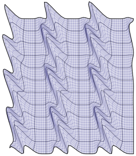


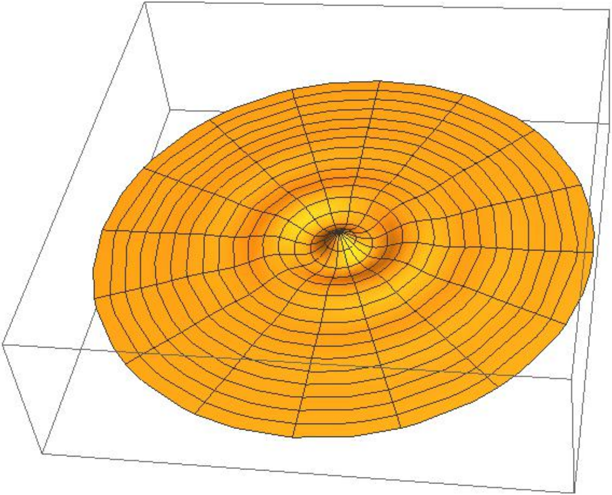
Counter-Intuitive Acousto-Elasticity

Author:
Artur L. Gower

Co-Authors:
Prof. Michel Destrade
Prof. Ray Ogden

National University of Ireland Galway

Surface waves



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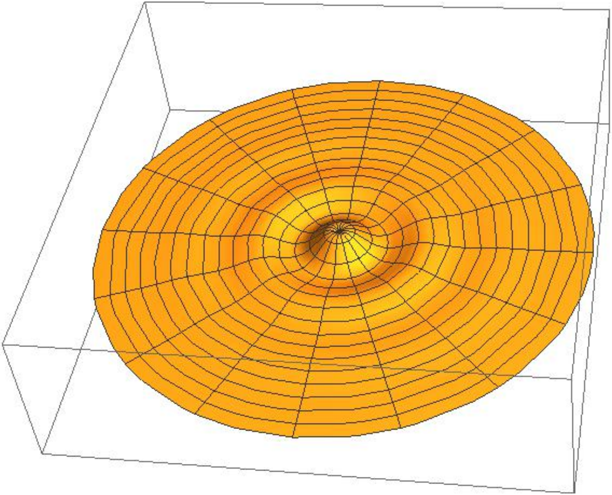

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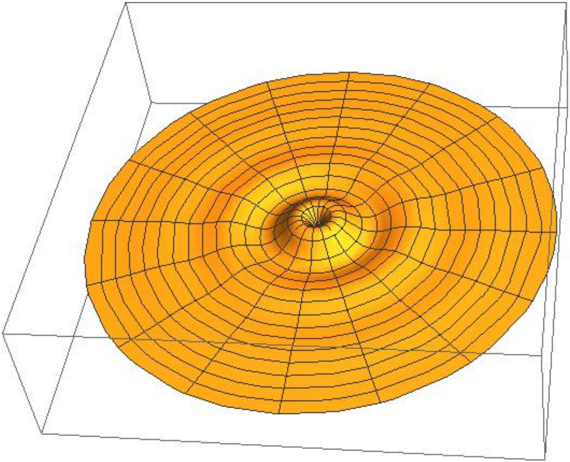

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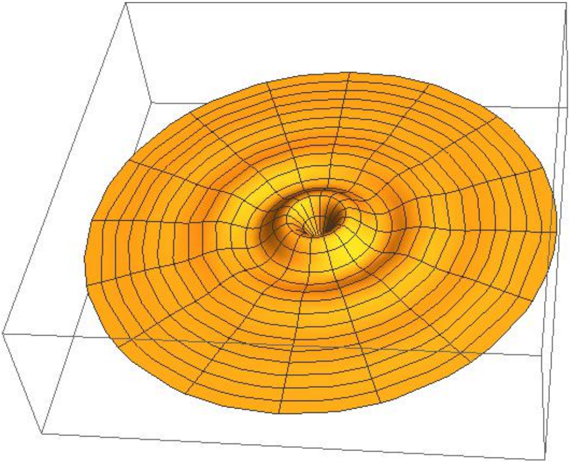

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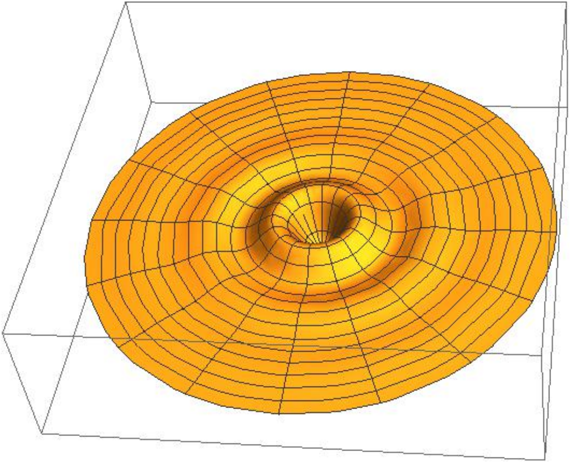

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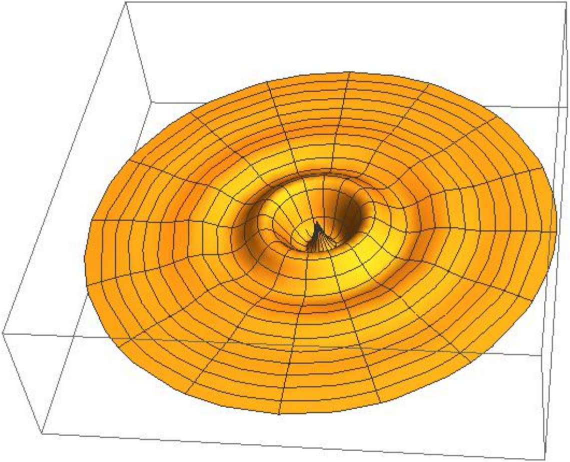

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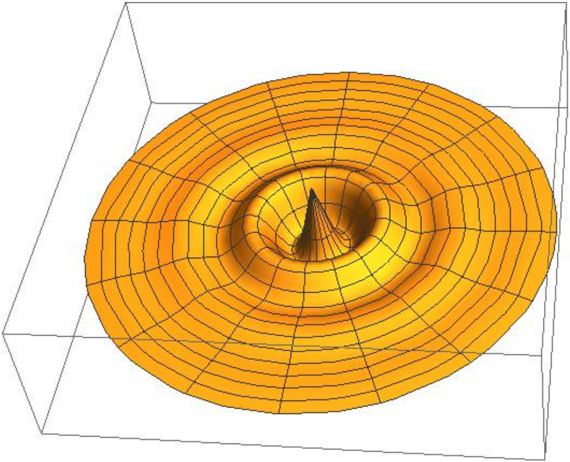

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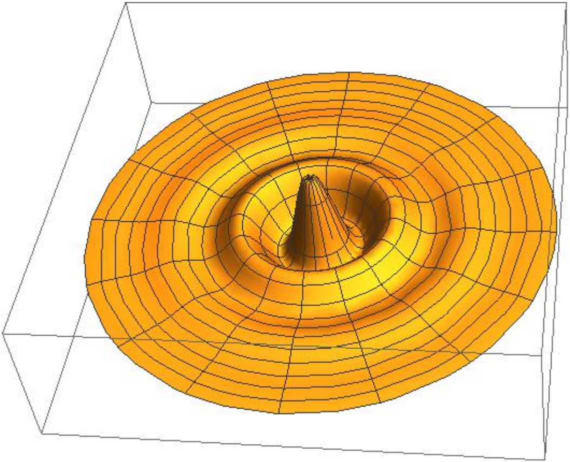

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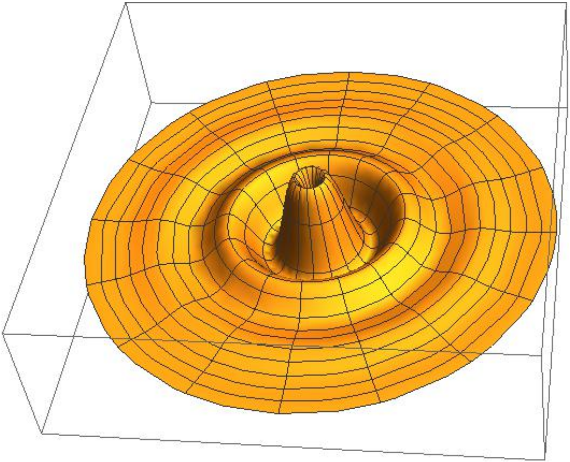

