

$$S = a \oplus b \oplus c$$

$$C_o = (a \oplus b)c + ab$$

Mapa Karnaugh:

ab \ c	00	01	11	10
0	0	1	0	1
1	1	0	1	0

$$S = a\bar{b}\bar{c} + \bar{a}b\bar{c} + ab\bar{c} + \bar{a}b\bar{c}$$

$$= \bar{c}(a\bar{b} + \bar{a}b) + c(ab + \bar{a}\bar{b})$$

XOR

$$a \oplus b = a\bar{b} + \bar{a}b$$

XNOR

$$\neg(a \oplus b) = ab + \bar{a}\bar{b}$$

$$S = \bar{c}(a \oplus b) + c[\neg(a \oplus b)] = a \oplus b \oplus c$$

ab \ c	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$$C_o = ac + bc + ab$$

$$= c(a+b) + ab$$

różni się od $c(a \oplus b) + ab$

tylko $a \oplus b$ i $a+b$ ale te wyrażenia są różne tylko dla $a=b=1$, a wtedy $ab=1$ więc mimo wszystko $C_o=1$.

Zatem układy są równoważne bo wyrażenia algebry Boole'a je opisujące są logicznie równoważne.

a_k	b_k	c_k	s_k	$a_k \oplus b_k$	$(a_k \oplus b_k) \oplus s_k$
0	0	0	0	0	0
0	1	0	1	1	0
1	0	0	1	1	0
1	1	0	0	0	0
0	0	1	1	0	1
0	1	1	0	1	1
1	0	1	0	1	1
1	1	1	1	0	1

2

Formuły c_k i $(a_k \oplus b_k \oplus c_k)$ przyjmują te same wartości dla wszystkich wartościów a_k, b_k, c_k są więc równoważne

③ $\text{for } n=4 \text{ 8-bit}$

$$C_{k+1} = g_k + p_k C_k$$

$$C_n = \sum_{i=0}^{n-1} g_i \prod_{j=i+1}^{n-1} p_j + C_0 \prod_{j=0}^{n-1} p_j$$

$$C_1 = g_0 + p_0 C_0 \quad \textcircled{2} \text{ branches (AND, OR)}$$

$$C_2 = g_1 + p_1 C_1 = g_1 + p_1 (g_0 + p_0 C_0) = g_1 + p_1 g_0 + p_1 p_0 C_0 \quad \textcircled{2} \text{ AND, 1 OR} \quad \textcircled{3} \text{ branches}$$

$$C_3 = g_2 + p_2 C_2 = g_2 + p_2 p_1 g_0 + p_2 p_1 p_0 C_0 \quad \textcircled{3} \text{ AND, 1 OR} \quad \textcircled{4} \text{ branches}$$

$$C_4 = g_3 + p_3 g_2 + p_3 p_2 g_1 + p_3 p_2 p_1 g_0 + p_3 p_2 p_1 p_0 C_0 + (p_3 p_2 p_1 p_0) C_0$$

2 OR 5 AND $\textcircled{7}$ branches

$$C_5 = g_4 + p_4 g_3 + p_4 p_3 g_2 + p_4 p_3 p_2 g_1 + p_4 p_3 p_2 p_1 g_0 + p_4 p_3 p_2 p_1 p_0 C_0 + p_4 p_3 p_2 p_1 p_0 C_0$$

-||- + -||- + -||- + -||- + $p_4 p_3 p_2 p_1 (g_0 + p_0 C_0)$

6 AND 3 OR $\textcircled{9}$ branches

$$C_6 = g_5 + p_5 g_4 + p_5 p_4 g_3 + p_5 p_4 p_3 g_2 + p_5 p_4 p_3 p_2 g_1 + p_5 p_4 p_3 p_2 p_1 g_0 + p_5 p_4 p_3 p_2 p_1 p_0 C_0 + p_5 p_4 p_3 p_2 p_1 p_0 C_0$$

