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Practical portfolio performance measurement and attribution

Second Edition

CARL BACON

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Carl R. Bacon

John Wiley & Sons, Ltd

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This book is dedicated to Alex for her continued love and support

Contents

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This book is based on a series of performance measurement training courses I have had the pleasure of running around the world over the last decade. I have learnt so much and continue to learn from the questions and observations of the participants over the years, all of whom must be thanked.

I should also like to thank the many individuals I've had the pleasure to work with at various institutions, those I've met at conferences and at numerous GIPS committee meetings that have influenced my views over the years.

Naturally from the practitioner's perspective I've favoured certain methodologies over others. My strong preferences are difficult to disguise, nevertheless I've attempted to present each methodology as fairly as possible – apologies to those who may feel their methods have been unfairly treated.

Of course all errors and omissions are my own.

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1
Introduction

The more precisely the position is determined, the less precisely the momentum is known in this instant, and vice versa.

Heisenberg (1901–1976) *The Uncertainty Principle* (1927)

Learn as much by writing as by reading.

Lord Acton (1834–1902)

WHY MEASURE PORTFOLIO PERFORMANCE?

Whether we manage our own investment assets or choose to hire others to manage the assets on our behalf we are keen to know "how well" our collection or portfolio of assets is performing.

The process of adding value via benchmarking, asset allocation, security analysis, portfolio construction, and executing transactions is collectively described as the investment decision process. The measurement of portfolio performance should be part of the investment decision process, not external to it.

Clearly, there are many stakeholders in the investment decision process; this book focuses on the investors or owners of capital and the firms managing their assets (asset managers or individual portfolio managers). Other stakeholders in the investment decision process include independent consultants tasked with providing advice to clients, custodians, independent performance measurers and audit firms.

Portfolio performance measurement answers the three basic questions central to the relationship between asset managers and the owners of capital:

(1) *What* is the return on their assets?
(2) *Why* has the portfolio performed that way?
(3) *How* can we improve performance?

Portfolio performance measurement is the quality control of the investment decision process providing the necessary information to enable asset managers and clients to assess exactly how the money has been invested and the results of the process. The US Bank Administration Institute (BAI, 1968) laid down the foundations of the performance measurement process as early as 1968. The main conclusions of their study hold true today:

(1) Performance measurement returns should be based on asset values measured at market value not at cost.
(2) Returns should be "total" returns, that is, they should include both income and changes in market value (realised and unrealised capital appreciation).
(3) Returns should be time-weighted.
(4) Measurement should include risk as well as return.

THE PERFORMANCE MEASUREMENT PROCESS

Performance measurement is essentially a three-stage process:

(1) *Measurement*:
Calculation of returns, benchmarks and peer groups
Distribution of information
(2) Attribution:
Return attribution
Risk analysis (*ex post* and *ex ante*)
(3) Evaluation:
Feedback
Control

THE PURPOSE OF THIS BOOK

The writing of any book is inevitably a selfish activity, denying precious time from family who suffer in silence but also work colleagues and friends driven by the belief that you have something to contribute in your chosen subject.

The motivation to write the first edition was simply to provide the book I most wanted to read as a performance analyst which did not exist at the time.

The vocabulary and methodologies used by performance analysts worldwide are extremely varied and complex. Despite the development and global success of performance measurement standards there are considerable differences in terminology, methodology and attitude to performance measurement throughout the world.

The main aims of the first edition were:

(1) Provide a reference of the available methodologies and to hopefully provide some consistency in their definition.
(2) Promote the role of performance measurers.
(3) Provide some insights into the tools available to performance measurers.
(4) Share my practical experience.

Since the first edition I'm pleased to say the CFA Institute have launched the CIPM designation which further reinforces the role of performance measurement and is a major step in developing performance measurement as a professional activity. I can certainly recommend the CIPM course of study and I'm delighted to have successfully achieved the CIPM designation. The CIPM curriculum has also to some degree influenced the content of this second edition.

With practical examples this book should meet the needs of performance analysts, portfolio managers, senior management within asset management firms, custodians, verifiers and the ultimate clients. I'm particularly pleased that this second edition includes a CD including many of the practical examples used throughout the book.

ROLE OF PERFORMANCE MEASURERS

Performance measurement is a key function in an asset management firm, it deserves better than to be grouped with the back office. Performance measurers provide real added value, with feedback into the investment decision process and analysis of structural issues. Since their role is to understand in full, make transparent and communicate the sources of return within portfolios they are often the only independent source equipped to understand the performance of all the portfolios and strategies operating within the asset management firm.

Performance measurers are in effect alternative risk controllers able to protect the firm from rogue managers and the unfortunate impact of failing to meet client expectations.

BOOK STRUCTURE

The chapters of this book are structured in the same order as the performance measurement process itself, namely:

Chapter 2 Calculation of portfolio returns
Chapter 3 Comparison against an appropriate benchmark
Chapter 4 Proper assessment of the reward received for the risk taken
Chapters 5 to 10 Attribution of the sources of excess return
  Chapter 5 Fundamentals of attribution
  Chapter 6 Multi-currency attribution
  Chapter 7 Fixed income attribution
  Chapter 8 Multi-period attribution
  Chapter 9 Attribution issues
  Chapter 10 Attribution for derivatives
Chapter 11 Presentation and communicating the results

Inevitably, due to time constraints the first edition was not complete; the second edition affords me the opportunity to make substantial additions and improvements.

In Chapter 2 the "what" of performance measurement is introduced describing the many forms of return calculation including the relative merits of each method together with calculation examples.

Performance returns in isolation add little value; we must compare these returns against a suitable benchmark. Chapter 3 discusses the merits of good and bad benchmarks and examines the detailed calculation of commercial and customised indexes. I've added a section on random portfolios, some additional remarks, a few benchmark statistics and extended the section on performance fees.

Chapter 4 is substantially enhanced to include risk measures for hedge funds in an attempt to catalogue all the available risk measures used by performance analysts including suggestions for consistent definition were such definitions are lacking.

The original Chapter 5 in the first edition was perhaps too long. Attribution is a broad subject; Chapter 5 is now focused on the fundamentals of attribution, principally the Brinson model and its adaptation to arithmetic and geometric approaches.

Chapter 6 focuses on multi-currency attribution, including the important work of Karnosky and Singer plus a detailed description of geometric multi-currency attribution.

Chapter 7 is largely new material focusing on fixed income attribution. Since the investment decision process of fixed income managers is fundamentally different the unadjusted Brinson model is not appropriate.

Attribution analysis is useful for analysing performance not only for the most recent period but also for the longer-term requiring the linking of multi-period attribution results. The issues of multi-period attribution are discussed in Chapter 8.

Chapter 9 is new material covering a variety of technical attribution issues including security-level analysis, off benchmark investments, and balanced and multi-level attribution.

Chapter 10 is also new material covering the measurement and attribution of derivative instruments and attribution for various alternative asset strategies such as market neutral and 130:30 funds.

Finally, in Chapter 11 we turn to the presentation of performance and consider the global development of performance presentation standards. The second edition is updated for the latest version of the GIPS standards published in 2006.

2
The Mathematics of Portfolio Return

Mathematics is the gate and key of the sciences.... Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world.

Roger Bacon, *Doctor Mirabilis, Opus Majus* (1214–1294)

Mathematics has given economics rigour, alas also mortis.

Robert Helibroner (1919–2005)

SIMPLE RETURN

In measuring the performance of a "portfolio" or collection of investment assets we are concerned with the increase or decrease in the value of those assets over a specific time period, in other words the change in "wealth".

This change in wealth can be expressed either as a "wealth ratio" or a "rate of return".

The wealth ratio describes the ratio of the end value of the portfolio relative to the start value, mathematically:

$$\frac{V_E}{V_S} \tag{2.1}$$

where: $V_E =$ the end value of the portfolio
$V_S =$ the start value of the portfolio.

A wealth ratio greater than 1.0 indicates an increase in value, a ratio less than 1.0 a decrease in value.

Starting with a simple example, take a portfolio valued at £100m initially and valued at £112m at the end of the period. The wealth ratio is calculated as follows:

Exhibit 2.1 Wealth ratio

$$\frac{112}{100} = 1.12$$

The value of a portfolio of assets is not always easy to obtain, but should represent a reasonable estimate of the current economic value of the assets. Firms should ensure internal valuation policies are in place and consistently applied over time. A change in valuation policy may generate spurious performance over a specific time period.

Economic value implies that the traded market value, rather than the settlement value of the portfolio, should be used. For example, if an individual security has been bought but the trade has not been settled (i.e. paid for) then the portfolio is economically exposed to any change in price of that security. Similarly, any dividend declared and not yet paid or interest accrued on a fixed income asset is an entitlement of the portfolio and should be included in the valuation. Since it is a potential asset of the portfolio any reclaimable withholding tax should also be accrued in the market value. Although it may take some time before any withholding tax is recovered it should remain in the market value until either it is recovered or written off thus capturing performance in the appropriate period.

The rate of return denoted r describes the gain (or loss) in value of the portfolio relative to the starting value, mathematically:

$$r = \frac{V_E - V_S}{V_S} \tag{2.2}$$

Rewriting Equation (2.2):

$$r = \frac{V_E}{V_S} - \frac{V_S}{V_S} = \frac{V_E}{V_S} - 1 \tag{2.3}$$

Using the previous example the rate of return is:

Exhibit 2.2 Rate of return

$$\frac{112}{100} - 1 = 12\%$$

Equation (2.3) can be conveniently rewritten as:

$$1 + r = \frac{V_E}{V_S} \tag{2.4}$$

Hence, the wealth ratio is actually the rate of return plus one.

Where there are no "external cash flows" it is easy to show that the rate of return for the entire period is the "compounded return" over multiple subperiods.

Let V_t equal the value of the portfolio after the end of period t then:

$$\frac{V_1}{V_S} \times \frac{V_2}{V_1} \times \frac{V_3}{V_2} \times \cdots \frac{V_{n-1}}{V_{n-2}} \times \frac{V_E}{V_{n-1}} = \frac{V_E}{V_S} = 1 + r \tag{2.5}$$

External cash flow is defined as any new money added to or taken from the portfolio, whether in the form of cash or other assets. Dividend and coupon payments, purchases and sales and corporate transactions funded from within the portfolio are not considered external cash flows. Income from a security or stock lending programme initiated by the portfolio manager is not considered to be external cash flow, while if initiated by the client such income should be treated as external cash flow; the performance does not belong to the portfolio manager.

Substituting Equation (2.4) into Equation (2.5) we establish Equation (2.6):

$$(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \cdots (1 + r_{n-1}) \times (1 + r_n) = (1 + r) \tag{2.6}$$

Exhibit 2.3 Chain linking

	Market value (£ m)	Return (%)
Start value V_S	100	
End of period 1 V_1	112	12.0
End of period 2 V_2	95	−15.18
End of period 3 V_3	99	4.21
End of period 4 V_4	107	8.08
End value V_E	115	7.48

$$\frac{112}{100} \times \frac{95}{112} \times \frac{99}{95} \times \frac{107}{99} \times \frac{115}{107} = \frac{115}{100} = 1.15 \quad or \quad 15.0\%$$

$$1.12 \times 0.8482 \times 1.0421 \times 1.0808 \times 1.0748 = 1.15 \quad or \quad 15.0\%$$

This process (demonstrated in Exhibit 2.3) of compounding a series of subperiod returns to calculate the entire period return is called "geometric" or "chain" linking.

MONEY-WEIGHTED RETURNS

Unfortunately, in the event of external cash flows we cannot continue to use the ratio of market values to calculate wealth ratios and hence rates of return. The cash flow itself will make a contribution to the valuation. Therefore we must develop alternative methodologies that adjust for external cash flow.

Internal rate of return (IRR)

To make allowance for external cash flow we can borrow a methodology used throughout finance, the "internal rate of return" or IRR.

The internal rate of return has been used for many decades to assess the value of capital investment or other business ventures over the future lifetime of a project. Normally, the initial outlay, estimated costs and expected returns are well known and the internal rate of return of the project can be calculated to determine if the investment is worth undertaking. IRR is also used to calculate the future rate of return on a bond and called the yield to redemption.

Simple internal rate of return

In the context of the measurement of investment assets for a single period the IRR method in its most simple form requires that a return r be found that satisfies the following equation:

$$V_E = V_S \times (1 + r) + C \times (1 + r)^{0.5} \tag{2.7}$$

where: C = external cash flow.

In this form we are making an assumption that all cash flows are received at the midpoint of the period under analysis. To calculate the simple IRR we need only the start and end market values and the total external cash flow as shown in Exhibit 2.4:

Exhibit 2.4 Simple IRR

Market start value	\$74.2 m
Market end value	\$104.4 m
External cash flow	\$37.1 m

$$104.4 = 74.2 \times (1 + r) + 37.1 \times (1 + r)^{0.5}$$

We can see $r = -7.41\%$ satisfies the above equation:

$$74.2 \times (0.9259) + 37.1 \times (0.9259)^{0.5} = 104.4$$

Modified internal rate of return

Making the assumption that all cash flows are received midway through the period of analysis is a fairly crude estimate. The midpoint assumption can be modified for all cash flows to adjust for the fraction of the period of measurement that the cash flow is available for investment as follows:

$$V_E = V_S \times (1 + r) + \sum_{t=1}^{t=T} C_t \times (1 + r)^{W_t} \tag{2.8}$$

where: $C_t =$ the external cash flow on day t
$W_t =$ weighting ratio to be applied on day t.

Obviously, there will be no external cash flow for most days:

$$W_t = \frac{TD - D_t}{TD} \tag{2.9}$$

where: $TD =$ total number of days within the period of measurement
$D_t =$ number of days since the beginning of the period including weekends and public holidays.

In addition to the information in Exhibit 2.4, to calculate the modified internal rate of return shown in Exhibit 2.5 we need to know the date of the cash flow and the length of the period of analysis:

Exhibit 2.5 Modified IRR

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m

Assuming the cash flow is at the end of day 14 is:

$$104.4 = 74.2 \times (1 + r) + 37.1 \times (1 + r)^{\frac{17}{31}}$$

We can see $r = -7.27\%$ satisfies the above equation:

$$74.2 \times (0.9273) + 37.1 \times (0.9273)^{\frac{17}{31}} = 104.4$$

The standard internal rate of return method in Equation (2.8) is often described by performance measurers as the modified internal rate of return method to differentiate it from the simple rate of return method described in Equation (2.7) which assumes midpoint cash flows.

This method assumes a single, constant force of return throughout the period of measurement, an assumption we know not to be true since the returns of investment assets are rarely constant. This assumption also means we cannot disaggregate the IRR into different asset categories since we cannot continue to use the single constant rate.

For project appraisal or calculating the redemption yield of a bond this assumption is not a problem since we are calculating a future return for which we must make some assumptions.

IRR is an example of a money-weighted return methodology; each amount or dollar invested is assumed to achieve the same effective rate of return irrespective of when it was invested. In the US the term "dollar-weighted" rather than "money-weighted" is more often used.

The weight of money invested at any point of time will ultimately impact the final return calculation. Therefore if using this methodology it is important to perform well when the amount of money invested is largest.

To calculate the "annual" internal rate of return rather than the "cumulative" rate of return for the entire period we need to solve for r the using following formula:

$$V_E = V_S \times (1+r)^Y + \sum_{t=1}^{t=T} C_t \times (1+r)^{W_t^y} \tag{2.10}$$

where: $Y =$ length of time period to be measured in years
$W_t^y =$ factor to be applied to external cash flow on day t.

This factor is the time available for investment after the cash flow given by:

$$W_t^y = Y - Y_t \tag{2.11}$$

where: $Y_t =$ number of years since the beginning of the period of measurement.

For example, assume cash flow occurs on the 236th day of the 3rd year for a total measurement period of 5 years. Then:

$$W_t^y = 5 - 2\frac{236}{365} = 2\frac{129}{365}$$

Although today spreadsheets offer easy solutions to IRR calculations, historically solutions were more difficult to obtain, even taxing Sir Isaac Newton in the 17th century. Controversy surrounds the authorship of various iterative methods, in particular the Newton–Raphson method which perhaps should be attributed to Thomas Simpson (1740), (Kollerstrom 1992).

For the simple internal rate of return only, the solution of the quadratic equation can be used:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{2.12}$$

where: $ax^2 + bx + c = 0$.

Using data from Exhibit 2.4 in Exhibit 2.6:

Exhibit 2.6 Simple IRR – using quadratic formula

Market start value	\$74.2 m
Market end value	\$104.4 m
External cash flow	\$37.1 m

$$74.2 \times (1+r) + 37.1 \times (1+r)^{0.5} - 104.4 = 0$$

Let $x = (1+r)^{0.5}$:

$$(1+r)^{0.5} = \frac{-37.1 \pm \sqrt{37.1^2 - 4 \times 74.2 \times -104.4}}{2 \times 74.2} = \frac{-37.1 + 179.895}{148.4} = 0.96223$$

Therefore $r = -7.41\%$, same as Exhibit 2.4.

Simple Dietz

Even in its simple form the internal rate of return is not a particularly practical calculation, especially over longer periods with multiple cash flows. Peter Dietz (1966) suggested as an alternative the following simple adaptation to Equation (2.2) to adjust for external cash flow, let's call this the simple (or original) Dietz method:

$$r = \frac{V_E - V_S - C}{V_S + \dfrac{C}{2}} \tag{2.13}$$

where: C represents external cash flow.

The numerator of Equation (2.13) represents the investment gain in the portfolio. In the denominator replacing the initial market value we now use the average capital invested represented by the initial market value plus half the external cash flow. An assumption has been made that the external cash flow is invested midway through the period of analysis and has been weighted accordingly. The average capital invested is absolutely not the average of the start and end values which would factor in an element of portfolio performance into the denominator.

This method is also a money-(or dollar-) weighted return, and is in fact the first-order approximation of the internal rate of return method.

To calculate a simple Dietz return, like the simple IRR, only the start market value, end market value and total external cash flow are required.

Exhibit 2.7 Simple Dietz

Using the existing example data:

Market start value	\$74.2 m
Market end value	\$104.4 m
External cash flow	\$37.1 m

The simple Dietz rate of return is:

$$\frac{104.4 - 74.2 - 37.1}{74.2 + \dfrac{37.1}{2}} = \frac{-6.9}{92.75} = -7.44\%$$

Dietz originally described his method as assuming one half of the net contributions are made at the beginning of the time interval and one half at the end of the time interval:

$$r = \frac{V_E - \dfrac{C}{2}}{V_S + \dfrac{C}{2}} - 1 \tag{2.14}$$

This simplifies to the more common description:

$$r = \frac{V_E - \dfrac{C}{2}}{V_S + \dfrac{C}{2}} - \frac{V_S + \dfrac{C}{2}}{V_S + \dfrac{C}{2}} = \frac{V_E - V_S - C}{V_S + \dfrac{C}{2}} \tag{2.13}$$

The Dietz method is easier to calculate, easier to visualise than the IRR method and can also be disaggregated, that is to say the total return is the sum of the individual parts.

ICAA method

The Investment Counsel Association of America (ICAA, 1971) proposed a straightforward extension of the simple Dietz method as follows:

$$r = \frac{V'_E - V_S - C' + I}{V_S + \dfrac{C'}{2}} \tag{2.15}$$

where: I = total portfolio income
C' = external cash flow including any reinvested income
V'_E = market end value including any reinvested income.

Extending our previous example in Exhibit 2.8:

Exhibit 2.8 ICAA method

Market start value	\$74.2 m
Market end value	\$104.2 m
External cash flow	\$37.1 m
Total income	\$0.4 m
Income reinvested	\$0.2 m

$$\frac{104.2 - 74.2 - (37.1 + 0.2) + 0.4}{74.2 + \dfrac{(37.1 + 0.2)}{2}} = \frac{-6.9}{92.85} = -7.43\%$$

In this method income (equity dividends, interest or coupon payments) is not automatically assumed to be available for reinvestment. The gain in the numerator is appropriately adjusted for any reinvested income included in the final value by including reinvested income in the definition of external cash flow.

Interestingly, although the average capital is increased for any reinvested income in the denominator there is no negative adjustment for any income not reinvested. This is perhaps not unreasonable from the perspective of the client if the income is retained and not paid until the end of period.

However, from the portfolio manger's viewpoint if this income is not available for reinvestment it should be treated as a negative cash flow as follows:

$$r = \frac{V_E - V_S - C + I}{V_S + \dfrac{(C - I)}{2}} \tag{2.16}$$

Extending our previous example again in Exhibit 2.9:

Exhibit 2.9 Income unavailable

Market start value	\$74.2 m
Market end value	\$104.0 m
External cash flow	\$37.1 m
Total Income	\$0.4 m

$$\frac{104.0 - 74.2 - 37.1 + 0.4}{74.2 + \dfrac{(37.1 - 0.4)}{2}} = \frac{-6.9}{92.55} = -7.46\%$$

In Equation (2.16) any income received by the portfolio is assumed to be unavailable for investment by the portfolio manager and transferred to a separate income account for later payment or alternatively paid directly to the client.

Obviously, income paid or transferred is no longer included in the final value V_E of the portfolio. In effect, in this methodology income is treated as negative cash flow. Since income is normally always positive, this method has the effect of reducing the average capital employed, decreasing the size of the denominator, and thus leveraging (or gearing) the final rate of return.

Consequently, this method should only be used if portfolio income is genuinely unavailable to the portfolio manager for further investment. Typically, this method is used to calculate the return of an asset category (sector or component) within a portfolio.

Modified Dietz

Making the assumption that all cash flows are received midway through the period of analysis is a fairly crude estimate. The simple Dietz method can be further modified by day weighting each cash flow by the following formula to establish a more accurate average capital employed:

$$r = \frac{V_E - V_S - C}{V_S + \sum C_t \times W_t} \tag{2.17}$$

where: C = total external cash flow within period
C_t = external cash flow on day t
W_t = weighting ratio to be applied to external cash flow on day t.

Recall from Equation (2.9):

$$W_t = \frac{TD - D_t}{TD}$$

where: TD = total number of days within the period of measurement
D_t = number of days since the beginning of the period including weekends and public holidays.

In determining D_t the performance analyst must establish if the cash flow is received at the beginning or end of the day. If the cash flow is received at the start of the day then it is reasonable to assume that the portfolio manager is aware of the cash flow and able to respond to it, therefore it is reasonable to include this day in the weighting calculation. On the other hand, if the cash flow is received at the end of the day the portfolio manager is unable to take any action at that point and therefore it is unreasonable to include the current day in the weighting calculation.

For example, take a cash flow received on the 14th day of a 31-day month. If the cash flow is at the start of the day, then there are 18 full days including the 14th day available for investment and the weighting factor for this cash flow should be $(31-13)/31$. Alternatively, if the cash flow is at the end of the day then there are 17 full days remaining and the weighting factors should be, $(31-14)/31$.

Performance analysts should determine a company policy and apply this consistently to all cash flows.

Extending our standard example in Exhibit 2.10:

Exhibit 2.10 Modified Dietz

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m

Assuming the cash flow is at the end of day 14:

$$\frac{104.4 - 74.2 - 37.1}{74.2 + \dfrac{(31-14)}{31} \times 37.1} = \frac{-6.9}{94.55} = -7.30\%$$

Assuming the cash flow is at the beginning of day 14 with 18 full days in the month left:

$$\frac{104.4 - 74.2 - 37.1}{74.2 + \dfrac{(31-13)}{31} \times 37.1} = \frac{-6.9}{95.74} = -7.21\%$$

TIME-WEIGHTED RETURNS

True time-weighted

Time-weighted rate of returns provide a popular alternative to money-weighted returns in which each time period is given equal weight regardless of the amount invested, hence the name "time-weighted".

In the "true or classical time-weighted" methodology performance is calculated for each subperiod between cash flows using simple wealth ratios. The subperiod returns are then chain linked as follows:

$$\frac{V_1 - C_1}{V_S} \times \frac{V_2 - C_2}{V_1} \times \frac{V_3 - C_3}{V_2} \times \cdots \frac{V_{n-1} - C_{n-1}}{V_{n-2}} \times \frac{V_E - C_n}{V_{n-1}} = 1 + r \tag{2.18}$$

where: V_t is the valuation immediately after the cash flow C_t at the end of period t.

Since $(V_t - C_t)/V_{t-1} = 1 + r_t$ is the wealth ratio immediately prior to receiving the external cash flow, Equation (2.18) simplifies to the familiar Equation (2.6) from before:

$$(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \cdots (1 + r_{n-1}) \times (1 + r_n) = (1 + r)$$

In Equation (2.18) we have made the assumption that any cash flow is only available for the portfolio manager to invest at the end of the day. If we make the assumption that the cash flow is available from the beginning of the day we must change Equation (2.18) to:

$$\frac{V_1}{V_S + C_1} \times \frac{V_2}{V_1 + C_2} \times \frac{V_3}{V_2 + C_3} \times \cdots \frac{V_{n-1}}{V_{n-2} + C_{n-1}} \times \frac{V_E}{V_{n-1} + C_n} = 1 + r \qquad (2.19)$$

Alternatively, we may wish to make the assumption that the cash flow is available for investment midday and use a half weight assumption as follows:

$$\frac{V_1 - \frac{C_1}{2}}{V_S + \frac{C_1}{2}} \times \frac{V_2 - \frac{C_2}{2}}{V_1 + \frac{C_2}{2}} \times \cdots \times \frac{V_E - \frac{C_n}{2}}{V_{n-1} + \frac{C_n}{2}} = 1 + r \qquad (2.20)$$

Note from Equation (2.13):

$$r_t = \frac{V_t - V_{t-1} - C_t}{V_{t-1} + \frac{C_t}{2}} = \frac{V_t - \frac{C_t}{2}}{V_{t-1} + \frac{C_t}{2}} - 1$$

Equation (2.20) is really a hybrid methodology combining both time weighting and a money-weighted return for each individual day and therefore ceases to be a true time-weighted rate of return.

Using our standard example data we now need to know the value of the portfolio immediately after the cash flow as shown in Exhibits 2.11, 2.12 and 2.13:

Exhibit 2.11 True time-weighted end of day cash flow

End of day cash flow assumption:

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m
Market value end of	14 January	\$103.1 m

$$\frac{103.1 - 37.1}{74.2} \times \frac{104.4}{103.1} - 1 = 0.8895 \times 1.0126 - 1 = -9.93\%$$

Unit price method

The "unit price" or "unitised" method is a useful variant of the true time-weighted methodology. Rather than use the ratio of market values between cash flows, a standardised unit price or "net asset value" price is calculated immediately before each external cash flow by dividing the market value by the number of units previously allocated. Units are then added or subtracted

Exhibit 2.12 True time-weighted start of day cash flow

Start of day cash flow assumption:

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m
Market value start of	14 January	\$67.0 m

$$\frac{67.0}{74.2} \times \frac{104.4}{67.0 + 37.1} - 1 = 0.9030 \times 1.0029 - 1 = -9.44\%$$

Exhibit 2.13 Time-weighted midday cash flow

Midday cash flow assumption:

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m
Market value start of	14 January	\$67.0 m
Market value end of	14 January	\$103.1 m

$$\frac{67.0}{74.2} \times \frac{103.1 - \dfrac{37.1}{2}}{67.0 + \dfrac{37.1}{2}} \times \frac{104.4}{103.1} - 1 = 0.9030 \times 0.9883 \times 1.0126 - 1 = -9.63\%$$

(bought or sold) in the portfolio at the unit price corresponding to the time of the cash flow – the unit price is in effect a normalised market value.

The starting value of the portfolio is also allocated to units, often using a notional, starting unit price of say 1 or 100.

The main advantage of the unit price method is that, the ratio of the end of period unit price with the start of period unit price always provides the rate of return irrespective of the change of value in the portfolio due to cash flow. Therefore to calculate the rate of return between any two points the only information you need to know is the start and end unit prices.

Let NAV_i equal the net asst value unit price of the portfolio after the end of period i. Then:

$$\frac{NAV_1}{NAV_S} \times \frac{NAV_2}{NAV_1} \times \frac{NAV_3}{NAV_2} \times \cdots \frac{NAV_{n-1}}{NAV_{n-2}} \times \frac{NAV_E}{NAV_{n-1}} = \frac{NAV_E}{NAV_S} = 1 + r \tag{2.21}$$

The unitised method is so convenient for quickly calculating performance that returns calculated using other methodologies are often converted to unit prices for ease of use, particularly over longer time periods.

The unitised method is a variant of the true or classical time-weighted return and will always give the same answer as can be seen in Exhibit 2.14.

Typically, the performance of mutual funds is calculated using net asset value prices for external presentation and with a true time-weighted methodology for internal analysis. It can be a challenge to reconcile both sets of returns; the performance analyst must ensure that

Exhibit 2.14 Unit price method

		$m
Market value of portfolio at start of period	(31/12)	74.2
Market value of portfolio at end of period	(31/01)	104.4
Cash flow at end of day 14	(14/01)	37.1
Market value of portfolio immediately prior to cash flow	(14/01)	66.0
Emerging market index return in January		−7.92%
Index return 31/12 to 14/01		−10.68%
Index return 14/1 to 31/01		+3.09%

	Valuation	Unit price	Units allocated	Total units
Start value	74.2	1.0000	74.20	74.2
Valuation 14/1	66.0	0.8895		74.2
Cash flow 14/1	37.1	0.8895	41.71	115.9
End value	104.4	0.9007		115.9

$$\frac{90.07}{100.00} - 1 = -9.93\%$$

the internal valuations are aligned with valuations produced by the unit pricing accountant including the timing of external cash flows. Unless there are no external cash flows it will be impossible to reconcile the time-weighted unit price method with a money-weighted return.

TIME-WEIGHTED VERSUS MONEY-WEIGHTED RATES OF RETURN

Time-weighted returns measure the returns of the assets irrespective of the amount invested. This can generate counterintuitive results as shown in Exhibit 2.15:

Exhibit 2.15 (Time-weighted returns versus money-weighted returns)

Start period 1 Market value	£100
End period 1 Market value	£200
Cash flow	£1000
Start period 2 Market value	£1200
End period 2 Market value	£700

Time-weighted return:

$$\frac{1200 - 1000}{100} \times \frac{700}{1200} - 1 = 16.67\%$$

Money-weighted return:

$$\frac{700 - 100 - 1000}{100 + \dfrac{1000}{2}} = -67.67\%$$

In Exhibit 2.15 the client has lost £400 over the entire period, yet the time-weighted return is calculated as a positive 16.67%. The money-weighted return reflects this loss, −66.67% of the average capital employed. It is important to perform well in the second period when the majority of client money is invested.

If the client had invested all the money at the beginning of the period of measurement then a 16.67% return would have been achieved. The difference in return calculated is due to the timing of cash flow. Over a single period of measurement the money-weighted rate of return will always reflect the cash gain and loss over the period.

The time-weighted rate of return adjusts for cash flow and weights each time period equally, measuring the performance that would have been achieved had there been no cash flows. Clearly, this return is most appropriate for comparing the performance of different portfolio managers with different patterns of cash flows and with benchmark indexes, which for the most part are calculated using a time-weighted approach.

In effect, the time-weighted rate of return measures the portfolio manager's performance adjusting for cash flows and the money-weighted rate of return measures the performance of the client's invested assets including the impact of cash flows.

With such large potential differences between methodologies, which method should be used and in what circumstances?

Most performance analysts would prefer time-weighted returns. By definition time-weighted returns weight each time period equally, irrespective of the amount invested, therefore the timing of external cash flows or the amount of money invested does not affect the calculation of return. In the majority of cases portfolio managers do not determine the timing of external cash flows, nor does the amount of money invested normally change the investment decision process, therefore it is desirable to use a methodology that is not impacted by the timing of cash flow.

A few performance analysts argue that from a presentational perspective, particularly when dealing with private clients, money-weighted returns are preferred since over a single period a loss always results in a negative return and a gain in a positive return. While this is indeed true it may not be a true reflection of the portfolio manager's performance, and we are certain that the central assumption of money-weighted returns, that of a constant force of return, is most unlikely to be correct.

A major drawback of true time-weighted returns is that accurate valuations are required at the date of each cash flow. This is an onerous and expensive requirement for some asset managers. The manager must make an assessment of the benefits of increased accuracy against the costs of frequent valuations for each external cash flow and the potential for error. Asset management firms must have a daily valuation mindset to succeed with daily performance calculations. Exhibit 2.16 demonstrates the impact of a valuation error on the return calculation:

Exhibit 2.16 Valuation error

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m
Erroneous market value	14 January	\$101.1 m

$$\frac{101.1 - 37.1}{74.2} \times \frac{104.4}{101.1} - 1 = 0.8625 \times 1.0326 - 1 = -10.93\%$$

A significant and permanent difference from the accurate time-weighted return of −9.93% calculated in Exhibit 2.11.

Not unreasonably, institutional clients such as large pension funds paying significant fees might expect that the asset manager has sufficient quality information on a daily basis to manage their portfolio accurately. Most large managers will also have mutual or other pooled funds in their stable which in most cases will already require daily valuations (not just at the date of each external cash flow). The industry, driven by performance presentation standards and the demand for more accurate analysis, is gradually moving to daily calculations as standard.

In terms of statistical analysis daily calculation adds more noise than information; however, in terms of return analysis, daily calculation (or at the least valuation at each external cash flow which practically amounts to the same thing) is essential to ensure the accuracy of long-term returns.

I do not believe in the daily analysis of performance, which is far too short term for long-term investment portfolios, but I do believe in accurate returns, which require daily calculation. It is also useful for the portfolio manager or performance measurer to analyse performance between any two dates other than the standard calendar period ends.

APPROXIMATIONS TO THE TIME-WEIGHTED RETURN

Asset managers without the capability or unwilling to pay the cost of achieving accurate valuations on the date of each cash flow may still wish to use a time-weighted methodology and can use methodologies that approximate to the "true" time-weighted return by estimating portfolio values on the date of cash flow, such as the methodologies outlined in the next three subsections.

Index substitution

Assuming an accurate valuation is not available, an index return may be used to estimate the valuation on the date of the cash flow thus approximating the "true" time-weighted return as demonstrated in Exhibit 2.17:

Exhibit 2.17 Index substitution

Given an assigned benchmark performance of −10.68% up to the point of cash flow and using the data from Exhibit 2.11 the estimated valuation at the date of the cash flow is:

$$74.2 \times (1 - 10.68\%) = 66.28$$

Therefore the approximate time-weighted return is:

$$\frac{66.28}{74.2} \times \frac{104.4}{66.28 + 37.1} - 1 = 0.8933 \times 1.0099 - 1 = -9.79\%$$

In Exhibit 2.17 the index is a good estimate of the portfolio value and therefore the resultant return is a good estimate of the true time-weighted rate of return. However, if the index is a poor estimate of the portfolio value, see Exhibit 2.18, then the resultant return may be inaccurate, although in this case a better estimate of underlying return than say the modified Dietz or IRR.

Exhibit 2.18 Index substitution

Using an index return of −7.90% to estimate the portfolio value at the point of cash flow:

$$74.2 \times (1 - 7.9\%) = 68.34$$

Therefore the approximate time-weighted return is:

$$\frac{68.34}{74.2} \times \frac{104.4}{68.34 + 37.1} - 1 = 0.9210 \times 0.9901 - 1 = -8.81\%$$

Regression method (or β method)

The regression method is an extension of the index substitution method. A theoretically more accurate estimation of portfolio value can be calculated adjusting for the systematic risk (as represented by the portfolio's beta) normally taken by the portfolio manager.

Exhibit 2.19 Regression method

Again using the data from Exhibit 2.17 but assuming a portfolio beta of 1.05 in comparison with the benchmark, the revised estimated valuation at the time of cash flow is:

$$74.2 \times (1 - 10.68\% \times 1.05) = 65.88$$

Therefore the approximate time-weighted return is:

$$\frac{65.88}{74.2} \times \frac{104.4}{65.88 + 37.1} - 1 = 0.8879 \times 1.0138 - 1 = -9.99\%$$

The index substitution method is only as good as the resultant estimate of portfolio value; making further assumptions about portfolio beta need not improve accuracy.

Analyst's test

A further more accurate approximation was proposed by a working group of the UK's Society of Investment Analysts (SIA, 1972). They demonstrated that the ratio of the money weighted return of the portfolio against the money weighted return of the notional fund (portfolio market values and cash flows invested in the benchmark) approximates to the ratio of the time weighted return of the portfolio with the time weighted return of the notional fund, mathematically:

$$\frac{(1 + MWA)}{(1 + MWN)} = \frac{V_A - (C_T - C_W)}{V_N - (C_T - C_W)} \cong \frac{(1 + TWA)}{(1 + TWN)} \tag{2.22}$$

where:
MWA = money-weighted return of actual portfolio
MWN = money-weighted return of notional fund
V_A = value of portfolio at end of period
V_N = value of notional fund at end of period
C_T = total external cash flow in period
C_W = weighted external cash flow in period
TWA = time-weighted return of actual portfolio
TWN = time-weighted return of notional fund.

Rearranging Equation (2.22):

$$TWA \cong \frac{(1+MWA)}{(1+MWN)} \times (1+TWN) - 1 \tag{2.23}$$

or

$$TWA \cong \frac{V_A - (C_T - C_W)}{V_N - (C_T - C_W)} \times (1+TWN) - 1 \tag{2.24}$$

In other words, the time-weighted return of the portfolio can be approximated by the ratio of the money-weighted return of the portfolio divided by the money-weighted return of notional fund and then multiplied by the notional fund time-weighted rate of return. Since all commercial indexes are time weighted (they don't suffer cash flows and are therefore useful for comparative purposes) we can use an index return for the time-weighted notional fund.

Again using the standard example in Exhibit 2.20:

Exhibit 2.20 Analyst's test

Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External cash flow	14 January	\$37.1 m
Index return in January		−7.92%
Index return 31/12 to 14/01		−10.68%
Index return 14/1 to 31/01		+3.09%

Final value of notional fund:

$$V_N = (74.2 \times (1 - 0.1068) + 37.1) \times 1.0309 = 106.57$$

$$C_T = 37.1$$

$$C_W = 37.1 \times \left(\frac{31-14}{31}\right) = 20.35$$

$$TWA = \frac{104.4 - (37.1 - 20.35)}{106.57 - (37.1 - 20.35)} \times (1 - 0.0792) - 1$$

$$TWA = \frac{87.65}{89.82} \times 0.9208 - 1 = -10.14\%$$

The advantage of these three approximate methods is that a time-weighted return may be estimated even without sufficient data to calculate an accurate valuation and hence an accurate time-weighted return. The disadvantages are clear: if the index, regression and notional fund assumptions respectively are incorrect or inappropriate the resultant return calculated will also be incorrect. Additionally, the actual portfolio return appears to change if a different index is applied which is counterintuitive (surely the portfolio return ought to be unique) and is very difficult to explain to the lay trustee.

HYBRID METHODOLOGIES

In practice, many managers use neither true time-weighted nor money-weighted calculations exclusively but rather a hybrid combination of both.

If the standard period of measurement is monthly, it is far easier and quicker to calculate the modified (or even simple) Dietz return for the month and then chain link the resulting monthly returns. This approach treats each monthly return with equal weight and is therefore a version of time weighting. All of the methods mentioned previously can be calculated for a specific time period and then chain linked to create a time-weighted type of return for that time period.

Linked modified Dietz

Currently, the standard approach for institutional asset managers is to chain link monthly modified Dietz returns. Often described as a time-weighted methodology, in fact it is a hybrid chain-linked combination of monthly money-weighted returns. Each monthly time period is given equal weight and therefore time weighted, but within the month the return is money weighted.

BAI method (or linked IRR)

The US Bank Administration Institute (BAI, 1968) proposed an alternative hybrid approach first proposed by Fisher, (1966), which essentially links simple internal rates of return rather than linking modified Dietz returns.

Because of the difficulties in calculating internal rates of return this is not a popular method and is virtually unknown outside of the US.

For clarification both the BAI method and the linked modified Dietz methods can be described as a type of time-weighted methodology because each standard period (normally monthly) is given equal weight. True time weighting requires the calculation of performance between each cash flow.

The index substitution, regression and analyst tests methods are approximations to the true time-weighted rate of return. The simple Dietz, modified Dietz and ICAA methods are approximations of the internal rate of return and are therefore money weighted.

WHICH METHOD TO USE?

Determining which methodology to use will ultimately depend on the requirements of the client, the degree of accuracy required, the type and liquidity of assets, availability of accurate valuations and cost and convenience factors.

Time-weighted returns neutralise the impact of cash flow. If the purpose of the return calculation is to measure and compare the portfolio manager's performance against other managers and commercially published indexes then time weighting is the most appropriate. On the other hand, if there is no requirement for comparison and only the performance of the client's assets are to be analysed then money weighting may be more appropriate.

As demonstrated in Exhibit 2.15, a time-weighted return, which does not depend on the amount of money invested, may lead to a positive rate of return over the period in which the client may have lost money. This may be difficult to present to the ultimate client although in truth the absolute loss of money in this example is due to the client giving the portfolio manager more money to manage prior to a period of poor performance in the markets. If there had been no cash flows the client would have made money.

Confidence in the accuracy of asset valuation is crucial in determining which method to use. If accurate valuations are available only on a monthly basis, then a linked monthly modified Dietz methodology may well be the most appropriate. The liquidity of assets is also a key determinant of methodology. If securities are illiquid it may be difficult to establish an accurate valuation at the point of cash flow. For illiquid assets such as private equity any perceived accuracy in the true time-weighted return could be quite spurious.

Internal rates of return are traditionally using for venture capital and private equity asset categories for a number of reasons:

(1) The initial investment appraisal for non-quoted investments often uses an IRR approach.
(2) Assets are difficult to value accurately and are illiquid.
(3) The venture capital manager often controls the timing of cash flow.

It is interesting to note that like private equity, real estate performance has been traditionally calculated using a money-weighted methodology; however, with increased frequency of property valuations (albeit not independent) real estate has strived to be treated with equivalence to other assets and more recently time-weighted approaches have become more common. It is often said that private equity uses a money-weighted methodology because the asset manager has more control of the timing of cash flows; that may well be true but I believe the more likely reason is that historically accurate valuations at the time of cash flows are simply not available, and therefore a time-weighted methodology cannot be applied.

Money-weighted rates of return are often used for private clients to avoid the difficulty of explaining why a loss could possibly lead to a positive time-weighted rate of return. Perhaps the most significant advantage of money-weighted returns is that if the portfolio generates a profit, the rate of return with be positive (and vice versa) regardless of the pattern of cash flows.

Because the recognised advantage of time-weighted returns is that they neutralise the impacts of cash flows they are clearly favoured when the portfolio manager does not influence the timing of cash flows. This does not imply that if the portfolio manager influences the timing of cash flows then money-weighted returns should be preferred, the resultant return is simply the constant force of return required when applied to the pattern of cash flow that results in the end market value of the portfolio. This return is unique and not really comparable with other portfolios enjoying a different pattern of cash flow or indeed benchmark indexes; there are other ways of measuring the impact of timing, notably attribution analysis.

Mutual funds suffer a particular performance problem caused by using backdating unit prices as illustrated in Exhibit 2.21.

This is in effect what happened in the "late trading and market timing" scandal in US mutual funds revealed in 2003. Privileged investors were allowed to buy or sell units in international funds at slightly out-of-date prices with the knowledge that overseas markets had risen or fallen significantly already, resulting in small but persistent dilution of performance for existing unit holders.

SELF-SELECTION

With the choice of so many different, acceptable calculation methodologies, managers should establish an internal policy to avoid both intentional and unintentional abuse.

Table 2.1 illustrates the range of different returns calculated for our standard example in just the one period.

The fundamental reason for the difference in all of these returns is the assumptions relating to external cash flow. Without cash flow all these methods – money-weighted, time-weighted and approximations to time-weighted – will give the same rate of return.

Exhibit 2.21 Late trading

Start portfolio value	$5 000 000
Units in issue	10 000 000
Start unit price	0.50
End portfolio value	$5 250 000
Units in issue	10 000 000
End period unit price	0.525

Assume because of administrative error that $500 000 should have been allocated at the start of period. The administrator determines the client should not suffer and allocates the $500 000 in 1 000 000 units at 0.50:

The final price is now	$5 750 000
Units in issue	11 000 000
Erroneous unit price	0.5227

In effect existing unit holders have been diluted by 0.44%. Units should only be issued at the current price, 1 000 000 at 0.525 = $525 000. The administrator should inject $25 000 to correct the error.

The reason for the difference effectively lies in the denominator (or average capital invested) of the return calculation. Each of these methods makes different assumptions about the impact of external cash flow on the denominator: the greater the cash flow the greater the impact.

The differences in the simple Dietz and the modified Dietz returns in Table 2.1 are so significant in this example because the cash flow is large relative to the starting value. If the cash flow is not large then the assumptions used to weight the cash flow will not have a measurable effect.

Because this effect is often not significant it is not always worth revaluing the portfolio for each cash flow. Many institutional asset managers employ a standard modified Dietz method and only revalue for a large external cash flow above a set percentage limit (10% is common). Asset management firms should set a limit and apply it rigorously. The limit may be defined to apply to a single cash flow during the period or the total cash flow during the period.

Table 2.1 Return variations due to methodology

Method	Return (%)
Simple Dietz	−7.44
Modified Dietz (end of day)	−7.30
Modified Dietz (beginning of day)	−7.21
Simple IRR	−7.41
Modified IRR	−7.27
True time-weighted (end of day)	−9.93
True time-weighted (beginning of day)	−9.63
Time-weighted midday cash flow	−9.40
Index substitution	−9.79
Regression	−9.99
Analyst's test	−10.14

If multiple returns are routinely calculated for each methodology and the best return chosen for each period, even poor performing portfolios could appear to be performing quite well. Clearly, it is unethical to calculate performance using multiple methods and then choose the best return.

Intentional self-selection of the best methodology is easy to avoid but unintentional abuse can occur. Portfolio managers are well aware that cash flow can impact performance and often they have a good feel for the performance of their own portfolios. If they have underperformed their expectations by say 0.2% they may require the performance measurer to investigate the return. The measurer identifying that a cash flow has occurred (but less than the normal limit) may conclude that the return has been adversely impacted by the cash flow. It would be entirely inappropriate for the performance measurer to adjust the return (even though it is theoretically more accurate) because the portfolio manager is unlikely to require the same analysis if the return is 0.2% above expectations, resulting in only positive adjustments taking place.

Table 2.2 lists the advantages and disadvantages of each return methodology available to the performance measurer together with my personal preference from the asset manager's perspective. My preferences are consistent with the evolution of performance returns methodologies shown in Figure 2.1.

Different asset categories and different product types have progressed through this evolution of return methodologies at different rates – I would suggest driven by the availability of accurate valuations. Mutual funds evolved to true time-weighted return first because an accurate

Table 2.2 Calculation methodologies

Method	Advantages	Disadvantages	Author's ranking
Simple Dietz	(i) Simple to calculate (ii) Not sensitive to valuation errors (iii) Can be disaggregated (iv) Net gain = positive return (v) Net loss = negative return	(i) Crude estimate for timing of cash flows (ii) Not suitable for comparison against other funds or published indexes	4
Modified Dietz	(i) Simple to calculate (ii) Not sensitive to valuation errors (iii) Can be disaggregated (iv) Net gain = positive return (v) Net loss = negative return	(i) Not suitable for comparison against other funds or published indexes	8
ICCA	(i) Simple to calculate (ii) Not sensitive to valuation errors (iii) Can be disaggregated (iv) Net gain = positive return (v) Net loss = negative return	(i) Not suitable for comparison against other funds or published indexes (ii) Potential gearing effect because of treatment of income	7
Simple IRR	(i) Reflects the value added from the client's perspective (ii) Not sensitive to valuation errors (iii) Net gain = positive return (iv) Net loss = negative return	(i) Crude estimate for timing of cash flows (ii) Cannot be disaggregated (iii) Not suitable for comparison against other funds or published indexes	3

Table 2.2 (*Continued*)

Method	Advantages	Disadvantages	Author's ranking
Internal rate of return	(i) Reflects the value added from the client's perspective (ii) Not sensitive to valuation errors (iii) Common measure in finance. Frequently used in private equity/venture capital	(i) Relatively difficult to calculate (ii) Cannot be disaggregated (iii) Not suitable for comparison against other funds or published indexes	5
BAI	(i) Hybrid time-weighted, money-weighted return (ii) Not sensitive to valuation errors	(i) Relatively difficult to calculate (ii) Cannot be disaggregated	6
Linked modified Dietz	(i) Hybrid time-weighted, money-weighted return (ii) Easier to calculate than BAI method (iii) Can be disaggregated within single periods (iv) Well established and in common usage	(i) Can be impacted by large cash flows	9
True time-weighted	(i) Measures the true performance of the portfolio manager adjusting for cash flows (ii) Suitable for comparison with other asset managers and benchmarks	(i) Sensitive to incorrect valuations	10
Analyst's test	(i) Good estimate of true time-weighted rate of return (ii) Does not require accurate daily valuations	(i) Change in benchmark appears to change portfolio return	2
Index substitution	(i) Time-weighted methodology (ii) Does not require accurate daily valuations	(i) Approximation only as good as index assumption. Less accurate than analyst's test (ii) Change in benchmark appears to change portfolio return	1
Regression	(i) Time-weighted methodology (ii) Does not require accurate daily valuations (iii) Use of systematic risk may provide a more accurate valuation approximation than index substitution method	(i) Approximation only as good as index and systematic risk assumption. Low R^2 implies volatile systematic risk (ii) Change in benchmark or systematic risk calculation appears to change portfolio return	0

valuation must be derived to allow investors to enter and exit the mutual fund at a fair price. Pension funds are perhaps one step behind in evolutionary development seeking time-weighted returns to neutralise the impact of cash flows and allow fair comparison of their managers which each other and benchmarks. In fact, pension funds are still in the process of evolving to the final stage with many institutional asset managers still using linked modified Dietz; they are not

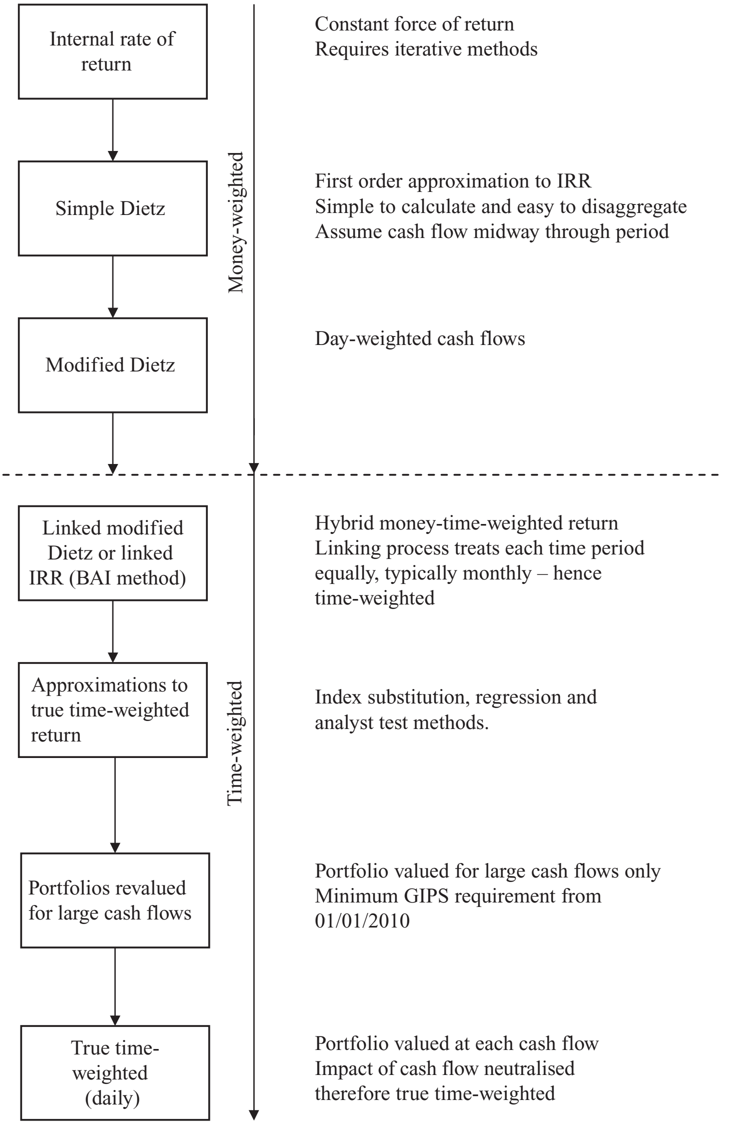

Figure 2.1 Evolution of performance returns

able, or unwilling because of cost, to calculate daily valuations. In reality, the GIPS standards mandate from 2010 a half-way house between linked modified Dietz and true time weighting which requires assets managers to value for large cash flows only.

Internal rate of return is clearly the starting point followed by the simple and modified Dietz approximations to the internal rate of return. Private equity, because of the difficulty of obtaining accurate valuations, is unable to evolve further. Interestingly, real estate has evolved from money-weighted to time-weighted more recently facilitated by more frequent property valuations.

ANNUALISED RETURNS

When comparing returns over long periods it is easier to think in terms of standardised periods; annual returns being the most convenient. The average annual return over a number of years can be calculated arithmetically or geometrically as follows:

$$\text{Arithmetic average or average return} \quad r_A = \frac{f}{n} \times \sum_{i=1}^{i=n} r_i \tag{2.25}$$

$$\text{Geometric average or annualised return} \quad r_G = \left(\prod_{i=1}^{n} (1 + r_i)\right)^{\frac{f}{n}} - 1 \tag{2.26}$$

where: $n =$ the number of periods under analysis
$f =$ the number of periods within the year (monthly $f = 12$, quarterly $f = 4$).

The geometric average or annualised return is the return which, if compounded with itself for the cumulative period, will result in the cumulative return.

Average and annualised returns are calculated in Exhibit 2.22:

Exhibit 2.22 Average and annualised returns

Annual Returns:

2003	10.5%
2002	−5.6%
2001	23.4%
2000	−15.7%
1999	8.9%

Cumulative return:

$$(1.105 \times 0.944 \times 1.234 \times 0.843 \times 1.089) - 1 = 18.2\%$$

Arithmetic average:

$$\frac{10.5\% - 5.6\% + 23.4\% - 15.7\% + 8.9\%}{5} = 4.3\%$$

If the annual average return is compounded for 5 years the cumulative return is considerably greater than 18.2%:

$$(1.043 \times 1.043 \times 1.043 \times 1.043 \times 1.043) - 1 = 23.4\%$$

Geometric average or annualised return:

$$(1.105 \times 0.944 \times 1.234 \times 0.843 \times 1.089)^{\frac{1}{5}} - 1 = 3.4\%$$

As expected, if the annualised return is compounded for 5 years the cumulative return remains the same:

$$(1.034 \times 1.034 \times 1.034 \times 1.034 \times 1.034) - 1 = 18.2\%$$

It is poor performance practice to annualise returns for short periods less than one year. It is inappropriate to assume that the rate of return achieved in the year to date will continue for the remainder of the year.

The terms "arithmetic" and "geometric" are common in the field of performance measurement: arithmetic reflects additive relationships and geometric reflects multiplicative or compounding relationships.

