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Working Draft, Technical Specification for C++ Extensions for Parallelism, Revision 1

Note: this is an early draft. It's known to be incomplet and incorrekt, and it has lots of bad formatting.

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1 General [general]

1.1 Scope [general.scope]

This Technical Specification describes requirements for implementations of an interface that computer programs written in the C++ programming language may use to invoke algorithms with parallel execution. The algorithms described by this Technical Specification are realizable across a broad class of computer architectures.

This Technical Specification is non-normative. Some of the functionality described by this Technical Specification may be considered for standardization in a future version of C++, but it is not currently part of any C++ standard. Some of the functionality in this Technical Specification may never be standardized, and other functionality may be standardized in a substantially changed form.

The goal of this Technical Specification is to build widespread existing practice for parallelism in the C++ standard algorithms library. It gives advice on extensions to those vendors who wish to provide them.

1.2 Normative references

[general.references]

The following reference document is indepensible for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

• ISO/IEC 14882:2011, Programming Languages – C++

ISO/IEC 14882:2011 is herein called the C++ Standard. The library described in ISO/IEC 14882:2011 clauses 17-30 is herein called the C++ Standard Library. The C++ Standard Library components described in ISO/IEC 14882:2011 clauses 25 and 26.7 are herein called the C++ Standard Algorithms Library.

Unless otherwise specified, the whole of the C++ Standard Library introduction [lib.library] is included into this Technical Specification by reference.

1.3 Namespaces and headers

[general.namespaces]

Since the extensions described in this Technical Specification are experimental and not part of the C++ Standard Library, they should not be declared directly within namespace std. Unless otherwise specified, all components described in this Technical Specification are declared in namespace std::experimental::parallel.

[Note: Once standardized, the components described by this Technical Specification are expected to be promoted to name space ${\tt std.}$ – end note]

Unless otherwise specified, references to other entities described in this Technical Specification are assumed to be qualified with std::experimental::parallel, and references to entities described in the C++ Standard Library are assumed to be qualified with std::.

Extensions that are expected to eventually be added to an existing header <meow> are provided inside the <experimental/meow> header, which shall include the standard contents of <meow> as if by

#include <meow>

1.4 Terms and definitions

[general.defns]

For the purposes of this document, the terms and definitions given in the C++ Standard and the following apply.

A parallel algorithm is a function template described by this Technical Specification declared in namespace std::experimental::parallel with a formal template parameter named ExecutionPolicy.

2 Execution policies

[execpol]

2.1 In general

[execpol.general]

This subclause describes classes that represent *execution policies*. An *execution policy* is an object that expresses the requirements on the ordering of functions invoked as a consequence of the invocation of a standard algorithm. Execution policies afford standard algorithms the discretion to execute in parallel.

[Example:

```
std::vector<int> vec = ...
// standard sequential sort
std::sort(vec.begin(), vec.end());
using namespace std::experimental::parallel;
// explicitly sequential sort
sort(seq, vec.begin(), vec.end());
// permitting parallel execution
sort(par, vec.begin(), vec.end());
// permitting vectorization as well
sort(vec, vec.begin(), vec.end());
// sort with dynamically-selected execution
size t threshold = ...
execution_policy exec = seq;
if(vec.size() > threshold)
{
  exec = par;
sort(exec, vec.begin(), vec.end());
- end example]
```

[Note: Because different parallel architectures may require idiosyncratic parameters for efficient execution, implementations of the Standard Library are encouraged to provide additional execution policies to those described in this Technical Specification as extensions. - end note]

2.2 Header <experimental/execution_policy> synopsis

[execpol.synop]

```
#include <type_traits>
namespace std {
namespace experimental {
namespace parallel {
    // 2.3, execution policy type trait
    template<class T> struct is_execution_policy;
```

```
// 2.4, sequential execution policy
  class sequential execution policy;
  // 2.5, parallel execution policy
  class parallel_execution_policy;
  // 2.6, vector execution policy
  class vector execution policy
  // 2.7, dynamic execution policy
  class execution_policy;
  // 2.8, specialized algorithms
  void swap(sequential_execution_policy &a, sequential_execution_policy &b);
  void swap(parallel_execution_policy &a, parallel_execution_policy &b);
  void swap(vector_execution_policy &a, vector_execution_policy &b);
  void swap(execution_policy &a, execution_policy &b);
  // 2.9, standard execution policy objects
  extern const sequential_execution_policy seq;
  extern const parallel_execution_policy
  extern const vector_execution_policy
                                            vec;
}
}
}
```

1. An implementation may provide additional execution policy types besides parallel_execution_policy, sequential_execution_policy, vector_execution_policy, or execution_policy.

