The Bouncing Mass Problem - Numerical Solution

Assignment IV
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Hybrid Dynamical Systems

Hybrid dynamical systems are systems that exhibit both continuous dynamics (described by differential equations) and discrete events (described by logic or switching). These systems switch between different modes of behavior based on conditions or events, making them essential for modeling real-world systems like thermostats, automated control systems, or cyber-physical systems. In the case of the falling mass-spring system considered in this assignment, the system exhibits hybrid behavior due to the transition between the flight phase and the bouncing phase, each governed by different dynamics.

Problem Specification

In the following, the motion of a mass mounted on top of a vertical, massless spring dropped from an initial height of 1.2 meters will be analyzed. The mass is released from rest. Air resistance is neglected, and only the effects of gravity and the spring force are considered. The goal is to model the motion of the mass, derive the governing equations of motion, and evaluate how the mechanical energy evolves over time.

Parameter	**Symbol**	**Value**	**Unit**
Mass	m	1.0	kg
Initial drop height	y_0	1.2	m
Initial velocity	v_0	0	m/s
Gravitational acceleration	g	9.81	m/s^2
Spring stiffness	k	100	N/m
Spring length	l	1.0	m

Table 1: System Parameters

Figure 1 shows the two states of the mass–spring system. In the **flight phase**, the mass is falling freely under gravity, and the spring has not yet been compressed, so no spring force acts on the system. Once the mass reaches the spring's unstretched length, it enters the **bouncing phase**, during which the spring is compressed and applies a force upward. The motion is constrained to the vertical y-axis.

Simulink Implementation

The two differential equations based on Newton's second law—one describing the flight phase (1) and the other the bouncing phase (2)—were modeled in Simulink using Integrator blocks.A Switch block, governed by a logical signal from a Relational Operator, selected the active equation depending on whether the mass had reached the spring's rest position, i.e., when $y \leq 1$. The initial condition for the position was set to 1.2 meters in the Integrator block to represent the mass being released from that height.

$$\ddot{y}_{flight}(t) = -g \tag{1}$$

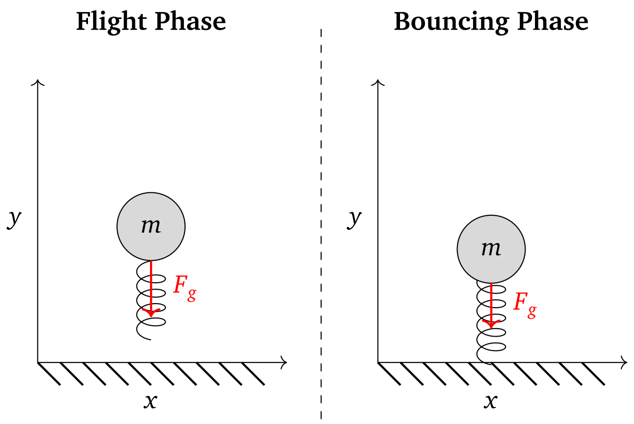

Figure 1: Free-Body Diagrams for Flight and Bouncing Phase.