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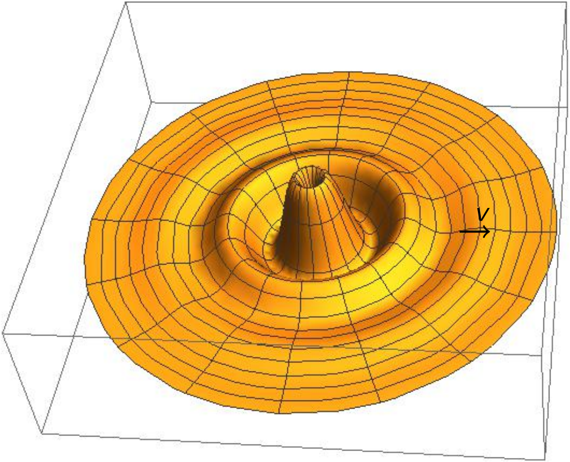
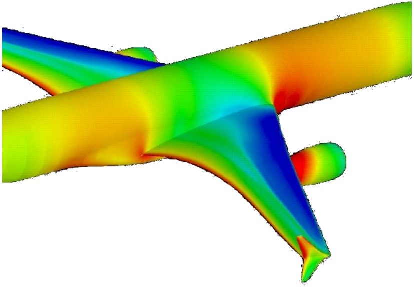


Optical Coherence Tomography (Dundee School of Medicine).

and

Lazer Vibrometers (Ultrasound Imaging Laboratory, Mayo Clinic).

Non destructive evaluation of the surface



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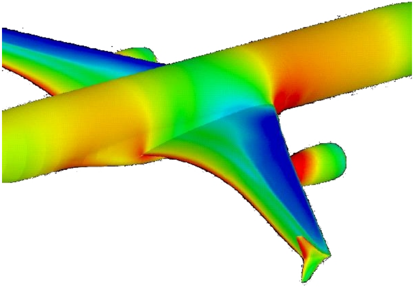

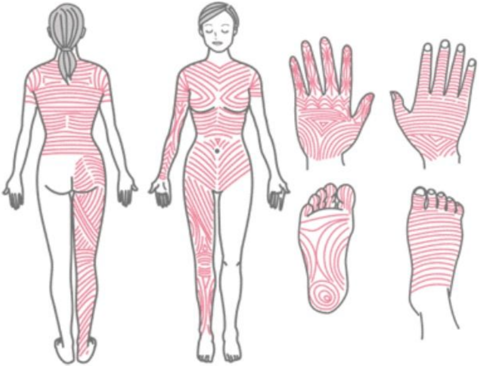

langer lines

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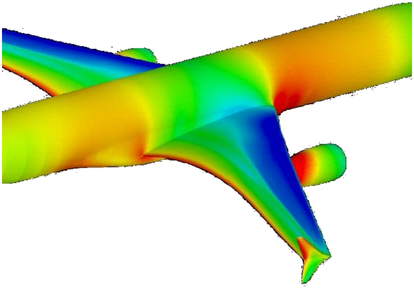
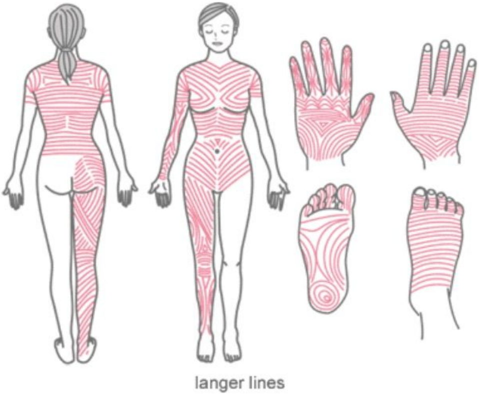
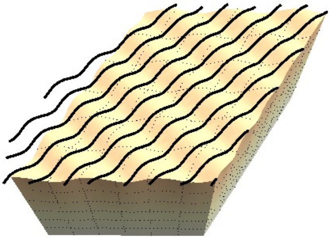




Measure stress and discover defects.

Testing ground for soft matter models



Fibre reinforced?

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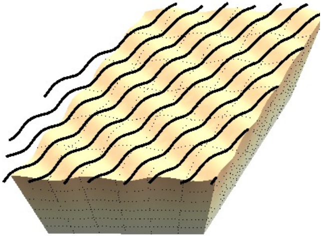

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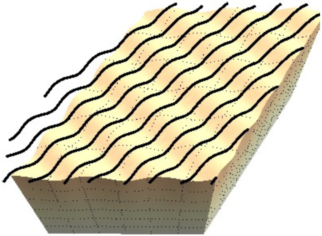

