TUM SCHOOL OF COMPUTATION, INFORMATION AND TECHNOLOGY (CIT)

TECHNICAL UNIVERSITY OF MUNICH

Report Submitted to Praktikum Robot Modelling and Identification (IN2106)

Experiment 5: Adaptive Control of a Planar Manipulator

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We confirm that this report is my our own work and we have documented all sources and material used.

Munich, June 12, 2025
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Contents

1 ACM - Adaptive Control of a Planar Manipulator

Consider a two DoF robot whose dynamics are given by $\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}p_1 + 2p_3c(q_2) & p_2 + p_3c(q_2)\\ p_2 + p_3c(q_2) & p_2\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix}-p_3s(q_2)\dot{q}_2 & -p_3s(q_2)(\dot{q}_1 + \dot{q}_2)\\ p_3s(q_2)\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + \begin{bmatrix}f_{d1} & 0\\ 0 & f_{d2}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}$,

where $c(q_i) = \cos(q_i)$ and $s(q_i) = \sin(q_i)$. The parameter values are as follows: $p_1 = 3.473\,\mathrm{kg}\cdot\mathrm{m}^2$, $p_2 = 0.196\,\mathrm{kg}\cdot\mathrm{m}^2$, $p_3 = 0.242\,\mathrm{kg}\cdot\mathrm{m}^2$, $f_{d1} = 5.3\,\mathrm{Nms}$, and $f_{d2} = 1.1\,\mathrm{Nms}$.

It is desired that its joints follow this trajectory:

$$q_d = \begin{bmatrix}\sin(3t)\\ 2\cos(t)\end{bmatrix}$$

1.1 Task 1

We are to design a **gradient-based adaptive controller** using the known desired trajectory and the measurable joint angles and velocities. The update law should be driven by **tracking error** and use a constant gain matrix.

The tracking error is defined:

$$\begin{aligned} e &= q_d - q,\\ \dot{e} &= \dot{q}_d - \dot{q},\\ \ddot{e} &= \ddot{q}_d - \ddot{q} \end{aligned}$$

The filtered tracking error is defined:

$$\begin{aligned} r &= \dot{e} + \alpha e,\\ \dot{r} &= \ddot{e} + \alpha\dot{e}. \end{aligned}$$

Rewriting dynamics and grouping terms to form the regressor:

$$M(\cdot)\dot{r} = M(\cdot)\ddot{q}_d + \alpha M(\cdot)\dot{e} + C(\cdot)(\dot{q}_d + \alpha e) + g(q) - C(\cdot)r - \tau = \mathbf{Y}(\cdot)\theta - C(\cdot)r - \tau$$

Where we choose filtering parameter alpha=1, controller gain k=diag[30;30], adaptation gain

Gamma=diag([50,10,100,30,30])

Defining the controller as $\tau = kr + \mathbf{Y}(\cdot)\hat{\theta}$ and substituting in the dynamics equation:

$$M(\cdot)\dot{r} = \mathbf{Y}(\cdot)\tilde{\theta} - C(\cdot)r - kr, \quad \text{where } \tilde{\theta} = \hat{\theta} - \theta$$

Defining the Lyapunov function as:

$$V = \frac{1}{2}r^T M(\cdot)r + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1}\tilde{\theta}.$$

The dynamics of the estimated parameters, defined by $\dot{\hat{\theta}} = \Gamma \mathbf{Y}^T(\cdot)r$, are selected to ensure that the time derivative of the Lyapunov function, $\dot{V}$, is negative semi-definite. This facilitates the proof of stability using Lyapunov's direct method.

