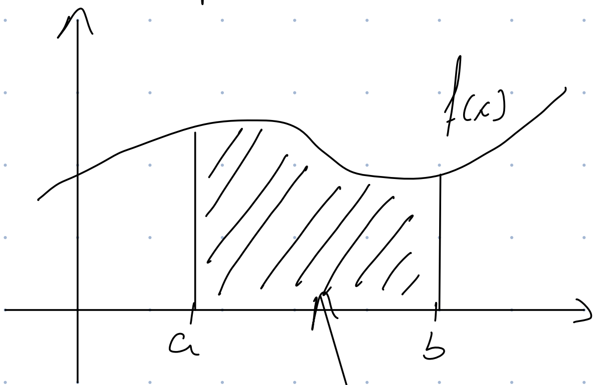
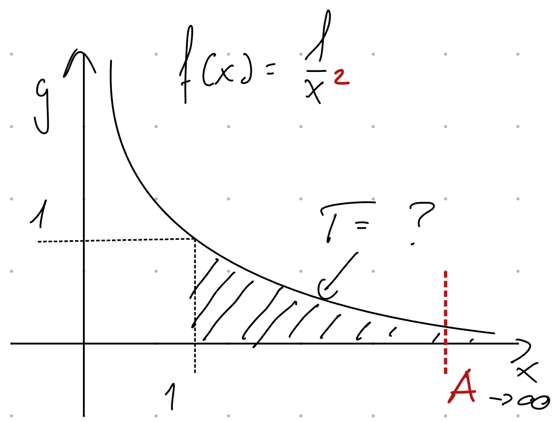
Impropius integrál:



$$\int_a^b f(x)\,dx = \text{előjeles terület}$$

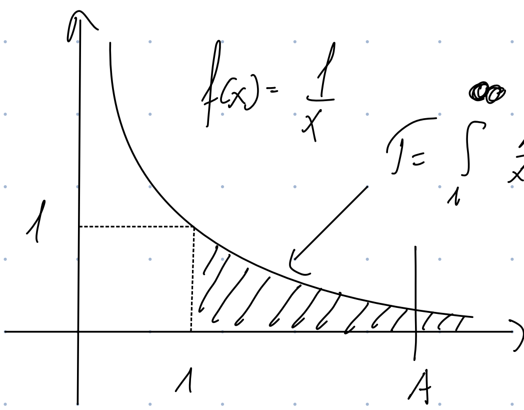


Nincs értelme területről beszélni! $\left(\frac{1}{x}\right)$

$$T = \int_1^\infty \underbrace{\frac{1}{x^2}}_{=x^{-2}}\,dx = \left[\frac{x^{-1}}{-1}\right]_1^\infty = \left[-\frac{1}{x}\right]_1^\infty$$

$$\to \lim_{A\to\infty}\left[-\frac{1}{x}\right]_1^A = \lim_{A\to\infty}\left(\underbrace{-\frac{1}{A}}_{0} \underbrace{-\left(-\frac{1}{1}\right)}_{+1}\right) = \underline{\underline{1}}$$

$\int_a^\infty f(x)\,dx$ csak akkor lehet véges, ha $\lim_{x\to\infty} f(x) = 0$!



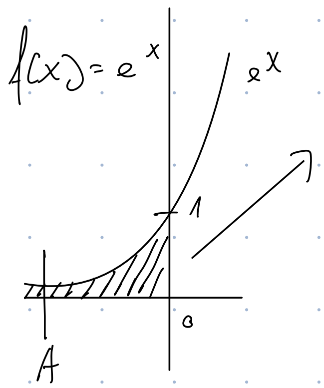
$$T = \int_1^\infty \frac{1}{x}\,dx = \lim_{A\to\infty}\int_1^A \frac{1}{x}\,dx = \lim_{A\to\infty}\left[\ln|x|\right]_1^A$$

$$= \lim_{A\to\infty}\left(\underbrace{\ln A}_{\infty} - \underbrace{\ln 1}_{=0}\right) = \infty$$

Ez most $g(A)$ fgv!

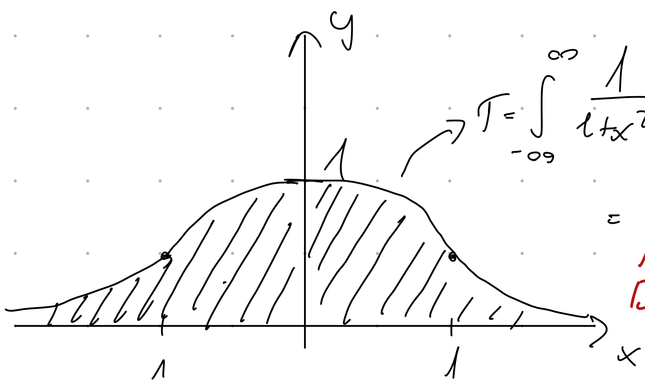
Az integrál divergens!

