ANL - Lista 3

(1) a) $\underbrace{4\cos^2 x - 3}_{=0,\ \cos x = \pm\frac{\sqrt{3}}{2},\ x\in\{-\frac{\pi}{6}+2k\pi,\ \frac{\pi}{6}+2k\pi : k\in\mathbb{Z}\}}$

$4\cos^2 x - 3 = \underbrace{\cos^2 x - \sin^2 x}_{\cos 2x} - 2\sin^2 x =$

$2\sin x\cos x = \sin 2x$

$\cos 2x - 2\sin^2 x = \frac{\cos 2x\cos x - 2\sin^2 x\cos x}{\cos x} =$

$\frac{\cos 2x\cos x - \sin 2x\sin x}{\cos x} = \frac{\cos(2x+x)}{\cos x} = \frac{\cos 3x}{\cos x}$

b) $x^{-3}\left(\frac{\pi}{2} - x - \operatorname{arcctg}(x)\right) =$

$x^{-3}\left(\frac{\pi}{2} - x - \left(\frac{\pi}{2} - \operatorname{arctg}(x)\right)\right) =$

$x^{-3}\underbrace{\left(\operatorname{arctg}(x) - x\right)}_{\cancel{x}\to 0 \text{ problem}}$

Rozwijamy $\operatorname{arctg}(x)$ $(-1\leq x\leq 1)$: $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$

$\left(\frac{\operatorname{arctg}(x) - x}{x^3}\right)$: $-\frac{1}{3} + \frac{x^2}{5} - \frac{x^4}{7}$

(2) a) $b > 0$; $ac \ll b^2$

$$-b + \sqrt{b^2 - 4ac} \approx 0$$

b) $b < 0$ $ac >> b^2$

$$-b - \sqrt{b^2 - 4ac} \approx 0$$

Wzory Viete'a : $x_1 x_2 = \frac{c}{a}$, $x_1 + x_2 = \frac{-b}{a}$

a) $b > 0$

$$x_1 = \frac{-b \overset{\text{normalnie}}{-} \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{c}{a x_1}$$

b) $b < 0$

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{c}{a x_1}$$

$x_2 = \quad a x_1 \qquad\qquad a x_1$

(3) $$x = \Big(r + \overbrace{\sqrt{q^3+r^2}}^{b=\sqrt{q^3+r^2}}\Big)^{1/3} + \Big(r - \sqrt{q^3+r^2}\Big)^{1/3}$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$(r+b)^{1/3} + (r-b)^{1/3} = \frac{r+b + (r-b)}{(r+b)^{2/3} - (r+b)^{1/3}(r-b)^{1/3} + (r-b)^{2/3}}$$

$$= \frac{2r}{(r+b)^{2/3} - \underbrace{[(r+b)(r-b)]}_{r^2-b^2}^{1/3} + (r-b)^{2/3}} = \frac{2r}{\left(r+\sqrt{q^3+r^2}\right)^{2/3} - \left(r^2-q^3-r^2\right)^{1/3} + \left(r-\sqrt{q^3+r^2}\right)^{2/3}}$$

$$\boxed{r - \sqrt{q^3+r^2} = \frac{r^2-q^3-r^2}{r+\sqrt{q^3+r^2}}}$$

$$= \frac{2r}{\left(\sqrt[3]{r+\sqrt{q^3+r^2}}\right)^2 + q + \left(\sqrt[3]{\frac{-q^3}{r+\sqrt{q^3+r^2}}}\right)^2} =$$

$$= \frac{2r}{\left(\sqrt[3]{r+\sqrt{q^3+r^2}}\right)^2 + q + q^2\left(\frac{1}{\sqrt[3]{r+\sqrt{q^3+r^2}}}\right)^2}$$

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(5) $\left|\frac{(x+h)-x}{x}\right| = \left|\frac{h}{x}\right|$ — względna zmiana danych

$\left|\frac{f(x+h)-f(x)}{f(x)}\right|$ — -//- wyniku

$$K(x) = \left|\frac{f(x+h)-f(x)}{f(x)}\right| : \left|\frac{h}{x}\right| = \left|\frac{f(x+h)-f(x)}{f(x)}\right| \cdot \left|\frac{x}{h}\right| =$$

$$\underbrace{\left|\frac{f(x+h)-f(x)}{h}\right|}_{f'(x)} \left|\frac{x}{f(x)}\right| = |f'(x)| \left|\frac{x}{f(x)}\right| = \left|\frac{f'(x)\,x}{f(x)}\right|$$

(5) a) $f(x) = x^3 + 2020, \quad f'(x) = 3x^2$

$$k(x) = \left| \frac{x \cdot 3x^2}{x^3 + 2020} \right| = \left| \frac{3}{\underbrace{1 + \frac{2020}{x^3}}_{\approx 0}} \right|$$