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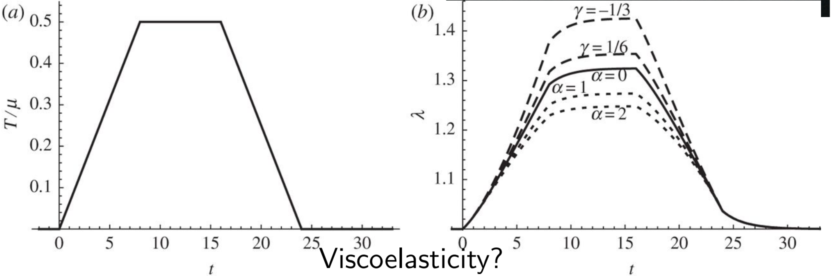
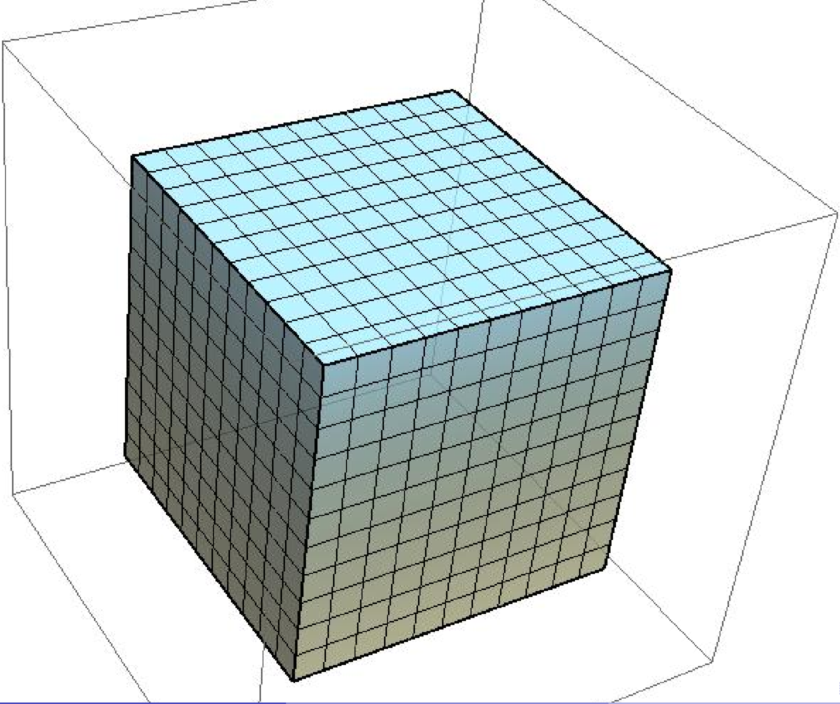
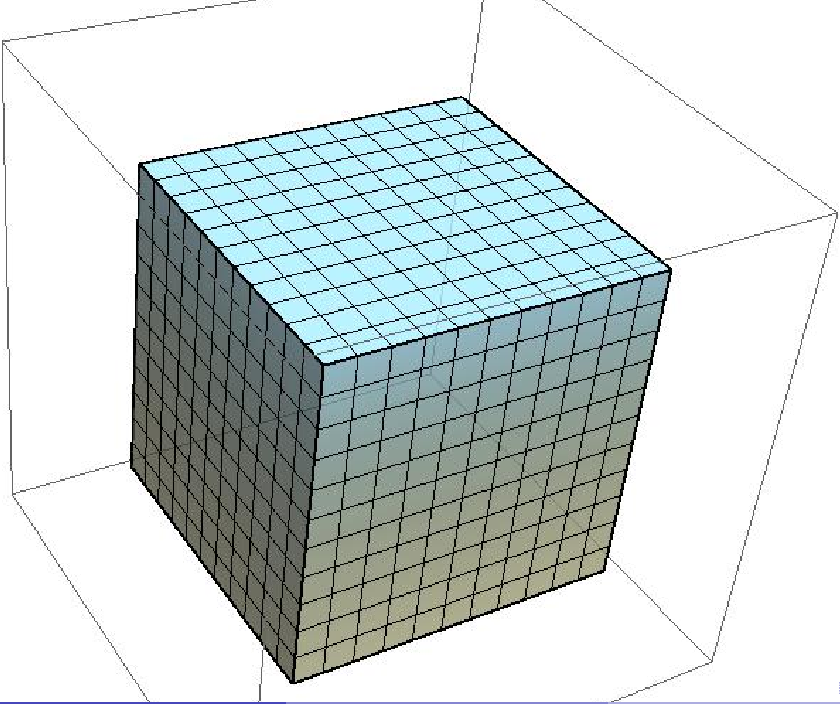
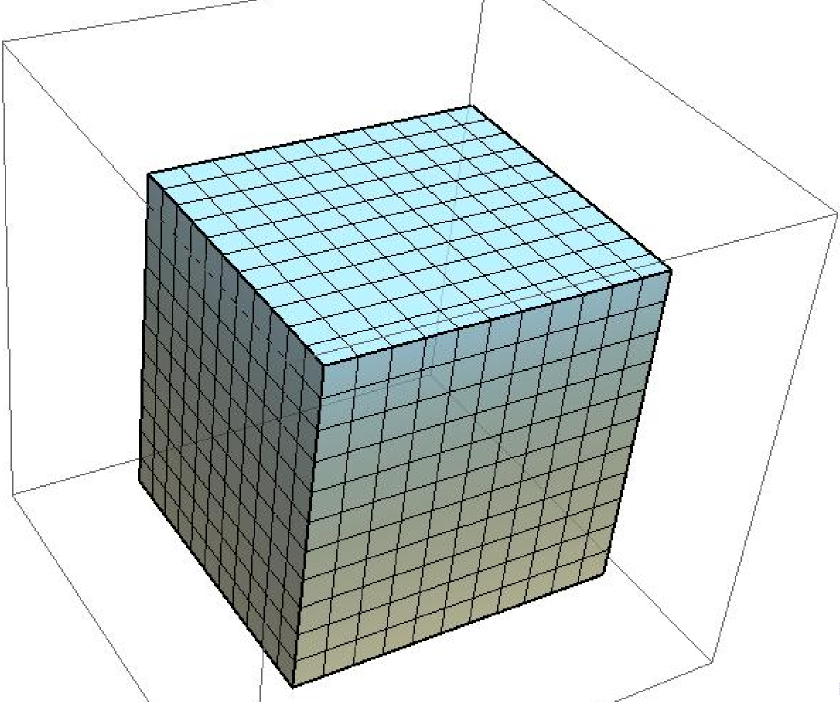
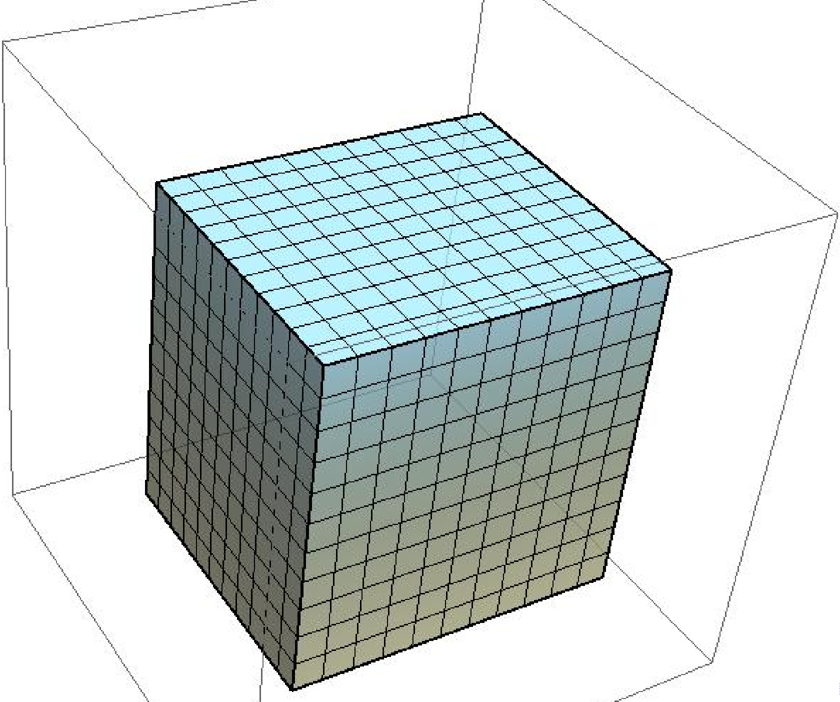


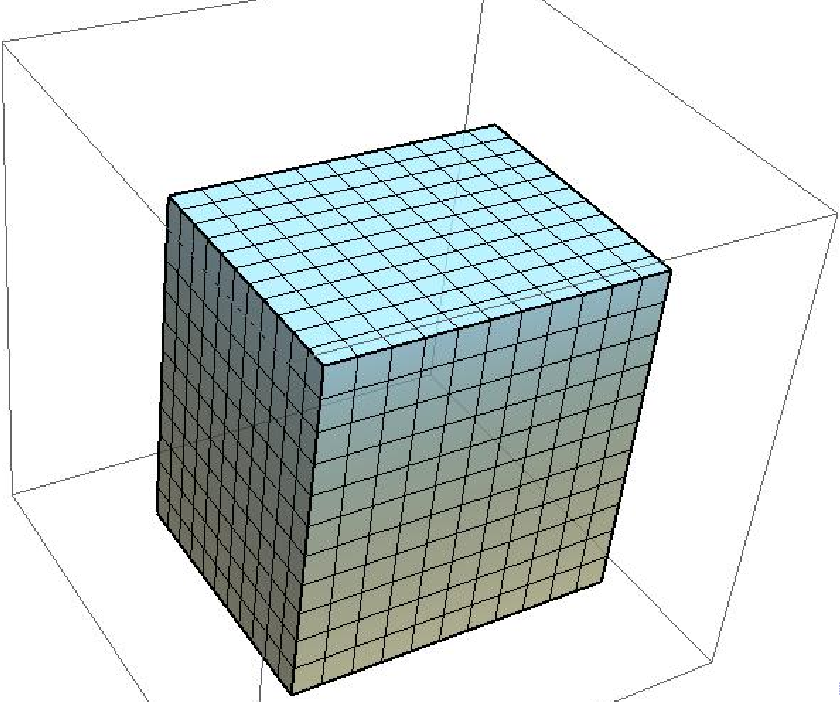
Viscoelasticity?