Investment returns compound. When assessing historic performance it is essential to use the constant rate of return that will compound to the same value as the historic series of returns as shown in Exhibit 2.23:

Exhibit 2.23 Positive bias

Assume the rate of return is +20% in the 1st period and −20% in the 2nd period. Assume also a start value of £100:

The value at the end of the 1st period is £120

The value at the end of the 2nd period is £96

The arithmetic average is $\dfrac{20.0\% - 20.0\%}{2} = 0\%$

The annualised return is $(1.2 \times 0.8)^{\frac{1}{2}} - 1 = -2.02\%$

−2.02% compounded over the two periods will generate the accurate end value of £ 96.

Arithmetic averages are positively biased; if returns are not constant the annualised return will always be less than the arithmetic average. Note that the annualised return when compounded with itself over the entire period will reconcile to the original cumulative return; the arithmetic average return will not. Hence, the annualised return provides a better indicator of wealth at the end of the period than the arithmetic average. Performance analysts should use annualised rather than average returns.

Return hiatus

If there is any hiatus or gap in the performance track record, however short, it is impossible to bridge the gap and link performance returns to produce cumulative and hence annualised returns.

Some analysts might argue that it is possible to substitute an index return for the gap but I would argue that it is not best practice.

CONTINUOUSLY COMPOUNDED RETURNS

While simple returns are positively biased, continuously compounded returns are not.

We observe from the operation of our bank accounts that interest paid into our accounts compounds over time, in other words we receive interest on our interest payments. The more frequent the payments the higher the compounded return at the end of the year.

For example, to achieve an equivalent rate of 12% in one year we need only obtain a rate of return in each half year period of 5.83% ($1.0583 \times 1.0583 = 1.12$). The nominal rate of return in each half period required to achieve an effective rate of return of 12% is therefore $5.83\% \times 2 = 11.66\%$.

For n periods in the year we can calculate the effective return r as follows:

$$r = \left(1 + \frac{\tilde{r}}{n}\right)^n - 1 \tag{2.27}$$

where: $\tilde{r}$ = nominal return.

The nominal rate of return in each monthly period required to achieve an effective rate of return of 12% is 11.39%.

If we continue to break down the periods into smaller and smaller periods eventually we find the continuously compounded return or in effect the "force of return":

$$1 + r = \lim_{n \to \infty} \left(1 + \frac{\tilde{r}}{n}\right)^n = e^{\tilde{r}} \tag{2.28}$$

$$\tilde{r} = \ln(1 + r) \tag{2.29}$$

The continuously compounded return required to achieve an effective return of 12% is therefore the natural logarithm of the wealth ratio $\ln(1.12) = 11.33\%$.

The main advantage of continuously compounded returns is that they are additive. The total return can be calculated as follows:

$$\ln(1 + r) = \ln(1 + r_1) + \ln(1 + r_2) + \cdots + (1 + r_n) \tag{2.30}$$

Continuously compounded returns should be used in statistical analysis because unlike simple returns they are not positively biased.

GROSS- AND NET-OF-FEE CALCULATIONS

A key component in long-term investment performance is the fee charged by the asset manager. Fees are charged in many different ways by several different parties, in evaluating and comparing the performance of a portfolio manager it is essential that the impact of fees be appropriately assessed.

There are three basic types of fees or costs incurred in the management of an investment portfolio:

(1) Transaction fees – the costs directly related to buying and selling assets including broker's commission, bid/offer spread, and transaction-related regulatory charges and taxes (stamp duty, etc.), But excluding transaction-related custody charges.
(2) Portfolio management fee – the fees charged by the asset manager for the management of the account.
(3) Custody and other administrative fees including audit fees, performance measurement fees, legal fees and any other fee.

Portfolio managers should be evaluated against those factors that are under their control. Clearly, the portfolio manager has a choice whether or not to buy or sell securities; therefore performance should always be calculated after (or net of) transaction costs. This is naturally reflected in the valuations used in the methods described previously and no more action need be taken.

Portfolio management fees are traditionally taken directly from the account, but need not be; the portfolio manager may invoice the client directly, thereby receiving payment from another source.

If payments were not taken directly from the portfolio then any return calculated would be before or "gross" of fees.

The gross-of-fee effect can be replicated if the fee is deducted from the account by treating the management fee as a negative external cash flow. If the fee is not treated as an external cash flow, then the return calculated is after or "net" of fees.

The gross return is the investment return achieved by the portfolio manager and normally the most appropriate return to use for comparison purposes since institutional clients are normally able to negotiate fees.

Custody and other administration fees are not normally in the control of the portfolio manager and hence should not be reflected in the calculation of performance return for evaluation purposes. It should be noted, however, that the "client return" after administration fees is the real return delivered to the client.

Portfolio managers may bundle all of their services together and charge a "bundled fee". If the bundled fee includes transaction costs that cannot be separated, then the entire fee must be subtracted to obtain the investment return.

In most countries local regulators will require mutual funds to report and advertise their performance net of all fees.

If calculating performance net of fees then to reflect the correct economic value of the portfolio, fees (including performance fees) should be accrued negatively in the valuation.

Estimating gross- and net-of-fee returns

The most accurate way to calculate the gross and net series of returns for a portfolio would be to calculate each set of returns separately, treating the fee as an external cash flow for the gross return but making no adjustment for the net return.

Alternatively, if only one series is calculated (either gross or net) the other can be estimated using the fee rate as follows:

(1) "Grossing up" net returns:

$$r_g = (1 + r_n) \times (1 + f) - 1 \tag{2.31}$$

(2) "Netting down" gross returns:

$$r_n = \frac{(1 + r_g)}{(1 + f)} - 1 \tag{2.32}$$

where: r_g = return gross of portfolio management fee
r_n = return net of portfolio management fee
f = nominal period portfolio management fee rate.

The grossing up of return calculation is normally applied to mutual fund returns which are normally calculated net of all fees (including custody and other administrative fees). The total cost in fees and expenses of a mutual fund is normally expressed as the total expense ratio or TER. The total net return can be grossed up for custody and administration fees only and/or the portfolio management fees depending on the requirement, taking care that regulatory requirements to advertise net of all expenses (or client return) are met. The netting down of return calculation is normally applied to institutional funds, although it is probably far easier and certainly more accurate to calculate net returns directly.

In Table 2.3 actual and estimated grossed-up returns are shown over a period of 6 months. Note the underlying growth in the portfolio has appeared to exaggerate the arithmetic difference between gross and net returns $35.8\% - 35.0\% = 0.8\%$. The expected fee difference for a 1.2% per annum fee over 6 months would be 0.6%. In fact, the geometric difference $1.358/1.35 = 0.6\%$ between gross and net returns will provide a better representation of the fees charged. In this example the estimated gross return is a good estimate of the actual gross return. Timing of fee cash flows, payment in advance or in arrears and the frequency of calculation (i.e. monthly or quarterly) will all have minor impacts on the gross return calculation.

Table 2.3 Gross- and net-of-fees calculations, based on fees at 1.2% per annum calculated monthly in arrears (assume paid at the midpoint of next month)

	Market value (£ m)	Fee (£ m)	Net-of-fee return (%)	Gross-of-fee return (%)	Estimated gross-of-fee (%)
Start Value	100	0.1			
End of month 1	112	0.112	$\frac{112}{100} - 1 = 12.0$	$\frac{112 - 100 + 0.1}{100 - \frac{0.1}{2}} = 12.106$	$1.12 \times 1.001 - 1 = 12.112$
End of month 2	95	0.095	$\frac{95}{112} - 1 = -15.178$	$\frac{95 - 112 + 0.112}{112 - \frac{0.112}{2}} = -15.086$	$0.8482 \times 1.001 - 1 = -15.095$
End of month 3	99	0.099	$\frac{99}{95} - 1 = 4.211$	$\frac{99 - 95 + 0.095}{95 - \frac{0.095}{2}} = 4.313$	$1.04211 \times 1.001 - 1 = 4.315$
End of month 4	107	0.107	$\frac{107}{99} - 1 = 8.081$	$\frac{107 - 99 + 0.099}{99 - \frac{0.099}{2}} = 8.185$	$1.08081 \times 1.001 - 1 = 8.189$
End of month 5	115	0.115	$\frac{115}{107} - 1 = 7.477$	$\frac{115 - 107 + 0.107}{107 - \frac{0.107}{2}} = 7.580$	$1.07477 \times 1.001 - 1 = 7.584$
End of month 6	135		$\frac{135}{115} - 1 = 17.391$	$\frac{135 - 115 + 0.115}{115 - \frac{0.115}{2}} = 17.500$	$1.17391 \times 1.001 - 1 = 17.508$

Six-month return net of fees $135/100 = 35.0\%$.
Actual gross-of-fees return $1.12106 \times 0.84914 \times 1.04313 \times 1.08185 \times 1.0758 \times 1.175 - 1 = 35.794\%$.
Estimated gross-of-fees return $1.12112 \times 0.84905 \times 1.04315 \times 1.08189 \times 1.07584 \times 1.17508 - 1 = 35.810\%$.

Initial fees

Initial fees are normally associated with investment product wrappers around a portfolio of assets. While initial fees will certainly impact the ultimate client return in terms of making judgements about the portfolio manager's performance, they can normally be ignored for both gross- and net-of-fee calculation. Their impact will vary with the time period measured and in many cases initial fees are wholly or partially rebated. Appropriate disclosure should be sufficient.

PORTFOLIO COMPONENT RETURNS

Calculating the performance of the total portfolio is only part of the analytical process. If we are to understand all the sources of return in a portfolio we must calculate the returns of subsets (sectors or components) of assets that contribute to the total return of a portfolio.

The calculation methodologies for component returns are the same as for the total portfolio; however, internal cash flows between components or sectors should be treated as external cash flows. Dividend and coupon payments should be treated as cash flow out of the relevant sector into an appropriate cash sector, that is, if these sectors are defined separately.

Provided return calculation methodologies are consistent the sum of component returns should equal the total portfolio return. This is a key requirement for performance return attribution analysis. Because internal rates of return assume a constant rate of return for all assets within the portfolio it is not appropriate to use internal rates of return to calculate component returns.

Component weight

Both simple and modified Dietz total returns can be broken down or disaggregated into component returns.

Let r_i be the return of the portfolio in the ith component, sector or asset category. Then using modified Dietz:

$$r_i = \frac{{}^iV_E - {}^iV_S - {}^iC}{{}^iV_S + \sum {}^iC_t \times {}^iW_t} \tag{2.33}$$

where: ${}^iV_E$ = end value of sector i
${}^iV_S$ = start value of sector i
${}^iC$ = total cash flow in sector i
${}^iC_t$ = cash flow in sector i on day t
${}^iW_t$ = weighting ratio in sector i on day t.

Then the total portfolio return:

$$r = \sum_{i=1}^{i=n} w_i \times r_i \tag{2.34}$$

where: w_i = weight of the portfolio in the ith asset class.

Now:

$$w_i = \frac{{}^iV_S + \sum {}^iC_t \times {}^iW_t}{V_S + \sum C_t \times W_t} \tag{2.35}$$

Note that:

$$\sum_{i=1}^{i=n} w_i = 1$$

since

$$\sum_{i=1}^{i=n} {}^iV_S = V_S \quad \text{and} \quad \sum_{i=1}^{i=n} \sum {}^iC_t \times {}^iW_t = \sum C_t \times W_t$$

Time-weighted returns can be disaggregated as well. The weight allocated to transactions within the time-weighted period must be the same as that used for the overall portfolio (i.e. beginning of day, middle of day or end of day). Because cash flows will exist between categories within a portfolio it is no longer sufficient to revalue only at the date of an external cash flow. To calculate the time-weighted return for a component or sector, valuations are required for internal cash flows, which in effect require daily valuations.

In rare circumstances, due to transaction activity, the effective weight in a sector, may total zero but still contribute a gain or loss to the overall portfolio. The most common instance of this is a buy transaction (an inflow) in a sector with no current holding but using the end of day cash flow assumption. In these circumstances it is acceptable to use the size of cash flow as the weight for that sector ensuring there is a cancelling cash flow in the cash sector.

It is this problem of apparent zero weights for individual sectors, and most particularly for individual securities, that has led some firms to use a midday weighting assumption, which of course guarantees that the cash flow will be represented in the denominator of the return calculation.

An alternative solution treats inflows at the start of day and outflows at the end of day for all transactions. The impact on the overall return for our standard example is shown in Exhibit 2.24. Unsurprisingly, given the net external cash flow is predominately inflow, the resultant return of −9.42% is close to the start of day assumption return of −9.44%.

Exhibit 2.24 Time-weighted inflow start of day, outflow end of day

Inflow start of day, outflow end of day:		
Market start value	31 December	\$74.2 m
Market end value	31 January	\$104.4 m
External inflow	14 January	\$39.5 m
External outflow	14 January	\$2.4 m
Net external cash flow		\$37.1 m
Market value start of	14 January	\$67.0 m (before inflow)
Market value end of	14 January	\$103.1 m (after outflow)

$$\frac{67.0}{74.2} \times \frac{103.1 + 2.4}{67.0 + 39.5} \times \frac{104.4}{103.1} - 1 = 0.9030 \times 0.9906 \times 1.0126 - 1 = -9.42\%$$

Although both midday weighting and inflow start of day and outflow end of day approaches remove the zero weighting problem they are systems-inspired solutions. I prefer end of day as a logical timing for all cash flows. If this method results in a zero weight in an individual category it is appropriate to default to start of day for that sector only with a balancing adjustment in the cash sector. Therefore the overall return is unaffected by this adjustment and the sum of the parts still equals the total because of the equalising adjustment in the cash sector.

A zero weight can also arise in a modified Dietz calculation on the last and first day of trading for outflows and inflows respectively. A similar default to that described above is appropriate in such circumstances.

Another typical problem occurs when a single line of stock or whole sector is entirely sold when dividend entitlements still exist for payment in the next period.

Short positions

Portfolio managers are increasing using derivatives and shorting assets as they seek additional ways to add value to their portfolios. When a portfolio manager takes a short position in a stock they are taking the view that the price will fall. The performance of the underlying asset is unchanged whether or not the holding is long or short. For presentational purposes portfolio managers often prefer to show a falling price as a positive return when the holding is short. This is incorrect, in reality the negative weight of the short position together with the negative return of the falling asset price combine to produce a positive contribution to performance. This convention must be maintained particularly when calculating component performance.

If the weight of the holding changes sign it is impossible to chain link (or compound) the performance of the underlying holding.

Overlay strategies

In traditional investment strategies the client will provide funds to be physically invested across a universe of assets. In addition, clients may wish to enhance returns or manage risk of these core physically invested assets by using other independent asset managers.

Overlay strategies utilise derivative instruments with reference to the underlaid or core assets. The overlay manager may have little or no physical assets to invest resulting in small, zero or negative asset values. Traditional measurement methodologies using negative asset values are clearly inappropriate. The value of the assets underlaid or their notional value should be introduced into the denominator of the return calculation. Thus, the denominator of the return calculation for overlaied assets will include both the notional value of the overlaid assets plus the value including unrealised gains and losses of the overlaid assets themselves. Naturally, the numerator is derived from the overlay assets exclusively.

Carve-outs

Subsector returns of larger portfolios are often calculated and presented separately by asset managers to demonstrate competence in managing assets of that type, particularly if the manager does not manage that type of asset in stand-alone portfolios. These subsector returns are often called “carve-outs”.

Cash equivalents in a portfolio should be measured using the same methodology as any other asset within the portfolio. Because the cash sector naturally suffers large and frequent cash flows the calculation assumptions with regard to cash flow may result in a return which on face value is most unlike a cash return: crucially, the combination of average weight and return will replicate the contribution of the cash component to total return.

Multi-period component returns

Portfolio managers will make asset allocation decisions into different portfolio components over time. The timing of these decisions will impact the overall return of the portfolio. It is entirely possible that the overall portfolio return is less or more than all of the component returns, see Exhibit 2.25:

Exhibit 2.25 Multi-period component returns

Period	Equity weight (%)	Fixed income weight (%)	Equity return (%)	Fixed income return (%)	Total return (%)
Q1	20	80	10.4	2.3	*3.9*
Q2	60	40	3.5	1.3	*2.6*
Q3	90	10	−15.7	1.4	*−14.0*
Q4	30	70	12.7	8.7	*9.9*
Year	50	50	8.6	14.2	*0.8*

In Exhibit 2.25 both the equity and fixed income returns exceed the total return for the full year by a considerable margin. The total return is so low because of high weight in equities in the quarter 3. The timing of asset allocation decisions can make a significant difference.

BASE CURRENCY AND LOCAL RETURNS

Clearly, international portfolios will include assets denominated in foreign currencies. The methods of calculation previously described are still appropriate to calculate returns provided the securities denominated in foreign currencies are converted at appropriate exchange rates.

The impact of currency on a single asset portfolio is shown in Exhibit 2.26:

Exhibit 2.26 Currency returns

	Portfolio value ($)	Exchange rate ($:€)	Portfolio value (€)
Start value	100*	1:0	$100 × 1.0 = €100
End value	110#	1:1	$110 × 1.1 = €121

* 100 units of stock priced at $1.0
\# 100 units of stock priced at $1.1

Note that the dollar buys €1.1 at the end of period compared to €1.0 at the start; the dollar has increased in value.

The return of the portfolios in dollars is:

$$\frac{110}{100} = 1.1 \quad \text{or a return of 10\%}$$

The currency return is:

$$\frac{1.1}{1.0} = 1.1 \quad \text{or a return of 10\%}$$

The return of the portfolio in euros is:

$$\frac{110 \times 1.1}{100 \times 1.0} = \frac{121}{100} = 1.21 \quad \text{or a return of 21\%}$$

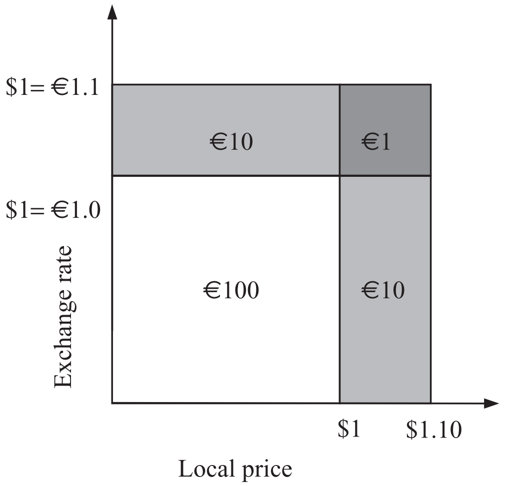

Figure 2.2 Components of base currency return

Note that the portfolio return in Exhibit 2.26 is not the addition of the local return and currency return but the compound of returns:

$$(1 + r_L) \times (1 + r_C) = (1 + r) \tag{2.36}$$

whers: r_L = return in local currency
r_C = currency return.

r is the return of the portfolio in the base currency, the currency denomination or reference currency of the portfolio. Figure 2.2 illustrates the currency impact on portfolio valuation. The shaded top right-hand quadrant (€1) represents the combined impact of market returns with currency return

The base currency returns of the portfolio can be readily converted to any other currency return for presentation purposes as follows:

$$(1 + r) \times (1 + c) - 1 = r_C \tag{2.37}$$

where: c = currency return relative to the base currency
r_C = portfolio return expressed in currency c.

Local currency returns are calculated ignoring the impact of changes in the currency exchange rate. Although in reality the local currency return of a portfolio consisting of assets in multiple currencies does not exist, it is useful to make an intermediate calculation. The local return of a multi-currency portfolio is defined as the weighted average local return for assets in each currency as follows:

$$r_L = \sum_{j=1}^{j=n} w_i \times r_{Li} \tag{2.38}$$

where: w_i = weight of sector i
r_{Li} = local return of sector i.

The ratio of the base currency return with the local return of the portfolio will calculate the implicit return due to currency in the portfolio.

3
Benchmarks

Business is a good game – lots of competition and a minimum of rules. You keep score with money.
Atari founder Nolan Bushnell (1943–)

BENCHMARKS

Measuring the return of a portfolio in isolation provides only part of the story; we need to know if the return is good or bad. In other words, we need to evaluate performance return and risk against an appropriate benchmark.

Benchmark is defined in the *Oxford English Dictionary* as a standard or point of reference. Not readily translated into other languages it derives from a surveyor's mark cut in a wall and used as a reference point in measuring altitudes.

Benchmark attributes

Good benchmarks should have the following attributes:

(1) *Appropriate*. The chosen benchmark must be relevant to the appropriate investment strategy. It is essential that the benchmark matches the client's requirements.
(2) *Investible*. The portfolio manager should be able to invest in all the securities included in the benchmark. If not there will always be an element of relative performance for which the portfolio manager has no control.
(3) *Accessible*. To allow the portfolio manager to construct the portfolio against the benchmark it is essential that there is access not only to the returns of the benchmark but to the constituent elements and their weights at the start of the period of measurement.
(4) *Independent*. An independent third party should calculate all benchmark return to ensure a fair comparison.
(5) *Unambiguous*. The chosen benchmark should be clear and unambiguous. It is bad performance measurement practice to measure performance against more than one benchmark and to change benchmarks retrospectively.

There are three main forms of benchmark used in the evaluation of portfolio performance, "indexes", "peer groups" and "random portfolios", a cross between indexes and peer groups. Students of Latin may disagree but according to the *Oxford English Dictionary* both indexes and indices are acceptable plurals of index, I prefer to use indexes.

Commercial indexes

Commercial indexes and averages have been available for many decades and continue to be invented to the present day. The first indexes were calculated to measure price inflation. Today, index providers create indexes for a variety of reasons, to promote investment in certain markets (for example, emerging markets), to provide added services for trading clients, to provide a broad indicator of market movements, to promote the use of derivative instruments and to generate revenue in their own right.

Index providers create "intellectual property" in the construction of indexes in terms of the selection of securities to be included, calculation formulae, weighting schemes and construction rules. The relevance of the index to the asset manager or ultimate client will depend very much on the investment strategy and the original purpose of the index.

Calculation methodologies

For effective portfolio construction managers should be cognisant of the calculation methodologies and the rules for including and excluding various securities.

Indexes may be calculated excluding income (price index) or including income (total return). Performance measurers should always use total return indexes.

Aggregate price index (price-weighted index)

The simplest index can be constructed by adding the share prices of the constituent securities together and dividing by a divisor.

The index at time t is:

$$I_t = \frac{I_0 \times \sum_{i=1}^{n} P_{t,i}}{D_t} \tag{3.1}$$

where: I_0 = the base index number (e.g. 100 or 1000)
$P_{t,i}$ = the price of security i at time t
D_t = the value of the divisor at time t after adjusting for past capital and constituent changes.

Note that:

$$D_0 = \sum_{i=1}^{n} P_{0,i} \tag{3.2}$$

If there is a capital or constituent change at time t, the new divisor D'_t is calculated from the former divisor as follows:

$$D'_t = \frac{\sum_{i=1}^{i=n} P'_{t,i}}{\sum_{i=1}^{i=n} P_{t,i}} \times D_t \tag{3.3}$$

where: $P'_{t,i}$ = is the price of constituent i immediately after the change.

Thus, the notional price of the security determines the weighting of the security in the index. The Dow Jones Industrial Average first published in 1928 with 30 securities and the Nikkei Stock Average first published in 1950 with 225 securities are examples of aggregate price indexes.

Geometric (or Jevons-type) index

Geometric indexes are calculated by multiplying the wealth ratios of each security and taking the nth root of the product and multiplying by the base index value:

$$I_t = I_0 \times \left(\frac{P_{t,1}}{P_{0,1}} \times \frac{P_{t,2}}{P_{0,2}} \times \ldots \frac{P_{t,n}}{P_{0,n}} \right)^{\frac{1}{n}} \tag{3.4}$$

A percentage change in the price of any constituent affects the index to the same extent. Although a form of equal weighting, geometric indexes are impossible for portfolio managers to replicate in real portfolios and therefore not appropriate indexes.

In theory, if one constituent falls to zero the whole index falls to zero. In practice, a failing security would be removed from the index before it reached zero. The FT30 index established in 1935 is an example of a geometric index.

Market capitalisation index

Price and geometric indexes are neither investible nor accessible and hence most indexes used in the context of performance measurement are market capitalisation indexes in which the weight of each security in an index is normally determined by its market capitalisation (price multiplied by number of shares in issue); however, this may be adjusted by a free float factor if an index provider determines that not all the market capitalisation of a security is available to the general investing public.

For international indexes the total weight of respective countries is determined by the sum of securities in that country qualified for entry into the index. In some indexes entire country weights might be adjusted to better reflect the economic strength of that country by reweighting in proportion to GDP.

Laspeyres index

The majority of market capitalisation indexes are a form of Laspeyres index, a methodology proposed by the German economist Étienne Laspeyres (1834–1913) for measuring price inflation. In a Laspeyres index the quantity of each product, in this case the weight of a security, is fixed at the start of the period. The index return is calculated for each finite period, typically one month and then monthly returns are chain linked.

Therefore a Laspeyres index is calculated for period t as follows:

$$\text{Laspeyres index } b = \sum_{i=1}^{i=n} W_i \times \frac{P_{t,i}}{P_{t-1,i}} \tag{3.5}$$

where: W_i = weight of security i at the beginning of period.

Replacing the return performance of securities with the return of each component sector return within the benchmark we obtain the standard formula for most commercial indexes:

$$b = \sum_{i=1}^{i=n} W_i \times b_i \tag{3.6}$$

Paasche index

Paasche indexes developed by the German economist Hermann Paasche (1851–1925) are also used for measuring price inflation but use the end of period rather than start of period weight for the quantity of each product. In the context of price inflation the distinction between start and end of period is important since consumers will typically react to price changes by changing the quantities they buy; if prices go up then quantity should fall. Hence, Laspeyres indexes systematically overstate inflation and Paasche indexes understate inflation. Therefore Paasche indexes are not investible from a portfolio manager's point of view since the quantity is unknown until the end of the period; Laspeyres indexes are more relevant in the context of performance measurement:

$$\text{Paasche index } b_p = \sum_{i=1}^{i=n} W_i^E \times \frac{P_{t,i}}{P_{t-1,i}} \tag{3.7}$$

where: W_i^E = weight of security i at the beginning of period.

Marshall–Edgeworth index

There are a few methods that attempt to negate the natural bias of Laspeyres and Paasche indexes. The Marshall–Edgeworth Index (proposed by Alfred Marshall and Francis Ysidro Edgeworth) takes the arithmetic average between the start and end weights as follows:

$$\text{Marshall–Edgeworth index } b_{M-E} = \sum_{i=1}^{i=n} \frac{(W_i + W_i^E)}{2} \times \frac{P_{t,i}}{P_{t-1,i}} \tag{3.8}$$

Fisher index

Also wishing to negate the natural bias of Laspeyres and Paasche indexes Irving Fisher* suggests the geometric mean of the of the Laspeyres and Paasche indexes as follows:

$$\text{Fisher index } b_F = \sqrt{b_P \times b} \tag{3.9}$$

Neither Marshall–Edgeworth nor Fisher indexes are investible and hence are rarely used in the context of portfolio performance measurement.

Equal-weighted indexes

Some would argue that equal-weighted indexes, in which each security is given equal weight, represent a fair index for portfolio managers because they have an equal opportunity to buy each stock; arguing that capitalisation weights are arbitrary. Unfortunately, such indexes give equal weight to relatively illiquid smaller companies by definition and therefore are not practical for the entire industry to use.

Equal-weighted indexes are also difficult for passive managers to track. To maintain equal weight positively performing stocks must be sold and negatively performing stocks must be bought, generating unnecessary transaction costs and of course bad news if stock returns are

* Irving Fisher (1867–1947) president of the American Statistical Association in 1932, not to be confused with Sir Ronald Aylmer Fisher (1890–1962) Professor of Genetics at University College London 1943–1957.

persistent; continuously selling a good performing stock is not necessarily a good strategy if the stock continues to perform well.

Fundamental indexes

Yet again others would argue that market capitalisation is a very poor choice of weight, tending to overweight overvalued stocks and underweight undervalued stocks. Alternative indexes known as fundamental indexes use other characteristics of stocks to establish appropriate weights including book value, sales, free cash flow or dividends.

Ultimately, the key determinant will be the client's requirements. Clients will be concerned about the coverage and concentration of indexes. Coverage is the percentage by market capitalisation of securities included in the index compared to the total market capitalisation of all securities in that market. Concentration measures the percentage weight of the top few securities in the index. A highly concentrated index may well introduce significant specific risk for the client.

Currency effects in benchmark

Benchmark base currency and local returns are linked using a similar approach to Equation (2.36):

$$(1 + b_L) \times (1 + b_C) = (1 + b) \tag{3.10}$$

where: b_L = return in local currency
b_C = currency return.

Hedged indexes

Many index providers calculate index returns in a specific base currency, in local currency terms and hedged back to the base currency.

Hedged calculation methods differ, one approach being to sell notional 1-month forward contracts at the start of the period of measurement and calculating a return based on the gain or loss on those contracts in conjunction with the gain or loss in the underlying assets. This method does not hedge dynamically. Currency positions caused by the market gains of the underlying assets during the month are therefore not hedged leading paradoxically to a residual currency element in the hedged return.

Alternatively, the local currency return can be compounded with the interest rate differential (between the base currency and the currency of the underlying asset) as in Equation (3.11):

$$\text{Hedged return } b_{Hi} = (1 + b_{Li}) \times (1 + d_i) - 1 \tag{3.11}$$

where: b_{Li} = local benchmark return for category i
b_{Hi} = benchmark return for category i hedged back to the base currency
d_i = interest rate differential.

Since there is no currency exposure in a hedged return the only contributing factor to the difference between the local and hedged returns is the interest rate differential. The hedged return will be greater than the local return if interest rates are lower in the underlying currency than the base return, if d_i is positive.

Alternatively, the hedged return can be derived from the base return as follows:

$$\text{Hedged return } b_{Hi} = \frac{(1+b_i)}{(1+f_i)} - 1 \tag{3.12}$$

where: f_i = return on forward currency contract (forward currency return).

Customised (or composite) indexes

Generic indexes are irrelevant to clients that have specific requirements based on their liabilities. Increasingly clients require benchmarks that are customised to their own requirements.

Benchmarks derived or customised from multiple indexes are often called composite indexes (not to be confused with indexes allocated to a composite of portfolio manager returns).

Index providers in the past may have been more relaxed about the reuse of their data in customised form but are increasingly seeking to leverage their intellectual capital and will often charge for the extended use of their data. Asset managers should ensure that they are licensed to use the index data in the form they want to use it.

Index information is normally provided in a specified base currency with local and possibly hedged returns. Given base local and hedged returns it is relatively simple to convert these returns into any alternative base currency.

The sample index in Table 3.1 is customised to exclude Australia in Exhibit 3.1:

Table 3.1 Sample index data

	Weight (%)	Base currency return (£)	Local return (%)	Hedged return (%)
UK	20	15.00	15.00	15.00
Norway	4	−4.21	−7.00	−7.19
Sweden	3	−3.10	−5.00	−5.10
France	15	−10.75	−15.00	−14.75
US	35	6.70	10.00	10.55
Japan	20	26.50	15.00	15.81
Australia	3	20.75	5.00	4.48
Total	*100*	*9.39*	*6.97*	*7.34*

From Table 3.1 currency returns and hedge differentials can be calculated for each currency as shown in Table 3.2. Exhibit 3.2 demonstrates how to convert the base currency of an index:

Exhibit 3.1 Customised index

Contribution to total return excluding Australia:

$20\% \times 15.0\% + 4\% \times -4.21\% + 3\% \times -3.1\% + 15\% \times -10.75\%$

$35\% \times 6.7\% + 20\% \times 26.5\% = 8.77\%$

Customised index excluding Australia:

$$\frac{8.77}{1-3\%} = 9.04\%$$

Table 3.2 Index currency returns

	Currency return $\frac{b_i}{b_{Li}} - 1$	Interest differential $\frac{b_{Hi}}{b_{Li}} - 1$
Norwegian krona	$\frac{0.9579}{0.93} - 1 = 3.0\%$	$\frac{0.9281}{0.93} - 1 = -0.2\%$
Swedish krone	$\frac{0.929}{0.95} - 1 = 2.0\%$	$\frac{0.949}{0.95} - 1 = -0.1\%$
Euro	$\frac{0.8925}{0.85} - 1 = 5.0\%$	$\frac{0.8525}{0.85} - 1 = 0.3\%$
US dollar	$\frac{1.067}{1.1} - 1 = -3.0\%$	$\frac{1.1055}{1.1} - 1 = 0.5\%$
Yen	$\frac{1.265}{1.15} - 1 = 10.0\%$	$\frac{1.1581}{1.15} - 1 = 0.7\%$
Australian dollar	$\frac{1.2075}{1.05} - 1 = 15.0\%$	$\frac{1.0448}{1.05} - 1 = -0.5\%$

Exhibit 3.2 Benchmark currency conversions

Converting the sterling-based customised index in Exhibit 3.1 to an Australian dollar return:

$$\frac{1.0904}{1.15} - 1 = -5.18\%$$

Table 3.3 Fixed weight and dynamised benchmarks

Fixed weights	1st quarter		2nd quarter		3rd quarter		4th quarter		Year
	Weight (%)	Return	Weight (%)	Return	Weight (%)	Return	Weight (%)	Return	Return
Equities	50	10.4	50	3.5	50	−15.7	50	12.7	*8.56*
Bonds	50	2.3	50	1.3	50	1.4	50	8.7	*14.22*
Total		*6.35*		*2.4*		*−7.15*		*10.70*	*11.94*
Floating weights									
Equities	50	10.4	51.90	3.5	52.44	−15.7	47.83	12.7	*8.56*
Bonds	50	2.3	48.10	1.3	47.56	1.4	52.17	8.7	*14.22*
Total		*6.35*		*2.44*		*−7.57*		*10.61*	*11.39*

Fixed weight and dynamised benchmarks

Customised indexes are often defined using fixed weights for certain asset categories. The performance of assets categories will diverge over time thus impacting the original strategic asset allocation. It is essential that the frequency of fixed weight rebalancing is established within the benchmark definition. If the initial fixed weight is allowed to float with the performance of the individual categories the impact compared to a genuine fixed weight can be significant as demonstrated in Table 3.3.

Exhibit 3.3 illustrates how the floating weights are calculated for each quarter. The quarterly balanced fixed weight index outperforms the floating weight index. This outperformance is for the most part generated in the third quarter; the fixed weight index is required to reduce the

Exhibit 3.3 Dynamised benchmark

Applying the 50%:50% weight to the annual returns the total benchmark return is:

$$50\% \times 8.56 + 50\% \times 14.22 = 11.39$$

To achieve this return using quarterly data we need to reweight each quarter to reflect underlying market movements:

2nd quarter weights:

$$50\% \times 10.4\% = 55.2\% \quad 50\% \times 2.3\% = 51.15\% \quad \text{Total } 106.35\%$$

Revised weights $\dfrac{55.2\%}{106.35\%} = 51.9\% \quad \dfrac{51.15\%}{106.35\%} = 48.1\%$

3rd quarter weights:

$$51.9\% \times 3.5\% = 53.72\% \quad 48.1\% \times 1.3\% = 48.72\% \quad \text{Total } 102.44\%$$

Revised weights $\dfrac{53.72\%}{102.44\%} = 52.44\% \quad \dfrac{48.72\%}{102.44\%} = 47.56\%$

4th quarter weights:

$$52.44\% \times -15.7\% = 44.21\% \quad 47.56\% \times 1.4\% = 48.23\% \quad \text{Total } 92.43\%$$

Revised weights $\dfrac{44.21\%}{92.43\%} = 47.83\% \quad \dfrac{48.72\%}{92.43\%} = 51.17\%$

With the revised weights the quarterly benchmark returns compound to 11.39%, using fixed weights the benchmark return compounds to 11.94% – a significant difference.

exposure to equities at the beginning of the period immediately before a big fall in the market. Note that to replicate the annual return resulting from applying the 50%:50% fixed weight to the category annual returns, floating weights must be used each quarter.

Capped indexes

Due to regulatory requirements or the specific requirements of the clients, customised indexes often include maximum limits for securities, countries or industrial sectors, etc. These limits should be reflected by fixed weights in the customised index, either at the maximum limit or to allow the portfolio manager an overweight allocation at a fixed level lower than the limit.

Blended (or spliced) indexes

It is possible to change the index associated with the measurement of a specific portfolio over time, a good example being after a change in investment strategy. It is bad performance measurement practice to change the associated index retrospectively; therefore a blended or spliced index should be calculated in order to maintain the long-term return series of the associated benchmark. This can be achieved by chain linking the respective indexes.

Money-weighted benchmarks

Generally speaking, all commercially produced indexes are time-weighted; there is no need to adjust for cash flow between performance periods (although from time to time indexes are subject to corporate actions of various types that in effect generate cash flow). Money-weighted portfolio returns are impacted by cash flow. To ensure a like-for-like comparison a commercial index return can be recalculated to reflect the cash flow experience of the portfolio. Clearly, the money-weighted benchmark will differ from the commercially published return and is unique to the portfolio's cash flow experience.

BENCHMARK STATISTICS

Index turnover

Securities are constantly added and deleted from commercial indexes caused by takeover, business failure or simply major changes in market capitalisation. Most indexes suffer no transaction costs to effect these changes; however, portfolio managers will suffer costs:

$$\text{Index turnover} = \frac{\text{Market cap (additions + deletions)}}{\text{Average total market} \times 2} \tag{3.13}$$

Turnover will be greatest in mid-cap indexes due to traffic in both directions including securities promoted into the large-cap index and previously large-cap securities falling into the mid-cap index.

All other things being equal an index with high turnover will be more difficult for a portfolio manager to outperform because of potentially higher transaction costs. Portfolio managers start with a small structural disadvantage when compared to indexes.

Up capture indicator

$$\text{Up capture indicator} = \frac{\bar{r}^{+}}{\bar{b}^{+}} \tag{3.14}$$

where: $\bar{b}^{+}$ = average positive benchmark return
$\bar{r}^{+}$ = average portfolio return for each period in which the benchmark return is positive.

The up capture indicator divides the average portfolio return by the average benchmark return for each period for which the benchmark return is positive. The greater the value the better.

Down capture indicator

$$\text{Down capture indicator} = \frac{\bar{r}^{-}}{\bar{b}^{-}} \tag{3.15}$$

where: $\bar{b}^{-}$ = average negative benchmark return
$\bar{r}^{-}$ = average portfolio return for each period in which the benchmark return is negative.

The down capture indicator divides the average portfolio return by the average benchmark return for each period for which the benchmark return is negative. Lower values are preferred.

Up number ratio

The up number ratio measures the percentage of returns in each measurement period for which the portfolio returns are greater than zero when the benchmark returns are greater than zero.

Ideally the ratio should by 100%, the closer to 100% the better.

Down number ratio

The down number ratio measures the percentage of returns in each measurement period for which the portfolio returns are less than zero when the benchmark returns are less than zero.

Ideally, but very rarely, the ratio should be 0%. The lower the ratio the better, although for highly correlated returns ratios close to 100% should be expected.

Up percentage ratio

More interestingly the up percentage ratio measures the percentage of periods in which the excess return of the portfolio against the benchmark is greater than zero in each measurement period when the benchmark return is greater than zero.

In other words, how often does the portfolio manager outperform a rising market?

Down percentage ratio

The down percentage ratio measures the percentage of periods in which the excess return is greater than zero in each measurement period when the benchmark return is less than zero.

Or in other words, how often does the portfolio manager outperform a falling market.

Percentage gain ratio

$$\text{Percentage gain ratio} = \frac{n_r^+}{n_b^+} \tag{3.16}$$

where: n_r^+ = number of portfolio returns greater than zero
n_b^+ = number of benchmark returns greater than zero.

Clearly, higher percentage gain ratios are better.

PEER GROUPS AND UNIVERSES

Peer groups are collections of competitor portfolios of similar strategies grouped together to provide both an average and range of competitor returns.

Some would argue that peer groups offer a more appropriate comparison than indexes for a portfolio manager because they offer a genuine alternative to the client, and because they consist of real portfolios they suffer transaction costs, therefore comparisons are on a like-for-like basis. Portfolio managers are always up against an inbuilt disadvantage in that indexes never suffer transaction costs.

There are a number of disadvantages associated with peer groups; first, you are reliant on the independent peer group compiler to control the quality of the peer group. Comparisons are only relevant between similar strategies if the peer group compiler adopts loose entry criteria; the peer group may be larger but may consist of widely divergent strategies.

Peer groups generate different challenges for portfolio managers, not only are they required to make good investment decisions but they must have a good understanding of what their competitors are doing. For example, if a portfolio manager likes IBM, against the index it is easy to ensure the position is overweight, but in the peer group there is an element of guesswork involved, the portfolio manager must guess the average weight in the competitor portfolios and then determine the weight of IBM required. A peer group benchmark requires an additional competency from the portfolio manager.

Peer groups suffer "survivorship bias"; poor performing portfolios are either closed or removed from the universe because the asset manager is unwilling to keep a poor performing portfolio in the survey. The result is increasing, good, long-term performance of the peer group as poor performing portfolios cease to belong to the long-term track record.

Percentile rank

One way of describing the relative performance of a portfolio or fund in a peer group is to provide the rank (or position) of the portfolio compared to the total number of portfolios in the universe.

Raw ranks, however, are very hard to compare against peer groups of different size. To allow comparison we can convert the raw rank to an equivalent rank based on a total peer group size of one hundred using the following formula:

$$\text{Percentile rank } \frac{n-1}{N-1} \tag{3.17}$$

where: n = the raw rank of the portfolio in its peer group universe
N = the total number of portfolios in the peer group universe.

The percentile rank's function is to rank portfolios between 0% and 100%, 0% being the top-ranked portfolio and 100% being the bottom-ranked portfolio. There are other methodologies for calculating percentile rank but this method at least ensures that the middle rank portfolio, the median, has a percentile rank of 50%. For example, the percentile rank of the portfolio ranked 8 out of a peer group size of 15 (the median portfolio) is calculated as follows:

$$\frac{8-1}{15-1} = \frac{7}{14} = 50\%$$

To test if the percentile ranking methodology is sound simply calculate the percentile rank of the middle rank fund in an odd sample size; if it is not exactly 50% it is a poor methodology.

Percentile ranks are often banded as follows:

0%–25%	1st quartile
25%–50%	2nd quartile
50%–75%	3rd quartile
75%–100%	4th quartile

Quintiles (20% bands) and deciles (10% bands) are also common.

An extremely useful way of showing peer group information is in the form of a floating bar chart as shown in Figure 3.1

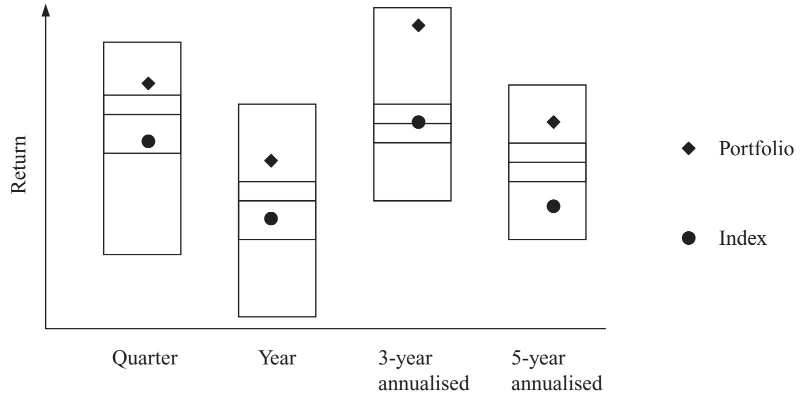

Figure 3.1 Floating bar chart

The bars represent the range of returns of the peer group, which are banded by quartiles. Note the bandwidth for the 2nd and 3rd quartiles are much narrower than the 1st and 4th quartiles indicating a normal distribution of returns. For convenience many peer group managers ignore the top and bottom 5% of returns to ensure the distribution can fit onto the page.

In this example it is easy to see that the portfolio has performed well within the 1st quartile for all periods as well as outperforming the index.

RANDOM PORTFOLIOS

Peer groups and indexes both have their critics – peer groups because of survivorship bias, inefficient construction, investability and achievability issues and indexes because of the choice of stock weightings and the tendency for portfolio managers to be closet indexers, that is to say reducing the asset manager's business risk by taking only small deviations from index weights. Closet indexers will neither underperform nor outperform significantly since they are taking little relative risk; clients presumably pay portfolio managers to take risk on their behalf, and hence closet indexers are not acting in the best interests of their clients.

Random portfolios avoid most of these issues. Using Monte Carlo simulations we can create all of the possible outcomes given the restraints of the investment management mandate. If the actual performance of the portfolio manager is toward the top of all the possibilities the performance is good; if toward the bottom performance is bad. Since the construction of the median portfolio is unknown beforehand it is impossible to closely follow the index and of course there are no competitor construction or survivorship bias issues. Whether a random portfolio is investible or achievable is debatable; it is certainly impossible to construct the portfolio to match the median return of all possible outcomes at the outset of the period; the manager is required to take positions from the very start.

NOTIONAL FUNDS

Notional funds are specific benchmark calculations used to replicate or simulate the impact of unique effects or calculation conditions within real portfolios. The analyst test method in Equation (2.23) utilises notional funds. In this method the benchmark return is recalculated with the cash flow experience of the real portfolio thus replicating the error in the money-weighted return calculation.

Notional funds can be used as alternative benchmarks that include the cost of transactions resulting from external cash flow.

The main disadvantage of notional funds is that the adjusted return is clearly different from the published index return and unique to the cash flow or transaction experience of that portfolio. It makes the comparison of portfolio to notional fund performance more accurate but does not aid the comparison of performance of multiple different portfolios.

Normal portfolio

Commercial indexes may not adequately reflect the investment options open to individual portfolio managers. Normal portfolios provide an alternative benchmark consisting of specific securities available for investment (for example, the recommended list of securities from the in-house research team).

Although a useful comparison against the real alternatives of the portfolio manager they lack the stamp of independence required in the definition of a good benchmark.

Growth and value

Growth investors look for companies that are likely to have strong earnings growth. Strong earnings growth will lead to higher dividends encouraging other investors to buy leading to higher prices. Growth investors are less concerned about the current value and more concerned about future prospects.

On the other hand, value investors look for companies whose value is not reflected in the current price. Value investors will use valuation tools like discounted cash flow to calculate a current fair price – if the market price is less than the calculated fair price the value investor is likely to buy assuming other investors will eventually realise the company is undervalued.

In the technology boom growth investors fared best because they bought loss-making companies on the expectation of future profits with valuations often based on multiples of revenue (real or manipulated) not profits. Value investors are unlikely to buy new loss-making companies.

Portfolios managed with a growth or value strategy should be measured against growth or value indexes. The index should be appropriate to the portfolio manager's style.

EXCESS RETURN

Given portfolio and benchmark returns it is only natural to make a comparison and calculate the difference in performance or "excess return".

There are two common measures of the excess return, "arithmetic" and "geometric"

Arithmetic excess return

Arithmetic excess return is the profit in excess of a notional or benchmark fund expressed as a percentage of the initial amount invested:

$$a = r - b \tag{3.18}$$

where: a = arithmetic excess return
b = benchmark return.

Geometric excess return

Geometric excess return is the profit in excess of a notional or benchmark fund expressed as a percentage of the final value of the notional or benchmark fund:

$$g = \frac{1+r}{1+b} - 1 \tag{3.19}$$

where: g = geometric excess return.

In both these definitions the added value (or profit) is identical. Both definitions attempt to explain the same added value in cash terms. The arithmetic excess return explains the added value relative to the initial amount invested and the geometric excess return explains the same added value but relative to the notional fund or the amount expected if the client had invested in the benchmark.

This simple difference is crucial, as demonstrated in Exhibit 3.4:

Exhibit 3.4 Arithmetic and geometric excess returns

Portfolio start value = \$1 000 000

Portfolio end value = \$1 070 000

$$\frac{1\,070\,000 - 1\,000\,000}{1\,000\,000} = 7\%$$

Notional fund start value = \$1 000 000 (i.e. same as the portfolio start value). For benchmark performance assume a 5% return. Therefore notional fund end value = \$1 050 000. The portfolio's added value above the benchmark or notional fund is:

$$\$70\,000 - \$50\,000 = \$20\,000$$

The arithmetic excess return:

$$\frac{\$20\,000}{\$1\,000\,000} = 2\%$$

or, alternatively:

$$7\% - 5\% = 2\%$$

The geometric excess return or the increase in value of the portfolio compared to what would have been achieved if invested in the benchmark is:

$$\frac{\$1\,070\,000 - \$1\,050\,000}{\$1\,050\,000} = 1.9\%$$

or

$$\$20\,000/\$1\,050\,000 \quad \text{or} \quad \frac{1.07}{1.05} - 1 = 1.9\%$$

In other words the portfolio is 1.9% larger than it would have been had it been invested in the benchmark.

Both versions of excess return are used worldwide with neither dominating globally. I strongly prefer the geometric version and fully expect this to be standard in the long term. The three main arguments for using geometric excess returns are quite persuasive:

(1) Proportionality.
(2) Convertibility.
(3) Compoundablity.

Most arguments in favour of using arithmetic excess returns centre on its ease of use, simplicity and intuitive feel. It is clearly easier to subtract two numbers, rather than to calculate a wealth ratio. Even proponents of the use of geometric excess returns will occasionally resort to arithmetic excess returns when reporting to clients. They argue that it's simply not worth valuable time discussing why the ratio of returns is favoured over a more simple subtraction, the message of the report is more important.

To some extent I would agree, but the arithmetic or geometric debate is not a continuous discussion. Once understood, you need not have the same debate with the client again.

In fact, the geometric excess return is *more*, not less intuitive to the layperson.

The pension fund client is most concerned about the value of the portfolio at the end of period (not the start), and quite rightly should ask how much larger the portfolio is now, than it would have been had the portfolio invested in the benchmark. This is the natural question of pension fund trustees, particularly before calculating rates of return confuses the issue.

Rearranging Equation (3.19) we see that there is a relationship between arithmetic and geometric excess return:

$$\frac{1+r}{1+b} - 1 = \frac{1+r}{1+b} - \frac{1+b}{1+b} = \frac{r-b}{1+b} \tag{3.20}$$

This relationship is important, it demonstrates that in rising markets the arithmetic excess return is always greater than the geometric excess return and in falling markets the reverse is true. If I were a cynic I would suggest that asset managers prefer arithmetic excess returns because they look better in most market conditions.

The geometric excess return represents a better measure of the relative added value of the asset manager's performance – it is proportionate as demonstrated in Exhibit 3.5:

Exhibit 3.5 Proportionality

Portfolio start value \$1 000 000

Portfolio end value \$500 000

Portfolio return -50%

Notional fund start value \$1 000 000

Notional fund end value \$250 000

Notional or benchmark return -75%

Arithmetic excess return $= -50\% - (-75\%) = +25\%$

Geometric excess return $= \dfrac{0.5}{0.25} - 1 = +100\%$

The geometric excess return correctly demonstrates that the portfolio is double the size that would have been achieved by investing in the benchmark.

The most convincing argument for using geometric excess returns is their convertibility across different currencies. Exhibit 3.6 illustrates the impact of reporting excess returns in different currencies:

Exhibit 3.6 Convertibility

In an extension of Exhibit 3.4 let's look at the same portfolio from the perspective of a client whose base currency is euros (€). Assume a beginning of period exchange rate of \$1 = €1 and an end of period exchange rate of \$1 = €1.1.

Portfolio start value	€1 000 000
Portfolio end value	1 070 000 × 1.1 = €1 177 000
Portfolio return in euros	17.7%
Notional start value	€1 000 000
Notional end value	1 050 000 × 1.1 = €1 155 000

The portfolio's added value above the notional fund is:

$$€177\,000 - €155\,000 = €22\,000$$

The arithmetic excess return:

$$\frac{€22\,000}{€1\,000\,000} = 2.2\%$$

alternatively

$$17.7\% - 15.5\% = 2.2\%$$

The geometric excess return or increase in value of the portfolio compared to what would have been achieved if invested in the benchmark is:

$$\frac{€1\,177\,000 - €1\,155\,000}{€1\,150\,000} = 1.9\%$$

or

$$€22\,000/€1\,155\,000 \quad \text{or} \quad \frac{1.177}{1.155} - 1 = 1.9\%$$

In Exhibit 3.6 we can see that simply by expressing the same performance in a different currency the arithmetic excess return appears to have increased from 2.0% in Exhibit 3.4 to 2.2%

We cannot add more value simply by presenting returns in a different currency; the underlying increase in the benchmark, compounded with the currency return, has increased the added value relative to the initial amount invested. Crucially, because the portfolio and notional fund have been compounded by the same currency return the added value relative to the final value of the notional fund remains the same regardless of the currency in which the report is denominated. The geometric excess return is always the same regardless of the currency used to calculate performance.

This explains why geometric excess returns are more popular in the UK and Europe than say in the US and Australia. In Europe, particularly in the UK, the asset management community has been more international in nature for some time. The problem of presenting the same portfolio or composite returns in many currencies was addressed many years ago.

This relationship holds since the currency return c is identical for both the portfolio and the benchmark.

Let r_L = the portfolio return in local currency and b_L = the benchmark return in local currency. Then the portfolio return in currency:

$$c = (1 + r_L) \times (1 + c) - 1 = r_c \tag{3.21}$$

and the benchmark return in currency:

$$c = (1 + b_L) \times (1 + c) - 1 = b_c \tag{3.22}$$

It follows that:

$$\frac{(1 + r_c)}{(1 + b_c)} = \frac{(1 + r_L) \times (1 + c)}{(1 + b_L) \times (1 + c)} = \frac{(1 + r_L)}{(1 + b_L)} \tag{3.23}$$

Geometric excess returns are also compoundable over time. This is an extremely useful property in the measurement of portfolio performance.

In Chapter 2 we established by definition that time-weighted rates of return are calculated by "chain linking" each finite performance period within the overall period as follows:

$$(1 + r_1) \times (1 + r_2) \times (1 + r_3) \times \ldots (1 + r_{n-1}) \times (1 + r_n) = (1 + r) \tag{2.6}$$

Similarly the total benchmark return b can be derived in a similar calculation:

$$(1 + b) = (1 + b_1) \times (1 + b_2) \times \ldots (1 + b_{n-1}) \times (1 + b_n) \tag{3.24}$$

It can be seen that the geometric excess return for the total period g can be calculated by chain linking the geometric excess returns of the subperiods g_i as follows:

$$(1 + g) = \frac{(1 + r)}{(1 + b)} = \frac{(1 + r_1)}{(1 + b_1)} \times \frac{(1 + r_2)}{(1 + b_2)} \times \ldots \times \frac{(1 + r_n)}{(1 + b_n)} \tag{3.25}$$

or

$$(1 + g) = (1 + g_1) \times (1 + g_2) \times \ldots \times (1 + g_n) \tag{3.26}$$

Compoundability is demonstrated in Exhibit 3.7:

PERFORMANCE FEES

A performance-based fee is a variable fee based on the performance of the portfolio. Performance fees may be symmetrical or asymmetrical.

Symmetrical performance fees (or fulcrum fees)

Symmetrical performance fees as their name suggests pay a performance fee for outperformance and a performance rebate for underperformance; they have both a downside and an upside element. Base fees may be similar or even higher than normal to compensate the asset manager firm for the increased volatility of its income stream.

