ANL

(1) a) $[a_n, b_n] \supset [a_{n+1}, b_{n+1}]$, $n \in (0,1,\dots)$

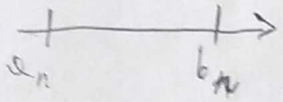
$f(a_0) f(b_0) < 0$

$f(a_0) < 0$

$$m_{n+1} = \frac{a_n + b_n}{2}$$

$f(m_{n+1}) = 0 \to$ koniec

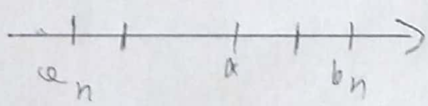
$$[a_{n+1}, b_{n+1}] = \begin{cases} [m_{n+1}, b_n], & a_n < m_{n+1} \Rightarrow [a_n, b_n] \supset [a_{n+1}, b_{n+1}] \\ [a_n, m_{n+1}], & m_{n+1} < b_n \; \cancel{b_n < m_{n+1}} \Rightarrow [a_n, b_n] \supset [a_{n+1}, b_{n+1}] \end{cases}$$



b) $$|b_n - a_n| = \left| \frac{b_{n-1} - a_{n-1}}{2} \right| = \dots = \left| \frac{b_0 - a_0}{2^n} \right|$$

Zaczynamy: $|b_0 - a_0|$, bez straty ogólności $b_1 = b_0$, $a_1 = \frac{a_0 + b_0}{2} \Rightarrow |b_1 - a_1| = \left| \frac{b_0 - a_0}{2} \right|$ itd.

c) $|e_n| \leq 2^{-n-1}(b_0 - a_0)$, $n \geq 0$

d) 

$$|b_{n+1} - a_{n+1}| = \left| \frac{b_0 - a_0}{2^{n+1}} \right| = \frac{b_0 - a_0}{2^{n+1}}$$

$a_0 < 0$ $b_0 < 0$, $a_0 < b_0$, $b_0 - a_0 > 0$

$a_0 > 0$ $b_0 < 0$ X

1° $m_{n+1} = a_{n+1} \Rightarrow \alpha > m_{n+1}^{a}$, $\alpha < b_{n+1}$

$|\alpha - m_{n+1}| = |\alpha - a_{n+1}| \leq |b_{n+1} - a_{n+1}|$

2° $m_{n+1} = b_{n+1} \Rightarrow \alpha < m_{n+1}^{b}$, $\alpha > a_{n+1}$

$|\alpha - m_{n+1}| = |\alpha - b_{n+1}| = |b_{n+1} - \alpha| \leq |b_{n+1} - a_{n+1}|$

Więc $|e_n| = |\alpha - m_{n+1}| \leq |b_{n+1} - a_{n+1}| = \frac{b_0 - a_0}{2^{n+1}}$ ∎

e) Skończony V

Nieskończony X $\xrightarrow[n \to \infty]{} b_0$

ANL - 4

(2) $|e_n| \leq 2^{-n-1}(b_0 - a_0)$ z zad 1

Dla danego ε, szukamy takiego n, że $|e_n| \leq |\varepsilon|$:

$$\frac{(b_0 - a_0)}{2^{n+1}} \leq |\varepsilon| \quad / \cdot \left(\frac{2^n}{\varepsilon}\right)$$

$$\frac{(b_0 - a_0)}{2\varepsilon} \leq 2^n \quad / \log_2(\,) \,,\ 2>1$$

$$\log_2 \frac{(b_0 - a_0)}{2\varepsilon} \leq n \quad , \text{ale } n \in \mathbb{N}$$

$$n = \left\lceil \log_2\left[\frac{(b_0 - a_0)}{2\varepsilon}\right] \right\rceil$$