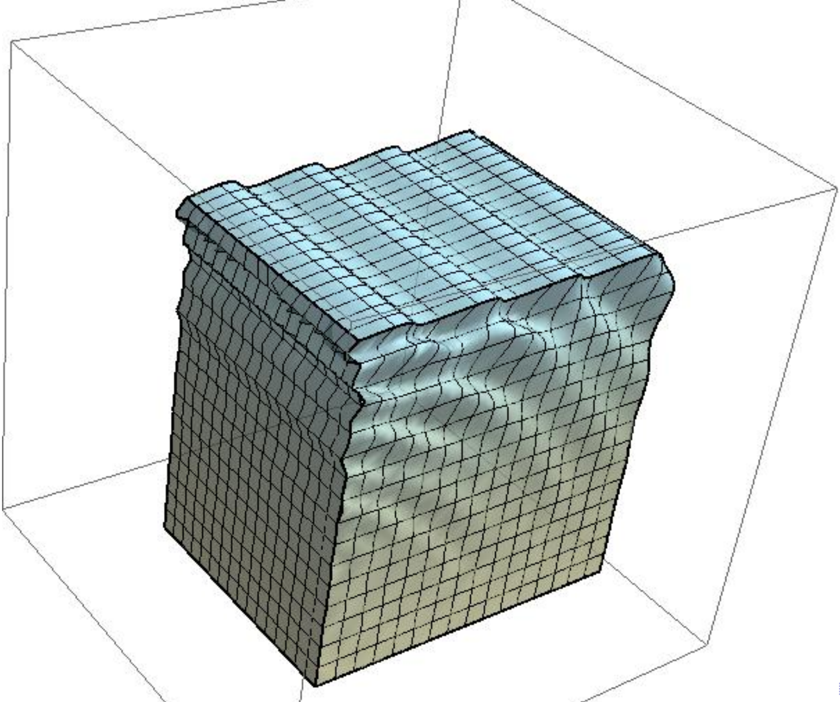


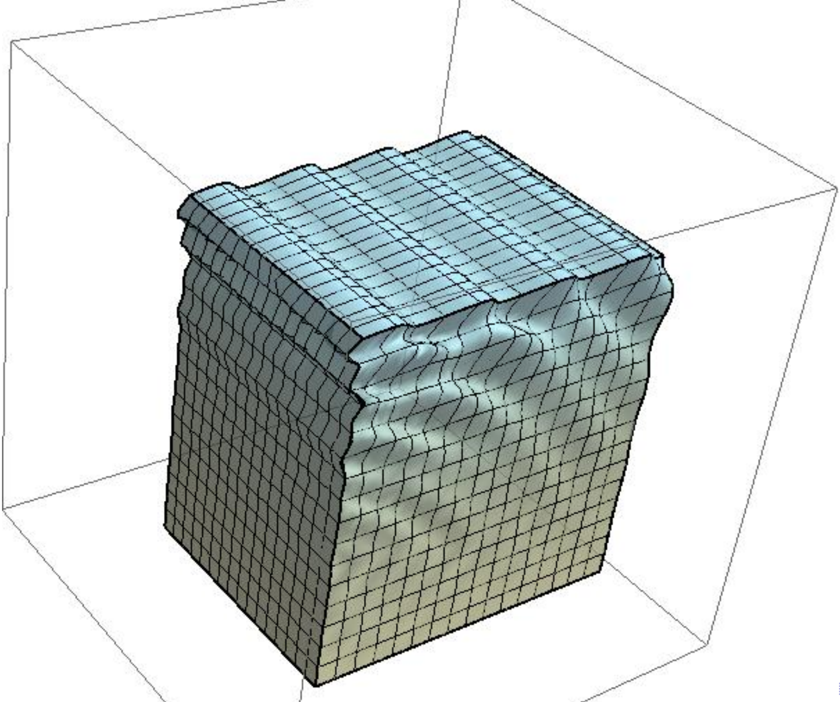


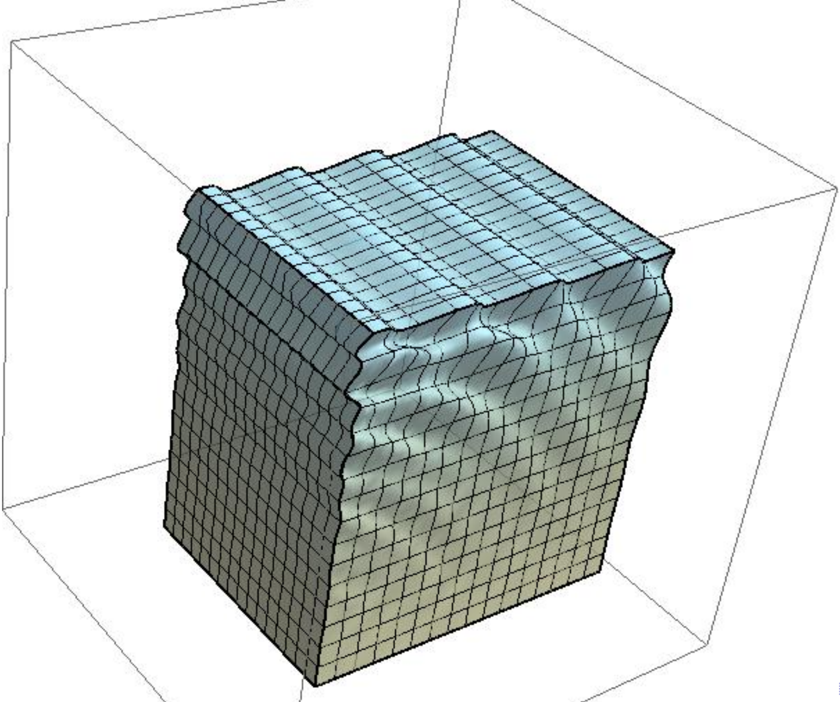


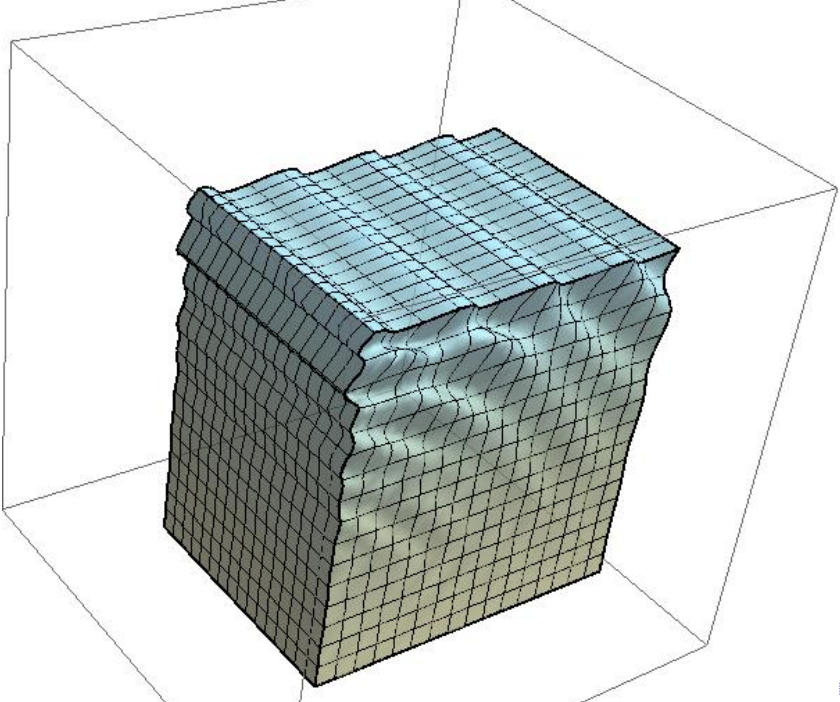


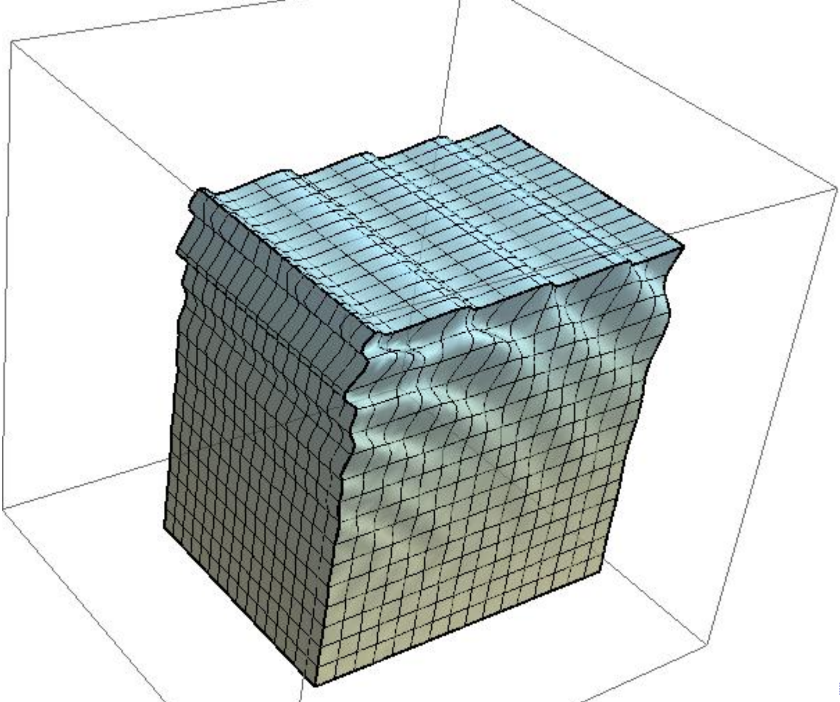


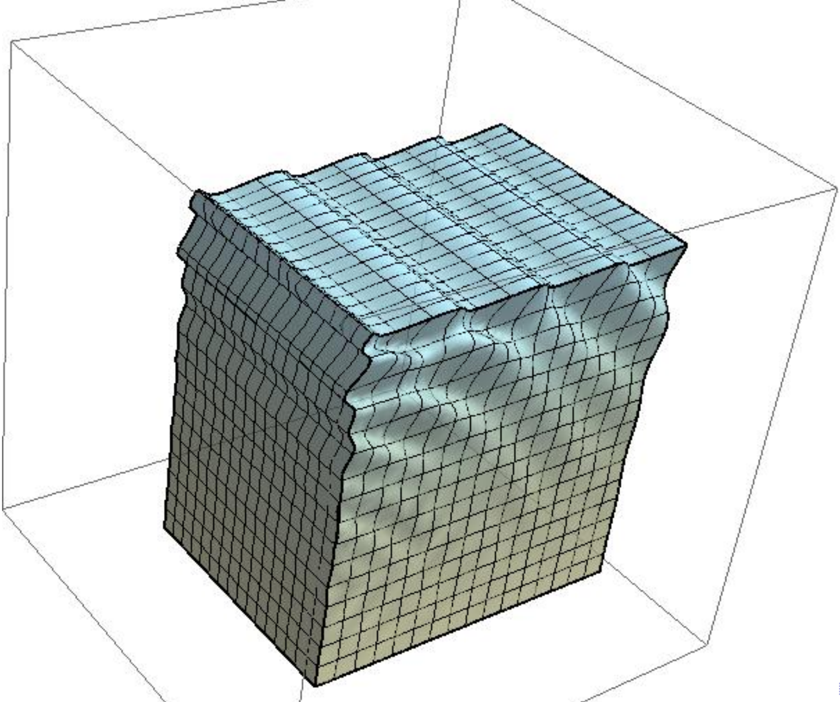


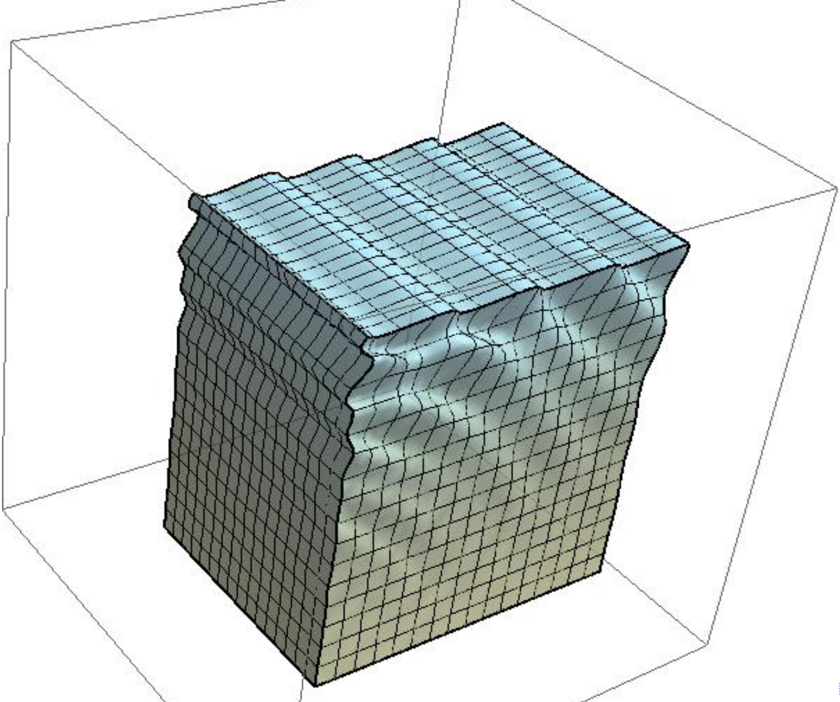


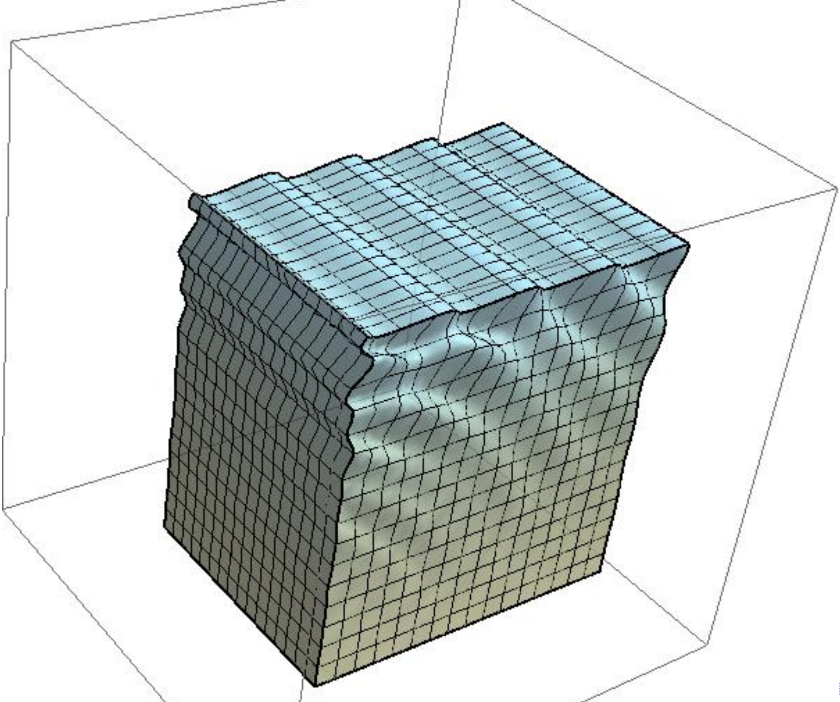


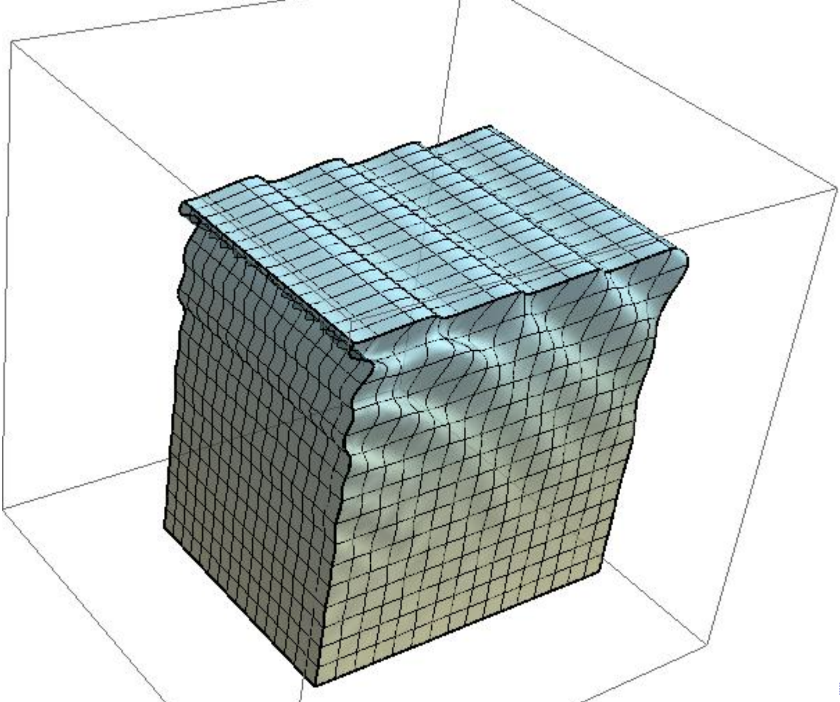




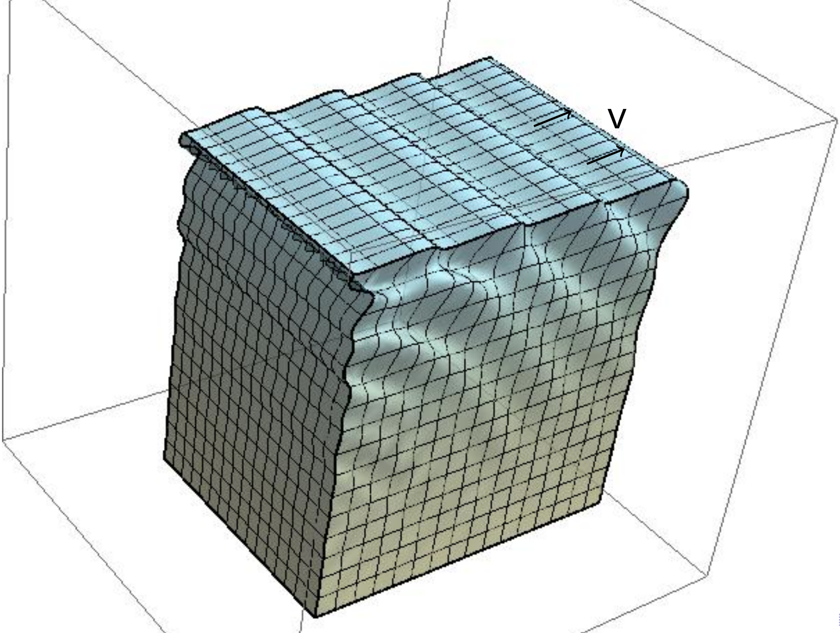




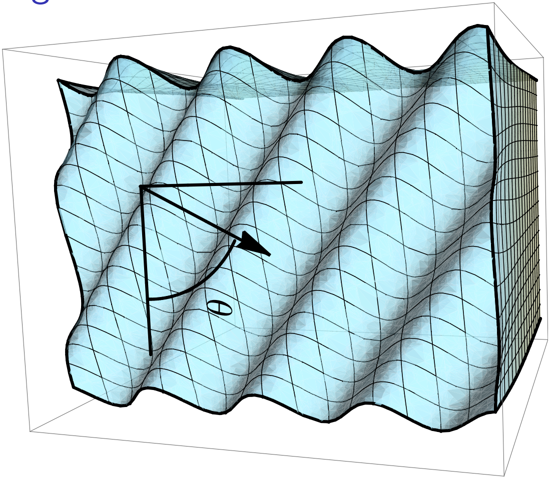




Incremental surface wave on top of large deformation



Wavefront Angle from direction of Greatest Compression



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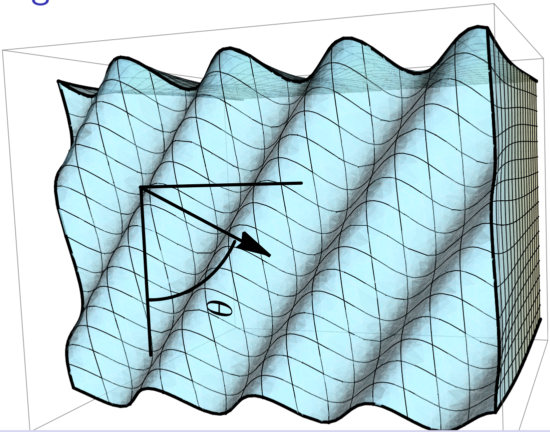


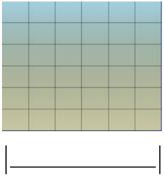