In the US under the Investment Advisers Act (1940) any performance fee charged to a mutual fund can only utilise symmetrical performance fee structures. Consequently, performance fees are rare in US mutual funds; asset managers do not like rebating fees to clients. In the US symmetrical fee structures are often called fulcrum fees.

Exhibit 3.7 Compondability

Assume that the same returns in Exhibit 3.4 are repeated over four quarters. The portfolio return is:

$$(1.07) \times (1.07) \times (1.07) \times (1.07) - 1 = 31.1\%$$

The benchmark return is:

$$(1.05) \times (1.05) \times (1.05) \times (1.05) - 1 = 21.6\%$$

The arithmetic excess return over the entire period is:

$$31.1\% - 21.6\% = 9.5\%$$

There is no apparent straightforward link between the individual arithmetic excess returns for each quarter and the total arithmetic excess return:

$$2\% + 2\% + 2\% + 2\% \neq 9.5\%$$
$$(1.02) \times (1.02) \times (1.02) \times (1.02) - 1 \neq 9.5\%$$

The geometric excess return over the entire period is:

$$\frac{1.311}{1.216} - 1 = 7.8\%$$

The geometric excess return for each finite period can be compounded to calculate the geometric excess return for the entire period:

$$(1.019) \times (1.019) \times (1.019) \times (1.019) - 1 = 7.8\%$$

Asymmetrical performance fees

Typically, asymmetrical performance fees pay a performance fee for outperformance only with no rebates for underperformance. Base fees are normally lower to accommodate a higher expectation of fees from the performance fee element.

Performance-based fees are becoming more and more popular, favoured both by the asset manager and the owners of capital. Investors don't like paying fees for underperformance but appear happy to pay fees when the asset manager has outperformed. Supporters of performance fees, both asset managers and investors, claim that performance fees align the interests of clients and asset managers.

Investors should think through the implications before establishing a performance fee structure for their asset managers. Theoretically, the existence of a performance fee should not alter the actual performance enjoyed. The asset manager cannot favour performance fee clients over and above other clients without a performance fee; they have a fiduciary duty to treat all clients equally. Sufficient incentives already exist for asset managers to deliver good performance. For continued success they will need to refer to their historic performance track record and many will invest their own cash in the funds that they manage.

Why pay a performance fee for performance you might expect to receive anyway? The only possible reason to enter a performance fee arrangement is because the desired and demonstrably better asset manager will only accept a performance fee arrangement to manage the client's assets.

Obviously, the client chooses the asset management firm; is it not rather perverse that the client is then rewarded by a lower fee (i.e. no performance fee) for choosing an underperforming asset manager and penalised (by paying a performance fee) for choosing an outperforming asset manager?

The flawed logic often applied by the owners of capital is that since there is more money available to pay fees then the asset manager should share in this success. This absurdity is taken to extremes with hedge fund managers that charge high base fees and very high performance fees, potentially 20% or more of any gain. These managers are incentivised to take risks with their client's capital.

Asset managers certainly view performance fees as a way of increasing average revenue expectations; this can only result in clients paying higher fees for ultimately the same level of performance.

Most performance fee arrangements are also badly written. Often the original authors have long since left their respective roles and the original rationale for the performance fee is lost or forgotten. At a time when both clients and asset managers should be celebrating good performance there are frequently unpleasant disputes about the calculation of the performance fee.

The existence of a performance fee may adversely affect the asset manager's decision processes, increasing the risk profile inappropriately to increase the chance of gaining performance fees or alternatively locking in outperformance by inappropriately decreasing risk after a period of good performance. If performance fee structures are to be used at all, the most appropriate measures are risk-adjusted.

Ideally, performance based fees should be:

(1) *Fair*. The performance fee structure should be fair and equitable to all participants; asset managers, clients and unitholders. The performance fee structure should be consistent with the portfolio's objective and should provide reward commensurate with the skill exercised in achieving the performance.
(2) *Unambiguous*. The agreement should be clear and unambiguous, preferably including a worked example. Calculation methodologies should be agreed, mechanisms for adjusting for cash flows, changing benchmarks and arbitration processes should be approved at the start of the relationship. The performance fee calculation should be verifiable. A worked example of the performance fee calculation should be included in the performance fee agreement.
(3) *Risk-adjusted*. The performance fee structure should not influence the risk strategy of the asset manager. Ideally, performance calculations should be risk-adjusted.
(4) *Simple*. The more complex the performance fee structure the more likely the interests of the asset manager and the client will be misaligned.

PERFORMANCE FEE STRUCTURES

Sliding scale

This is the most basic of all performance fee structures. For achieving certain minimum targets of performance the asset manager is simply paid at a higher rate. There may be several tiers of performance fees for different levels of performance.

Performance fee caps

To avoid encouraging asset managers to take excessive risk, performance fee structures can be capped either by a fixed maximum cash value or a fixed maximum ad valorum calculation based on absolute return or relative to a benchmark. If restrained by a maximum fee, asset managers might legitimately negotiate a minimum payment or collar.

Hurdle rate

Normally applying to absolute return strategies, performance fees are only paid in excess of a predetermined hurdle rate.

Crystallisation

Performance fees by their very nature are variable and paid on an infrequent basis; entitlements can build up over a period of three years or longer. Because base fees are inevitably lower it would be inappropriate not to accrue for any performance fee element earned in a net of fee calculation. The performance fee accrual should reflect the performance of the portfolio at that point in time and unlike other accruals can be quite volatile. A reducing performance fee accrual may, over shorter time periods, give the impression that the performance net-of-fees is greater than gross-of-fees performance.

The eventual payment of the performance fee accrual is described as crystallisation.

Crystallisation may be smoothed over a rolling three- or longer-year period – one third of the entitlement paid in the first year with subsequent payments deferred based on performance.

Some performance fees may only be crystallised when an investor leaves a fund in terms of an exit fee.

High water mark

Similar to a hurdle rate, the high water mark represents the performance level achieved at which the last performance fee was crystallised, the concept being that asset managers should only be paid for exceeding this high water mark in future.

Although at face value perfectly reasonable, if markets have subsequently collapsed a high water mark may represent an unachievable target and act as a disincentive to the current portfolio manager.

Because of unachievable high water marks they are often reset after a suitable predefined time limit or rolling high water marks may be used over say a one-year period.

Equalisation

Equalisation is designed to be fair to all investors in a collective mutual or hedge fund who inevitable invest at different times and are therefore subject to different performance returns. An investor who joins in the middle of a good period of performance may be charged a full performance fee even if they only participated for part of that performance; alternatively, an investor may enjoy a free ride if they invest at a low point after a high water mark and avoid paying a performance fee as the fund recovers to previous highs.

One obvious solution would be to issue a different class of share for investors investing at different times each with its own performance fee calculation. Clearly, several different classes of share can soon become a heavy administrative burden; alternatively, equalisation can be used. Any performance fee accrued but not yet crystallised will be included in the net asset value price of the fund; any investor buying units at this price in effect buys an equalisation credit equal to the accrual for the performance fee ultimately to be paid at the next crystallisation. The investor should not pay for performance not received. At crystallisation, if any performance fee is due an amount is rebated either directly or in the form of additional units up to the amount of the equalisation credit.

If the unit price is below a high water mark an equalisation debit (or depreciation deposit) is established equal to the equivalent performance fee entitlement between the current net asset value price and the high water mark level. If the high water mark is achieved a performance fee is charged normally by redeeming units in the fund. Example calculations are shown in Exhibit 3.8:

Exhibit 3.8 Equalisation

Assume a performance fee of 25%.

	Gross assets	Net assets	Performance fee accural
Unit price 31/12/06	100	100	0
Unit price 31/03/07	90	90	0
Unit price 30/06/07	120	115	5
Unit price 30/09/07	140	130	10
Unit price 31/12/07	130	122.5	7.5

First investor invests 31/12/06:

Performance return $130 - 100 = 30$
Performance fee due $30 \times 25\% = 7.5$ (distributed from the fund)

Second investor invests 31/03/07:

Performance return $130 - 90 = 40$
Performance fee due $40 \times 25\% = 10$

Distributed from the fund = 7.5

Depreciation deposit $(100 - 90) \times 25\% = 2.5$ (paid by redeemed units)
Total distribution $7.5 + 2.5 = 10$

Third investor invests 30/06/07:

Performance return $130 - 120 = 10$
Performance fee due $10 \times 25\% = 2.5$

Distributed from the fund = 7.5

Equalisation credit $20 \times 25\% = 5$ (rebated to client)
Total distribution $7.5 - 5 = 2.5$

Fourth investor invests 30/09/07:

Performance return $130 - 140 = -10$
Performance fee due nil

Distributed from the fund = 7.5

Equalisation credit $40 \times 25\% = 10$ (rebated to client)
Total distribution $7.5 - 7.5 = 0$
Equalisation credit retained $10 - 7.5 = 2.5$

4
Risk

Money is like muck, not good except it be spread.
Francis Bacon (1561–1626)

DEFINITION OF RISK

Risk is defined as the uncertainty of expected outcomes.

Within asset management firms there are many types of risk that should concern portfolio managers and senior management, for convenience I've chosen to classify risk into four main categories:

- Compliance risk.
- Operational risk.
- Counterparty or credit risk.
- Portfolio risk.

Although a major concern of all asset managers, reputational risk does not warrant a separate category; a risk failure in any category can cause significant damage to a firm's reputation.

Compliance or regulatory risk is the risk of breaching a regulatory, client or internally imposed limit. I draw no distinction between internal or external limits; the breach of an internal limit indicates a control failure, which could just have easily been a regulatory, or client mandated limit.

Operational risk, often defined as a residual catch-all category to include risks not defined elsewhere, actually includes the risk of human error, fraud, system failure, poor controls, management failure and failed trades. Risks of this type are more common but often less severe. Nevertheless, it is important to continuously monitor errors of all types, even those that don't result in financial loss. An increase in the frequency of errors regardless of size or sign may indicate a more serious problem that requires further investigation and corrective action.

Counterparty risk occurs when counterparties are unwilling or unable to fulfil their contractual obligations. This could include profits on a derivatives contract, unsettled transactions and even with the comfort of appropriate collateral the failure to return stock that has been used for stock lending.

In performance measurement we are most concerned with portfolio risk, which I define as the uncertainty of meeting client expectations.

Risk management versus risk control

There is a clear distinction between risk management and risk control. Portfolio managers are risk managers, they are paid to take risk, and they need to take risk to achieve higher returns.

Risk controllers on the other hand are paid to monitor risk (or often from their perspective to reduce risk). The risk controller's objective is to reduce the probability or eliminate entirely a major loss event on their watch. Risk managers 'and risk controllers' objectives are in conflict.

To resolve this conflict we need measures that assess the quality of return and answer the question: "Are we achieving sufficient return for the risk taken?"

Risk aversion

It is helpful to assume that investors are risk averse, that is to say, that given portfolios with equal rates of return they will prefer the portfolio with the lowest risk.

Investors will only accept additional risk if they are compensated by higher returns.

RISK MEASURES

Ex post and ex ante

Risk is calculated in two fundamentally different ways, *ex post* and *ex ante*. *Ex post* or historical risk is the analysis of risk after the event; it answers the question: "How risky has the portfolio been in the past?"

On the other hand, *ex ante* risk or prospective risk is forward looking, based on a snapshot of the current securities and instruments within the portfolio; it is an estimate or forecast of the future risk of the portfolio.

Ex post and *ex ante* risk calculations are substantially different and therefore can lead to completely different results and conclusions. Differences between *ex post* and *ex ante* risk calculations provide significant additional information, although as performance measurers we are more concerned about analysing past performance and are therefore more concerned with *ex post* risk.

Variability

In considering risk we are concerned with the variability (or dispersion) of returns from the average or mean return. Mean absolute deviation, variance and standard deviation are three related measures used to calculate variability.

Mean absolute deviation

Clearly, if added together, the positive and negative differences of each return from the average return would cancel; however, using the absolute difference (i.e. ignore the sign) we are able to calculate the mean or average absolute deviation as follows:

$$\text{Mean absolute deviation} = \frac{\sum_{i=1}^{i=n} |r_i - \bar{r}|}{n} \tag{4.1}$$

where: n = number of observations
r_i = return in month i
$\bar{r}$ = mean return.

Variance

The variance of returns is the average squared deviation of returns from the mean return calculated as:

$$\text{Variance}\, \sigma^2 = \frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n} \tag{4.2}$$

$$\text{Sample variance}\, \sigma_s^2 = \frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n-1} \tag{4.3}$$

Deviations from the mean $(r_i - \bar{r})$ are squared; this avoids the problem of negative deviations cancelling with positive deviations and also penalises larger deviations from the mean.

Standard deviation

For analysis it is more convenient to use our original non-squared units of return; therefore, we take the square root of the variance to obtain the standard deviation:

$$\text{Standard deviation}\, \sigma = \sqrt{\frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n}} \tag{4.4}$$

A higher standard deviation would indicate greater uncertainty, variability or risk.

In this version of standard deviation n not $n - 1$ is used in the denominator. The use of $n - 1$ would calculate the sample standard deviation. For large n it will make little difference if n or $n - 1$ is used. Since the majority of performance analysts tend to use n, for the sake of consistency and comparability I prefer to use n. The CFA Institute (previously the Association for Investment Management and Research) effectively reinforced the standard use of n in the 1997, second edition of the *AIMR Performance Presentation Standards Handbook*, stating:

> The use of n in the denominator of standard deviation (as opposed to $n - 1$) is supported because using n yields the maximum likelihood estimate of standard deviation. The use of $n - 1$ in the denominator of the sample variance makes sample variance an unbiased estimate of the true variance. When the square root of sample variance is taken to obtain the sample standard deviation, however, the result is not an unbiased estimate of population standard deviation. The seldom used unbiased estimate of standard deviation has a cumbersome constant based on sample size, which needs to be calculated. Because the unbiased estimate of standard deviation is not practical, it is wise to use the maximum likelihood estimate of standard deviation. Further compounding the issue is the fact that the use of $n - 1$ (unbiased) hinges on the assumptions that random and independent samples are taken from a normal distribution. The sample data (in this case, the manager's returns) are not random, arguably not independent, and may not be normally distributed.

$$\text{Sample standard deviation}\, \sigma_S = \sqrt{\frac{\sum_{i=1}^{i=n} (r_i - \bar{r})^2}{n-1}} \tag{4.5}$$

Equations (4.4) and (4.5) calculate standard deviation based on the periodicity of the data used – daily, monthly, quarterly, etc. For comparison, standard deviation is normally annualised for presentation purposes.

To annualise standard deviation we need to multiply by the square root of the number of observations in the year:

$$\text{Annualised standard deviation } \sigma^A = \sqrt{t} \times \sigma \tag{4.6}$$

where: t = number of observations in year (quarterly = 4, monthly = 12, etc.).

For example, to annualise a monthly standard deviation multiply by $\sqrt{12}$ and for a quarterly standard deviation multiply by $\sqrt{4}$ or 2.

Basic risk calculations are actually very straightforward and relatively simple to compute as shown in Tables 4.1 and 4.2. It is perhaps unfortunate that risk is considered a complex subject that requires an understanding of advanced mathematics. It is the role of both the performance analyst and risk controller to ensure the broadest understanding of the statistics presented.

Frequency and number of data points

If variability is stable then clearly the more observations, the higher number of data points, the better to maximise the accuracy of the estimation process. If variability is not stable then we must find a balance between long measurement periods that are more accurate but slow to reflect structural changes and short measurement periods that reflect recent market conditions but are less accurate. The industry standard requires a minimum of 36 monthly periods and 20 quarterly periods. If absolutely pushed I would provide risk statistics calculated using 24 months of data but never less; the resulting information is meaningless. Daily information is too noisy for long-term investment portfolios and should be ignored although very tempting for short time periods. Although it is easy to calculate annualised standard deviations it is never appropriate to compare portfolios with risk statistics calculated using different frequencies. For daily valued mutual funds you may wish to calculate the daily, weekly, monthly and quarterly annualised standard deviations over say 5 years; you might be shocked by the range of results and thus encouraged to use the dominating monthly standard.

Sharpe ratio (reward to variability)

Investors are risk averse. Given the same return they would prefer the portfolio with less risk or less variability; therefore, how do we evaluate portfolios with different returns and different levels of risks?

With two variables it is natural to resort to a graphical representation with return represented by the vertical axis and risk represented by the horizontal axis as shown in Figure 4.1 using data from Table 4.3.

A straight line is drawn from a fixed point on the vertical axis to points A and B representing the annualised returns and annualised variability (risk) of portfolios A and B, respectively.

The fixed point represents the natural starting point for all investors: the risk-free rate, the return I should expect on a riskless asset, for example the interest return on cash or Treasury

Table 4.1 Portfolio variability

Portfolio monthly return r_i (%)	Deviation from average $(r_i - \bar{r})$ (%)	Absolute deviation $\lvert r_i - \bar{r} \rvert$ (%)	Deviation squared $(r_i - \bar{r})^2$ (%)
0.3	−0.6	0.6	0.36
2.6	1.7	1.7	2.89
1.1	0.2	0.2	0.04
−1.0	−1.9	1.9	3.61
1.5	0.6	0.6	0.36
2.5	1.6	1.6	2.56
1.6	0.7	0.7	0.49
6.7	5.8	5.8	33.64
−1.4	−2.3	2.3	5.29
4.0	3.1	3.1	9.61
−0.5	−1.4	1.4	1.96
8.1	7.2	7.2	51.84
4.0	3.1	3.1	9.61
−3.7	−4.6	4.6	21.16
−6.1	−7.0	7.0	49.0
1.7	0.8	0.8	0.64
−4.9	−5.8	5.8	33.64
−2.2	−3.1	3.1	9.61
7.0	6.1	6.1	37.21
5.8	4.9	4.9	24.01
−6.5	−7.4	7.4	54.76
2.4	1.5	1.5	2.25
−0.5	−1.4	1.4	1.96
−0.9	−1.8	1.8	3.24

Average monthly return $\bar{r} = 0.9\%$

Mean absolute difference $\frac{74.6\%}{24} = 3.1\%$

Monthly standard deviation $\sigma_P = \sqrt{\frac{359.74}{24}} = 3.87\%$ $\quad \sum_{i=1}^{i=n} \lvert r_i - \bar{r} \rvert = 74.6\%$

Annualised standard deviation $\sigma_p^A = 3.87\% \times \sqrt{12} = 13.4\%$ $\quad \sum_{i=1}^{i=n} (r_i - \bar{r})^2 = 359.74\%$

bills. An investor can achieve this return without any variability or risk. It is important to ensure the same risk-free rate is used for all portfolios for comparison purposes.

Clearly, the investor will prefer to be in the top left-hand quadrant of this graph representing high return and low risk. The gradient of the line determines how far toward the left-hand quadrant each portfolio is represented: the steeper the gradient the further into the top left-hand side the investor goes.

This gradient is called the Sharpe ratio named after William Sharpe (1966) and is calculated as follows:

$$SR = \frac{r_P - r_F}{\sigma_P} \tag{4.7}$$

Table 4.2 Benchmark variability

Benchmark monthly return b_i (%)	Deviation from average $(b_i - \bar{b})$ (%)	Absolute deviation $\lvert b_i - \bar{b} \rvert$ (%)	Deviation squared $(b_i - \bar{b})^2$ (%)
0.2	−0.8	0.8	0.64
2.5	1.5	1.5	2.25
1.8	0.8	0.8	0.64
−1.1	−2.1	2.1	4.41
1.4	0.4	0.4	0.16
1.8	0.8	0.8	0.64
1.4	0.4	0.4	0.16
6.5	5.5	5.5	30.25
−1.5	−2.5	2.5	6.25
4.2	3.2	3.2	10.24
−0.6	−1.6	1.6	2.56
8.3	7.3	7.3	53.29
3.9	2.9	2.9	8.41
−3.8	−4.8	4.8	23.04
−6.2	−7.2	7.2	51.84
1.5	0.5	0.5	0.25
−4.8	−5.8	5.8	33.64
2.1	1.1	1.1	1.21
6.0	5.0	5.0	25.0
5.6	4.6	4.6	21.16
−6.7	−7.7	7.7	59.29
1.9	0.9	0.9	0.81
−0.3	−1.3	1.3	1.69
0.0	−1.0	1.0	1.0

Average monthly return $\bar{b} = 1.0\%$

Mean absolute difference $\dfrac{69.7\%}{24} = 2.9\%$

Monthly standard deviation $\sigma_M = \sqrt{\dfrac{338.83}{24}} = 3.76\%$ $\quad \sum_{i=1}^{i=n} \lvert b_i - \bar{b} \rvert = 69.7\%$

Annualised standard deviation $\sigma_M^A = 3.76\% \times \sqrt{12} = 13.0\%$ $\quad \sum_{i=1}^{i=n} (b_i - \bar{b})^2 = 338.83\%$

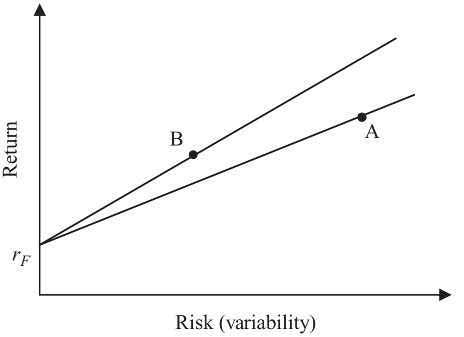

Figure 4.1 Sharpe ratio

Table 4.3 Sharpe ratio

	Portfolio A	Portfolio B	Benchmark
Annualised return	7.9%	6.9%	7.5%
Annualised risk	5.5%	3.2%	4.5%
Sharpe ratio (risk-free rate = 2%)			
$SR = \dfrac{r_P - r_F}{\sigma_P}$	$\dfrac{7.9\% - 2.0\%}{5.5\%} = 1.07$	$\dfrac{6.9\% - 2.0\%}{3.2\%} = 1.53$	$\dfrac{7.5\% - 2.0\%}{4.5\%} = 1.22$

where: r_P = portfolio return
r_F = risk-free rate
σ_P = portfolio risk (variability, standard deviation of return) normally annualised.

The higher the Sharpe ratio, the steeper the gradient and therefore the better combination of risk and return. The Sharpe ratio can be described as the return (or reward) per unit of variability (or risk).

Both graphically in Figure 4.1 and in the Sharpe ratios calculated in Table 4.3 we can see that portfolio B has a better risk-adjusted performance than either portfolio A or the benchmark.

Negative returns will generate negative Sharpe ratios, which despite the views of some commentators still retain meaning. Perversely for negative returns, it is better to be more variable not less! For those that think higher variability is always less desirable, negative Sharpe ratios are difficult statistics to interpret.

Risk-adjusted return: M^2

The Sharpe ratio is sometimes erroneously described as a risk-adjusted return; actually it's a ratio. We can rank portfolios in order of preference with the Sharpe ratio but it is difficult to judge the size of relative performance. We need a risk-adjusted return measure to gain a better feel of risk-adjusted outperformance.

In Figure 4.2 a straight line is drawn vertically through the risk of the benchmark σ_M. The intercept with the Sharpe ratio line of portfolio B would give the return of the portfolio with the same Sharpe ratio of portfolio B but at the risk of the benchmark. This return is called

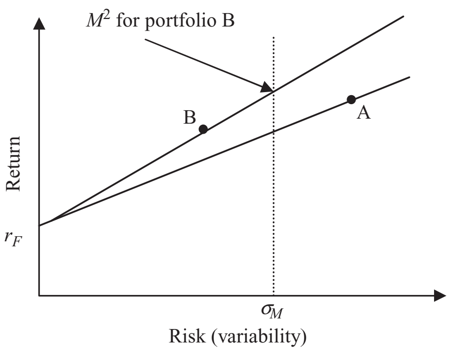

Figure 4.2 M^2

M^2, a genuinely risk-adjusted return extremely useful for comparing portfolios with different levels of risk:

$$M^2 = r_P + SR \times (\sigma_M - \sigma_P) \tag{4.8}$$

where: σ_M = market risk (variability, standard deviation of benchmark return).

The statistic is called M^2 not because any element of the calculation is squared but because it was first proposed by the partnership of Leah Modigliani (1997) and her grandfather Professor Franco Modigliani.

Alternatively, you might see the equation for M^2 expressed as:

$$M^2 = (r_P - r_F) \times \frac{\sigma_M}{\sigma_P} + r_F \tag{4.9}$$

Using the data from Table 4.3 M^2 is calculated in Table 4.4.

Table 4.4 M^2

	Portfolio A	Portfolio B
Annualised return	7.9%	6.9%
Annualised risk	5.5%	3.2%
Sharpe ratio	1.07	1.53
$M^2 = r_p + SR \times (\sigma_M - \sigma_P)$	7.9% + 1.07 × (4.5% − 5.5%) = 6.83%	6.9% + 1.53 × (4.5% − 3.2%) = 8.74%

M^2 excess return

Exactly the same arguments apply to geometric or arithmetic M^2 excess returns as they do to normal excess returns. Simple geometry from Figure 4.2 might suggest arithmetic excess return would be more appropriate; however, it is easy to argue that continuously compounded returns should be used. For consistency I prefer the geometric definition:

$$M^2 \text{ excess return} = \frac{\left(1 + M^2\right)}{(1 + b)} - 1 \tag{4.10}$$

or, arithmetically:

$$M^2 \text{ excess return} = M^2 - b \tag{4.11}$$

Differential return

The differential return is similar in concept to M^2 excess return except that the benchmark return is adjusted to the risk of the portfolio. The differential return is the difference between the portfolio return and the adjusted benchmark return. For the same portfolio the M^2 excess return and the differential return will differ because the Sharpe ratio lines of the portfolio and benchmark will diverge over time.

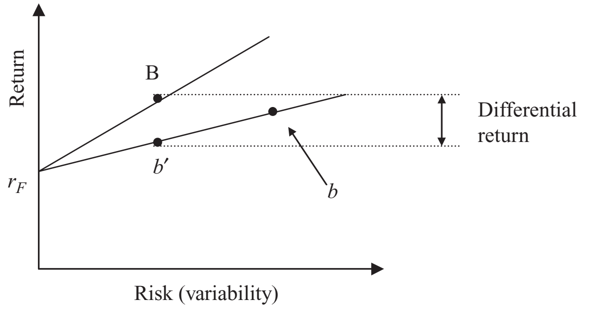

Figure 4.3 Differential return

The adjusted benchmark return b' is calculated as follows:

$$b' = r_F + \left(\frac{b - r_F}{\sigma_M}\right) \times \sigma_P \tag{4.12}$$

Therefore subtracting the adjusted benchmark return from the portfolio return we derive the differential return:

$$\text{Differential return } DR = r_P - r_F - \left(\frac{b - r_F}{\sigma_M}\right) \times \sigma_P \tag{4.13}$$

Differential returns are calculated in Table 4.5 based on the data from Table 4.3.

Differential return is less useful for comparing multiple portfolios because multiple risk-adjusted benchmark returns are required to be calculated, whereas for M^2 the benchmark returns are consistent for all portfolios. M^2 is a demonstrably better measure than either Sharpe ratio from which it is derived or differential return. It is possible to calculate differential return geometrically.

Table 4.5 Differential return

	Portfolio A	Portfolio B
Annualised return	7.9%	6.9%
Annualised risk	5.5%	3.2%
M^2	6.83%	8.74%
M^2 excess return (arithmetic) $= M^2 - b$	$6.8\% - 7.5\% = -0.7\%$	$8.7\% - 7.5\% = +1.2\%$
Differential return $r_P - r_F - \left(\frac{b - r_F}{\sigma_M}\right) \times \sigma_P$	$7.9\% - 2.0\% - \frac{7.5\% - 2.0\%}{4.5\%} \times 5.5\% = -0.8\%$	$6.9\% - 2.0\% - \frac{7.5\% - 2.0\%}{4.5\%} \times 3.2\% = +1.0\%$

GH1 (Graham and Harvey 1)

GH1, suggested by John Graham and Campbell Harvey (1997), is similar to the differential return but utilising an efficient frontier created by combining various weights of the benchmark

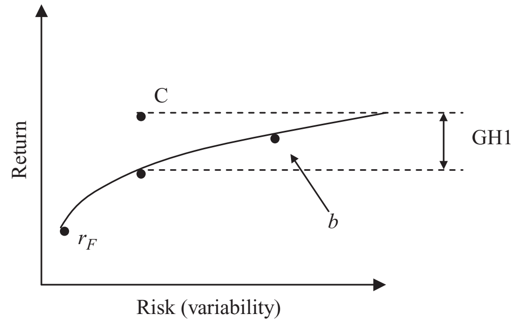

Figure 4.4 GH1

and risk-free rate (noting that the variability of the risk-free rate is not necessarily zero). The difference between the return of the portfolio and the combined benchmark and risk-free rate notional portfolio with identical risk (or variability) is GH1 as shown in Figure 4.4.

GH2 (Graham and Harvey 2)

GH2 is similar to M^2; instead of combining the benchmark and risk-free rate, increasing weights of actual portfolio returns are combined with reducing weights of the risk-free rate. GH2 is calculated at the risk of the benchmark as shown in Figure 4.5.

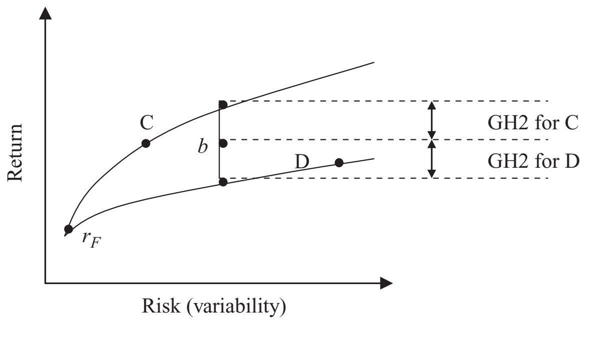

Figure 4.5 GH2

REGRESSION ANALYSIS

We can gain further information from a portfolio by plotting the portfolio returns against the corresponding benchmark returns in a scatter diagram, see Figure 4.6.

We might expect portfolio returns to move in line with benchmark returns; if so, we can fit a line of best fit through these points, the aim of which is to minimise the vertical distance of any one point from this line.

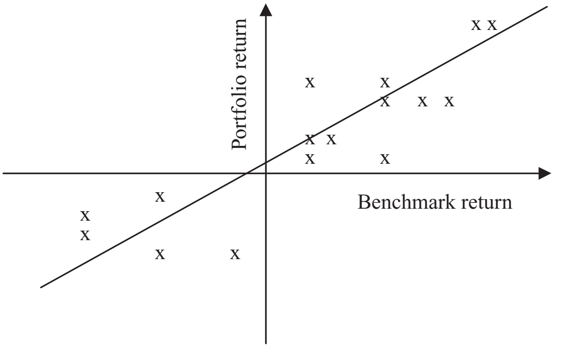

Figure 4.6 Regression analysis

Regression equation

The formula of any straight line is given by the slope or gradient of the line plus the intercept with the vertical access. Thus the return of the portfolio might be described as:

$$r_p = \alpha_R + \beta_R \times b + \varepsilon_R \tag{4.14}$$

This equation is called the regression equation.

Regression alpha (α_R)

The regression alpha is the intercept of the regression equation with the vertical axis

Regression beta (β_R)

The regression beta is the slope or gradient of the regression equation. The slope of the regression equation is given by:

$$\beta_R = \frac{\sum_{i=1}^{i=n} \left[(r_i - \bar{r}) \times \left(b_i - \bar{b} \right) \right]}{\sum_{i=1}^{i=n} \left(b_i - \bar{b} \right)^2} \tag{4.15}$$

where: b_i = benchmark return in month i
$\bar{b}$ = mean benchmark return.

Regression epsilon (ε_R)

The regression epsilon is an error term measuring the vertical distance between the return predicted by the equation and the real result.

Capital Asset Pricing Model (CAPM)

In the CAPM model we can factor in the risk-free rate and use the following revised regression equation to calculate a new beta and alpha (Jensen's alpha):

$$r_p - r_F = \alpha + \beta \times (b - r_F) + \varepsilon \tag{4.16}$$

Beta (β) (systematic risk or volatility)

I prefer the term volatility, not the more commonly used term systematic risk, to describe beta. Unfortunately, although originally used in the context of volatility it is now almost universally used to describe standard deviation and used interchangeably with variability. Needless to say I prefer the term variability when used in the context of standard deviation:

$$\beta = \frac{\sum_{i=1}^{i=n} \left[(r_i - r_{Fi} - \bar{r} - \bar{r}_F) \times \left(b_i - r_{Fi} - \bar{b} - \bar{r}_F \right) \right]}{\sum_{i=1}^{i=n} \left(b_i - r_{Fi} - \bar{b} - \bar{r}_F \right)^2} \tag{4.17}$$

where: $\bar{r}_F$ = mean risk-free rate
r_{Fi} = risk-free rate in month i
$\bar{b}$ = mean benchmark return.

Jensen's alpha (or Jensen's measure or Jensen's differential return or *ex post* alpha)

Jensen's alpha is the intercept of the regression equation in the Capital Asset Pricing Model and is in effect the excess return adjusted for systematic risk.

Ignoring the error term for *ex post* calculations and using Equation (4.15):

$$\alpha = r_P - r_F - \beta_P \times (b - r_F) \tag{4.18}$$

Note the similarities to the related formula for differential return (Equation 4.11), hence the alternative name, Jensen's differential return.

Portfolio managers often talk in terms of alpha to describe their added value, rarely are they referring to either the regression or even Jensen's alpha; in all probability they are referring to their excess return above the benchmark. Confusingly, academics also frequently refer to excess return as the return above the risk-free rate.

In fact, both these terms are frequently abused; beta is often used by the financial media and portfolio managers alike to describe market returns when of course it is the systematic risk relative to the market.

Bull beta (β^+)

We need not restrict ourselves to fitting lines of best fit to all market returns, positive and negative. If we calculate a regression equation for only positive market returns we gain information on the behaviour of the portfolio in positive or "bull" markets.

Bear beta (β^-)

The beta for negative market returns is described as the "bear" beta.

Beta timing ratio

Ideally, we would prefer a portfolio manager with a beta greater than 1 in rising markets and less than 1 in falling markets. In all likelihood such a manager would be a good timer of asset allocation decisions:

$$\text{Beta timing ratio} = \frac{\beta^+}{\beta^-} \tag{4.19}$$

Covariance

Covariance measures the tendency of the portfolio and benchmark returns moving together:

$$\text{Covariance} = \frac{\sum_{i=1}^{i=n} (r_{Pi} - \bar{r}_P) \times \left(b_i - \bar{b}\right)}{n} \tag{4.20}$$

Equation (4.20) multiplies the period portfolio return above the average portfolio return with the same period benchmark return above the average benchmark return. If both are positive or negative this will make a positive contribution; if they are of different signs it will make a negative contribution.

Therefore a positive covariance indicates the returns are associated, they move together. A negative covariance indicates the returns move in opposite directions. A low or near-zero covariance would indicate no relationship between portfolio and benchmark returns.

Correlation (ρ)

In isolation covariance is a difficult statistic to interpret. We can standardise the covariance to a value between 1 and –1 by dividing by the product of the portfolio standard deviation and the benchmark standard deviation as follows:

$$\text{Correlation } \rho_{P,M} = \frac{\text{Covariance}}{\sigma_P \times \sigma_M} \tag{4.21}$$

Note that correlation is also:

$$\rho_{P,M} = \frac{\text{Systematic risk}}{\text{Total risk}} \tag{4.22}$$

or

$$\rho_{P,M} = \frac{\beta_P \times \sigma_M}{\sigma_P}$$

Therefore beta and correlation are linked by the formula:

$$\beta_p = \rho_{P,M} \times \frac{\sigma_P}{\sigma_M} \tag{4.23}$$

Correlation measures the variability in the portfolio that is systematic compared to the total variability.

Correlation and risk-adjusted return: M^3

Muralindar (2000) proposed an extension to M^2, M^3, in which returns are adjusted for correlation to achieve a target tracking error as well as adjusting the portfolio variability to that of the benchmark. Tracking error and absolute return targets are normally viewed as being inconsistent.

Covariance, correlation and regression beta are calculated for our standard set of example date in Table 4.6. A CAPM beta would require monthly risk-free rates; however, if the risk-free rate is constant the regression beta is the same as the CAPM beta.

Table 4.6 Covariance, correlation and regression beta

Portfolio monthly return (%)	Deviation from average $(r_i - \bar{r})$ (%)	Benchmark monthly return b_i (%)	Deviation from average $(b_i - \bar{b})$ (%)	Portfolio deviation × benchmark deviation $(r_i - \bar{r}) \times (b_i - \bar{b})$	Benchmark deviation squared $(b_i - \bar{b})^2$
0.3	−0.6	0.2	−0.8	0.48	0.64
2.6	1.7	2.5	1.5	2.55	2.25
1.1	0.2	1.8	0.8	0.16	0.64
−1.0	−1.9	−1.1	−2.1	3.99	4.41
1.5	0.6	1.4	0.4	0.24	0.16
2.5	1.6	1.8	0.8	1.28	0.64
1.6	0.7	1.4	0.4	0.28	0.16
6.7	5.8	6.5	5.5	31.90	30.25
−1.4	−2.3	−1.5	−2.5	5.75	6.25
4.0	3.1	4.2	3.2	9.92	10.24
−0.5	−1.4	−0.6	−1.6	2.24	2.56
8.1	7.2	8.3	7.3	52.56	53.29
4.0	3.1	3.9	2.9	9.00	8.41
−3.7	−4.6	−3.8	−4.8	22.08	23.04
−6.1	−7.0	−6.2	−7.2	50.40	51.84
1.7	0.8	1.5	0.5	0.40	0.25
−4.9	−5.8	−4.8	−5.8	33.64	33.64
−2.2	−3.1	2.1	1.1	−3.41	1.21
7.0	6.1	6.0	5.0	30.50	25.00
5.8	4.9	5.6	4.6	22.54	21.16
−6.5	−7.4	−6.7	−7.7	56.98	59.29
2.4	1.5	1.9	0.9	1.35	0.81
−0.5	−1.4	−0.3	−1.3	1.82	1.69
−0.9	−1.8	0.0	−1.0	1.80	1.00
				Total = 338.44	*Total* = 338.83

Covariance $\frac{338.44\%}{24} = 14.1$

Correlation $\frac{14.1}{3.87 \times 3.76} = 0.97$

Regression beta $\frac{338.44}{338.83} = 1.0$

Regression alpha $0.9 - 1.0 \times 1.0 = -0.1$

R^2 (or coefficient of determination)

R^2 is the proportion of variance in fund returns that is related to the variance of benchmark returns; it is a measure of portfolio diversification. Note that variance is the square of standard deviation or variability.

The closer R^2 is to 1 the more portfolio variance is explained by benchmark variance. A low R^2 would indicate that returns are more scattered and would indicate a less reliable line of best fit leading to unstable alphas and betas. Therefore if a portfolio has a low R^2 (say much less than 0.8) then any alphas and betas and their derivative statistics should probably be ignored:

$$R^2 = \frac{\text{Systematic variance}}{\text{Total variance}} = \text{Correlation}^2 \tag{4.24}$$

Systematic risk

Michael Jensen (1969) described beta as systematic risk. If we multiply beta by market risk we obtain a measure of systematic risk calculated in the same units as variability. In my view this is a better definition of systematic risk:

$$\text{Systematic risk}\, \sigma_S = \beta \times \sigma_M \tag{4.25}$$

Specific or residual risk

Residual or specific risk is not attributed to general market movements but is unique to the particular portfolio under consideration. It is represented by the standard deviation of the error term in the regression equation σ_ε.

Since specific risk and systematic risk are by definition independent we can calculate total risk by using Pythagoras's theorem:

$$\text{Total risk}^2 = \text{systematic risk}^2 + \text{specific risk}^2 \tag{4.26}$$

Table 4.7 demonstrates that Equation (4.26) holds for our standard example.

Treynor ratio (reward to volatility)

Treynor ratio (Figure 4.7) is similar to Sharpe ratio, the numerator (or vertical axis graphically speaking) is identical but in the denominator (horizontal axis) instead of total risk we have systematic risk as calculated by beta:

$$\text{Treynor ratio}\, TR = \frac{r_P - r_F}{\beta_P} \tag{4.27}$$

Presumably, because it is included in most MBA studies, the Treynor ratio is extremely well known but perhaps less frequently used because it ignores specific risk. If a portfolio is fully diversified with no specific risk the Treynor and Sharpe ratios will give the same ranking. Sharpe actually favoured the Treynor ratio because he felt any value gained from being not fully diversified was transitory. Unfortunately, the performance analyst does not have the luxury of ignoring specific risk when assessing historic returns.

Table 4.7 Specific risk

Portfolio monthly return r_i (%)	Regression residual or error term $(r_i - b_i \times \beta - \alpha)$ (%)
0.3	0.20
2.6	0.20
1.1	−0.60
−1.0	0.20
1.5	0.20
2.5	0.80
1.6	0.30
6.7	0.31
−1.4	0.20
4.0	−0.10
−0.5	0.20
8.1	−0.09
4.0	0.20
−3.7	0.19
−6.1	0.19
1.7	0.30
−4.9	−0.01
−2.2	−4.20
7.0	1.11
5.8	0.31
−6.5	0.29
2.4	0.60
−0.5	−0.10
−0.9	−0.80
Specific risk (annualised standard deviation of error term σ_ε)	3.29
Systematic risk $(\beta \times \sigma_M)$	$1.0 \times 13.0 = 13.0\%$
Total risk^2 = systematic risk^2 + specific risk^2	
Total risk$^2 = 13.0\%^2 + 3.29\%^2 = 13.4\%^2$	

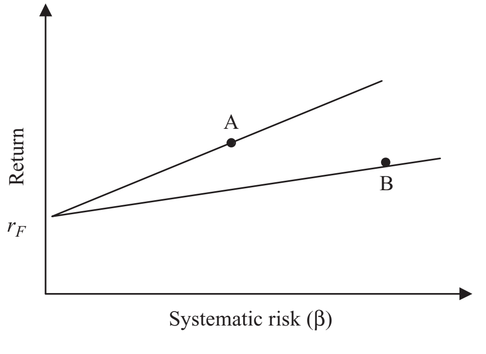

Figure 4.7 Treynor ratio

Modified Treynor ratio

A logical alternative form of the Treynor ratio might use systematic risk σ_S in the denominator, which is more consistent with the Sharpe ratio for convenience called the modified Treynor ratio:

$$MTR = \frac{r_P - r_F}{\sigma_S} \tag{4.28}$$

Appraisal ratio (or Treynor–Black ratio)

The appraisal ratio first suggested by Treynor and Black (1973) is similar in concept to the Sharpe ratio; but, using Jensen's alpha, excess return adjusted for systematic risk in the numerator is divided by specific risk, not total risk:

$$\text{Appraisal ratio} = \frac{\alpha}{\sigma_\varepsilon} \tag{4.29}$$

This measures the systematic risk-adjusted reward for each unit of specific risk taken. Although seldom used I must say this statistic appeals to me and perhaps should be given more consideration by investors.

Modified Jensen

Smith and Tito (1969) suggested the use of modified Jensen to rank portfolio performance. Similar to the appraisal ratio, Jensen's alpha is divided by systematic risk rather than specific risk:

$$\text{Modified Jensen} = \frac{\alpha}{\beta} \tag{4.30}$$

This measures the systematic risk-adjusted return per unit of systematic risk.
Although not suggested in their paper a logical alternative might be:

$$\text{Alternative modified Jensen} = \frac{\alpha}{\sigma_S} \tag{4.31}$$

Fama decomposition

Fama (1972) extended the concept of Treynor's ratio in his paper "Components of investment performance" to further breakdown the return of a portfolio.

The excess return above risk-free rate can be expressed as the selectivity (or Jensen's alpha) plus the return due to systematic risk as follows:

$$\underbrace{r_P - r_F}_{\text{Excess return}} = \underbrace{r_P - \beta_P \times (b - r_F) - r_F}_{\text{Selectivity}} + \underbrace{\beta_P \times (b - r_F)}_{\text{Systematic risk}} \tag{4.32}$$

If a portfolio is completely diversified there is no specific risk and the total portfolio risk will equal the systematic risk. Portfolio managers will give up diversification seeking additional return. Selectivity can be broken down into net selectivity and the return required to justify the diversification given up.

Selectivity

Isolating selectivity in Equation (4.31) we notice that it is equivalent to Jensen's alpha from Equation (4.16):

$$\alpha = r_P - r_F - \beta_P \times (b - r_F) \tag{4.16}$$

Diversification

Diversification is always positive and is the measure of return required to justify the loss of diversification for the specific risk taken by the portfolio manager.

To calculate the loss of diversification we have to calculate the effective beta required so that the systematic risk is equivalent to the total portfolio risk. Call this the Fama beta, calculated as follows:

$$\beta_F = \frac{\sigma_P}{\sigma_M} \tag{4.33}$$

Therefore the return required to justify not being fully diversified is calculated as follows:

$$d = (\beta_F - \beta_P) \times (b - r_F) \tag{4.34}$$

Net selectivity

Net selectivity is the remaining selectivity after deducting the amount of return required to justify not being fully diversified:

$$\text{Net selectivity } S_{Net} = \alpha - d \tag{4.35}$$

Obviously, if net selectivity is negative the portfolio manager has not justified the loss of diversification.

Fama decomposition is a useful analysis if we only have access to total fund returns and are unable to perform more detailed analysis on the components of return, for example mutual funds.

Figure 4.8 illustrates Fama's decomposition for portfolio A. A' represents the return from systematic risk plus the risk-free rate and A'' represents the return from the Fama equivalent systematic risk plus risk-free rate.

RELATIVE RISK

The risk measures we have discussed so far are examples of absolute rather than relative risk measures: that is to say, the returns and risks of the portfolio and benchmark are calculated separately and then used for comparison.

Relative risk measures on the other hand focus on the excess return of portfolio against benchmark. The variability of excess return calculated using standard deviation is called tracking error, tracking risk, relative risk or active risk.

Tracking error

Tracking error is often forecast and, since the calculation methods and meaning are quite different, it is essential to clearly label whether you are using an *ex post* or *ex ante*

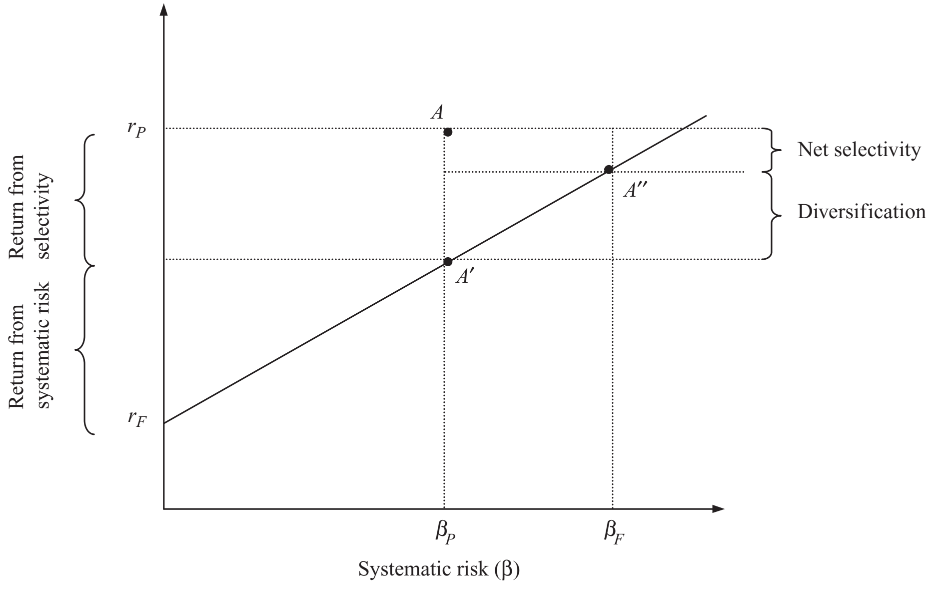

Figure 4.8 Fama decomposition

tracking error:

$$\text{Tracking error} = \sqrt{\frac{\sum_{i=1}^{i=n}(a_i - \bar{a})^2}{n}} \tag{4.36}$$

where: a_i = arithmetic excess return in month i
$\bar{a}$ = mean arithmetic excess return.

Or, if you prefer geometric excess returns:

$$\text{Tracking error} = \sqrt{\frac{\sum_{i=1}^{i=n}(g_i - \bar{g})^2}{n}} \tag{4.37}$$

where: g_i = geometric excess return in month i
$\bar{g}$ = mean geometric excess return.

Tracking error is a function of the portfolio standard deviation and the correlation between portfolio and benchmark returns and can also be calculated by:

$$\text{Tracking error} = \sigma_P \times \sqrt{\left(1 - \rho_{P,M}^2\right)} \tag{4.38}$$

This is an important relationship; if variability increases then tracking error increases.

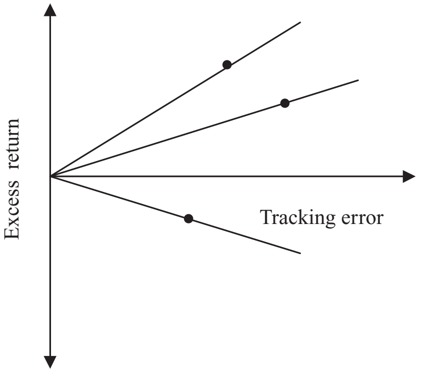

Figure 4.9 Information ratio

Information ratio

In exactly the same way we compared absolute return and absolute risk in the Sharpe ratio you can compare excess return and tracking error (the standard deviation of excess return) graphically, see Figure 4.9.

The information ratio is extremely similar to the Sharpe ratio except that, instead of absolute return, on the vertical excess we have excess return and, instead of absolute risk, on the horizontal access we have tracking error or relative risk (the standard deviation of excess return), hence the alternative name modified Sharpe ratio.

We have no need for a risk-free rate since we are dealing with excess returns; information ratio lines always originate from the origin. The gradient of the line is simply the ratio of excess return and tracking error as follows:

$$\text{Information ratio } IR = \frac{\text{Annualised excess return}}{\text{Annualised tracking error}} \tag{4.39}$$

Normally, information ratios are calculated using annualised excess returns and annualised tracking errors. To aid comparison it is absolutely essential to disclose the method of calculation, i.e. frequency of data, overall time period, arithmetic or geometric excess returns, arithmetic or geometric means, n or $n-1$, *ex post* or *ex ante*.

The method of calculation can certainly generate different results and therefore it is essential to ensure information ratios are calculated in the same way if comparisons are to be made.

If *ex ante* tracking errors are used in the denominator, please remember it is only a forecast based on a current snapshot of the portfolio. The portfolio manager will be able to "window dress" the portfolio by reducing any "bets" at the point of measurement thus reducing the forecast tracking error and therefore apparently improving the information ratio.

Information ratio is a key statistic, used extensively by institutional asset managers; it's often described as the measure of a portfolio manager's skill.

Views vary on what constitutes a good information ratio. In his research commentary Thomas Goodwin (1998) quotes Grinold and Kahn stating that an information ratio of 0.5 is good, 0.75 is very good and 1.0 is exceptional. These numbers certainly accord with my personal experience if sustained over a substantial period (3 to 5 years). Clearly, a positive information ratio indicates outperformance and a negative information ratio indicates underperformance. Unlike the Sharpe ratio, there is more consensus that if you going to underperform, consistent underperformance (as indicated by a low tracking error) is worse than inconsistent underperformance (high tracking error).

It is quite easy to obtain a good information ratio for a single period; like all statistics the development of the information ratio should be viewed over time. Goodwin suggests that sustaining a high information ratio above 0.5 is more difficult than Grinold and Kahn suggest in their article.

Tracking error and information ratios are calculated for our standard example data in Table 4.8 .

Table 4.8 Information ratio arithmetic excess return

Portfolio monthly return r_i (%)	Benchmark monthly return b_i (%)	Arithmetic excess return $a_i = r_i - b_i$ (%)	Deviation from average $(a_i - \bar{a})$	Deviation squared $(a_i - \bar{a})^2$
0.3	0.2	0.1	0.2	0.04
2.6	2.5	0.1	0.2	0.04
1.1	1.8	−0.7	−0.6	0.36
−1.0	−1.1	0.1	0.2	0.04
1.5	1.4	0.1	0.2	0.04
2.5	1.8	0.7	0.8	0.65
1.6	1.4	0.2	0.3	0.09
6.7	6.5	0.2	0.3	0.09
−1.4	−1.5	0.1	0.2	0.04
4.0	4.2	−0.2	−0.1	0.01
−0.5	−0.6	0.1	0.2	0.04
8.1	8.3	−0.2	−0.1	0.01
4.0	3.9	0.1	0.2	0.04
−3.7	−3.8	0.1	0.2	0.04
−6.1	−6.2	0.1	0.2	0.04
1.7	1.5	0.2	0.3	0.09
−4.9	−4.8	−0.1	0.0	0.00
−2.2	2.1	−4.3	−4.2	17.61
7.0	6.0	1	1.1	1.22
5.8	5.6	0.2	0.3	0.09
−6.5	−6.7	0.2	0.3	0.09
2.4	1.9	0.5	0.6	0.37
−0.5	−0.3	−0.2	−0.1	0.01
−0.9	0.0	−0.9	−0.8	0.63
Annualised portfolio return 10.37%	Annualised benchmark return 11.80%			*Total = 21.69*

Tracking error $\sqrt{\dfrac{21.69}{24}} = 0.95$

Annualised tracking error (close to specific risk in this example because $0.95 \times \sqrt{12} = 3.293$

Or annualised tracking error $= \sigma_P \times \sqrt{\left(1 - \rho_{P,M}^2\right)}$ $\quad 13.4 \times \sqrt{\left(1 - 0.97^2\right)} = 3.293$

Information ratio $\dfrac{(10.37\% - 11.80\%)}{3.293} = -0.43\%$

Table 4.9 Information ratio geometric excess return

Portfolio monthly return r_i (%)	Benchmark monthly return b_i (%)	Geometric excess return $g_i = \frac{(1+r_i)}{(1+b_i)} - 1$ (%)	Deviation from average $(g_i - \bar{g})$	Deviation squared $(g_i - \bar{g})^2$
0.3	0.2	0.1	0.2	0.04
2.6	2.5	0.1	0.2	0.04
1.1	1.8	−0.7	−0.6	0.35
−1.0	−1.1	0.1	0.2	0.04
1.5	1.4	0.1	0.2	0.04
2.5	1.8	0.7	0.8	0.62
1.6	1.4	0.2	0.3	0.09
6.7	6.5	0.2	0.3	0.08
−1.4	−1.5	0.1	0.2	0.04
4.0	4.2	−0.2	−0.1	0.01
−0.5	−0.6	0.1	0.2	0.04
8.1	8.3	−0.2	−0.1	0.01
4.0	3.9	0.1	0.2	0.04
−3.7	−3.8	0.1	0.2	0.04
−6.1	−6.2	0.1	0.2	0.04
1.7	1.5	0.2	0.3	0.09
−4.9	−4.8	−0.1	0.0	0.00
−2.2	2.1	−4.2	−4.1	16.92
7.0	6.0	0.9	1.0	1.09
5.8	5.6	0.2	0.3	0.08
−6.5	−6.7	0.2	0.3	0.10
2.4	1.9	0.5	0.6	0.35
−0.5	−0.3	−0.2	−0.1	0.01
−0.9	0.0	−0.9	−0.8	0.64
Annualised portfolio return 10.37%	Annualised benchmark return 11.80%			*Total = 20.78*

Tracking error $\sqrt{\frac{20.78}{24}} = 0.93$

Annualised tracking error $0.93 \times \sqrt{12} = 3.223$

Information ratio $\frac{\left(\frac{1.1037}{1.1180} - 1\right) \times 100}{3.223} = -0.40\%$

RETURN DISTRIBUTIONS

Normal (or Gaussian) distribution

A distribution is said to be normal if there is a high probability that an observation will be close to the average and a low probability that an observation is far away from the average. A normal distribution curve peaks at the average value.