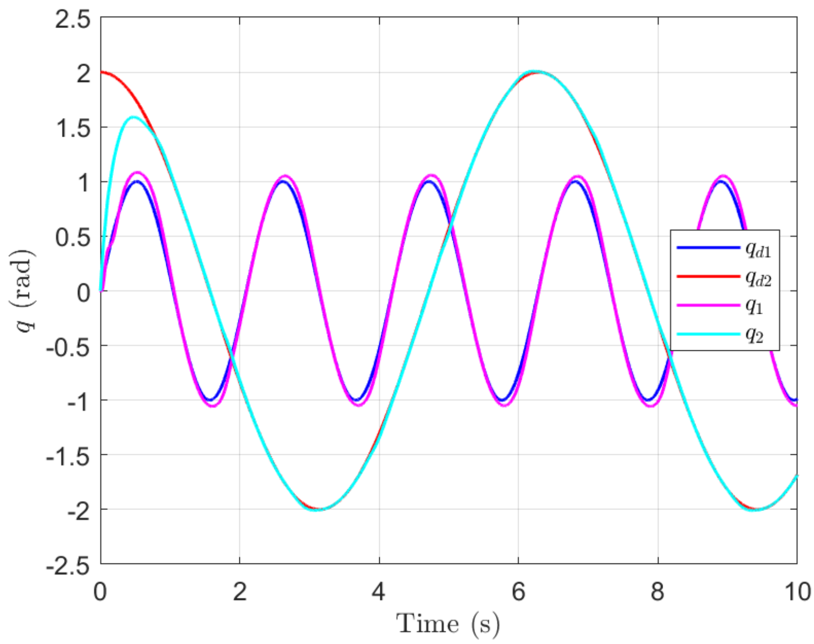

Figure 1.1: Tracking performance of the 2-link manipulator under the proposed controller.

As shown in Figure 1.6, the controller enables the 2-link robotic manipulator to effectively track the desired joint trajectories. The desired signals q_{d1} and q_{d2} are represented in blue and red, respectively, while the actual joint responses q_1 and q_2 are shown in magenta and cyan. The close match between actual and desired trajectories confirms the controller's accuracy and effectiveness in ensuring smooth, precise tracking.

1.2 Task 2

Under the same model, initial conditions and control gains as task 1, an adaptive controller needs to be designed in task 2. The controller uses a composite adaptive update law, which contains a time-varying adaptive gain matrix based on least squares and uses both tracking error and prediction error.

According to the following two formulas:

$$f = \beta e^{-\beta t}$$

$$\tau_f = f * \tau = f * \dot{h} + f * g = \dot{f} * h + f(0)h - fh(0) + f * g$$

We can deduce the expression of the filter torque term $\tilde{\tau}_f$ as:

$$\tilde{\tau}_f = \tau_f - \hat{\tau}_f = \tau_f - \mathbf{Y}_f \hat{\theta}$$

$$\text{with } \tau_f = f * \tau \quad \Rightarrow \quad \dot{\tau}_f = -\beta \tau_f + \beta \tau$$

The filtered torque τ_f can also be expressed as:

$$Y_f \theta = \left(\dot{f} * Y_a + Y_b + f * Y_c\right) \theta$$

Compared to Equation:

$$\tau_f = \dot{f} * h + f(0)h - fh(0) + f * g$$

We can match each term and determine the equivalent matrix relations:

- $Y_a\theta$ corresponds to the inertial term $h(t)$, so $\dot{f} * Y_a\theta$ corresponds to $\dot{f} * h$;
- $Y_b\theta$ corresponds to the correction term from initial conditions, i.e., $Y_b\theta \approx f(0)h - fh(0)$;
- $Y_c\theta$ corresponds to the nonlinear and velocity-dependent term $g(t)$, so $f * Y_c\theta$ corresponds to $f * g$.

The controller is defined as:

$$\tau = kr + Y(\cdot)\,\hat{\theta}$$

The adaptation law is defined as:

$$\dot{\hat{\theta}} = \Gamma_{LS}(t) Y^T(\cdot) r + \Gamma_{LS}(t) Y_f^T(\cdot) \tilde{\tau}_f$$

$$\dot{\Gamma}_{LS}(t) = -\Gamma_{LS}(t) Y_f^T Y_f \Gamma_{LS}(t)$$

We calculate Yf as follows

$$Y_f = \left(\dot{f} * Y_a + Y_b + f * Y_c\right)$$

with

$$\dot{Y}_{a,f} = -\beta Y_{a,f} - \beta^2 Y_a$$

$$\dot{Y}_{c,f} = -\beta Y_{c,f} + \beta Y_c$$

We build the Simulink model based on the above mathematical relationship and select the parameter β=1. After running, we get the following image of the actual parameter estimate and the true parameter value:

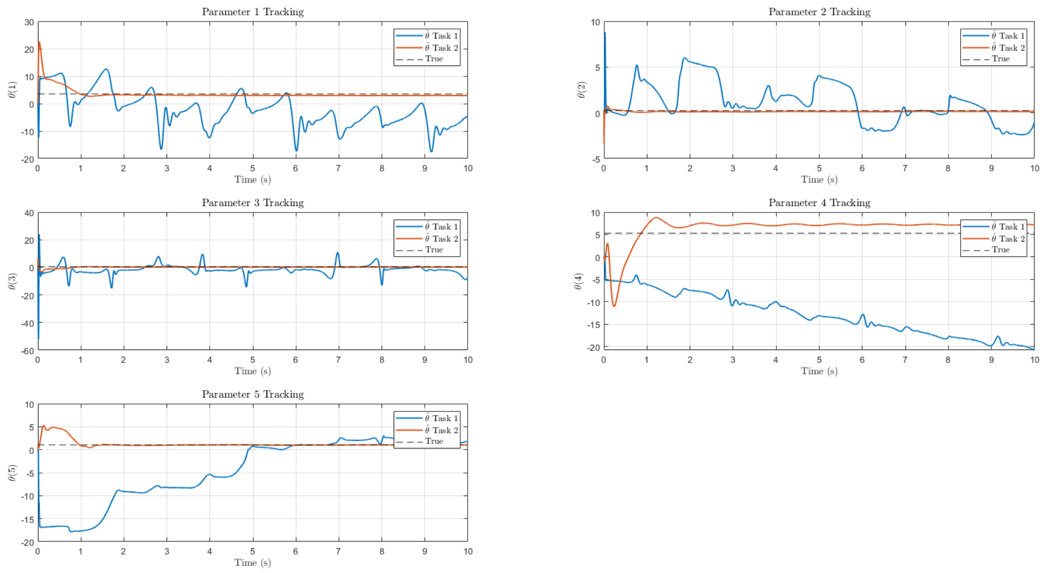

Figure 1.2: Actual parameter estimate and the true parameter value when β=1 alpha=1, k=diag[30;30], Gamma=diag([50,10,100,30,30]).

As can be seen from the figure, under this gain setting, the actual parameter estimate for task 1 is quite different from the true parameter value, which means there may be still a lot of room for optimization through parameter adjustment.. But for task 2, the difference is little and the estimation is relative accuracy.

The plot of the tracking error e(t) is shown below:

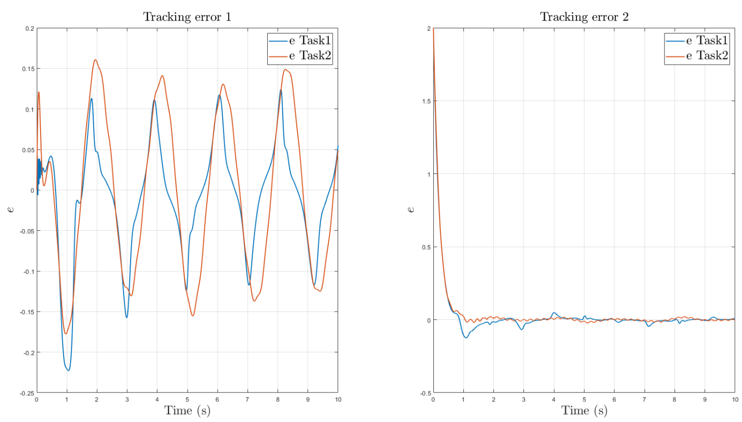

Figure 1.3: Tracking errors e

From the figure, we can see that the tracking error e1 of q1 keeps fluctuating, while the tracking error e2 of q2 quickly stabilizes.

