TUM SCHOOL OF COMPUTATION, INFORMATION AND TECHNOLOGY (CIT)

TECHNICAL UNIVERSITY OF MUNICH

Report Submitted to Praktikum Robot Modelling and Identification (IN2106)

Experiment 3: Robot Inertial Parameter Identification

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Experiment 3: Robot Inertial Parameter Identification

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We confirm that this report is my our own work and we have documented all sources and material used.

Munich, May 21, 2025
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1 IPI - Robot Inertial Parameter Identification

1.1 Task 1

In this task, the goal is to get a numerical comparison between the robot dynamics from the first lab experiment, the LIP form, and the minimal parameter form.
The standard inverse dynamics model torque:

$$\tau_{\mathrm{id}} = M(\mathbf{q})\,\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\,\dot{\mathbf{q}} + G(\mathbf{q}) \tag{1.1}$$

To more clearly illustrate the dependence on the inertial parameters, the LIP (Linear-in-the-Parameter) regression form is to be used:

$$\tau_{\mathrm{reg}} = C(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \cdot \mathbf{X} \tag{1.2}$$

After the process of elimination and regrouping, we have the minimum parameter form:

$$\tau_{\mathrm{mp}} = C_b(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \cdot \boldsymbol{\beta}_b \tag{1.3}$$

Our results for the reduced regressor matrix (for formatting purposes, the matrix is presented in its transposed form) and the minimal parameter vector are

$$\mathbf{C}_b^\top = \begin{bmatrix} 2\ddot{q}_1 + \ddot{q}_2 & \ddot{q}_1 + \ddot{q}_2 \\ g\cos(q_1) & 0 \\ -g\sin(q_1) & 0 \\ \sigma_1 + \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) - \dot{q}_2\sin(q_2)(2\dot{q}_1 + \dot{q}_2) & \sin(q_2)\dot{q}_1^2 + \sigma_1 + \ddot{q}_1\cos(q_2) \\ -\sigma_2 - \sin(q_2)(2\ddot{q}_1 + \ddot{q}_2) - \dot{q}_2\cos(q_2)(2\dot{q}_1 + \dot{q}_2) & \cos(q_2)\dot{q}_1^2 - \sigma_2 - \ddot{q}_1\sin(q_2) \\ \ddot{q}_1 + g\cos(q_1) & 0 \end{bmatrix} \tag{1.4}$$

