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(1) zał:
$$x_{n+1} = x_n - f_n \frac{x_n - x_{n-1}}{f_n - f_{n-1}}$$

$$= \frac{x_n(f_n - f_{n-1})}{f_n - f_{n-1}} - \frac{f_n x_n - f_n x_{n-1}}{f_n - f_{n-1}}$$

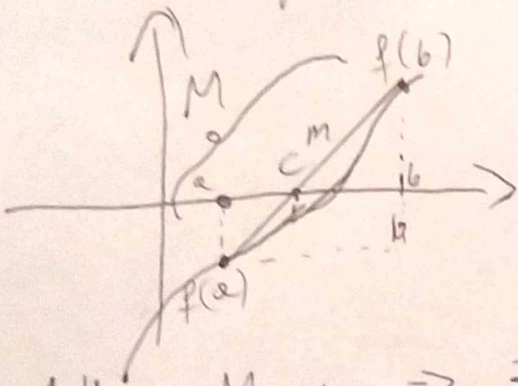
T:
$$= \frac{f_n x_{n-1} - f_{n-1} x_n}{f_n - f_{n-1}} \qquad \blacksquare$$

$f_n \approx f_{n-1}$, blisko rozwiązania $f_n \approx 0$ i redukuje błąd; x_{n-1}, x_n tych samych znaków

pierwszy wzór lepszy

(2): $\alpha,\ f(\alpha)=0,\ [a,b],\ f(a)f(b)<0$

$(a,f(a)),\ \underbrace{(c,0)}_{\text{przecięcie z } OX},\ (b,f(b))$ ~~styczny~~ sieczny



$f(a)<0$

1° $f(c)<0 \Rightarrow [c,b]$

2° $f(c)>0 \Rightarrow [a,c]$

3° $f(c)=0 \Rightarrow$ koniec

$(|f(c)|<\varepsilon)$

$$\frac{\Delta y}{\Delta x} \Rightarrow M=m \Rightarrow \frac{f(b)-f(a)}{b-a} = \frac{f(b)-0}{b-c}$$

$$\Rightarrow c = b - f(b)\frac{(b-a)}{f(b)-f(a)}$$

$$c_n = \frac{a_n f(b_n) - b_n f(a_n)}{f(b_n)-f(a_n)},$$

Zalety:

- zwarte przesłanki, zbiega
- $\operatorname{sgn}(f(a_n)) \neq \operatorname{sgn}(f(b_n)) \to$ można w dowolnym momencie

Wady:

- może być wolniejsza od bisekcji

ANL-5

3) $x_{n+1} = F(x_n)$

$p-1$-razy różniczkowalna, δ pomiędzy x_n, α

otoczenie α: $= F(\alpha) + (x_n-\alpha)\overset{=0}{F'(\alpha)} + (x_n-\alpha)^2\dfrac{\overset{=0}{F''(\alpha)}}{2!} + \dots + \dfrac{(x_n-\alpha)^{p-1}\overset{=0}{F^{(p-1)}(\alpha)}}{(p-1)!} + \dfrac{(x_n-\alpha)^p F^{(p)}(\delta)}{p!}$

$$x_{n+1} - \alpha = \frac{(x_n-\alpha)^p\, F^{(p)}(\delta)}{p!}$$

$$\frac{x_{n+1}-\alpha}{(x_n-\alpha)^p} = \frac{F^{(p)}(\delta)}{p!}$$

$F^{(p)}(\alpha) \neq 0 \Rightarrow F^{(p)}(x) \neq 0$ w otoczeniu α

$$\lim_{n\to\infty} \frac{|x_{n+1}-\alpha|}{|x_n-\alpha|^p} = \cancel{F^{(p)}(\alpha)} \left|\frac{F^{(p)}(\alpha)}{p!}\right| = C \neq 0$$

(4) zał: $f(\alpha)=0$, $f'(\alpha)\neq 0$

$1^\circ\ F(\alpha)=\alpha \quad 2^\circ\ F'(\alpha)=0 \quad 3^\circ\ F''(\alpha)\neq 0$

$$1^\circ\ F(x) = x - \frac{f(x)}{f'(x)}$$

$$F(\alpha) = \alpha - \frac{\overset{=0}{f(\alpha)}}{f'(\alpha)} = \alpha$$

$$2^\circ\ F'(x) = 1 - \frac{[f'(x)]^2 - \overset{0|_{x=\alpha}}{f(x)} f^{(2)}(x)}{[f'(x)]^2}\Bigg|_{x=\alpha} = 1 - 1 = 0$$

$$3^\circ\ F'(x) = 1 - \left[\frac{f'(x)}{f'(x)}\right]^2 + \frac{f(x) f^{(2)}(x)}{[f'(x)]^2} = \frac{f(x) f^{(2)}(x)}{[f'(x)]^2}$$

$$F''(x) = \frac{[f'(x) f^{(2)}(x) + \underbrace{f(x)}_{0|_{x=\alpha}} f^{(3)}(x)][f'(x)]^2 - \underbrace{f(x)}_{0|_{x=\alpha}} f^{(2)}(x) \cdot 2f'(x) \cdot f^{(2)}(x)}{[f'(x)]^4}$$

$$F''(x) = \frac{[f'(x)]^3 f^{(2)}(x)}{[f'(x)]^4}\Bigg|_{x=\alpha} = \frac{f^{(2)}(\alpha)}{f'(\alpha)} \neq 0$$

