Valuation and Risk Models

Valuation and Risk Models (1/2)

- Value at risk
- EWMA Model
- Factors affecting portfolio risk
- GARCH Estimation Model
- Linear vs Nonlinear derivatives
- Types of VaR
- Methods of VAR calculation
- Project & CF VAR
- Risk budgeting

Valuation and Risk Models (2/2)

- One factor Risk Metrics
- Multi factor Risk Metrics
- Country Risk Assessment
- Internal and External Ratings
- Capital structure of banks
- Operational Risk
- Stress Testing

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How to measure VAR

- **VaR** $_{\text{(daily VaR)}}$ (in%) = $Z_{X\%} * \sigma$
  - $Z_{X\%}$: the normal distribution value for the given probability (x%) (normal distribution has mean as 0 and standard deviation as 1)
  - σ: standard deviation (volatility) of the asset (or portfolio)
- **VAR (X%) dollar basis** = VAR (X%) * asset value
- **VAR for n days** using 1day VAR:

$$VAR(X\%)_{n\text{-days}} = (VAR(X\%)_{1\text{-days}})*\sqrt{n}$$

$\sigma_{port} = \sqrt{(w_a^2\,\sigma_a^2 + w_b^2\,\sigma_b^2 + 2w_aw_b*\,\sigma_a*\,\sigma_b*\text{ correlation }(a,b))}$

VaR$_{\text{port (daily VaR)}}$ (in%): $=\sqrt{(w_a^2\,(\%VaR_a)^2 + w_b^2(\%VaR_b)^2 + 2w_aw_b*(VaR_a)*(\%VaR_b)*\,\sigma_{ab})}$

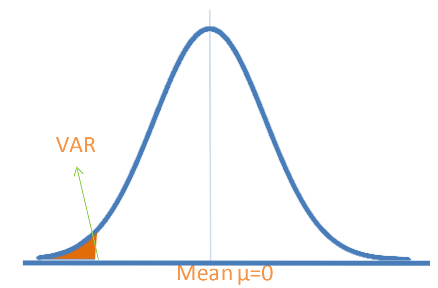
\$ VAR portfolio $= \sqrt{(\$VAR_a^2 + \$VAR_b^2 + 2\$VAR_a*\$VAR_b*\,\sigma_{a,b})}$

VAR of uncorrelated positions: VAR portfolio $= \sqrt{(VAR_1^{\,2} + VAR_2^{\,2})}$

Q.
A portfolio is composed of 2 securities. Calculate VAR at 95% confidence level.
Investment in security A & B are USD 1.5 mn and 3 mn respectively. Volatility of security A & B are 7% & 3% respectively. Correlation A & B is 10%
Ans.
σ portfolio $= \sqrt{(1/3)^2\,(7\%)^2 + (2/3)^2\,(3\%)^2 + 2*(1/3)*(2/3)*10\%*7\%*3\%} = 0.0316$
VAR = 1.65 * 0.0316 * 4,500,000 = 234,630

Value at Risk (VaR) has become the standard measure that financial analysts use to quantify this risk.VAR represents maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain horizon.
Example: The daily 5% VAR is $10,000, it indicates that there is only 5% chance that on any given day, the portfolio will experience a loss of $10,000 or more.
VAR Benefits:

- Aggregates all the risks in a portfolio into a single number provides an approach to arrive at economical capital.
- Relates capital with the expected losses
- Scaled to time



Approximately Normal Curve Representing VAR

The area under the normal curve for confidence value is:

Confidence (X%)	$Z_{x\%}$
90%	1.28
95%	1.65
97.5%	1.96
99%	2.32

Q.
If the assets has a daily σ of returns equal to 1.4% and asset has a current value of $5.3 mn, calculate the VAR (5%) on both percentage & dollar basis.
Ans.
$Z_{5\%}*\sigma = 1.65*1.4\% = 2.31\%$, and 0.0231* $5,300,000 = $122,430

