Machine Learning Homework 08

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1 Problem 3

a) Assume that the optimal W^*_{NN} you obtain are non-negative. What will the relation $(<, \leq, =, \geq, >)$ between the neural network loss $L_{NN}(W^*_{NN})$ and the linear regression loss $L_{LS}(w^*_{LS})$ be? Provide a mathematical argument to justify your answer.

The neural network can be defined
$f_W(x_n) = W^T_{(L+1)} ReLu\big(W^T_{(L)} \cdots ReLu(W^T_{(1)} x_n)\big)$.

Since $x_n \geq$, $ReLu = x$
$f_W(x_n) = W^T_{(L+1)} \cdot W^T_{(L)} \cdots W^T_{(1)} x_n$.

Since W^*_{NN} and W^*_{LS} are global minimum of $\arg\min_W \mathcal{L}_{\text{NN}}(W)$ and $\arg\min_W \mathcal{L}_{\text{LS}}(W)$, we have $\mathcal{L}_{\text{NN}}(W^*_{\text{NN}}) \leq \mathcal{L}_{\text{LS}}(W^*_{\text{LS}})$.

$\mathcal{L}_{\text{NN}}(W^*_{\text{NN}}) = \mathcal{L}_{\text{LS}}(W^*_{\text{LS}})$ if $W_1^\top, \ldots, W_L^\top = I,\ W_{L+1}^\top = W$

b) In contrast to a), now assume that the optimal weights w^*_{LS} you obtain are non-negative. What will the relation $(<, \leq, =, \geq, >)$ between the linear regression loss $L_{LS}(w^*_{LS})$ and the neural network loss $L_{NN}(W^*_{NN})$ be? Provide a mathematical argument to justify your answer.

a) shows that linear regression is a special case of the neural network,
for $W^*_{\text{LS}} \geq 0, \quad \mathcal{L}_{\text{NN}}(W^*_{\text{NN}}) \leq \mathcal{L}_{\text{LS}}(W^*_{\text{LS}})$ since the neural network can potentially find a better solution.