TUM SCHOOL OF COMPUTATION, INFORMATION AND TECHNOLOGY (CIT)

TECHNICAL UNIVERSITY OF MUNICH

Report Submitted to Praktikum Robot Modelling and Identification (IN2106)

Experiment 4: Computed Torque Control of a Planar Manipulator

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Experiment 4: Computed Torque Control of a Planar Manipulator

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We confirm that this report is my our own work and we have documented all sources and material used.

Munich, May 29, 2025
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1 CTC - Computed Torque Control of a Planar Manipulator

Consider the 2-link planar manipulator model and the desired trajectory provided. The goal is to design a joint trajectory tracking controller.

Consider the initial condition to be:

$$q_0 = \begin{bmatrix} -\frac{\pi}{2} \\ 0 \end{bmatrix}$$

1.1 Task 1

In this task, a classical Proportional-Integral-Derivative (PID) controller is implemented to regulate the joint positions of a planar manipulator. The goal is to drive the joints to the desired setpoint :

$$q_d = \begin{bmatrix} 0 \\ -\frac{3}{4}\pi \end{bmatrix}$$

Figure 1.1 shows the joint regulation response of the two-link manipulator using a PID controller with gains set to $K_p = [50,\ 50]$, $K_d = [25,\ 25]$, and $K_i = [23,\ 23]$. As observed, both joints reach their desired setpoints quickly and with minimal transient behavior. There is no overshoot, and the system achieves steady-state within approximately 10 seconds, demonstrating effective regulation performance.

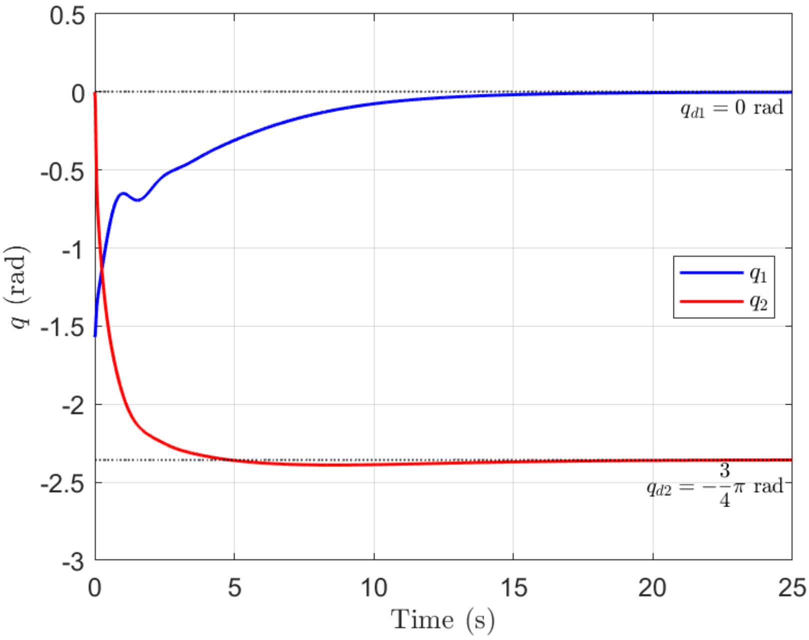

Figure 1.1: Joint Regulation Response Using PID Controller

As shown in Figure 1.2, the PID controller is tasked with following a sinusoidal desired trajectory. However, the response shows that the controller struggles to accurately track the reference. The actual joint positions exhibit noticeable phase lag and amplitude attenuation compared to the desired signal. This is due to the limitations of a traditional PID controller when applied to time-varying trajectories, especially in nonlinear systems like the two-link manipulator.