$x \to \sqrt[3]{2020}$ žle [illegible]

vtedy $k(x) \to \infty$

b) $f(x) = \frac{\ln x}{x} \quad f'(x) = \frac{1 - \ln x}{x^2}$

$$k(x) = \left| \frac{x \cdot \left(\frac{1 - \ln x}{\cancel{x^2}} \right)}{\frac{\ln x}{\cancel{x}}} \right| = \left| \frac{1 - \ln x}{\ln x} \right|$$

$x \to 1$ ← žle

$\ln x \to 0$

$k(x) \to \infty$

c) $f(x) = \cos(5x) \quad f'(x) = -5 \sin 5x$

$$k(x) = \left| \frac{x \cdot (-5 \sin 5x)}{\cos 5x} \right| = \left| -5x \, tg(5x) \right|$$

$tg \frac{\pi}{2} \quad \lim_{x \to \frac{\pi}{2}} tg(x) = +\infty$

$5x = \frac{\pi}{2} + k\pi$

$x \to \frac{\pi}{10} + \frac{1}{5} k\pi$ žle

d) $f(x) = \left(\sqrt{x^4 + 2020} + x \right)^{-1}$

$$f'(x) = -\left(\sqrt{x^4 + 2020} + x \right)^{-2} \cdot \left(1 + \frac{1}{2\sqrt{x^4 + 2020}} \cdot 4x^3 \right)$$

$$k(x) = \left| \frac{x \left(1 + \frac{2x^3}{\sqrt{x^4 + 2020}} \right) \cancel{\left(\sqrt{x^4 + 2020} + x \right)}}{\left(\sqrt{x^4 + 2020} + x \right)^{\cancel{2}}} \right|$$

$$= \left| \frac{\frac{2x^4}{\sqrt{x^4 + 2020}} + x}{\sqrt{x^4 + 2020} + x} \right| \quad \frac{2x^2}{x^2}$$

$x \to \pm\infty \quad k(x) \to 2$

dobre [illegible]

7 $\left(x + \frac{1}{x}(1+\varepsilon_1)\right)(1+\varepsilon_2) = \left(x+\frac{1}{x}\right)(1+y)(1+\varepsilon_2)$

$\cancel{x} + \cancel{\frac{1}{x}} + \frac{\varepsilon_1}{x} = \cancel{x} + \cancel{\frac{1}{x}} + y\left(x+\frac{1}{x}\right)$

$|y| = \left|\frac{\varepsilon_1}{x^2+1}\right| \leq |\varepsilon_1| \leq 2^{-t}$

$|y| \leq 2^{-t} \quad |\varepsilon_2| \leq 2^{-t} \quad |E| \leq 2 \cdot 2^{-t}$

$\left(x + \frac{1}{x}(1+\varepsilon_1)\right)(1+\varepsilon_2) = \left(x + \frac{1}{x}\right)(1+E)$

② ANL - Lista 3

$rd(x_i) = x_i$

$$I = x_1(1+\varepsilon_1) \cdot x_2(1+\varepsilon_2) \cdot x_3(1+\varepsilon_3) \cdots x_n(1+\varepsilon_n)$$

$|\varepsilon_i| \le 2^{-t}$ (for $i>1$), $\varepsilon_1 = 0$

$$I = \prod_{i=1}^{n} x_i(1+\varepsilon_i) = \left(\prod_{i=1}^{n} x_i\right)\underbrace{\left(\prod_{i=1}^{n}(1+\varepsilon_i)\right)}_{(1+E)}{}_{*} = \left(\prod_{i=1}^{n} x_i\right)(1+E)$$

$(1+E)^{*}$: $|E| \le (n-1)\cdot 2^{-t}$

bez założenia o $rd(x_i) = x_i$

$$I = x_1(1+\alpha_1)(1+\varepsilon_1)\, x_2(1+\alpha_2)(1+\varepsilon_2) \cdot \ldots \cdot x_n(1+\alpha_n)(1+\varepsilon_n)$$

$$I = \prod_{i=1}^{n} x_i(1+\alpha_i)(1+\varepsilon_i) = \left(\prod_{i=1}^{n} x_i(1+\alpha_i)\right)\underbrace{\prod_{i=1}^{n}(1+\varepsilon_i)}_{(1+E)_*} = \left(\prod_{i=1}^{n} x_i(1+\alpha_i)\right)(1+E)$$

$|\varepsilon_0| \le 0$ $\quad |\varepsilon_i| \le 2^{-t}$ (for $i>1$) $\quad |\alpha_i| \le 2^{-t}$ $\quad |E| \le (n-1)2^{-t}$