2.3 Execution policy type trait

[execpol.type]

```
namespace std {
namespace experimental {
namespace parallel {
  template < class T > struct is_execution_policy
    : integral_constant < bool, see below > { };
}
}
}
```

- 1. is_execution_policy can be used to detect parallel execution policies for the purpose of excluding function signatures from otherwise ambiguous overload resolution participation.
- 2. If T is the type of a standard or implementation-defined non-standard execution policy, is_execution_policy<T> shall be publicly derived from integral_constant<bool,true>, otherwise from integral_constant<bool,false>.
- 3. The effect of specializing is_execution_policy for a type which is not defined by the library is unspecified.

[Note: This provision reserves the privilege of creating non-standard execution policies to the library implementation. - end note.]

2.4 Sequential execution policy

[execpol.seq]

```
namespace std {
namespace experimental {
namespace parallel {
   class sequential_execution_policy
   {
      void swap(sequential_execution_policy &other);
   };
}
```

- 1. The class sequential_execution_policy provides a mechanism to require a standard algorithm invocation to execute in a sequential order.
- 2. Implementations of sequential_execution_policy are permitted to provide additional non-standard data and function members.

[Note: This provision permits sequential_execution_policy objects to be stateful. - end note.]

void swap(sequential_execution_policy &other);

2. Effects: Swaps the state of *this and other.

2.5 Parallel execution policy

[execpol.par]

```
namespace std {
namespace experimental {
namespace parallel {
    class parallel_execution_policy
    {
       void swap(parallel_execution_policy &other);
    };
}
```

- 1. The class parallel_execution_policy provides a mechanism to allow a standard algorithm invocation to execute in an unordered fashion when executed on separate threads, and indeterminately sequenced when executed on a single thread.
- 2. Implementations of parallel_execution_policy are permitted to provide additional non-standard data and function members.

[Note: This provision permits parallel_execution_policy objects to be stateful. - end note.]

void swap(parallel_execution_policy &other);

2. Effects: Swaps the state of *this and other.

2.6 Vector execution policy

[execpol.vec]

```
namespace std {
namespace experimental {
namespace parallel {
   class vector_execution_policy
   {
     void swap(vector_execution_policy &other);
   };
}
}
```

- 1. The class vector_execution_policy provides a mechanism to allow a standard algorithm invocation to execute in an unordered fashion when executed on separate threads, and unordered when executed on a single thread.
- 2. Implementations of vector_execution_policy are permitted to provide additional non-standard data and function members.

[Note: This provision permits vector_execution_policy objects to be stateful. - end note.]

```
void swap(vector_execution_policy &other);
```

2. Effects: Swaps the state of *this and other.

2.7 Dynamic execution policy

[execpol.dynamic]

```
namespace std {
namespace experimental {
namespace parallel {
  class execution_policy
    public:
      // 2.7.1, construct/assign/swap
      template<class T> execution_policy(const T &exec);
      template<class T> execution_policy &operator=(const T &exec);
      void swap(execution_policy &other);
      // 2.7.2, object access
      const type_info& target_type() const;
      template<class T> T *target();
      template<class T> const T *target() const;
  };
}
}
}
```

- 1. The class execution_policy is a dynamic container for execution policy objects.
- 2. execution_policy allows dynamic control over standard algorithm execution.

[Example:

```
std::vector<float> sort_me = ...
   std::execution_policy exec = std::seq;
   if(sort_me.size() > threshold)
     exec = std::par;
   std::sort(exec, sort_me.begin(), sort_me.end());
- end example]
  3. The stored dynamic value of an execution_policy object may be retrieved.
     [Example:
     void some_api(std::execution_policy exec, int arg1, double arg2)
       if(exec.target_type() == typeid(std::seq))
         std::cout << "Received a sequential policy" << std::endl;</pre>
         auto *exec_ptr = exec.target<std::sequential_execution_policy>();
       }
       else if(exec.target_type() == typeid(std::par))
         std::cout << "Received a parallel policy" << std::endl;</pre>
         auto *exec_ptr = exec.target<std::parallel_execution_policy>();
       else if(exec.target_type() == typeid(std::vec))
         std::cout << "Received a vector policy" << std::endl;</pre>
         auto *exec_ptr = exec.target<std::vector_execution_policy>();
       }
       else
         std::cout << "Received some other kind of policy" << std::endl;</pre>
       }
     }
     - end example]
```

4. Objects of type execution_policy shall be constructible and assignable from any additional non-standard execution policy provided by the implementation.

