TUM SCHOOL OF COMPUTATION, INFORMATION AND TECHNOLOGY (CIT)

TECHNICAL UNIVERSITY OF MUNICH

Report Submitted to Praktikum Robot Modelling and Identification (IN2106)

Experiment 1: Serial Robot Modelling

Stefan Ahner, Jesus Arturo Sol Navarro, Sarah Weber

TECHNICAL UNIVERSITY OF MUNICH

Report Submitted to Praktikum Robot Modelling and Identification (IN2106)

Experiment 1: Serial Robot Modelling

Author:	Stefan Ahner, Jesus Arturo Sol Navarro, Sarah Weber
Lecturers:	Prof. Dr. M.Sc. Alexander König, M.Sc. Moritz Eckhoff, Moein Forouhar
Submission Date:	May 8, 2025

We confirm that this report is my our own work and we have documented all sources and material used.

Munich, May 8, 2025
Stefan Ahner, Jesus Arturo Sol Navarro, Sarah Weber

Contents

1 SRM - Serial Robot Modelling

Consider the two-link planar manipulator shown in Fig. 1.1. Here the gravity vector points in the negative y direction.

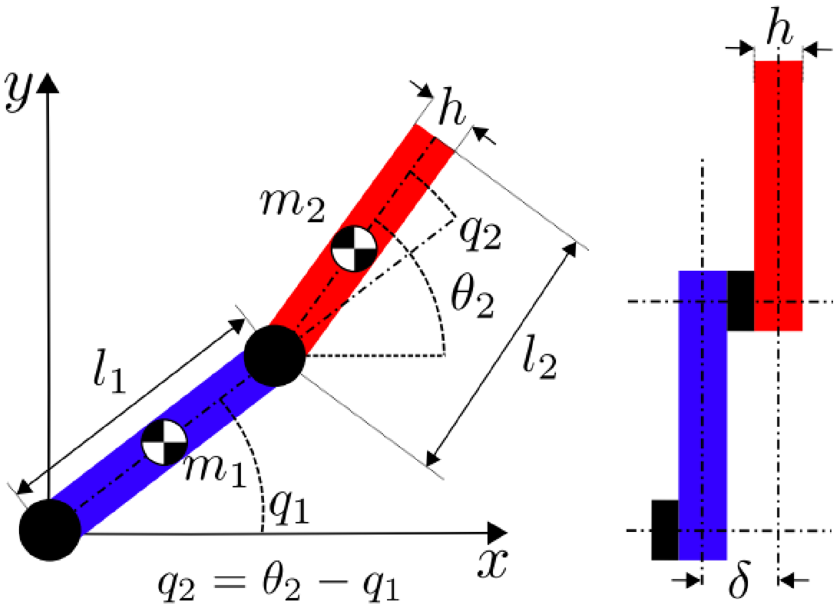

Figure 1.1

In the following experiment, the parameters shown in Fig. 1.1 are assigned the following values:

- $l_1 = l_2 = 1\,\mathrm{m}$: Lengths of the links
- $m_1 = m_2 = 1\,\mathrm{kg}$: Masses of the links
- $h = 0.1\,\mathrm{m}$: Link thickness
- $\delta = 0.05\,\mathrm{m}$
- $g = 9.81\,\mathrm{m/s^2}$

1.1 Task 1

Get the MDH parameters of the two-link arm in Fig. 1.1.

i	Link 1	Link 2
$a_i[m]$	0	l_1
$d_i[m]$	0	0
$\alpha_i[rad]$	0	0
$\theta_i[rad]$	q_1	q_2

Table 1.1: MDH-Parameters for the planar manipulator in Fig. 1.1

1.2 Task 2

Find the homogeneous transformation matrices $^0T_1$ and $^1T_2$ and the Jacobians from the center of mass of each link to the base frame, i.e., $^0J(q)_{1,\mathrm{CoM}_1}$ and $^0J(q)_{2,\mathrm{CoM}_2}$.

