(2) $f(x) = ax(2021x - 2020) + 1977$

$\mathcal{X} = \{f(x) : a \in \mathbb{R}\}$

$$\|f - w^*\|_2 = \min_{w \in \mathcal{X}} \|f - w\|_2 = \min_{a \in \mathbb{R}} \sqrt{\underbrace{\sum_{k=0}^{n} (y_k - w(x_k))^2}}$$

$$E(a) = \sum_{k=0}^{n} \left(y_k - ax_k(2021x_k - 2020) - 1977\right)^2$$

$$E'(a) = \sum_{k=0}^{n} 2\left(y_k - ax_k(2021x_k - 2020) - 1977\right)\left(x_k[2021x_k - 2020]\right) = 0$$

$$\sum_{k=0}^{n} \left(y_k - ax_k(2021x_k - 2020) - 1977\right)\left(x_k(2021x_k - 2020)\right) = 0$$

$$\sum_{k=0}^{n} (y_k - 1977)\left(x_k(2021x_k - 2020)\right) - \sum_{k=0}^{n} a\left[x_k(2021x_k - 2020)\right]^2 = 0$$

$$\triangle \quad a = \frac{\sum_{k=0}^{n} (y_k - 1977)\left(x_k(2021x_k - 2020)\right)}{\sum_{k=0}^{n} \left[x_k(2021x_k - 2020)\right]^2}$$

element optymalny w sensie aproksymacji: $\square$ śr

$w^* = \cancel{ax^*(2021} \; a^* x(2021x - 2020) + 1977, \quad a^* = \triangle$

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③ $E(a) = \sum_{k=0}^{r} \underbrace{\frac{e^{x_k}+2020}{1+\ln(x_k^2+1)}}_{l_k = \text{const.} - a} \left[y_k - a \underbrace{\left(\cos(2x_k+2020)+x_k^3\right)}_{c_k}\right]^2$

$$E(a) = \sum_{k=0}^{r} l_k \left[y_k - a c_k\right]^2$$

$$E'(a) = -2 \sum_{k=0}^{r} l_k (y_k - a c_k)(c_k) = 0 \quad // -2$$

$$\sum_{k=0}^{r} l_k y_k c_k - \sum_{k=0}^{r} l_k a c_k^2 = 0$$

$$\sum_{k=0}^{r} l_k y_k c_k - a \sum_{k=0}^{r} l_k c_k^2 = 0$$

$$a = \frac{\sum_{k=0}^{r} l_k y_k c_k}{\sum_{k=0}^{r} l_k c_k^2} = \frac{\sum_{k=0}^{r} y_k \cdot \frac{e^{x_k}+2020}{1+\ln(x_k^2+1)} \cdot \left(\cos(2x_k+2020)+x_k^3\right)}{\sum_{k=0}^{r} \frac{e^{x_k}+2020}{1+\ln(x_k^2+1)} \cdot \left(\cos(2x_k+2020)+x_k^3\right)^2}$$

(5) $S = aT + b$

k	0	1	2	3	4	5	6	7
T_k	0	10	20	30	40	80	90	95
S_k	68.0	67.1	66.4	65.6	64.6	61.8	61.0	60.0

Z wykładu:

$$\begin{cases} a = \dfrac{(N+1)s_4 - s_1 s_3}{(N+1)s_2 - s_1^2} \\ b = \dfrac{s_2 s_3 - s_1 s_4}{(N+1)s_2 - s_1^2} \end{cases}$$

$s_1 = \sum_{k=0}^{7} T_k = 365$

$s_2 = \sum_{k=0}^{7} T_k^2 = 26525$

$s_3 = \sum_{k=0}^{7} S_k = 514,5$

$s_4 = \sum_{k=0}^{7} T_k S_k = 0 + 671 + 1328 + 1968$

$2584 + 4944 + 5490 + 5700$

$= 22685$

$$a = \frac{\overset{181480}{8 \cdot (22685)} - \overset{187792,5}{365 \cdot 514,5}}{8(26525) - 365^2}$$

$$= \frac{-6312,5}{78975} \approx -0,07993$$

$$b = \frac{26525 \cdot 514,5 - 365 \cdot 22685}{8 \cdot 26525 - 365^2} = \frac{5367087,5}{78975}$$

$\approx 67,95932$

⑥ (x_k, y_k) $k \in \{0, 1, \dots r\}$

$y \approx e^{ax+b}$, szukamy a i b

$\downarrow$

Chcemy kombinację liniową

$\ln y \approx ax + b$ $\quad x_k \xrightarrow{f} \ln(y_k)$

Przybliżamy $\ln y$

Z wykładu (lub macierzy) $\begin{bmatrix} \langle 1, 1 \rangle & \langle 1, x \rangle \\ \langle x, 1 \rangle & \langle x, x \rangle \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \langle 1, \ln y \rangle \\ \langle x, \ln y \rangle \end{bmatrix}$

$S_1 = \sum_{k=0}^{r} x_k \quad S_2 = \sum_{k=0}^{r} x_k^2 \quad S_3 = \sum_{k=0}^{r} f(x_k) = \sum_{k=0}^{r} \ln y_k$

$S_4 = \sum_{k=0}^{r} x_k f(x_k) = \sum_{k=0}^{r} x_k \ln y_k$

Wtedy $b = \dfrac{S_2 S_3 - S_1 S_4}{(r+1) S_2 - S_1^2}$

$a = \dfrac{(r+1) S_4 - S_1 S_3}{(r+1) S_2 - S_1^2}$

$y = e^{ax+b}$

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$$H(t) = h_0 + a_1 \sin\frac{2\pi t}{12} + a_2 \cos\frac{2\pi t}{12}$$

$$g_0 = 1, \; g_1 = \sin\frac{2\pi t}{12}, \; g_2 \cos\frac{2\pi t}{12}$$

$$H(t) = g_0 h_0 + g_1 a_1 + g_2 a_2$$

$$\begin{bmatrix} \langle g_0 g_0\rangle & \langle g_0 g_1\rangle & \langle g_0 g_2\rangle \\ \langle g_1 g_0\rangle & \langle g_1 g_1\rangle & \langle g_1 g_2\rangle \\ \langle g_2 g_0\rangle & \langle g_2 g_1\rangle & \langle g_2 g_2\rangle \end{bmatrix} \begin{bmatrix} h_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \langle g_0, H(t)\rangle \\ \langle g_1, H(t)\rangle \\ \langle g_2, H(t)\rangle \end{bmatrix}$$

$$Ax = B$$

$\langle a, b\rangle = \langle b, a\rangle$

6 pomiarów $\to k = 5$

Trzeba policzyć $\langle 1, 1\rangle = \sum_{k=0}^{5} 1 = 6$, $\langle 1, H(t)\rangle = \langle H(t), 1\rangle = \sum_{k=0}^{5} H(t_k)$ itp.

$$\langle g_1, g_2\rangle = \sum_{k=0}^{n} \sin\frac{2\pi t_k}{12} \cos\frac{2\pi t_k}{12}$$

$h_0 \approx 0{,}933$, $a_1 = 0{,}577$, $a_2 \approx 0{,}267$