2.7.1 execution_policy construct/assign/swap

[execpol.con]

template<class T> execution_policy(const T &exec);

1. Effects: Constructs an execution_policy object with a copy of exec's state.

2. Remarks: This signature does not participate in overload resolution if is_execution_policy<T>::value is false.

template<class T> execution_policy &operator=(const T &exec);

- 3. Effects: Assigns a copy of exec's state to *this.
- 4. Returns: *this.
- Remarks: This signature does not participate in overload resolution if is_execution_policy<T>::value is false.

void swap(execution_policy &other);

1. Effects: Swaps the stored object of *this with that of other.

2.7.2 execution_policy object access

[execpol.access]

```
const type_info &target_type() const;
```

1. Returns: typeid(T), such that T is the type of the execution policy object contained by *this.

```
template<class T> T *target();
template<class T> const T *target() const;
```

- 2. Returns: If target_type() == typeid(T), a pointer to the stored execution policy object; otherwise a null pointer.
- 3. Remarks: This signature does not participate in overload resolution if is_execution_policy<T>::value is false.

2.8 Execution policy specialized algorithms

[execpol.algorithms]

```
void swap(sequential_execution_policy &a, sequential_execution_policy &b);
void swap(parallel_execution_policy &a, parallel_execution_policy &b);
void swap(vector_execution_policy &a, vector_execution_policy &b);
void swap(execution_policy &a, execution_policy &b);
```

1. Effects: a.swap(b).

2.9 Standard execution policy objects

[execpol.objects]

```
namespace std {
namespace experimental {
namespace parallel {
  extern const sequential_execution_policy seq;
  extern const parallel_execution_policy par;
  extern const vector_execution_policy vec;
}
}
}
```

- 1. The header <execution_policy> declares a global object associated with each standard execution policy.
- 2. An implementation may provide additional execution policy objects besides seq, par, or vec.
- 3. Concurrent access to these objects shall not result in a data race.

const sequential_execution_policy seq;

4. The object seq requires a standard algorithm to execute sequentially.

const parallel_execution_policy par;

5. The object par allows a standard algorithm to execute in an unordered fashion when executed on separate threads, and indeterminately sequenced when executed on a single thread.

const vector_execution_policy vec;

6. The object vec allows a standard algorithm to execute in an unordered fashion when executed on separate threads, and unordered when executed on a single thread.

3 Parallel exceptions

[exceptions]

3.1 Exception reporting behavior

[exceptions.behavior]

- 1. During the execution of a standard parallel algorithm, if the application of a function object terminates with an uncaught exception, the behavior of the program is determined by the type of execution policy used to invoke the algorithm.
- If the execution policy object is of type vector execution policy, std::terminate shall be called.
- If the execution policy object is of type sequential_execution_policy or parallel_execution_policy, the execution of the algorithm terminates with an exception_list exception. All uncaught exceptions thrown during the application of user-provided function objects shall be contained in the exception_list, however the number of such exceptions is unspecified.

[Note: For example, the number of invocations of the user-provided function object in for_each is unspecified. When for_each is executed sequentially, only one exception will be contained in the exception_list object - end note]

[Note: These guarantees imply that, unless the algorithm has failed to allocate memory and terminated with $std::bad_alloc$, all exceptions thrown during the execution of the algorithm are communicated to the caller. It is unspecified whether an algorithm implementation will "forge ahead" after encountering and capturing a user exception. - end note]

- If the execution policy object is of any other type, the behavior is implementation-defined.
- 2. If temporary memory resources are required by the algorithm and none are available, the algorithm may terminate with an std::bad_alloc exception.

