TUM SCHOOL OF COMPUTATION, INFORMATION AND TECHNOLOGY (CIT)

TECHNICAL UNIVERSITY OF MUNICH

Report Submitted to Praktikum Robot Modelling and Identification (IN2106)

Experiment 6: Controller and Observer Design for a Robot Link

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Experiment 6: Controller and Observer Design for a Robot Link

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We confirm that this report is my our own work and we have documented all sources and material used.

Munich, June 19, 2025
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1 COD - Controller and Observer Design for a Robot Link

The objective of this experiment is to design a continuous-time Luenberger observer for the states of the robot link, shown in Fig. 1.1. The dynamics of the real system are provided by equations 1.1 and 1.2. The physical parameters used in this experiment are summarized in Table 1.1.

$$u(t) = Ri(t) + L\dot{i}(t) + k_e\dot{\varphi}(t) \tag{1.1}$$

$$(J + ml^2)\ddot{\varphi}(t) = k_m i(t) - d\dot{\varphi}(t) - mgl\sin(\varphi(t)) \tag{1.2}$$

Parameter	Description	Value	Unit
J	motor moment of inertia	0.001	$\mathrm{kg\,m^2}$
d	damping constant	0.1	$\mathrm{N\,m\,s}$
k_e	electrical motor constant	0.1	$\mathrm{V\,s/rad}$
k_m	mechanical motor constant	0.1	$\mathrm{N\,m/A}$
L	inductance	0.01	H
R	electrical resistance	0.1	Ω
m	mass of the link	1	kg
l	lever arm of the link mass	0.1375	$\mathrm{m\,s}$

Table 1.1: Physical parameters of the robot link system

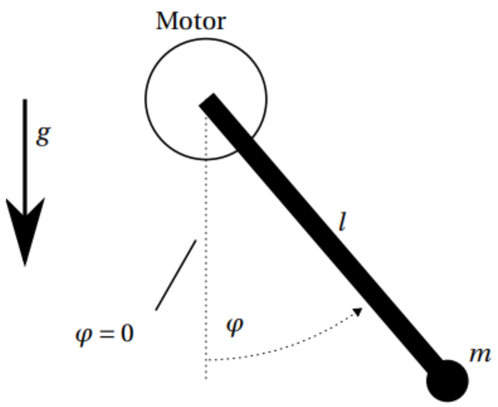

Figure 1.1: Sketch of the robot link.

The nonlinear dynamics given in Equations 1.1 and 1.2 were linearized around the equilibrium point $\varphi_0 = 0$. The resulting linear state-space representation was obtained as follows:

$$\dot{x}(t) = Ax(t) + b\,u(t) \tag{1.3}$$
$$y(t) = c^T x(t) + d\,u(t) \tag{1.4}$$

where:

- $x(t) = \begin{bmatrix}\varphi(t) & \dot{\varphi}(t) & i(t)\end{bmatrix}^T$ is the state vector,
- $u(t)$ is the input (motor voltage),
- $y(t)$ is the output (measured motor position),
- A is the system matrix (see Equation 1.5),
- b is the input vector (see Equation 1.6),
- c^T is the output vector (see Equation 1.7),
- d is the throughput scalar (see Equation 1.8).

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{mgl}{J+ml^2} & -\frac{d}{J+ml^2} & \frac{k_m}{J+ml^2} \\ 0 & -\frac{k_e}{L} & -\frac{R}{L} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -67.761 & 5.023 & 5.023 \\ 0 & -10 & -10 \end{bmatrix} \tag{1.5}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix} \tag{1.6}$$

$$c^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \tag{1.7}$$

$$d = 0 \tag{1.8}$$

1.1 Task 1

In task 1, we need to design a Luenberger observer and test whether the observer output is consistent with the actual system. We build the linearized system in Simulink as shown in the figure 1.2 and apply a constant voltage of u =1V to the system, set the external torque to 0 Nm. The Luenberger observer formula is:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + h\left(y(t) - C\hat{x}(t)\right) \tag{1.9}$$