$$\sigma_1 = g\cos(q_1 + q_2), \quad \sigma_2 = g\sin(q_1 + q_2) \tag{1.5}$$

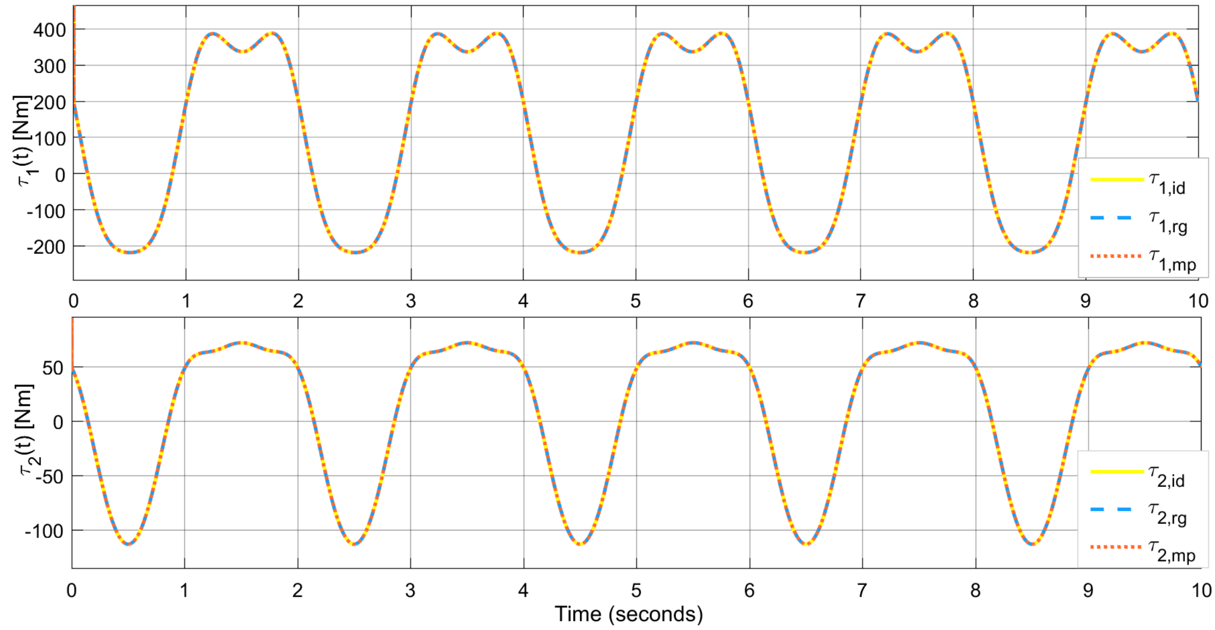

Figure 1.1: The plot compares $\tau_{id} = M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q)$, $\tau_{reg} = C(q,\dot{q},\ddot{q})X$ and $\tau_{mp} = C_b(q,\dot{q},\ddot{q})\beta_b$ for joint 1 (upper graph) and 2 (lower graph).

$$\beta_b = \begin{pmatrix} \mathrm{ZZ}_2 \\ \mathrm{mX}_2 \\ \mathrm{mY}_2 \\ \mathrm{mX}_1 \\ \mathrm{mY}_1 \\ m_1 \end{pmatrix}. \tag{1.6}$$

In Fig. 1.1, the comparison of the differently computed joint torques is shown. From the figure, it can be observed that the three curves almost completely overlap in both subplots, indicating that all three methods yield equivalent joint torque results. The orange curve τ_{mp} is nearly indistinguishable from the others, demonstrating that using fewer parameters ($\boldsymbol{\beta}_b$) does not compromise the accuracy of the dynamic computation.

1.2 Task 2

In this task, the goal is to generate optimal joint trajectories for a 2 DoF planar manipulator, which will be used for identifying the robot's parameters. The quality of the generated trajectories is determined by how well they excite the parameters β_b (1.6), which is critical for achieving accurate identification results.

In this approach, the goal is to improve the robustness of parameter identification by minimizing the condition number $\kappa(F)$ of the information matrix F. The matrix F depends on the joint trajectories and their derivatives. Therefore, by parameterizing the trajectories (using a combination of Fourier series and polynomials) and formulating the condition number as a

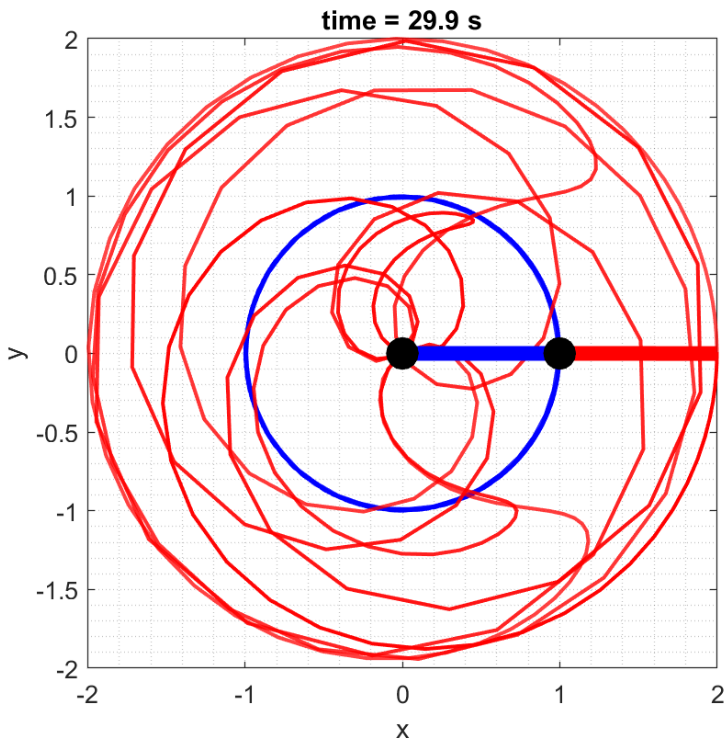

Figure 1.2: Optimized Joint Trajectories: Planar Manipulator Motion in Cartesian Space

function of these parameters, the optimization becomes a trajectory optimization problem.

