TÉCNICO LISBOA

LAB 1: CIRCUIT ANALYSIS METHODS

Circuit Theory and Electronics Fundamentals

Ana Bárbara Azevedo (96504), Artur Gonçalves (96513), Diogo Soares (96519)

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1 Introduction

In this laboratory assignment, a circuit containing two voltage sources (independent source V_a and current controlled source V_c), and two current sources (independent source I_d and voltage controlled source I_b), connected to multiple resistors (from R_1 to R_7), is going to be studied. The described circuit can be observed in detail in Figure 1.

Although the circuit is composed by somewhat simple, linear components, these are the core of electronic circuits. Because of that, it is essential for one to understand the way they behave to be able to build and apply their knowledge in other more complex circuits. Therefore, its study is still of maximum importance.

In Section 2, a theoretical analysis of the circuit - using the mesh and nodal methods - is presented. After that, in Section 3, the circuit is analysed via simulation, using the software Ngspice. The obtained results are then compared, explaining the reasons behind the differences and similarities found. Finally, one can find the conclusions of this study outlined in Section 4.

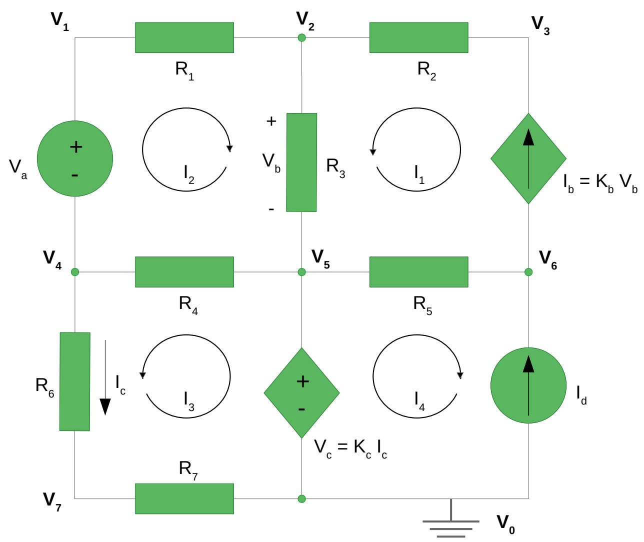

Figure 1: Circuit diagram with node (V_0 to V_7) and mesh current (I_1 to I_4) representation.

2 Theoretical Analysis

In this section, the previously shown circuit is theoretically analised. Even though there are many ways to proceed, two methods are emphasized: Mesh Analysis and Nodal Analysis.

First of all, it is important to refer that it is considered the Lumped Elements Aproximation. In other words, there is no notion of physical distance, so there is no wave propagation or radiation. The conductors that connect branches (circuit components) by means of nodes do not dissipate any energy and do not offer resistance to the current flux. It is this work's aim to find values for two very important physical quantities. Current (I) is the measurement of the charge that passes through a branch per unit of time. It is measured in Ampere $(1A = 1C/s)$. On the other hand, Voltage/Eletric Tension (V) is a difference of eletrical potential measured between

2 points of the circuit. It is measured in Volt ($1V = 1J/C$). It is common to attribute a voltage to the nodes by comparison to a reference one (GND), defined as having $V_{GND} = V_0 = 0V$.

Along the analysis, some laws are considered, such as Ohm's Law (eq. 1), the Kirchhoff Current Law (KCL, eq. 2) and the Kirchhoff Voltage Law (KVL, eq. 3). Ohm's law is applied to resistors as a way to linearly relate their voltage and current. KCL is applied to nodes and states that the sum of currents converging (resp. diverging) in a node is null, whereas KVL tells that the sum of voltages in a circuit loop is null:

$$V = RI \tag{1}$$

$$\sum_{i=1}^{N} I_i = 0 \tag{2}$$

$$\sum_{i=1}^{N} V_i = 0 \tag{3}$$

During the analysis, the currents were considered to flow from the positive to the negative terminal of each component.

2.1 Mesh Analysis

For this method, it is required a definition of "Mesh Currents". As the studied circuit has four meshes (loops with no other loops inside), disposed like a cartesian plane xOy with the origin as its centre, each one of them circulated by a current I_x, with x being the number of the quadrant respecting to the referential. I_1 and I_3 are circulating anti-clockwise and I_2 and I_4 are circulating clockwise (as shown in Figure 1). Knowing mesh currents, we can know any node voltages or branch currents, using Ohm's law.

It is worth noting that some replacements have to be made, so the dependent sources can be considered on the previously refered laws:

$$V_b = R_3(I_1 + I_2) \tag{4}$$

$$I_c = I_3 \tag{5}$$

Therefore, it is presented the following system of equations with $I_x, x \in \{1, 2, 3, 4\}$ as the unknown variables:

$$\begin{cases} K_b R_3(I_1 + I_2) - I_1 = 0 \\ -V_a + R_1 I_2 + R_3(I_1 + I_2) + R_4(I_2 + I_3) = 0 \\ R_6 I_3 + R_7 I_3 - K_c I_3 + R_4(I_2 + I_3) = 0 \\ I_4 = -I_d \end{cases}$$

By solving this system using Octave (free available software), we obtain the solution presented on Table 1.

