(2) $\underbrace{\square\,\square\,\square}_{X}\;000\;\underbrace{\square}_{Y}$

0 0 1

0 1 0

0 1 1

1 0 0

1 0 1

1 1 0

1 1 1

2^{-1}	2^{0}	2^{1}	
$X=0.01$	$X=0.1$	$X=1.$	
$Y=-$	$Y=-$	$Y-0$	
8	8	8	$=24$
		i 24 ujemne	

48 liczb

$$\min(X) = (0.01000)_2 = \frac{1}{4}$$

$$\max(X) = (1.1110)_2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 1\frac{7}{8} = 1.875$$

$$[A, B] = \left[-1\frac{7}{8}, 1\frac{7}{8}\right]$$

3) zd.: $x = s m 2^c$, $s = \operatorname{sgn} x$, $c \in \mathbb{Z}$, $m \in [\frac{1}{2}, 1)$

$rd(x) = s m_t^r 2^c$

$m_t^r \in [\frac{1}{2}, 1)$

$|m - m_t^r| \leq \frac{1}{2} \cdot 2^{-t}$ (*)

T: $\left|\frac{rd(x) - x}{|x|}\right| \leq 2^{-t}$

$$L = \frac{|s m_t^r 2^c - s m 2^c|}{|s m 2^c|} = \frac{\cancel{2^c s}}{\cancel{2^c s}} \frac{|m_t^r - m|}{|m|} \overset{m \in [\frac{1}{2}, 1)}{=} \frac{|m_t^r - m|}{m}$$

$$\overset{(*)}{\leq} \frac{1}{m} \cdot \frac{1}{2} \cdot 2^{-t} \leq 2 \cdot \frac{1}{2} \cdot 2^{-t} = 2^{-t} \quad \blacksquare$$

$\frac{1}{2} \leq m < 1 \Rightarrow 1 < \frac{1}{m} \leq 2$

⑥ $\sqrt{x^2+y^2}$, problem z x^2

np: $X_{fl} = [2^{-10}, 2^{10}]$

$\sqrt{(2^8)^2+(2^8)^2} = 2^8\sqrt{2} < 2^{10}$, ale $(2^8)^2 = 2^{16} \not< 2^{10}$

Załóżmy że $|x| \geq |y|$ (jak nie to $\begin{matrix} x := y \\ y := x \end{matrix}$)

$$\sqrt{x^2\left(1+\frac{y^2}{x^2}\right)} = |x|\sqrt{\left(1+\underset{\overset{\wedge\!\!\shortparallel}{1}}{\frac{y^2}{x^2}}\right)} \leq \sqrt{2}|x|$$

założyliśmy $|x| \geq |y|$ więc $\max(|x|, |y|) = |x|$, więc spełnia założenia zadania

y - analogicznie

$$||v|| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

Sortujemy malejąco V_i w ciąg w tj. $w_1 \geq w_2 \geq w_3 \geq \ldots \geq w_i \geq \ldots \geq w_n$

$\max(w_i) = w_1$

$$||v|| = \sqrt{w_1^2\left(1+\frac{w_2^2}{w_1^2}+\ldots+\frac{w_i^2}{w_1^2}+\ldots+\frac{w_n^2}{w_1^2}\right)} =$$

$$= |w_1|\sqrt{1+\sum_{i=2}^{n}\underbrace{\frac{w_i^2}{w_1^2}}_{\left(\frac{w_i}{w_1}\right)^2 \leq 1 \text{ bo } w_1 \geq w_i}} \leq w_1\sqrt{n}$$

$L \approx R$

7 a) $x^3 - \sqrt{x^6+2020}$

$$\frac{(x^3 - \sqrt{x^6+2020})(x^3 + \sqrt{x^6+2020})}{x^3 + \sqrt{x^6+2020}} = \frac{\cancel{x^6} - 2020}{x^3 + \sqrt{x^6+2020}} \to 0$$

b) $x \approx 0 \to \cos x \approx 1$, $\cos x - 1 + \frac{x^2}{2} \approx 0$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$$

$$L = x^{-4}\left(\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots\right) - 1 + \frac{x^2}{2}\right) =$$

$$x^{-4}\left(\frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \right) =$$

$$\frac{1}{4!} - \frac{x^2}{6!} + \frac{x^4}{8!} + \dots \to \frac{1}{24}$$

c) $\log_5 x - 6$

$6 - \log_5 (5)^6$

$\log_5 x - \log_5 (5^6) = \log_5 \left(\frac{x}{5^6}\right)$

8 $$4040\,\frac{\sqrt{x^{11}+1}-1}{x^{11}} = 4040\,\frac{1}{\sqrt{x^{11}+1}+1} \Rightarrow \frac{1}{2}$$

9) $k \approx 28$ nie działa

$$\left(\frac{x_k}{2^k}\right)^2 \approx 0 \Rightarrow 1 - \sqrt{1 - \frac{x_k}{2^k}} \approx 0$$

sprzężenie

$$x_{k+1} = 2^k \sqrt{2 \cdot \frac{\left(\frac{x_k}{2^k}\right)^2}{1 + \sqrt{1 - \left(\frac{x_k}{2^k}\right)^2}}}$$