A normal distribution shown in Figure 4.10 has special properties that are useful if we can assume returns or excess returns are normally distributed. If returns are normally distributed we can use the average return and variability or standard deviation of returns to describe the distribution of returns, such that:

- Approximately 68% of returns will be within a range of 1 standard deviation above and below the average return.
- Approximately 95% of returns will be within a range of 2 standard deviations above and below the average return.
- Approximately 99.7% of returns will be within a range of 3 standard deviations above and below the average return.*

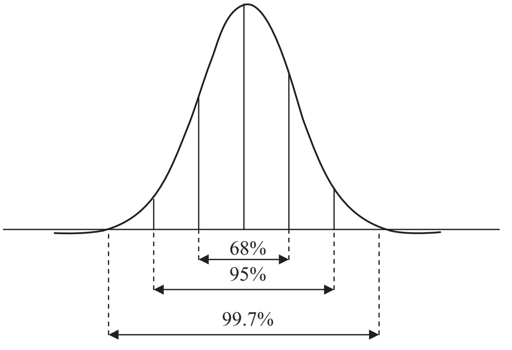

Figure 4.10 Normal distribution

This property is obviously very useful for calculating the probability of an event occurring outside a specified range of returns. Normal distributions are popular because of these statistical properties and because many random events can be approximated by a normal distribution.

The central limit theorem

The central limit theorem states that if a number of observations of an independent random variable are taken:

(1) The mean of the resulting sample mean is equal to the mean of the underlying population.
(2) The standard deviation of the resulting sample mean is the standard deviation of the underlying population divided by the square root of the number of observations.
(3) Even if the underlying distribution is strongly non-normal the sampling distribution of means will increase approximate equal to a normal distribution as the sample size increases.

We rely on the central limit theorem to annualise standard deviations calculated from different frequencies. For monthly data we multiply by 12 and then divide by $\sqrt{12}$, which is equivalent to multiplying by $\sqrt{12}$.

Skewness (Fisher's or moment skewness)

Not all distributions are normal distributed; if there are more extreme returns extending to the right tail of a distribution it is said to be positively skewed and if there are more returns extending to the left it is said to be negatively skewed.

* The "emperical or 68-95-99.7" rule; actually more accurately 68.2689%, 95.4499% and 99.7300%.

We can measure the degree of skewness (or more accurately Fisher's skewness) in the following formula:

$$\text{Skewness } S = \sum\left(\frac{r_i - \bar{r}}{\sigma_P}\right)^3 \times \frac{1}{n} \tag{4.40}$$

A normal distribution will have a skewness of 0. Note that in Equation (4.40) extreme values carry greater weight since they are cubed while maintaining their initial sign positive or negative.

We can use skewness to make a judgement about the possibility of large negative or positive outliers when comparing portfolio returns. Skewness provides more information about the shape of return distribution, where deviations from the mean are greater in one direction than the other; this measure will deviate from zero in the direction of the larger deviations. Positive and negative skew are illustrated in Figure 4.11.

Sample skewness

Sample skewness is calculated as follows:

$$\text{Sample skewness } S_S = \sum\left(\frac{r_i - \bar{r}}{\sigma_{S_P}}\right)^3 \times \frac{n}{(n-1)\times(n-2)} \tag{4.41}$$

where: σ_{S_P} = portfolio sample standard deviation.

Note that the standard Excel function called skewness is in fact sample skewness.

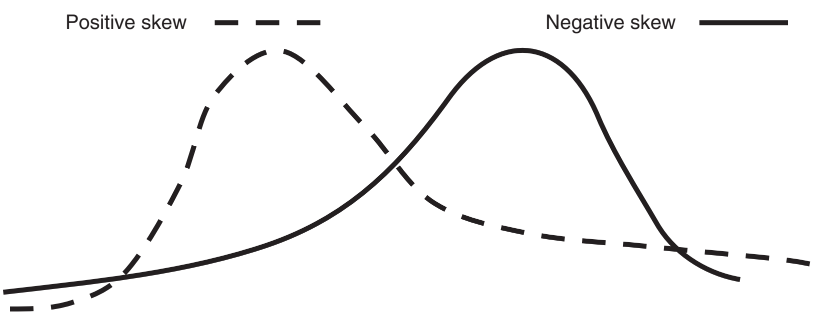

Figure 4.11 Positive and negative skew

Kurtosis (Pearson's kurtosis)

Kurtosis (Pearson, 1905) (or more correctly Pearson's kurtosis) provides additional information about the shape of a return distribution; formally, it measures the weight of returns in the tails of the distribution relative to standard deviation but is more often associated as a measure of flatness or peakedness of the return distribution:

$$\text{Kurtosis } K = \sum\left(\frac{r_i - \bar{r}}{\sigma_p}\right)^4 \times \frac{1}{n} \tag{4.42}$$

The kurtosis of a normal distribution is 3 (called mesokurtic); greater than 3 would indicate a peaked distribution with fat tails (called platykurtic) and less than 3 would indicate a less peaked distribution with thin tails (called leptokurtic).

Subtracting 3 from Equation (4.42) we obtain a measure of excess kurtosis (or Fisher's kurtosis). The terms kurtosis and excess kurtosis are often confused:

$$\text{Excess kurtosis } K_E = \sum\left(\frac{r_i - \bar{r}}{\sigma_p}\right)^4 \times \frac{1}{n} - 3 \tag{4.43}$$

High and low kurtosis are illustrated in Figures 4.12 and 4.13 respectively.

Sample kurtosis

Sample kurtosis is calculated as follows:

$$\text{Sample kurtosis } K_S = \sum\left(\frac{r_i - \bar{r}}{\sigma_{Sp}}\right)^4 \times \frac{n \times (n+1)}{(n-1) \times (n-2) \times (n-3)} \tag{4.44}$$

Sample excess kurtosis is calculated as follows:

$$K_{SE} = \sum\left(\frac{r_i - \bar{r}}{\sigma_{Sp}}\right)^4 \times \frac{n \times (n+1)}{(n-1) \times (n-2) \times (n-3)} - \frac{3 \times (n-1)^2}{(n-2) \times (n-3)} \tag{4.45}$$

The standard Excel function labelled kurtosis is in fact sample excess kurtosis.

A better understanding of the shape of the distribution of returns will aid in assessing the relative qualities of portfolios. Whether we prefer higher or lower kurtosis (or for that matter positive or negative skewness) will depend on the type of return series we want to see.

Equity markets tend to have fat tails. When markets fall portfolio managers tend to sell and when they rise portfolio managers tend to buy; there is a higher probability of extreme events than the normal distribution would suggest. Therefore tracking error and *VaR* statistics calculated using normal assumptions may underestimate risk.

Investors will tend to seek positive skew and lower kurtosis or thinner tails.

The variance of a distribution is known as the 2nd moment about the mean, skewness the 3rd moment and kurtosis the 4th moment.

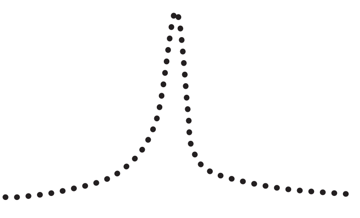

Figure 4.12 Kurtosis >3 peaked with fat tails

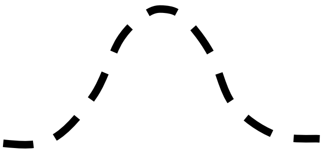

Figure 4.13 Kurtosis <3 less peaked with thin tails

Bera–Jarque statistic

Since normal distributions should have skewness near 0 and kurtosis near 3 we can test for normality using the Bera–Jarque test:

$$\text{Bera–Jarque statistic } BJ = \frac{n}{6} \times \left(S^2 + \frac{K_E^2}{4} \right) \tag{4.46}$$

With a confidence level of 5% we will reject a distribution as normal if the Bera–Jarque statistic exceeds 5.99, and with a confidence level of 1% if it exceeds 9.21. Perfectly normal distributions will have a $BJ = 0$.

Skewness, kurtosis and the Bera–Jarque statistic are calculated in Table 4.10.

RISK-ADJUSTED PERFORMANCE MEASURES FOR HEDGE FUNDS

There are numerous definitions of hedge funds; my favourite is from Ineichen (2003):

> A hedge fund constitutes an investment program whereby the managers or partners seek absolute returns by exploiting investment opportunities while protecting principal from potential financial loss.

Predominately, hedge fund management styles are designed to be asymmetric in their return patterns. If successful this leads to variability of returns on the upside but not on the downside. Investors are less concerned with variability on the upside but of course are extremely concerned about variability on the downside. This leads to an extended family of risk-adjusted measures reflecting the downside risk tolerances of investors seeking absolute not relative returns. Investors should prefer high average returns, lower variance or standard deviation, positive skewness and lower kurtosis.

In addition, hedge funds can be further differentiated by:

(1) More flexible investment strategies and the ability to employ leverage.
(2) Substantial personal holding by the portfolio manager and performance fees.
(3) Light regulation.
(4) Liquidity – investors are typically subject to a lock-up period.

Table 4.10 Skewness and kurtosis

Portfolio monthly return r_i (%)	Deviation from average $(r_i - \bar{r})$	Deviation squared $(r_i - \bar{r})^2$	Deviation cubed $(r_i - \bar{r})^3$	4th power deviation $(r_i - \bar{r})^4$
0.3	−0.6	0.36	−0.22	0.13
2.6	1.7	2.89	4.91	8.35
1.1	0.2	0.04	0.01	0.00
−1.0	−1.9	3.61	−6.86	13.03
1.5	0.6	0.36	0.22	0.13
2.5	1.6	2.56	4.10	6.55
1.6	0.7	0.49	0.34	0.24
6.7	5.8	33.64	195.11	1131.65
−1.4	−2.3	5.29	−12.17	27.98
4.0	3.1	9.61	29.79	92.35
−0.5	−1.4	1.96	−2.74	3.84
8.1	7.2	51.84	372.25	2687.39
4.0	3.1	9.61	29.79	92.35
−3.7	−4.6	21.16	−97.34	444.75
−6.1	−7.0	49.00	−343.00	2401.00
1.7	0.8	0.64	0.51	0.41
−4.9	−5.8	33.64	−195.11	1131.65
−2.2	−3.1	9.61	−29.79	92.35
7.0	6.1	37.21	226.98	1384.58
5.8	4.9	24.01	117.65	576.48
−6.5	−7.4	54.76	−405.22	2998.66
2.4	1.5	2.25	3.38	5.06
−0.5	−1.4	1.96	−2.74	3.84
−0.9	−1.8	3.24	−5.83	10.50
	Sum	*359.74*	*−114.99*	*13 116.28*

Monthly standard deviation $\sqrt{\dfrac{359.74}{24}} = 3.87\%$

Skewness $S = \sum\left(\dfrac{r_i - \bar{r}}{\sigma_P}\right)^3 \times \dfrac{1}{n} \quad \dfrac{-114.99}{3.87^3 \times 24} = -0.08$

Sample standard deviation $\sqrt{\dfrac{359.74}{24-1}} = 3.95\%$

Kurtosis $= \sum\left(\dfrac{r_i - \bar{r}}{\sigma_p}\right)^4 \times \dfrac{1}{n}\dfrac{13\,116.28}{3.87^4 \times 24} = 2.43$

Excess kurtosis $2.43 - 3 = -0.57$

Bera–Jacque $BJ = \dfrac{n}{6} \times \left(S^2 + \dfrac{K_E^2}{4}\right) \quad \dfrac{24}{6} \times \left(-0.08^2 + \dfrac{-0.57^2}{4}\right) = 0.35$

Sample skewness $S_S = \sum\left(\dfrac{r_i - \bar{r}}{\sigma_{Sp}}\right)^3 \times \dfrac{n}{(n-1)\times(n-2)} = -0.09$

Sample excess kurtosis $K_{SE} = \sum\left(\dfrac{r_i - \bar{r}}{\sigma_{Sp}}\right)^4 \times \dfrac{n \times (n+1)}{(n-1)\times(n-2)\times(n-3)} - \times \dfrac{3 \times (n-1)^2}{(n-2)\times(n-3)}$
$= -0.41$

DRAWDOWN

Perhaps the simplest measure of risk in a return series from an absolute return investor's perspective, wishing to avoid losses, is any continuous losing return period or drawdown.

Average drawdown

As its name suggests, the average drawdown is the average continuous negative return over an investment period, 3 years being a typical period of measurement:

$$\text{Average drawdown } \bar{D} = \left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right| \tag{4.47}$$

where: D_j = jth drawdown over entire period
d = total number of drawdowns in entire period.

Some investors take the view that only the largest drawdowns in the return series are of any consequence and therefore restrict d to a predetermined maximum limit of say 3 or 5 thus enabling fair comparison between portfolios.

Maximum drawdown

The maximum drawdown (D_{Max}), not to be confused with the largest individual drawdown, is the maximum potential loss over a specific time period, typically 3 years. Maximum drawdown represents the maximum loss an investor can suffer in the fund buying at the highest point and selling at the lowest. Like any other statistic it is essential to compare performance over the same time period.

Largest individual drawdown

As its name suggests the largest individual drawdown (D_{Lar}) is the largest individual uninterrupted loss in a return series.

Recovery time (or drawdown duration)

The recovery time or drawdown duration is the time taken to recover from an individual or maximum drawdown to the original level.

Individual and maximum drawdowns and recovery time are illustrated in Figure 4.14. The largest individual loss is D_3.

Drawdown deviation

Drawdown deviation calculates a standard deviation-type statistic using individual drawdowns as follows:

$$\text{Drawdown deviation } DD = \sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}} \tag{4.48}$$

Ulcer index

The ulcer index developed by Peter G. Martin in 1987 (Martin and McCann, 1987) (so called because of the worry suffered by both the portfolio manager and investor) is similar to drawdown deviation with the exception that the impact of the duration of drawdowns is incorporated by selecting the negative return for each period below the previous peak or high water mark. The impact of long, deep drawdowns will have a significant impact since the underperformance since the last peak is squared:

$$\text{Ulcer index } UI = \sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}} \tag{4.49}$$

where: D_i' = drawdown since pervious peak in period i.
This approach is clearly sensitive to the frequency of time period and clearly penalises managers that take time to recover to previous highs, taking into account both the depth and duration of drawdowns.

Pain index

If the drawdowns are not squared then the resulting pain index is very similar to the Zephyr pain index in discrete form as proposed by Thomas Becker in 2006:

$$\text{Pain index } PI = \sum_{i=1}^{i=n} \frac{|D_i'|}{n} \tag{4.50}$$

Calmar ratio

The Calmar ratio is a Sharpe-type measure that uses maximum drawdown rather than standard deviation to reflect the investor's risk. In the context of hedge fund performance it is easy to understand why investor's might prefer the maximum possible loss from peak to valley as an appropriate measure of risk:

$$\text{Calmar ratio } CR = \frac{r_P - r_T}{D_{Max}} \tag{4.51}$$

Sterling ratio

The Sterling ratio replaces maximum drawdown in the Calmar ratio with the average drawdown over the period of analysis.

There are multiple variations of the Sterling ratio in common usage, perhaps reflecting its use across a range of differing asset categories and outside the field of finance.

The original definition attributed to Deanne Sterling Jones (McCafferty, 2003) appears to be:

$$\text{Original Sterling ratio } OSR = \frac{r_p}{\bar{D}_{Lar} + 10\%} \tag{4.52}$$

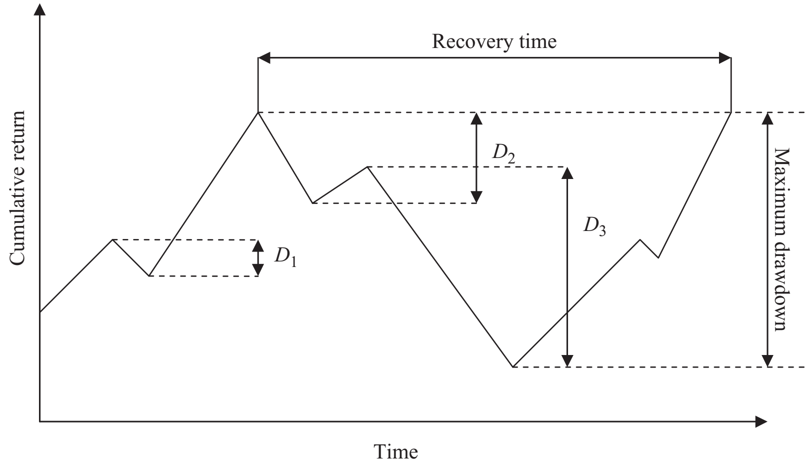

Figure 4.14 Drawdown statistics

The denominator is defined as the average largest drawdown plus 10%. The addition of 10% is arbitrary compensation for the fact that the average largest drawdown is inevitably smaller than the maximum drawdown. Typically, only a fixed number of the largest drawdowns are averaged with apologies to Deanne Sterling Jones, I suggest the definition is standardised to exclude the 10% but in Sharpe form as follows:

$$\text{Sterling ratio } SR = \frac{r_p - r_F}{\left|\sum_{j=1}^{j=d} \frac{D_j}{d}\right|} \tag{4.53}$$

The number of observations d fixed to the investor's preference.

Sterling–Calmar ratio

Perhaps the most common variation of the Sterling ratio uses the average annual maximum drawdown in the denominator over 3 years. A combination of both Sterling and Calmar concepts, to avoid confusion and to encourage consistent use across the industry I suggest the following standardised definition:

$$\text{Sterling–Calmar ratio } SCR = \frac{r_p - r_F}{\bar{D}_{\max}} \tag{4.54}$$

Given the variety of Sterling ratio definitions great care should be taken to ensure the same definition is used over the same time period using the same frequency of data when ranking portfolio performance.

Burke ratio

Burke (1994) in his article "A sharper Sharpe ratio" suggested using the familiar concept of the square root of the sum of the squares of each drawdown in order to penalise major drawdowns as opposed to many mild ones:

$$\text{Burke ratio } BR = \frac{r_P - r_F}{\sqrt{\sum_{j=1}^{j=d} D_j^2}} \tag{4.55}$$

Just like the Sterling ratio the number of drawdowns used can be restricted to a set number of the largest drawdowns.

Modified Burke ratio

For consistency with other Shape-type statistics it might be more appropriate to define the modified Burke ratio using drawdown deviation in the denominator as follows:

$$\text{Modified Burke ratio } MBR = \frac{r_P - r_F}{\sqrt{\sum_{j=1}^{j=d} \frac{D_j^2}{n}}} \tag{4.56}$$

Clearly, both the modified and standard Burke ratios will generate identical portfolio rankings if the number of drawdowns is not restricted to the largest drawdowns.

Martin ratio (or ulcer performance index)

If the duration of drawdowns is a concern for investors the Martin ratio is similar to the modified Burke index but using the ulcer index in the denominator:

$$\text{Martin ratio } MR = \frac{r_P - r_F}{\sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}} \tag{4.57}$$

Pain ratio

The equivalent to the Martin ratio but using the pain index is the pain ratio:

$$\text{Pain ratio } PR = \frac{r_P - r_F}{\sum_{i=1}^{i=m} \frac{D_i'}{n}} \tag{4.58}$$

Lake ratio

Seykota (2003) suggests an alternative drawdown statistic combining drawdown magnitude with drawdown duration. The lake ratio is constructed from the ratio of the volume in the valleys (or lakes) of the performance track record with the total volume under the cumulative return. Lower lake ratios are preferred. Both the pain and Martin ratios penalise managers with early high water marks in their track record, therefore it is far better to peak later rather than earlier when applying these ratios. To some degree the lake ratio compensates for high early peaks in performance by taking into account the duration and timing of the peaks.

Peak ratio

The volume of lakes in the lake ratio is equivalent to the pain index. The lake ratio is presented in inverted form, with the pain index in the numerator. The inverse of the lake index, if I might suggest a name, is the peak ratio, which is more consistent with our standard return/risk-type ratios.

Examples of drawdown, maximum drawdown, pain index, ulcer index, Sterling, Calmar, Burke, pain and Martin ratios are contained in Table 4.11.

DOWNSIDE RISK (OR SEMI-STANDARD DEVIATION)

Standard deviation and the symmetrical normal distribution are the foundations of modern portfolio theory. Post-modern portfolio theory recognises that investors prefer upside risk rather than downside risk and utilises semi-standard deviation.

Semi-standard deviation measures the variability of underperformance below a minimum target rate. The minimum target rate could be the risk-free rate, the benchmark or any other fixed threshold required by the client. All positive returns are included as zero in the calculation of semi-standard deviation or downside risk as follows:

$$\text{Downside risk } \sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\min\left[(r_i - r_T), 0\right]^2}{n}} \tag{4.59}$$

where: r_T = minimum target return.

Clearly, since positive returns are excluded there are potentially fewer or in some cases no observations less than the target return. Therefore great care must be taken to ensure there are sufficient returns to ensure the calculation is meaningful.

Alternatively, a distribution curve can be fitted to the data points and integral calculus used to model the probability of returns below the minimum target return (Sortino and Satchell, 2001), a type of semi-*ex ante* analysis.

As should be expected downside variance is the square of downside risk and downside potential is simply the average sum of returns below target:

$$\text{Downside variance } \sigma_D^2 = \sum_{i=1}^{n} \frac{\min\left[(r_i - r_T), 0\right]^2}{n} \tag{4.60}$$

$$\text{Downside potential } DP = \sum_{i=1}^{n} \frac{\min\left[(r_i - r_T), 0\right]}{n} \tag{4.61}$$

Downside variance is the second lower partial moment of return $LPM_2\,(r_T)$, and downside potential is the first lower partial moment $LPM_1\,(r_T)$

Upside risk

The equivalent upside statistics are as expected:

$$\text{Upside risk } \sigma_U = \sqrt{\sum_{i=1}^{n} \frac{\max\left[(r_i - r_T), 0\right]^2}{n}} \tag{4.62}$$

Table 4.11 Drawdown statistics (assume risk-free rate $r_F = 0\%$)

Portfolio monthly return (%)	Continuous drawdown D_j	Continuous drawdown squared D_j^2	Drawdown from peak D'_i	Drawdown from peak squared $D_i'^2$
0.3			0.00	0.00
2.6			0.00	0.00
1.1			0.00	0.00
−1.0	1.0	1.00	1.00	1.00
1.5			0.00	0.00
2.5			0.00	0.00
1.6			0.00	0.00
6.7			0.00	0.00
−1.4	1.4	1.96	1.40	1.96
4.0			0.00	0.00
−0.5	0.5	0.25	0.50	0.25
8.1			0.00	0.00
4.0			0.00	0.00
−3.7			3.70	13.69
−6.1	9.6	91.67	9.57	91.67
1.7			8.04	64.59
−4.9			12.54	157.33
−2.2	7.0	48.89	14.47	209.30
7.0			8.48	71.91
5.8			3.17	10.06
−6.5	6.5	42.25	9.47	89.60
2.4			7.29	53.19
−0.5			7.76	60.16
−0.9	1.4	1.95	8.59	73.73
Annualised portfolio return 10.37%		*Total* = *187.97*	*Total* = *95.98*	*Total* = *898.44*

Maximum drawdown 15.47

Largest drawdown 9.57

Average largest drawdown (3) $\dfrac{9.57 + 6.99 + 6.50}{3} = 7.69$

Sterling ratio $\dfrac{10.37 - 0}{9.57} = 1.08$

Calmar ratio $\dfrac{10.37 - 0}{15.47} = 0.67$

Burke ratio $\dfrac{10.37 - 0}{\sqrt{187.97}} = 0.76$

Pain index $\dfrac{95.98}{24} = 4.0$

Ulcer index $\sqrt{\dfrac{898.44}{24}} = 6.12$

Pain ratio $\dfrac{10.37 - 0}{4.0} = 2.59$

Martin ratio $\dfrac{10.37 - 0}{6.12} = 1.69$

$$\text{Upside variance } \sigma_U^2 = \sum_{i=1}^{n} \frac{\max[(r_i - r_T), 0]^2}{n} \tag{4.63}$$

$$\text{Upside potential } UP = \sum_{i=1}^{n} \frac{\max[(r_i - r_T), 0]}{n} \tag{4.64}$$

Note that in all the above equations the total number of observations n is always used irrespective of the number of observations above or below target.

Loss and gain standard deviation measures the variability of returns above or below target only and are typically not used in performance analysis:

$$\text{Loss standard deviation } \sigma_d = \sqrt{\sum_{i=1}^{n} \frac{\min[(r_i - r_T)]^2}{n_d}} \tag{4.65}$$

$$\text{Gain standard deviation } \sigma_u = \sqrt{\sum_{i=1}^{n} \frac{\max[(r_i - r_T)]^2}{n_u}} \tag{4.66}$$

where: n_d = number of returns less than target
n_u = number of returns greater than target.

Shortfall risk (or downside frequency)

Shortfall risk is the ratio (or probability) of returns below target compared to the total number of returns:

$$\text{Shortfall risk} = \frac{n_d}{n} \tag{4.67}$$

Omega ratio (Ω)

In their article "A universal performance measure" Shadwick and Keating (2002) suggest a gain–loss ratio that captures the information in the higher moments of a return distribution as follows:

$$\text{Omega ratio } \Omega = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} \tag{4.68}$$

Note:

$$\sum_{i=1}^{i=n} \max(r_T - r_i, 0) = -1 \times \sum_{i=1}^{i=n} \min(r_i - r_T, 0) \tag{4.69}$$

The omega ratio can be used as a ranking statistic; the higher the better. It equals 1 when r_T is the mean return.

The omega ratio implicitly adjusts for both skewness and kurtosis in the return distribution.

Bernardo and Ledoit (or gain–loss) ratio

The Bernardo and Ledoit (1996) ratio is a special case of the omega ratio with $r_T = 0$:

$$\text{Bernado} - \text{Ledoit ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(0 - r_i, 0)} \tag{4.70}$$

***d* ratio**

The *d* ratio (Lavinio, 1999) is similar to the Bernado Ledoit ratio but inverted and taking into account the frequency of positive and negative returns:

$$d \text{ ratio} = \frac{n_d \times \sum_{i=1}^{i=n} \max(0 - r_i, 0)}{n_u \times \sum_{i=1}^{i=n} \max(r_i, 0)} \tag{4.71}$$

where: n_d = number of returns less than zero
n_u = number of returns greater than zero.

The *d* ratio will have values between zero and infinity and can be used to rank the performance of portfolios. The lower the *d* ratio the better the performance, a value of zero indicating there are no returns less than zero and a value of infinity indicating there are no returns greater than zero. Portfolio managers with positively skewed returns will have lower *d* ratios.

Omega–Sharpe ratio

The omega ratio can be converted to a ranking statistic in familiar form to the Sharpe ratio. Clearly, the average portfolio return less the target return is equal to the sum of the upside and downside potential:

$$\text{Omega–Sharpe ratio} = \frac{r_P - r_T}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} \tag{4.72}$$

Noting again Equation (4.69):

$$r_P - r_T = \frac{1}{n} \times \sum_{i=1}^{i=n} r_i - r_T = \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0) - \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0) \tag{4.73}$$

Substituting Equation (4.73) into Equation (4.72):

$$= \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0) - \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} \tag{4.74}$$

$$= \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} - \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)} = \Omega - 1 \tag{4.75}$$

$$\text{Omega–Sharpe ratio} = \Omega - 1 \tag{4.76}$$

Hence, the omega–Sharpe ratio will rank portfolios in the same order as the omega ratio.

Sortino ratio

A natural extension of the Sharpe and Sharpe–omega ratios is suggested by Sortino and van der Meer (1991) (Figure 4.15) which uses downside risk in the denominator as follows:

$$\text{Sortino ratio} = \frac{(r_P - r_T)}{\sigma_D} \tag{4.77}$$

Clearly, investors should be seeking returns greater than the risk-free rate (why take any risk otherwise), therefore the minimum accepted return in most cases should be greater than the risk-free rate.

Kappa (κ_l)

Kaplan and Knowles (2004) in their paper "Kappa: a generalized downside risk-adjusted performance measure", demonstrate that both the Sortino ratio and the Sharpe–omega ratio are special cases of kappa:

$$K_l = \frac{r_p - r_T}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)^l} \tag{4.78}$$

For $l = 1$ K_1 is the Sharpe–omega ratio and for $l = 2$ K_2 is the Sortino ratio. It is difficult to interpret the meanings of K_3 and K_4 but they would equate to the 3rd and 4th moments of skewness and kurtosis, respectively.

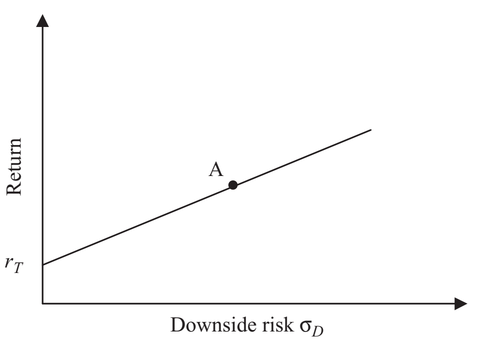

Figure 4.15 Sortino ratio

Note that in continuous form the lth lower partial moment function is defined as:

$$LPM_l(r_T) = \int_{-\infty}^{r_T} (r_T - r)^l dF(r) \tag{4.79}$$

The lth lower partial moment can be estimated from a sample of discrete returns as follows:

$$LPM_l(r_T) = \frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)^l \tag{4.80}$$

Some would argue that analysis using just discrete values tells only part of the story and that the continuous form is preferred. However, the discrete form is both easier to understand and calculate, doesn't require a probability function to be estimated and focuses on the return periods (nomally monthly) of most concern to the investor. Admittedly, reaching a fixed amount each month is a different proposition than reaching the annualised equivalent each year but then again many investors are concerned about monthly performance below target.

Upside potential ratio

The upside potential ratio suggested by Sortino *et al.* (1999) can also be used to rank portfolio performance and combines upside potential with downside risk as follows:

$$\text{Upside potential ratio } UPR = \frac{\sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\sigma_D} \tag{4.81}$$

Portfolio downside risk, Sortino ratio, upside and downside potential, omega, omega–Sharpe ratio and upside potential ratio are calculated in Table 4.13.

Investors retain an interest in the actual return delivered. I take the view that the actual return in the numerator is relevant to most investors and hence my preference for the Sortino ratio.

Volatility skewness

A similar measure to omega but using the second partial moment is volatility skewness (Rom and Ferguson, 2001), the ratio of the upside variance compared to the downside variance. Values greater than 1 would indicate positive skewness and values less than 1 would indicate negative skewness:

$$\text{Volatility skewness} = \frac{\sigma_U^2}{\sigma_D^2} \tag{4.82}$$

This measure, in particular, rewards extreme positive events and penalises extreme negative events. For those like me that are unsure about the merits of overrewarding extreme, potentially one-off positive events then perhaps the upside potential ratio is more appropriate.

Table 4.12 Portfolio downside risk (monthly minimum target return = 0.5%, annual minimum target return = 6.17%)

Portfolio monthly return r_i (%)	Deviation against target (downside only) $(r_i - r_T)$ (%)	Squared deviation $(r_i - r_T)^2$	Upside only $(r_i - r_T)$ (%)
0.3	−0.2	0.04	
2.6			2.1
1.1			0.6
−1.0	−1.5	2.25	
1.5			1.0
2.5			2.0
1.6			1.1
6.7			6.2
−1.4	−1.9	3.61	
4.0			3.5
−0.5	−1.0	1.00	
8.1			7.6
4.0			3.5
−3.7	−4.2	17.64	
−6.1	−6.6	43.56	
1.7			1.2
−4.9	−5.4	29.16	
−2.2	−2.7	7.29	
7.0			6.5
5.8			5.3
−6.5	−7.0	49.00	
2.4			1.9
−0.5	−1.0	1.00	
−0.9	−1.4	1.96	
Annualised portfolio return *10.37%*	*Total = 32.9%*	*Total = 156.51*	*Total = 42.5%*
	Monthly downside risk	$\sqrt{\frac{156.51}{24}} = 2.55\%$	
	Annualised downside risk	$2.55.\% \times \sqrt{12} = 8.85\%$	
		Sortino ratio	$\frac{10.37\% - 6.17\%}{8.85\%} = 0.48$
Downside potential	$\frac{32.9\%}{24} = 1.37\%$	*Upside potential*	$\frac{42.5\%}{24} = 1.77\%$
		Upside potential ratio	$\frac{1.77\%}{8.85\%} = 0.20$
Omega	$\frac{1.77\%}{1.37\%} = 1.29$	*Omega–Sharpe ratio*	$1.29 - 1 = 0.2$

Variability skewness

Although the rankings will be identical for consistency with other measures I prefer the square root of volatility skewness. To differentiate the term I use the name variability skewness:

$$\text{Variability skewness} = \frac{\text{Upside risk}}{\text{Downside risk}} = \frac{\sigma_U}{\sigma_D} \tag{4.83}$$

Table 4.13 Benchmark downside risk (monthly minimum target return = 0.5%, annual minimum target return = 6.17%)

Benchmark monthly return b_i (%)	Deviation against target (downside only) $(b_i - r_T)$ (%)	Squared deviation $(b_i - r_T)^2$
0.2	−0.3	0.09
2.5		
1.8		
−1.1	−1.6	2.56
1.4		
1.8		
1.4		
6.5		
−1.5	−2.0	4.00
4.2		
−0.6	−1.1	1.21
8.3		
3.9		
−3.8	−4.3	18.49
−6.2	−6.7	44.89
1.5		
−4.8	−5.3	28.09
2.1		
6.0		
5.6		
−6.7	−7.2	51.84
1.9		
−0.3	−0.8	0.64
0.0	−0.5	0.25
		Total = 152.06
	Monthly downside risk	$\sqrt{\frac{152.06}{24}} = 2.52\%$
	Annualised downside risk	$2.52\% \times \sqrt{12} = 8.72\%$
$M_S^2 = r_P + \text{Sortino Ratio} \times (\sigma_{DM} - \sigma_D)$		$10.42\% + 0.48 \times (8.72\% - 8.84\%) = 10.36\%$

Adjusted Sharpe ratio

Pézier and White (2006) suggests using the adjusted Sharpe ratio (ASR) which explicitly adjusts for skewness and kurtosis by incorporating a penalty factor for negative skewness and excess kurtosis as follows:

$$\text{Adjusted Sharpe ratio } ASR = SR \times \left[1 + \left(\frac{S}{6}\right) \times SR - \left(\frac{K-3}{24}\right) \times SR^2\right] \tag{4.84}$$

Skewness–kurtosis ratio

Watanabe (2006) also explicitly adjusts for skewness and kurtosis by suggesting using the skewness–kurtosis ratio in conjunction with the Sharpe ratio, ranking portfolios using the addition of the two rather than the Sharpe ratio in isolation. Again, higher rather than lower ratios are preferred:

$$\text{Skewness} - \text{kurtosis ratio} = \frac{S}{K} \tag{4.85}$$

Prospect ratio

Watanabe notes that people have a tendency to feel loss greater than gain – a well-known phenomena described by prospect theory (Kahneman and Tversky, 1979). He suggests penalising loss as follows in the prospect ratio:

$$\text{Prospect ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} (\text{Max}\,(r_i, 0) + 2.25 \times \text{Min}\,(r_i, 0)) - r_T}{\sigma_D} \tag{4.86}$$

VALUE AT RISK (VaR)

VaR measures the worst expected loss over a given time interval under normal market conditions at a given confidence level. For example, an annual value of risk of £5m at a 95% confidence level for a portfolio would suggest that only once in 20 years would the annual loss exceed £ 5m. Therefore it is far from the maximum possible loss. Value at Risk measures the downside; upside potential measures the equivalent best-expected gain.

VaR, like tracking error, can be calculated *ex post* or *ex ante*. Typically, VaR is calculated *ex ante* although like tracking error it is useful to calculate *ex post* as well to monitor risk efficiency.

VaR can be calculated in conjunction with tracking error, since tracking error is a 1 standard deviation measure covering approximately 68% of returns; within 1 standard deviation of the average it is entirely possible that a change in strategy may reduce tracking error while increasing VaR in the tails of the distribution. Client preferences will determine which of these measures is the most relevant. There are three different methods for calculating VaR.

Variance–covariance (or parametric)

The variance–covariance method assumes that portfolio returns are normally distributed. Only the mean return and standard deviation are required. With 95% confidence we know the VaR is:

$$\text{VaR} = \bar{r} - 1.65 \times \sigma \tag{4.87}$$

With 99% confidence we know the VaR is:

$$\text{VaR} = \bar{r} - 2.33 \times \sigma \tag{4.88}$$

Historical simulation (or non-parametric)

Historical simulation simply reorganises actual historical returns, putting them in order from worst to best. The Value at Risk is determined at the 95th percentile.

Monte Carlo simulation

Monte Carlo simulation is similar to historical simulation, but rather than use observed changes in market factors a model is chosen for future portfolio returns and a random number generator is used to calculate thousands of hypothetical portfolio returns. The Value at Risk is then determined from this distribution.

The variance–covariance method is possibly the easiest to implement but it relies on a normal assumption of returns. We know portfolio returns, and hedge fund returns in particular, are not normally distributed.

Historical simulation is perhaps the most accurate; it's suitable for all asset types and certainly conceptually easier to explain to pension fund trustees but requires the processing of tremendous amounts of historical data.

Monte Carlo simulation is more complex and suffers from a greater model risk.

VaR ratio

VaR ratio is the ratio of Value of Risk divided by the total size of the portfolio, essentially the percentage of the portfolio at risk:

$$\frac{\text{VaR}}{\text{Assets}} \tag{4.89}$$

Reward to VaR ratio

Reward to VaR is a Sharpe-type measure but with VaR ratio replacing standard deviation as the measure of risk in the denominator:

$$\text{RVaR} = \frac{r_P - r_F}{\text{VaR ratio}} \tag{4.90}$$

Conditional VaR (or expected shortfall)

VaR does not provide any information about the shape of the tail or the expected size of loss beyond the confidence level. In this sense it is a very unsatisfactory risk measure; of more interest is conditional VaR (Figure 4.16), otherwise known as expected shortfall, mean expected loss tail VaR and tail loss which takes into account the shape of the tail. Historical simulation methods, which make no assumptions of normality, are particularly suitable for calculating conditional VaR.

Conditional Sharpe ratio

Conditional Sharpe ratio replaces VaR with conditional VaR in the denominator of the reward to VaR ratio. Clearly, if expected shortfall is the major concern of the investor then the conditional Sharpe ratio is demonstrably favourable to the reward to VaR ratio:

$$\text{Conditional Sharpe ratio} = \frac{r_P - r_F}{\text{CVaR}} \tag{4.91}$$

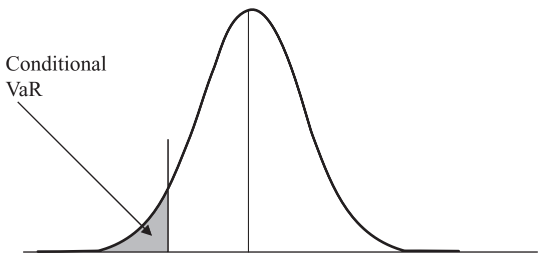

Figure 4.16 Conditional VaR

Modified VaR

Alternatively, VaR can be modified (Favre and Galeano, 2002) to adjust for kurtosis and skewness using a Cornish–Fisher expansion as follows:

$$\text{MVaR} = \bar{r}_p + \left[z_c + \frac{z_c^2 - 1}{6} \times S + \frac{z_c^3 - 3z_c}{24} \times K_E - \frac{2z_c^3 - 5z_c}{36} \times S^2 \right] \times \sigma \tag{4.92}$$

where: $z_c = -1.96$ with 95% confidence
$z_c = -2.33$ with 99% confidence.

Note that if the return distribution is normal, S and K_E are zero and Equation (4.92) reduces to the equivalent of Equation (4.87):

$$\text{VaR} = \bar{r} + z_c \times \sigma \tag{4.93}$$

This method works less well for distributions with more extreme skewness and excess kurtosis.

Modified Sharpe ratio

Similar to the adjusted Sharpe ratio, the modified Sharpe ratio uses modified VaR adjusted for skewness and kurtosis:

$$\text{Modified Sharpe ratio} = \frac{r_P - r_F}{\text{MVaR}} \tag{4.94}$$

RETURN ADJUSTED FOR DOWNSIDE RISK

Like all ranking statistics it is easy to rank portfolios in order of preference using the Sortino ratio but it is rather more difficult to answer the question exactly how much better one portfolio is than the other.

M^2 for Sortino

M^2 can be calculated for downside risk in the same way as it is calculated for total risk.

In Figure 4.17 a straight line is drawn vertically through the downside risk of the benchmark; the intercept with the Sortino ratio line of portfolio A would give the return of the

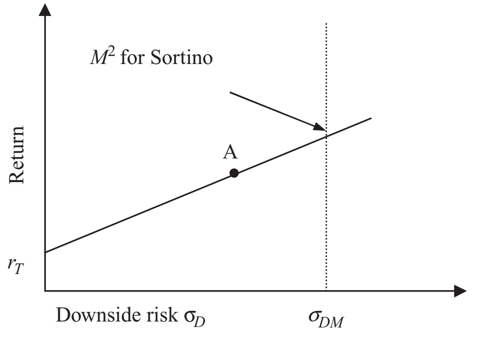

Figure 4.17 M^2 for Sortino

portfolio with the same Sortino ratio of portfolio A but at the same downside risk of the benchmark:

$$M_S^2 = r_P + \text{Sortino ratio} \times (\sigma_{DM} - \sigma_D) \tag{4.95}$$

Benchmark downside risk, omega, omega–Sharpe ratio and M_S^2 are calculated in Table 4.13.

Omega excess return

Another form of downside risk-adjusted return is omega excess return (Sortino *et al.*, 1997) (not related to the omega ratio). Similar to differential return the downside risk-adjusted benchmark return is calculated by multiplying the downside variance of the style benchmark by 3 times the style beta.

The 3 is arbitrary and assumes the investor requires 3 units of return for one unit of variance ($3 \times \sigma_{MD}^2$ effectively takes the place of the benchmark excess return above the risk-free rate in the differential return calculation). The style beta adjusts for the downside risk taken by the portfolio manager by taking the ratio of the downside risk of the portfolio divided by the downside risk of the style benchmark:

$$\text{Downside risk-adjusted-style benchmark } 3 \times \beta_S \times \sigma_{MD}^2 \tag{4.96}$$

$$\text{Omega excess return } \omega = r_p - 3 \times \beta_S \times \sigma_{MD}^2 \tag{4.97}$$

where: σ_{MD}^2 = style benchmark variance.

$$\text{Style beta } \beta_S = \frac{\sigma_D}{\sigma_{MD}} \tag{4.98}$$

For much the same reasons as I prefer M^2 excess return above differential return I prefer M^2 for Sortino to omega excess return.

Hurst index

The Hurst index[†] is a useful statistic for detecting if a portfolio manager's returns are mean reverting (anti-persistent), totally random or persistent. It is calculated as follows:

$$H = \frac{\log(m)}{\log(n)} \tag{4.99}$$

where: $$\text{m} = \frac{[\max(r_i) - \min(r_i)]}{\sigma_p} \tag{4.100}$$

n = number of observations.

A Hurst index between 0 and 0.5 would suggest a portfolio manager's series of returns are mean reverting (anti-persistent).

A Hurst index of 0.5 would suggest the series of returns was totally random.

A Hurst index between 0.5 and 1 would suggest the series of returns are persistent, i.e. there is memory in the return series.

Clearly, persistent positive returns or excess returns would be a desirable property for an asset manager.

FIXED INCOME RISK

Duration (or volatility)

In many ways fixed income securities or bonds are easier to measure than equities; they consist (for the most part) of predictable future cash flows in the form of coupon payments and a final redemption value.

Duration is defined as the average life of the present values of all future cash flows from a fixed income security. Duration is a systematic risk or volatility measure for bonds. In calculating the present value of the future cash flows, a discount rate equal to the redemption yield is used. This measures the fixed income securities' sensitivities to changes in interest rates.

Macaulay duration

Frederick Macaulay (1938) was among the first to suggest the use of duration for studying the returns on bonds:

$$D = \frac{\sum_{i=1}^{n} F_i \times t_i \times d^{t_i}}{\sum_{i=1}^{n} F_i \times d^{t_i}} \tag{4.101}$$

where: n = number of future coupon and capital repayments
F_i = ith future coupon or capital repayment
t_i = time in years to the ith coupon or capital repayment
d = discount factor.

[†] H.E. Hurst originally developed the Hurst index to help in the difficult task of establishing optimal water storage along the Nile. Nile floods are extremely persistent as evidenced by a Hurst index of 0.9. Equity markets have a Hurst index of around 0.7. See Clarkson (2001).

Note that:

$$d = \frac{1}{(1+y)} \tag{4.102}$$

where: y = yield to maturity or redemption yield.

The denominator in Equation (4.101) is equal to the present value of future coupon and capital repayments of the securities or, in the other words, the price P:

$$P = \sum_{i=1}^{n} F_i \times d^{ti} \tag{4.103}$$

Substituting Equation (4.103) into Equation (4.101) we have the formula for Macaulay duration:

$$D = \frac{\sum_{i=1}^{n} F_i \times t_i \times d^{t_i}}{P} \tag{4.104}$$

Modified duration

A slight variant of Macaulay duration is modified duration:

$$MD = \frac{D}{1 + \frac{y}{k}} \tag{4.105}$$

where: k = number of cash flows or coupons per year.

Duration measures the sensitivity of a bond's price to changes in yield. Price changes can be estimated by:

$$\Delta P = -MD \times \Delta y \tag{4.106}$$

Macaulay–Weil duration

There is no reason why the discount rate should be constant for all coupon payments; in fact, interest rates are likely to be different for different time periods. Macaulay–Weil duration uses the appropriate spot rate for each cash flow and is therefore more accurate.

Price, Macaulay, modified and Macaulay–Weil duration are calculated for a simple bond in Table 4.14.

Portfolio duration

Durations in a fixed income portfolio or benchmark are additive. The entire duration of the portfolio can be computed directly from the weighted sum of each bond in the portfolio:

$$\text{Portfolio duration} = \sum_{i=1}^{i=n} w_i \times MD_i \tag{4.107}$$

Table 4.14 Price, Macaulay, modified and Macaulay–Weil duration

20-year bond 6% coupon		Macaulay		Macaulay–Weil		
Period	Cash flow	Present value	Time × present value	Spot rate	Present value	Time × present value
1	6	5.78	5.78	3.5	5.80	5.80
2	6	5.56	11.12	3.5	5.60	11.20
3	6	5.35	16.05	3.5	5.41	16.23
4	6	5.15	20.60	3.5	5.23	20.91
5	6	4.96	24.79	3.5	5.05	25.26
6	6	4.77	28.63	3.5	4.88	29.29
7	6	4.59	32.15	3.5	4.72	33.01
8	6	4.42	35.36	3.5	4.56	36.45
9	6	4.25	38.29	3.75	4.31	38.77
10	6	4.1	40.95	3.75	4.15	41.52
11	6	3.94	43.36	3.75	4.00	44.02
12	6	3.79	45.53	3.75	3.86	46.29
13	6	3.65	47.48	3.75	3.72	48.33
14	6	3.52	49.21	3.75	3.58	50.17
15	6	3.38	50.75	3.75	3.45	51.81
16	6	3.26	52.11	3.75	3.33	53.27
17	6	3.13	53.29	4.0	3.08	52.36
18	6	3.02	54.31	4.0	2.96	53.31
19	6	2.9	55.18	4.0	2.85	54.11
20	106	49.39	987.75	4.0	48.38	967.54
	Price	*128.91*	*1692.70*	*Price*	*128.91*	*1679.67*
	IRR (y)	3.89%		IRR (y)	3.89%	
Macaulay duration	$\frac{1692.7}{128.91} = 13.13$			*Macaulay–Weil duration*		$\frac{1679.67}{128.91} = 13.03$
Modified duration	$\frac{13.13}{1.0389} = 12.64$			*Modified duration*		$\frac{13.03}{1.0389} = 12.54$

Effective duration (or option-adjusted duration)

Modified duration does not calculate the effective duration of the bond if there is any optionality in future payments. To calculate the effective duration the estimate price must be calculate for both a positive and negative change in interest rates:

$$ED = \frac{P_- - P_+}{2 \times P \times \Delta y} \tag{4.108}$$

where: Δy = change in interest rates
P_- = estimated price if interest rate is decreased by Δy
P_+ = estimated price if interest rate is increased by Δy.

If there is no optionality in future payments effective duration will be identical to modified duration.

The effective duration of the bond used in Table 4.14 is calculated in Table 4.15 based on a parallel change of 25 basis points. Notice that the answer is close to modified Macaulay–Weil duration as expected.

Duration to worst

Duration to worst is identical to modified duration but if there is optionality using the cash flow this results in the worst yield for the investor.

Table 4.15 Effective duration

20 year bond 6% coupon	+0.25% parallel shift		−0.25% parallel shift	
Cash flow	Spot rate (%)	Present value	Spot rate (%)	Present value
6	3.75	5.78	3.25	5.81
6	3.75	5.57	3.25	5.63
6	3.75	5.37	3.25	5.45
6	3.75	5.18	3.25	5.28
6	3.75	4.99	3.25	5.11
6	3.75	4.81	3.25	4.95
6	3.75	4.64	3.25	4.80
6	3.75	4.47	3.25	4.65
6	4.0	4.22	3.5	4.40
6	4.0	4.05	3.5	4.25
6	4.0	3.90	3.5	4.11
6	4.0	3.75	3.5	3.97
6	4.0	3.60	3.5	3.84
6	4.0	3.46	3.5	3.71
6	4.0	3.33	3.5	3.58
6	4.0	3.20	3.5	3.46
6	4.25	2.96	3.75	3.21
6	4.25	2.84	3.75	3.09
6	4.25	2.72	3.75	2.98
106	4.25	46.11	3.75	50.76
	Price	*124.96*	*Price*	*133.04*

Effective duration

$$ED = \frac{P_- - P_+}{2 \times P \times \Delta y} \quad \frac{133.04 - 124.96}{2 \times 128.91 \times 0.25\%} = 12.54$$

Convexity

Duration is only the first order approximation in the change of the fixed income securities' price. The approximation is due to the fact that a curved line represents the relationship between bond prices and interest rates. Duration assumes there is a linear relationship. This approximation can be improved by using a second approximation, convexity:

$$C = \frac{\sum_{i=1}^{n} F_i \times t_i \times (t_i + 1) \times d^{t_i}}{P} \tag{4.109}$$

Modified convexity

$$MC = d^2 \times \frac{\sum_{i=1}^{n} F_i \times t_i \times (t_i + 1) \times d^{t_i}}{P} \tag{4.110}$$

Effective convexity

Again, modified convexity does not calculate the effective convexity of the bond if there is any optionality in future payments. Using estimated prices to calculate effective convexity:

$$EC = \frac{P_{-} + P_{+} - 2 \times P}{P \times (\Delta y)^2} \tag{4.111}$$

Duration beta

The ratio of the portfolio's sensitivity to yield changes with that of the benchmark provides a measure equivalent to beta:

$$D_\beta = \frac{D_P}{D_M} \tag{4.112}$$

Reward to duration

Reward to duration is the Treynor ratio for fixed income portfolios with modified duration replacing beta as the measure of systematic risk.

$$\text{Reward to duration} = \frac{r_P - r_F}{MD} \tag{4.113}$$

WHICH RISK MEASURES TO USE?

Risk like beauty is very much in the eye of the beholder. Determining which risk measure to use is determined by the objectives and preferences of the investor. Although most risk measures are easy to calculate they are not all easy to interpret and are often contradictory.

My advice is to calculate only a few risk measures which are consistent with the investment objectives and easily understood by all parties.

Whichever risk measure you decide to focus on it is important to monitor the change in that measure over time. Risk controllers can never assume the measure is accurate; far better to analyse the change over time and investigate any sudden changes.

The measure may be incorrect but any sudden change provides additional information. The change may result from data errors, system or model errors, change in model assumptions, or an intentional or unintentional change in portfolio risk. Whatever the reason for the change, it should be discussed with the portfolio manager and fully understood.

Most risk measures take the same form, that of a ratio of reward to risk. A summary of reward to risk ratios is shown in Table 4.16.

These measures are most suitable for ranking portfolio performance. The most appropriate ratios take account of investors' risk tolerances. In most circumstances it would be inappropriate to use more than one ratio. For downside risk measures the minimum acceptable or target return should be consistent with the asset manager's investment objectives and strategy. There is relatively little value in using a range of minimum target returns to analyse portfolio performance if the asset manager is targeting relative return.

Ratios all suffer the same problem, they allow us to rank portfolios in order of preference but do not inform us in terms so that we can understand the return advantage of a higher ranking. Risk-adjusted returns on the other hand convert ratios to returns we can understand. Risk-adjusted returns are shown in Table 4.17. Although M^2 is shown for total risk and downside risk, in theory M^2 can be calculated for any risk statistic.

Ex post risk measures will not change dramatically but for effective risk control it is essential to compare the predictive risk calculated by internal systems with the actual realised risk of portfolios.

A number of statistics summarised in Table 4.18 are descriptive only and cannot be used for ranking portfolios but do provide additional information about the nature of portfolio returns.

Risk efficiency ratio

It is important to monitor changes over time in both the *ex ante* and *ex post* tracking errors. It is also important to compare the forecast tracking error with the realised tracking error to gauge how close the forecast is to reality.