Incremental

$$\mathbf{u}(x,y,z) = \boldsymbol{U}(y)e^{ik(x\cos\theta + z\sin\theta - vt)} \qquad \text{(Displacement)}$$

$$\lim_{y\to\infty} \boldsymbol{U}(y) = 0 \quad \text{and} \quad \boldsymbol{\sigma}\cdot\boldsymbol{e}_y = \mathbf{0} \text{ on } y = 0 \qquad \text{(Boundary conditions)}$$

What is intuitive about a deformed isotropic material?

$\theta = 90^\circ$



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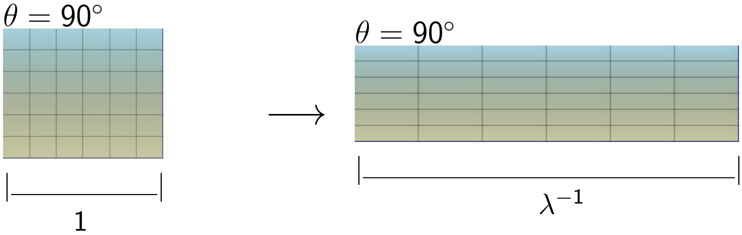

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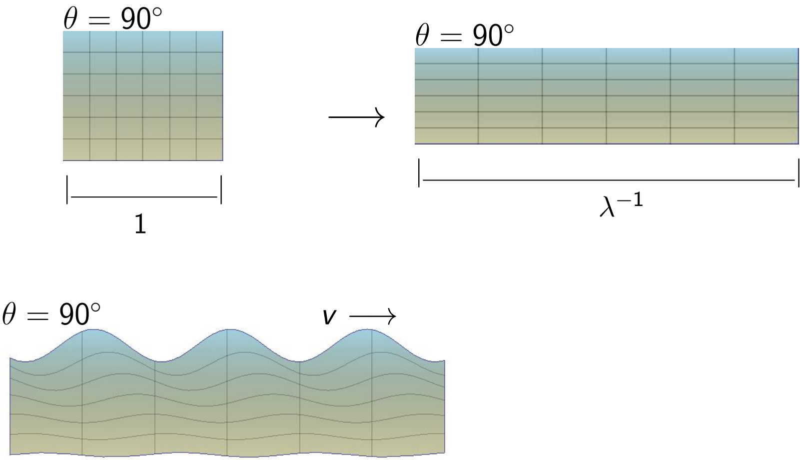

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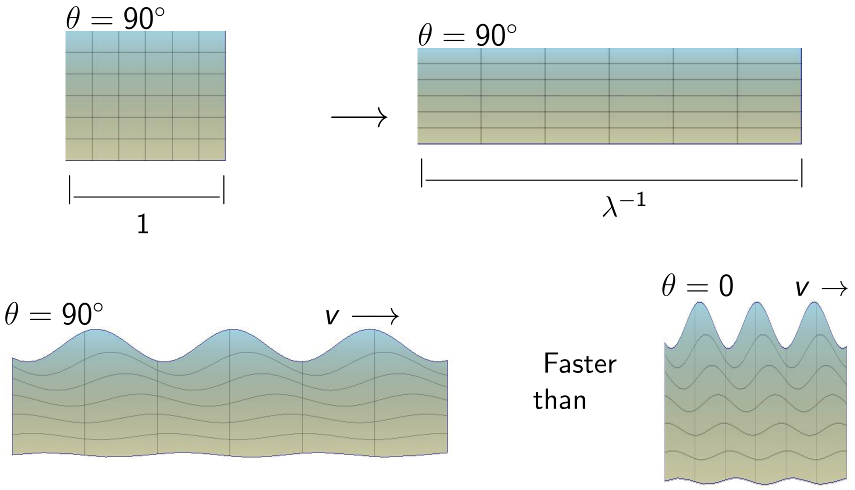

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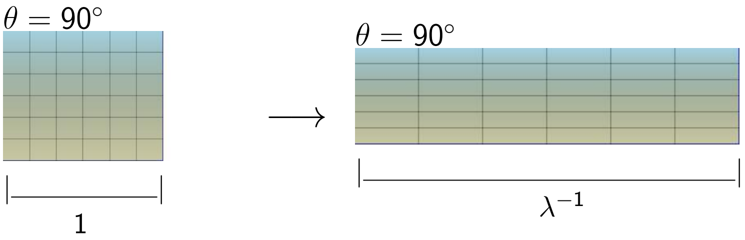