[Note: The algorithm may terminate with the std::bad_alloc exception even if one or more user-provided function objects have terminated with an exception. For example, this can happen when an algorithm fails to allocate memory while creating or adding elements to the exception_list object—end note]

3.2 Header <experimental/exception> synopsis

[exceptions.synop]

```
namespace std {
namespace experimental {
namespace parallel {
  class exception_list : public exception
    public:
      typedef exception_ptr
                                 value_type;
      typedef const value_type& reference;
      typedef const value_type& const_reference;
      typedef size t
                                 size_type;
      typedef unspecified
                                 iterator;
      typedef unspecified
                                 const_iterator;
      size_t size() const;
      iterator begin() const;
      iterator end() const;
    private:
      std::list<exception_ptr> exceptions_; // exposition only
  };
}
}
```

1. The class exception_list is a container of exception_ptr objects parallel algorithms may use to communicate uncaught exceptions encountered during parallel execution to the caller of the algorithm.

```
size_t size() const;
```

2. Returns: The number of exception_ptr objects contained within the exception_list.

```
exception_list::iterator begin() const;
```

3. Returns: An iterator pointing to the first exception_ptr object contained within the exception_list.

```
exception_list::iterator end() const;
```

4. Returns: An iterator pointing to one position past the last exception_ptr object contained within the exception_list.

4 Parallel algorithms

[alg]

4.1 In general [alg.general]

This clause describes components that C++ programs may use to perform operations on containers and other sequences in parallel.

4.1.1 Effect of execution policies on parallel algorithm execution [alg.general.exec]

1. Parallel algorithms have template parameters named ExecutionPolicy which describe the manner in which the execution of these algorithms may be parallelized and the manner in which they apply user-provided function objects.

- 2. The applications of function objects in parallel algorithms invoked with an execution policy object of type sequential_execution_policy execute in sequential order in the calling thread.
- 3. The applications of function objects in parallel algorithms invoked with an execution policy object of type parallel_execution_policy are permitted to execute in an unordered fashion in unspecified threads, or indeterminately sequenced if executed on one thread. [Note: It is the caller's responsibility to ensure correctness, for example that the invocation does not introduce data races or deadlocks. end note]

[Example:

```
using namespace std::experimental::parallel;
int a[] = {0,1};
std::vector<int> v;
for_each(par, std::begin(a), std::end(a), [&](int i) {
  v.push_back(i*2+1);
});
```

The program above has a data race because of the unsynchronized access to the container v - end example

[Example:

```
using namespace std::experimental::parallel;
std::atomic<int> x = 0;
int a[] = {1,2};
for_each(par , std::begin(a), std::end(a), [](int n) {
    x.fetch_add(1 , std::memory_order_relaxed);
    // spin wait for another iteration to change the value of x
    while(x.load(std::memory_order_relaxed) == 1);
});
```

The above example depends on the order of execution of the iterations, and is therefore undefined (may deadlock). — end example

[Example:

```
using namespace std::experimental::parallel;
int x;
std::mutex m;
int a[] = {1,2};
for_each(par, std::begin(a), std::end(a), [&](int) {
    m.lock();
    ++x;
    m.unlock();
});
```

The above example synchronizes access to object \mathbf{x} ensuring that it is incremented correctly. — end example]

4. The applications of function objects in parallel algorithms invoked with an execution policy of type vector_execution_policy are permitted to execute in an unordered fashion in unspecified threads, or unsequenced if executed on one thread. [Note: as a consequence, function objects governed by the vector_execution_policy policy must not synchronize with each other. Specifically, they must not acquire locks. — end note]

[Example:

```
using namespace std::experimental::parallel;
int x;
std::mutex m;
int a[] = {1,2};
for_each(vec, std::begin(a), std::end(a), [&](int) {
    m.lock();
    ++x;
    m.unlock();
});
```

The above program is invalid because the applications of the function object are not guaranteed to run on different threads.

[Note: the application of the function object may result in two consecutive calls to m.lock on the same thread, which may deadlock — end note]

```
- end example]
```

[Note: The semantics of the parallel_execution_policy or the vector_execution_policy invocation allow the implementation to fall back to sequential execution if the system cannot parallelize an algorithm invocation due to lack of resources. — end note.]

5. If they exist, a parallel algorithm invoked with an execution policy object of type parallel_execution_policy or vector_execution_policy may apply iterator member functions of a stronger category than its specification requires. In this case, the application of these member functions are subject to provisions 3. and 4. above, respectively.

[Note: For example, an algorithm whose specification requires InputIterator but receives a concrete iterator of the category RandomAccessIterator may use operator[]. In this case, it is the algorithm caller's responsibility to ensure operator[] is race-free. — end note.]

- 6. Algorithms invoked with an execution policy object of type execution_policy execute internally as if invoked with instances of type sequential_execution_policy, parallel_execution_policy, or a non-standard implementation-defined execution policy depending on the dynamic value of the execution_policy object.
- 7. The semantics of parallel algorithms invoked with an execution policy object of type other than those described by this Technical Specification are unspecified.