7 AND 3 OR $\textcircled{10}$ branches

$$C_7 = g_6 + p_6 g_5 + p_6 p_5 g_4 + p_6 p_5 p_4 g_3 + p_6 p_5 p_4 p_3 g_2 + p_6 p_5 p_4 p_3 p_2 g_1 + p_6 p_5 p_4 p_3 p_2 p_1 g_0 + p_6 p_5 p_4 p_3 p_2 p_1 p_0 C_0 + p_6 p_5 p_4 p_3 p_2 p_1 p_0 C_0$$

-||- + $p_6 p_5 p_4 p_3 (g_2 + p_2 g_1 + p_2 p_1 g_0 + p_2 p_1 p_0 C_0)$

8 AND 3 OR $\textcircled{12}$ branches

$$C_8 = [\overline{g_7} + \overline{p_7}g_6 + \overline{p_7}p_6g_5 + \overline{p_7}p_6p_5g_4 + \overline{p_7}p_6p_5p_4g_3 + \overline{p_7}p_6p_5p_4p_3g_2 + \overline{p_7}p_6p_5p_4p_3p_2g_1 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_0 + \overline{p_7}p_6p_5p_4p_3p_2p_1p_0C_0]$$

A: 3 AND 1 OR

$$B: \overline{p_7}p_6p_5p_4p_3p_2p_1p_0C_0 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_0 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_1 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_2 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_3 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_4 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_5 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_6 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_7$$

~~$\overline{p_7}p_6p_5p_4p_3p_2p_1g_0 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_1 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_2 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_3 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_4 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_5 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_6 + \overline{p_7}p_6p_5p_4p_3p_2p_1g_7$~~

10 AND 2 OR

10 AND 2 OR

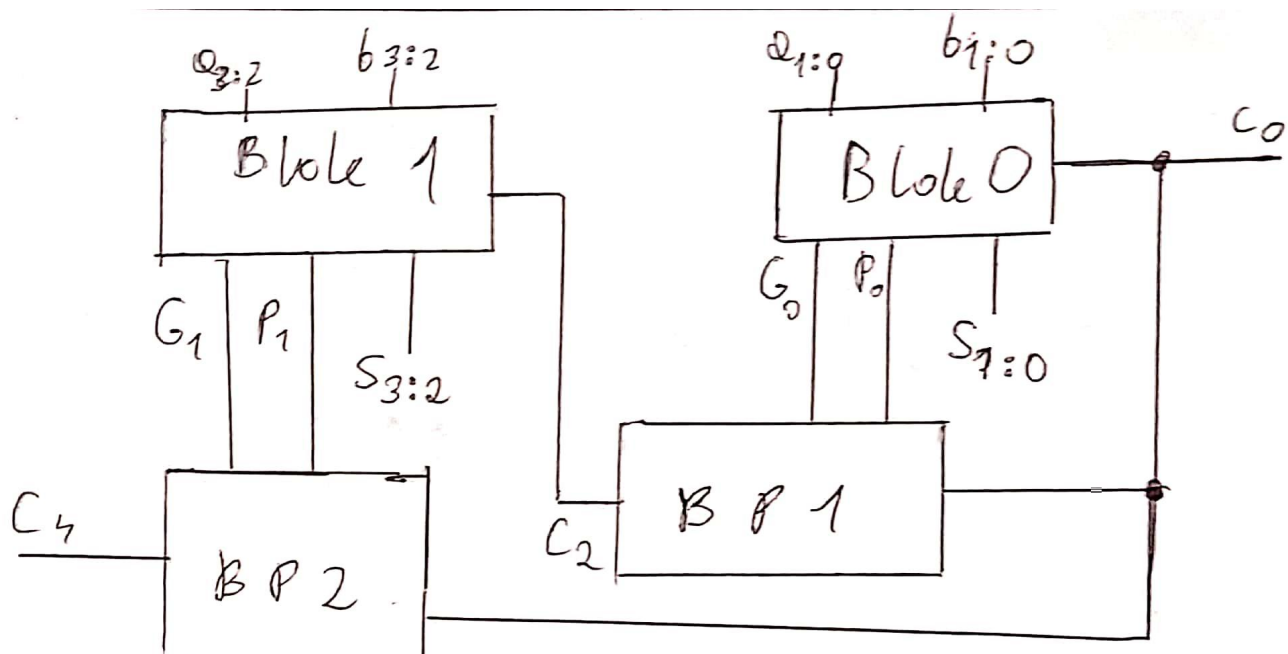
10 AND 2 OR (14) branches

So to get each XOR on each bit (8) branches

$\left. \begin{matrix} g_0 \\ p_0 \end{matrix} \right\}$ 2 branches on bit (16) branches