I_1	I_2	I_3	I_4
-0.200589	0.191571	0.973158	-1.017967

Table 1: Current values in miliAmpere.

Note that the values for the currents that pass through each component can be deducted from the results shown above, for example:

- $I_{R_1} = I_2 = 0.191571$ mA
- $I_{R_4} = I_2 + I_3 = 1.164729$ mA
- $I_b = I_1 = -0.200589$ mA

2.2 Nodal Analysis

For this method, it is required to attribute a number to every node in the circuit (numbers visible in Figure 1).

Once again, some replacements have to be made:

$$V_b = V_2 - V_5 \tag{6}$$

$$I_c = (V_4 - V_7)G_6 \tag{7}$$

Therefore, it is presented a system of equations with $V_x, x \in \{1,2,3,4,5,6,7\}$ as the unkown variables (note that $G_x = \frac{1}{R_x}$):

$$\begin{cases} V_1 - V_4 = V_a \\ (V_2 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0 \\ (V_3 - V_2)G_2 - K_b(V_2 - V_5) = 0 \\ (V_1 - V_2)G_1 + (V_4 - V_5)G_4 + (V_4 - V_7)G_6 = 0 \\ V_5 - K_cG_6V_4 + K_cG_6V_7 = 0 \\ (V_6 - V_5)G_5 + K_b(V_2 - V_5) = I_d \\ (V_7 - V_4)G_6 + V_7G_7 = 0 \end{cases}$$

By solving this system using Octave, the following solution is obtained:

V_1	V_2	V_3	V_4	V_5	V_6	V_7
8.030092	7.833907	7.417135	2.989101	7.861734	11.632840	0.995129

Table 2: Voltage values in Volt.

Once again, note that the values for the voltages between the terminals of each component can be deducted from the results shown above, for example:

- $V_{R_3} = V_b = V_2 - V_5 = -0.027827$ V
- $V_c = V_5 - V_0 = 7.861734$ V

3 Simulation Analysis

3.1 Operating Point Analysis

Table 3 shows the simulated operating point results (obtained from Ngspice) for the circuit under analysis, considering that current directions I_1, I_2, I_3 and I_4 (and, accordingly, the positive and negative terminals of the components) were defined based on the same model as before.

Variable name	Value [mA or V]
@gb[i]	-2.00589e-01
@id[current]	1.017967e+00
@r1[i]	1.915709e-01
@r2[i]	-2.00589e-01
@r3[i]	-9.01783e-03
@r4[i]	1.164729e+00
@r5[i]	-1.21856e+00
@r6[i]	9.731579e-01
@r7[i]	9.731579e-01
v(1)	8.030092e+00
v(2)	7.833907e+00
v(3)	7.417135e+00
v(4)	2.989101e+00
v(5)	7.861734e+00
v(6)	1.163284e+01
v(7)	9.951293e-01
v(8)	2.989101e+00

Table 3: Operating point analysis - a variable preceded by @ is of type *current* and expressed in miliAmpere; other variables are of type *voltage* and expressed in Volt. gb refers to the controlled current source I_b and the rest is defined as before.

Compared to the theoretical analysis results, one notices a few differences.

Focusing on the obtained nodal voltage levels, an almost exact resemblance is shown, except for the fact that Octave's used result precision is of 6 decimal places, whereas Ngspice always presents 7 significant digits (hence the small differences). It's worth noting that node 8 (introduced exclusively on the simulation analysis) was created so a zero valued voltage source (working as an ammeter) could be placed in series with resistor 6, allowing the algorithm to measure the current between nodes 4 and 7.

The same issue takes place with the current values, altough we can only directly compare four of them: I_1 with $@r2[i]$, I_2 with $@r1[i]$, I_3 with both $@r6[i]$ and $@r7[i]$ as well as I_4 with the symmetric of I_d, as stated on section 2 (even if it was rather an input value and not quite an output one). For further comparisons, proceed as indicated in sections 2.1 and 2.2.

Other than that, we could say there's a perfect match on the results obtained from both analysis methods.

4 Conclusion

In this laboratory assignment, the goal of analysing the circuit has been achieved. Theoretically, both node and mesh analyses have been performed, and the results were obtained using the Octave maths tool. Besides that, a circuit simulation, using the Ngspice tool, was also executed. The simulation results matched the theoretical results almost precisely, as explained in Section 3. The excellent match relies on the fact that this is a quite straightforward circuit, containing only linear and simple components. So, eventhough the theoretical and simulation models could differ - when using more complex components- this is not the case in this work.

In conclusion, we confirmed that this was a very important circuit to study, since it allowed us to expand our understanding of the basics of electronic circuits. It was also pretty curious for us to apply some physics concepts we have learned in other disciplines when studying this circuit and being able to testify that they work in practise.