Figure 1.2 illustrates the optimized joint trajectories across the entire workspace for 30 s. Testing a wide range of joint configurations, including stretched positions, ensures the robustness of the solution.

Figure 1.3 shows the joint positions $q_1(t), q_2(t)$, velocities $\dot{q}_1(t), \dot{q}_2(t)$, and accelerations $\ddot{q}_1(t), \ddot{q}_2(t)$ over a duration of 30 seconds. The plots confirm that the generated trajectories remain within the defined boundary limits for both joints. The smooth acceleration profiles demonstrate that the resulting trajectories are continuous and physically feasible.

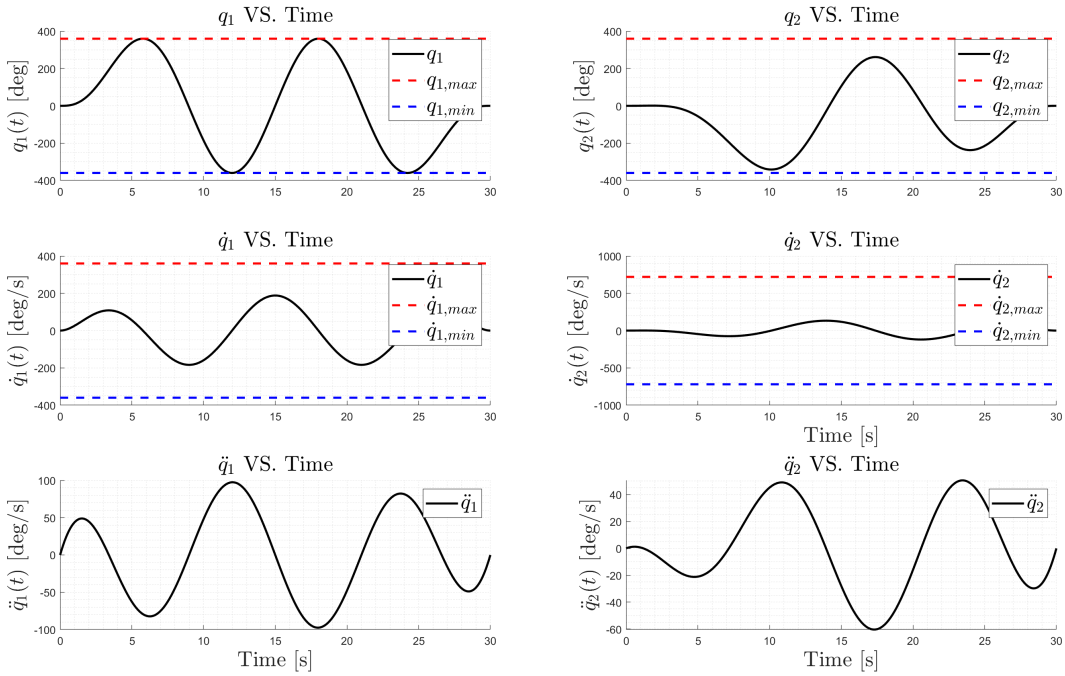

Figure 1.3: Optimized Joint Trajectories: Position, Velocity, and Acceleration of q_1 and q_2

1.3 Task 3

In this task, a virtual identification experiment is to be executed. With the collected measurement data(joint positions, velocities, and accelerations as well as joint torques), the minimal parameters of the planar manipulator will be estimated. Since the measured values are affected by noise, we perform mean filtering on the data to obtain a more accurate minimum parameter vector in subsequent estimation.