$$^0T_1 = T_{Rot,x}(\alpha_1)T_{Trans,x}(a_1)T_{Rot,z}(\theta_1)T_{Trans,z}(d_1)$$
$$= \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}cos(q_1)&-sin(q_1)&0&0\\sin(q_1)&cos(q_1)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$
$$= \begin{bmatrix}cos(q_1)&-sin(q_1)&0&0\\sin(q_1)&cos(q_1)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$^1T_2 = T_{Rot,x}(\alpha_2)T_{Trans,x}(a_2)T_{Rot,z}(\theta_2)T_{Trans,z}(d_2)$$
$$= \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&l1\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}cos(q_2)&-sin(q_2)&0&0\\sin(q_2)&cos(q_2)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$
$$= \begin{bmatrix}cos(q_2)&-sin(q_2)&0&l1\\sin(q_2)&cos(q_2)&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$^0J(q)_{1,\mathrm{CoM}_1} = \begin{bmatrix}-lc_1\sin(q_1)&0\\lc_1\cos(q_1)&0\\0&0\\0&0\\0&0\\1&0\end{bmatrix}$$

$$
{}^{0}J(q)_{2,\mathrm{CoM}_2} = \begin{bmatrix} -\mathrm{lc}_2\,\sin(q_1+q_2) - l_1\,\sin(q_1) & -\mathrm{lc}_2\,\sin(q_1+q_2) \\ \mathrm{lc}_2\,\cos(q_1+q_2) + l_1\,\cos(q_1) & \mathrm{lc}_2\,\cos(q_1+q_2) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}
$$

1.3 Task 3

Get the required terms to describe the forward dynamics of the two-link arm using the Lagrangian method and the MATLAB Symbolic Toolbox (use the provided MATLAB Live Script). Verify that the properties in Section 2.4.5 from the script are satisfied.

$$
M(q) =
$$

$$
\begin{bmatrix} m_2\,{l_1}^2 + 2\,m_2\cos(q_2)\,l_1\,\mathrm{lc}_2 + m_1\,{\mathrm{lc}_1}^2 + m_2\,{\mathrm{lc}_2}^2 + \mathrm{I1zz} + \mathrm{I2zz} & m_2\,{\mathrm{lc}_2}^2 + l_1\,m_2\cos(q_2)\,\mathrm{lc}_2 + \mathrm{I2zz} \\ m_2\,{\mathrm{lc}_2}^2 + l_1\,m_2\cos(q_2)\,\mathrm{lc}_2 + \mathrm{I2zz} & m_2\,{\mathrm{lc}_2}^2 + \mathrm{I2zz} \end{bmatrix}
$$

$$
C(q, dq) = \begin{bmatrix} -\dot{q}_2\,l_1\,\mathrm{lc}_2\,m_2\,\sin(q_2) & -l_1\,\mathrm{lc}_2\,m_2\,\sin(q_2)\,(\dot{q}_1+\dot{q}_2) \\ \dot{q}_1\,l_1\,\mathrm{lc}_2\,m_2\,\sin(q_2) & 0 \end{bmatrix}
$$

$$
g(q) = \begin{bmatrix} g\,m_2\,(\mathrm{lc}_2\cos(q_1+q_2) + l_1\cos(q_1)) + g\,\mathrm{lc}_1\,m_1\cos(q_1) \\ g\,\mathrm{lc}_2\,m_2\cos(q_1+q_2) \end{bmatrix}
$$

1.4 Task 4

Use Simulink to simulate the motion of the pendulum using the found dynamics. Consider the initial conditions to be zero, i.e. $q(0) = 0$ and $\dot{q}(0) = 0$.

Figure 1.2 shows the joint angles $q_1(t)$ and $q_2(t)$ over time for the two-link planar manipulator without friction. Since no damping or resistance is present, the system exhibits continuous, high-energy motion.
To simulate the dynamic behavior of the two-link planar manipulator, a Simulink block diagram (Figure 1.3) was developed based on the equations of motion derived from the Lagrangian method.