The risk efficiency ratio compares realised risk with forecast risk–ideally we would like the ratio to be 1 indicating that our forecasting tools were efficient. If the ratio is much greater than 1 then we are aware that our forecasting tool is underestimating relative risk:

$$\frac{\textit{Ex post}\text{ tracking error}}{\textit{Ex ante}\text{ tracking error}} \text{ or } \frac{\text{Realised risk}}{\text{Forecast risk}} \tag{4.114}$$

Or alternatively:

$$\frac{\textit{Ex post}\text{ VaR}}{\textit{Ex ante}\text{ VaR}} \tag{4.115}$$

Fund rating systems

Noel Amenc and Veronique Le Sourd (2007) wrote a very interesting article on fund rating systems critiquing the methods of Standard & Poor's Micropal, Morningstar and Lipper, essential

Table 4.16 Reward to risk ratios

Ratio	Return measure vertical axis numerator	Risk measure horizontal axis denominator	Comment
Traditional			
Sharpe	Return above risk-free rate	Total risk variability Standard deviation	Reward to variability. The grandfather of all reward to risk ratios
Treynor	Return above risk-free rate	Systematic risk β or volatility	Reward to volatility. Actually defined before the Sharpe ratio but rarely used in practice
Information	Excess return	Tracking error Relative risk	Frequently used. Favoured by institutional asset managers
Appraisal	Jensen's alpha	Specific risk	Rarely used
Modified Jensen	Jensen's alpha	Systematic risk β or volatility	Rarely used
Drawdown			
Calmar	Return above risk-free rate	Maximum drawdown	Quite common for commodity, future and hedge funds
Burke	Return above risk-free rate	Drawdown deviation	Penalises deep drawdowns in particular
Sterling–Calmar	Return above risk-free rate	Average annual maximum drawdown	Most common form of Sterling ratio
Sterling	Return above risk-free rate	Average drawdown	Similar to Sharpe–omega ratio. Multiple definitions of average drawdown used
Martin	Return above risk-free rate	Ulcer index	Similar to Burke. Penalises both depth and duration of drawdowns
Pain	Return above risk-free rate	Pain index	Similar to Martin index but not squared
Downside			
Sortino	Return above risk-free rate	Downside risk	The most appropriate downside risk statistic
Upside potential	Upside potential	Downside risk	Combines upside potential with downside risk
Omega	Upside potential	Downside potential	Similar to upside potential but extreme losses are less harshly penalised

Sharpe–omega	Return above risk-free rate	Downside potential	Identical ranking statistic to omega
Prospect	Utility-adjusted return	Downside risk	Gives greater weight to negative returns
Volatility skewness	Upside variance	Downside variance	Similar to omega but extreme returns are more significant on both the upside and downside
Variability skewness	Upside risk	Downside risk	Square root of volatility skewness
VaR			
Reward to VaR	Return above risk-free rate	VaR	Unconcerned with shape of tail
Conditional Sharpe	Return above risk-free rate	Conditional VaR Tail loss or tail VaR Expected shortfall	Takes account of tail shape
Modified Sharpe	Return above risk-free rate	Modified VaR	Adjusts for Skewness and kurtosis
Other			
Adjusted Sharpe	Return above risk-free rate	Risk, skewness and kurtosis	For those concerned takes account of higher moments. Surprisingly easy to calculate
Reward to duration	Return above risk-free rate	Duration Systematic risk volatility	For fixed income only. Equivalent to Treynor ratio
Kappa	Return above risk-free rate	lth lower moment	Generalised downside measure. For $l = 1$ equivalent to Sharpe–omega ratio; for $l = 2$ equivalent to Sortino ratio

Table 4.17 Risk-adjusted return measures

Measure	Risk adjusted for	Comment
Total return		
M^2	Total risk variability	The original and my preferred measure. Under-used throughout the asset management industry
M^2 for downside risk	Downside risk	Rarely used although demonstrably better than Sortino ratio
Excess return		
Jensen's alpha	Systematic risk	By far the most common. Term often confused with simple excess return. Jensen's alpha is more accurately excess return adjusted for systematic risk
Regression alpha	Systematic risk	Similar to Jensen's alpha ignoring risk-free rate
M^2 excess	Depends on the M^2 measure used	Most often expressed arithmetically but geometric is more appropriate
Net selectivity	Systematic risk and diversification	Good measure for mutual finds. Early version of attribution
Differential return	Total risk	Benchmark return adjusted to risk of portfolio. Less effective than M^2 excess
GH1	Total risk	Efficient frontier equivalent of differential return
GH2	Total risk	Efficient frontier equivalent of M^2
Omega excess	Downside-style variance	Rarely used

reading for any portfolio manager benchmarked against one of these peer group providers. Good ratings from these organisations will influence retail investors and drive sales, therefore it is essential that portfolio managers understand how these ratings are calculated. In effect, these organisations select the most appropriate risk measures on behalf of the retail investor.

The Standard & Poor's Micropal star rating is based on the information ratio relative to the sector average (or median) performance of the funds belonging to the same peer group over a 36-month period. Stars are attributed as follows:

Fund percentile	Rating
Top 10%	*****
10% to 30%	****
30% to 50%	***
50% to 75%	**
Bottom 25%	*

Apart from the problems of peer groups already discussed in Chapter 3, the fact that the benchmark used for the calculation of the information ratio is the average return of the peer group can lead to the counterintuitive result that a fund with less absolute variability is less well regarded simply because it has greater variability against the competition; low absolute risk is not necessarily rewarded in this rating system.

Table 4.18 Descriptive statistics

Statistic	Comment
Regression statistics	
Covariance	Tendency of the portfolio and benchmark returns to move together
Correlation	Standardised covariance
R^2	Square of correlation
Beta	Systematic risk or volatility
Specific risk	Standard deviation of the error term
Other	
Tracking error, relative risk, active risk	Standard deviation of excess returns. The dominant measure of risk for long only institutional asset managers
Total risk, standard deviation variability	Standard deviation of portfolio returns
Downside risk	Semi-standard deviation of returns below target
Kurtosis	Measures flatness of distribution
Skewness	Determines if returns are positively or negatively skewed
Hurst index	Persistency
Average drawdown	
Maximum drawdown	Maximum loss over any investment period. Buy at the top, sell at the bottom
Fixed income measures	
Modified duration	Systematic risk for bonds
Convexity	Rate of change of duration
Value at Risk statistics	
VaR	Expected maximum loss with certain level of confidence
Conditional VaR, Expected shortfall	
Tail loss	
Modified VaR	VaR modified for kurtosis and skewness
Potential gain	Positive (right-hand) equivalent of Value at Risk

Morningstar created their own risk-adjusted performance measure MRAR(γ):

$$\mathrm{MRAR}(\gamma) = \left[\frac{1}{T} \times \sum_{t=1}^{t=T} \left(\frac{1+r_t}{1+r_{Ft}}\right)^{-\gamma}\right]^{-\frac{12}{\gamma}} - 1 \tag{4.116}$$

where: γ = a parameter reflecting the investor's degree of risk aversion.

This measure is subtracted from the annualised excess return relative to the risk-free rate over 36 months. Morningstar set $\gamma = 2$ thus more variable excess returns relative to the risk-free rate are penalised. Stars are attributed as follows:

Fund percentile	Rating
Top 10%	*****
10% to 32.5%	****
32.5% to 67.5%	***
67.5% to 90%	**
Bottom 10%	*

Lipper Micropal provides several different rankings using total return, persistence, capital preservation and expense ratios. [For persistence the Hurst index is used, for capital preservation the sum of negative returns over the cumulative period]. Lipper Micropal scores are attributed as follows:

Fund percentile	Score
Top 20%	Lipper leaders
20% to 40%	2
40% to 60%	3
60% to 80%	4
Bottom 20%	5

Scores are allocated for 3, 5, 10 years and overall. The overall calculation uses an equal weighted average percentile for each period thus giving greater weight to recent performance.

The usefulness of these rankings will very much depend on the quality and monitoring of each sector to ensure consistency by the peer group provider and the relevance of the risk measure to the investor.

RISK CONTROL STRUCTURE

Performance measurers have a key role to play in the risk control environment of asset managers. In the ideal asset management organisation I would have performance measurement, risk control, the legal department and the internal audit function reporting to the head of middle office. Performance measurers should never report to the front office or the marketing department.

For effective risk management in an asset management firm the following should be in place:

(1) *Written risk policy*. To provide a framework for the risk control environment every asset management firm should have a risk policy that clearly articulates the firm's attitude to risk.
(2) *Independence*. For effective risk control it is essential there are appropriate checks and balances within the firm with a clear front, middle and back office structure with clear areas of responsibility and reporting lines.

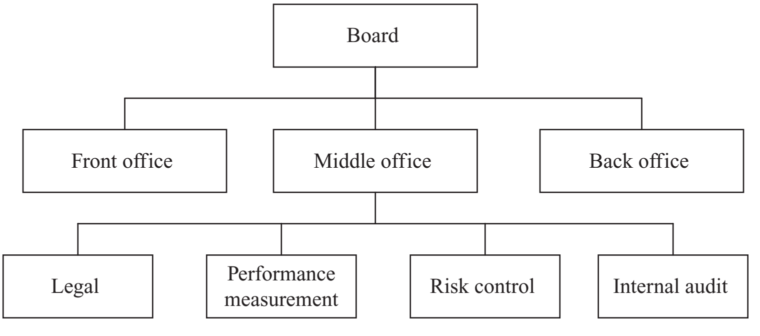

Figure 4.18 Risk control structure

(3) *Risk awareness*. Risk awareness should be embedded within the firm. All employees will own some aspect of risk, it is essential they understand the risks they own and are constantly self-assessing their own risks. Once identified there are four possible responses to risk:
   (a) Ignore the identified risk. A risk although identified maybe ignored if the cost (including opportunity cost) of controlling is greater than the potential cost of risk failure.
   (b) Mitigate. Arranging appropriate insurance cover may mitigate the cost of risk failure.
   (c) Control. Risk may be controlled by establishing risk limits and establishing monitoring procedures.
   (d) Eliminated. It may be appropriate to eliminate an identified risk by ceasing that type of activity.
(4) *Clear risk limits*. Risk limits should be clear and unambiguous and agreed by the client and asset manager in the investment management agreement.
(5) *Risk and performance attribution*. The sources of risk and return should be identified and monitored independently using performance return attribution and other techniques. Accurate return and risk attribution can be used to identify the consistency of added value across the firm and consistency with the agreed investment objectives.
(6) *Appropriate risk-adjusted measures*. Risk and reward should be combined in risk-adjusted performance measures appropriate to the investment management strategy.
(7) *Review process for new products, instruments and strategies*. All new products, instruments and strategies should be rigorously reviewed. For example, a new derivative instrument may meet the needs of the portfolio manager but could generate significant operational and counterparty risks that must be assessed and approved.

An appropriate risk control infrastructure for an asset management firm would include:

(1) *Risk management committee*. Reporting to the board, probably chaired by the head of risk control. Coordinating all risk control activity including senior representatives from front, middle and back office. Responsible for portfolio risk, counterparty risk, compliance risk, review of insurance arrangements disaster recovery and systems change control.
(2) *Portfolio risk committee*. Reporting to the risk management committee, probably chaired by the head of front office. Review that portfolios are managed within client expectations and mandate restrictions. Approve new products, strategies and instruments.
(3) *Credit risk committee*. Reporting to risk management committee. Approve counterparties and limits and monitor firm exposures.
(4) *Operational risk committee*. Reporting to risk management committee. Responsible for error monitoring and information quality.

5 Performance Attribution

Never try to walk across a river just because it has an average depth of four feet.
Milton Friedman (1912–2006)

Definition *Performance attribution is a technique used to quantify the excess return of a portfolio against its benchmark into the active decisions of the investment decision process.*

Performance return attribution is a key management tool for several key stakeholders in the asset management process.

Above all it is the key tool for performance analysts; it allows them to participate in the investment decision process and demonstrably add value, thus justifying their salary. Performance return attribution, together with risk analysis, is the key tool that allows the analyst to understand the sources of return in a portfolio and to communicate that understanding to portfolio managers, senior management and clients.

Effective attribution requires that the analyst thoroughly understands the investment decision process. The task of the analyst is to quantify the decisions taken by the portfolio manager. If the analyst can demonstrate an understanding of the decision process and accurately quantify the decisions taken then the confidence of the portfolio manager will soon be gained. There is little value in analysing factors that are not part of the decision process.

Portfolio managers are obviously major users of attribution analysis. Clearly, they will have a good qualitative understanding of the portfolio but not necessarily a good quantitative understanding. It is all too easy to overestimate the impact of good performing securities and underestimate failures. It is even harder, sometimes, to consider the impact of stocks not held in the portfolio. Stocks represented in the index but not in the portfolio are often large "bets" and may have a significant impact on relative performance, positive or negative.

Attribution analysis provides a good starting point for a dialogue with clients entering a discussion on the positive and negative aspects of recent performance. It is possible to use attribution analysis extremely aggressively, identifying underperformance early and visiting clients with a thorough explanation of underperformance. It is crucial to gain the confidence of the client by demonstrating a good understanding of the drivers of performance.

Senior management take an active interest in attribution analysis to provide them with a tool to monitor their portfolio managers. They will be keen to identify performance outliers – good or bad – and to ensure that value is added consistently across the firm.

ARITHMETIC ATTRIBUTION

The foundations of performance attribution were established in two articles published by Brinson *et al.* (1986) and Brinson and Fachler (1985) now collectively know as the Brinson model.

These articles build on the assumption that the total portfolio returns and benchmark returns are the sum of their parts; in other words, both portfolio and benchmark returns can be disaggregated as follows:

$$\text{Portfolio return} \quad r = \sum_{i=1}^{i=n} w_i \times r_i \tag{5.1 or 2.33}$$

where w_i = weight of the portfolio in the ith asset class (note $\sum_{i=1}^{i=n} w_i = 1$)

r_i = return of the portfolio assets in the ith asset class:

$$\text{Benchmark return} \quad b = \sum_{i=1}^{i=n} W_i \times b_i \tag{5.2 or 3.6}$$

where W_i = weight of the benchmark in the ith asset class (note also $\sum_{i=1}^{i=n} W_i = 1$)

b_i = return of the benchmark in the ith asset class.

The challenge for single period attribution is to quantify each of the portfolio manager's active decisions that contribute to the difference between the portfolio return r and the benchmark return b.

Brinson, Hood and Beebower

Brinson, Hood and Beebower suggested a model to break down the arithmetic excess return $(r - b)$ assuming a standard investment decision process in which the portfolio manager seeks to add value through both asset allocation and security selection.

In asset allocation the portfolio manager (or asset allocator) will seek to add value by taking different asset category (or sector) weights in the portfolio in comparison to category benchmark weights. A category weight in the portfolio greater than the equivalent benchmark category weight would be described as *overweight* and a lesser weight would be described as *underweight*.

Clearly, the asset allocator will aim to overweight good performing categories and underweight poor performing categories. In their original article Brinson, Hood and Beebower called this impact timing; asset or market allocation is now a more common and appropriate description.

In security selection the portfolio manager (or stock selector) will seek to add value by selecting individual securities within the asset category.

Again the stock selector will aim to be overweight in good-performing securities and underweight in poor-performing securities

Asset allocation

To identify the added value from asset allocation we must calculate the return of an intermediate fund called the "allocation or semi-notional fund", which is one step away from the benchmark portfolio, one step towards the actual portfolio.

In the semi-notional fund the asset allocation weights of the actual fund are applied to index returns within each category. By definition the return on this notional fund reflects the portfolio manager's asset allocation "bets" but since index returns are used within the asset category it includes no stock selection:

$$\text{Allocation or semi-notional fund} \quad b_S = \sum_{i=1}^{i=n} w_i \times b_i \tag{5.3}$$

The contribution from asset allocation is therefore the difference between the semi-notional fund and the benchmark fund or:

$$b_S - b = \sum_{i=1}^{i=n} w_i \times b_i - \sum_{i=1}^{i=n} W_i \times b_i = \sum_{i=1}^{i=n} (w_i - W_i) \times b_i \tag{5.4}$$

The contribution to asset allocation in the ith category is:

$$A_i = (w_i - W_i) \times b_i \tag{5.5}$$

Note that:

$$\sum_{i=1}^{i=n} A_i = b_S - b \tag{5.6}$$

Security (or stock) selection

Similarly, to identify the added value from security selection we must calculate the return of a different intermediate fund called the "selection notional fund", which is also by definition one step away from the benchmark return. In the selection notional fund the asset allocation weights of the benchmark are kept static and applied to the category returns within the actual portfolio. By definition the return on this notional fund reflects the portfolio manager's stock selection since real returns are applied to index weights, but excludes any contribution from asset allocation:

$$\text{Selection notional fund} \quad r_S = \sum_{i=1}^{i=n} W_i \times r_i \tag{5.7}$$

The contribution from stock selection is therefore the difference between the selection notional fund and the benchmark fund or:

$$r_S - b = \sum_{i=1}^{i=n} W_i \times r_i - \sum_{i=1}^{i=n} W_i \times b_i = \sum_{i=1}^{i=n} W_i \times (r_i - b_i) \tag{5.8}$$

The contribution to stock selection in category i is:

$$S_i = W_i \times (r_i - b_i) \tag{5.9}$$

Note that:

$$\sum_{i=1}^{i=n} S_i = r_S - b \tag{5.10}$$

Interaction

In this the "classical" definition of attribution, stock selection and asset allocation do not explain the arithmetic difference completely – a third term is required:

$$\text{Stock selection} + \text{Asset allocation} = r_S - b + b_S - b \tag{5.11}$$

or

$$= r_S + b_S - 2 \times b$$

To achieve $r - b$ we must add a third term called interaction:

$$\underbrace{r_S - b}_{\substack{\text{Stock}\\\text{selection}}} + \underbrace{b_S - b}_{\substack{\text{Asset}\\\text{allocation}}} + \underbrace{r - r_S - b_S + b}_{\text{Interaction}} = r - b \tag{5.12}$$

In their article Brinson, Hood and Beebower described this term as other; interaction is perhaps a better description and is in common usage today:

$$r - r_s - b_s + b = \sum_{i=1}^{i=n} w_i \times r_i - \sum_{i=1}^{i=n} W_i \times r_i - \sum_{i=1}^{i=n} w_i \times b_i + \sum_{i=1}^{i=n} W_i \times b_i \tag{5.13}$$

This simplifies to:

$$\sum_{i=1}^{i=n} (w_i - W_i) \times (r_i - b_i) \tag{5.14}$$

It can be seen from Equation (5.14) that interaction is the combination of asset allocation and stock selection effects.

The contribution to interaction in category i is:

$$I_i = (w_i - W_i) \times (r_i - b_i) \tag{5.15}$$

Note that:

$$\sum_{i=1}^{i=n} I_i = r - r_S - b_S + b \tag{5.16}$$

Figure 5.1 illustrates the Brinson framework for return attribution.

Figure 5.2 graphically illustrates the attribution factors for each category i. The contribution to total portfolio return from category i is the area $r_i \times w_i$, the contribution from the benchmark is area $b_i \times W_i$.

The contribution to excess return in category i is the sum of the areas representing selection $W_i \times (r_i - b_i)$, allocation $(w_i - W_i) \times b_i$ and interaction $(w_i - W_i) \times (r_i - b_i)$.

Table 5.1 provides the data for a simple numerical example of a three-category portfolio consisting of UK, Japanese and US equities.

Selection

Allocation	Actual	Passive
Actual	**QuadrantIV** Portfolio return $r=\sum_{i=1}^{i=n} w_i \times r_i$	**QuadrantII** Semi-notional $b_S=\sum_{i=1}^{i=n} w_i \times b_i$
Passive	**QuadrantIII** Selection notional $r_S=\sum_{i=1}^{i=n} W_i \times r_i$	**QuadrantI** Benchmark return $b=\sum_{i=1}^{i=n} W_i \times b_i$

Excess returns due to:

Asset allocation	II – I
Security selection	III – I
Interaction	IV – III – II + I
Total	IV – I

Figure 5.1 Brinson framework for return attribution

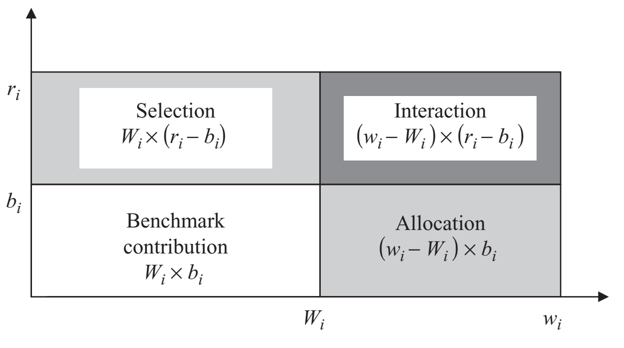

Figure 5.2 Brinson, Hood and Beebower attribution model

Table 5.1 Three category portfolio

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)
UK equities	40	40	20	10
Japanese equities	30	20	–5	–4
US equities	30	40	6	8
Total	*100*	*100*	*8.3*	*6.4*

Total portfolio, benchmark and notional funds are calculated in Exhibit 5.1:

Exhibit 5.1 Return calculations

Using the data from Table 5.1 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 40\% \times 20\% + 30\% \times -5\% + 30\% \times 6\% = 8.3\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 40\% \times 10\% + 20\% \times -4\% + 40\% \times 8\% = 6.4\%$$

The allocation notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 40\% \times 10\% + 30\% \times -4\% + 30\% \times 8\% = 5.2\%$$

The selection notional return $r_S = \sum_{i=1}^{i=n} W_i \times r_i$ is:

$$r_S = 40\% \times 20\% + 20\% \times -5\% + 40\% \times 6\% = 9.4\%$$

The challenge of attribution analysis is to break down and quantify the decisions made by the portfolio manager contributing to the arithmetic excess return of 1.9%.

Using Equation (5.5) we can derive the asset allocation effects shown in Exhibit 5.2 for the data in Table 5.1:

Exhibit 5.2 Asset allocation

Total asset (or country) allocation:

$$b_S - b = 5.2\% - 6.4\% = -1.2\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 40\%) \times 10\% = 0.0\%$
Japanese equities	$(30\% - 20\%) \times -4.0\% = -0.4\%$
US equities	$(30\% - 40\%) \times 8\% = -0.8\%$
Total	$0.0\% - 0.4\% - 0.8\% = -1.2\%$

In Exhibit 5.2 the portfolio weight in UK equities is exactly in line with the benchmark weight, therefore there is no contribution to asset allocation in this category.

There is, however, an overweight position of 10% in Japanese equities which when applied to the negative market return in Japanese equities of −4.0% results in a negative contribution of −0.4%.

If there is an overweight position in a portfolio it follows there must be a least one underweight position. In Table 5.1 there is a 10% underweight position in US equities resulting in a negative contribution of −0.8% when applied to the positive market return in the US market of 8.0%.

The total contribution to arithmetic excess return from asset allocation is −1.2%.

Using Equation (5.9) stock selection effects are calculated in Exhibit 5.3:

Exhibit 5.3 Stock selection

Total stock selection:

$$r_S - b = 9.4\% - 6.4\% = 3.0\%$$

Individual country stock selection effects are:

UK equities	$40\% \times (20\% - 10\%) = 4.0\%$
Japanese equities	$20\% \times (-5.0\% + 4.0\%) = -0.2\%$
US equities	$40\% \times (6.0\% - 8.0\%) = -0.8\%$
Total	$4.0\% - 0.2\% - 0.8\% = 3.0\%$

UK equity performance is very strong, outperforming the benchmark by 10%; the benchmark suggests that 40% of the portfolio should be invested in this category resulting in a 4.0% contribution to arithmetic excess return.

Japanese equities underperformed by 1%; the benchmark suggested a 20% weighting, therefore resulting in a negative contribution of −0.2% from Japanese stock selection.

US equity performance is also poor, underperforming by 2%; the benchmark suggests a 40% weighting, therefore generating a negative contribution of −0.8% from US stock selection.

Total contribution to arithmetic excess return from stock selection is +3.0%. Combining asset allocation of −1.2% and stock selection of +3.0%, 1.8% of added value is explained. The remaining term is interaction calculated by Equation (5.15) as demonstrated in Exhibit 5.4:

Exhibit 5.4 Interaction

Total interaction in Table 5.1 is:

$$r - r_S - b_S + b = 8.3\% - 9.4\% - 5.2\% + 6.4\% = 0.1\%$$

Individual interaction effects are:

UK equities	$(40\% - 40\%) \times (20\% - 10\%) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times (-5.0\% + 4.0\%) = -0.1\%$
US equities	$(30\% - 40\%) \times (6.0\% - 8.0\%) = 0.2\%$
Total	$0.0\% - 0.1\% + 0.2\% = 0.1\%$

The overall contribution from interaction is small. For UK equities the portfolio weight is in line with the benchmark weight and therefore there is no contribution to interaction.

For Japanese equities there is an asset allocation bet of 10%; we have 10% more of this underperforming asset category than suggested by the benchmark, therefore causing a further negative impact of −0.1%.

In US equities there is an underweight bet of 10% in this underperforming category. There is less of this underperforming category than the benchmark suggests, therefore the combined effect of an underweight position in an underperforming category is an added value of +0.2%. Total contribution from interaction is +0.1%

The attribution results are summarised in Table 5.2.

Table 5.2 Brinson, Hood and Beebower attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times b$ (%)	Stock selection $W_i \times (r_i - b_i)$ (%)	Interaction $(w_i - W_i) \times (r_i - b_i)$ (%)
UK equities	40	40	20	10	0.0	4.0	0.0
Japanese equities	30	20	−5	−4	−0.4	−0.2	−0.1
US equities	30	40	6	8	−0.8	−0.8	0.2
Total	*100*	*100*	*8.3*	*6.4*	*−1.2*	*3.0*	*0.1*

Clearly, this attribution model successfully breaks down the sources of arithmetic excess return but does it reflect the investment decision process of the portfolio manager?

For the most part asset allocation decisions are taken in the context of an overall benchmark return; the asset allocator is not seeking to be overweight in positive markets but rather to be overweight in markets that outperform the overall benchmark. The asset allocator will have lost value by being overweight in a market with a positive return that nevertheless returns less than the overall benchmark. We therefore need an attribution model that follows this decision process.

BRINSON AND FACHLER

In the Brinson, Hood and Beebower* model all overweight positions in positive markets will generate positive attribution factors irrespective of the overall benchmark return while all overweight positions in negative markets will generate negative attribution factors.

Clearly, if the asset allocator is overweight in a negative market that has outperformed the overall benchmark, then there should be a positive effect.

The Brinson and Fachler model solves this problem by modifying the asset allocation factor to compare returns against the overall benchmark as follows:

$$b_s - b = \sum_{i=1}^{i=n} (w_i - W_i) \times b_i = \sum_{i=1}^{i=n} (w_i - W_i) \times (b_i - b) \tag{5.17}$$

Since $\sum_{i=1}^{i=n} w_i = \sum_{i=1}^{i=n} W_i = 1$

The contribution to asset allocation in the ith category is now:

$$A_i = (w_i - W_i) \times (b_i - b) \tag{5.18}$$

Graphically extending Figure 5.2 to include the benchmark return in Figure 5.3 we observe no change to the areas representing selection and interaction, but allocation is now described by the area $(w_i - W_i) \times (b_i - b)$.

Equation (5.17) demonstrates that the sum of allocation areas in Figure 5.3 for all categories is equal to the sum of allocation areas in Figure 5.2.

Since $\sum_{i=1}^{n=1} W_i = 1$ the benchmark return is derived by the sum of areas $W_i \times b$ for all categories:

$$\sum_{i=1}^{i=n} W_i \times b = b \tag{5.19}$$

and since both $\sum_{i=1}^{n=1} W_i = 1$ and $\sum_{i=1}^{n=1} w_i = 1$ the sum of areas $(w_i - W_i) \times b$ reduces to zero:

$$\sum_{i=1}^{n=1} (w_i - W_i) \times b = 0 \tag{5.20}$$

*In fact the original Brinson, Hood and Beebower article does not attempt to attribute returns to individual categories. In all likelihood I do not believe the authors intended their top-level formulae to be applied to individual categories as shown; however, over the years many practitioners have done just that.

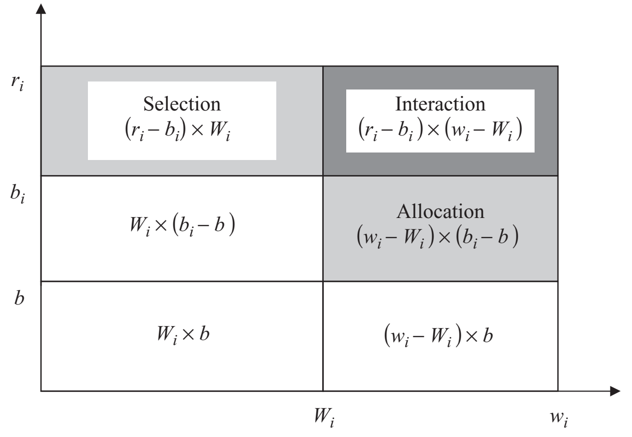

Figure 5.3 Brinson and Fachler attribution model

Revised Brinson and Fachler asset allocation effects are calculated in Exhibit 5.5:

Exhibit 5.5 Brinson and Fachler asset allocation

$$b_S - b = 5.2\% - 6.4\% = -1.2\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 40\%) \times (10.0\% - 6.4\%) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times (-4.0\% - 6.4\%) = -1.04\%$
US equities	$(30\% - 40\%) \times (8.0\% - 6.4\%) = -0.16\%$
Total	$0.0\% - 1.04\% - 0.16\% = -1.2\%$

The impact in Japanese equities is much greater. In addition to being overweight in a negative market which costs −0.4%, we are also rightly penalised the opportunity cost of not being invested in the overall market return of 6.4%, generating a further cost of $10\% \times -6.4\% = -0.64\%$ resulting in a total impact of −1.04%.

The impact in US equities is much smaller. Although being underweight in a positive market which costs −0.8%, we must add back the opportunity cost of being invested in the overall market return of 6.4%, generating a contribution of $-10\% \times -6.4\% = 0.64\%$ resulting in a total impact of −0.16%

The revised attribution effects are summarised in Table 5.3.

INTERACTION

A flaw of both Brinson models is the inclusion of the interaction or other term. Interaction is not part of the investment decision process; you are unlikely to identify in any asset management firm individuals responsible for adding value through interaction.

While it is true that interaction reflects the combined effect of asset allocation bets with stock selection decisions, portfolio managers simply do not seek to add value through interaction. For most investment decision processes the asset allocation decision comes first and stock selection decisions are taken after the cash has been allocated.

Table 5.3 Brinson and Fachler attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times (b_i - b)$ (%)	Stock selection $W_i \times (r_i - b_i)$ (%)	Interaction $(w_i - W_i) \times (r_i - b_i)$ (%)
UK equities	40	40	20	10	0.0	4.0	0.0
Japanese equities	30	20	−5	−4	−1.04	−0.2	−0.1
US equities	30	40	6	8	−0.16	−0.8	0.2
Total	*100*	*100*	*8.3*	*6.4*	*−1.2*	*3.0*	*0.1*

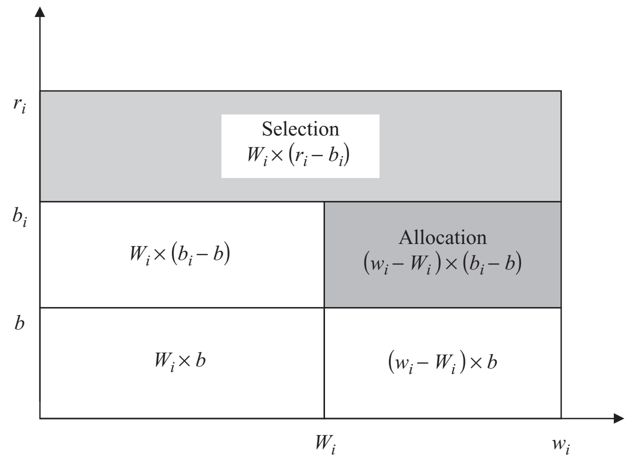

Figure 5.4 Interaction combined with selection

For genuine bottom-up stock pickers, asset allocation decisions are not made, therefore the attribution model should reflect this process and measure the contribution of each stock decision to the overall performance ignoring asset allocation.

Because interaction is not well understood, presumably because it is not intuitively part of the investment decision process, it is often abused. It may be ignored and not shown, randomly allocated to other factors, split proportionally or simply split 50:50 between stock selection and asset allocation and therefore potentially misleading the user.

Assuming asset allocation decisions are taken first, then the contribution from stock selection must be:

$$r - b_s = \sum_{i=1}^{i=n} w_i \times r_i - \sum_{i=1}^{i=n} w_i \times b_i = \sum_{i=1}^{i=n} w_i \times (r_i - b_i) \tag{5.21}$$

Equivalent to quadrant IV – quadrant II from Figure 5.1.

The contribution to stock selection in the ith category is now:

$$S_i = w_i \times (r_i - b_i) \tag{5.22}$$

Figure 5.4 graphically demonstrates the revised impact on individual categories.

Revised stock selection effects including interaction are calculated in Exhibit 5.6:

Exhibit 5.6 Stock selection including interaction

Total stock selection including interaction using the data from Table 5.1 is:

$$r - b_S = 8.3\% - 5.2\% = 3.1\%$$

Individual country stock selection effects are:

UK equities	$40\% \times (20\% - 10\%) = 4.0\%$
Japanese equities	$30\% \times (-5.0\% + 4.0\%) = -0.3\%$
US equities	$30\% \times (6.0\% - 8.0\%) = -0.6\%$
Total	$4.0\% - 0.3\% - 0.6\% = 3.1\%$

Table 5.4 Brinson and Fachler (stock selection and interaction combined)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times (b_i - b)$ (%)	Stock selection $w_i \times (r_i - b_i)$ (%)
UK equities	40	40	20	10	0.0	4.0
Japanese equities	30	20	−5	−4	−1.04	−0.3
US equities	30	40	6	8	−0.16	−0.6
Total	*100*	*100*	*8.3*	*6.4*	*−1.2*	*3.1*

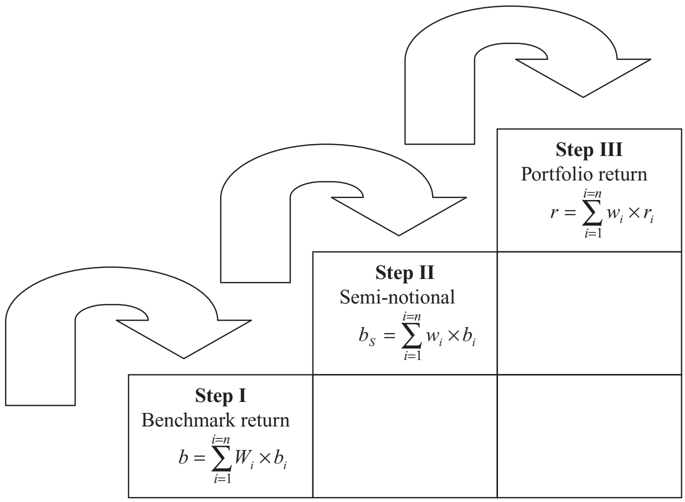

Figure 5.5 Brinson steps

Actual portfolio weights are now used to calculate stock selection effects. Stock pickers rightly point out that their contribution to stock selection is impacted by the weight of assets chosen by the asset allocator. This is true for either portfolio or benchmark weight; there is no particular advantage in calculating the stock selection impact had the portfolio been at the benchmark weight. The individual stock picker should be judged by the performance within the asset category not their contribution to overall performance. The revised results are summarised in Table 5.4.

Rather than the Brinson quadrants of Figure 5.1 I prefer to think in terms of the steps of the investment decision process illustrated in Figure 5.5.

GEOMETRIC EXCESS RETURN ATTRIBUTION

The Brinson models described so far quantify arithmetic excess return. In Chapter 3 an alternative geometric definition of excess return was proposed to measure the added value of portfolio managers.

A number of geometric excess return attribution models (geometric methods) have been developed over the years (Allen, 1991; Bain, 1996; Burnie *et al.*, 1998; Bacon, 2002). These methods are similar and in most cases were in use long before being published externally.

The Brinson model can be easily extended to break down the geometric excess return:

$$\frac{(1+r)}{(1+b)} - 1 \tag{3.3}$$

Asset allocation

To identify the contribution from asset allocation we can use the same intermediate or semi-notional fund we used in the Brinson method, but this time using the geometric rather than the arithmetic difference:

$$\frac{(1+b_S)}{(1+b)} - 1 \tag{5.23}$$

The contribution to geometric asset allocation in the ith category is now:

$$A_i^G = (w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right) \tag{5.24}$$

Note the total geometric asset allocation A^G:

$$A^G = \sum_{i=1}^{i=n} A_i^G = \frac{(1+b_S)}{1+b} - 1 \tag{5.25}$$

Formally, this type of method might be described as a mixed geometric arithmetic method since the individual category allocations sum to the total geometric asset allocation. In a pure geometric model the individual category allocations would compound to the total geometric allocation.

Equation (5.24) is analogous to Equation (5.18); the geometric difference of the category return against the overall benchmark is used rather than the arithmetic difference. A more detailed proof can be found in Appendix A.

Stock selection

Similarly, to identify the total contribution to stock selection we can use the ratio of the portfolio return compared to the semi-notional return:

$$\frac{(1+r)}{(1+b_S)} - 1 \tag{5.26}$$

The contribution to geometric stock selection in the ith category is now:

$$S_i^G = w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)} \tag{5.27}$$

Equation (5.27) is not quite as expected on extension from Equation (5.22); there is an unexpected term $(1+b_i)/(1+b_S)$. This term is required because outperformance in a category whose benchmark return is already performing well will add more value geometrically than the equivalent outperformance in a category whose benchmark return is not performing well. Again, a more detailed proof can be found in Appendix A.

Equation (5.27) simplifies to Equation (5.28) the arithmetic difference divided by the semi-notional fund preferred by Burnie *et al.*:

$$S_i^G = w_i \times \frac{(r_i - b_i)}{(1+b_S)} \tag{5.28}$$

Note the total geometric stock selection S^G:

$$S^G = \sum_{i=1}^{i=n} S_i^G = \frac{(1+r)}{(1+b_S)} - 1 \tag{5.29}$$

The total stock selection and asset allocation effects compound together to produce the geometric excess return:

$$\frac{(1+r)}{(1+b_S)} \times \frac{(1+b_S)}{(1+b)} - 1 = \frac{(1+r)}{(1+b)} - 1 \tag{5.30}$$

or

$$\left(1+S^G\right) \times \left(1+A^G\right) - 1 = \frac{(1+r)}{(1+b)} - 1 = g \tag{5.31}$$

The contribution to geometric excess return from asset allocation is calculated in Exhibit 5.7:

Exhibit 5.7 Geometric asset allocation

Geometric asset allocation for data in Table 5.1 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.052}{1.064} - 1 = -1.13\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(30\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.15\%$
Total	$0.0\% - 0.98\% - 0.15\% = -1.13\%$

Given that the benchmark return is positive the geometric excess return is less than the arithmetic excess return. The contributions to asset allocation are of the same order but slightly less than the arithmetic asset allocation. The sign will always be the same. The contribution to geometric excess return from stock selection is calculated in Exhibit 5.8:

Exhibit 5.8 Geometric stock selection

Geometric stock selection including interaction for data in Table 5.1 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$$

Individual country stock selection effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.052} = 3.80\%$
Japanese equities	$30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.052} = -0.29\%$
US equities	$30\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.052} = -0.57\%$
Total	$3.80\% - 0.29\% - 0.57\% = 2.95\%$

Again, as expected, the geometric stock selection effects are similar to the arithmetic stock selection effects but slightly smaller in magnitude. The geometric attribution effects are summarised in Table 5.5.

Table 5.5 Geometric attribution (stock selection and interaction combined)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	40	40	20	10	0.0	3.8
Japanese equities	30	20	−5	−4	−0.98	−0.28
US equities	30	40	6	8	−0.15	−0.57
Total	*100*	*100*	*8.3*	*6.4*	*−1.13*	*2.95*

SECTOR WEIGHTS

In the Brinson-type models (arithmetic or geometric) we have assumed from Equation (5.1) that the sum of the weights and returns for each asset category will equal the total portfolio return and that the weights will sum to 100%.

To ensure Equation (5.1) holds we must use return calculation methodologies from Chapter 2 that can be disaggregated, i.e. broken down into the contributions to return from each active decision of the investment process.

The return methodology for each category, segment or sector must be identical to that used for the overall return. Because internal rates of return assume a single constant force of return throughout the period of measurement this methodology is not suitable for attribution.

The Dietz methodologies readily fit into the Brinson model since the returns can be disaggregated. Although the total return will be completely explained, transactions during the period of measurement may result in the contributions to asset allocation and stock selection being calculated incorrectly due to the Dietz method weighting assumptions. The more detailed analysis of our standard attribution in Table 5.6 illustrates this point.

The portfolio returns used to calculate the attribution effects summarised in Table 5.5 have actually been calculated using a simple Dietz formula. Using time-weighted returns, summarised for each subperiod in Table 5.6, it would appear that US equities outperform rather than underperform the benchmark.

To calculate the true attribution we must calculate attribution effects for the period immediately before the cash flow and then after the cash flow, summarised in Table 5.7.

Over the entire period asset allocation effects are minimal; all markets performed equally well in the first period and the bet sizes were small for the second period.

Our first attribution did not capture the portfolio's move to a neutral asset allocation position at the time of the cash flow. Although the original attribution reconciled, in this example the Dietz-type returns do not provide the full picture as there has been a transfer of effects between asset allocation and stock selection caused by the choice of methodology.

Table 5.6 More detailed analysis of our standard attribution

	UK equities	Japanese equities	US equities
Start value	£400	£400	£200
End value	£480	£185	£418
Cash flow	£0	−£200	£200
Market value at time of cash flow	n/a	£420	£220
Simple Dietz	$\frac{480-400}{400} = 20.0\%$	$\frac{185-400+200}{400-\frac{200}{2}} = -5\%$	$\frac{418-200-200}{200+\frac{200}{2}} = 6.0\%$
Time-weighted	$\frac{480}{400} - 1 = 20.0\%$	$\frac{420}{400} \times \frac{185}{220} - 1 = -11.7\%$	$\frac{220}{200} \times \frac{418}{420} - 1 = 9.5\%$

Table 5.7 Attribution effects for the period before and after the cash flow

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
1st period						
UK equities	40	40	10	10	0.0	0.0
Japanese equities	40	20	5	10	0.0	−1.8
US equities	20	40	10	10	0.0	0.0
Total	*100*	*100*	*8.0*	*10.0*	*0.0*	*−1.8*
2nd period						
UK equities	40.74	40	9.1	0.0	0.0	3.8
Japanese equities	20.37	20	−15.9	−12.7	−0.0	−0.7
US equities	38.89	40	−0.5	−1.8	−0.0	0.5
Total	*100*	*100*	*0.3*	*−3.3*	*−0.0*	*3.7*
Combined period						
UK equities	n/a	n/a	20.0	10	0.0	3.8
Japanese equities	n/a	n/a	−11.7	−4.0	0.0	−2.5
US equities	n/a	n/a	9.5	8.0	0.0	0.5
Total			*8.3*	*6.4*	*−0.0*	*1.8*

In reality, US stock selection is much better than initially presented and Japanese stock selection much worse. The more frequent the attribution, ideally daily, the more accurate the result.

6
Multi-currency Attribution

The secret of being a bore is to tell everything.
Voltaire (1694–1778) *Sept discours en vers sur l'homme,* 1738

Multi-currency attribution is generally accepted to add several layers of additional complexity to an already complex and controversial area. Many of us have the luxury of ignoring multi-currency issues in single currency portfolios or perhaps the currency decision is simply an implicit decision within the asset allocation process. However, for genuinely multi-currency investment decision processes we must face these issues head on. These issues include but are not limited to:

(1) The compounding effects of market and currency returns.
(2) Interest rate differentials.
(3) Unrealised gains losses in forward currency contracts.
(4) Securities denominated in currencies other than the currency of economic exposure.

Early papers addressing multi-currency attribution include Allen (1991) which offers a geometric approach and Ankrim and Hensel (1992) which is arithmetic.

ANKRIM AND HENSEL

Ankrim and Hensel recognised that the currency return is comprised of two components: the unpredictable "currency surprise" and the predictable interest rate differential or "forward premium" between the appropriate currencies.

Let S_i^t = the spot rate of currency i at time t and F_i^{t+1} = the forward exchange rate of currency i at time t for conversion through a forward contract at time $t+1$. Then the return of currency i is:

$$c_i = \frac{S_i^{t+1} - S_i^t}{S_i^t} = \frac{S_i^{t+1}}{S_i^t} - 1 \tag{6.1}$$

Expanding Equation (6.1):

$$c_i = \frac{S_i^{t+1} - F_i^{t+1} + F_i^{t+1} - S_i^t}{S_i^t} \tag{6.2}$$

We can break down the currency return into:

$$\text{Currency surprise in currency } i \quad e_i = \frac{S_i^{t+1} - F_i^{t+1}}{S_i^t} \tag{6.3}$$

and

$$\text{Forward premium in currency } i \quad d_i = \frac{F_i^{t+1} - S_i^t}{S_i^t} = \frac{F_i^{t+1}}{S_i^t} - 1 \tag{6.4}$$

The currency return is the sum of the currency surprise plus the forward premium or interest rate differential:

$$c_i = e_i + d_i \tag{6.5}$$

We can now expand Equation (5.1) as follows:

$$r = \sum_{i=1}^{i=n} w_i \times (r_i - e_i - d_i) + \sum_{i=1}^{i=n} w_i \times e_i + \sum_{i=1}^{i=n} w_i \times d_i \tag{6.6}$$

If the portfolio includes forward currency contracts these can be isolated separately as shown in Equation (6.7):

$$r = \sum_{i=1}^{i=n} w_i \times (r_i - e_i - d_i) + \sum_{i=1}^{i=n} w_i \times e_i + \sum_{i=1}^{i=n} w_i \times d_i + \sum_{i=1}^{i=n} \tilde{w}_i \times f_i \tag{6.7}$$

where: $\tilde{w}_i =$ weight of currency forward contact in currency i.

The return to forward currency contracts is:

$$f_i = \frac{S_i^{t+1} - F_i^{t+1}}{F_i^{t+1}} = \frac{S_i^{t+1}}{F_i^{t+1}} - 1 \tag{6.8}$$

Note that the forward return is linked to the currency surprise by the formula:

$$f_i = \frac{e_i}{(1 + d_i)} \tag{6.9}$$

In effect, forward currency contracts economically consist of two currencies, one with a positive weight and one negative.

Assuming that the currency returns, currency forward returns, currency surprise and forward premiums are the same in the portfolio and the benchmark then it follows that the benchmark return can be constructed as:

$$b = \sum_{i=1}^{i=n} W_i \times (b_i - e_i - d_i) + \sum_{i=1}^{i=n} W_i \times e_i + \sum_{i=1}^{i=n} W_i \times d_i + \sum_{i=1}^{i=n} \tilde{W}_i \times f_i \tag{6.10}$$

where: $\tilde{W}_i =$ benchmark weight of currency forward contact in currency i.

Applying the standard Brinson approach to Equation (6.7) and Equation (6.10) we derive the following attribution effects:

Asset allocation:

$$A_i = (w_i - W_i) \times (l_i - l) \tag{6.11}$$

where: $l_i = b_i - e_i - d_i = b_i - c_i$ (6.12) which is the arithmetic difference between the benchmark return in base currency and the currency return, but not quite the local return, which should only be derived by using the geometric difference. The revised weighted average benchmark return adjusting for currency is now:

$$l = \sum_{i=1}^{i=n} W_i \times l_i \tag{6.13}$$

Security selection excluding interaction.

$$S_i = W_i \times (k_i - l_i) \tag{6.14}$$

where:

$$k_i = r_i - e_i - d_i = r_i - c_i \tag{6.15}$$

Interaction:

$$I_i = (w_i - W_i) \times (k_i - l_i) \tag{6.16}$$

Or, if you prefer, security selection including stock selection:

$$S_i = w_i \times (k_i - l_i) \tag{6.17}$$

The contribution from currency is analogous to asset allocation:

$$C_i = \underbrace{(w_i - W_i) \times (e_i - e)}_{\text{Underlying assets}} + \underbrace{\left(\tilde{w}_i - \tilde{W}_i\right) \times (f_i - e)}_{\text{Currency forwards}} \tag{6.18}$$

where: C_i = contribution to currency from currency i

$$e = \sum_{i=1}^{i=n} W_i \times e_i \text{ (weighted average currency surprise)} \tag{6.19}$$

The final term, forward premium, is also analogous to asset allocation:

$$D_i = (w_i - W_i) \times (d_i - d) \tag{6.20}$$

where: D_i = contribution to forward premium in currency i.

$$d = \sum_{i=1}^{i=n} W_i \times d_i \quad \text{(weighted average benchmark forward premium)} \tag{6.21}$$

Extending the basic data in Table 5.1 to Table 6.1 creates a multi-currency account including currency forward contracts, with a base currency of sterling.

Exhibit 6.1 calculates the portfolio and benchmark returns for the data in Table 6.1.

Ankrim and Hensel asset allocation effects are calculated in Exhibit 6.2. The asset allocation bets are exactly as before but the individual country returns and the revised benchmark return now include the compounding effect of market and currency returns thus impacting the calculation of asset allocation effects. This compounding effect has changed the total asset allocation effect in Exhibit 5.5 from −1.2% to −1.4%.

Security selection effects are calculated in Exhibit 6.3. There is no compounding effect for UK equities, therefore the result is the same as in Exhibit 5.6. However, the Japanese and US

Table 6.1 Ankrim and Hensel

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Currency return c_i (%)	Currency surprise e_i (%)	Forward premium d_i (%)
UK equities	40	40	20	10.0	0	0	0
Japanese equities	30	20	4.5	5.6	10	9	1
US equities	30	40	27.2	29.6	20	18	2
	$\tilde{w}_i$ (%)	$\tilde{W}_i$ (%)		f_i (%)			
Sterling forward contracts	+20	+30		0			
Yen forward contracts	−15	−10		8.91			
US dollar forward contracts	−10	−20		17.65			
Total	*100*	*100*	*15.29*	*12.54*			

Exhibit 6.1 Return calculations for Ankrim and Hansel

Using Table 6.1 the portfolio return is:

$$r = 40\% \times 20\% + 30\% \times 5.6\% + 30\% \times 29.6\% - 15\% \times 8.91\% - 10\% \times 17.65 = 15.29\%$$

The benchmark return is:

$$b = 40\% \times 10\% + 20\% \times 5.6\% + 40\% \times 29.6\% - 10\% \times 8.91 - 20\% \times 17.65 = 12.54\%$$

The weighted average benchmark currency surprise is:

$$e = 40\% \times 0.0\% + 20\% \times 9.0\% + 40\% \times 18\% = 9.0\%$$

The weighted average benchmark forward premium is:

$$d = 40\% \times 0.0\% + 20\% \times 1.0\% + 40\% \times 2.0\% = 1.0\%$$

The revised weighted average benchmark return premium is:

$$l = 40\% \times (10.0\% - 0.0\%) + 20\% \times (5.6\% - 10.0\%) + 40\% \times (29.6\% - 20.0\%) = 6.96\%$$

Exhibit 6.2 Asset allocation for Ankrim and Hensel

	$(w_i - W_i) \times (l_i - l)$
UK equities	$(40\% - 40\%) \times [(10.0\% - 0.0\%) - 6.96\%] = 0.0\%$
Japanese equities	$(30\% - 20\%) \times [(5.6\% - 10.0\%) - 6.96\%] = -1.14\%$
US equities	$(30\% - 40\%) \times [(29.6\% - 20\%) - 6.96\%] = -0.26\%$
Total	$0.0\% - 1.14\% - 0.26\% = -1.4\%$

Exhibit 6.3 Stock selection for Ankrim and Hensel

	$W_i \times (k_i - l_i)$
UK equities	$40\% \times [(20.0\% - 0.0\%) - (10.0\% - 0.0\%)] = 4.0\%$
Japanese equities	$20\% \times [(4.5\% - 10.0\%) - (5.6\% - 10.0\%)] = -0.22\%$
US equities	$40\% \times [(27.2\% - 20.0\%) - (29.6\% - 20.0\%)] = -0.96\%$
Total	$4.0\% - 0.22\% - 0.96\% = 2.82\%$

equities stock selection has been impacted by currency compounding – dramatically in the case of US equities, changing the effect from −0.6% to −0.96.

For completeness, the revised interaction effects are shown in Exhibit 6.4:

Exhibit 6.4 Interaction for Ankrim and Hensel

	$(w_i - W_i) \times (k_i - l_i)$
UK equities	$(40\% - 40\%) \times [(20.0\% - 0.0\%) - (10.0\% - 0.0\%)] = 0.0\%$
Japanese equities	$(30\% - 20\%) \times [(4.5\% - 10\%) - (5.6\% - 10.0\%)] = -0.11\%$
US equities	$(30\% - 40\%) \times [(27.2\% - 20\%) - (29.6\% - 20\%)] = 0.24\%$
Total	$0.0\% - 0.11\% + 0.24\% = 0.13\%$

Currency allocation is calculated in Exhibit 6.5. From the underlying assets there is no contribution from sterling since there is no bet and no contribution from yen since although there is a 10% overweight position the currency surprise in yen is equal to the weighted average benchmark surprise. There is, however, a −0.9% contribution from a 10% underweight position in the strongly performing US dollar.

The currency forward contracts also contribute to the added value from currency management. Even though this is a sterling-based account it is possible to add value by being underweight sterling – in this example caused by being 10% underweight the benchmark hedged position, sterling underperforming both the yen and US dollar. In the benchmark the currency positions caused by the index weights in the underlying markets have been 50% hedged. Very little impact is caused by the underweight yen bet, but the overweight US dollar forward contract more than offsets the underweight underlying assets' position and contributes +1.3%.

Exhibit 6.5 Currency management for Ankrim and Hensel

Underlying assets	$(w_i - W_i) \times (e_i - e)$
Sterling	$(40\% - 40\%) \times (0.0\% - 9.0\%) = 0.0\%$
Yen	$(30\% - 20\%) \times (9.0\% - 9.0\%) = 0.0\%$
US dollar	$(30\% - 40\%) \times (18.0\% - 9.0\%) = -0.9\%$
Currency forwards	$\left(\tilde{w}_i - \tilde{W}_i\right) \times (f_i - e)$
Sterling	$(20\% - 30\%) \times (0.0\% - 9.0\%) = 0.9\%$
Yen	$(-15\% - 10\%) \times (8.91\% - 9.0\%) = 0.0\%$
US dollar	$(-5\% - 20\%) \times (17.65\% - 9.0\%) = 1.3\%$
Total	$0.0\% + 0.0\% - 0.9\%$ $+0.9\% + 0.0\% + 1.3\% = 1.3\%$

Forward premium effects tend to be small particularly over short periods.

In Exhibit 6.6 the impact of being underweight the US dollar caused by the underlying asset allocations contributes −0.1%:

Exhibit 6.6 Forward premium for Ankrim and Hensel

	$(w_i - W_i) \times (d_i - d)$
Sterling	$(40\% - 40\%) \times (0.0\% - 1.0\%) = 0.0\%$
Yen	$(30\% - 20\%) \times (1.0\% - 1.0\%) = 0.0\%$
US dollar	$(30\% - 40\%) \times (2.0\% - 1.0\%) = -0.1\%$
Total	$0.0\% + 0.0\% - 0.1\% = -0.1\%$

Total attribation effects are summarised in Table 6.2.

There are a number of problems with the Ankrim and Hensel approach:

(1) The main problem is the use of an arithmetic return premium k. This ignores the compounding effect between market and currency returns and distributes this effect across asset allocation, stock selection and interaction.
(2) The forward premium returns are isolated as a separate factor. In reality, this effect is always a consequence of asset allocation decisions. The asset allocator should be cognisant of forward premium effects to avoid abuse; these effects should always be included with asset allocation.
(3) The reference benchmark currency effect e is unaffected by hedging changes to the benchmark.