4.1.2 ExecutionPolicy algorithm overloads

[alg.overloads]

- 1. Parallel algorithms coexist alongside their sequential counterparts as overloads distinguished by a formal template parameter named ExecutionPolicy. This template parameter corresponds to the parallel algorithm's first function parameter.
- 2. Unless otherwise specified, the semantics of ExecutionPolicy algorithm overloads are identical to their overloads without.
- 3. Parallel algorithms have the requirement is_execution_policy_v<ExecutionPolicy> is true.
- 4. The algorithms listed in table 1 have parallel overloads.

uninitialized_copy	uninitialized_copy_n	uninitialized_fill	uninitialized_fill_n
all_of	any_of	none_of	find
find	find_if	find_if_not	find_end
find_first_of	adjacent_find	count	count_if

mismatch	equal	search	search_n
сору	copy_n	copy_if	move
swap_ranges	transform	replace	replace_copy
replace_copy_if	fill	fill_n	generate
generate_n	remove	remove_if	remove_copy
remove_copy_if	unique	unique_copy	reverse
reverse_copy	rotate	rotate_copy	is_partitioned
partition	stable_partition	partition_copy	sort
stable_sort	partial_sort	partial_sort_copy	is_sorted
is_sorted_until	nth_element	merge	inplace_merge
includes	set_union	set_intersection	set_difference
set_symmetric_difference	min_element	max_element	minmax_element
lexicographical_compare	reduce	inclusive_scan	exclusive_scan
	for_each	for_each_n	

Table 1: Table of parallel algorithms

4.2 Novel algorithms

[alg.novel]

This subclause describes novel algorithms introduced by this technical specification.

```
4.2.1 Header <experimental/algorithm> synopsis
```

[alg.novel.algorithm.synop]

4.2.2 For each

[alg.novel.foreach]

- 1. Effects: Applies f to the result of dereferencing every iterator in the range [first,last). [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions through the dereferenced iterator. end note]
- 2. Complexity: O(last first).
- 3. Remarks: If f returns a result, the result is ignored.
- 4. *Note:* Unlike its sequential form, the parallel overload of for_each does not return a copy of its Function parameter, since parallelization does not permit sequential state accumulation.

```
XXX fix return type
```

- 1. Requires: Function shall meet the requirements of MoveConstructible [Note: Function need not meet the requirements ofCopyConstructible'. end note]
- 2. Effects: Applies f to the result of dereferencing every iterator in the range [first,first + n), starting from first and proceeding to first + n 1. [Note: If the type of first satisfies the requirements of a mutable iterator, f may apply nonconstant functions throughthe dereferenced iterator. end note]
- 3. Returns: first + (last first).
- 4. Complexity: Applies f exactly n times.
- 5. Remarks: If f returns a result, the result is ignored.

4.2.3 Header <experimental/numeric> synopsis

[alg.novel.numeric.synop]

```
namespace std {
namespace experimental {
namespace parallel {
  template < class InputIterator >
    typename iterator traits<InputIterator>::value type
      reduce(InputIterator first, InputIterator last);
  template<class InputIterator, class T>
    T reduce(InputIterator first, InputIterator last T init);
  template<class InputIterator, class T, class BinaryOperation>
    T reduce(InputIterator first, InputIterator last, T init,
             BinaryOperation binary_op);
  template < class InputIterator, class OutputIterator >
    OutputIterator
      exclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result);
  template < class InputIterator, class OutputIterator,
           class T>
   OutputIterator
      exclusive_scan(InputIterator first, InputIterator last,
                     OutputIterator result,
                     T init);
  template < class InputIterator, class OutputIterator,
```

```
class T, class BinaryOperation>
    OutputIterator
      exclusive_scan(InputIterator first, InputIterator last,
                       OutputIterator result,
                      T init, BinaryOperation binary_op);
  template < class InputIterator, class OutputIterator >
    OutputIterator
      inclusive_scan(InputIterator first, InputIterator last,
                       OutputIterator result);
  template<class InputIterator, class OutputIterator,</pre>
            class BinaryOperation>
    OutputIterator
      inclusive_scan(InputIterator first, InputIterator last,
                       OutputIterator result,
                       BinaryOperation binary_op);
  template < class InputIterator, class OutputIterator,
            class T, class BinaryOperation>
    OutputIterator
       inclusive_scan(InputIterator first, InputIterator last,
                       OutputIterator result,
                       T init, BinaryOperation binary_op);
}
}
}
4.2.4 Reduce
                                                                               [alg.novel.reduce]
template < class InputIterator >
  typename iterator_traits<InputIterator>::value_type
    reduce(InputIterator first, InputIterator last);
  1. Returns: The result of the sum of T{} and the elements in the range [first,last).
     The order of operands of the sum is unspecified.
  2. Requires: Let T be the type of InputIterator's value type, then T(0) shall be a valid expression. The
     operator+ function associated with T shall have associativity and commutativity.
     operator+ shall not invalidate iterators or subranges, nor modify elements in the range [first,last).
  3. Complexity: O(last - first).
  4. Note: The primary difference between reduce and accumulate is that the behavior of reduce may be
     non-deterministic for non-associative or non-commutative binary op.
template<class InputIterator, class T>
  T reduce(InputIterator first, InputIterator last, T init);
template < class InputIterator, class T, class BinaryOperation>
  T reduce(InputIterator first, InputIterator last, T init,
            BinaryOperation binary_op);
  1. Returns: The result of the generalized sum of init and the elements in the range [first,last).
```