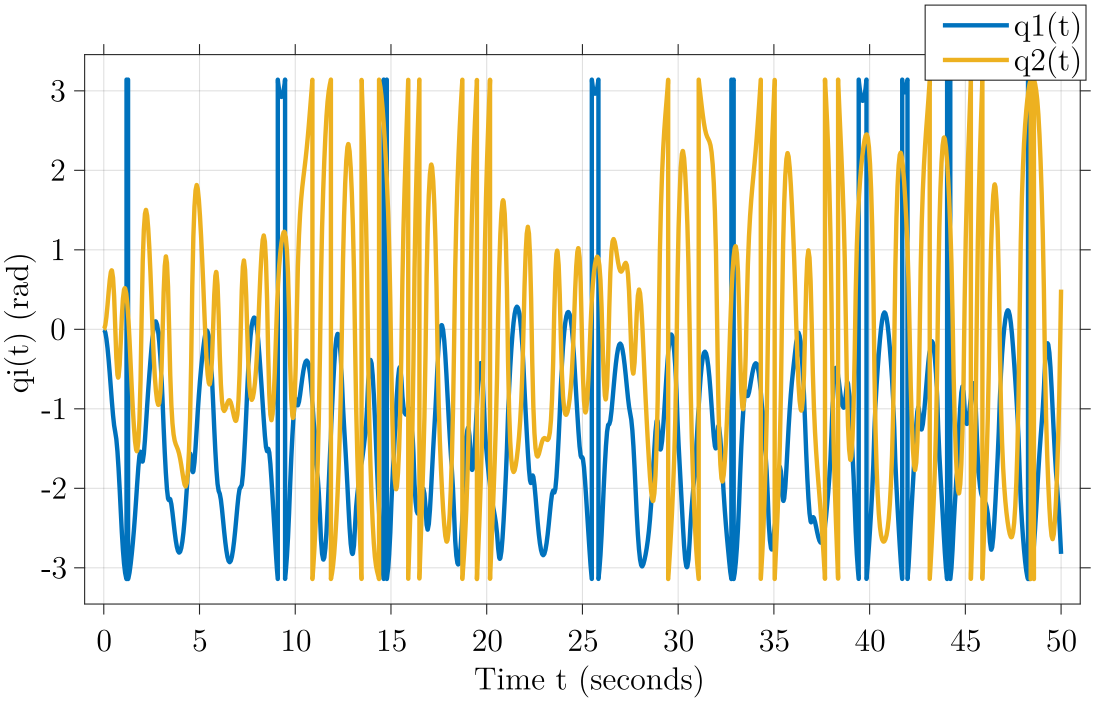

Figure 1.2: Joint angle trajectory over time

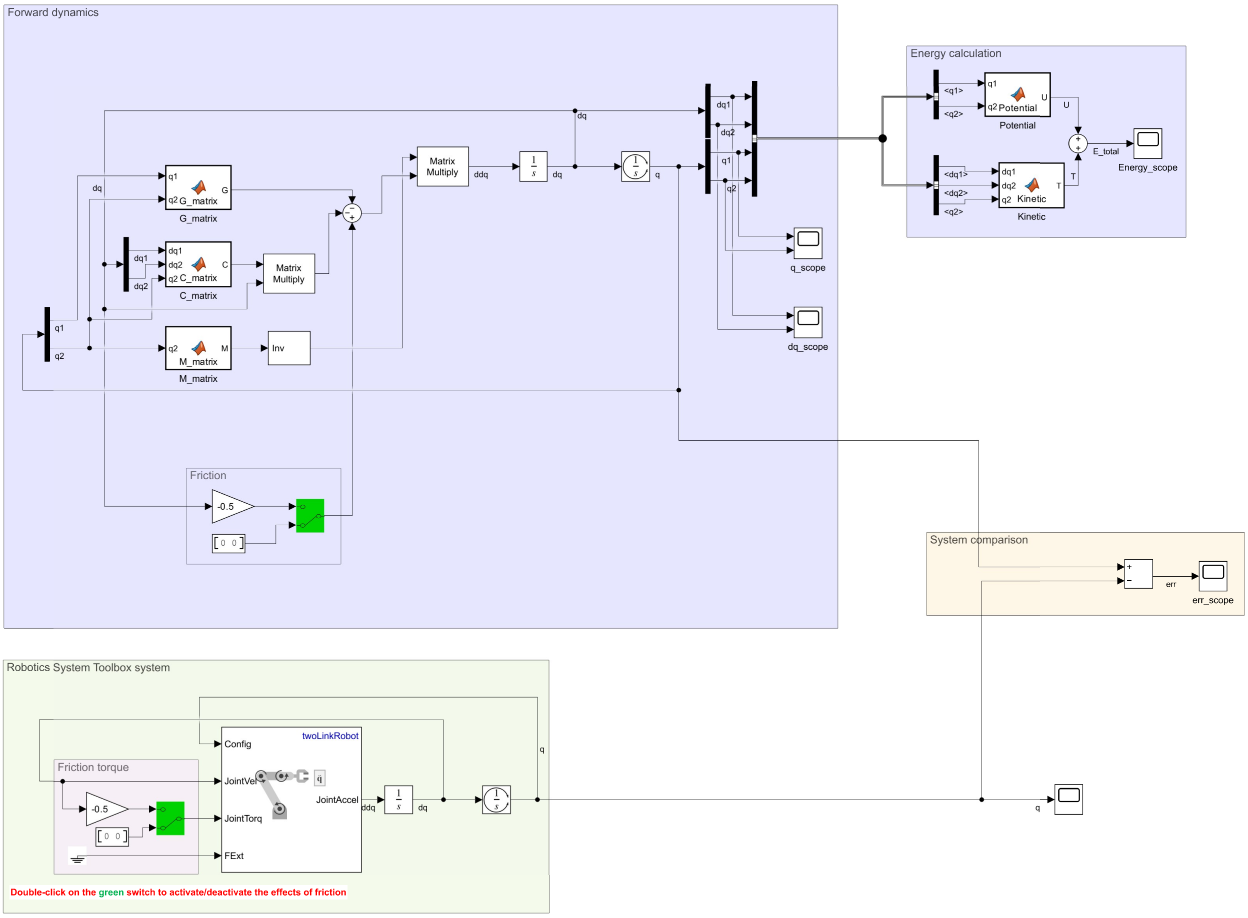

Figure 1.3: Simulink block diagram

1.5 Task 5

Verify that energy is conserved.

Figure 1.4 demonstrates that the total energy of the system remains within a very narrow range, fluctuating, on average, between 0 and -1×10^{-7}. Thus, it can be concluded that the total energy is conserved throughout the simulation. It is assumed that the minor deviation from zero arises from numerical inaccuracies.

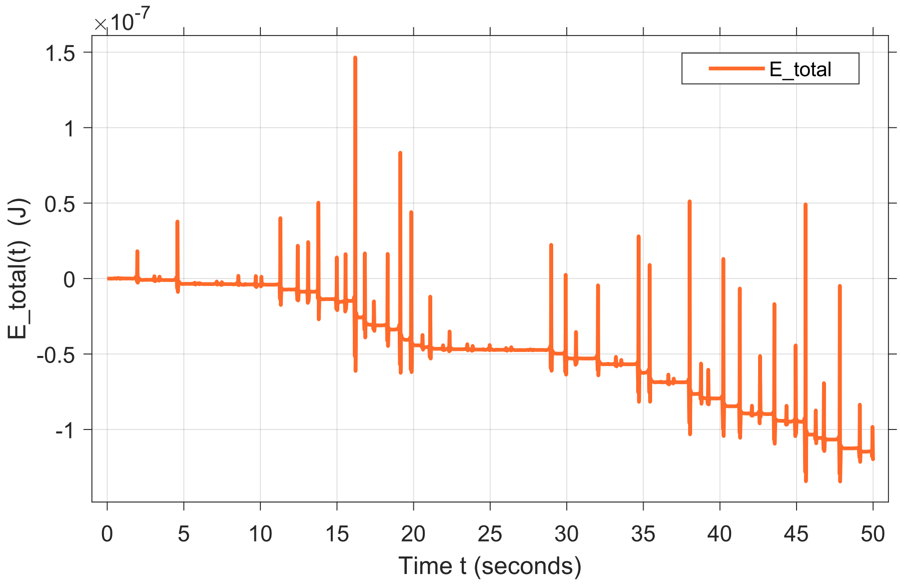

Figure 1.4: Total energy (kinetic + potential energy) of the model over time

1.6 Task 6

Add viscous friction to the joints.

The following viscous friction was used

$$\tau_{f,i} = \nu_i \dot{q}_i,$$

where $\nu_i = 0.5\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}$ is the viscous friction coefficient for joint i.

As a result of the viscous friction, the kinetic energy of the robot arm gradually decreases and eventually reaches zero. Meanwhile, the potential energy stabilizes at approximately $-19.62\,\mathrm{J}$, corresponding to the system's steady state configuration.

$$U_{\mathrm{SteadyState}} = m_1 g(-lc1) + m_2 g(-l_1 - lc2) = -4.905 - 14.715 = -19.62\,\mathrm{J}$$

This behavior is clearly visible in Figure 1.5, where the total mechanical energy converges to the final value of the potential energy, confirming the expected dissipative dynamics. In Fig. 1.6 we show the result of the corresponding joint angle trajectory.