We calculate the observer gain h as following:

$$h = place(A', C', q)'; \tag{1.10}$$

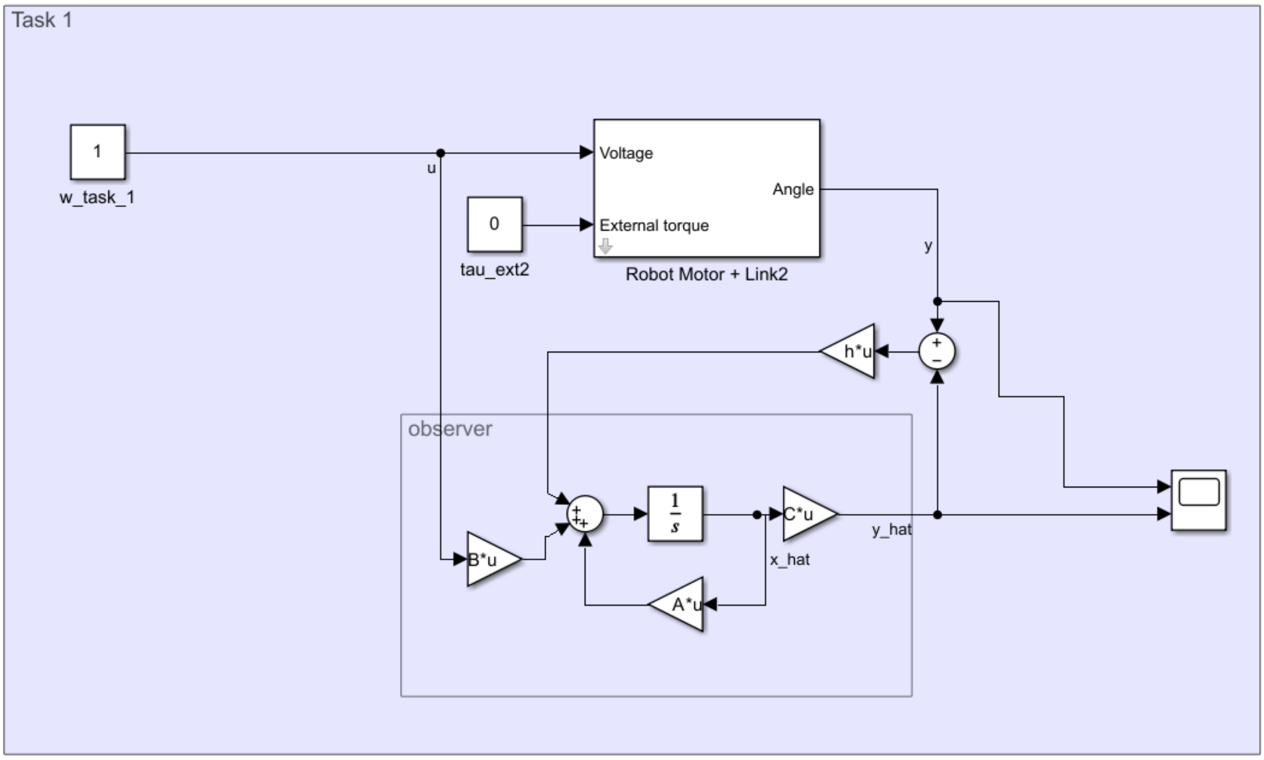

Figure 1.2: The Simulink model of the linearized system with observer.

The parameter q represents the observer pole we need to choose. The observer should be faster than the controlled system, so we should choose a pole that is further to the left than the original system pole to make the observer error decay quickly. At the same time, we should also avoid the pole being too far to the left to avoid excessively amplifying the noise and causing system instability. Therefore we choose q = [-15, -16, -17] and the observer gain is

$$h = \begin{bmatrix} 32.976 \\ 103.34 \\ -187.96 \end{bmatrix}$$

We run the model to detect the error of the observed value $\hat{y}$ relative to the true value y. We get the plot of y and $\hat{y}$ about time in figure 1.3. From we can see that the observer match the real system almost perfectly, because the simulation assumes an ideal linear system without noise or disturbances, and the observer poles are placed to ensure fast convergence. The system is fully observable, so the Luenberger observer can accurately reconstruct the states. As a result, the estimation error quickly goes to zero, and the observer output matches the real system very closely.

