Experiment Procedure
Robot Modelling, Identification, and Control

ACM - Adaptive Control of a Planar Manipulator

June 5, 2025

⚠ **IMPORTANT:** It is essential that you carry out the following steps before starting the experiment!

1.) Select "`Fixed-Step`" as the solver for your Simulink model with a variable Sample Time $T_s = 0.001$. You will select this later depending on the task. You can set this under "`Model Configuration Parameters`" in the upper bar.

2.) Avoid hardcoded values, i.e. only use variables within Simulink and define them outside in a central script which is called by the simulation via callback[1].

3.) Deactivate the check mark at "`Limit data points to...`" in Scopes in order not to lose any data points during longer simulation times.

4.) If you need to compare two systems, the easiest way is to copy the original system and make the changes to the copy. So you always have both versions available.

5.) For "`To Workspace`" blocks, select "`Array`" as the storage format, since they are the easiest to handle.

6.) If the function of a command is not clear, use MATLAB Help.

7.) Use the "`clear`" command in your main script to clean up your workspace before performing a task and avoid errors due to old data.

⚠ **IMPORTANT:** In Moodle you are provided with a MATLAB script and a Simulink model.
Items marked with a ★ must be included in your experiment report.

1 Experimental procedure

Consider a two DoF robot whose dynamics are given by

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix} p_1+2p_3c(q_2) & p_2+p_3c(q_2)\\ p_2+p_3c(q_2) & p_2\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix}-p_3s(q_2)\dot{q}_2 & -p_3s(q_2)(\dot{q}_1+\dot{q}_2)\\ p_3s(q_2)\dot{q}_1 & 0\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + \begin{bmatrix} f_{d1} & 0\\ 0 & f_{d2}\end{bmatrix}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}, \quad (1)$$

where $c(q_i) = \cos(q_i)$ and $s(q_i) = \sin(q_i)$. The parameter values are as follows $p_1 = 3.473\ kgm^2$, $p_2 = 0.196\ kgm^2$, $p_3 = 0.242\ kgm^2$, $f_{d1} = 5.3\ Nms$ and $f_{d2} = 1.1\ Nms$.
It is desired that its joints follow this trajectory

$$\boldsymbol{q}_d = \begin{bmatrix}\sin(3t)\\ 2\cos(t)\end{bmatrix} \quad (2)$$

T1 (**10 P**) Given the model and the given desired trajectory for a simple point mass two link robot provided in Moodle, implement a traditional adaptive controller with a gradient-based (i.e., constant diagonal adaptation gain matrix $\boldsymbol{\Gamma}$) tracking-error-driven adaptive update law and a proportional element with gain k.
⚠ **IMPORTANT:** do not use numerical differentiation blocks.
⚠ **IMPORTANT:** here it is assumed that **only** the joint angles and velocities are measurable.

- Define the tracking error as $\boldsymbol{e}(t) = \boldsymbol{q}_d(t) - \boldsymbol{q}(t)$
- Define the filtered tracking error as $\boldsymbol{r}(t) = \dot{\boldsymbol{e}}(t) + \alpha \boldsymbol{e}(t)$
- Make a change of variable and express the pendulum dynamics equations in terms of $\boldsymbol{r}(t)$, $\boldsymbol{e}(t)$, $\dot{\boldsymbol{e}}(t)$ and the desired trajectory and its derivatives.
  Hint: The desired joint trajectory $\boldsymbol{q}_d$ is known, what about its first and second derivatives?
  ⚠ **IMPORTANT:** only substitute terms that are **outside** the matrices; i.e. do not change $\boldsymbol{M}(\boldsymbol{q})$.
- Group the terms depending on the error, the desired trajectory and the filtered tracking error and find the regressor matrix $\boldsymbol{Y}(\boldsymbol{e}, \dot{\boldsymbol{e}}, \boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d, \boldsymbol{r})$
  ⚠ **IMPORTANT:** you will come across the term $\boldsymbol{C}(\cdot)\boldsymbol{r}$, **DO NOT** include it in the regressor.
  Hint: At this point your equation should look like this $\boldsymbol{M}(\cdot)\dot{\boldsymbol{r}} = \boldsymbol{Y}(\cdot)\boldsymbol{\theta} - \boldsymbol{C}(\cdot)\boldsymbol{r} - \boldsymbol{\tau}$
- Define the controller as $\boldsymbol{\tau} = k\boldsymbol{r} + \boldsymbol{Y}(\cdot)\hat{\boldsymbol{\theta}}$ and substitute it in the dynamics equation

  ★ **What would be a reasonable choice of Lyapunov function for this system? Write it in your report.**

  Hint: look at equation (2.12) in the script and use $\boldsymbol{M}(\boldsymbol{q})$ as your choice for $\boldsymbol{P}$
- Based upon your Lyapunov function define the dynamics for the estimated parameters $\hat{\boldsymbol{\theta}}$, i.e. $\dot{\hat{\boldsymbol{\theta}}} = ?$
  Hint: It should look very similar to the eq. (2.4) in the script.
- Create a Simulink model to test the tracking performance of your controller on the desired trajectory

T2 (**10 P**) For the **same model**, **same initial conditions**, and **same control gains** (i.e. k and $\boldsymbol{\Gamma}$) implement an adaptive controller that uses a composite adaptive update law that contains a least-squares based time varying adaptation gain matrix and uses both a tracking error and a prediction error.