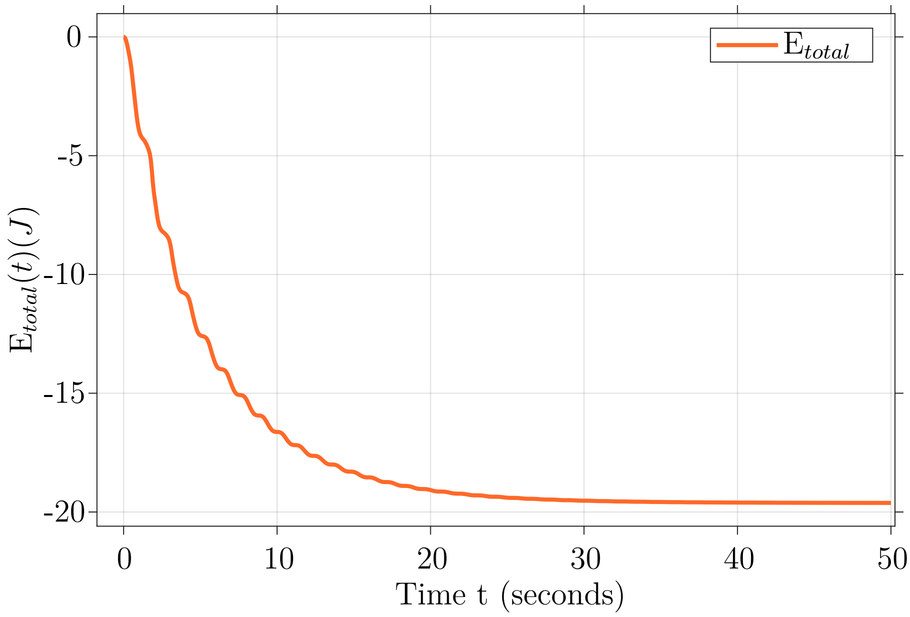

Figure 1.5: Total energy of the model over time with viscous friction

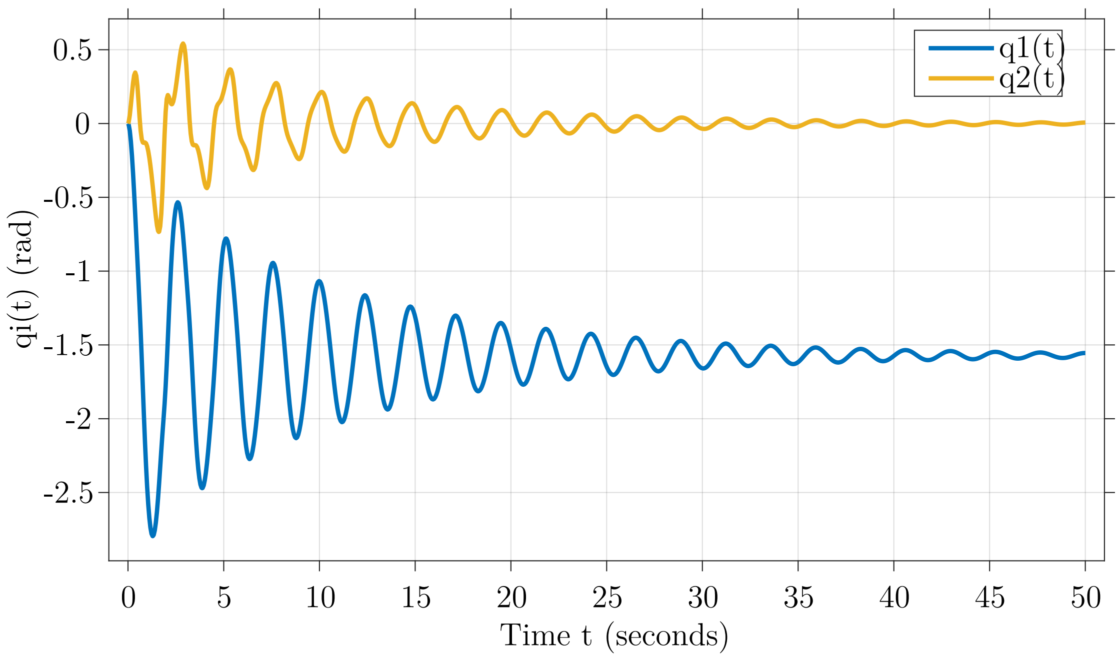

Figure 1.6: Joint angle trajectory over time with viscous friction