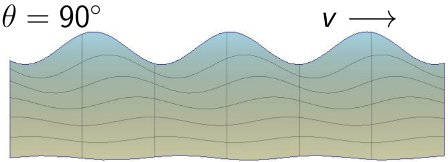
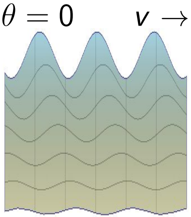




Faster than



ISOTROPIC: $\boxed{\text{Direction of greatest stress} = \text{Direction of greatest strain}}$

Inuitive Infinitesimal Prestress

- K. Tanuma, C.-S. Man, W. Du., (2013), perturbation of phase velocity of Rayleigh waves :

$$\boxed{v(\theta) = v^0 + C_1(\sigma_1 + \sigma_2) - C_2(\sigma_1 - \sigma_2)\cos 2\theta},$$

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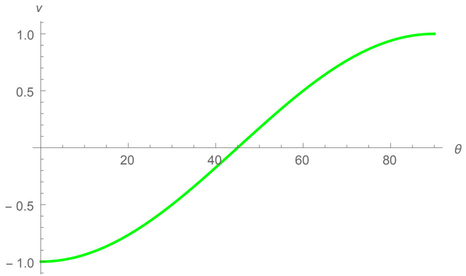

Nonlinear Elastic Results

$$W = \frac{\lambda_0}{2} i_1^2 + \mu_0 i_2 + \frac{A}{3} i_3 + B i_1 i_2 + \frac{C}{3} i_1^3 \quad \text{(Landau coefficients)}$$

For nonlinear elasticity, all bets are off...

