

ANL-15

$$\textcircled{1} A = \begin{bmatrix} 780 & 563 \\ 813 & 658 \end{bmatrix} \quad b = \begin{bmatrix} 217 \\ 254 \end{bmatrix} \rightarrow x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\tilde{x} = \begin{bmatrix} 0.888 \\ -1.001 \end{bmatrix} \quad \hat{x} = \begin{bmatrix} 0.341 \\ -0.087 \end{bmatrix}$$

$$A\tilde{x} = \begin{bmatrix} 780 \cdot 0.888 + 563 \cdot (-1.001) \\ 813 \cdot 0.888 + 658 \cdot (-1.001) \end{bmatrix} = \begin{bmatrix} 216.999 \\ 252.418 \end{bmatrix}$$

$$A\hat{x} = \begin{bmatrix} 216.999 \\ 254 \end{bmatrix}$$

$$\tilde{r} = A\tilde{x} - b = \begin{bmatrix} -1.343 \\ -1.572 \end{bmatrix} \quad \hat{r} = A\hat{x} - b = \begin{bmatrix} -0.001 \\ 0 \end{bmatrix}$$

$$\tilde{e} = \begin{bmatrix} -0.001 \\ -0.001 \end{bmatrix}$$

$$\hat{e} = \begin{bmatrix} -0.653 \\ 0.913 \end{bmatrix}$$

② $A = \begin{bmatrix} 2 & 0 & 2 & 1 \\ -4 & 2 & -4 & 0 \\ -6 & 8 & -4 & 5 \\ -10 & 12 & -24 & 8 \end{bmatrix}$ $\begin{cases} u_{ij} = a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj} \quad (1) \quad i \leq j \\ l_{ij} = \frac{a_{ij} - \sum_{k=1}^{j-1} l_{ik} u_{kj}}{u_{jj}} \quad (2) \quad i > j \end{cases}$

$A = LU$ $u_{jj} L = \begin{bmatrix} 2 & 0 & 2 & 1 \\ -4 & 2 & -4 & 0 \\ -6 & 8 & -4 & 5 \\ -10 & 12 & -24 & 8 \end{bmatrix} u = \begin{bmatrix} 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

$j=1$ $i \downarrow$

$u_{11} = a_{11} = 2$

$l_{21} = \frac{a_{21}}{u_{11}} = \frac{-4}{2} = -2$

$l_{31} = \frac{a_{31}}{u_{11}} = \frac{-6}{2} = -3$

$l_{41} = \frac{a_{41}}{u_{11}} = \frac{-10}{2} = -5$

$j=2$ $i \downarrow$

$u_{22} = a_{22} = 2$

$u_{22} = a_{22} - l_{21} u_{12} = 2$

$l_{32} = \frac{a_{32} - l_{31} u_{12}}{u_{22}} = \frac{8 - (-3) \cdot 2}{2} = \frac{14}{2} = 7$

$l_{42} = \frac{a_{42} - l_{41} u_{12}}{u_{22}} = \frac{12 - (-5) \cdot 2}{2} = \frac{22}{2} = 11$

$j=3$

$u_{13} = a_{13} = 2$

$u_{23} = a_{23} - l_{21} u_{13} = -4 + 4 = 0$

$u_{33} = a_{33} - l_{31} u_{13} - l_{32} u_{23} = -4 - (-3) \cdot 2 - 7 \cdot 0 = 2$

$l_{43} = \frac{a_{43} - l_{41} u_{13} - l_{42} u_{23}}{u_{33}} = \frac{-10 - (-5) \cdot 2 - 11 \cdot 0}{2} = \frac{0}{2} = 0$