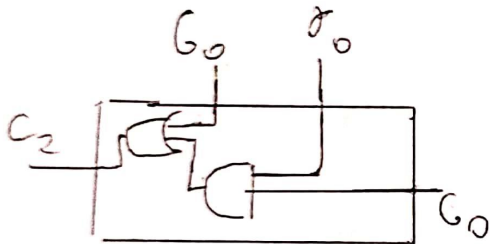
$$\text{SUMA: } 2 + 3 + 4 + 7 + 8 + 10 + 12 + 14 + 8 + 16 = \underline{85 \text{ branches}}$$

4



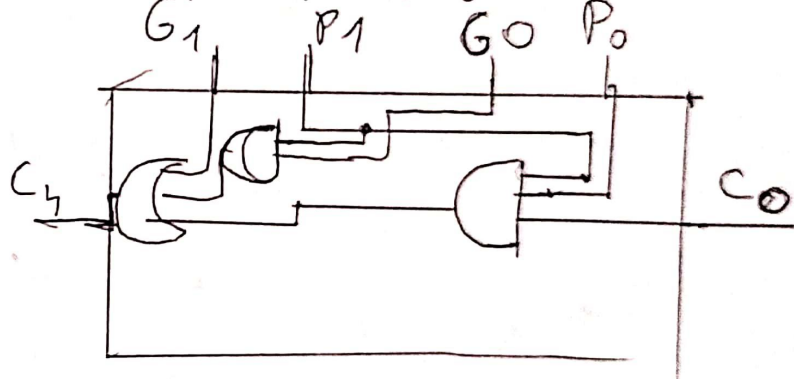
Blok ~~przeniesienia~~ 1
przenośnikowa jednoczesna