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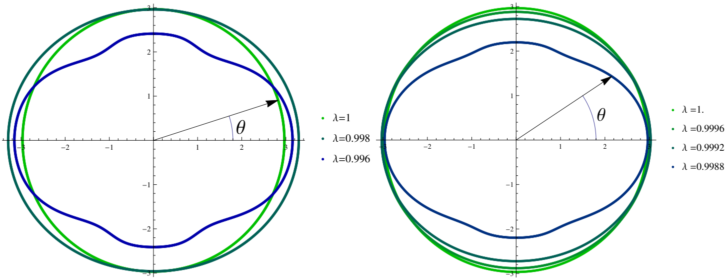

Figure: Speed profiles for surface waves (plotted as $v\sqrt{\rho}$) in Concrete subject to Uniaxial stress (left) and to Shear Stress (right).

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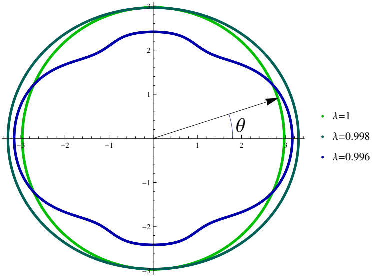


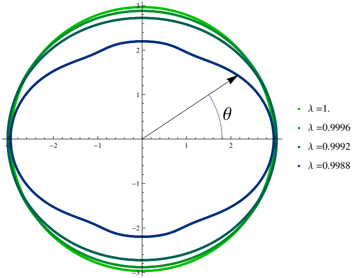


Figure: Speed profiles for surface waves (plotted as $v\sqrt{\rho}$) in Concrete subject to Uniaxial stress (left) and to Shear Stress (right).

- The sinusoidal regularity was lost early, for strains less than 1% (though the stress is reasonable.)

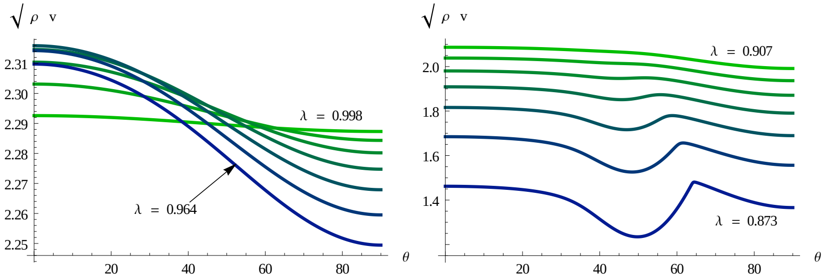
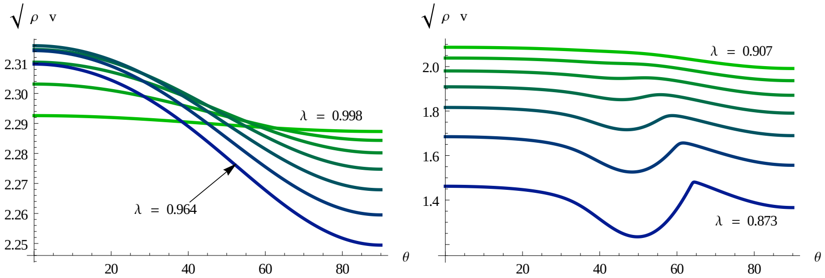


Figure: Nickel Uniaxial for varying strains λ.



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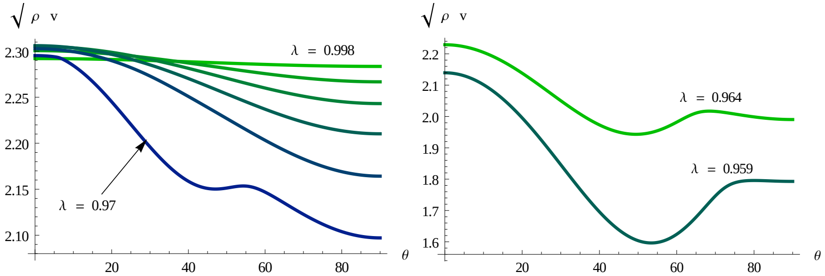

Figure: Nickel Pure Shear for varying strains λ

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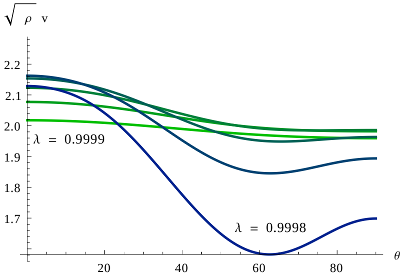

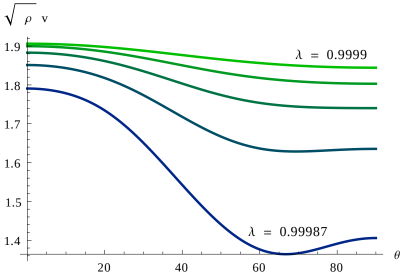

Figure: Uniaxial and Pure Shear Berea for varying strains λ

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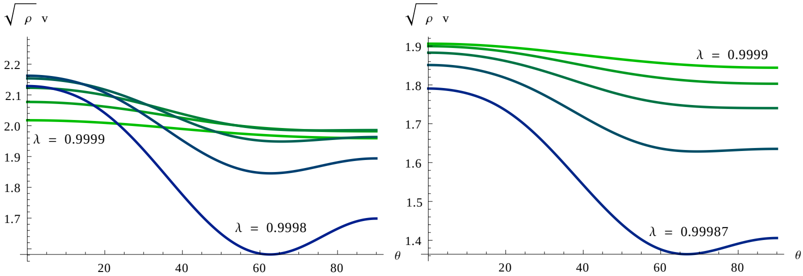

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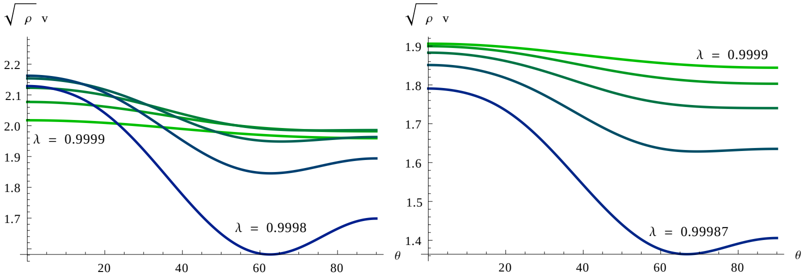

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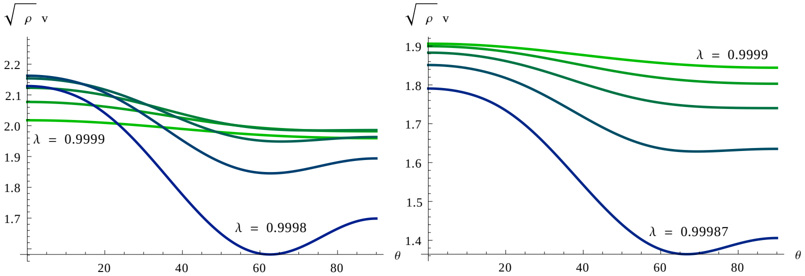

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- These results complicate protocols for finding the directions of greatest and least stress.
- K.Y. Kim, W. Sachse, (2001):
  "The principal stress direction is found where the variations of the SAW speeds show symmetry about the direction".

Matrix Impedance Method

take:

$$\mathbf{u}(x, y, t) = \underbrace{\boldsymbol{U}(y)\mathrm{e}^{\mathrm{i}kx-\mathrm{i}vt}}_{\textit{Displacement}} \rightarrow \underbrace{\rho u_{i,tt} = \mathcal{C}_{ipjq} u_{j,pq}}_{\textit{Balance of Momentum}}$$

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and Im Spec $\boldsymbol{E} > 0$,

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$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = \mathrm{div}\ \boldsymbol{\sigma} \implies -v^2 \rho \mathbf{u}^* \cdot \mathbf{u} = \mathbf{u}^* \cdot \mathrm{div}\ \boldsymbol{\sigma},$$

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$$\boldsymbol{U}^*(0) \cdot \frac{\partial \boldsymbol{Z}(v)}{\partial v} \boldsymbol{U}(0) = -2v \int_0^\infty \rho \boldsymbol{U}^*(y) \cdot \boldsymbol{U}(y) dy,$$

positive definite $\boldsymbol{Z}(0)$ with monotone decreasing Eigenvalues!