$j=4$

$u_{14} = a_{14} = 1$

$u_{24} = a_{24} - l_{21} u_{14} = 0$

$u_{34} = a_{34} - l_{31} u_{14} - l_{32} u_{24} = 0$

$u_{44} = a_{44} - l_{41} u_{14} - l_{42} u_{24} - l_{43} u_{34} = 2$

e) $\det(A) = \det(L) \cdot \det(U) = \det(U) = 2^4 = 16$

\uparrow \uparrow \uparrow \uparrow

Cramer's \uparrow \uparrow \uparrow \uparrow

trigonal \uparrow \uparrow \uparrow \uparrow

trigonal \uparrow \uparrow \uparrow \uparrow

2. best

ANL-13

2/3

$$AA^{-1} = I$$

$$A[c_i] = [e_i]$$

holen wir A^{-1}

weltig
jedenfalls
↓

$$LVA^{-1} = I \rightarrow LVA^{-1} \cdot [c_1 \ c_2 \ c_3 \ c_4] = [e_1 \ e_2 \ e_3 \ e_4]$$

$$A = \begin{bmatrix} 2 & 0 & 2 & 1 \\ -4 & 2 & -4 & 0 \\ -6 & 8 & -4 & 5 \\ -10 & 12 & -24 & 8 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & & \\ -2 & 1 & & \\ -3 & 4 & 1 & \\ -5 & 6 & -7 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 0 & 2 & 1 \\ & 2 & 0 & 2 \\ & & 2 & 0 \\ & & & 2 \end{bmatrix}$$

$$A \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \\ c_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$Ly = [e_1] \Rightarrow y = \begin{bmatrix} 1 \\ 2 \\ -5 \\ -42 \end{bmatrix}$$

$$Ux = y \Rightarrow x = \begin{bmatrix} 27/2 \\ 22 \\ -5/2 \\ -21 \end{bmatrix} = c_1$$

$$(c_2): Ly = [e_2] \Rightarrow y = \begin{bmatrix} 0 \\ 1 \\ -4 \\ -34 \end{bmatrix}$$

$$Ux = y \Rightarrow x = \begin{bmatrix} 21/2 \\ 35/2 \\ -2 \\ -17 \end{bmatrix}$$

$$(c_3): Ly = [e_3] \Rightarrow y = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 7 \end{bmatrix}$$

$$Ux = y \Rightarrow x = \begin{bmatrix} -3/4 \\ -7/2 \\ 1/2 \\ 7/2 \end{bmatrix}$$

$$(c_4): Ly = [e_4] \Rightarrow y = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Ux = y \Rightarrow x = \begin{bmatrix} -1/4 \\ -1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

$$A^{-1} = [c_1 | c_2 | c_3 | c_4] = \begin{bmatrix} 27/2 & 21/2 & -3/4 & -1/4 \\ 22 & 35/2 & -7/2 & -1/2 \\ -5/2 & -2 & 1/2 & 0 \\ -21 & -17 & 7/2 & 1/2 \end{bmatrix}$$

$$AA^{-1} = I \checkmark$$

③

ANL-14

3/3

$$A = \begin{bmatrix} 1 & 1 & 1 & -10 \\ -1 & -3 & 0 & 11 \\ -2 & -10 & 5 & 25 \\ -3 & -13 & -16 & 25 \end{bmatrix} \quad b = \begin{bmatrix} -6 \\ 8 \\ 31 \\ -13 \end{bmatrix}$$

$$Ax = b$$

$$A = LU \quad A \cdot I \rightarrow LU$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ -1 & -3 & 0 & 11 \\ -2 & -10 & 5 & 25 \\ -3 & -13 & -16 & 25 \end{bmatrix} \begin{array}{l} w_2 - (-w_1) \\ w_3 - (-2w_1) \\ w_4 - (-3w_1) \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -3 & 0 & 0 & 1 \end{bmatrix} \downarrow \begin{bmatrix} 1 & 1 & 1 & -10 \\ 0 & -2 & 1 & 1 \\ 0 & -8 & 7 & 5 \\ 0 & -10 & -13 & -5 \end{bmatrix} \begin{array}{l} w_3 - (4w_2) \\ w_4 - (5w_2) \end{array}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -2 & 4 & 1 & 0 \\ -3 & 5 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ 0 & -2 & 1 & 1 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & -18 & -10 \end{bmatrix} w_4 - (6w_3)$$

$$\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ -2 & 4 & 1 \\ -3 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & -10 \\ -2 & 1 & 1 & 1 \\ 0 & 3 & -1 & 1 \\ 0 & 3 & -1 & 1 \end{bmatrix} = L \cdot U$$

$$Ly = b \Rightarrow y = \begin{bmatrix} -6 \\ 3 \\ 7 \\ -4 \end{bmatrix}$$

$$Ux = y \Rightarrow x = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$