The plot of the control input τ is shown below:

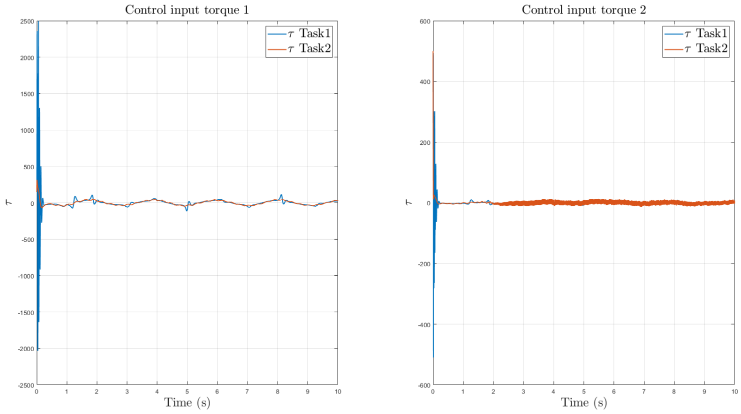

Figure 1.4: Control input τ

As can be seen from the figure, the control inputs of the two tasks (Task 1 and Task 2) eventually stabilize and show a high degree of consistency.

1.3 Task 3

In task 3, we change the gains on the composite adaptive controller to see if you can achieve improved performance. We choose alpha=5, k=diag[50;50], initial of Gamma=diag([50,10,50,30,10]), and plot the actual parameter estimate and the true parameter value again.

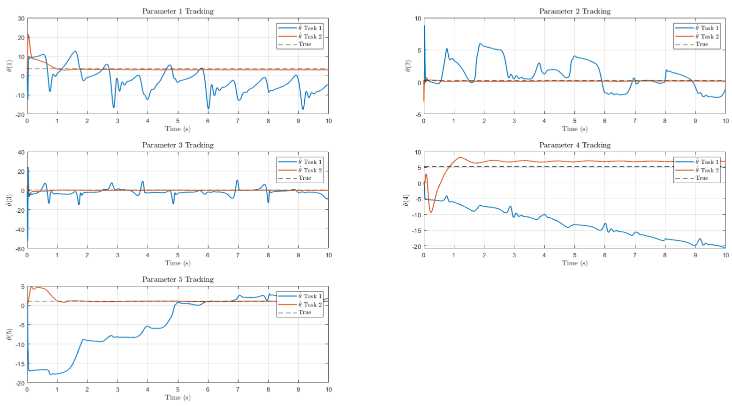

Figure 1.5: Actual parameter estimate and the true parameter value when β=1 alpha=5, k=diag[50;50], Gamma=diag[10,10,10,1,1].

From the image we can see after the gain adjustment the oscillation of the task2 parameters has slightly decreased, but the overall accuracy has not changed much.

1.4 Task 4

In task 4, we add the following probing signal to both controllers:

$$prob_1 = e^{-0.05t}\tanh(10t)\left(-10\sin(7\pi t) + 5\sin(e^t)\right)$$
$$prob_2 = e^{-0.05t}\tanh(10t)\left(-10\sin(9\pi t) + 10\sin(e^{2t})\right)$$

The controller is now defined as:

$$\tau = kr + \mathbf{Y}(\cdot)\,\hat{\theta} + \begin{bmatrix} prob_1 \\ prob_2 \end{bmatrix}.$$

The plot of the actual parameter estimate and the true parameter value is shown below:

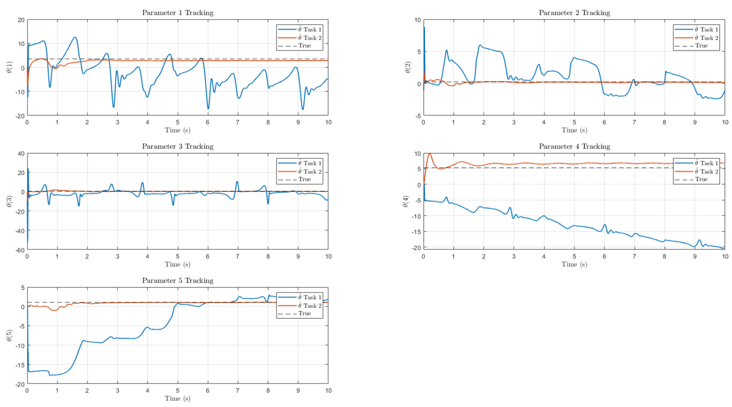

Figure 1.6: Actual parameter estimate and the true parameter value when β=1 alpha=5, k=diag[50;50], Gamma=diag[10,10,10,1,1], with probing signal

From the figure, we can see that after we add the probing signal, the parameter convergence speed of the composite adaptive controller becomes faster and the overshoot is reduced.