ANL-4

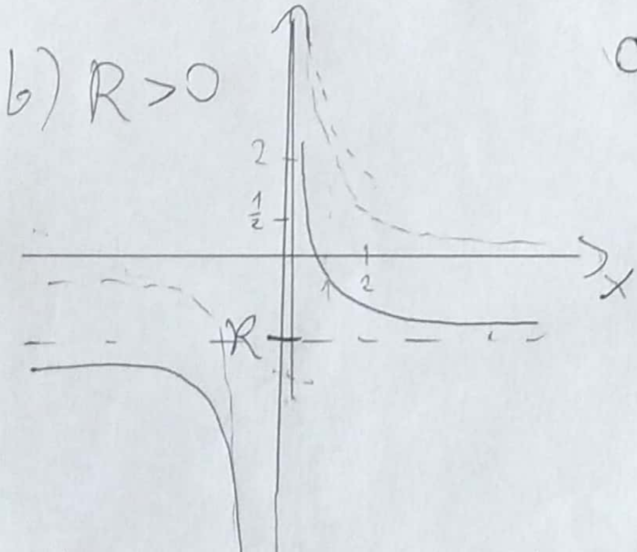
(5) $x_{n+1} = x_n(2 - x_n R)$

a) $f(x) = \frac{1}{x} - R \qquad x = \frac{1}{R} > 0$

$f'(x) = \frac{-1}{x^2}$

$$x_{n+1} = x_n - \frac{\frac{1}{x_n} - R}{-\frac{1}{x_n^2}} = x_n + (x_n - Rx_n^2) = 2x_n - Rx_n^2 = x_n(2 - x_n R)$$

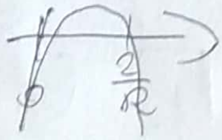
b) $R > 0$



c) $x_n \Rightarrow x_{n+1} < 0$

$x_n(2 - x_n R)$

$x_n = \frac{2}{R}$



$x \in (-\infty, 0) \cup (\frac{2}{R}, +\infty)$

d) $x_n < 0 \Rightarrow x_{n+1} < x_n$

$2x_n - x_n^2 R < x_n$

$x_n - x_n^2 R < 0 \Rightarrow x_n(1 - x_n R) < 0$

e) $\underbrace{0 < \underbrace{x_n(2 - x_n R)} < \frac{1}{R}}$

c)
$x_n \in (0, \frac{2}{R})$

$2x_n - x_n^2 R - \frac{1}{R} < 0 \quad / \cdot R$

$-x_n^2 R^2 + 2x_n R - 1 < 0$

$-(x_n R - 1)^2 < 0$

$x_n \in \mathbb{R} \setminus \{\frac{1}{R}\}$

$x \in (0, \frac{1}{R}) \cup (\frac{1}{R}, \frac{2}{R})$

ANL-4

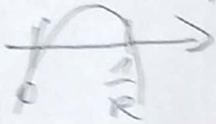
f) $x_n \in (0, \frac{1}{R}) \Rightarrow x_{n+1} \in (x_n, \frac{1}{R})$

$x_n < 2x_n - x_n^2 R \wedge \underbrace{2x_n - x_n^2 R < \frac{1}{R}}_{e)}$

$0 < x_n - x_n^2 R$

$0 < x_n(1 - x_n R)$

$x_n \in (0, \frac{1}{R})$



g) $\lim_{n \to \infty} x_n = \frac{1}{R}$

z e) ograniczone przez $\frac{1}{R}$

skoro $x_{n+1} \in (x_n, R^{-1})$ to $x_{n+1} > x_n$ dla każdego wyrazu

ograniczone, rosnące

$$1^\circ \left.\begin{array}{l} f(a) = \alpha \\ |f'(a)| < 1 \end{array}\right\} \text{zbieżność}$$

$f(\frac{1}{R}) = 2 \cdot \frac{1}{R} - \frac{1}{R^2} \cdot R = \frac{1}{R}$ ✓

$|f'(x)| = |2 - 2xR| < 1$

$|1 - xR| < \frac{1}{2} \Rightarrow R|\frac{1}{R} - x| < \frac{1}{2}$

$|\frac{1}{R} - x| < \frac{1}{2R}$ $\quad x \in (-\frac{1}{2R} + \frac{1}{R}, \frac{3}{2R})$