Fu, Y. B. and Mielke, A. (2002). In: Proc. R. Soc. A **458**, 25232543.

More results

This procedure works for any elastic strain-energy function, for example...

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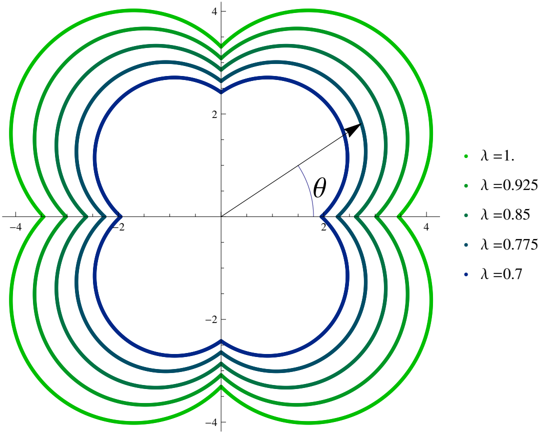

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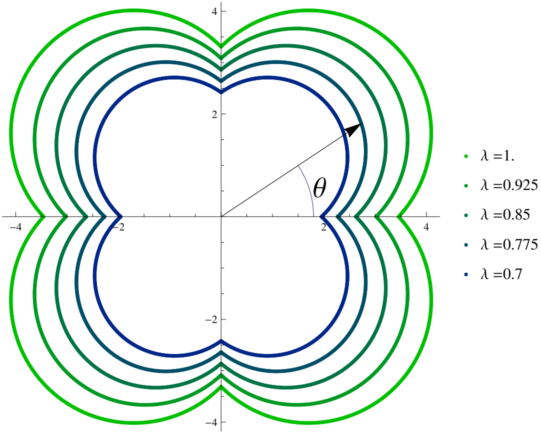
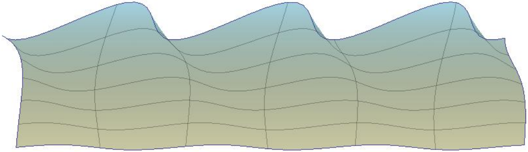


A model for skin, that has a neo-hookean matrix with fibers. This is an example of shear against the skin fibers.

What happened to our intuition?



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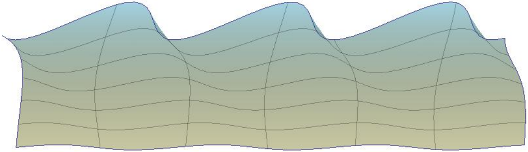

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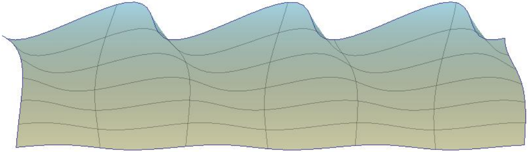

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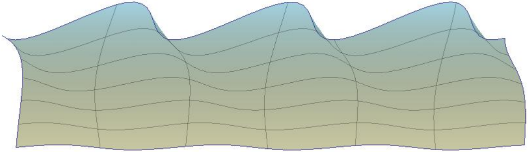

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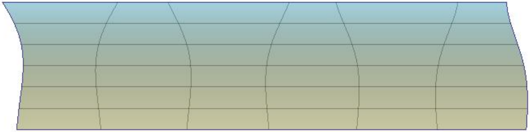

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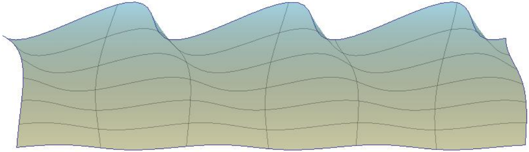

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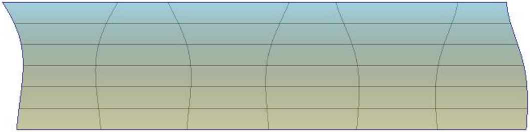

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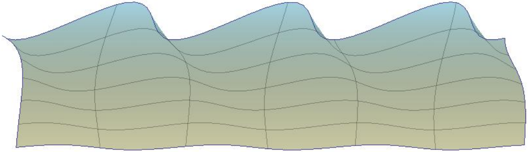

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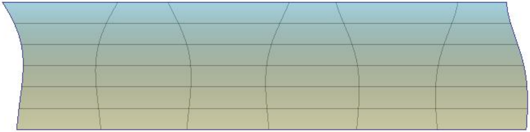

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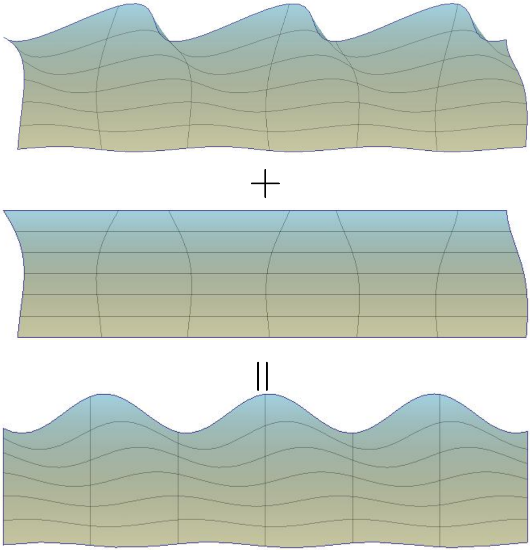

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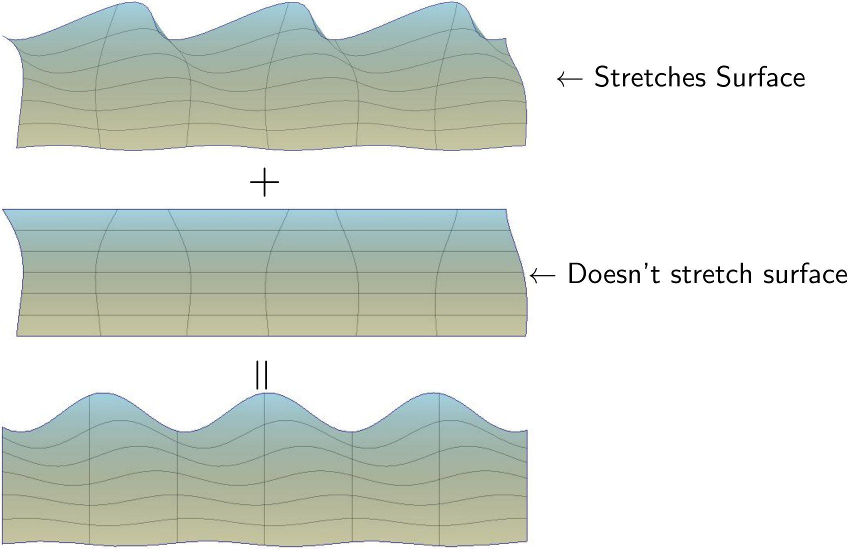

Any questions?

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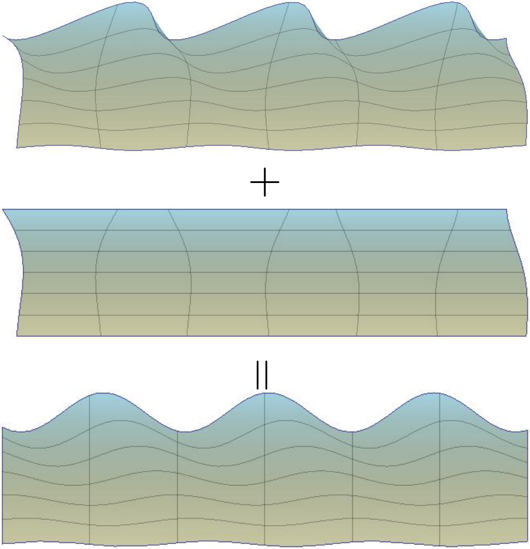

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Thanks for listening

Hope you enjoyed the talk!

Any questions?

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