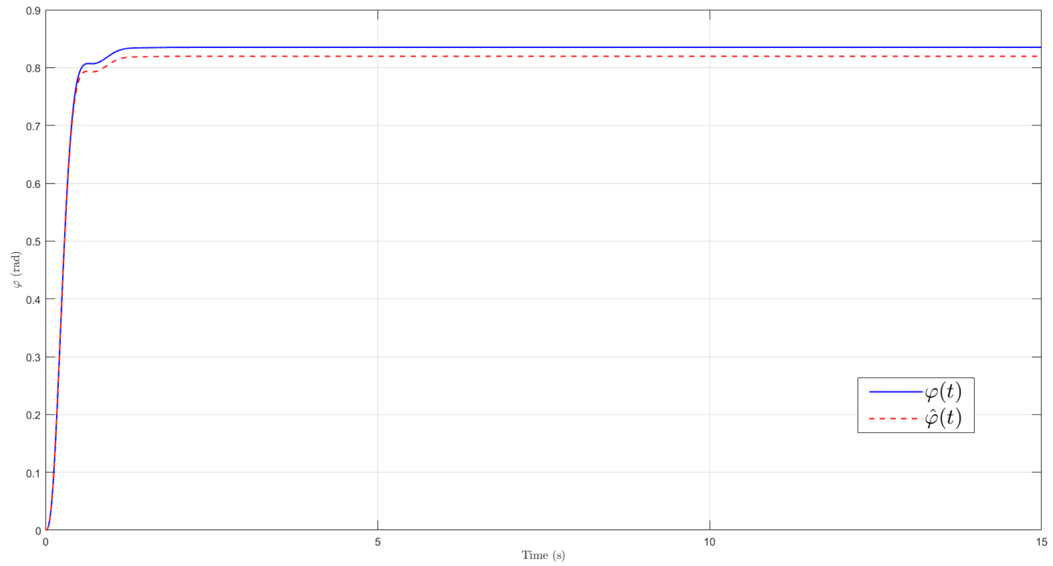

Figure 1.3: Comparison between the real motor position $\varphi(t)$ and the estimated position $\hat{\varphi}(t)$.

1.2 Task 2

In the following, the model developed in Task 1 is extended by incorporating state feedback based on the observer estimates. The estimated states are fed back into the system via the gain k. No external torque is applied to the link in this task.

To verify whether the linearized system is controllable, the controllability matrix Q_s is constructed as shown in Equation 1.11. Since Q_s has rank 3 , the system is confirmed to be fully controllable.

$$Q_s = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} 0 & 0 & 502.3 \\ 0 & 502.3 & -7547.2 \\ 100 & -1000 & 4976.5 \end{bmatrix} \tag{1.11}$$

The characteristic polynomial of the linearized system describes the dynamics of the open-loop system and is determined by the eigenvalues of the system matrix A, as shown in Equation 1.12.

$$\det(sI - A) = s^3 + 15.02s^2 + 168.23s + 677.61 \tag{1.12}$$

To facilitate the design of a state feedback controller using the canonical form, the auxiliary matrix W is matrix is defined as:

$$W = \begin{bmatrix} 168.23 & 15.02 & 1 \\ 15.02 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \tag{1.13}$$

The transformation matrix T is is defined as:

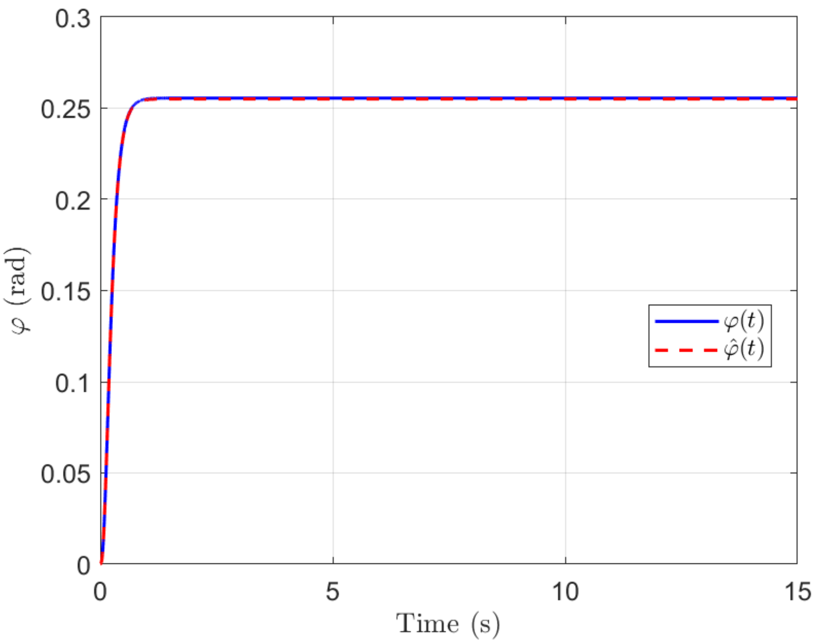

Figure 1.4: Comparison between the real motor position $\varphi(t)$ and the estimated position $\hat{\varphi}(t)$.

$$T = Q_s W = \begin{bmatrix} 502.35 & 0 & 0 \\ 0 & 502.35 & 0 \\ 677.61 & 502.35 & 100 \end{bmatrix} \tag{1.14}$$

The state feedback gain k is computed to place the closed-loop poles of the system at the desired locations $p_{1,2} = -10$ and $p_3 = -20$, as shown in Equation 1.2.

$$k^T = \begin{bmatrix} -0.7366 \\ 0.4107 \\ 0.2498 \end{bmatrix}$$

To verify the correctness of the computed gain k, the eigenvalues of the closed-loop system matrix $A - Bk$ were calculated using the `eig` function. The result was $-10.0000 \pm 0.0000i$ and $-20.0000 + 0.0000i$, confirming that the desired pole locations were successfully achieved.

Figure 1.4 shows the response of the real system $\varphi(t)$ and the estimated state $\hat{\varphi}(t)$. As seen, the observer tracks the system output with high accuracy. However, the system does not reach the desired setpoint of 1 rad. This is likely due to the absence of reference tracking compensation in the controller, which prevents the output from converging to the input reference.

1.3 Task 3

Figure 1.7 shows the Simulink model used for Task 3, where an integrator compensator was added to eliminate the steady-state error observed in the previous task. Normally, one would choose to use the estimated angle $\hat{\varphi}$ for feedback and integrator compensation, as it is typically