We use the optimal trajectory obtained in Experiment 2 to calculate the information matrix F and perform parameter identification to get the minimum parameter vector β_b^* by the following formula:

$$\boldsymbol{\beta}_b^* = (F^\top F)^{-1} F^\top \mathbf{b}, \quad \text{where} \quad F \in \mathbb{R}^{n \cdot m \times p}, \quad \mathbf{b} \in \mathbb{R}^{n \cdot m \times 1}, \quad \boldsymbol{\beta}_b^* \in \mathbb{R}^{p \times 1} \tag{1.7}$$

The minimum vector matrix is

$$\beta_b^* = \begin{pmatrix} 3.2333 \\ 4.8442 \\ 0.0224 \\ 4.9885 \\ 0.0085 \\ 10.1563 \\ 0.3662 \\ -2.2049 \\ -0.1182 \\ -1.8982 \end{pmatrix}. \tag{1.8}$$

Based on this minimum parameter vector, the torques of the two joints can be estimated and compared with the measured torques:

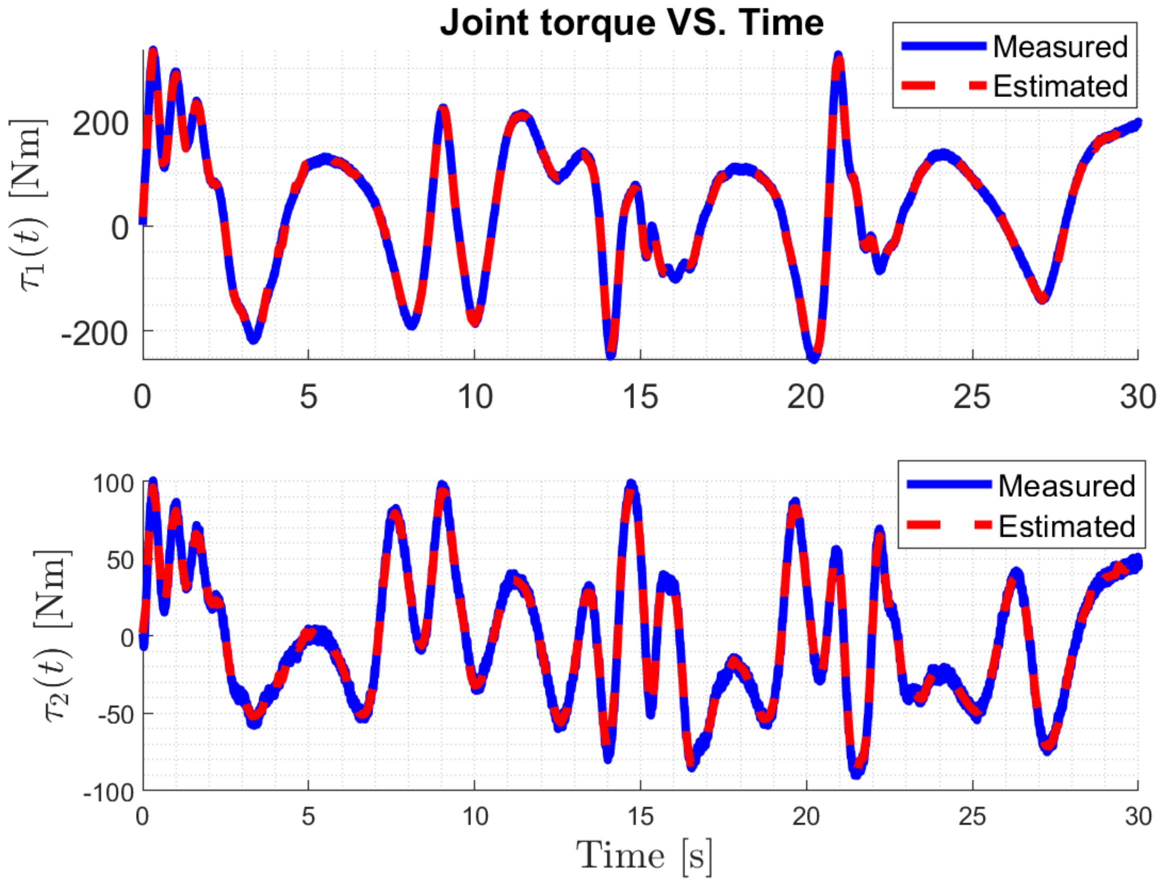

Figure 1.4: Joint torque estimated by identification model with optimal trajectory and measured torque

To evaluate the generalization capability of the identified parameters, a new test trajectory provided by the system is used. The simulation is performed using the newly identified minimal parameter vector $\boldsymbol{\beta}_b^*$, and a comparison is made between the model-based torque and the measured torque under the new trajectory.