KARNOSKY AND SINGER

Perhaps the single most important paper published on attribution is Karnosky and Singer (1994). Not only does this paper explain why managing multi-currency portfolios is suboptimal if currency is not managed independently, it also provides a framework for calculating attribution effects, taking into account interest rate differentials. Karnosky and Singer resolve the issue of compounding by using continuously compounded returns in their model and solve the forward premium concern by thinking in terms of "return premium" above local interest rates.

They defined the total return on the portfolio as:

$$r = \sum_{i=1}^{i=n} w_i \times r_{Li} + \sum_{i=1}^{i=n} w_i \times c_i \tag{6.22}$$

where: r_{Li} = return in local currency for currency i.

Expanding Equation (6.22):

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{6.23}$$

where: x_i = interest rate in currency i.

Table 6.2 Ankrim and Hensel attribution

	Asset allocation $(w_i - W_i) \times (l_i - l)$ (%)	Stock selection $W_i \times (k_i - l_i)$ (%)	Forward premium $(w_i - W_i) \times (d_i - d)$ (%)	Currency management $(w_i - W_i) \times (e_i - e)$ (%)	Interaction $(w_i - W_i) \times (k_i - l_i)$ (%)
UK equities	0.0	4.0	0.0	0.0	0.0
Japanese equities	−1.14	−0.22	0.0	0.0	−0.11
US equities	−0.26	−0.96	−0.10	−0.9	0.24
				$\left(\tilde{w}_i - \tilde{W}_i\right) \times (f_i - e)$	
				(%)	
Sterling forward contracts				0.9	
Yen forward contracts				0.0	
US dollar forward contracts				1.3	
Total	*−1.40*	*2.82*	*−0.10*	*1.30*	*0.13*
				Total excess return	2.75

If the portfolio includes forward currency contracts these can be isolated separately as shown in Equation (6.24):

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) + \sum_{i=1}^{i=n} \tilde{w}_i \times f_i \tag{6.24}$$

Note that using continuously compounded returns:

$$f_i = c_i + x_i - x_B$$

where: x_B = interest rate in base currency.

If $\sum_{i=1}^{i=n} \tilde{w}_i = 0$ then $\sum_{i=1}^{i=n} \tilde{w}_i \times f_i = \sum_{i=1}^{i=n} \tilde{w}_i \times (c_i + x_i)$ and Equation (6.24) simplifies to:

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} (w_i + \tilde{w}_i) \times (c_i + x_i) \tag{6.25}$$

It follows that the benchmark can be produced in similar form:

$$b = \sum_{i=1}^{i=n} W_i \times (b_{Li} - x_i) + \sum_{i=1}^{i=n} \left(W_i + \tilde{W}_i\right) \times (c_i + x_i) \tag{6.26}$$

where: b_{Li} = benchmark return in local currency for currency i.

Subtracting Equation (6.26) from Equation (6.25):

$$r - b = \underbrace{\sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) - \sum_{i=1}^{i=n} W_i \times (b_{Li} - x_i)}_{\text{Local premium attribution}}$$

$$+ \underbrace{\sum_{i=1}^{i=n} (w_i + \tilde{w}_i) \times (c_i + x_i) - \sum_{i=1}^{i=n} \left(W_i + \tilde{W}_i\right) \times (c_i + x_i)}_{\text{Currency attribution}} \tag{6.27}$$

Applying the standard Brinson and Fachler* approach separately to the local premium part and currency part of Equation (6.27) as represented in Figure 6.1 we derive the following attribution effects:

Asset allocation:

$$A_i = (w_i - W_i) \times \left(l_i' - l'\right) \tag{6.28}$$

where: $l_i' = b_{Li} - x_i$ (the benchmark return premium)

$$l' = \sum_{i=1}^{i=n} W_i \times l_i' \text{ (the average benchmark return premium)} \tag{6.29}$$

This definition of asset allocation includes the forward premium effect in the benchmark return premium.

Security selection excluding interaction:

$$S_i = W_i \times \left(k_i' - l_i'\right) \tag{6.30}$$

* In their original 1994 paper Karnosky and Singer used a Brinson, Hood and Beebower approach. I've used Brinson and Fachler since it is more relevant to the way most asset managers manage money.

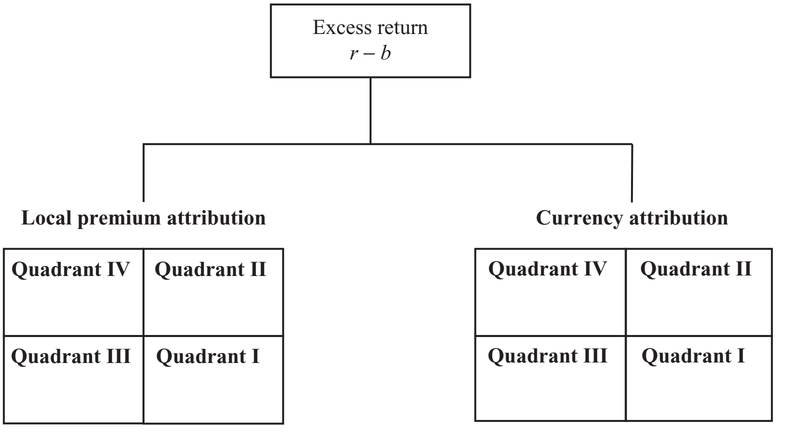

Figure 6.1 Karnosky and Singer multi-currency attribution

where: $k_i' = r_{Li} - x_i$ (the portfolio return premium) (6.31)

Interaction:

$$I_i = (w_i - W_i) \times \left(k_i' - l_i'\right) \tag{6.32}$$

Or, if you prefer, security selection including interaction:

$$S_i = w_i \times \left(k_i' - l_i'\right) \tag{6.33}$$

The contribution from currency is analogous to asset allocation:

$$C_i = \underbrace{(w_i - W_i) \times \left(c_i + x_i - c'\right)}_{\text{Underlying assets}} + \underbrace{\left(\tilde{w}_i - \tilde{W}_i\right) \times \left(c_i + x_i - c'\right)}_{\text{Currency forwards}} \tag{6.34}$$

where:

$$c' = \sum_{i=1}^{i=n} \left(W_i + \tilde{W}_i\right) \times (c_i + x_i) \tag{6.35}$$

From the data in Table 6.3 we can calculate total portfolio and benchmark returns and the local benchmark return premium and currency benchmark in Exhibit 6.7.

Asset allocation effects using the return premium are calculated in Exhibit 6.8:

Stock selection effects, effectively based on local portfolio returns, are calculated in Exhibit 6.9:

The currency management effect including forward currency contracts is calculated in Exhibit 6.10. Note that the currency benchmark is essentially currency plus cash.

The Karnosky and Singer attribution effects are summarised in Table 6.4. This portfolio added value through stock selection and asset allocation but lost value via currency management. Although the physical assets were in line with the benchmark the currency manager chose to use currency forwards to overweight sterling resulting in a loss of –0.23%. Even though the base currency of the account is sterling it is still possible to overweight sterling against

Table 6.3 Karnosky and Singer

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Local interest rates x_i (%)	Currency return c_i (%)
UK equities	40	40	15.0	10.0	1.5	0
Japanese equities	30	20	5.0	6.0	0.5	5
US equities	30	40	−8.0	−6.0	1.0	10
	$\tilde{w}_i$ (%)	$\tilde{W}_i$ (%)				
Sterling forward contracts	+40	+30			1.5	0
Yen forward contracts	−30	−10			0.5	5
US dollar forward contracts	−10	−20			1.0	10
Total	*100*	*100*	*5.1*	*2.8*		
		Total base currency return	7.45	5.5		

Exhibit 6.7 Total portfolio and benchmark returns for Karnosky and Singer

Using Table 6.3 the portfolio return is:

$$r = 40\% \times (15.0\% + 0.0\%) + 30\% \times (6.0\% + 5.0\%) + 30\% \times (-6.0\% + 10.0\%)$$
$$+40\% \times (1.5\% + 0.0\%) - 30\% \times (0.5\% + 5.0\%) - 10\% \times (1.0\% + 10.0\%) = 7.45\%$$

The benchmark return is:

$$b = 40\% \times (10.0\% + 0.0\%) + 20\% \times (6.0\% + 5.0\%) + 40\% \times (-6.0\% + 10.0\%)$$
$$+30.0\% \times (1.5\% + 0.0\%) - 10\% \times (0.5\% + 5.0\%) - 20\% \times (1.0\% + 10.\%) = 5.5\%$$

The average local benchmark return premium is:

$$l' = 40\% \times (10.0\% - 1.5\%) + 20\% \times (6.0\% - 0.5\%) + 40\% \times (-6.0\% - 1.0\%) = 1.7\%$$

The currency plus interest benchmark return is:

$$c' = 40\% \times (1.5\% + 0.0\%) + 20\% \times (0.5\% + 5.0\%) + 40\% \times (1\% + 10\%)$$
$$+30\% \times (1.5\% + 0.0\%) - 10\% \times (0.5\% + 5\%) - 20\% \times (1\% + 10\%) = 3.8\%$$

Exhibit 6.8 Asset allocation for Karnosky and Singer

	$(w_i - W_i) \times \left(l'_i - l'\right)$
UK equities	$(40\% - 40\%) \times [(10.0\% - 1.5\%) - 1.7\%] = 0.0\%$
Japanese equities	$(30\% - 20\%) \times [(6.0\% - 0.5\%) - 1.7\%] = 0.38\%$
US equities	$(30\% - 40\%) \times [(-6.0\% - 1.0\%) - 1.7\%] = 0.87\%$
Total	$0.0\% + 0.38\% + 0.87\% = 1.25\%$

Exhibit 6.9 Stock selection (including interaction) for Karnosky and Singer

	$w_i \times (k'_i - l'_i)$
UK equities	40% × [(15.0% − 1.5%) − (10.0% − 1.5%)] = 2.0%
Japanese equities	30% × [(5.0% − 0.5%) − (6.0% − 0.5%)] = −0.3%
US equities	30% × [(−8.0% − 1.0%) − (−6.0% − 1.0%)] = −0.6%
Total	2.0% − 0.3% − 0.6% = 1.1%

Exhibit 6.10 Currency management for Karnosky and Singer

Underlying assets	$(w_i - W_i) \times (c_i + x_i - c')$
Sterling	(40% − 40%) × [(0.0% + 1.5%) − 3.8%] = 0.0%
Yen	(30% − 20%) × [(5.0% + 0.5%) − 3.8%] = 0.17%
US dollar	(30% − 40%) × [(10.0% + 1.0%) − 3.8%] = −0.72%
Currency forwards	$(\tilde{w}_i - \tilde{W}_i) \times (c_i + x_i - c')$
Sterling	(40% − 30%) × [(0.0% + 1.5%) − 3.8%] = −0.23%
Yen	(−30% − 10%) × [(5.0% + 0.5%) − 3.8%] = −0.34%
US dollar	(−10% − 20%) × [(10.0% + 1.0%.) − 3.8%] = 0.72%
Total	0.0% + 0.17% − 0.72% − 0.23% − 0.34% + 0.72% = −0.40%

a benchmark containing other currency exposures; the other currencies both outperformed sterling even after taking into account higher sterling interest rates. The US dollar currency forward overweight position of 10% exactly matched the underweight US equity physical asset position, therefore the attribution effects cancelled. Unfortunately, the overweight Japanese equity position was more than matched by the underweight yen currency forward position leading to a net loss of −0.17%. The total loss due to currency management is therefore −0.4%.

Table 6.4 Karnosky and Singer attribution

	Asset allocation $(w_i - W_i) \times (l'_i - l')$ (%)	Stock selection $w_i \times (k'_i - l'_i)$ (%)	Currency management $(w_i - W_i) \times (c_i + x_i - c')$ (%)
UK equities	0.0	2.0	0.0
Japanese equities	0.38	−0.30	0.17
US equities	0.87	−0.60	−0.72
			$(\tilde{w}_i - \tilde{W}_i) \times (c_i + x_i - c')$ (%)
Sterling forward contracts			−0.23
Yen forward contracts			−0.34
US dollar forward contracts			0.72
Total	*1.25*	*1.1*	*−0.40*
		Total excess return	*1.95*

GEOMETRIC MULTI-CURRENCY ATTRIBUTION

Karnosky and Singer overcame the arithmetic deficiencies evident in Ankrim and Hansel by using continuously compounded returns. If we wish to continue using simple returns, because of the geometric relationship between market and currency returns, it is essential to use the geometric definition of excess return for multi-currency attribution.

Naïve currency attribution

Deriving the contribution from currency attribution at top level is a relatively straightforward calculation.

We have already established the total portfolio returns in the base currency as:

$$r = \sum_{i=1}^{i=n} w_i \times r_i \qquad \text{(2.34) or (5.1)}$$

and in local currency as:

$$r_L = \sum_{i=1}^{i=n} w_i \times r_{Li} \qquad (2.38)$$

Similarly, for the benchmark:

$$b = \sum_{i=1}^{i=n} W_i \times b_i \qquad \text{(5.2) or (3.6)}$$

and in local currency the "weighted average local benchmark return":

$$b_L = \sum_{i=1}^{i=n} W_i \times b_{Li} \qquad (6.36)$$

and defining in local currency the semi-notional local return:

$$b_{SL} = \sum_{i=1}^{i=n} w_i \times b_{Li} \qquad (6.37)$$

By definition the currency performance of the portfolio must be the relative difference between the portfolio performance in base currency and the weighted total local return of the portfolio:

$$r'_C = \frac{1+r}{1+r_L} - 1 \qquad (6.38)$$

Likewise, the currency performance of the benchmark is defined as the relative difference between the performance of the benchmark in base currency and the performance of the benchmark in local currency:

$$b'_C = \frac{1+b}{1+b_L} - 1 \qquad (6.39)$$

The ratio of the currency return of the portfolio relative to the currency return of the benchmark, called naïve attribution, is:

$$\left(\frac{\dfrac{1+r}{1+r_L}}{\dfrac{1+b}{1+b_L}}\right) - 1 \tag{6.40}$$

This can be rewritten as:

$$\text{Naïve currency attribution} \quad \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1 \tag{6.41}$$

This simple version of currency attribution is described as naïve because it does not take into account interest rate differentials in the manner so excellently described by Karnosky and Singer to which we will return later.

In this naïve version of currency attribution we can calculate our normal attribution effects of stock selection and allocation in local currency as follows:

$$\text{Stock selection} \quad \frac{1+r_L}{1+b_{SL}} - 1 \tag{6.42}$$

$$\text{Asset allocation} \quad \frac{1+b_{SL}}{1+b_L} - 1 \tag{6.43}$$

$$\text{Total currency effects} \quad \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1 \tag{6.44}$$

We can then see that these factors compound to give:

$$\left(\frac{1+r_L}{1+b_{SL}}\right) \times \left(\frac{1+b_{SL}}{1+b_L}\right) \times \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1 = \frac{1+r}{1+b} - 1 \tag{6.45}$$

extending our existing example in Table 6.5, including currency but with no forward currency contracts and expressed in the base currency of sterling. Base currency returns are calculated in Exhibit 6.11.

Table 6.5 Multi-currency geometric

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Currency return c_i (%)
UK equities	40	40	20	10	20	10.0	0
Japanese equities	30	20	−5	−4	4.5	5.6	10
US equities	30	40	6	8	27.2	29.6	20
Total	*100*	*100*	*8.3*	*6.4*	*17.5*	*17.0*	

Exhibit 6.11 Multi-currency attribution returns

Using Table 6.5 the local portfolio return is:

$$r_L = 40\% \times 20\% + 30\% \times -5\% + 30\% \times 6\% = 8.3\%$$

The weighted average local benchmark return is:

$$b = 40\% \times 10\% + 20\% \times -4\% + 40\% \times 8\% = 6.4\%$$

The local semi-notional return is:

$$b_{SL} = 40\% \times 10\% + 30\% \times -4\% + 30\% \times 8\% = 5.2\%$$

The base currency (£) return of the portfolios:

$$r = 40\% \times 20\% + 30\% \times 4.5\% + 30\% \times 27.2\% = 17.51\%$$

The base currency (£) return of the benchmarks:

$$b = 40\% \times 10\% + 20\% \times 5.6\% + 40\% \times 29.6\% = 16.96\%$$

Naïve currency attribution is a fairly straightforward calculation geometrically as shown in Exhibit 6.12:

Exhibit 6.12 Naïve currency attribution

The currency return in the portfolio is:

$$r'_C = \frac{1+r}{1+r_L} - 1 = \frac{1.1751}{1.083} - 1 = 8.50\%$$

The currency return in the benchmark is:

$$b'_C = \frac{1+b}{1+b_L} - 1 = \frac{1.1696}{1.064} - 1 = 9.92\%$$

The naïve currency allocation is therefore:

$$\frac{1+r'_C}{1+b'_C} - 1 = \frac{1+r}{1+r_L}\Bigg/\frac{1+b}{1+b_L} - 1 = \frac{1.0850}{1.0992} - 1 = -1.29\%$$

Using the data in Table 6.5 the return from currency in the portfolio is 8.5% and the return from currency in the benchmark is 9.92%, therefore the added value from currency management must be the ratio of the portfolio currency return with the benchmark currency return, −1.29%.

The asset allocation and stock selection shown in Exhibits 6.13 and 6.14 is unchanged from Exhibits 5.7 and 5.8:

Note that the currency, asset allocation and stock selection compound to provide the total geometric excess return as shown in Exhibit 6.15:

The Naïve currency attribution results are summarised in Table 6.6.

Compounding effects

An unavoidable but calculable complication in multi-currency portfolios is the impact of changing currency exposure due to the changing market values of the underlying assets.

Genuine "currency overlay" managers are unaware of changing market values (either in the portfolio or benchmark) and are only obliged to respond when they are informed of market value changes. Therefore if the currency management is independent we must isolate these compounding effects.

Exhibit 6.13 Multi-currency asset allocation (naïve)

From Exhibit 5.7 the geomatric asset allocation is:

$$\frac{(1+b_{SL})}{(1+b)} - 1 = \frac{1.052}{1.064} - 1 = -1.13\%$$

Individual country asset allocation effects are:

UK equities	$(w_i - W_i) \times \left(\frac{1+b_{Li}}{1+b_L} - 1\right) = (40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(30\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.15\%$
Total	$0.0\% - 0.98\% - 0.15\% = -1.13\%$

Exhibit 6.14 Multi-currency stock selection

From before in Exhibit 5.8 total stock selection in local currency is:

$$\frac{1+r_L}{1+b_{SL}} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$$

Individual country stock selection effects are:

UK equities	$w_i \times \left(\frac{1+r_{Li}}{1+b_{Li}} - 1\right) \times \frac{1+b_{Li}}{1+b_{SL}} = 40\% \times \left(\frac{1.2}{1.1} - 1\right) \times \frac{1.1}{1.052} = 3.8\%$
Japanese equities	$30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.052} = -0.29\%$
US equities	$30\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.052} = -0.57\%$
Total	$3.8\% - 0.29\% - 0.57\% = 2.95\%$

Exhibit 6.15 Naïve currency attribution

$$\left(\frac{1+r_L}{1+b_{SL}}\right) \times \left(\frac{1+b_{SL}}{1+b_L}\right) \times \left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1$$
$$= \frac{1.083}{1.052} \times \frac{1.052}{1.064} \times \frac{1.1751}{1.083} \times \frac{1.064}{1.1696} - 1$$
$$= \frac{1+r}{1+b} - 1 = \frac{1.1751}{1.1696} - 1 = 0.47\%$$

or

$$1.0295 \times 0.9887 \times 0.9871 - 1 = 0.47\%$$

Table 6.6 Naïve currency attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_{Li}}{1+b_L} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_{Li}}{1+b_{Li}} - 1\right) \times \frac{1+b_{Li}}{1+b_{SL}}$ (%)	Naïve currency allocation $\left(\frac{1+r}{1+r_L}\right) \times \left(\frac{1+b_L}{1+b}\right) - 1$ (%)
UK equities	40	40	20	10	0.0	3.8	
Japanese equities	30	20	4.5	5.6	−0.98	−0.28	
US equities	30	40	27.2	29.6	−0.15	−0.57	
Total	*100*	*100*	*17.51*	*16.96*	*−1.13*	*2.95*	*−1.29*

The currency return of the benchmark in the *i*th currency can be calculated as follows:

$$c_i = \frac{1 + b_i}{1 + b_{Li}} - 1 \tag{6.46}$$

Because most commercial international indexes use the same spot rates,† benchmark currency returns for each currency will be consistent and can be derived from base and local index returns using Equation (6.46).

Currency returns from Table 6.4 are calculated in Exhibit 6.16:

Exhibit 6.16 Multi-currency attribution returns

$$\text{Yen} \quad \frac{1.056}{0.96} - 1 = 10.0\%$$

$$\$ \quad \frac{1.296}{1.08} - 1 = 20.0\%$$

At this stage, for simplicity I've assumed the currency return within the portfolio is the same as within the benchmark:

$$\text{Yen} \quad \frac{1.045}{0.95} - 1 = 10.0\%$$

$$\$ \quad \frac{1.272}{1.06} - 1 = 20.0\%$$

We might expect the implied currency return in the benchmark to be equal to the sumproduct of the benchmark weights and currency returns:

$$\text{Implied benchmark currency return} \quad b_C = \sum_{i=1}^{i=n} W_i \times c_i \tag{6.47}$$

Similarly, we might expect the implied currency return in the portfolio to be equal to the sumproduct of the portfolio weights and currency returns:

$$\text{Implied portfolio currency return} \quad b_{SC} = \sum_{i=1}^{i=n} w_i \times c_i \tag{6.48}$$

Implied currency returns are calculated in Exhibit 6.17:

Both the implied benchmark and portfolio currency returns in Exhibit 6.17 appear to differ from the actual currency returns calculated in Exhibit 6.12 which at first seems somewhat unexpected. This is caused by the changing market values of the underlying assets during the period of measurement. In this example the falling Japanese market reduces the exposure to

† WM Reuters 4 o'clock London close.

Exhibit 6.17 Implied currency returns

Implied benchmark currency return:

$$b_C = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% = 10.0\%$$

Implied portfolio currency return:

$$b_{SC} = 40\% \times 0\% + 30\% \times 10.0\% + 30\% \times 20.0\% = 9.0\%$$

the rising yen, which is only partially offset, by the increased exposure to the US dollar caused by the rising US market. The derivation of the real benchmark and portfolio currency returns adjusting for changing market values are shown in Exhibit 6.18:

Exhibit 6.18 Currency return:

Real benchmark currency return:	
	$b'_C = \dfrac{1+b}{1+b_L} - 1 = 9.92\%$
Sterling	$40\% \times 0.0\% \times \left(\dfrac{1.1}{1.064}\right) = 0\%$
Japanese equities	$20\% \times 10.0\% \times \left(\dfrac{0.96}{1.064}\right) = 1.80\%$
US equities	$40\% \times 20.0\% \times \left(\dfrac{1.08}{1.064}\right) = 8.12\%$
	$0.0\% + 1.8\% + 8.12\% = 9.92\%$
Real portfolio currency return:	
	$r'_C = \dfrac{1+r}{1+r_L} - 1 = 8.5\%$
Sterling	$40\% \times 0.0\% \times \left(\dfrac{1.20}{1.083}\right) = 0\%$
Yen	$30\% \times 10.0\% \times \left(\dfrac{0.95}{1.083}\right) = 2.63\%$
US dollar	$30\% \times 20.0\% \times \left(\dfrac{1.06}{1.083}\right) = 5.87\%$
	$0.0\% + 2.63\% + 5.87\% = 8.50\%$

The difference between implied and real currency returns is called compounding.

The total impact of compounding within the benchmark is:

$$\frac{1+b_C}{1+b'_C} - 1 \tag{6.49}$$

The total impact of compounding within the portfolio is:

$$\frac{1 + r'_C}{1 + b_{SC}} - 1 \tag{6.50}$$

The total compounding effects in our sample portfolio using data from Table 6.5 are calculated in Exhibit 6.19:

Exhibit 6.19 Compounding effects

Benchmark compounding:

$$\frac{1 + b_C}{1 + b'_C} - 1 = \frac{1.1}{1.0992} - 1 = 0.07\%$$

Portfolio compounding:

$$\frac{1 + r'_C}{1 + b_{SC}} - 1 = \frac{1.085}{1.09} - 1 = -0.45\%$$

The combined compounding impact in both the portfolio and benchmark is therefore:

$$0.9954 \times 1.0007 = -0.39\%$$

Clearly, it is inappropriate to allocate this effect to currency overlay managers who are unaware of market movements between points of measurement. The effects in this sample portfolio are unusually large and normally only amount to a few basis points, particularly if measurement periods are maintained as short as possible, preferably daily.

Other geometric attribution multi-currency methodologies (Bain, 1996; McLaren, 2001) recognise this impact but suggest addressing this issue by adjusting market weights to calculate revised stock and allocation effects.[‡]

Geometric currency allocation

Using the implied currency returns implicit in Equation (6.48) and Equation (6.47) we can calculate currency allocation effects from the perspective of the currency overlay manager excluding compounding effects.

Currency allocation is analogous to asset allocation and we can use a similar formula:

$$(w_i - W_i) \times \left(\frac{1 + c_i}{1 + b_C} - 1\right) \tag{6.51}$$

So the total currency allocation performance is:

$$\frac{1 + b_{SC}}{1 + b_C} - 1 \tag{6.52}$$

[‡] McLaren provides a choice of which factor to impact determined by the order of the investment decision process.

Currency allocation effects are calculated in Exhibit 6.20:

Exhibit 6.20 Geometric currency allocation

Total currency allocation:

$$\frac{1+b_{SC}}{1+b_C}-1=\frac{1.09}{1.10}=-0.91\%$$

Calculating for individual currencies:

Sterling	$(40\% - 40\%) \times \left(\frac{1.0}{1.10}-1\right) = 0.00\%$
Yen	$(30\% - 20\%) \times \left(\frac{1.1}{1.1}-1\right) = 0.00\%$
US dollar	$(30\% - 40\%) \times \left(\frac{1.20}{1.10}-1\right) = -0.91\%$
Total	$0.00\% + 0.00\% - 0.91\% = -0.91\%$

The currency allocation effects and both compounding effects combine to produce the total naïve currency attribution as follows:

$$\underbrace{\left(\frac{1+r'_C}{1+b_{SC}}\right)}_{\text{Portfolio compounding}} \times \underbrace{\left(\frac{1+b_C}{1+b'_C}\right)}_{\text{Benchmark compounding}} \times \underbrace{\left(\frac{1+b_{SC}}{1+b_C}\right)}_{\text{Currency allocation}} - 1 = \frac{1+r'_C}{1+b'_c}-1 \tag{6.53}$$

Currency timing

Up to now we have assumed that the return for each currency in the portfolio is the same as the benchmark; in real portfolios this is rarely the case. Transactions will take place at exchange rates other than the exchange rates used to calculate indexes. This effect is analogous to stock selection and can be called currency selection or currency timing.

Most performance measurement systems actually derive local returns directly from the base return using benchmark currency returns, therefore a currency timing effect is never observed (one could say lost). More accurately, the local and base currency returns should be calculated separately using market values and cash flows in local and base currencies, respectively. From the point of view of underlying assets this will most likely reflect issues in calculation methodology and data rather than genuine currency timing effects but for forward currency contracts genuine timing effects are likely to be observed and are likely to be important.

Note that b_{SC} is in effect a semi-notional return for currency in that it does not reflect any currency timing effects.

Including currency timing effects the implied currency return of the portfolio should be:

$$r_C=\sum_{i=1}^{i=n} w_i \times c'_i \tag{6.54}$$

where the portfolio currency return in each market is defined as:

$$c'_i = \frac{1 + r_i}{1 + r_{Li}} - 1 \tag{6.55}$$

The formula for currency timing is analogous to stock selection in the *i*th currency as follows:

$$w_i \times \left(\frac{1 + c'_i}{1 + c_i} - 1\right) \times \left(\frac{1 + c_i}{1 + b_{SC}}\right) \tag{6.56}$$

So the total currency timing effect is:

$$\frac{1 + r_C}{1 + b_{SC}} - 1 \tag{6.57}$$

Equation (6.53) must be expanded to include currency timing:

$$\begin{aligned}\underbrace{\left(\frac{1 + r'_C}{1 + r_C}\right)}_{\text{Portfolio compounding}} \times \underbrace{\left(\frac{1 + b_C}{1 + b'_C}\right)}_{\text{Benchmark compounding}} \times \underbrace{\left(\frac{1 + b_{SC}}{1 + b_C}\right)}_{\text{Currency allocation}} \times \underbrace{\left(\frac{1 + r_C}{1 + b_{SC}}\right)}_{\text{Currency timing}} - 1 &= \frac{1 + r'_C}{1 + b'_c} - 1 \\ &= \frac{1 + r}{1 + r_L} \times \frac{1 + b_L}{1 + b} - 1\end{aligned} \tag{6.58}$$

The combination of currency allocation and currency timing can be genuinely allocated to the currency overlay manager.

Equation (6.45) can now be expanded to:

$$\underbrace{\frac{1 + r_L}{1 + b_{SL}}}_{\text{Stock selection}} \times \underbrace{\frac{1 + b_{SL}}{1 + b_L}}_{\text{Asset allocation}} \times \underbrace{\frac{1 + r_C}{1 + b_{SC}} \times \frac{1 + b_{SC}}{1 + b_C}}_{\text{Currency overlay}} \times \underbrace{\frac{1 + r'_C}{1 + r_C} \times \frac{1 + b_C}{1 + b'_C}}_{\text{Compounding}} - 1 = \frac{1 + r}{1 + b} - 1 \tag{6.59}$$

The data in Table 6.7 is adapted from Table 6.5 to illustrate the impact of portfolio currency returns differing from benchmark currency returns. Revised portfolio and portfolio implied currency returns are calculated in Exhibit 6.21; benchmark returns are unchanged.

The currency timing effect can now be calculated in Exhibit 6.22:

The impact of currency timing will change both the portfolio currency return and the compounding effects as demonstrated in Exhibit 6.23:

Revised attribution effects are summarised in Exhibit 6.24.

INTEREST RATE DIFFERENTIALS

A further complicating factor in multi-currency portfolios is the exposure of currency managers to interest rate differentials between currencies as they take currency "bets".

Ankrim and Hensel recognised the impact of interest rate differentials by identifying a separate forward premium effect; Karnosky and Singer recognised this effect by using the return premium of the local market return above local interest rates.

Table 6.7 Multi-currency geometric

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Portfolio currency return c'_i (%)	Benchmark currency return c_i (%)
UK equities	40	40	20	10	20	10.0	0	0
Japanese equities	30	20	−5	−4	4.7	5.6	10.2	10
US equities	30	40	6	8	28.0	29.6	20.8	20
Total	*100*	*100*	*8.3*	*6.4*	*17.8*	17.0		

Exhibit 6.21 Revised portfolio returns

Portfolio base currency return:

$$r = 40\% \times 20\% + 30\% \times 4.7\% + 30\% \times 28.0\% = 17.81\%$$

Note that in Table 6.7 the portfolio currency return in Japanese equities is:

$$\frac{1.047}{0.95} - 1 = 10.2\%$$

This differs from the benchmark currency of 10.0%. Likewise the portfolio currency in US dollars is:

$$\frac{1.28}{1.06} - 1 = 20.8\%$$

This differs from the benchmark currency of 20.0%. The implied portfolio currency return is now:

$$r_C = 40\% \times 0\% + 30\% \times 10.2\% + 30\% \times 20.8\% = 9.29\%$$

Exhibit 6.22 Currency timing effect

$$\frac{1 + r_C}{1 + b_{SC}} - 1 = \frac{1.0929}{1.09} - 1 = 0.27\%$$

Calculating the impact per currency:

Sterling	$40\% \times \left(\frac{1.0}{1.0} - 1\right) \times \frac{1.0}{1.09} = 0\%$
Yen	$30\% \times \left(\frac{1.102}{1.10} - 1\right) \times \frac{1.10}{1.09} = 0.06\%$
US dollar	$30\% \times \left(\frac{1.208}{1.20} - 1\right) \times \frac{1.20}{1.09} = 0.21\%$
Total	$0\% + 0.06\% + 0.21\% = 0.27\%$

Exhibit 6.23 Impact of currency timing

Portfolio currency return:

$$\frac{1 + r}{1 + r_L} - 1 = \frac{1.178}{1.083} - 1 = 8.78\%$$

The total impact of compounding within the portfolio is:

$$\frac{1 + r'_C}{1 + r_C} - 1 = \frac{1.0878}{1.0929} - 1 = -0.47\%$$

From before, the compounding impact in the benchmark is:

$$\frac{1 + b_C}{1 + b'_C} = \frac{1.1}{1.0992} - 1 = 0.07\%$$

Exhibit 6.24 Attribution summary

Stock selection:

$$\frac{1+r_L}{1+b_{SL}}-1=\frac{1.083}{1.052}-1=2.95\%$$

Asset allocation:

$$\frac{1+b_{SL}}{1+b_L}-1=\frac{1.052}{1.064}-1=-1.13\%$$

Naïve currency attribution:

$$\frac{1+r}{1+r_L}\times\frac{1+b_L}{1+b}-1=\frac{1.178}{1.083}\times\frac{1.064}{1.1696}=-1.04\%$$

We can then see that these factors compound again to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}}\times\underbrace{\frac{1+b_{SL}}{1+b_L}}_{\text{Asset}}\times\underbrace{\frac{1+r}{1+r_L}\times\frac{1+b_L}{1+b}}_{\substack{\text{Naïve}\\ \text{currency}\\ \text{attribution}}}-1=\frac{1+r}{1+b}-1$$

$$\underbrace{\frac{1.083}{1.052}}_{\text{Stock}}\times\underbrace{\frac{1.052}{1064}}_{\text{Asset}}\times\underbrace{\frac{1.1781}{1083}\times\frac{1.064}{1.1696}}_{\substack{\text{Naïve}\\ \text{currency}\\ \text{attribution}}}-1=\frac{1.1781}{1.1696}-1=0.73\%$$

or

$$1.0295\times0.9887\times0.9896-1=0.73\%$$

The naïve currency effect can be broken down further to:

$$\underbrace{\frac{1+r_C}{1+b_{SC}}\times\frac{1+b_{SC}}{1+b_C}}_{\substack{\text{Currency}\\ \text{overlay}}}\times\underbrace{\frac{1+r'_C}{1+r_C}\times\frac{1+b_C}{1+b'_C}}_{\text{Compounding}}-1$$

$$\underbrace{\frac{1.0929}{1.09}\times\frac{1.09}{1.1}}_{\substack{\text{Currency}\\ \text{overlay}}}\times\underbrace{\frac{1.0878}{1.0929}\times\frac{1.1}{1.0992}}_{\text{Compounding}}-1=-1.04\%$$

One way to understand why interest rate differentials are important is to focus on asset allocation within the investment decision process. Assuming there is a separate country and currency allocation process, if the country manager decides to overweight Japanese equities that will inevitably create a long position in yen; the currency manager wishing to keep a neutral yen position may decide to hedge the exposed yen position using currency forwards. If the base currency of the portfolio is sterling the currency manager will sell yen and buy sterling. In effect, the currency manager is borrowing yen to buy sterling–there is a cost or benefit attached to this depending on the interest rate differential between the two currencies at the time; currently sterling interest rates are higher than yen, hence it's a benefit. This is why the "carry trade" is so beneficial for Japanese investors at present; they have been selling yen and buying high yielding currencies such as the New Zealand dollar to great advantage, clearly an effect that cannot be ignored in attribution analysis. The process of maintaining a neutral currency position within the portfolio is described as "hedged to neutral".

Apart from hedging exposed positions caused by country managers or achieving hedged positions implied by hedged benchmarks, the currency manager may well be seeking to generate active currency positions–in effect this can only be achieved by forward currency contacts or other derivative instruments that implicitly include interest rate differentials.

Forward currency contracts are priced by reference to the interest rate differential between the relevant currencies. Therefore any currency manager wishing to take a currency allocation "bet" must be exposed to the costs (or benefits) of these interest rate differentials. In other words, the forward currency return rather than the spot currency return must be used to measure currency allocation effects. Crucially, this cost or benefit should be borne by the country allocator not the currency manager.

The currency return between spot rates can be broken down into the forward currency return and the forward premium (or interest rate differential) as follows:

$$c_i = \frac{S_i^{t+1}}{S_i^t} - 1 \tag{6.60}$$

Defining the return on a benchmark forward currency contract as:

$$f_i = \frac{S_i^{t+1}}{F_i^{t+1}} - 1 \tag{6.61}$$

Note that the interest rate differential or forward premium in between currency i and the base currency of the portfolio is:

$$d_i = \frac{F_i^{t+1}}{S_i^t} - 1 \tag{6.62}$$

The currency return is therefore:

$$\frac{S_i^{t+1}}{S_i^t} = \frac{S_i^{t+1}}{F_i^{t+1}} \times \frac{F_i^{t+1}}{S_i^t} = (1 + f_i) \times (1 + d_i) = (1 + c_i) \tag{6.63}$$

The hedged benchmark return b_{Hi} in currency i is defined as:

$$b_{Hi} = \frac{(1 + b_i)}{(1 + f_i)} - 1 \tag{6.64}$$

Note that base, local, hedged, currency, forward returns and interest rate differentials are linked as follows:

$$b_{Hi} = (1 + b_{Li}) \times (1 + d_i) - 1 \tag{6.65}$$

$$b_{Hi} = \frac{(1 + b_i)}{(1 + f_i)} - 1 = (1 + b_{Li}) \times (1 + d_i) - 1 \tag{6.66}$$

Thus the hedged return is either the local return compounded with the interest rate differential or equivalently the base return divided by the forward currency return:

$$\frac{(1 + b_i)}{(1 + b_{Li})} - 1 = (1 + f_i) \times (1 + d_i) - 1 = c_i \tag{6.67}$$

Hedged benchmark returns are normally calculated slightly differently in commercial indexes (and therefore incorrectly). Typically, monthly hedged returns are calculated based on a notional forward contract at the start of the month. Markets will rise or fall but the forward contract is unaltered thus causing residual currency exposure in the calculation.

Revised currency allocation

We must recognise that any "bet" caused by the currency manager must be generated by a forward currency contract (notional or actual) or other currency derivative; the forward currency rate not the spot rate must then be used to measure the contribution of that decision.

Therefore the implied currency return or semi-notional currency return must be adjusted for any variation from the benchmark as follows:

$$b_{SC} = \sum_{i=1}^{i=n} W_i \times c_i + \sum_{i=1}^{i=n} (w_i - W_i) \times f_i \tag{6.68}$$

Equation (6.51) must be adapted to use forward rates rather than spot rates:

$$(w_i - W_i) \times \left(\frac{1 + f_i}{1 + b_C} - 1\right) \tag{6.69}$$

The total currency allocation effect remains:

$$\frac{1 + b_{SC}}{1 + b_C} - 1 \tag{6.52}$$

To continue our analysis we must supplement the data in Table 6.7 with the additional forward currency data* included in Table 6.8. Revised currency allocation is calculated in Exhibit 6.25.

* Note that the interest rate differentials of sterling against the yen and US dollar are: Yen 1.10/1.0887 − 1 or 0.97/0.96 − 1 = 1.04% (UK interest rates 1.04% higher than Japan) US $ 1.20/1.1782 − 1 or 1.1/1.08 − 1 = 1.85% (UK interest rates 1.85% higher than US)

Table 6.8 Forward currency data

	Benchmark hedged return b_{Hi} (%)	Forward currency	Benchmark forward return f_i (%)
UK equities	10.0	Sterling	0.0
Japanese equities	−3.0	Yen	8.87
US equities	10.0	US $	17.82

Revised country allocation

The cost or benefit caused by physical currency positions must be borne by the country allocator. To do this we can use hedged indexes rather than local indexes to measure the true impact of the country allocator. The cost or benefit is in effect transferred from the currency overlay manager to the country allocator.

The revised semi-motional return including the cost of hedge to neutral is:

$$b_{SH} = \sum_{i=1}^{i=n} W_i \times b_{Li} + (w_i - W_i) \times b_{Hi} \tag{6.70}$$

This represents a minor departure from the Karnosky and Singer approach in which any deviation from the base currency is hedged. In this approach only deviations from the benchmark currency position are hedged.

Equation (5.24) must be adapted to use hedged indexes as follows:

$$(w_i - W_i) \times \left(\frac{1 + b_{Hi}}{1 + b_L} - 1\right) \tag{6.71}$$

Exhibit 6.25 Revised currency allocation using forward rates

Revised implied currency return (or semi-notional currency return):

$$\begin{aligned} b_{SC} &= 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% \\ &\quad + (40\% - 40\%) \times 0\% + (30\% - 20\%) \times 8.87\% + (30\% - 40\%) \times 17.82\% = 9.10\% \end{aligned}$$

Revised total currency allocation:

$$\frac{1 + b_{SC}}{1 + b_C} - 1 = \frac{1.091}{1.10} = -0.81\%$$

Calculating for individual currencies:

Sterling $(40\% - 40\%) \times \left(\frac{1.0}{1.10} - 1\right) = 0.00\%$

Yen $(30\% - 20\%) \times \left(\frac{1.089}{1.10} - 1\right) = -0.10\%$

US dollar $(30\% - 40\%) \times \left(\frac{1.178}{1.10} - 1\right) = -0.71\%$

Total $0.00\% - 0.10\% - 0.71\% = -0.81\%$

Exhibit 6.26 Revised asset allocation

$$\begin{aligned} b_{SH} &= 40\% \times 10.0\% + 20\% \times -4.0\% + 40\% \times 8.0\% \\ &\quad +(40\% - 40\%) \times 10\% + (30\% - 20\%) \times -3.0\% + (30\% - 40\%) \times 10.0\% = 5.1\% \end{aligned}$$

Revised country allocation:

$$\frac{1 + b_{SH}}{1 + b_L} - 1 = \frac{1.051}{1.064} - 1 = -1.22\%$$

UK allocation $\quad (40\% - 40\%) \times \left(\frac{1.1}{1.064} - 1\right) = 0.0\%$

Japanese allocation $\quad (30\% - 40\%) \times \left(\frac{0.97}{1.064} - 1\right) = -0.88\%$

US allocation $\quad (30\% - 40\%) \times \left(\frac{1.1}{1.064} - 1\right) = -0.34\%$

$0.0\% - 0.88\% - 0.34\% = -1.22\%$

Total revised asset allocation including the cost of hedging:

$$\frac{1 + b_{SH}}{1 + b_L} - 1 \tag{6.72}$$

The revised semi-notional fund and revised asset allocation is calculated in Exhibit 6.26. The revised asset allocation effect is -1.22%. This differs very slightly from the original asset allocation effect of -1.13% earlier. The difference between the two is simply the cost of hedging the exposed asset allocation bets of the country allocator. Generally, there is no requirement to break down this cost per currency but for completeness this calculation is shown in Exhibit 6.27. There is a benefit in Japan because of the "carry trade"; in effect, low yielding yen are sold for high yielding sterling. In this example low yielding dollars must be borrowed from high yielding sterling. The interest rate differential between dollars and sterling is greater than the interest rate differential between yen and sterling therefore the overall cost is negative.

Exhibit 6.27 Cost of hedging to neutral

$$\frac{1 + b_{SH}}{1 + b_{SL}} - 1 = \frac{1.051}{1.052} - 1 = -0.10\%$$

The cost in each market can be calculated as follows although this level of detail is rarely required:

UK $\quad (40\% - 40\%) \times \left(\frac{1.10}{1.10} - 1\right) \times \left(\frac{1.10}{1.052}\right) = 0.00\%$

Japan $\quad (30\% - 20\%) \times \left(\frac{0.97}{0.96} - 1\right) \times \left(\frac{0.96}{1.052}\right) = 0.10\%$

US $\quad (30\% - 40\%) \times \left(\frac{1.10}{1.08} - 1\right) \times \left(\frac{1.08}{1.052}\right) = -0.19\%$

Total $\quad 0.00\% + 0.10\% - 0.19\% = -0.10\%$

Stock selection effects are calculated in local currency and remain exactly as before. We now have attributed relative performance to stock selection, asset allocation and currency using forward not spot rates as summarised in Exhibit 6.28:

Exhibit 6.28 Revised attribution effects including the cost of hedging

Stock selection:

$$\frac{1+r_L}{1+b_{SL}} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$$

Revised asset allocation:

$$\frac{1+b_{SH}}{1+b_L} - 1 = \frac{1.051}{1.064} - 1 = -1.22\%$$

Total currency effects:

$$\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r'_C}{1+b'_C} - 1 \quad \text{or} \quad \frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_{SL}}{1+b_{SH}}}_{\text{Hedging cost transferred}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\text{Naïve currency attribution}} -1 = \frac{1+r}{1+b} - 1$$

$$\underbrace{\frac{1.083}{1.052}}_{\text{Stock}} \times \underbrace{\frac{1.051}{1.064}}_{\text{Asset}} \times \underbrace{\frac{1.052}{1.051}}_{\text{Hedging cost transferred}} \times \underbrace{\frac{1.1781}{1.083} \times \frac{1.064}{1.170}}_{\text{Naïve currency attribution}} -1 = \frac{1.178}{1.1696} - 1 = 0.73\%$$

Incorporating forward currency contracts

Actual portfolio and notional benchmark forward contracts can be introduced in our standard return formula as follows:

$$r = \sum_{i=1}^{i=n} w_i \times r_i + \sum_{i=1}^{i=n} \tilde{w}_i \times f'_i \tag{6.73}$$

and

$$b = \sum_{i=1}^{i=n} W_i \times b_i + \sum_{i=1}^{i=n} \tilde{W}_i \times f_i \tag{6.74}$$

where: f'_i = return on portfolio forward currency contracts in currency i
f_i = return on benchmark forward currency contracts in currency i
$\tilde{w}_i$ = weight of portfolio forward currency contracts in currency i
$\tilde{W}_i$ = weight of benchmark forward currency contracts in currency i (or, put another way, the benchmark hedged weight).

Forward currency contracts are included for our standard example in Table 6.9. Since we have only introduced physical forward contracts into our example, the stock selection,

Table 6.9 Forward currency contracts

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio local return r_{Li} (%)	Benchmark local return b_{Li} (%)	Portfolio base return r_i (%)	Benchmark base return b_i (%)	Benchmark currency return w_i (%)
UK equities	40	40	20.0	10.0	20.0	10.0	0.0
Japanese equities	30	20	−5.0	−4.0	4.7	5.6	10
US equities	30	40	6.0	8.0	28.0	29.6	20
	$\tilde{w}_i$ (%)	$\tilde{W}_i$ (%)			Portfolio forward return f_i' (%)	Benchmark forward return f_i (%)	
Sterling forward contracts	+20	+30	n/a	n/a	0.0	0.0	
Yen forward contracts	−15	−10	n/a	n/a	9.5	8.9	
US dollar forward contracts	−5	−20	n/a	n/a	17.0	17.8	
3pt] *Total*	100	100	8.3	6.4	15.5	12.5	

Exhibit 6.29 Revised returns incorporating forward currency contracts

The base currency (£) return of the portfolio is now:

$$r = 40\% \times 20\% + 30\% \times 4.7\% + 30\% \times 28.0\%$$
$$+ 20\% \times 0\% - 15\% \times 9.5\% - 5\% \times 17.0\% = 15.54\%$$

The base currency (£) return of the benchmark is now:

$$b = 40\% \times 10\% + 20\% \times 5.6\% + 40\% \times 29.6\%$$
$$+30\% \times 0\% - 10\% \times 8.87\% + -20\% \times 17.82\% = 12.51\%$$

The revised benchmark currency return is now:

$$b_C = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\%$$
$$+30\% \times 0\% - 10\% \times 8.87\% - 20\% \times 17.82\% = 5.55\%$$

The revised currency semi-notional return is now:

$$b_{SC} = 40\% \times 0\% + 20\% \times 10.0\% + 40\% \times 20.0\% + (40\% - 40\% + 20\%) \times 0\%$$
$$+(30\% - 20\% - 15\%) \times 8.87\% + (30\% - 40\% - 5\%) \times 17.82\% = 6.88\%$$

asset allocation and cost of hedging results remain as before. Clearly, there are new currency allocations and currency timing effects but also revised compounding effects.

Extending Equations (6.47) and (6.48) to include currency forwards, the revised implied currency benchmark return and implied portfolio currency return are:

$$b_C = \sum_{i=1}^{i=n} W_i \times c_i + \sum_{i=1}^{i=n} \tilde{W}_i \times f_i \tag{6.75}$$

$$b_{SC} = \sum_{i=1}^{i=n} W_i \times c_i + \sum_{i=1}^{i=n} (w_i - W_i + \tilde{w}_i) \times f_i \tag{6.76}$$

Revised portfolio, benchmark and currency benchmark and currency semi-notional returns are calculated in Exhibit 6.29.

Note that spot rate returns are used for the equity market exposures; physical assets generate the exposure not forward currency contracts. Forward currency rates are only used if there is an element of hedging in the benchmark description. This differs slightly from the Karnosky and Singer method which assumes any investment in overseas assets generates a return premium, i.e. must be hedged back to the base currency; in most investment strategies only the variance from benchmark should be measured this way. The currency exposure in the benchmark is obtained without the use of forward currency contracts and not exposed to interest rate differential – unless of course the benchmark includes an element of hedging.

Equation (6.68) can be extended to specifically include forward contracts:

$$\left((w_i + \tilde{w}_i) - \left(W_i + \tilde{W}_i\right)\right) \times \left(\frac{1 + f_i}{1 + b_C} - 1\right) \tag{6.77}$$

Revised currency allocation is calculated in Exhibit 6.30. In this example the portfolio has benefited from being underweight sterling and overweight both the US dollar and Japanese yen.

Exhibit 6.30 Currency allocation including forwards

Total currency allocation:

$$\frac{1+b_{SC}}{1+b_C}-1=\frac{1.0688}{1.0555}=1.26\%$$

Calculating for individual currencies:

$$\left((w_i+\tilde{w}_i)-\left(W_i+\tilde{W}_i\right)\right)\times\left(\frac{1+f_i}{1+b_C}-1\right)$$

Sterling	$(60\%-70\%)\times\left(\frac{1.0}{1.0555}-1\right)=0.53\%$
Yen	$(15\%-10\%)\times\left(\frac{1.089}{1.0555}-1\right)=0.16\%$
US dollar	$(25\%-20\%)\times\left(\frac{1.178}{1.0555}-1\right)=0.58\%$
Total	$0.53\%+0.16\%+0.58\%=1.26\%$

To calculate currency timing effects we need to adjust Equation (6.73) to measure pure timing effects excluding the impact of hedging to neutral:

$$r_{SC}=\sum_{i=1}^{i=n}w_i\times c_i+\sum_{i=1}^{i=n}\tilde{w}_i\times f_i \tag{6.78}$$

Portfolio currency returns and pure currency semi-notional returns are calculated in Exhibit 6.31:

Exhibit 6.31 Revised portfolio currency returns including forwards

The portfolio currency return is now:

$$\begin{aligned}r_C&=40\%\times 0\%+30\%\times 10.2\%+30\%\times 20.8\%\\&\quad+20\%\times 0\%-15\%\times 9.5\%+-5\%\times 17.0\%=7.01\%\end{aligned}$$

From the perspective of the currency overlay manager the semi-notional currency return is:

$$\begin{aligned}r_{SC}&=40\%\times 0\%+30\%\times 10.0\%+30\%\times 20.0\%+20\%\times 0\%\\&\quad-15\%\times 8.87\%-5\%\times 17.82\%=6.78\%\end{aligned}$$

The timing effect of forward currency contracts and physical assets can be included by adapting Equation (6.56).

For physical assets:

$$w_i\times\left(\frac{1+c_i'}{1+c_i}-1\right)\times\left(\frac{1+c_i}{1+r_{SC}}\right) \tag{6.79}$$

For forward contracts:

$$\tilde{w}_i\times\left(\frac{1+f_i'}{1+f_i}-1\right)\times\left(\frac{1+f_i}{1+r_{SC}}\right) \tag{6.80}$$

Exhibit 6.32 Currency timing effects including forwards

$$\frac{1+r_C}{1+r_{SC}} = \frac{1.0701}{1.0678} - 1 = 0.22\%$$

$$w_i \times \left(\frac{1+c_i'}{1+c_i} - 1\right) \times \left(\frac{1+c_i}{1+r_{SC}}\right)$$

Sterling	$40\% \times \left(\frac{1.0}{1.0} - 1\right) \times \frac{1.0}{1.0678} = 0\%$
Yen	$30\% \times \left(\frac{1.102}{1.10} - 1\right) \times \frac{1.10}{1.0678} = 0.06\%$
US dollar	$30\% \times \left(\frac{1.208}{1.20} - 1\right) \times \frac{1.20}{1.0678} = 0.21\%$

The timing effects in forward contracts are:

$$\tilde{w}_i \times \left(\frac{1+f_i'}{1+f_i} - 1\right) \times \left(\frac{1+f_i}{1+b_{SC}}\right)$$

Sterling forwards	$20\% \times \left(\frac{1.0}{1.0} - 1\right) \times \frac{1.0}{1.0678} = 0\%$
Yen forwards	$-15\% \times \left(\frac{1.095}{1.089} - 1\right) \times \frac{1.089}{1.0678} = -0.09\%$
US dollar forwards	$-5\% \times \left(\frac{1.17}{1.178} - 1\right) \times \frac{1.17}{1.0678} = 0.04\%$
Total	$0\% + 0.06\% + 0.21\% + 0\% - 0.09\% + 0.04\% = 0.22\%$

Timing attribution is calculated in Exhibit 6.32. The timing effects in forward currency are genuine currency timing effects caused by the timing decisions of the currency overlay manager. To the extent that local returns of underlying assets are derived in performance systems, not calculated separately, typically, timing decisions are only observed in forward currency contracts and therefore appropriately attributed to the overlay manager.

