Assignment X

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1 Answers

Question D To compute the limiting distribution we first need to construct the transition probability matrix $P = P_{xy}$, with the transition probabilities given by ??? (Question b). This will be a 400×400 matrix, where the rows indicate the intial state and the columns the final state. The probability distribution vector $\vec{\pi}$ will be of size 400 and each component wlil correspond to one of the possible states of the system, so that $\vec{\pi}^T = (\pi_{11} \dots \pi_{2020})$. Each subscript represents the number of items left for each item type. The detailed construction of this matrix can be found in thte Appendix. We do:

$$\vec{\pi}_{t+1}^T = \vec{\pi}_t^T P$$

with an arbitrary $\vec{\pi}_0$ until convergence. The result is $\vec{\pi}_*$, which is the limiting distribution. The long run average costs can be computed now easily by multiplying $\vec{\pi}_*$ by the expected costs $c(\mathbf{x})$. The expected costs for each state $\mathbf{x}$ are just the holding costs for that state plus an extra cost of 5 if either x_1 or x_2 is equal to one. The costs can also be stored in a vector $\mathbf{c}$ of the same dimensions as $\vec{\pi}_*$ so the long-run average costs are then $\phi_* = \mathbf{c} \cdot \vec{\pi}_*$. We get a value of 10.172 for ϕ_*.

Question E Before defining the Poisson equation we need to introduce the value function for this problem, $V_t(\mathbf{x})$. This gives the total expected costs for a given state $\mathbf{x}$ when there are t time steps left. As before, we can construct a vecot $\mathbf{V}$ with the same dimensions as $\vec{\pi}$ that stores, for a given t, the total expected costs of every state, so that $\mathbf{V} = (V_{11} \dots V_{2020})$. With this, the Poisson equation is:

$$\mathbf{V} + \vec{\phi} = \mathbf{c} + P\mathbf{V}$$

Where $\vec{\phi}$ is just a vector of the right dimensions wiht all its entries equal to ϕ. We solve the equation using value iteration, this is, we do $\mathbf{V}_{t+1} = \mathbf{c} + P\mathbf{V}_t$ until $\mathbf{V}_{t+1} - \mathbf{V}_t \leqslant \epsilon$ for $\epsilon = 10^{-5}$. When convergence is reached, $\mathbf{V}_* = \mathbf{V} - \min\{\mathbf{V}\}$ and $\phi_* = \mathbf{c} + P\mathbf{V} - \mathbf{V}$. We get a value of 10.172 for ϕ_*.

A Appendix