Q.
If the value of stock is 100 and the value of the put option at 110 is 20. 10 units change in the underlying brings in change of 4 units change in the option premium. If the annual volatility is 0.25. Calculate daily VaR at 97.5% assuming 250 days?
Ans.
Delta = 0.4 STDEV(annual) =0.25
Days = 50 daily STDEV= 0.015811
Z at 97.5% =1.96
Options Value = 20 units VAR for option = 0.247923 units

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EWMA Model

$\sigma_n^2 = \lambda * \sigma^2_{n-1} + (1-\lambda) r^2_{n-1}$

Where λ = weight on previous volatility and (1- λ) weight on squared return

Q.
The current estimate of daily volatility is 1.5 %. The closing price of an asset yesterday & today are $30 and $30.5 respectively. Using the EWMA model with $\lambda = 0.94$, the updated estimate of volatility is?

Ans. 1.5096

Factors affecting portfolio risk

Assets concentration
Assets volatility
Assets correlation
Systematic risk

GARCH Estimation Model

$\sigma_n^2 = \omega + \alpha r^2_{n-1} + \beta \sigma^2_{n-1}$

Implicitly assumes variance reverts to a long run average level

Sum of $(\alpha+\beta) <= 1$ for the model to be stationary

Q.
Let h_t be the variance at t and $r^2(t-1)$ the squared return at t-1. Which of the following GARCH models will take the shortest time to revert to its mean? a. $h_t = 0.02 + 0.06r^2(t-1) + 0.93h_{t-1}$
b. $h_t = 0.03 + 0.04r^2(t-1) + 0.94h_{t-1}$
c. $h_t = 0.04 + 0.05r^2(t-1) + 0.95h_{t-1}$
d. $h_t = 0.05 + 0.01r^2(t-1) + 0.96h_{t-1}$

Ans.
(D) The speed of mean reversion is defined by $\alpha_1 + \beta$, which is lowest for D, which is 0.97

Q.
Which of the following is false?
a. EWMA approach of Risk Metrics is a particular case of a GARCH process.
b. GARCH allows for time-varying volatility.
c. GARCH can produce fat tails in the return distribution.
d. GARCH imposes a positive conditional mean return.

Ans. D

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Relationship b/w an underlying factor and the derivative's value are linear in nature

VaR for Linear and Non Linear Derivatives

1. **Linear Assets**: When the value of the delta is constant for all changes in the underlying. Example: Forwards, futures.

   Delta (1st derivative or duration in bonds) can be used to estimate the VAR for linear derivatives. **The delta-normal approach** (generally) does not work for portfolios of nonlinear securities.
   VAR Linear Derivative = Delta * VAR Underlying risk factor

2. **Non Linear Assets**: When the value of the delta keeps on changing with the change in the underlying asset. Examples: Options, Credit Derivatives, Swaps.

   Taylor Approximation: large changes can be explained by the 2nd derivative i.e. gamma expected change in the delta of an option (or convexity in bonds).Taylor approximation is ineffective for callable bonds & mortgage backed securities.

Q.
A bond of $10 mn, with modified duration of 3.6 yrs and annualized yield of 2%. calculate the 10 day holding period VaR of the position with 99% confidence interval, assuming there are 252 days in a year.
Ans.
VAR = $10,000,000* 0.02*3.6* [√10/ (√252)]* 2.33 = $334,186

Q.
A 6 month call option with a strike price of $10 is currently trading for $1.41, the market price of the underlying stock is $11. A 1% decrease in the stock to $10.89 results in a 6.35% decrease in the call option with a value of $1.32. If the annual volatility of the stock is s = 0.1975 and the risk free rate of return is 5%, calculate the 1 -day 5% VAR for this call option. **Ans**.
The daily volatility is = 1.25% (0.1975/√250); VAR_{stock}(5%) = 1.65 *1.25%= 2.06%;
Delta of the call = 0.0635/.01 = 6.35; $VAR_{call} = \Delta\ VAR_{stock}$ = 6.35*2.06%= 13.1%,

