    bin := (bin · (n-i)) / i+1
```

zwróć $w = P_n(t)$

ANL - 8

(7)

$$R_n(t) = \frac{\sum_{i=0}^{n} w_i W_i B_i^n(t)}{\sum_{i=0}^{n} w_i B_i^n(t)}$$

$0 \leq t \leq 1$

$W_0, W_1, \dots, W_n \in E^2$

$w_0, w_1, w_2, \dots, w_n \in R_+$ wagi

T: dla każdego $t \in [0,1]$ $R_n(t)$ - punkt będący kombinacją barycentryczną punktów kontrolnych $W_0, \dots, W_n$

$$R_n(t) = \sum_{i=0}^{n} \underbrace{\frac{w_i B_i^n(t)}{\sum_{j=0}^{n} w_j B_j^n(t)}}_{\alpha_i} W_i$$

Komb. barycentryczna: $\alpha_0 W_0 + \alpha_1 W_1 + \dots + \alpha_n W_n$

gdzie $\sum_{i=0}^{n} \alpha_i = 1$

Czyli

$$\sum_{i=0}^{n} \alpha_i = \sum_{i=0}^{n} \frac{w_i B_i^n(t)}{\sum_{j=0}^{n} w_j B_j^n(t)} = \frac{\sum_{i=0}^{n} w_i B_i^n(t)}{\sum_{i=0}^{n} w_i B_i^n(t)} = 1$$

$\blacksquare$