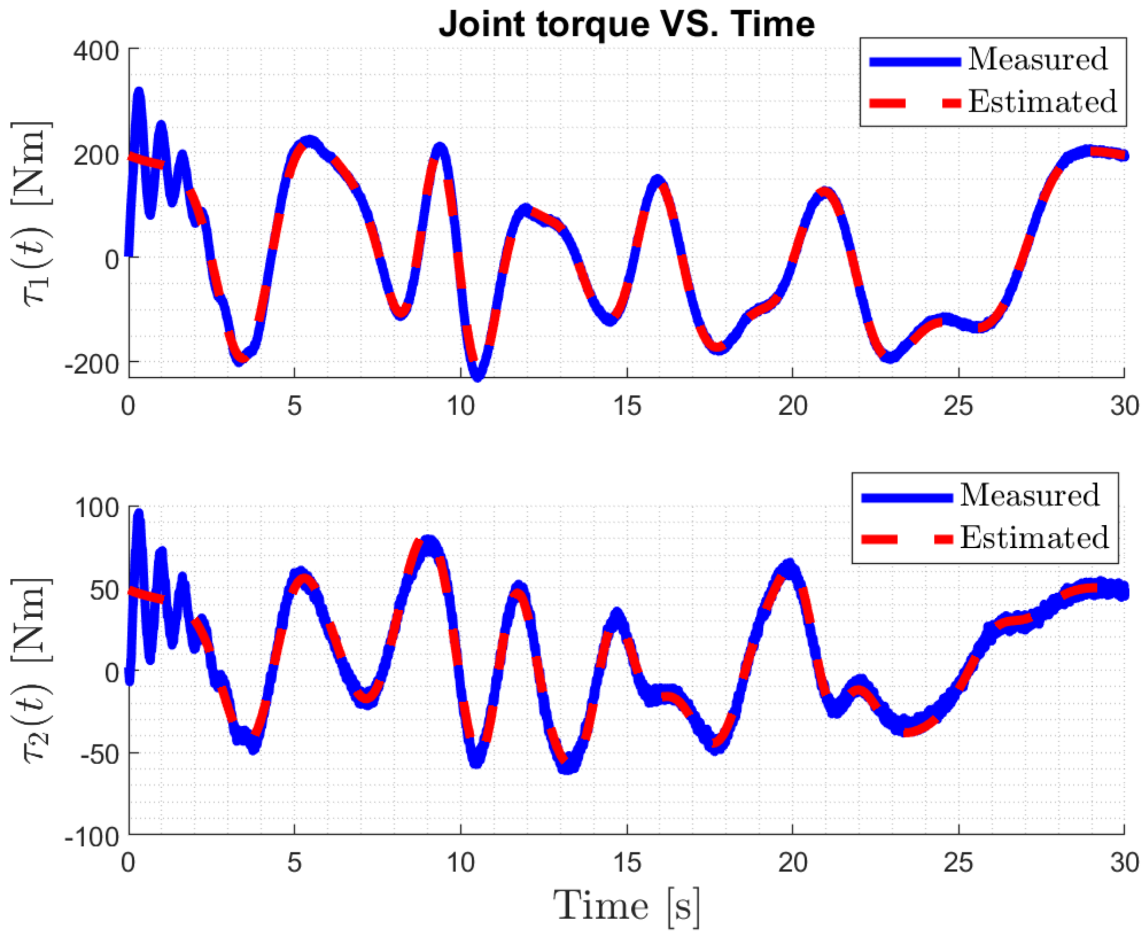

Figure 1.5: Joint torque estimated by identification model with the second trajectory and measured torque

It can be concluded from Figures 1.4 and 1.5 that the identified minimal parameter vector $\boldsymbol{\beta}_b^*$ demonstrates excellent performance in both the training (optimal) and test trajectories. In the optimal excitation trajectory, the estimated torques match the measured values almost perfectly, indicating high model accuracy. Under the provided test trajectory, the model still provides a close approximation, showing good generalization capability of the identified parameters. Minor deviations are observed in high dynamic segments, which can be attributed to noise or unmodeled dynamics.