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Types of VaR

Diversified VAR: accounts for diversification effects.
$DVAR_P$= z* std dev* portfolio value
$=\sqrt{(VAR_1{}^2+VAR_2{}^2)}$

Undiversified VAR: sum of the individual VARs for each risk factor. It assumes that all prices will move in the worst direction simultaneously, which is unrealistic.
VAR_P
$= \sqrt{(VAR_1{}^2+VAR_2{}^2+2VAR_1VAR_2)}$
$= VAR_1+VAR_2$

Marginal VAR is the change in VaR of the portfolio with one unit change in the components = DVAR *β_A /portfolio value

Incremental VAR: The change in VAR from the addition of a new position in a portfolio.

Component VAR is the Amount a portfolio VAR would change by deleting either of the assets from a portfolio = DVAR *β_A * weight of asset A.

Q.
Weight of asset A & B are 0.6 & 0.4 in a portfolio. The value of the total portfolio is USD1 million and its σ is 0.060606; if the betas of asset A and asset B are 1.3 and 0.8 respectively, the respectively. What is the MVAR of Asset B and CVAR of Asset B at a 95%.
Ans.
DVAR = 1.95*0.060606*1,000,000 = 99,999.90
MVAR = 99,999.90*.8/1,000,000 = $0.08 CVAR = 99,999.90*.8 *0.4 = $32,000.

Q.
A portfolio has an equal amount invested in X and Y. The expected excess return of X is 9% and that of Y is 12%. The MVAR are 0.06 and 0.075 respectively. What should manager do to move towards the optimal portfolio?
Ans.
The Expected excess Return ratio for X and Y are 1.5 and 1.6 respectively. Therefore portfolio weight in Y should increase to move the portfolio towards the optimal portfolio.

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Full valuation: to take into account nonlinear relationships

Worst Case Scenario (EVT)

Stress testing:

Local valuation: for Linear derivatives

Historical Simulation: simply re-organizes actual historical returns, putting them in order from worst to best. Assumes history repeats itself.
Pitfalls: Time variation in risk, unusual events

Q.
Calculate VAR for an S&P 500 futures contract using the HS approach. The current price is 935 and the multiplier is 250. The historical price data for the previous 300 days, What is the VAR of the position at 99%.
Returns: -6.1%,-6%,-5.9%,-5.7%, -5.5%, -5.1%.........4.9%, 5%, 5.3%, 5.6%, 5.9%
Ans.
The 99% return among 300 observations would be the 3rd worst observation i.e. 5.9%; Therefore (935)*250* (0.059) = $13,791.

MC Simulation: The price returns are subjected to simulation using certain Simulation Models to generate a set of random numbers which are mapped to particular statistical distributions and hence the tail events are calculated to arrive at the VaR; **Pitfalls**: Model risk

Q.
Delta-normal, historical simulation, & MCS are methods available to compute VAR. If underlying returns are normally distributed (ND), then

a. Delta-normal method VAR will be identical to the HS VAR.
b. Delta-normal method VAR will be identical to the MCVAR.
c. MCVAR will approach the delta-normal VAR as the number of replications increases.
d. MCVAR will be identical to the HS VAR.

Ans.
C, In finite samples, the HS VAR will be in different from the delta-normal method, as the sample size increases, they converge when the returns are ND.

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Worst Case Scenario (EVT): focuses on distribution of worst possible outcomes given in an unfavourable event, expected loss is then determined from distribution.

- EVT stresses on the extreme value.
- EVT is calculation of the tail events & captures expected value of the fat-tail.
- The expected shortfall is mean of the observations exceeding VaR value.
- **Back Testing**: process of testing a trading strategy on prior time periods.