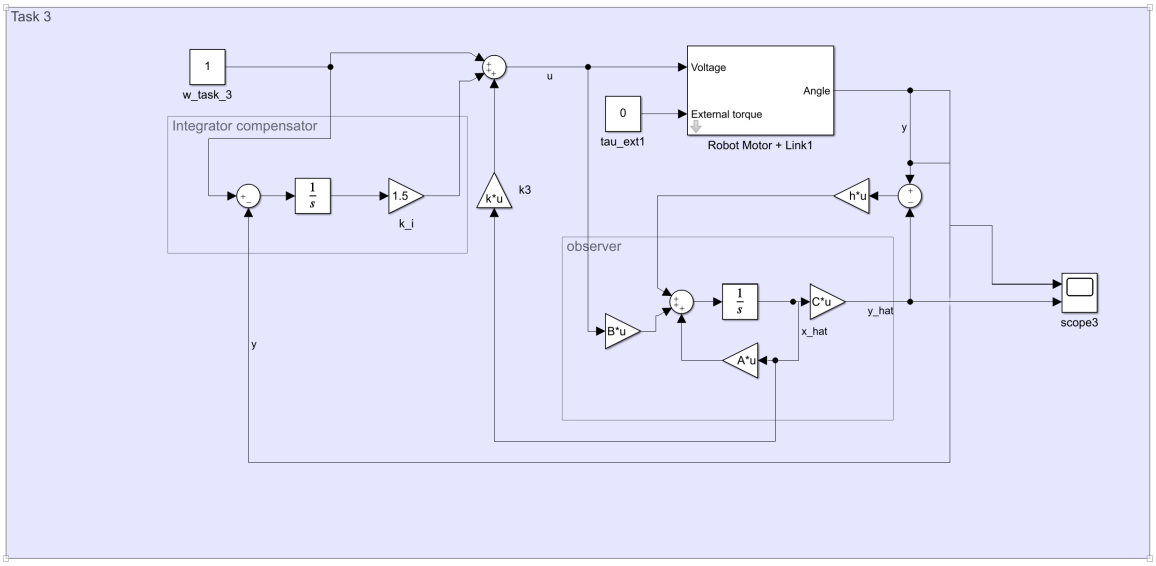

Figure 1.5: The block diagram shows the Simulink model with state feedback, integrator compensator, and observer.

free of measurement noise and more robust in practical applications. In this setup, the control error for the integrator compensator is computed using the system output $y = \varphi$, rather than the estimated angle $\hat{\varphi}$ from the observer. This choice was made because the measured output is not affected by observer estimation error or noise in this simulation, and it provides a more accurate representation of the true system state. The observer, based on a linearized model, inherently underestimates the true angle slightly due to its inability to fully capture the nonlinear dynamics of the real system (around the desired angle values $\varphi_d = 1$). The integrator output is scaled by a gain $k_I = 1.2$ and added to the state feedback control input. With this setup, we are able to achieve a zero steady-state error, as it can be seen in Figure 1.6. The plot illustrates that the observer is able to track the system dynamics with high accuracy. However, a small steady offset can be observed, which is expected due to the use of a linear observer to estimate the states of a nonlinear system. This mismatch becomes more noticeable for larger deviations from the linearization point. Nevertheless, the overall shape and timing of the estimated signal closely follow the true trajectory, confirming the correct functioning of the observer design.

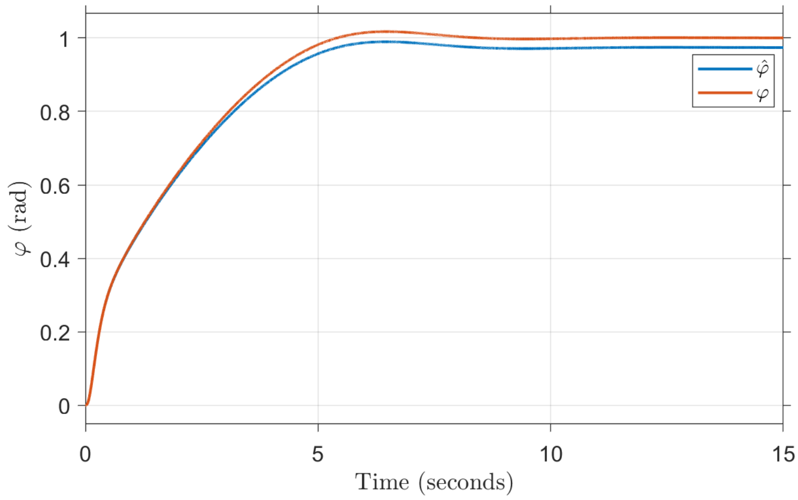

Figure 1.6: The plot shows a comparison between the trajectory of the real joint angle φ of the robot link and its estimate $\hat{\varphi}$ provided by the observer over time.