- Use the dynamics expressed as function of the filtered tracking error $\boldsymbol{r}(t)$ and the tracking error $\boldsymbol{e}(t)$ (and their corresponding derivatives)
- Get the corresponding expression for the filtered torque, use eqs. (2.22) and (2.23) from the script.
  Hint: Get the *filtered* regressor matrix form as follows:

$$\boldsymbol{Y}_f\boldsymbol{\theta} = \left(\dot{f} * \boldsymbol{Y}_a + \boldsymbol{Y}_b + f * \boldsymbol{Y}_c\right)\boldsymbol{\theta}$$

Compare the terms in parenthesis to eq. (2.23) to see what the corresponding matrices are equivalent to. Use the example in sec. 2.5.2 as guide.

Hint: you should arrive to the following (possibly further simplified) matrices

$$\boldsymbol{Y}_a = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \cos(q_2)\left(2\dot{q}_1+\dot{q}_2\right) & 0 & 0 \\ 0 & \left(\dot{q}_1+\dot{q}_2\right) & \cos(q_2)\dot{q}_1 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{Y}_b = \begin{bmatrix} \beta\dot{q}_1 - f\dot{q}_1(0) & \beta\dot{q}_2 - f\dot{q}_2(0) & \beta\cos(q_2)\left(2\dot{q}_1+\dot{q}_2\right) - f\cos(q_2(0))\left(2\dot{q}_1(0)+\dot{q}_2(0)\right) & 0 & 0 \\ 0 & \beta\left(\dot{q}_1+\dot{q}_2\right) - f\left(\dot{q}_1(0)+\dot{q}_2(0)\right) & \beta\cos(q_2)\dot{q}_1 - f\cos(q_2(0))\dot{q}_1(0) & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{Y}_c = \begin{bmatrix} 0 & 0 & \overbrace{\left(2\sin(q_2)\dot{q}_1\dot{q}_2 + \sin(q_2)\dot{q}_2\dot{q}_2 - \sin(q_2)\dot{q}_1\dot{q}_2 - \sin(q_2)(\dot{q}_1+\dot{q}_2)\dot{q}_2\right)}^{=0} & \dot{q}_1 & 0 \\ 0 & 0 & \sin(q_2)\left(\dot{q}_1\dot{q}_2+\dot{q}_1^2\right) & 0 & \dot{q}_2 \end{bmatrix}$$

- Define the controller as $\boldsymbol{\tau} = k\boldsymbol{r} + \boldsymbol{Y}(\cdot)\hat{\boldsymbol{\theta}}$
- Define the adaption law using

$$\dot{\hat{\boldsymbol{\theta}}} = \boldsymbol{\Gamma}_{LS}(t)\boldsymbol{Y}^T(\cdot)\boldsymbol{r} + \boldsymbol{\Gamma}_{LS}(t)\boldsymbol{Y}_f(\cdot)^T\tilde{\boldsymbol{\tau}}_f \tag{3a}$$

$$\dot{\boldsymbol{\Gamma}}_{LS}(t) = -\boldsymbol{\Gamma}_{LS}(t)\boldsymbol{Y}_f^T\boldsymbol{Y}_f\boldsymbol{\Gamma}_{LS}(t) \tag{3b}$$

Hint: Compute the filtered torque error $\tilde{\boldsymbol{\tau}}_f$ as described in sec. 2.5.5

★ **For both controllers in T1 and T2 plot the actual versus estimated adaptive parameters, the tracking errors, and the control input. Provide the control gains you have used.**

T3 (**3 P**) Change the gains on the composite adaptive controller to see if you can achieve improved performance..

★ **Show in your report the comparison plots as before for the new gains. Provide the new/tuned gain values.**

T4 (**3 P**) Add the following probing signal to both controllers

$$prob_1 = e^{-0.05t}\tanh(10t)\left(-10\sin(7\pi t) + 5\sin(e^t)\right) \tag{4}$$
$$prob_2 = e^{-0.05t}\tanh(10t)\left(-10\sin(9\pi t) + 10\sin(e^{2t})\right) \tag{5}$$

Hint: it should look like $\boldsymbol{\tau} = k\boldsymbol{r} + \boldsymbol{Y}(\cdot)\hat{\boldsymbol{\theta}} + \begin{bmatrix} prob_1 \\ prob_2 \end{bmatrix}$.

★ **Provide in your report plots of the actual and estimated parameters. Briefly discuss what was the impact of adding the probing signals.**