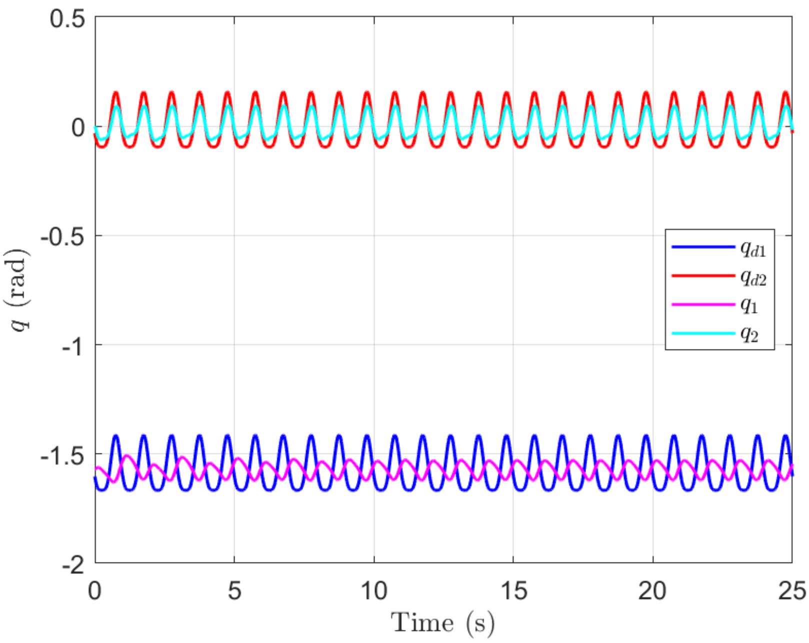

Figure 1.2: Joint Tracking Response Using PID Controller

1.2 Task 2

In this task, a computed torque controller with a PD outer loop is implemented for the same two-link manipulator model used previously. The objective is to improve trajectory tracking performance by explicitly compensating for the system's nonlinear dynamics.

The PD gains for the outer loop of the computed torque controller were selected based on the standard second-order system form. A damping ratio of $\xi = 0.7$ and a natural frequency of $\omega_n = 8\,\text{rad/s}$ were chosen to ensure a fast and well-damped response. This results in proportional gains $K_p = \omega_n^2 = 64$ and derivative gains $K_d = 2\xi\omega_n = 11.2$, applied identically to both joints.

By explicitly compensating for the manipulator's nonlinear dynamics through computed torque control, the tracking performance improved substantially compared to the previous PID-based approach. As shown in Figure 1.3, the actual joint positions closely follow the desired sinusoidal trajectories. The response is significantly improved, and the desired and actual curves are nearly indistinguishable, demonstrating the effectiveness of feedback linearization in trajectory tracking.

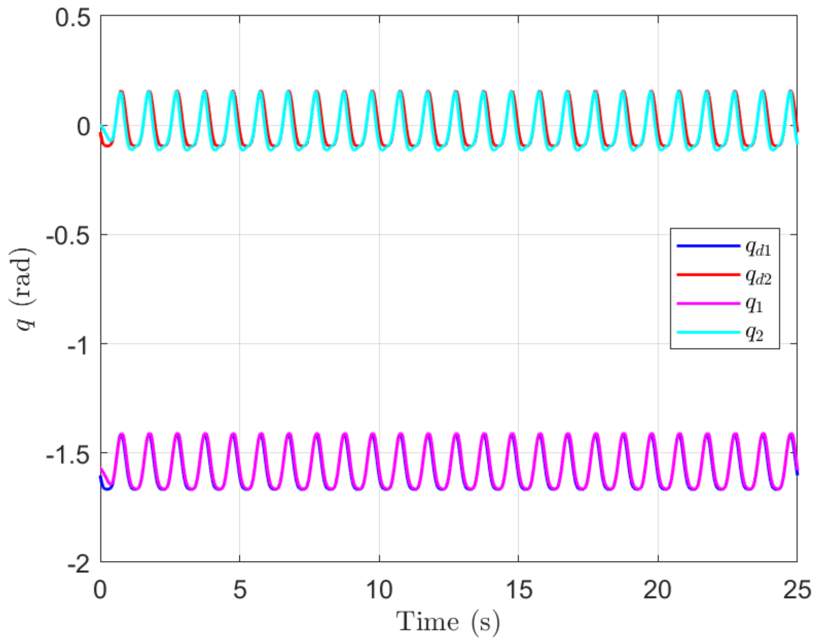

Figure 1.3: Joint Tracking Response Using Computed Torque Controller with a PD outer loop

Figure 1.4 illustrates that while the controller initially achieves excellent tracking—with the actual and desired joint trajectories nearly overlapping—a disturbance torque of $10\,\text{Nm}$ applied at $t = 5\,\text{s}$ causes a clear deviation. Following the disturbance, the system fails to return to the original trajectory, and the actual joint positions diverge from the desired sinusoidal paths, revealing limited disturbance rejection capability of the controller.