$$C_2 = G_0 + P_0 C_0$$



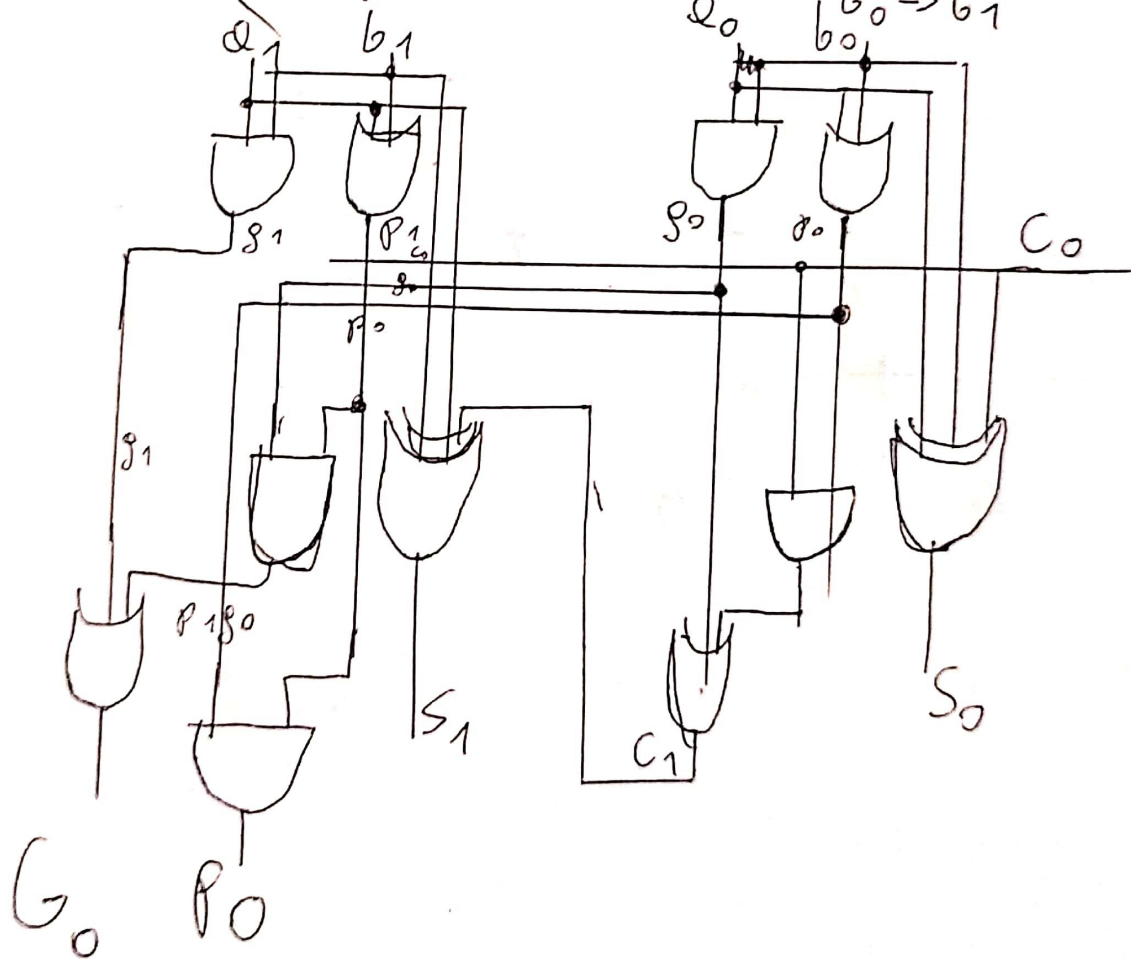
Blok ~~przeniesienia~~ 2
przenośnikowa jednoczesna

$$C_4 = G_1 + P_1 G_0 + P_1 P_0 C_0$$



4) kont $p_0 = a_0 + b_0$ $p_1 = a_1 + b_1$ $p_0 = p_1 p_0$
 $g_0 = a_0 b_0$ $g_1 = a_1 b_1$ $G_0 = g_1 + p_1 g_0$

Block 0 (1 podobnie tylko $p_0 \rightarrow p_2, p_1 \rightarrow p_3$ i p_0)
 $a_0 \rightarrow b_0 \rightarrow b_1$



5) Nie licząc z sumatorów pełnych opóźnienie wynosi 2. Na najdłuższej ścieżce jest ich 6 oraz 2 bramki AND, co razem daje $6 \cdot 2 + 1 = 13$ bramek opóźnienie na 31 bramkowej ścieżce

6) W układach w których ważny jest niski pobór mocy lub niskie wydzielenie ciepła lub wtedy gdy niskie skompaktowanie układu jest ważniejsze od precyzyjności obliczeń.

⑦ $0 \rightarrow 8, 1 \rightarrow 8 \text{ if } \dots$

$i[3]$	$i[2]$	$i[1]$	$i[0]$	$d3$	$d2$	$d1$	$d0$	
0	0	0	0	1	0	0	1	8
0	0	0	1	1	0	0	0	8
0	0	1	0	0	1	1	1	7
0	0	1	1	0	1	1	0	6
0	0	0	0	0	1	0	1	5
0	1	0	1	0	1	0	0	4
0	1	1	0	0	0	1	1	3
0	1	1	1	0	0	1	0	2
0	0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0	0
...	x	x	x	x	