Summarising

We now have attributed relative performance to the following factors:

Stock selection:

$$\frac{1+r_L}{1+b_{SL}} - 1 \tag{6.42}$$

Asset allocation:

$$\frac{1+b_{SH}}{1+b_L} - 1 \tag{6.70}$$

Exhibit 6.33 Summary of attribution factor ratios

Stock selection:

$$\frac{1+r_L}{1+b_{SL}} - 1 = \frac{1.083}{1.052} - 1 = 2.95\%$$

Asset allocation:

$$\frac{1+b_{SH}}{1+b_L} - 1 = \frac{1.051}{1.064} - 1 = -1.22\%$$

Total currency effects:

$$\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+C_r}{1+C_b} - 1 \quad \text{or} \quad \frac{1+b_S}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$$

$$\frac{1.052}{1.051} \times \frac{1.155}{1.083} \times \frac{1.064}{1.125} - 1 = 0.98\%$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_{SL}}{1+b_{SH}}}_{\substack{\text{Hedging}\\\text{cost}\\\text{transferred}}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\substack{\text{Naïve}\\\text{currency}\\\text{attribution}}} -1 = \frac{1+r}{1+b} - 1$$

$$\underbrace{\frac{1.083}{1.052}}_{\text{Stock}} \times \underbrace{\frac{1.051}{1064}}_{\text{Asset}} \times \underbrace{\frac{1.052}{1.051}}_{\substack{\text{Hedging}\\\text{cost}\\\text{transferred}}} \times \underbrace{\frac{1.155}{1083} \times \frac{1.064}{1.125}}_{\substack{\text{Naïve}\\\text{currency}\\\text{attribution}}} -1 = \frac{1.155}{1.125} - 1 = 2.69\%$$

The total currency effects can be broken down further to:

$$\underbrace{\frac{1+r_C}{1+r_{SC}} \times \frac{1+b_{SC}}{1+b_C}}_{\substack{\text{Currency}\\\text{overlay}}} \times \underbrace{\frac{1+b_{SH}}{1+b_{SL}} \times \frac{1+r_{SC}}{1+b_{SC}}}_{\substack{\text{Hedging}\\\text{mismatch}}} \times \underbrace{\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}}_{\text{Compounding}} -1$$

$$\underbrace{\frac{1.0701}{1.0678} \times \frac{1.0688}{1.0555}}_{\substack{\text{Currency}\\\text{overlay}}} \times \underbrace{\frac{1.052}{1.051} \times \frac{1.0678}{1.0688}}_{\substack{\text{Hedging}\\\text{mismatch}}} \times \underbrace{\frac{1.0668}{1.0701} \times \frac{1.0555}{1.0574}}_{\text{Compounding}} -1 = 0.98\%$$

Total currency effects:

$$\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r'_C}{1+b'_C} - 1 \tag{6.81}$$

or

$$\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1 \tag{6.82}$$

Table 6.10 Currency attribution

	Portfolio weight $w_i + \tilde{w}_i$ (%)	Benchmark weight $W_i + \tilde{W}_i$ (%)	Portfolio currency c'_i (%)	Benchmark currency c_i (%)	Portfolio forwards f'_i (%)	Benchmark forwards f_i (%)	Currency allocation $(w_i + \tilde{w}_i - W_i - \tilde{W}_i) \times \left(\frac{1+f_i}{1+c_i} - 1\right)$ (%)	Timing (%)
Sterling	60	70	0.0	0.0	0.0	0.0	0.53	0.0
Yen	15	10	10.2	10.0	9.5	8.9	0.16	−0.03
US $	25	20	20.8	20.0	17.0	17.9	0.58	0.25
Total	*100*	*100*	*7.01*	*5.55*			1.26	0.22
							Compounding $\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}$	−0.5
							Hedging mismatch $\frac{1+b_S}{1+b_{SH}} \times \frac{1+r_{SC}}{1+b_{SC}}$	0.0

Table 6.11 Multi-currency geometric attribution

	Stock selection (%)	County allocation (%)	Currency timing (%)	Currency allocation (%)	Other effects* (%)	
UK equities	3.8	0	0	0.53		
Japanese equities	−0.29	−0.88	−0.03	0.16		Total excess
US equities	−0.57	−0.34	0.25	0.58		return (%)
Total	2.95	−1.22	0.22	1.26	−0.5	2.69

*In real portfolios the other effects tend to be no more than one or two basis points and are reduced if measurement periods are kept as short as possible, ideally daily.

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}} \times \underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}} \times \underbrace{\frac{1+b_{SL}}{1+b_{SH}}}_{\substack{\text{Hedging}\\\text{cost}\\\text{transferred}}} \times \underbrace{\frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b}}_{\substack{\text{Naïve}\\\text{currency}\\\text{attribution}}} -1 = \frac{1+r}{1+b} - 1 \tag{6.83}$$

The total currency effects can be broken down further to:

$$\underbrace{\frac{1+r_C}{1+r_{SC}} \times \frac{1+b_{SC}}{1+b_C}}_{\substack{\text{Currency}\\\text{overlay}}} \times \underbrace{\frac{1+b_{SL}}{1+b_{SH}} \times \frac{1+r_{SC}}{1+b_{SC}}}_{\substack{\text{Hedging}\\\text{mismatch}}} \times \underbrace{\frac{1+r'_C}{1+r_C} \times \frac{1+b_C}{1+b'_C}}_{\text{compounding}} -1 \tag{6.84}$$

Note that an addition ratio has been introduced, $(1+r_{SC})\big/(1+b_{SC})$, to ensure no residuals result from the calculation. This effect, when combined with the cost of hedging, is normally close to zero. This minor adjustment is required because the cost of hedging impact, implicitly included in currency overlay, is applied to a slightly different denominator from that of the country allocator. All of these ratios are re-created for the standard example in Exhibit 6.33:

Finally, we can summarise the currency attribution effects in Table 6.10 and overall attribution effects in Table 6.11.

Other currency issues

Unlike futures contracts in which gains and losses are channelled through the margin account, unrealised gains and losses build up in forward currency contracts. The practical consequence of this is that there will be a net forward position which damps performance in the event of unrealised gains and gears (provides leverage) performance in the event of unrealised losses. This net position is an attributable factor in its own right.

The denomination of a security does not necessarily coincide with the economic exposure of a security. A classic example are Japanese warrants and convertible bonds denominated in US \$, Swiss francs and other currencies to encourage international investors to buy. These instruments are ultimately linked to the yen price of a security. The prices of these instruments effectively adjust for the currency movements between the denomination currency and yen and therefore are economically exposed to yen. Other common examples include ADRs.

7
Fixed Income Attribution

His priority did not seem to teach them what he knew, but rather to impress upon them that nothing, not even. . .knowledge, was foolproof.

J.K. Rowling (1965–) *Harry Potter and the Order of the Phoenix*

Gentlemen prefer bonds.

Andrew Mellon (1855–1937)

The investment decision process for bond managers is very different from that of equity managers, therefore for most fixed income investment strategies the standard Brinson model is not suitable. Bonds are simply a series of defined future cash flows which are relatively easy to price. Fixed income performance is therefore driven by changes in the shape of the yield curve. Systematic risk in the form of duration is a key part of the investment process. Fixed income attribution is, in fact, a specialist form of risk-adjusted attribution

THE YIELD CURVE

The yield curve is a graph of various bond redemption yields against term to maturity; a snapshot of current market yields. Portfolio managers will seek to add value by anticipating the changing shape of the yield curve over time.

Figure 7.1 shows a typical yield curve. At the short end of the curve to the left the starting point is short-term interest rates, to the right yields gradually increase since we expect greater risk over longer periods; in this example the yield curve is sloping upwards. The actual shape of the yield curve will reflect any expected changes in short-term rates and the risk premium for holding long-term assets. An inverted yield curve may indicate a recession is expected.

Yield to maturity (or gross redemption yield)

The yield to maturity or gross redemption yield of a bond takes into account the pattern of future coupon payments, the term to maturity and the final capital redemption payment. The yield to maturity is equivalent to the internal rate of return of the bond, the rate that equates to the value of discounted cash flows on the bond to its current price.

Coupon yield curve

The coupon yield curve graphs only bonds with the same coupon rate.

Par yield curve

The par yield curve graphs only bonds that are currently trading close to par.

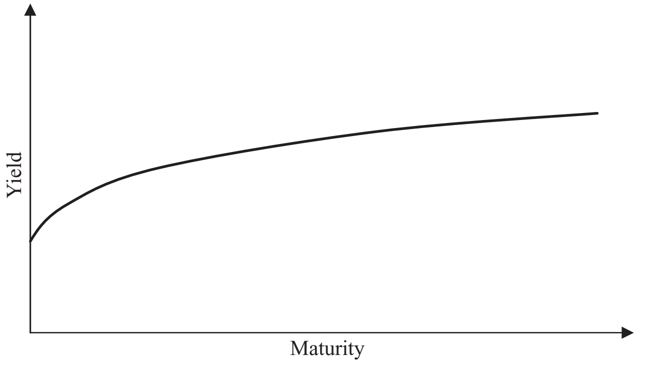

Figure 7.1 The yield curve

Zero-coupon (or spot) curve

The zero-coupon or spot yield curve uses the yield of zero-coupon bonds, bonds that have only a single cash flow at redemption on maturity.

Wagner and Tito

Wagner and Tito (1977) suggested a Fama-type decomposition for bonds using duration instead of *beta* as the measure of systematic risk, see Figure 7.2 which is similar to Figure 4.8.

In many ways Fama-type decomposition lends itself more to fixed income analysis than equity analysis since the measure of systematic risk, duration, is much more relevant in the fixed income decision process.

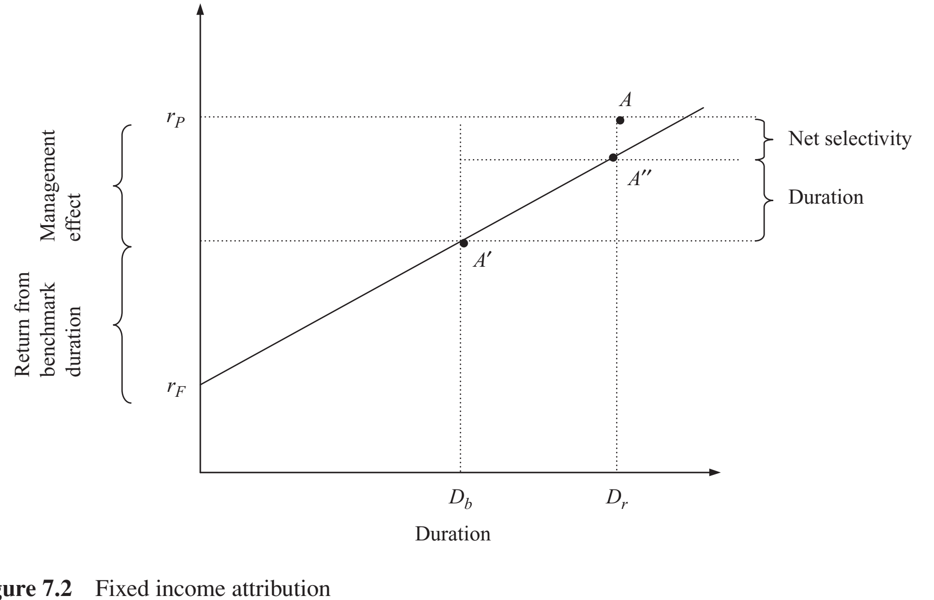

Figure 7.2 Fixed income attribution

The difference between A'' and A' represents the added value from interest rate changes–A'' is the return expected at the duration of the portfolio and A' is the return of benchmark. The difference between A and A' represents the added value from issue selection.

Weighted duration attribution

Van Breukelen (2000) suggested an approach to fixed interest attribution for "top-down" investment decision processes focused on the major risk factor of weighted duration which combines the approach of Wagnar and Tito and the Brinson model. Essentially, it is a risk-adjusted version of the Brinson model using duration as a measure of systematic risk.

Van Breukelen uses the following approximation formula for the return on bonds:

$$r_{L_i} = x_i + D_i \times (-\Delta y_i) \tag{7.1}$$

where D_i = modified duration in bond category i
Δy_i = change in yield for category i
x_i = interest rate in currency i (from Karnosky and Singer).

Using the Karnosky and Singer definition of portfolio return:

$$r = \sum_{i=1}^{i=n} w_i \times (r_{Li} - x_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{6.23}$$

It follows that:

$$b = \sum_{i=1}^{i=n} W_i \times (b_{Li} - x_i) + \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{7.2}$$

Substituting Equation (7.1) into Equation (6.23):

$$r = \sum_{i=1}^{i=n} w_i \times D_i \times (-\Delta y_i) + \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{7.3}$$

The factor $w_i \times D_i$ is equivalent to an equity weight; the bond manager can increase exposure by either increasing weight or increasing modified duration.

It follows that the benchmark return can be described as:

$$b = \sum_{i=1}^{i=n} W_i \times D_{bi} \times (-\Delta y_{bi}) + \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{7.4}$$

where D_{bi} = benchmark modified duration for category i
Δy_{bi} = change in benchmark yield for category i.

Applying the standard Brinson approach to Equations (7.3) and (7.4) the excess return we wish to attribute is:

$$\begin{aligned} r - b = &\sum_{i=1}^{i=n} w_i \times D_i \times -\Delta y_i - \sum_{i=1}^{i=n} W_i \times D_{bi} \times -\Delta y_{bi} \\ &+ \sum_{i=1}^{i=n} w_i \times (c_i + x_i) - \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \end{aligned} \tag{7.5}$$

Let:

$$c = \sum_{i=1}^{i=n} w_i \times (c_i + x_i) \tag{7.6}$$

$$c' = \sum_{i=1}^{i=n} W_i \times (c_i + x_i) \tag{7.35}$$

We are familiar with the last two terms which represent currency attribution in the Karnosky and Singer model (in this case without currency forwards which can easily be added).

Van Breukelen suggests creating two reference or notional funds to measure the contribution from fixed income management excluding currency, namely, overall duration, market selection and issue selection.

The overall duration notional fund is defined as:

$$b_D = \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times -\Delta y_{bi} + c' \tag{7.7}$$

where D_r = portfolio duration
D_b = benchmark duration

$$D_\beta = \frac{D_r}{D_b} = \text{duration beta} \tag{4.112}$$

The duration beta is equivalent to an equity beta and can be used in the same way.

Therefore the contribution from overall duration:

$$\begin{aligned} b_D - b &= \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times \Delta y_{bi} - \sum_{i=1}^{i=n} D_{bi} \times W_i \times \Delta y_{bi} \\ &= \left(D_\beta \times \sum_{i=1}^{i=n} D_{bi} \times W_i - \sum_{i=1}^{i=n} D_{bi} \times W_i \right) \times \Delta y_b \\ &= \sum_{i=1}^{i=n} D_{bi} \times W_i \times \left(D_\beta - 1\right) \times \Delta y_b \end{aligned} \tag{7.8}$$

The overall duration effect should only be measured if it is part of the investment decision process.

The duration-adjusted semi-notional fund is defined as:

$$r_S = \sum_{i=1}^{i=n} D_i \times w_i \times -\Delta y_{bi} + c' \tag{7.9}$$

Therefore the contribution from weighted duration allocation is:

$$r_S - b_D = \sum_{i=1}^{i=n} D_i \times w_i \times \Delta y_{bi} - \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times \Delta y_{bi} \tag{7.10}$$

Appling the same Brinson and Fachler approach as in Equation (5.17):

$$r_S - b_D = \sum_{i=1}^{i=n} (D_i \times w_i - D_\beta \times D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{7.11}$$

Table 7.1 Fixed income attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio modified duration D_i	Benchmark modified duration D_{bi}	Portfolio return r_i (%)	Benchmark return b_i (%)	Risk-free Rate x_i (%)
UK bonds	50	50	7.8	5.0	5.6	3.5	1.0
Japanese bonds	20	10	1.0	2.0	0.5	0.5	0.1
US bonds	30	40	4.0	3.0	3.2	3.0	0.2
Total	100	100	5.3	3.9	3.86	3.0	0.59

Market allocation for category i is therefore:

$$A_i' = \left(D_i \times w_i - D_\beta \times D_{bi} \times W_i\right) \times (-\Delta y_{bi} + \Delta y_b) \tag{7.12}$$

If the overall duration is not part of the investment decision process we can miss a step and move directly to:

$$r_S - b = \sum_{i=1}^{i=n} (D_i \times w_i - D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{7.13}$$

Market allocation for category i is therefore:

$$A_i = (D_i \times w_i - D_{bi} \times W_i) \times (-\Delta y_{bi} + \Delta y_b) \tag{7.14}$$

Issue selection is calculated by:

$$r - r_S - \left(c - c'\right) = \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{ri}) - \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{bi}) - c + c' \tag{7.15}$$

Issue selection for category i is therefore:

$$S_i = D_i \times w_i \times (-\Delta y_{ri} + \Delta y_{bi}) \tag{7.16}$$

Currency allocation using Karnosky and Singer without forward contracts is:

$$C_i = (w_i - W_i) \times \left(c_i + x_i - c'\right) \tag{6.34}$$

Table 7.1 provides the data for a simple numerical example of a three-category portfolio consisting of UK, Japanese and US bonds.

Using the data in Table 7.1 the portfolio and benchmark total returns and durations are verified and the duration beta calculated in Exhibit 7.1:

Exhibit 7.1 Total returns and duration

Portfolio modified duration:

$$D = \sum_{i=1}^{i=n} w_i \times D_i \quad 50\% \times 7.8 + 20\% \times 1.0 + 30\% \times 4.0 = 5.3$$

Benchmark modified duration:

$$D_b = \sum_{i=1}^{i=n} W_i \times D_{bi} \quad 50\% \times 5.0 + 10\% \times 2.0 + 40\% \times 3.0 = 3.9$$

(*Continued*)

Exhibit 7.1 *(Continued)*

Duration beta:

$$D_\beta = \frac{5.3}{3.9}$$

Portfolio return:

$$r = \sum_{i=1}^{i=n} w_i \times r_i \quad 50\% \times 5.6\% + 20\% \times 0.5\% + 30\% \times 3.2\% = 3.86\%$$

Benchmark return:

$$b = \sum_{i=1}^{i=n} W_i \times b_i \quad 50\% \times 3.5\% + 10\% \times 0.5\% + 40\% \times 3.0\% = 3.0\%$$

Portfolio risk-free rate:

$$x_r = \sum_{i=1}^{i=n} w_i \times x_i = 50\% \times 1.0\% + 20\% \times 0.1\% + 30\% \times 0.2\% = 0.58\%$$

or c if currency returns are 0. Benchmark risk-free rate:

$$x_b = \sum_{i=1}^{i=n} W_i \times x_i = 50\% \times 1.0\% + 10\% \times 0.1\% + 40\% \times 0.2\% = 0.59\%$$

or c' if currency returns are 0.

Using Equation (7.1) the implied yield changes are calculated directly from the portfolio and benchmark returns in Exhibits (7.2) and (7.3):

Exhibit 7.2 Implied portfolio yield changes

UK bonds	$\Delta y_i = \frac{-(r_i - x_i)}{D_i} = \frac{-(5.6\% - 1.0\%)}{7.8} = -0.59\%$
Japanese bonds	$\frac{-(0.5\% - 0.1\%)}{1.0} = -0.4\%$
US bonds	$\frac{-(3.2\% - 0.2\%)}{4.0} = -0.75\%$
Total portfolio	$\frac{-(3.86\% - 0.58\%)}{5.3} = -0.62\%$

Exhibit 7.3 Implied benchmark yield changes

UK bonds	$\frac{-(3.5\% - 1.0\%)}{5.0} = -0.5\%$
Japanese bonds	$\frac{-(0.5\% - 0.1\%)}{2.0} = -0.2\%$
US bonds	$\frac{-(3.0\% - 0.2\%)}{3.0} = -0.93\%$
Total benchmark	$\frac{-(3.00\% - 0.59\%)}{3.9} = -0.62\%$

In this particular example the overall duration is part of the decision process. The overall duration notional fund and duration-adjusted semi-notional funds are calculated in Exhibit 7.4:

Exhibit 7.4 Notional funds

Overall duration notional fund:

$$b_D = \sum_{i=1}^{i=n} D_\beta \times D_{bi} \times W_i \times -\Delta y_{bi} + c'$$

$$= \frac{5.3}{3.9} \times (5.0 \times 50\% \times 0.5\% + 2.0 \times 10\% \times 0.2\% + 3.0 \times 40\% \times 0.93) + 0.59\% = 3.87\%$$

Duration-adjusted semi-notional fund:

$$r_S = \sum_{i=1}^{i=n} D_i \times w_i \times -\Delta y_{bi} + c' = 7.8 \times 50\% \times 0.5\% + 1.0 \times 20\% \times 0.2\% + 4.0 \times 30\% \times 0.93$$
$$+ 0.59\% = 3.70\%$$

The overall duration effect is calculated by taking the difference between the duration notional fund and the benchmark return as shown in Exhibit 7.5. It is one decision and therefore one allocation number is calculated. In this example the portfolio duration is much greater than the benchmark duration; since markets are rising this is a positive effect, adding 0.87% of value.

Exhibit 7.5 Overall duration allocation

$$b_D - b = 3.87\% - 3.0\% = 0.87\%$$

Exhibit 7.6 calculates the market allocation effects for each category. Since we have adjusted for the overall duration we must adjust the benchmark-weighted duration using the duration beta to ensure the correct effect is calculated. The portfolio is effectively overweight UK bonds which underperformed the overall index slightly, losing 0.06% underweight Japanese bonds which added 0.03% but underweight US bonds which lost 0.14% of value.

Exhibit 7.6 Market allocation

$$r_S - b_D = \sum_{i=1}^{i=n} \left(D_i \times w_i - D_\beta \times D_{bi} \times W_i\right) \times (-\Delta y_{bi} + \Delta y_b) = 3.70\% - 3.87\% = -0.17\%$$

UK bonds	$\left(D_i \times w_i - D_\beta \times D_{bi} \times W_i\right) \times (-\Delta y_{bi} + \Delta y_b)$
	$= (7.8 \times 50\% - \frac{5.3}{3.9} \times 5.0 \times 50\%) \times (0.5 - 0.62) = -0.06\%$
Japanese bonds	$(1.0 \times 20\% - \frac{5.3}{3.9} \times 2.0 \times 10\%) \times (0.2 - 0.62) = 0.03\%$
US bonds	$(4.0 \times 30\% - \frac{5.3}{3.9} \times 3.0 \times 40\%) \times (0.93 - 0.62) = -0.14\%$
Total market allocation	$-0.06\% + 0.03\% - 0.14\% = -0.17\%$

Exhibit 7.7 calculates the security or issue selection effects. The portfolio outperformed in UK and Japanese bonds, evidenced by yield falls greater than benchmark, but underperformed in US bonds. Currency effects are calculated in Exhibit 7.8. In this example currency returns

are zero, therefore currency allocation is measuring the local interest rate allocation effects. The portfolio is overweight in low yielding Japanese interest rates, losing value, but this is almost offset against an underweight exposure to low US interest rates

Exhibit 7.7 Security (or issue) selection

$$r - r_S = \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{ri}) - \sum_{i=1}^{i=n} D_i \times w_i \times (-\Delta y_{bi}) - c + c'$$
$$= 3.86\% - 3.70\% - 0.58\% + 0.59\% = 0.17\%$$

UK bonds	$D_i \times w_i \times (-\Delta y_{ri} + \Delta y_{bi}) = 7.8 \times 50\% \times (0.59\% - 0.5\%) = 0.35\%$
Japanese bonds	$1.0 \times 20\% \times (0.4\% - 0.2\%) = 0.04\%$
US bonds	$4.0 \times 30\% \times (0.75\% - 0.93\%) = -0.22\%$
Total security selection	$0.35\% + 0.04\% - 0.22\% = 0.17\%$

Exhibit 7.8 Currency allocation

$$c - c' = \sum_{i=1}^{i=n} w_i \times (c_i + x_i) - \sum_{i=1}^{i=n} W_i \times (c_i + x_i) = 0.58\% - 0.59\%$$

UK bonds	$(w_i - W_i) \times (c_i + x_i - c') = (50\% - 50\%) \times (0.0\% + 1.0\% - 0.59\%) = 0.0\%$
Japanese bonds	$(20\% - 10\%) \times (0.0\% + 0.1\% - 0.59\%) = -0.05\%$
US bonds	$(30\% - 40\%) \times (0.0\% + 0.2\% - 0.59\%) = 0.04\%$
Total currency allocation	$0.0\% - 0.05\% + 0.04\% = -0.01\%$

The fixed income attribution effects are summarised in Table 7.2. This type of attribution is particularly suited to global bond portfolios and balanced portfolios. For balanced portfolios essentially the same Brinson approach is employed but the impact of duration can be factored into the fixed income portion of the portfolio. The risk factor for equities is category weight and the risk factor for bonds is weighted duration.

Geometric fixed income attribution

Although van Breukelen presented an arithmetic attribution model it can be easily adapted for geometric analysis. The four-step decision process is shown for the data in Table 7.1 in Figure 7.3; the geometric difference between each step represents the total effect of that factor. Exhibit 7.9 calculates the geometric overall duration effect:

Exhibit 7.9 Overall duration allocation (geometric)

$$\frac{1 + b_D}{1 + b} - 1 = \frac{1.0387}{1.03} - 1 = 0.84\%$$

Table 7.2 Weighted duration attribution

	Portfolio weighted duration $w_i \times D_i$	Benchmark weighted duration $W_i \times D_{bi}$	Portfolio change in yield Δy_i Δy_i (%)	Benchmark change in yield Δy_{bi} (%)	Market allocation $(D_i \times w_i - D_\beta$ $\times D_{bi} \times W_i)\times$ $(-\Delta y_{bi} + \Delta y_b)$ (%)	Issue selection $D_i \times w_i \times$ $(-\Delta y_{ri} + \Delta y_{bi})$ (%)	Currency allocation $(w_i - W_i) \times$ $(c_i + x_i - c')$ (%)
UK bonds	3.9	2.5	−0.59	−0.5	−0.06	0.35	0.0
Japanese bonds	0.2	0.2	−0.4	−0.2	0.03	0.04	−0.05
US bonds	1.2	1.2	−0.75	−0.93	−0.14	−0.22	0.04
Total	*100%*	*100%*	*−0.62*	*−0.62*	*−0.17*	0.17	−0.01
					Overall duration $b_D - b$		0.87

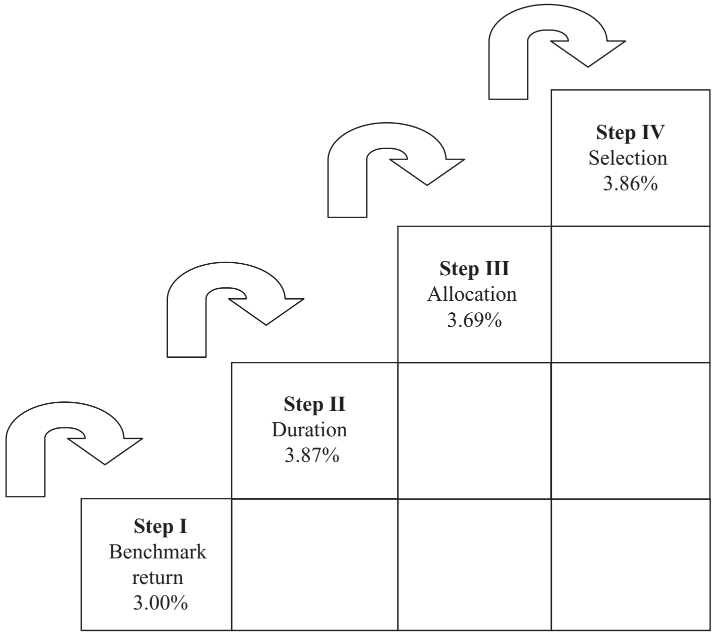

Figure 7.3 Geometric fixed income attribution

Exhibit 7.10 calculates the market allocation effect, adjusting for the overall duration effect already measured combined with the predictable contribution from allocation to the risk-free rate (currency effects in this example are ignored). Market allocation and risk–free rate allocation are combined in one step since portfolio weight (one part of the market allocation decision) determines the allocation to different interest yields.

Exhibit 7.10 Geometric market allocation

$$r'_S = \sum_{i=1}^{i=n} D_i \times w_i \times -\Delta y_{bi} + c = 7.8 \times 50\% \times 0.5\% + 1.0 \times 20\% \times 0.2\% + 4.0 \times 30\% \times 0.93$$
$$+0.58\% = 3.69\%$$

$$\frac{1+r'_S}{1+b_D} - 1 = \frac{1.0369}{1.0387} - 1 = -0.17\%$$

UK bonds $$\left(7.8 \times 50\% - \frac{5.3}{3.9} \times 5.0 \times 50\%\right) \times \frac{(0.5 - 0.62)}{1.0387} = -0.06\%$$

Japanese bonds $$\left(1.0 \times 20\% - \frac{5.3}{3.9} \times 2.0 \times 10\%\right) \times \frac{(0.20 - 0.62)}{1.0387} = 0.03\%$$

US bonds $$\left(4.0 \times 30\% - \frac{5.3}{3.9} \times 3.0 \times 40\%\right) \times \frac{(0.93 - 0.62)}{1.0387} = -0.13\%$$

UK risk-free rate $$(50\% - 50\%) \times \left(\frac{1.01}{1.0059} - 1\right) \times \frac{1.0059}{1.0387} = 0.00\%$$

Exhibit 7.10 (*Continued*)

Japanese risk-free rate	$(20\% - 10\%) \times \left(\frac{1.001}{1.0059} - 1\right) \times \frac{1.0059}{1.0387} = -0.05\%$
US risk-free rate	$(30\% - 40\%) \times \left(\frac{1.002}{1.0059} - 1\right) \times \frac{1.0059}{1.0387} = +0.04\%$
Total market allocation	$-0.06\% + 0.03\% - 0.13\% + 0.0\% - 0.05\% + 0.04\% = -0.17\%$

A revised semi-notional fund r'_S reflecting the combined step is calculated. Issue selection is calculated in Exhibit 7.11. As an alternative to using the change in yield applied to weighted duration it is possible to adjust the benchmark return to that expected at the portfolio duration in order to calculate issue selection. The interpretation of the attribution effects summarised in Table 7.3 remains identical to the arithmetic example.

Exhibit 7.11 Geometric issue selection

	$\frac{1+r}{1+r'_S} - 1 = \frac{1.0386}{1.0369} - 1 = 0.16\%$
Duration adjusted benchmark:	
UK bonds	$\frac{D_i}{D_{bi}} \times (b_i - x_i) + x_i = \frac{7.8}{5.0} \times (3.5\% - 1.0\%) + 1.0\% = 4.9\%$
Japanese bonds	$\frac{1.0}{2.0} \times (0.5\% - 0.1\%) + 0.1\% = 0.3\%$
US bonds	$\frac{4.0}{3.0} \times (3.0\% - 0.2\%) + 0.2\% = 3.93\%$
Individual issue selection:	
UK bonds	$50\% \times \left(\frac{1.056}{1.049} - 1\right) \times \frac{1.049}{1.0369} = 0.34\%$
Japanese bonds	$20\% \times \left(\frac{1.005}{1.003} - 1\right) \times \frac{1.003}{1.0369} = 0.04\%$
US bonds	$30\% \times \left(\frac{1.032}{1.0393} - 1\right) \times \frac{1.0393}{1.0369} = -0.21\%$
Total	$0.34\% - 0.04\% + 0.21\% = 0.16\%$

Campisi framework

Using similar concepts to van Breukelen and Wagnar and Tito, Campisi (2000) suggests breaking down fixed income returns into income and price return as illustrated in Figure 7.4.

In the Campisi framework the total return can be expressed as income plus price change:

$$\text{Total return} = \text{income return} + \text{price change} \tag{7.17}$$

Table 7.3 Geometric weighted duration attribution

	Portfolio weighted duration $w_i \times D_i$	Benchmark weighted duration $W_i \times D_{bi}$	Portfolio change in yield Δy_i (%)	Benchmark change in yield Δy_{bi} (%)	Market allocation (%)	Issue selection (%)	Risk-free rate (%)
UK bonds	3.9	2.5	−0.59	−0.5	−0.06	0.34	0.0
Japanese bonds	0.2	0.2	−0.4	0.2	0.03	0.04	−0.05
US bonds	1.2	1.2	−0.75	−0.93	−0.13	−0.21	0.04
Total	*100*	*100*	*−0.62*	*−0.62*	*−0.16*	0.16	−0.01
				Overall duration	$\frac{1+b_D}{1+b}-1$		0.84

where:

$$\text{Income return} = \frac{\text{Annual coupon rate}}{\text{Beginning market price}} \tag{7.18}$$

$$\begin{aligned}\text{Price return} &= \text{effect of yield changes}\\ &= \text{treasury effect} + \text{spread effect} + \text{selection effect}\end{aligned} \tag{7.19}$$

The Treasury effect represents the impact in the change of Treasury interest rates. The sensitivity to the change in yields of a portfolio is measured by modified duration, see Equation (4.106).

In effect, Campisi adapts Equation (7.1) replacing local interest rates with income return and including spread effects as follows:

$$r_i = I_i + D_i \times (-\Delta y_i) + D_i \times (-\Delta ys_i) + \varepsilon_i \tag{7.20}$$

where I_i = return from income in sector i
D_i = modified duration in sector i
Δy_i = change in Treasury interest rates at duration D_i
Δys_i = change in benchmark spreads in sector i
ε_i = residual return not explained by income, Treasury or spread effects.

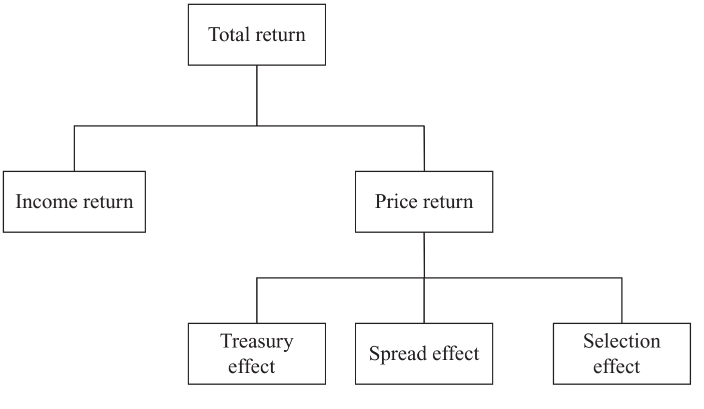

Figure 7.4 Campisi framework

It follows that the benchmark return is:

$$b_i = I_{bi} + D_{bi} \times (-\Delta y_i) + D_{bi} \times (-\Delta ys_i) \tag{7.21}$$

where I_{bi} = return from income in benchmark sector i
D_{bi} = benchmark modified duration in sector i.

Clearly, there should be no residual returns in the benchmark; it is assumed any return not explained by income or Treasury effects must be spread effects.

Attribution effects using the Campisi framework are calculated using data from Table 7.4.

The structural added value from income is simply the portfolio income return minus the benchmark income return (0.52% − 0.45%) which rounds to an added value of 0.08%. To calculate the Treasury effect we need the interest rate changes for each level of duration shown in Exhibit 7.12:

Exhibit 7.12 Treasury effect

Duration D_i	Interest change Δy_i (%)	Price effect $D_i \times (-\Delta y_i)$ (%)
3.6	−1.00	+3.60
4.0	−1.05	+4.20
4.3	−1.10	+4.73
4.75	−1.20	+5.70
5.25	−1.35	+7.09

Portfolio Treasury return is therefore:

$20\% \times 5.7\% + 65\% \times 3.6\% + 15\% \times 4.73\% = 4.19\%$

Benchmark Treasury return is therefore:

$50\% \times 5.7\% + 40\% \times 7.09\% + 10\% \times 4.2\% = 6.11\%$

Therefore the added value from the Treasury effect is $4.19\% - 6.11\% = -1.92\%$.

The benchmark spread effect can be calculated in each sector assuming that any return contribution that is neither income nor Treasury effect in the benchmark must be caused by spread. The implied yields calculated from the benchmark can be applied to the durations of the portfolio sectors to calculate the contribution from spread in the portfolio; any residual return must be a selection effect. Spread and selection effects are calculated in Exhibit 7.13 and attribution effects using the Campisi framework are summarised in Table 7.5.

Campisi does not suggest breaking down returns any further although it is relatively straightforward to apply a combination of both the Brinson model and van Breukelen to obtain a more detailed analysis. Income effects are calculated in Exhibit 7.14, Treasury effects in Exhibit 7.15, the spread effect in Exhibit 7.16 and results are summarised in Table 7.6.

Interpreting the results the portfolio was short duration and lost 118 basis points for a parallel shift in interest rates and another 74 basis points given that long rates fell more than short rates. Structurally, the portfolio benefited from slightly higher income and a tightening in credit spreads plus a slight contribution from issue selection.

Table 7.4 Campisi framework

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio modified duration D_i	Benchmark modified duration D_{bi}	Portfolio return r_i (%)	Portfolio income return I_i (%)	Benchmark return b_i (%)	Benchmark income return I_{bi} (%)
Treasuries	20	50	4.75	4.75	6.0	0.3	6.0	0.3
Corporates	65	40	3.6	5.25	4.4	0.52	8.0	0.57
High Yield	15	10	4.3	4.0	5.6	0.82	5.0	0.71
Total	100	100	3.94	4.88	4.9	0.52	6.7	0.45

Exhibit 7.13 Spread effects

Benchmark spread effect:

$$\Delta y_{Si} = \frac{-(b_i - I_{bi} - D_{bi} \times (-\Delta y_i))}{D_{bi}}$$

Corporates $\frac{-(8.0 - 0.57\% - 5.25 \times 1.35)}{5.25} = -0.07\%$

High yield $\frac{-(5.0 - 0.71\% - 4.0 \times 1.05)}{4.0} = -0.02\%$

The contribution to return from spread in the benchmark is therefore:

Corporates: Return $-\Delta y_{Si} \times D_{bi} = 0.07 \times 5.25 = 0.35\%$

Contribution $W_i \times -\Delta y_{Si} \times D_{bi} = 40\% \times 0.07 \times 5.25 = 0.14\%$

High yield: Return $0.02 \times 4.0 = 0.09\%$

Contribution $10\% \times 0.02 \times 4.0 = 0.01\%$

Total contribution $0.14\% + 0.01\% = 0.15\%$

The contribution to return from spread in the portfolio using the benchmark change in spreads is:

Corporates: Return $-\Delta y_{Si} \times D_i = 0.07 \times 3.6 = 0.24\%$

Contribution $w_i \times -\Delta y_{Si} \times D_i = 65\% \times 0.07 \times 3.6 = 0.15\%$

High yield: $0.02 \times 4.3 = 0.10\%$

Return Contribution $15\% \times 0.02 \times 4.3 = 0.01\%$

Total contribution $0.15\% + 0.01\% = 0.17\%$

Therefore the added value from spread effect is $0.17\% - 0.15\% = 0.02\%$.

Any contribution to return not derived from income, treasury effect or spread must be issue selection:

Corporates $4.4\% - 0.52\% - 3.6\% - 0.24\% = 0.04\%$ (Contribution $65\% \times 0.04\% = 0.03\%$)

High Yield $5.6\% - 0.82\% - 4.73\% - 0.10\% = -0.05\%$ (Contribution $15\% \times -0.05\% = -0.01\%$)

Therefore the added value from issue selection is $0.03\% - 0.01\% = 0.02\%$

For comparison, attribution effects using the normal Brinson model are calculated in Exhibit 7.17 and summarised in Table 7.7. The misleading conclusion from the standard approach would be significantly poor selection performance in corporates.

Table 7.5 Campisi framework – attribution summary

	Portfolio (%)	Benchmark (%)	Excess return (%)
Total return	*4.90*	*6.70*	*−1.80*
Income return	0.52	0.45	0.08
Treasury effect	4.19	6.11	−1.92
Spread	0.17	0.15	0.02
Issue selection	0.02	n/a	0.02

Exhibit 7.14 Income return attribution

Using the standard Brinson model to attribute the return from income.

Income allocation:

Treasuries	$(20\% - 50\%) \times (0.3\% - 0.45\%) = 0.04\%$
Corporates	$(65\% - 40\%) \times (0.57\% - 0.45\%) = 0.03\%$
High yield	$(15\% - 10\%) \times (0.71\% - 0.45\%) = 0.01\%$
Total	$0.04\% + 0.03\% + 0.01\% = 0.09\%$

Income selection:

Treasuries	$20\% \times (0.3\% - 0.3\%) = 0.0\%$
Corporates	$65\% \times (0.52\% - 0.57\%) = -0.03\%$
High yield	$15\% \times (0.82\% - 0.71\%) = 0.02\%$
Total	$0.00\% - 0.03\% + 0.02\% = -0.01\%$

Exhibit 7.15 Detailed Treasury effect attribution

Interest rate change at the benchmark duration of $4.88 = -1.25\%$.

Impact of a parallel shift in interest rates:

$(D_r - D_b) \times -y_b = (3.94 - 4.88) \times 1.25\% = -1.18\%$

Impact of non-parallel move in yield curve:

Treasuries	$w_i \times D_i \times (-y_i + y_b) = 20\% \times 4.75 \times (+1.20\% - 1.25\%) = -0.05\%$
Corporates	$65\% \times 3.6 \times (+1.0\% - 1.25\%) = -0.59\%$
High yield	$15\% \times 4.3 \times (+1.1\% - 1.25\%) = -0.10\%$
Total	$-0.05\% - 0.59\% - 0.10\% = -0.74\%$

Exhibit 7.16 Detailed spread effect attribution

Total benchmark spread change:

$$y_{Sb} = \frac{-(6.7 - 0.45\% - 6.11)}{4.88} = -0.03\%$$

Impact of overall spread changes at portfolio duration compared to benchmark duration:

$(D_r - D_b) \times -y_{Sb} = (3.94 - 4.88) \times 0.03\% = -0.03\%$

Spread allocation:

Treasuries	$(w_i \times D_i - W_i \times D_{bi}) \times (-y_{Si} + y_{Sb}) = (20\% \times 4.75 - 50\% \times 4.75) \times (0.0\% - 0.03\%) = 0.04\%$
Corporates	$(65\% \times 3.6 - 40\% \times 5.25) \times (0.07\% - 0.03\%) = 0.01\%$
High yield	$(15\% \times 4.3 - 10\% \times 4.0) \times (0.02\% - 0.03\%) = 0.0\%$
Total	$0.04\% + 0.01\% + 0.0\% = 0.05\%$

Table 7.6 Campisi framework – detailed attribution

	Income allocation (%)	Income selection (%)	Duration (%)	Other yield curve effects (%)	Spread duration (%)	Spread (%)	Issue selection (%)
Treasuries	0.04	0.00	n/a	−0.05	n/a	0.04	0.00
Corporates	0.03	−0.03	n/a	−0.59	n/a	0.01	0.03
High yield	0.01	0.02	n/a	−0.1	n/a	0.00	−0.01
Total	0.09	−0.01	−1.18	−0.74	−0.03	0.05	0.02
Total	*0.08*	*Total*	*−1.92*	*Total*	0.02	*Total*	−1.80

Exhibit 7.17 Standard Brinson model

Allocation:	
Treasuries	$(20\% - 50\%) \times (6.0\% - 6.7\%) = 0.21\%$
Corporates	$(65\% - 40\%) \times (8.0\% - 6.7\%) = 0.33\%$
High yield	$(15\% - 10\%) \times (5.0\% - 6.7\%) = -0.09\%$
Total	$0.21\% + 0.33\% - 0.09\% = 0.45\%$
Selection:	
Treasuries	$20\% \times (6.0\% - 6.0\%) = 0.0\%$
Corporates	$65\% \times (4.4\% - 8.0\%) = -2.34\%$
High yield	$15\% \times (5.6\% - 5.0\%) = 0.09\%$
Total	$0.00\% - 2.34\% + 0.09\% = -2.25\%$

Table 7.7 Attribution using the standard Brinson model

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i)$ $\times(b_i - b)$ (%)	Stock selection $w_i \times (r_i - b_i)$ (%)
Treasuries	20	50	6.0	6.0	0.21	0.0
Corporates	65	40	4.4	8.0	0.33	−2.34
High Yield	15	10	5.6	5.0	−0.09	0.09
Total	*100*	*100*	*4.9*	*6.7*	*0.45*	*−2.25*

Yield curve analysis

The changing shape of the yield curve can be decomposed into three different movements: shift, twist (or slope) and curvature (or butterfly).

Shift

The shift effect assumes a parallel shift of yields on all maturity points on the yield curve, see Figure 7.5. Parallel shift may be measured by referencing the shift at certain maturities or using the average shift across the yield curve.

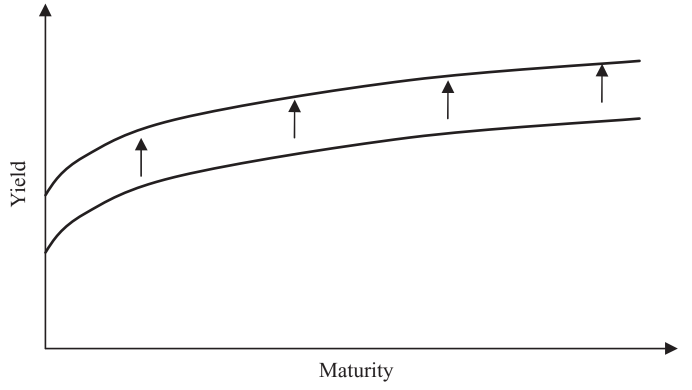

Figure 7.5 Parallel shift

Twist (or slope)

Twist or slope measures the impact from changes in the slope or non-parallel moves of the yield curve. The yield curve may have twisted around a pivot point or moved less dramatically at one end of the yield curve than the other, resulting in a steepening or flattening, see Figure 7.6.

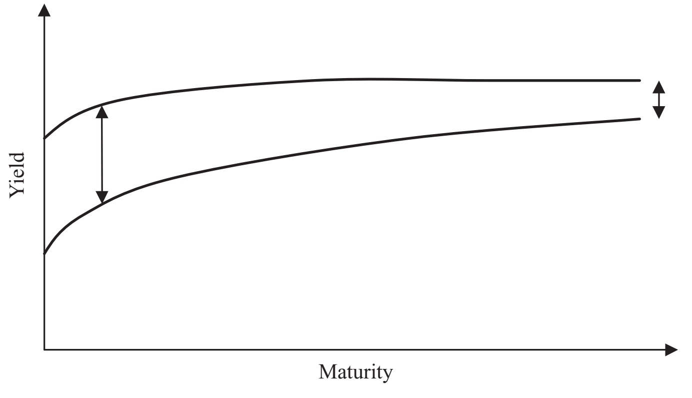

Figure 7.6 Slope (flattening yield curve)

Curvature (or butterfly)

Curvature measures the changing curvature of the yield curve. Perhaps there are larger changes in yields in the middle of the curve than either the long or short ends, see Figure 7.7.

Carry

An additional contribution to return is simply the result of time or carry. All other things being equal there will be an additional contribution from convergence or rolldown as the bond approaches maturity and from the coupon return as interest is accrued and paid.

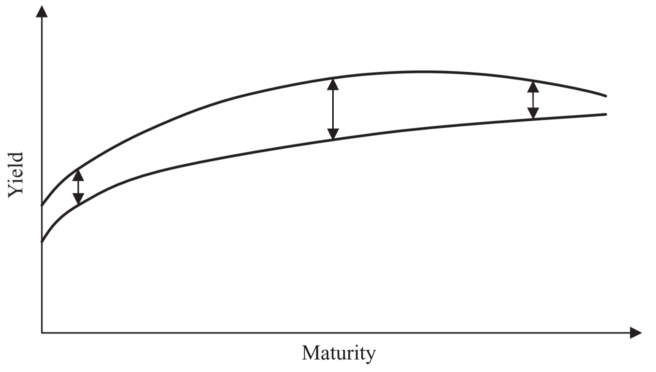

Figure 7.7 Curvature

Credit (or spread)

There is an additional risk that some borrowers will default. Investors will demand higher yields for greater risk. This risk premium to compensate for uncertainty is known as spread. An element of return may be due to the tightening or widening of spreads against Treasuries. Spreads decline during economic expansion and widen during economic downturns and periods of high risk. Credit rating agencies use a letter system to grade credit risk, see Table 7.8.

Yield curve decomposition

The total return of a bond portfolio can be decomposed into carry, movement in the yield curve (measured by shift, slope and curvature) and changing credit spreads.

Unlike the top-down approaches of van Breukelen and Campisi, portfolio managers are more inclined to the accuracy of bottom-up models that decompose the yield effects for each bond or instrument in the portfolio and the benchmark and aggregate results into attribution analysis consistent with their investment decision process.

Table 7.8 Credit ratings

S&P	Moody's	Fitch	
Investment grade			
AAA	Aaa	AAA	Highest quality
AA	Aa	AA	High quality
A	A	A	Upper quality
BBB	Baa	BBB	Medium upper quality
Non-investment grade			
BB	Ba	BB	Low quality
B	B1	B	Speculative
B−	B3	B−	Highly speculative
High yield or in default			
CCC	Caa	CCC	High risk
CC	Ca	CC	Extremely speculative
C	C	C	
D		DDD	In default

8

Multi-period Attribution

An expert is a person who has made all the mistakes that can be made in a very narrow field.

Niels Bohr (1885–1962)

We observed in Chapter 3 that the sum of the arithmetic excess return for each finite period does not sum to the total arithmetic excess return for the total period:

$$R - \bar{R} \neq \sum \left(R_t - \bar{R}_t\right) \tag{8.1}$$

Therefore over multiple periods we should not expect our arithmetic attribution factors which add over single periods to add up for the total period under analysis.

SMOOTHING ALGORITHMS

Accepting that it is desirable for multiple period arithmetic attribution factors to add up over time, a number of methodologies (known as smoothing algorithms) have been developed to achieve this:

Carino

Carino (1999) suggests that the results for the single period can be transformed into results that naturally cumulate over time. Continuously compounded returns may be summed as demonstrated in Equation (2.30).

Using this relationship Carino introduces the factor:

$$k_t = \frac{\ln(1 + r_t) - \ln(1 + b_t)}{r_t - b_t} \tag{8.2}$$

$$\text{if} \quad r_t = b_t \quad \text{set} \quad k_t = \frac{1}{(1 + r_t)}$$

Since from Chapter 2 for continuously compounded returns we known that:

$$\ln(1 + r) = \ln(1 + r_1) + \ln(1 + r_2) + \cdots + \ln(1 + r_n) \tag{2.30}$$

and similarly for the benchmark:

$$\ln(1 + b) = \ln(1 + b_1) + \ln(1 + b_2) + \cdots + \ln(1 + b_n) \tag{8.3}$$

Subtracting Equation (8.3) from Equation (2.30):

$$\ln(1 + r) - \ln(1 + b) = \ln(1 + r_1) - \ln(1 + b_1) \cdots + \ln(1 + r_n) - \ln(1 + b_n) \tag{8.4}$$

Substituting Equation (8.2) into Equation (8.4) and simplifying:

$$\ln(1 + r) - \ln(1 + b) = \sum_{t=1}^{T} k_t \times (r_t - b_t) \tag{8.5}$$

To transform back to the desired arithmetic difference Carino introduced a similar factor for the entire period:

$$k = \frac{\ln(1+r) - \ln(1+b)}{r-b} \tag{8.6}$$

Therefore substituting Equation (8.5) into Equation (8.4):

$$r - b = \sum_{t=1}^{t=n} \frac{k_t}{k} \times (r_t - b_t) \tag{8.7}$$

It follows that:

$$r - b = \sum_{t=1}^{t=n} \frac{k_t}{k} \times A_t + \sum_{t=1}^{t=n} \frac{k_t}{k} \times S_t + \sum_{t=1}^{t=n} \frac{k_t}{k} \times I_t \tag{8.8}$$

Table 8.1 extends the example data in Table 5.1 over four quarters. The portfolio return for the year is now 3.86% and the benchmark return is –9.41%. The objective is to calculate annual attribution effects that add to the arithmetic excess return of 13.27%.

Table 8.1 Example data over 4 quarters

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Asset allocation (%)	Stock selection (%)
1st quarter						
UK equities	40	40	20	10	0.0	4.0
Japanese equities	30	20	−5	−4	−1.04	−0.3
US equities	30	40	6	8	−0.16	−0.6
Total	*100*	*100*	*8.3*	*6.4*	*−1.2*	*3.1*
2nd quarter						
UK equities	70	40	−5	−7	−0.72	1.4
Japanese equities	20	30	3	4	−0.86	−0.2
US equities	10	30	−5	−10	1.08	0.5
Total	*100*	*100*	*−3.4*	*−4.6*	*−0.5*	*1.7*
3rd quarter						
UK equities	30	50	−20	−25	2.5	1.5
Japanese equities	50	40	8	5	1.75	1.5
US equities	20	10	−15	−20	−0.75	1.0
Total	*100*	*100*	*−5.0*	*−12.5*	*3.5*	*4.0*
4th quarter						
UK equities	30	40	10	5	−0.3	1.5
Japanese equities	50	40	−7	−5	−0.7	−1.0
US equities	20	20	25	10	0.0	3.0
Total	*100*	*100*	*4.5*	*2.0*	*−1.0*	*3.5*
Annual						
Total			*3.86*	*−9.41*		

Carino factors are calculated for each period and the overall period in Exhibit 8.1:

Exhibit 8.1 Carino factors

$$k_t = \frac{\ln(1+r_t) - \ln(1+b_t)}{r_t - b_t}$$

$$\text{1st quarter } k_1 = \frac{(\ln 1.083 - \ln 1.064)}{(8.3\% - 6.4\%)} = 0.93156$$

$$\text{2nd quarter } k_2 = \frac{(\ln 0.966 - \ln 0.954)}{(-3.4\% + 4.6\%)} = 1.04168$$

$$\text{3rd quarter } k_3 = \frac{(\ln 0.95 - \ln 0.875)}{(-5.0\% + 12.5\%)} = 1.09651$$

$$\text{4th quarter } k_4 = \frac{(\ln 1.045 - \ln 1.02)}{(4.5\% - 2.0\%)} = 0.96857$$

$$\text{Year } k = \frac{(\ln 1.0386 - \ln 0.9059)}{(3.86\% + 9.41\%)} = 1.03013$$

Using the Carino factors calculated in Exhibit 8.1 revised attribution effects can be calculated for the 1st quarter as shown in Exhibit 8.2:

Exhibit 8.2 Carino revised Q1 attribution factors

UK asset allocation	$0\% \times \frac{k_1}{k} = 0\% \times \frac{0.93156}{1.03013} = 0\%$
Japanese asset allocation	$-1.04\% \times \frac{0.93156}{1.03013} = -0.94\%$
US asset allocation	$-0.16\% \times \frac{0.93156}{1.03013} = -0.14\%$
UK stock selection	$4.0\% \times \frac{0.93156}{1.03013} = 3.62\%$
Japanese stock selection	$-0.3\% \times \frac{0.93156}{1.03013} = -0.27\%$
US stock selection	$-0.6\% \times \frac{0.93156}{1.03013} = -0.54\%$

Extending the process in Exhibit 8.2 to all four quarters we can calculate revised attribution factors that sum to the arithmetic difference for the entire 4-quarter period in Table 8.2.

The objective has been achieved in Table 8.2; our attribution factors are now additive. It should be noted that our revised attribution effects in each quarter are unique for that overall period. If we lengthen the period of analysis we will need to recalculate new revised attribution effects for each quarter. This is both counterintuitive and cumbersome, particularly if we wish to calculate attribution effects over a number of years.