Sums of elements are evaluated with operator+ or binary_op. The order of operands of the sum is

unspecified.

2. Requires: The operator+ function associated with InputIterator's value type or binary_op shall have associativity and commutativity.

Neither operator+ nor binary_op shall invalidate iterators or subranges, nor modify elements in the range [first,last).

3. Complexity: O(last - first).

4.2.5 Exclusive scan

[alg.novel.exclusive.scan]

1. Effects: For each iterator i in [result,result + (last - first)), produces a result such that upon completion of the algorithm, *i yields the generalized sum of init and the elements in the range [first, first + (i - result)).

During execution of the algorithm, every evaluation of the above sum is either of the corresponding form

```
(init + A) + B) or A + B or
binary_op(binary_op(init,A), B) or binary_op(A, B)
where there exists some iterator j in [first, last) such that:
```

- 1. A is the partial sum of elements in the range [j, j + n).
- 2. B is the partial sum of elements in the range [j + n, j + m).
- 3. n and m are positive integers and j + m < last.
- 2. Returns: The end of the resulting range beginning at result.
- 3. Requires: The operator+ function associated with InputIterator's value type or binary_op shall have associativity.

Neither operator+ nor binary_op shall invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last - first)).

- 4. Complexity: O(last first).
- 5. *Notes:* The primary difference between exclusive_scan and inclusive_scan is that exclusive_scan excludes the ith input element from the ith sum.

4.2.6 Inclusive scan

[alg.novel.inclusive.scan]

```
template<class InputIterator, class OutputIterator>
  OutputIterator
    inclusive_scan(InputIterator first, InputIterator last,
                   OutputIterator result);
template<class InputIterator, class OutputIterator,
         class BinaryOperation>
  OutputIterator
    inclusive_scan(InputIterator first, InputIterator last,
                   OutputIterator result,
                   BinaryOperation binary_op);
template < class InputIterator, class OutputIterator,
         class T, class BinaryOperation>
  OutputIterator
    inclusive_scan(InputIterator first, InputIterator last,
                   OutputIterator result,
                   T init, BinaryOperation binary_op);
```

1. Effects: For each iterator i in [result,result + (last - first)), produces a result such that upon completion of the algorithm, *i yields the generalized sum of init and the elements in the range [first, first + (i - result)].

During execution of the algorithm, every evaluation of the above sum is either of the corresponding form

```
(init + A) + B) or A + B or
```

binary_op(binary_op(init,A), B) or binary_op(A, B)

where there exists some iterator j in [first, last) such that:

- 1. A is the partial sum of elements in the range [j, j + n).
- 2. B is the partial sum of elements in the range [j + n, j + m).
- 3. n and m are positive integers and j + m < last.
- 2. Returns: The end of the resulting range beginning at result.
- 3. Requires: The operator+ function associated with InputIterator's value type or binary_op shall have associativity.

Neither operator+ nor binary_op shall invalidate iterators or subranges, nor modify elements in the ranges [first,last) or [result,result + (last - first)).

- 4. Complexity: O(last first).
- 5. *Notes:* The primary difference between exclusive_scan and inclusive_scan is that inclusive_scan includes the ith input element in the ith sum.