$f^{(2)}(\alpha) \neq 0 \rightarrow$ kwadratowe

$f^{(2)}(\alpha) = 0 \rightarrow$ sześcienne lub lepsze

$f(\alpha)=0,\ f'(\alpha)\neq 0,\ f''(\alpha)=0$

5 $f(\alpha) = f'(\alpha) = 0 \quad f''(\alpha) \neq 0$

$1^\circ\ F(\alpha) = \alpha \quad 2^\circ\ F'(\alpha) \neq 0 \quad 3^\circ\ |F'(\alpha)| < 1 \quad c \in (0,1)$

$1^\circ\ F(\alpha) = \alpha - \frac{f(\alpha)}{f'(\alpha)} \overset{\left[\frac{0}{0}\right]}{\underset{LH}{=}} \alpha - \frac{\overset{=0}{f'(\alpha)}}{f''(\alpha)} = \alpha$

$2^\circ\ F'(\alpha) = \frac{f(\alpha) \cdot f^{(2)}(\alpha)}{[f'(\alpha)]^2} \overset{\left[\frac{0}{0}\right]}{=} \frac{f'(\alpha) f^{(2)}(\alpha) + f(\alpha) f^{(3)}(\alpha)}{2 f'(\alpha) f^{(2)}(\alpha)} =$

$\frac{1}{2} + \underbrace{\frac{f(\alpha) f^{(3)}(\alpha)}{2f'(\alpha) f^{(2)}(\alpha)}}_{x \to \alpha} \overset{\left[\frac{0}{0}\right]}{=} \frac{1}{2} + \frac{\overset{=0}{f'(\alpha)} f^{(3)}(\alpha) + \overset{=0}{f(\alpha)} f^{(4)}(\alpha)}{2(\underset{\neq 0}{f^{(2)}(\alpha) f^{(2)}(\alpha)} + \underset{=0}{f'(\alpha) f^{(3)}(\alpha)})} \xrightarrow{x \to \alpha} \frac{1}{2}$

$F'(\alpha) = \underset{c}{\underset{\shortparallel}{\frac{1}{2}}} \quad |F'(\alpha)| < 1$ zb. liniowa

6) $x_n - \frac{f(x_n)}{f'(x_0)} = F(x_n)$

$f'(x_0) \neq 0$

$F(\alpha) = \alpha$, $\alpha - \frac{f(\alpha)^{=0}}{f'(x)} = \alpha$

$F'(x) = 1 - \frac{f'(x)}{\underbrace{f'(x_0)}_{\text{stała}}}$, $|F'(\alpha)| < 1$

liniowe:

kwadratowe: lub większe

$f'(x) = f'(x_0)$

$\Downarrow$

$F'(x) = 0$

$\left|1 - \frac{f'(\alpha)}{f'(x_0)}\right| < 1$ $\quad \frac{f'(\alpha)}{f'(x_0)} \in (0,2) \setminus \{1\}$

(7) $\varepsilon_n := x_n - \alpha$

$$|\varepsilon_{n+1}| \approx C \cdot |\varepsilon_n|^p \,, \quad |\varepsilon_n| \approx C \cdot |\varepsilon_{n-1}|^p$$

$$\frac{|\varepsilon_{n+1}|}{|\varepsilon_n|} = \frac{C \cdot |\varepsilon_{n}|^p}{C \cdot |\varepsilon_{n-1}|^p} = \left(\left| \frac{\varepsilon_n}{\varepsilon_{n-1}} \right| \right)^p$$

$$\log \left| \frac{\varepsilon_{n+1}}{\varepsilon_n} \right| = p \log \left| \frac{\varepsilon_n}{\varepsilon_{n-1}} \right| \qquad (\log a^b = b \log a)$$

$$p \approx \frac{\log \left| \frac{\varepsilon_{n+1}}{\varepsilon_n} \right|}{\log \left| \frac{\varepsilon_n}{\varepsilon_{n-1}} \right|} = \frac{\log \left| \frac{x_{n+1} - \alpha}{x_n - \alpha} \right|}{\log \left| \frac{x_n - \alpha}{x_{n-1} - \alpha} \right|}$$

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gdy nie znamy α:

$$\lim_{x_n \to \alpha} \left| \frac{\varepsilon_{n+1}}{\varepsilon_n} \right| = |F'(\alpha)|$$

$$(1)\quad x_{n+1} = \alpha + F'(\alpha)(x_n - \alpha) + \frac{F''(\delta_1)(x_n - \alpha)^2}{2}$$

$$(2)\quad x_n = \alpha + F'(\alpha)(x_{n-1} - \alpha) + \frac{F''(\delta_2)(x_{n-1} - \alpha)^2}{2}$$

$\frac{(1)-(2)}{x_n - x_{n-1}}$, kwadraty zbiegają szybciej do zera

$$\left| \frac{x_{n+1} - x_n}{x_n - x_{n-1}} \right| \xrightarrow{n \to \infty} |F'(\alpha)|$$

Czyli $\left| \frac{x_{n+1} - x_n}{x_n - x_{n-1}} \right| \approx \left| \frac{x_{n+1} - \alpha}{x_n - \alpha} \right|$ dla odpowiednio dużych n:

$$p \approx \frac{\log \left| \frac{x_{n+1} - x_n}{x_n - x_{n-1}} \right|}{\log \left| \frac{x_n - x_{n-1}}{x_{n-1} - x_{n-2}} \right|}$$