$f(x)=e^x$



$$T=\int_{-\infty}^{0} e^x\,dx=\lim_{A=-\infty}\int_{A}^{0} e^x\,dx=\lim_{A\to-\infty}\left[e^x\right]_A^0=\lim_{A\to-\infty}\left(\underbrace{e^0}_{1}-\underbrace{e^A}_{\to 0}\right)=1$$

$f(x)=\dfrac{1}{1+x^2}$

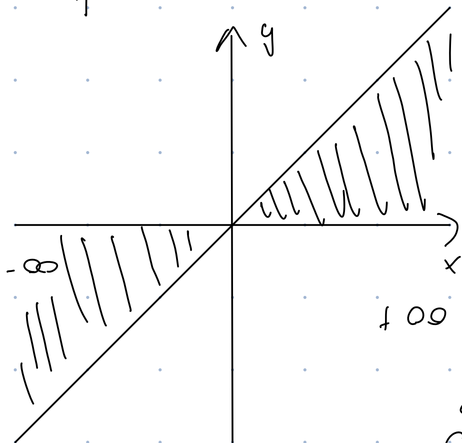


$$T=\int_{-\infty}^{\infty}\frac{1}{1+x^2}\,dx=\int_{-\infty}^{0}\frac{1}{1+x^2}\,dx+\int_{0}^{\infty}\frac{1}{1+x^2}\,dx$$

$$=\lim_{\substack{A\to-\infty\\ B\to+\infty}}\left[\mathrm{arctg}\,x\right]_A^B=\lim_{\substack{A\to-\infty\\ B\to+\infty}}\left(\mathrm{arctg}\,B-\mathrm{arctg}\,A\right)$$

$$=\underbrace{\lim_{B\to\infty}\mathrm{arctg}\,B}_{\pi/2}-\underbrace{\lim_{A\to-\infty}\mathrm{arctg}\,A}_{-\left(-\frac{\pi}{2}\right)}=\pi$$

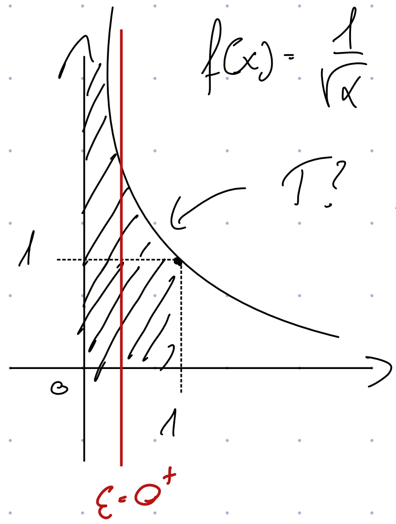
$f(x)=x$



$$\int_{-\infty}^{+\infty}x\,dx=\underbrace{\int_{-\infty}^{0}x\,dx}_{-\infty}+\underbrace{\int_{0}^{+\infty}x\,dx}_{+\infty}=\underbrace{\infty-\infty}_{?}$$

Ez HIBÁS!

$$\int_{-\infty}^{\infty}x\,dx=\lim_{A\to\infty}\int_{-A}^{A}x\,dx=\lim_{A\to\infty}\left[\frac{x^2}{2}\right]_{-A}^{A}=\lim_{A\to\infty}\left(\frac{A^2}{2}-\frac{(-A)^2}{2}\right)=0$$



$$f(x)=\frac{1}{\sqrt{x}} \qquad \lim_{x\to 0^+}\frac{1}{\sqrt{x}}=\infty$$

T? Egy 0-ban nem értelmezett!

$$T=\int_0^1 \frac{1}{\sqrt{x}}\,dx = \lim_{\varepsilon\to 0^+}\int_\varepsilon^1 \underbrace{\frac{1}{\sqrt{x}}}_{x^{-\frac{1}{2}}}\,dx = \lim_{\varepsilon\to 0^+}\left[\frac{x^{1/2}}{1/2}\right]_\varepsilon^1$$

$$= \lim_{\varepsilon\to 0^+}\Big(\underbrace{2\sqrt{1}}_{2}-\underbrace{2\sqrt{\varepsilon}}_{0}\Big) = \underline{\underline{2}}$$

$$\int_0^1 \frac{1}{x^2}\,dx = \lim_{\varepsilon\to 0^+}\int_\varepsilon^1 x^{-2}\,dx = \lim_{\varepsilon\to 0^+}\Big[\underbrace{\frac{x^{-1}}{-1}}_{-1/x}\Big]_\varepsilon^1 = \lim_{\varepsilon\to 0^+}\left(-\frac{1}{1}-\Big(-\frac{1}{\varepsilon}\Big)\right)=\infty$$

$$= -1+\underbrace{\frac{1}{\varepsilon}}_{\to +\infty}$$

Az integrál divergens

$0 \notin D_f$, és 0 közelében $f\to\infty$

$$\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}\,dx = \lim_{\substack{\alpha\to -1^+\\ \beta\to 1^-}}\int_\alpha^\beta \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{\substack{\alpha\to -1^+\\ \beta\to +1}}\big[\arcsin x\big]_\alpha^\beta =$$

Egyik sem eleme a D_f-nek

$$= \lim_{\beta\to +1^-}\underbrace{\arcsin\beta}_{\to\frac{\pi}{2}} - \lim_{\alpha\to -1^+}\underbrace{\arcsin\alpha}_{\to\frac{\pi}{2}} = \frac{\pi}{2}-\Big(-\frac{\pi}{2}\Big)=\underline{\underline{\pi}}$$

$$\int_{-1}^{1}\frac{1}{x}\,dx = \int_{-1}^{0}\frac{1}{x}\,dx + \int_0^1\frac{1}{x}\,dx = \lim_{\alpha\to 0^-}\int_{-1}^{\alpha}\frac{1}{x}\,dx + \lim_{\beta\to 0^+}\int_\beta^1\frac{1}{x}\,dx$$

de a $0 \notin D_f$ $\quad = \lim_{\alpha \to 0^-} \left[\ln|x|\right]_{-1}^{\alpha} + \lim_{\beta \to 0^+} \left[\ln|x|\right]_{\beta}^{1} =$

$$= \lim_{\alpha \to 0^-} \Big(\underbrace{\ln|\alpha|}_{\downarrow \atop -\infty} - \underbrace{\ln|-1|}_{=0}\Big) + \lim_{\beta \to 0^+} \left(\ln 1 - \ln \beta\right) = ? \; (\infty + \infty)$$

Divergente!