$d3:$

$xy \backslash zw$	00	01	11	10
00	1	0	x	0
01	1	0	x	0
11	0	0	x	x
10	0	0	x	x

$d2:$

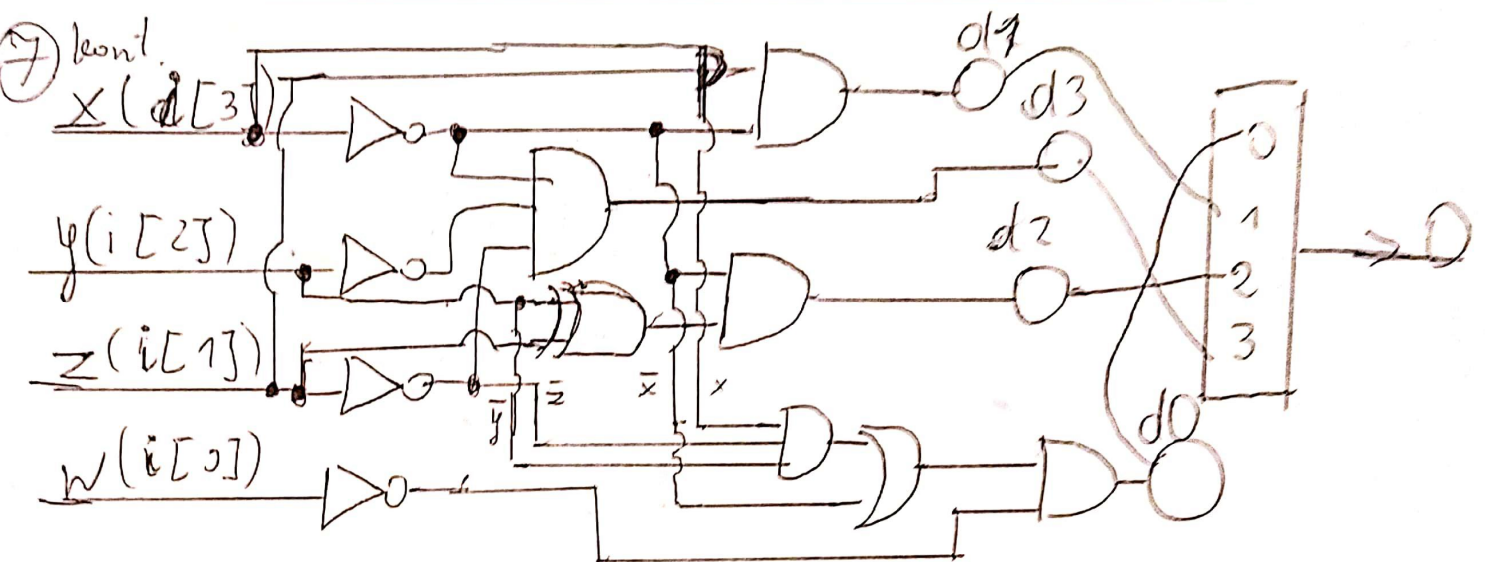
$xy \backslash zw$	00	01	11	10
00	0	1	x	0
01	0	1	x	0
11	1	0	x	x
10	1	0	x	x

$d1:$

$xy \backslash zw$	00	01	11	10
00	0	0	x	0
01	0	0	x	0
11	1	1	x	x
10	1	1	x	x

$d0:$

$xy \backslash zw$	00	01	11	10
00	1	1	x	1
01	0	0	x	0
11	0	0	x	x
10	1	1	x	x



$$d3 = \bar{x} \bar{y} \bar{z}$$

$$d2 = \bar{x} \bar{y} z + \bar{x} y \bar{z} = \bar{x} (\bar{y} z + y \bar{z}) = \bar{x} (\text{XOR}(y, z))$$

$$d1 = \bar{x} z$$

$$d0 = \bar{x} \bar{w} + x \bar{y} \bar{z} \bar{w} = \bar{w} (\bar{x} + x \bar{y} \bar{z})$$

8) $f_1 = 10$ $f_2 = 10$ $f_3 = 10$ $f_4 = 10$ $f_5 = 10$ $f_6 = 10$ $f_7 = 10$ $f_8 = 10$ $f_9 = 10$ $f_{10} = 10$

Wzrost: ABCD to linie w BCD

Wzrost: 0001 to 1

Wzrost: 0011 to 3 itp.

al. Ktore segmenty są zapalone dla odpowiednich liczb?