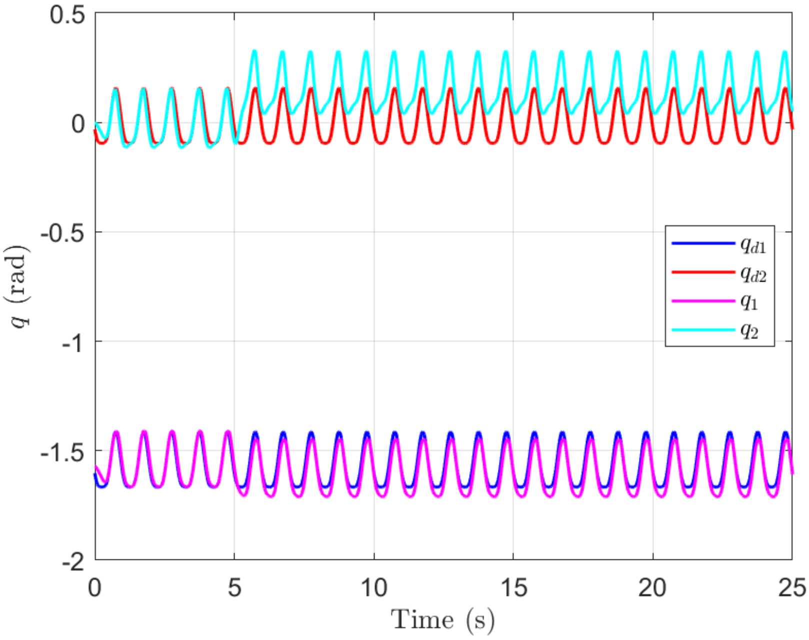

Figure 1.4: Joint Tracking Response Using Computed Torque Controller with a PD outer loop. A disturbance torque of 10 Nm was applied at $t = 5\,\mathrm{s}$

1.3 Task 3

In this task, a computed torque controller with a PID outer loop is implemented for the same two-link manipulator model used previously. The proportional and derivative gains K_p and K_d remain unchanged from Task 2. The same desired sinusoidal trajectory is used to evaluate the controller's performance.

Figure 1.5 shows the joint tracking performance using a computed torque controller with a PID outer loop. A disturbance is introduced at $t = 5\,\mathrm{s}$, and despite a brief deviation, the controller successfully restores the joint trajectories to follow the desired sinusoidal reference. For this task, an integral gain of $K_i = 11$ was selected empirically, applied identically to both joints. The inclusion of the integral term significantly enhances the controller's ability to eliminate steady-state error, which is particularly evident in the recovery observed after the disturbance.

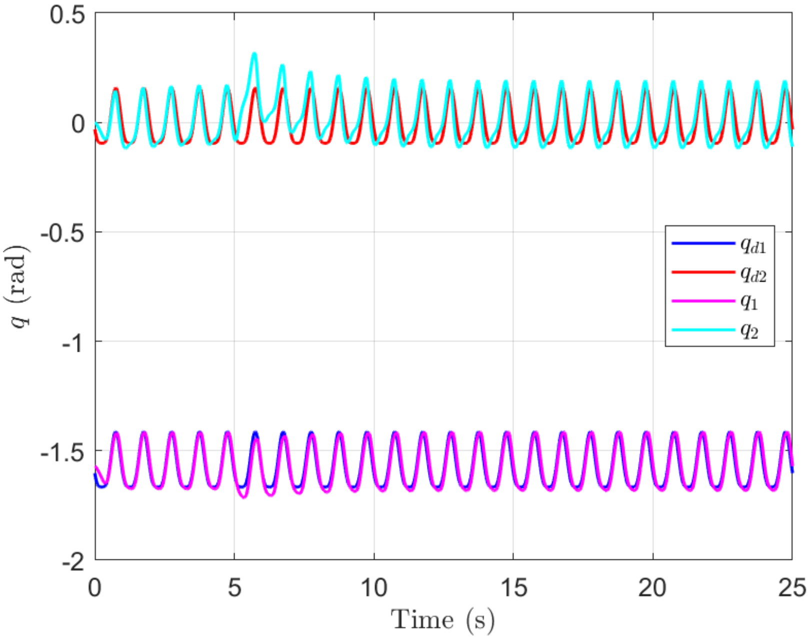

Figure 1.5: Joint Tracking Response Using Computed Torque Controller with a PID outer loop. A disturbance torque of 10 Nm was applied at $t = 5\,\text{s}$