$$\ddot{y}_{bouncing}(t) = -g + \frac{k}{m}(l - y(t)) \tag{2}$$

Results and Visualization

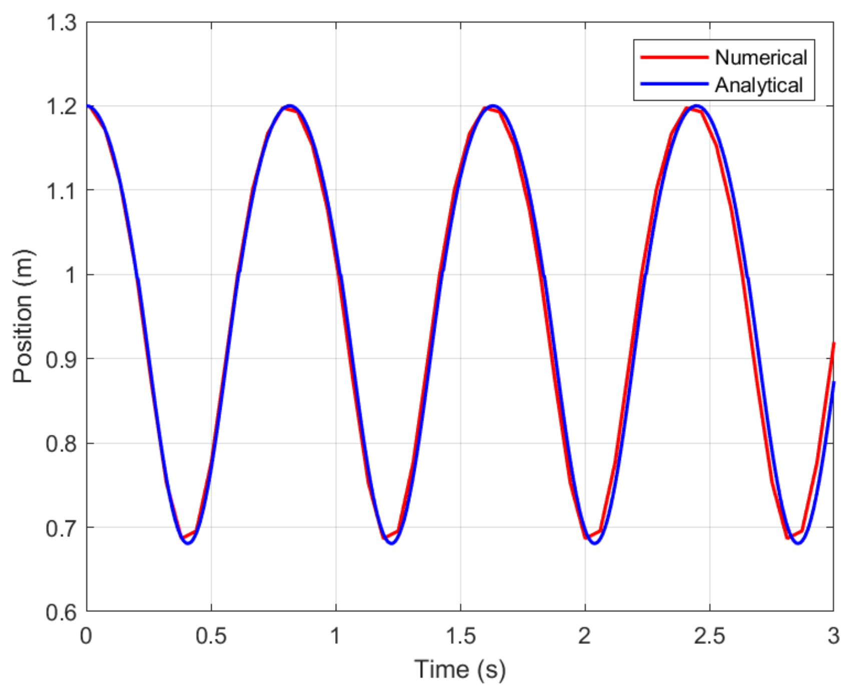

Figure 2: Comparison of Numerical and Analytical Solution : $y(t)$

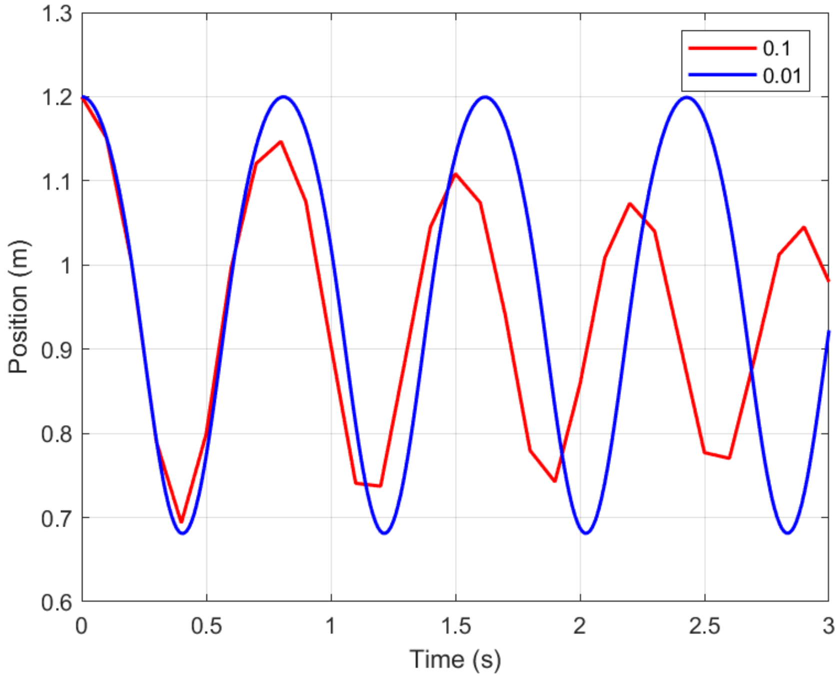

Figure 3: Comparison of Different Step Sizes for the Numerical Solution : $y(t)$

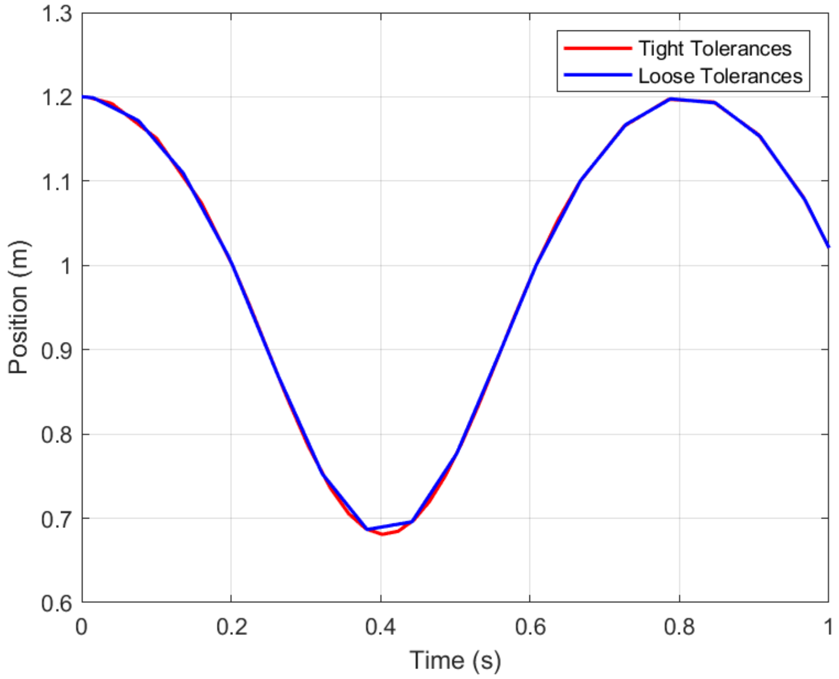

Figure 4: Comparison of Different Relative and Absolute Tolerances for the Numerical Solution : $y(t)$

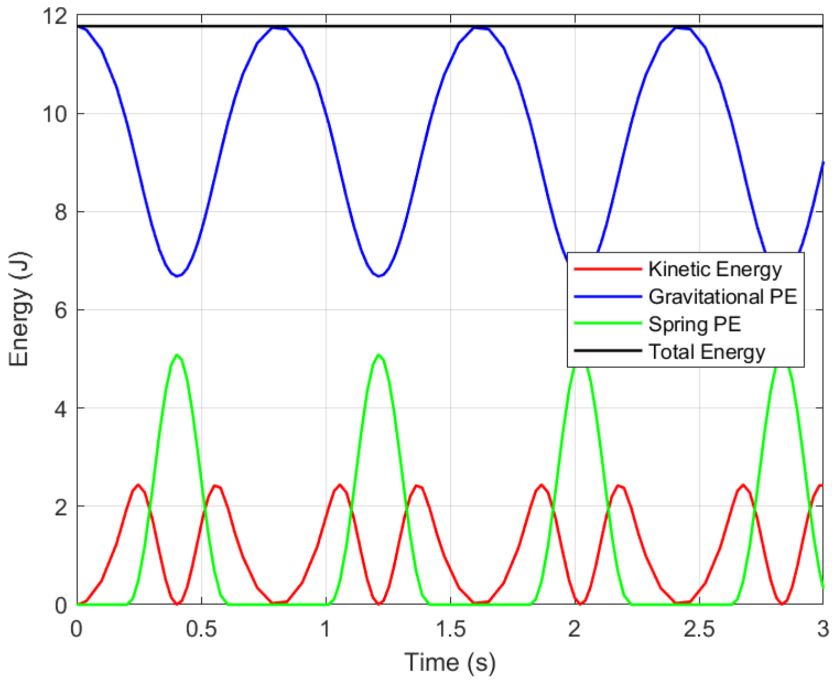

Figure 5: Energy Components in the Mass-Spring System

Discussion

As shown in Figure 2, the analytical and numerical solutions are largely equivalent, although the numerical result tends to lag slightly behind in time due to integration step effects.

Figure 3 compares the numerical solutions obtained with step sizes of 0.1 and 0.01. The simulation using a step size of 0.1 fails to capture the system's dynamics accurately. This highlights how an excessively large step size can lead to numerical errors and loss of important dynamic details, particularly in systems with rapid transitions or hybrid behavior.

Figure 4 presents a comparison of the numerical solution using two different solver tolerance settings. The simulation with looser tolerances (`RelTol = 1e-1`, `AbsTol = 1e-3`) produces a trajectory that is noticeably less smooth and less consistent. In contrast, the tighter tolerance configuration (`RelTol = 1e-6`, `AbsTol = 1e-8`) results in a more fluent and stable solution.

Figure 5 illustrates the time evolution of the mechanical energy components in the mass-spring system. The total mechanical energy remains constant throughout the simulation, as expected in this energy-conserving system. This confirms the correct transfer of energy between kinetic and potential forms during both the **flight and bouncing phases** .

AI usage declaration

tools used

ChatGPT

comments

This includes support with generation of the TikZ free-body diagram (Impossible to get an accurate diagram directly, but gave me a base to start and modify manually). Some aspects of the MATLAB plottingwere improved with the assistance of AI-based tools . As well to improve the grammar, structure, and readability of the text.