Stress testing: VAR tells the probability of exceeding a given loss but fails to incorporate the possible amount of a loss that results from an extreme event. Stress testing complements VAR by providing information about the magnitude of losses that may occur in extreme market conditions

Q.
Which of the following is NOT a drawback to stress testing?
a. Calculated losses may be extremely high relative to the 99% VAR significance level.
b. Historical correlations mix normal and hectic periods.
c. It identifies important factors not observed in historical data.
d. The number of scenarios increases greatly with additional risk factors.
Ans. C

Q.
Which of the following methods would be most appropriate for stress testing your portfolio?
a. Delta-gamma valuation
b. Full revaluation
c. Marked to market
d. Delta-normal VAR
Ans. B

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Delta-Normal or Variance-Covariance Method: assumes that the portfolio exposures are linear and that the risk factors are jointly normally distributed (ND); VAR(X%)=$z_{x\%}*\sigma$

- **Pitfall**: non linearity, fat tails underestimate the occurrence of large observations because of its reliance on a ND

Risk Metrics approach is similar to the delta-normal approach. The only difference is that the risk factor returns are measured as logarithms of the price ratios, instead of rates of returns.

Q.Under usually accepted rules of market behaviour, the relationship bw parametric delta-normal VAR and historical VAR will tend to be

a. Parametric VaR will be higher.
b. Parametric VaR will be lower.
c. It depends on the correlations.
d. none of the above are correct.

Ans.

B, parametric VAR at high confidence levels will generally underestimate VAR

Q.If you use delta-VAR for a portfolio of options, which of the following is *always correct*?

a. It necessarily understates the VaR because it uses a linear approximation.
b. It can sometimes overstate the VaR.
c. It performs most poorly for a portfolio of deep ITM options.
d. It performs most poorly for a portfolio of deep-OTM options.

Ans.

B, The delta-VAR could understate or overstate the true VAR, depending if the position is net long or short options, it is generally better for ITM options, because these have low gamma and for OTM options, delta is close to zero, so the delta-VAR would predict zero risk.

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Cash Flow at Risk: It is a measure of the expected cash flow at loss beyond a confidence level. If beta (β) of an asset is β_X with the portfolio then the cash flow at risk **(CFAR) = β_X* CFAR of portfolio**.

Project VAR: when considering a new project, you can explicitly calculate the dollar cost of the increase in CFAR and include it as an additional cost of the project.

Q.
A firm with existing projects have expected cash flow of $100 mn and cash flow volatility of $60 mn. New project with a cost of $30 mn and cash flow volatility of $20 mn. The correlation between two cash flows is 0.3. Calculate the volatility of the firm's projects with new projects at 95% confidence level and the additional project cost due to the increased cash flow volatility, if the cost of cash flow volatility is $0.12.
Ans.
δ projects = sqrt ($60^2 + 30^2 + 2*(.3)*60*20$) = $68.7 mn
CFAR (at 5%) existing = 1.65*60 = $99 mn
CFAR (at 5%) with new project = 1.65*68.7 = $113.4 mn
The additional project cost due to increased cash flow volatility is: ($113.4 mn - $68.7 mn)*.12=$1.73 mn

Q.
A trader has an allocation equal to 8% of the firm's capital; the beta of trader's return with the return of the firm is 0.90. The contribution of the trader to the Firm's VAR of $120 million is:
a. $7.8 mn
b. $8.6mn
c. $9.6 mn
d. $10.8mn
Ans.
0.08*0.9*120 million = 8.64 million

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Risk Budgeting involves choosing and managing exposure to risk,
1st step is to determine the total amount of risk, as measured by VAR,
Next is the optimal allocation of assets for that risk exposure.