A	B	C	D	a	b	c	d	e	f	g	h
0	0	0	0	1	1	1	1	1	1	0	0
0	0	0	1	0	1	1	0	0	0	0	1
0	0	1	0	1	1	0	1	1	0	1	2
0	0	1	1	1	1	1	0	0	0	1	3
0	1	0	0	0	1	1	0	0	1	1	4
0	1	0	1	1	0	1	1	0	1	1	5
0	1	1	0	1	0	1	1	1	1	1	6
0	1	1	1	1	1	1	0	0	0	0	7
1	0	0	0	1	1	1	1	1	1	1	8
1	0	0	1	1	1	1	1	0	1	1	9
				x	x	x	x	x	x	x	

AB	00	01	11	10
CD	00	1	0	1
01	0	1	X	1
11	1	1	X	X
10	1	1	X	X

$$Q = \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}\bar{D}$$

$$Q = (A+B+C+D)(\bar{A}\bar{B} + C+D)$$

AB	00	01	11	10
CD	00	1	X	1
01	1	0	X	1
11	1	1	X	X
10	1	0	X	X

$$B = \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D}$$

$$B = (\bar{A}\bar{B} + C+D)(\bar{A}\bar{B} + \bar{C}+D)$$

8 kont) ~~W~~ (C) $\overline{A}\overline{B}C\overline{D} = A + B + \overline{C} + D = C$

d

AB \ CD	00	01	11	10
00	1	1	X	1
01	1	1	X	1
11	1	1	X	X
10	1	1	X	X

$$\overline{d} = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D}$$

$$\overline{d} = (\overline{A} + \overline{B} + \overline{D})(\overline{A} + B + \overline{D})(\overline{A} + \overline{B} + \overline{C})$$

g

AB \ CD	00	01	11	10
00	1	1	X	1
01	1	1	X	1
11	1	1	X	X
10	1	1	X	X

e

AB \ CD	00	01	11	10
00	1	0	X	1
01	0	0	X	0
11	0	0	X	X
10	1	1	X	X

$$\overline{e} = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{B}C\overline{D}$$

f

AB \ CD	00	01	11	10
00	1	1	X	1
01	1	1	X	1
11	1	1	X	X
10	1	1	X	X

$$\overline{f} = \overline{A}\overline{B}\overline{C}\overline{D} + \overline{A}\overline{B}C\overline{D} + \overline{A}B\overline{C}\overline{D}$$

$$\overline{f} = (A + B + \overline{D})(A + B + \overline{C})(\overline{A} + \overline{B} + \overline{C})$$

$$\overline{g} = \overline{A}\overline{B}C\overline{D} + \overline{A}\overline{B}\overline{C}$$

$$\overline{g} = (\overline{A} + \overline{B} + \overline{D})(\overline{A} + \overline{B} + \overline{C})$$

$$\overline{g} = \overline{A}\overline{B}\overline{C} + B\overline{C}\overline{D}$$

$$g = (A + B + C)(\overline{B} + \overline{C} + \overline{D})$$

24 bity
 bez znaku

$(1, \dots, 1)_{10}$
 maximum: $\sum_{i=-12}^{10} 2^i = 4095 + \frac{4095}{4096}$

$(0, \dots, 0)_{10}$
 minimum: 0

24 bity
 ZM

$(0, 1, \dots, 1)_{10}$
 maximum: $\sum_{i=-12}^{10} 2^i = 2047 + \frac{4095}{4096}$

$(1, 1, \dots, 1)_{10}$
 minimum: $\left(\sum_{i=-12}^{10} 2^i \right) (-1) = -\left(2047 + \frac{4095}{4096} \right)$

24 bity
 U2

$(0, 1, \dots, 1)_{10}$
 maximum: $\sum_{i=-12}^{10} 2^i = 2047 + \frac{4095}{4096}$

$(1, 0, \dots, 0)_{10}$
 minimum: $-2^{11} = -2048$