1.4 Task 4

In the last task, we intend to estimate a constant disturbance applied to the system. The disturbance appears as anexternal torque ext in our system

$$u(t) = Ri(t) + L\dot{i}(t) + k_e\dot{\varphi}(t) \tag{1.15}$$

$$(J + ml^2)\ddot{\varphi}(t) = k_m i(t) - d\dot{\varphi}(t) - mlg\sin(\varphi) + \tau_{\text{ext}} \tag{1.16}$$

In this task, the system input is 0 and is subjected to a disturbance of 0.2 Nm. To estimate the external torque τ_{ext}, we augment the system by treating the torque as an additional state with zero dynamics. The augmented state-space model is:

$$A_{\text{dist}} = \begin{bmatrix} A & B_d \\ 0 & 0 \end{bmatrix}, \quad B_{\text{dist}} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_{\text{dist}} = \begin{bmatrix} C & 0 \end{bmatrix}$$

We choose the observer poles as q = [-20, -21, -22, -23] and design the observer gain using the pole placement method:

$$h_{\text{dist}} = \texttt{place}(A_{\text{dist}}^\top, C_{\text{aug}}^\top, q)^\top$$

If the last row of h_{dist} (corresponding to the disturbance state) becomes zero, we manually adjust it to ensure the observer can estimate the torque accurately. Finally, we get the parameters of the observer as follows

$$A_{\text{dist}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -67.7614 & -5.0235 & 5.0235 & 50.2355 \\ 0 & -10 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_{\text{dist}} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \end{bmatrix}$$

$$C_{\text{dist}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$h_{\text{dist}} = \begin{bmatrix} 70.976 \\ 1536.4 \\ -951.36 \\ 423.05 \end{bmatrix}$$

The figure below shows the estimated external torque $\hat{\tau}_{\text{ext}}$ compared to the true value of 0.2 Nm:

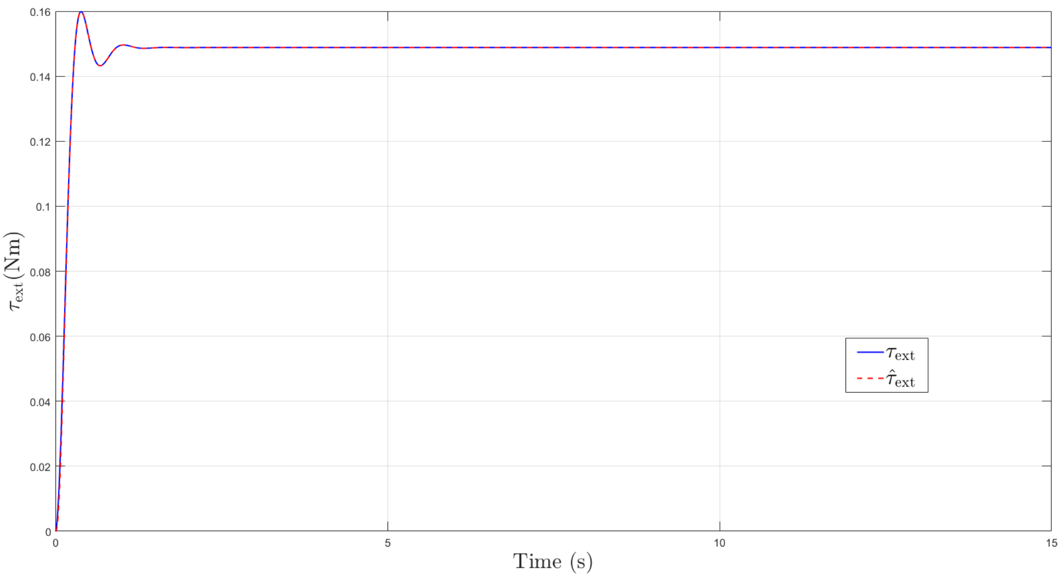

Figure 1.7: Comparison between the real motor position $\hat{\tau}_{\text{ext}}$ and the estimated position τ_{ext}.

We observe that the estimated torque closely matches the true value, confirming the accuracy of the augmented observer.