Funding Risk: is the risk that the value of the assets will not be sufficient to cover the liabilities of the fund

Q.
A Fund has $200 mn in assets and $180mn in liabilities. Expected return on the surplus, scales by assets is 4%, i.e. surplus is expected to grow by $8 mn over 1st year. The volatility of surplus is 10%. Use Z =1.65, what is the deficit with the loss associated with the VAR.
Ans.
Surplus = (200 - 180) = $20 mn, expected to grow by $8 mn to a value of $28 mn;
VaR = 1.65*20*0.1 = $33 mn
The deficit is: (33 - 28) = 5 mn

Risk Budgeting with Active Mangers: is done using Tracking error (Active Returns - benchmark return) & Information ratio (TE / volatility of managers TE)
Weight of portfolio managed by manager i
$= IR_i$*(portfolio's tracking error volatility)/ IR_i*(manager's tracking error volatility)

Q.
Determine the optimal weight ratio

	TE vol Ratio	IR
Manager A	5%	0.70
Manager B	5%	0.50
Benchmark	0%	0.00
Portfolio	3%	0.82

Ans.
A=51%, B=37% and remaining 12% in benchmark

Valuation and Risk Models (2/2)

One factor Risk Metrics

$$DV01 = \frac{-\Delta(\text{bond value})}{10000 \times \Delta(\text{interest rate})}$$

$$convexity = \frac{-1}{(\text{bond value})} \frac{d(\text{bond value})}{d(\text{interest rate})}$$

Multi factor Risk Metrics

$$DV01^{k} = \frac{-1}{10{,}000} \times \frac{\Delta(\text{bond value})}{\Delta(\text{key rate})}$$

$$D^{k} = \frac{-1}{(\text{bond value})} \times \frac{\Delta(\text{bond value})}{\Delta(\text{key rate})}$$

Country Risk Assessment

- Guidelines:

1. Politics can defy logic
2. Know your data sources
3. Question official statistics
4. Benefit from power of observation
5. Too much quantitative measures can be dangerous

Indicators used by rating agencies:

1. Macroeconomic performance
2. Public/external debt
3. External financing needs
4. Openness to trade and investment
5. Social pressures
6. Regime legiticimacy

Country risk assessment in practice:

•Country risk management framework

1. Identify exposures
2. Analyze exposures
3. Analyze risk management techniques
4. Select appropriate risk management technique
5. Implement chosen technique
6. Monitor results/ revise program

Selecting tools:

1. Grade based rating scheme; listing risk and mitigants; measuring event probability (A,B..E); Categorize by number and color
2. Measuring economic measures for economic growth, economic health and power sector

Internal and External Ratings

Capital structure of banks

Operational Risk

Stress Testing

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Internal and External Ratings

At-The-Point (ATP) / Through-The-Cycle (TTC)

Building Internal Ratings
- Identify most meaningful factor
- Assign weights to these factors

Issues in Back-Testing
- There is no database when an internal rating system is designed
- Generally a time of 10 to 18 years required to validate an internal rating system

Biases

- Time horizon bias
- Homogeneity bias
- Information bias
- Criteria bias
- Scale bias

Capital structure of banks

EL = Exposure *LGD*PD

$UL = AE*\sqrt{*EDF*\sigma^2_{D} + LGD^2*\sigma^2_{DF}}$]

Deviation from avg. EL is UL

Operational Risk

- Process Risk
- People Risk
- Systems Risk
- Business Strategy Risk
- External Risk

Top-down Approach:
- Operational risk could not be measured objectively,
- Overall cost is determined based on past data on internal operational failures

Bottom-up Approach:
Commonly used nowadays with rapid development of statistical methods and Information Technology
- Divide their whole business into various business lines
- Measure individual risk profiles in each business line

Stress Testing

- Supplement to VaR. E.g.,: SPAN
- Testing how well a portfolio performs under some of the most extreme market moves seen in the last 10 to 20 years
  1. Capital Allocation
  2. Exposure
  3. Ride out Turmoil

Advantages:
- Can take a large number of risk factors into consideration
- Can specifically focus on the tails (extreme losses)

Disadvantages:
- Highly subjective and can become overcautious
- Requires complete top management support

Thank you!

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