Table 8.2 Revised attribution factors for the four-quarter period (Carino)

	Original arithmetic attribution		Revised Carino attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
1st quarter				
UK equities	0.0	4.0	0.0	3.62
Japanese equities	−1.04	−0.3	−0.94	−0.27
US equities	−0.16	−0.6	−0.14	−0.54
Total	*−1.2*	*3.1*	*−1.09*	*2.80*
2nd quarter				
UK equities	−0.72	1.4	−0.73	1.42
Japanese equities	−0.86	−0.2	−0.87	−0.2
US equities	1.08	0.5	1.09	0.51
Total	*−0.5*	*1.7*	*−0.51*	*1.72*
3rd quarter				
UK equities	2.5	1.5	2.66	1.60
Japanese equities	1.75	1.5	1.86	1.60
US equities	−0.75	1.0	−0.8	1.06
Total	*3.5*	*4.0*	*3.73*	*4.26*
4th quarter				
UK equities	−0.3	1.5	−0.28	1.41
Japanese equities	−0.7	−1.0	−0.66	−0.94
US equities	0.0	3.0	0.0	2.82
Total	*−1.0*	*3.5*	*−0.94*	*3.29*
Four−quarter total				
UK equities			1.65	8.04
Japanese equities			−0.60	0.18
US equities			0.15	3.85
Total			*1.20*	*12.07*
	Portfolio return (%)	*Benchmark return (%)*	*Asset allocation (%)*	*Stock selection (%)*
Total	*3.86*	*−9.41*	*1.20*	*12.07*

Menchero

Menchero (2000) offers a similar if more sophisticated approach to Carino. He suggests the introduction of a constant factor M into Equation (8.1) that takes into account the characteristic scaling which arises from geometric compounding:

$$r - b \approx M \times \sum_{t=1}^{T} (r_t - b_t) \tag{8.9}$$

Logically, Menchero chooses for M the ratio of the difference of the arithmetic average between portfolio and benchmark returns with the difference of the geometric average portfolio and benchmark returns:

$$M = \frac{(r-b)\big/T}{\left[(1+r)^{1/T} - (1+b)^{1/T}\right]} \tag{8.10}$$

$$\text{If} \quad r = b \quad \text{set} \quad M = (1+r)^{\frac{(T-1)}{T}} \tag{8.11}$$

Unfortunately, this still leaves a residual in Equation (8.9) which means we must calculate a corrective term α_t such that:

$$r - b = \sum_{t=1}^{t=T} (M + \alpha_t) \times (r_t - b_t) \tag{8.12}$$

Calculating α_t as small as possible in order that the linking coefficients $(M + \alpha_t)$ be distributed as uniformly as possible to provide an "optimal solution", Menchero uses Lagrange multipliers to calculate:

$$\alpha_t = \left(\frac{r - b - M \times \sum_{t=1}^{t=T} (r_t - b_t)}{\sum_{t=1}^{t=T} (r_t - b_t)^2} \right) \times (r_t - b_t) \tag{8.13}$$

It follows that:

$$r - b = \sum_{t=1}^{t=T} (M + \alpha_t) \times A_t + \sum_{t=1}^{t=T} (M + \alpha_t) \times S_t + \sum_{t=1}^{t=T} (M + \alpha_t) \times I_t \tag{8.14}$$

Menchero factors are calculated for each period and the overall period for the data in Table 8.1 is shown in Exhibit 8.3.

Exhibit 8.3 Menchero factors

$$\alpha_t = \left(\frac{r - b - M \times \sum_{t=1}^{t=T} (r_t - b_t)}{\sum_{t=1}^{t=T} (r_t - b_t)^2} \right) \times (r_t - b_t)$$

$$M = \frac{(+3.86\% + 9.41\%)\big/4}{\left[(1.0386)^{\frac{1}{4}} - (0.9059)^{\frac{1}{4}}\right]} = 0.97813$$

$$\sum_{t=1}^{t=T} (r_t - b_t) = (8.3\% - 6.4\% - 3.4\% + 4.6\% - 5.0\% + 12.5\% + 4.5\% + 2\%) = 13.1\%$$

$$\sum_{t=1}^{t=T} (r_t - b_t)^2 = (1.9\%)^2 + (1.2\%)^2 + (7.5\%)^2 + (2.5\%)^2 = 0.6755\%$$

Therefore for the data in Table 8.1:

(*Continued*)

Exhibit 8.3 (*Continued*)

$$\alpha_t = \frac{(3.86\% + 9.41\% - 0.97813 \times 13.1\%)}{0.6755\%} \times (r_t - b_t) = 0.66928 \times (r_t - b_t)$$

$$\text{1st quarter } \alpha_1 = 0.66928 \times (8.3\% - 6.4\%) = 0.01272$$

$$\text{2nd quarter } \alpha_2 = 0.66928 \times (-3.2\% + 4.6\%) = 0.00803$$

$$\text{3rd quarter } \alpha_3 = 0.66928 \times (-5.0\% + 12.5\%) = 0.05020$$

$$\text{4th quarter } \alpha_4 = 0.66928 \times (4.5\% - 2\%) = 0.01673$$

Revised Menchero attribution effects for the 1st quarter are calculated in Exhibit 8.4:

Exhibit 8.4 Menchero revised attribution factors

UK asset allocation	$= 0\% \times (M + \alpha_1) = 0\% \times (0.97813 + 0.01272) = 0\%$
Japanese asset allocation	$= -1.04\% \times (0.97813 + 0.01272) = -1.03\%$
US asset allocation	$= -0.16\% \times (0.97813 + 0.01272) = -0.16\%$
UK stock selection	$= 4.0\% \times (0.97813 + 0.01272) = 3.96\%$
Japanese stock selection	$= -0.3\% \times (0.97813 + 0.01272) = -0.3\%$
US stock selection	$= -0.6\% \times (0.97813 + 0.01272) = -0.59\%$

Extending the process in Exhibit 8.4 to all four quarters we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period in Table 8.3.

Again, we have achieved our objective but with a slightly different solution from the Carino approach. It should be noted that our revised effects are unique for the overall period; it is necessary to recalculate each individual period's effects again as we lengthen the period of analysis. With the added disadvantage of complexity, this method is also counterintuitive and cumbersome.

GRAP method

Carino and Menchero are examples of smoothing algorithms in which the natural residual of the multi-period arithmetic attribution is structurally distributed across all the contributions to performance.

GRAP (1997) (Groupe de Recherche en Attribution de Performance), a Paris-based working group of performance experts, proposed a different type of "linking" approach as follows.

Let a_t = arithmetic excess return in period t. Then:

$$r_1 = b_1 + a_1 \quad \text{for period} \quad t = 1 \quad \text{and} \quad r_2 = b_2 + a_1 \quad \text{for period} \quad t = 2$$

Table 8.3 Revised attribution factors for the four-quarter period (Menchero)

	Original arithmetic attribution		Revised menchero attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
1st quarter				
UK equities	0.0	4.0	0.0	3.96
Japanese equities	−1.04	−0.3	−0.94	−0.30
US equities	−0.16	−0.6	−0.14	−0.59
Total	*−1.2*	*3.1*	*−1.19*	*3.07*
2nd quarter				
UK equities	−0.72	1.4	−0.71	1.38
Japanese equities	−0.86	−0.2	−0.85	−0.2
US equities	1.08	0.5	1.07	0.49
Total	*−0.5*	*1.7*	*−0.49*	*1.68*
3rd quarter				
UK equities	2.5	1.5	2.57	1.54
Japanese equities	1.75	1.5	1.80	1.54
US equities	−0.75	1.0	−0.77	1.03
Total	*3.5*	*4.0*	*3.60*	*4.11*
4th quarter				
UK equities	−0.3	1.5	−0.30	1.49
Japanese equities	−0.7	−1.0	−0.70	−0.99
US equities	0.0	3.0	0.0	2.98
Total	*−1.0*	*3.5*	*−0.99*	*3.48*
Four-quarter total				
UK equities			1.56	8.38
Japanese equities			−0.78	0.05
US equities			0.14	3.91
Total			*0.92*	*12.34*
	Portfolio return (%)	*Benchmark return (%)*	*Asset allocation (%)*	*Stock selection (%)*
Total	*3.86*	*−9.41*	*0.92*	*12.34*

The total return over the first two periods is therefore:

$$
\begin{aligned}
(1+r) &= (1+b_1+a_1)\times(1+b_2+a_2)\\
&= (1+b_1+a_1)\times(1+b_2)+(1+b_1+a_1)\times a_2\\
&= (1+b_1)\times(1+b_2)+a_1\times(1+b_2)+(1+r_1)\times a_2\\
&= (1+b)+a_1\times(1+b_2)+(1+r_1)\times a_2\\
(r-b) &= a = a_1\times(1+b_2)+(1+r_1)\times a_2
\end{aligned}
\tag{8.15}
$$

In effect, the excess return for the 1st period is reinvested in the benchmark return of the 2nd period and the excess return of the 2nd period is compounded by the actual portfolio return in the 1st period.

Over n periods we can generalise:

$$a = \sum_{T=1}^{n} a_T \times \prod_{t=1}^{T-1}(1+r_t) \times \prod_{t=T+1}^{n}(1+b_t) \qquad (8.16)$$

In effect, the excess return for period $t = T$ is compounded by the actual portfolio return up to period $t = T$ and reinvested in the benchmark thereafter.

It follows that:

$$r - b = \sum_{T=1}^{n}(A_T + S_T + I_T) \times \prod_{t=1}^{T-1}(1+r_t) \times \prod_{t=T+1}^{n}(1+b_t) \qquad (8.17)$$

GRAP revised 1st quarter attribution effects are calculated in Exhibit 8.5:

Exhibit 8.5 GRAP revised 1st quarter attribution factors

UK asset allocation	$0\% \times (1+b_2) \times (1+b_3) \times (1+b_4) = 0\% \times 0.954 \times 0.875 \times 1.02 = 0\%$
Japanese asset allocation	$-1.04\% \times 0.954 \times 0.875 \times 1.02 = -0.89\%$
US asset allocation	$-0.16\% \times 0.954 \times 0.875 \times 1.02 = -0.14\%$
UK stock selection	$4.0\% \times 0.954 \times 0.875 \times 1.02 = 3.41\%$
Japanese stock selection	$-0.3\% \times 0.954 \times 0.875 \times 1.02 = -0.26\%$
US stock selection	$-0.6\% \times 0.954 \times 0.875 \times 1.02 = -0.51\%$

GRAP revised 2nd quarter attribution effects are calculated in Exhibit 8.6:

Exhibit 8.6 GRAP revised 2nd quarter attribution factors

UK asset allocation	$-0.72\% \times (1+r_1) \times (1+b_3) \times (1+b_4) = -0.72\% \times 1.083 \times 0.875 \times 1.02 = -0.7\%$
Japanese asset allocation	$-0.86\% \times 1.083 \times 0.875 \times 1.02 = -0.83\%$
US asset allocation	$1.08\% \times 1.083 \times 0.875 \times 1.02 = 1.04\%$
UK stock selection	$1.4\% \times 1.083 \times 0.875 \times 1.02 = 1.35\%$
Japanese stock selection	$-0.2\% \times 1.083 \times 0.875 \times 1.02 = -0.19\%$
US stock selection	$0.5\% \times 1.083 \times 0.875 \times 1.02 = 0.48\%$

Extending the process in Exhibit 8.5 and Exhibit 8.6 to all four quarters we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period as shown in Table 8.4.

Table 8.4 Revised attribution factors for the four-quarter period (GRAP)

	Original arithmetic attribution		Revised GRAP attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
1st quarter				
UK equities	0.0	4.0	0.0	3.41
Japanese equities	−1.04	−0.3	−0.89	−0.26
US equities	−0.16	−0.6	−0.14	−0.51
Total	*−1.2*	*3.1*	*−1.04*	*2.64*
2nd quarter				
UK equities	−0.72	1.4	−0.70	1.35
Japanese equities	−0.86	−0.2	−0.83	−0.19
US equities	1.08	0.5	1.04	0.48
Total	*−0.5*	*1.7*	*−0.48*	*1.64*
3rd quarter				
UK equities	2.5	1.5	2.67	1.60
Japanese equities	1.75	1.5	1.87	1.60
US equities	−0.75	1.0	−0.80	1.07
Total	*3.5*	*4.0*	*3.73*	*4.27*
4th quarter				
UK equities	−0.3	1.5	−0.30	1.49
Japanese equities	−0.7	−1.0	−0.70	−0.99
US equities	0.0	3.0	0.0	2.98
Total	*−1.0*	*3.5*	*−0.99*	*3.48*
Four-quarter total				
UK equities			1.67	7.85
Japanese equities			−0.55	0.16
US equities			0.11	4.02
Total			*1.24*	*12.03*
	Portfolio return	*Benchmark return*	*Asset allocation*	*Stock selection*
Total	*3.86*	*−9.41*	*1.24*	*12.03*

Again the objective is achieved but with a different solution from either the Carino or Menchero approaches.

Frongello

Frongello (2002) suggested a linking algorithm using the same concept as the GRAP method with:

$$f_T = a_T \times \prod_{t=1}^{t=T-1} (1 + r_t) + b_T \times \sum_{t=1}^{t=T-1} f_t \tag{8.18}$$

where: f_t = the revised Frongello attribution effect for period t.

Exhibit 8.7 Frongello revised 2nd quarter attribution factors

Note that Frongello 1st period attribution effects are never adjusted.

UK asset allocation	$-0.72\% \times (1 + r_1) + b_2 \times f_1 = -0.72\% \times 1.083 - 0.046 \times 0.0\% = -0.78\%$
Japanese asset allocation	$-0.86\% \times 1.083 - 0.046 \times -1.04\% = -0.88\%$
US asset allocation	$1.08\% \times 1.083 - 0.046 \times -0.16\% = 1.18\%$
UK stock selection	$1.4\% \times 1.083 - 0.046 \times 4.0\% = 1.33\%$
Japanese stock selection	$-0.2\% \times 1.083 - 0.046 \times -0.30\% = -0.2\%$
US stock selection	$0.5\% \times 1.083 - 0.046 \times -0.6\% = 0.57\%$

The first part of this equation compounds the single period arithmetic excess return a_T with the cumulative return of the actual portfolio up to the prior period, while the second part is the gain on the sum of the prior period Frongello-adjusted excess returns generated by the current benchmark return as demonstrated in Exhibit 8.7.

It follows that:

$$a = r - b = \sum_{T=1}^{T=n} (A_T + S_T + I_T) \times \prod_{t=1}^{t=T-1} (1 + r_t) + b_T \times \sum_{t=1}^{t=T-1} f_t \tag{8.19}$$

Frongello revised 3rd quarter attribution effects are calculated in Exhibit 8.8:

Exhibit 8.8 Frongello revised 3rd quarter attribution factors

UK asset allocation	$2.5\% \times (1 + r_1) \times (1 + r_2) + b_3 \times (f_2 + f_1) = 2.5\% \times 1.083 \times 0.966 - 0.125 \times (-0.78 + 0.0\%) = 2.71\%$
Japanese asset allocation	$1.75\% \times 1.083 \times 0.966 - 0.125 \times (-0.88\% - 1.04\%) = 2.07\%$
US asset allocation	$-0.75\% \times 1.083 \times 0.966 - 0.125 \times (1.18\% - 0.16\%) = -0.91\%$
UK stock selection	$1.5\% \times 1.083 \times 0.966 - 0.125 \times (1.33\% + 4.0\%) = 0.9\%$
Japanese stock selection	$1.5\% \times 1.083 \times 0.966 - 0.125 \times (-0.2\% - 0.3\%) = 1.63\%$
US stock selection	$1.0\% \times 1.083 \times 0.966 - 0.125 \times (0.57\% - 0.6\%) = 1.05\%$

Extending the process in Exhibit 8.7 and Exhibit 8.8 to all four quarters we can calculate revised attribution factors that sum to the arithmetic difference for the entire four-quarter period in Table 8.5.

The Frongello method produces the same total period effects as the GRAP method for the total period.

Both these methods are order dependent – the smoothed results will alter if the order of quarters is rearranged. Despite being order dependent I prefer these methods to Carino and Menchero; they are slightly more intuitive and computationally less cumbersome. I have a naturally aversion to distribution residuals implicit in the Carino and Menchero methods and clearly recalculating attribution effects every time the period is extended is extremely inefficient.

Table 8.5 Revised attribution factors for the four-quarter period (Frongello).

	Original arithmetic attribution		Revised Frongello attribution	
	Asset allocation (%)	Stock selection (%)	Asset allocation (%)	Stock selection (%)
1st quarter				
UK equities	0.0	4.0	0.0	4.0
Japanese equities	−1.04	−0.3	−1.04	−0.3
US equities	−0.16	−0.6	−0.16	−0.6
Total	*−1.2*	*3.1*	*−1.2*	*3.1*
2nd quarter				
UK equities	−0.72	1.4	−0.78	1.33
Japanese equities	−0.86	−0.2	−0.88	−0.50
US equities	1.08	0.5	1.18	0.57
Total	*−0.5*	*1.7*	*−0.49*	*1.70*
3rd quarter				
UK equities	2.5	1.5	2.71	0.90
Japanese equities	1.75	1.5	2.07	1.63
US equities	−0.75	1.0	−0.91	1.05
Total	*3.5*	*4.0*	*3.87*	*3.58*
4th quarter				
UK equities	−0.3	1.5	−0.26	1.62
Japanese equities	−0.7	−1.0	−0.69	−0.97
US equities	0.0	3.0	0.0	3.00
Total	*−1.0*	*3.5*	*−0.95*	*3.65*
Four-quarter total				
UK equities			1.67	7.85
Japanese equities			−0.55	0.16
US equities			0.11	4.02
Total			*1.24*	*12.03*
	Portfolio return	*Benchmark return*	*Asset allocation*	*Stock selection*
Total	*3.86*	*−9.41*	*1.24*	*12.03*

Davies and Laker

Davies and Laker (2001) refer back to Brinson, Hood and Beebower's original article that suggested applying the Brinson model over multiple periods and also refer to the work of Kirievsky and Kirievsky (2000).

They suggest compounding each of the notional funds to derive the total attribution effects for multiple periods as follows.

Arithmetic excess return over total period:

$$r - b = \prod_{t=1}^{t=n}(1 + r_t) - \prod_{t=1}^{t=n}(1 + b_t) \tag{8.20}$$

Asset allocation:

$$\prod_{t=1}^{t=n}\left(1+b_{S,t}\right)-\prod_{t=1}^{t=n}(1+b_t) \tag{8.21}$$

where: $b_{S,t}$ = the semi-notional (allocation notional) fund in period t.

$$b_S=\prod_{t=1}^{t=n}\left(1+b_{S,t}\right)-1.$$

Stock selection:

$$\prod_{t=1}^{t=n}\left(1+r_{S,t}\right)-\prod_{t=1}^{t=n}(1+b_t) \tag{8.22}$$

where: $r_{S,t}$ = the selection notional fund in period t

$$r_S=\prod_{t=1}^{t=n}\left(1+r_{S,t}\right)-1.$$

Interaction:

$$\prod_{t=1}^{t=n}(1+r_t)-\prod_{t=1}^{t=n}\left(1+r_{S,t}\right)-\prod_{t=1}^{t=n}\left(1+b_{S,t}\right)+\prod_{t=1}^{t=n}(1+b_t) \tag{8.23}$$

The Davies and Laker method establishes total contributions for allocation, selection and interaction. Their 2001 article did not address individual sector returns although it did hint at the use of a Carino-type algorithm to calculate individual sector contributions that add up to the total for each factor.

Using the data from Table 8.1 again, the Brinson notional funds are calculated for each quarter and compounded in Exhibit 8.9:

Exhibit 8.9 Brinson notional funds

Quarter 1

The allocation notional return $b_{S,1}=\sum_{i=1}^{i=n} w_i\times b_i$ for quarter 1 is:

$$b_{S,1}=40\%\times 10\%+30\%\times -4\%+30\%\times 8\%=5.2\%$$

The selection notional return $r_{S,1}=\sum_{i=1}^{i=n} W_i\times r_i$ for quarter 1 is:

$$r_{S,1}=40\%\times 20\%+20\%\times -5\%+40\%\times 6\%=9.4\%$$

Quarter 2

$b_{S,2}=70\%\times -7.0\%+20\%\times 4.0\%+10\%\times 10.0\%=-5.1\%$

$r_{S,2}=40\%\times -5.0\%+30\%\times 3.0\%+30\%\times -5.0\%=-2.6\%$

Exhibit 8.9 (*Continued*)

Quarter 3

$$b_{S,3} = 50\% \times -20.0\% + 40\% \times 8.0\% + 10\% \times -15.0\% = -9.0\%$$
$$r_{S,3} = 30\% \times -25.0\% + 50\% \times 5.0\% + 20\% \times -20.0\% = -8.3\%$$

Quarter 4

$$b_{S,4} = 30\% \times 5.0\% + 50\% \times -5.0\% + 20\% \times 10.0\% = 1.0\%$$
$$r_{S,4} = 40\% \times 5.0\% + 40\% \times -5.0\% + 20\% \times 10.0\% = 6.2\%$$

Compounded notional funds

$$\prod_{t=1}^{t=n} \left(1 + b_{S,t}\right) = 1.052 \times 0.949 \times 0.91 \times 1.01 = 0.9176$$
$$\prod_{t=1}^{t=n} \left(1 + r_{S,t}\right) = 1.094 \times 0.974 \times 0.917 \times 1.062 = 1.0377$$

Total or "exact" attribution effects are calculated for the entire period in Exhibit 8.10:

Exhibit 8.10 Exact attribution effects

$$r = \prod_{t=1}^{t=n} (1 + r_t) - 1 = 3.86\%$$
$$b = \prod_{t=1}^{t=n} (1 + b_t) - 1 = -9.41\%\%$$
$$r_S = \prod_{t=1}^{t=n} \left(1 + r_{S,t}\right) - 1 = 3.77\%$$
$$b_S = \prod_{t=1}^{t=n} \left(1 + b_{S,t}\right) - 1 = -8.24\%$$

Excess return $3.86\% + 9.41\% = 13.27\%$
Stock selection $3.77\% + 9.41\% = 13.18\%$
Asset allocation $-8.24 + 9.41\% = 1.17\%$

Interaction $3.86\% - 3.77\% + 8.24\% - 9.41\% = -1.08\%$

$$\underbrace{13.27\%}_{\text{Excess return}} = \underbrace{13.18\%}_{\text{Stock selection}} + \underbrace{1.17\%}_{\text{Asset allocation}} + \underbrace{-1.08\%}_{\text{Interaction}}$$

Only the total level attribution effect can be calculated this way. To establish the contribution from each category a smoothing algorithm must be used on each effect separately.

This method combines arithmetic and geometric concepts and is really an evolution stage between the arithmetic and full geometric methodologies.

The Davies and Laker method literally compounds the basic flaw of the Brinson model, interaction, with the result that this factor is even less meaningful. It is perhaps more sensible to combine interaction with stock selection, defining stock selection as:

$$\prod_{t=1}^{t=n}(1+r_t)-\prod_{t=1}^{t=n}(1+b_{St}) \tag{8.24}$$

Therefore stock selection plus asset allocation fully explain excess return:

$$r-b=\underbrace{\prod_{t=1}^{t=n}(1+r_t)-\prod_{t=1}^{t=n}(1+b_{St})}_{\text{Stock selection}}+\underbrace{\prod_{t=1}^{t=n}(1+b_{St})-\prod_{t=1}^{t=n}(1+b_t)}_{\text{Asset allocation}} \tag{8.25}$$

Multi-period geometric attribution

Multi-period geometric attribution does not suffer the same linking challenges as multi-period arithmetic attribution. Chapter 3 demonstrated that geometric excess returns compound over time. Geometric attribution effects also compound to provide the single period excess return, therefore substituting Equation (5.31) into Equation (3.26):

$$\prod_{t=1}^{t=n}\left(1+S_t^G\right)\times\prod_{t=1}^{t=n}\left(1+A_t^G\right)-1=g \tag{8.26}$$

where: S_t^G = is the total geometric attribution to stock selection in period t
A_t^G = is the total geometric attribution to asset allocation in period t.

Table 8.6 summarises the geometric attribution effects for all four quarters.

Unlike for multi-period arithmetic attribution there is no need to make any continual adjustments as the total period of measurement extends. The total attribution effects compound as demonstrated in Exhibit 8.11:

Exhibit 8.11 Multi-period geometric attribution effects

Stock selection	$1.0295 \times 1.0179 \times 1.044 \times 1.0347 - 1 = 13.19\%$
Asset allocation	$0.9887 \times 0.9948 \times 1.04 \times 0.9902 - 1 = 1.29\%$
Geometric excess return	$\dfrac{1.0386}{0.9059} - 1 = 1.1319 \times 1.0129 - 1 = 14.64\%$

In fact the geometric total effects are the geometric equivalent of the Davies and Laker arithmetic effects as shown in Exhibit 8.12:

Exhibit 8.12 Geometric total effects

$$r = \prod_{t=1}^{t=n} (1 + r_t) - 1 = 3.86\%$$

$$b = \prod_{t=1}^{t=n} (1 + b_t) - 1 = -9.41\%\%$$

$$b_S = \prod_{t=1}^{t=n} \left(1 + b_{S,t}\right) - 1 = -8.24\%$$

Excess return $\dfrac{1.0386}{0.9059} - 1 = 14.64\%$

Stock selection $\dfrac{1.0386}{0.9176} - 1 = 13.19\%$

Asset allocation $\dfrac{0.9176}{0.9059} - 1 = 1.29\%$

$$\underbrace{14.64\%}_{\substack{\text{Geometric}\\ \text{excess}\\ \text{return}}} = \underbrace{1.1319}_{\substack{\text{Stock}\\ \text{selection}}} \times \underbrace{1.0129}_{\substack{\text{Asset}\\ \text{allocation}}} - 1$$

Table 8.6 Geometric attribution effects for all four quarters

	Geometric attribution	
	Asset allocation (%)	Stock selection (%)
1st quarter		
UK equities	0.0	3.8
Japanese equities	−0.98	−0.29
US equities	−0.15	−0.57
Total	*−1.13*	*2.95*
2nd quarter		
UK equities	−0.75	1.48
Japanese equities	−0.90	−0.21
US equities	1.13	0.53
Total	*−0.52*	*1.79*
3rd quarter		
UK equities	2.86	1.65
Japanese equities	2.00	1.65
US equities	−0.86	1.10
Total	*4.0*	*4.4*
4th quarter		
UK equities	−0.29	1.49
Japanese equities	−0.69	−0.99
US equities	0.0	2.97
Total	*−0.98*	*3.47*
	Asset allocation	*Stock selection*
Total	*1.29*	*13.19*

The total geometric attribution effects for each period need not be adjusted. Within each period the attribution effects for each category sum to the total geometric effect, therefore the unadjusted categories cannot be compounded in the same way as the totals. The individual category effects can be adjusted if desired so that they compound to the total effect by Equation (8.27):

$$\hat{S}_i = (1 + S_i) \times \left(\frac{1 + S}{\prod_{i=1}^{i=n} (1 + S_i)} \right)^{\left(\frac{|S_I|}{\sum |S_i|}\right)} - 1 \tag{8.27}$$

where: $\hat{S}_i$ = adjusted geometric effect for category i.

Adjusted geometric effects for the 1st quarter are calculated for stock selection in Exhibit 8.13. The adjustments are quite small and need not be changed if the period of measurement is extended.

Exhibit 8.13 Adjusted stock selection

$$\prod_{i=1}^{i=n} (1 + S_i) = 1.038 \times 0.9971 \times 0.9943 = 1.02916$$

$$\sum |S_i| = 3.8\% + 0.29\% + 0.53\% = 4.62\%$$

$$UK equities 1.038 \times \left(\frac{1.0295}{1.02916}\right)^{\frac{3.8\%}{4.62\%}} - 1 = 3.83\%$$

$$Japanese equities 0.9971 \times \left(\frac{1.0295}{1.02916}\right)^{\frac{0.29\%}{4.62\%}} - 1 = -0.28\%$$

$$US equities 0.9943 \times \left(\frac{1.0295}{1.02916}\right)^{\frac{0.57\%}{4.62\%}} - 1 = -0.57\%$$

$$\prod_{i=1}^{i=n} \left(1 + \hat{S}_i\right) = 1.0383 \times 0.9972 \times 0.9943 = 1.0295$$

For most scenarios the choice of linking method will not normally change the interpretation of results. I favour geometric excess returns and therefore I'm more comfortable with the geometric linking. For arithmetic excess returns I prefer the GRAP or Frongello methods.

Annualisation of excess return

Just like portfolio returns and benchmark returns, excess returns can (and should) be annualised. Geometric excess returns lend themselves to the natural compounding process of multi-period returns; arithmetic exercise returns do not. Benchmark data and therefore excess returns are introduced to sample data in Exhibit 2.22 for use in Exhibit 8.14:

Exhibit 8.14 Annualised excess return

	Porfolio	Benchmark	Arithmetic excess	Geometric excess
2003	10.5%	8.6%	+1.9%	+1.75%
2002	−5.6%	−8.1%	+2.5%	+2.72%
2001	23.4%	18.7%	+4.7%	+3.96%
2000	−15.7%	−13.2%	−2.5%	−2.88%
1999	8.9%	12.5%	−3.6%	−3.20%
Cumulative	18.2%	15.70%	+2.5%	+2.15%
Average	4.3%	3.70%	+0.60%	n/a
Annualised	3.4%	2.96%	+0.44%	+0.43%

There are three potential methods for calculating an annual excess return:

(1) The average of the cumulative arithmetic excess return:

$$\frac{2.5\%}{5} = 0.5\%$$

(2) The arithmetic difference of the average annual returns:

$$4.3\% - 3.7\% = 0.6\%$$

(3) The arithmetic difference of the annualised annual returns:

$$3.4\% - 2.96\% = 0.44\%$$

In my view both methods (1) and (2) are incorrect; if an annualised arithmetic excess return must be calculated use method (3).

Geometrically all three methods give the same answer, another reason for using geometric excess returns.

(1) The geometric average of the cumulative geometric excess return:

$$(1.0216)^{\frac{1}{5}} - 1 = 0.43\%$$

(2) The geometric average of the geometric excess returns:

$$(1.0175 \times 1.0272 \times 1.0396 \times 0.9712 \times 0.968.)^{\frac{1}{5}} - 1 = 0.43\%$$

(3) The geometric difference of the annualised returns:

$$\frac{1.034}{1.0296} - 1 = 0.43\%$$

Attribution annualisation

Expanding Equation (8.26) to include currency using Equation (6.23):

$$\prod_{t=1}^{t=n}\left(1+S_t^G\right) \times \prod_{t=1}^{t=n}\left(1+A_t^G\right) \times \prod_{t=1}^{t=n}\left(1+C_t^G\right) - 1 = g \tag{8.28}$$

where: $C_t^G =$ is the total geometric attribution to currency in period t.

Exhibit 8.15 Annualised contribution to excess return

	Porfolio	Benchmark	Geometric excess	Stock selection	Asset allocation	Currency
2003	10.5%	8.6%	+1.75%	+2.3%	+1.8%	−2.30%
2002	−5.6%	−8.1%	+2.72%	+1.1%	+0.5%	+1.10%
2001	23.4%	18.7%	+3.96%	+0.7%	+1.1%	+2.11%
2000	−15.7%	−13.2%	−2.88%	−2.5%	−0.9%	+0.51%
1999	8.9%	12.5%	−3.20%	−2.7%	+1.1%	−1.60%
Cumulative	18.17%	15.70%	+2.15%	−1.20%	+3.63%	−0.24%
Annualised	3.4%	2.96%	+0.43%	−0.24%	+0.72%	−0.05%

Stock selection $(1.023 \times 1.011 \times 1.007 \times 0.975 \times 0.973.)^{\frac{1}{5}} - 1 = -0.24\%$

Asset allocation $(1.018 \times 1.005 \times 1.011 \times 0.991 \times 1.011)^{\frac{1}{5}} - 1 = +0.72\%$

Currency $(0.977 \times 1.011 \times 1.0211 \times 1.0051 \times 0.984)^{\frac{1}{5}} - 1 = -0.05\%$

Note that:

$$\left(\underbrace{0.988}_{\text{Stock}} \times \underbrace{1.0363}_{\text{Asset}} \times \underbrace{0.9976}_{\text{Currency}}\right) - 1 = +2.15\% \quad \text{(cumulative)}$$

and

$$\left(\underbrace{0.9976}_{\text{Stock}} \times \underbrace{1.0072}_{\text{Asset}} \times \underbrace{0.9995}_{\text{Currency}}\right) - 1 = +0.43\% \quad \text{(annualised)}$$

It therefore follows that the annualised contribution to stock selection, asset allocation and currency in turn compound to produce the annualised geometric excess return, see Exhibit 8.15. Geometric excess returns are internally consistent and can be annualised for multiple periods. The logic of systematically distributing residuals in the form of Carino and Menchero becomes less appealing and geometric compounding more appealing the longer the period under analysis.

9
Further Attribution Issues

Prudens quaestio dimidium scientiae.
Half of science is asking the right questions.

Roger Bacon (1214–1294)

ATTRIBUTION VARIATIONS

The Brinson model remains the foundation of performance return attribution methodologies but there are many variations and some alternatives.

Contribution analysis (or absolute return attribution)

Portfolios without benchmarks or absolute return strategies do not readily fit into the definition of performance attribution since there is no benchmark to attribute against. Contribution analysis simply breaks down the total return of the portfolio by individual instrument or instrument types to provide some information of the sources of return in the portfolio.

Even though there is a zero or cash return benchmark, standard attribution analysis may still be applied. The portfolio manager may be employing a clear strategy to add value to an absolute return objective – that strategy may be converted to customised benchmark and hence attribution can be calculated. Market neutral-type strategies are a good example of this approach.

Return (or regression)-based attribution

Return-based attribution uses regression analysis to identify the sources of return of a portfolio. These sources of return or factors can be statistically derived from principal component analysis, macroeconomic factors or fundamental factors. Stock-specific factors are normally the residual of the chosen approach. Although useful information can be derived from factor models accuracy is never guaranteed.

Holding-based (or buy/hold) attribution

To simplify the attribution process some practitioners suggest an approximate approach to attribution (called buy and hold attribution) which ignores the impact of transactions during the period of measurement.

In this approach the beginning weights of securities and sectors together with their returns are used to calculate attribution effects. The returns are usually derived from another source and not derived from the actual portfolio return.

The advantage of this method is the ease of implementation; only the holdings need to be input into the attribution system. Proponents argue that returns are only estimates; valuations are uncertain and cannot be achieved if the portfolio manager wished to sell anyway. Why worry too much about the accuracy of the methodology if the valuation is incorrect?

The big disadvantage of this approach is that the buy/hold return will not reconcile with the real return (good estimate or not) of the portfolio. This will lead to a residual between the real portfolio return and the return explained by attribution. A small residual will be of no concern but often the residual will grow over time to become the single largest factor thus invalidating the entire analysis.

Residuals will tend to be larger:

(1) For more active managers.
(2) If IPOs are a key part of the portfolio manager's strategy. The buy–hold approach tends to pick up the end of day price not the float price.
(3) For Large cash flows.
(4) For Illiquid assets.
(5) For Longer measurement periods. The smaller the measurement period the better, but even daily analysis can result in significant residuals.

If transaction information is not captured there is no opportunity to investigate the value added by the trading department. Since performance analysts using buy/hold attribution are not expecting the attribution to reconcile to the portfolio return operational errors will not be spotted.

Spaulding (2003) in his article "Holding's vs. transaction-based attribution – an overview" provides a more balanced perspective on the various approaches. The alternative to buy/hold is transaction-based attribution in which holdings and transaction information is used to replicate the portfolio return.

The decision whether to use buy/hold or transaction-based attribution will ultimately depend on the purpose for which the attribution will be used and the requirements of the asset manager. Buy/hold attribution may be acceptable for the exclusive internal use of the portfolio manager but clients are not always tolerant of attribution residuals. Pragmatism is a key tool in the performance measurer's armoury; however, in this instance I would not compromise. Performance measurers rely on good quality data. Reconciled transaction-based attribution improves the quality of back office processes by providing a tool that quickly identifies operational errors and improves the general risk control environment of the firm.

Transaction-based attribution

Transaction-based attribution is the most complete form of attribution identifying all sources of return and reconciling exactly to the published return. Transaction-based attribution is defined as:

> *Performance return attribution calculated directly from transaction and holdings-based data.*

Because transaction data is sourced directly including the transaction price all the sources of return can be measured included timing effects within the measurement period, bid/offer spread and transaction costs.

Security-level attribution

If the investment decision process is genuinely "bottom up" then it is not appropriate to calculate asset or sector allocation effects. For security-level attribution the investment decision

Table 9.1 Security-level attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Stock allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Timing $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
Allianz	10.2	9.3	15.0	15.0	0.08	0.0
Marconi	5.2	0.1	−25.0	−25.0	−1.48	0.0
Vodafone	15.0	20.8	3.4	3.4	0.11	0.0
AOL	22.0	14.7	−5.2	−5.2	−0.74	0.0
Microsoft	47.6	55.1	6.9	7.5	−0.15	−0.28
Total	*100*	*100*	*2.88*	*5.46*	*−2.17*	*−0.28*

process is effectively over- or underweighting an individual security, therefore to measure the contribution at security level we can use the asset allocation formula in Equations (5.18) and (5.24), respectively, for arithmetic and geometric attribution. The traditional stock selection formula, Equations (5.22) and (5.27), effectively measures the ability to add value within that security by timing trades. This timing attribution effect includes transaction costs, the impact of bid/offer spread and gain or loss resulting from transacting the trade at prices other than end of period, typically within the day.

Security-level attribution is demonstrated using data from Table 9.1 which contains five securities measured against a simple five security benchmark.

Detailed calculations are shown in Exhibits 9.1 and 9.2:

Exhibit 9.1 Geometric "bottom-up" security-level allocation

Geometric stock allocation for data in Table 9.1 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.0317}{1.0546} - 1 = -2.17\%$$

Individual security (or stock) allocation effects are:

Allianz $\quad (10.2\% - 9.3\%) \times \left(\frac{1.15}{1.0546} - 1\right) = 0.08\%$

Marconi $\quad (5.2\% - 0.1\%) \times \left(\frac{0.75}{1.0546} - 1\right) = -1.48\%$

Vodafone $\quad (15\%.0 - 20.8\%) \times \left(\frac{1.034}{1.0546} - 1\right) = 0.11\%$

AOL Time Warner $\quad (22.0\%.0 - 14.7\%) \times \left(\frac{0.948}{1.0546} - 1\right) = -0.74\%$

Microsoft $\quad (47.6\% - 55.1\%) \times \left(\frac{1.075}{1.0546} - 1\right) = -0.15\%$

Total $\quad 0.08\% - 1.48\% + 0.11\% - 0.74\% - 0.15\% = -2.17\%$

Exhibit 9.2 Geometric "bottom up" security-level allocation

Geometric timing for data in Table 9.1 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.0288}{1.0317} - 1 = -0.28\%$$

Individual security timing effects are:

Allianz	$10.2\% \times \left(\frac{1.15}{1.15} - 1\right) \times \left(\frac{1.15}{1.0317}\right) = 0.0\%$
Marconi	$5.2\% \times \left(\frac{0.75}{0.75} - 1\right) \times \left(\frac{0.75}{1.0317}\right) = 0.0\%$
Vodafone	$15.0\% \times \left(\frac{1.034}{1.034} - 1\right) \times \left(\frac{1.034}{1.0317}\right) = 0.0\%$
AOL Time Warner	$22.0\% \times \left(\frac{0.948}{0.948} - 1\right) \times \left(\frac{0.948}{1.0317}\right) = 0.0\%$
Microsoft	$47.6\% \times \left(\frac{1.069}{1.075} - 1\right) \times \left(\frac{1.075}{1.0317}\right) = -0.28\%$
Total	$0.0\% + 0.0\% + 0.0\% + 0.0\% - 0.28\% = -0.28\%$

Interpretation of this result is fairly straightforward; geometrically, the portfolio has underperformed the benchmark by 2.44%, the largest contributor to which is an overweight position in underperforming Marconi contributing −1.48% of geometric excess return.

The contribution from timing is negative and relatively small. Without any transactions it should be expected that the return of each security within the portfolio will be the same as the return in the benchmark, particularly if the same pricing source has been used. In this example for Microsoft there is a timing contribution of −0.28%. This indicates some trading activity within the period; while it is possible to generate a positive trading or timing effect this would require trading at a sufficiently advantageous price to cover both transaction costs and the bid/offer spread. Therefore, typically timing effects will be negative.

It is inappropriate to roll up individual security allocation and security effects into sectors for genuine bottom-up decision processes. Sector attribution is calculated by reference to sector indexes; bottom-up stock selection is essentially a different investment process referencing individual security prices. The sum of individual stock allocations will not equal total stock selection within the sector.

Transaction costs

Almost all transaction-based attribution methodologies include transaction costs in the stock selection effect by default. Asset allocation effects are measured by reference to the category index and the overall benchmark only, with no allowance for transaction costs. Asset allocation decisions when implemented clearly generate transaction costs. These costs can be significant particularly for illiquid assets such as emerging markets, and should be allocated to the asset allocator not the stock selector.

Table 9.2 Off-benchmark investment – security selection only

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	40	40	20	10	0.0	3.81
Japanese equities	30	20	−5	−4	−0.98	−0.29
US equities	20	40	6	8	−0.30	−0.38
Emerging markets	10	0	8	6.4	0.00	0.15
Total	*100*	*100*	*8.5*	*6.4*	*−1.28*	*3.29*

Off-benchmark (or zero-weight sector) attribution

If decisions are taken to invest outside of the benchmark there are two possible solutions. The desired answer depends on the investment decision process. If the portfolio manager wishes to buy an individual security in a country outside the benchmark based solely on the merits of the security, then this is a security selection decision and the performance should be measured against the overall benchmark.

If, however, the manager wishes to invest in a sector or country outside the benchmark, this is an asset allocation decision and should be measured accordingly. A representative index must be chosen to measure the impact of this overweight decision. There is a second decision to determine which securities to buy with the allocated cash; this will generate a security selection effect against the chosen representative index. Alternative presentations with the same data are shown in Tables 9.2 and 9.3 with detailed calculations in Exhibits 9.3 and 9.4 and Exhibits 9.5 and 9.6, respectively.

The assumption implicit in the presentation of data in Table 9.2 is that there is no allocation to emerging markets. The 10% allocation to emerging markets results from individual stock picks that happen to come from emerging market countries. These stock picks clearly represent

Table 9.3 Off-benchmark investment – security selection and asset allocation

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	40	40	20	10	0.00	3.80
Japanese equities	30	20	−5	−4	−0.98	−0.28
US equities	20	40	6	8	−0.30	−0.38
Emerging markets	10	0	8	9.4	0.28	−0.13
Total	*100*	*100*	*8.5*	*6.4*	*−1.00*	*3.00*

Exhibit 9.3 Geometric off-benchmark asset allocation

Geometric asset allocation for data in Table 9.2 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.050}{1.064} - 1 = -1.28\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(20\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.30\%$
Emerging markets	$(10\% - 0\%) \times \left(\frac{1.064}{1.064} - 1\right) = 0.0\%$
Total	$0.0\% - 0.98\% - 0.30\% + 0.0\% = -1.28\%$

stock or security-level decisions and are measured as such generating 15 basis points of added value.

The assumption implicit in the presentation of data in Table 9.3 is that first there is an allocation to emerging markets of 10%. This separate decision must be measured; an emerging markets index is required to calculate an asset allocation effect, a positive contribution of 28 basis points. After allocating cash to emerging markets individual securities must be bought, in this case underperforming the emerging markets index resulting in a negative contribution of 13 basis points.

Exhibit 9.4 Geometric off-benchmark stock selection

Geometric stock selection including interaction for data in Table 9.2 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.085}{1.050} - 1 = 3.29\%$$

Individual country stock selection effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.05} = 3.81\%$
Japanese equities	$30\% \times \left(\frac{0.94}{0.95} - 1\right) \times \frac{0.95}{1.05} = -0.29\%$
US equities	$20\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.05} = -0.38\%$
Emerging markets	$10\% \times \left(\frac{1.08}{1.064} - 1\right) \times \frac{1.064}{1.05} = 0.15\%$
Total	$3.81\% - 0.29\% - 0.38\% + 0.15\% = 3.29\%$

Exhibit 9.5 Geometric asset allocation

Geometric asset allocation for data in Table 9.3 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.053}{1.064} - 1 = -1.00\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 0.0\%$
Japanese equities	$(30\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -0.98\%$
US equities	$(20\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = -0.30\%$
Emerging markets	$(10\% - 0\%) \times \left(\frac{1.094}{1.064} - 1\right) = 0.28\%$
Total	$0.0\% - 0.98\% - 0.30\% + 0.28\% = -1.00\%$

Exhibit 9.6 Geometric off-benchmark stock selection

Geometric stock selection including interaction for data in Table 9.3 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.085}{1.053} - 1 = 3.00\%$$

Individual country stock selection effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.053} = 3.80\%$
Japanese equities	$30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.053} = -0.29\%$
US equities	$20\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.053} = -0.38\%$
Emerging markets	$10\% \times \left(\frac{1.08}{1.094} - 1\right) \times \frac{1.094}{1.053} = -0.13\%$
Total	$3.80\% - 0.29\% - 0.38\% - 0.13\% = 3.00\%$

MULTI-LEVEL ATTRIBUTION

In Chapter 5 we used the Brinson model to measure a fairly simple three-step investment decision process:

- Step 1 Benchmark.
- Step 2 Asset allocation (represented by the semi-notional fund).
- Step 3 Stock selection.

Table 9.4 Multi-level attribution

		Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)
Level 1	Equities	*58.5*	*55.0*	*8.49*	*8.95*
Level 2	US	*19.0*	*15.0*	5.36	7.50
Level 3	Financial	*10.0*	*5.0*	8.38	9.60
Level 3	Software	*9.0*	*10.0*	2.00	6.45
Level 2	Europe	39.5	40	5.36	7.50
Level 3	Autos	*22.5*	*25.0*	16.05	15.60
Level 3	Chemicals	*17.0*	*15.0*	2.00	−0.67
Level 1	Bonds	*41.5*	*45.0*	*2.35*	*1.78*
Level 2	Govt	*12.5*	10.0	2.00	1.00
Level 2	Corp.	29.0	35.0	2.50	2.00
Total		*100.0*	*100.0*	*5.94*	*5.72*

For more complex multi-level decision processes, provided we can identify each step and importantly the order of the decision process, we can still employ the Brinson model.

Consider the balanced (equity and fixed income) portfolio in Table 9.4.

For this portfolio there are five clear steps in the decision process illustrated in Figure 9.1. Each intermediate step is represented by a semi-notional fund and the wealth ratio between each successive step represents the impact of that isolated decision:

$$\frac{1+{}^{1}b_S}{1+b} \times \frac{1+{}^{2}b_S}{1+{}^{1}b_S} \times \frac{1+{}^{3}b_S}{1+{}^{2}b_S} \times \frac{1+r}{1+{}^{3}b_S} = \frac{1+r}{1+b} \tag{9.1}$$

The semi-notional fund for each step d is calculated as follows using benchmark returns appropriate to the level of the decision process:

$${}^{d}b_S = \sum_{i=1}^{i=n} {}^{d}w_i \times {}^{d}b_i \tag{9.2}$$

The five-step investment decision process is as follows:

- Step 1 Benchmark.
- Step 2 Semi-notional return 1 (representing the allocation between equities and bonds).
- Step 3 Semi-notional return 2 (representing the allocation between equity countries and bond sectors).
- Step 4 Semi-notional return 3 (representing the allocation between equity industrial sectors – note there is no decision process for bonds at this level in this particular process).
- Step 5 The final step as always representing stock selection.

The contribution to geometric asset allocation in the ith category for the dth level in the decision process is:

$$\left({}^{d}w_i - {}^{d}W_i\right) \times \left(\frac{1+{}^{d}b_i}{1+{}^{d-1}b_i} - 1\right) \times \frac{1+{}^{d-1}b_i}{1+b_s^{d-1}} \tag{9.3}$$

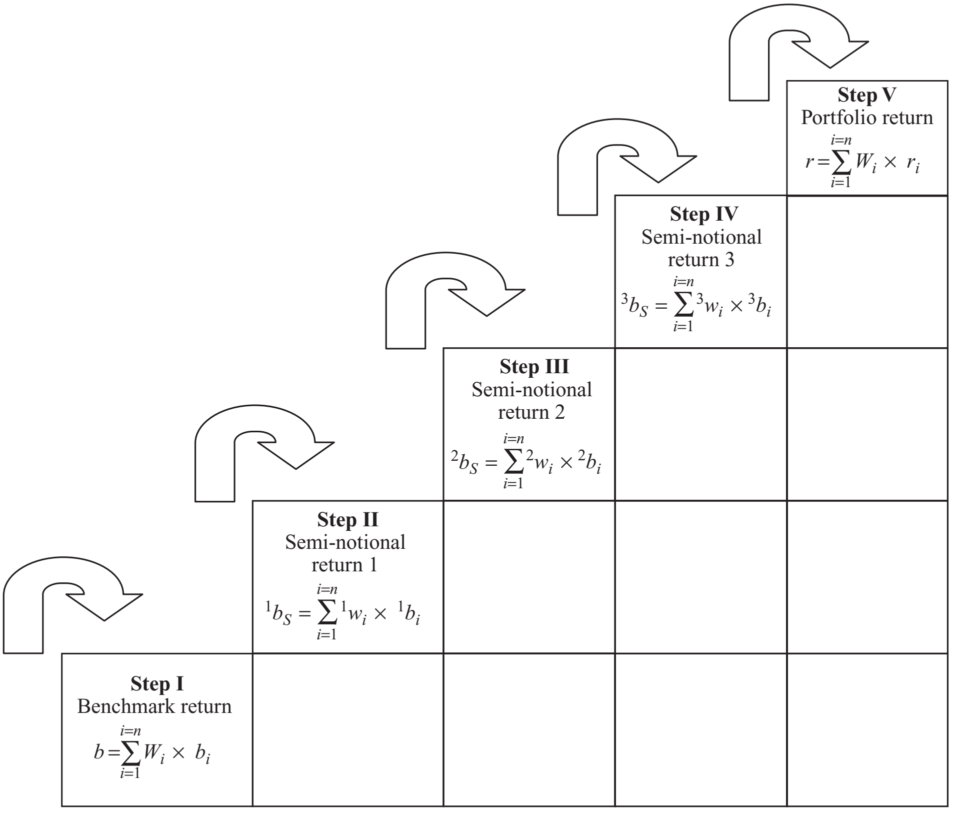

Figure 9.1 Multi-level attribution

b^{d-1} must be the subindex in which the category is present. At each asset allocation level the effect is adjusted for the impact of decisions already made. Note that ${}^{0}b_S = b$, the original benchmark. For the first step, since there are no prior effects, asset allocation is:

$$\left({}^{1}w_i - {}^{1}W_i\right) \times \left(\frac{1 + {}^{1}b_i}{1 + b} - 1\right) \tag{9.4}$$

The total attribution for each asset allocation step in the decision process is:

$$\frac{1 + {}^{d}b_S}{1 + {}^{d-1}b_S} - 1 \tag{9.5}$$

The final step, stock selection, is measured by:

$${}^{d}w_i \times \left(\frac{1 + r_i}{1 + {}^{d}b_i} - 1\right) \times \frac{1 + {}^{d}b_i}{1 + {}^{d}b_S} \tag{9.6}$$

Portfolio, benchmark and semi-notional returns for each level of data shown in Table 9.3 are calculated in Exhibit 9.7:

Exhibit 9.7 Calculation of multi-level portfolio benchmark and semi-notional returns

Using Table 9.3 the portfolio return is:

$$r = 10\% \times 8.38\% + 9.0\% \times 2.0\% \times 22.5\% \times 16.05\% + 17\% \times 2.0\%$$
$$12.5\% \times 2.0\% + 29\% \times 2.5\% = 5.94\%$$

or

$$r = 58.5\% \times 8.49\% + 41.5\% \times 2.35\% = 5.94\%$$

Note that each subsector is the sum of its parts. The benchmark return is:

$$b = 5\% \times 9.6\% + 10\% \times 6.45\% + 25\% \times 15.6\% + 15\% \times -0.67\%$$
$$10\% \times 1.0\% + 35\% \times 2.0\% = 5.72\%$$

or

$$b = 55\% \times 8.95\% + 45\% \times 1.78\% = 5.72\%$$

The semi-notional return 1 is:

$${}^1b_S = 58.5\% \times 8.95\% + 41.5\% \times 1.78\% = 5.98\%$$

The semi-notional return 2 is:

$${}^2b_S = 19\% \times 7.50\% + 39.5\% \times 9.50\% + 12.5\% \times 1.0\% + 29\% \times 2.0\% = 5.88\%$$

The semi-notional return 3 is:

$${}^3b_S = 10\% \times 9.60\% + 9\% \times 6.45\% + 22.5\% \times 15.6\% + 17\% \times -0.67\%$$
$$+ 12.5\% \times 1.0\% + 29.0\% \times 2.0\% = 5.64\%$$

Top-level attribution effects for each step in the decision process are calculated in Exhibit 9.8:

Exhibit 9.8 Multi-level attribution effects

$$\frac{1+{}^1b_S}{1+b} \times \frac{1+{}^2b_S}{1+{}^1b_S} \times \frac{1+{}^3b_S}{1+{}^2b_S} \times \frac{1+r}{1+{}^3b_S} - 1 = \frac{1+r}{1+b} - 1$$

Level 1 allocation (step II) $\quad \dfrac{1+{}^1b_S}{1+b} - 1 = \dfrac{1.0598}{1.0572} - 1 = 0.24\%$

Level 2 allocation (step III) $\quad \dfrac{1+{}^2b_S}{1+{}^1b_S} - 1 = \dfrac{1.0588}{1.0598} - 1 = -0.09\%$

Level 3 allocation (step IV) $\quad \dfrac{1+{}^3b_S}{1+{}^2b_S} - 1 = \dfrac{1.0564}{1.0588} - 1 = -0.23\%$

Stock selection (step V) $\quad \dfrac{1+r}{1+{}^3b_S} - 1 = \dfrac{1.0594}{1.0564} - 1 = 0.28\%$

Total excess return $\quad \dfrac{1+r}{1+b} - 1 = \dfrac{1.0594}{1.0572} - 1 = 0.21\%$

$$\frac{1.0598}{1.0572} \times \frac{1.0588}{1.0598} \times \frac{1.0564}{1.0588} \times \frac{1.0594}{1.0564} - 1 = \frac{1.0594}{1.0572} - 1$$

Exhibit 9.9 Level 1 multi-level attribution effects

$$\frac{1+{}^1b_S}{1+b}-1=\frac{1.0598}{1.0572}-1=0.24\%$$

Equities	$\left({}^1w_i-{}^1W_i\right)\times\left(\frac{1+{}^1b_i}{1+b}-1\right)=(58.5\%-55\%)\times\left(\frac{1.0895}{1.0572}-1\right)=0.11\%$
Bonds	$(41.5\%-45\%)\times\left(\frac{1.0178}{1.0572}-1\right)=0.13\%$
Total	$0.11\%+0.13\%=0.24\%$

Individual level 1 allocation effects are calculated in Exhibit 9.9, level 2 allocation effects in Exhibit 9.10, level 3 allocation effects in Exhibit 9.11 and security selection in Exhibit 9.12.

Interpretation of the attribution effects is fairly straightforward. The first decision, overweighting equities and underweighting bonds, is positive, adding 24 basis points of value. The level 2 decision, overweighting US equities and government bonds and underweighting corporates, lost 9 basis points. The level 3 decision, underweighting automobiles and overweighting chemicals, only partial offset by overweighting financials, lost 23 basis points.

Positive selection effects in automobiles, chemicals, government and corporate bonds outweighed negative selection effects in financials and software to add 28 basis points of geometric excess return.

Exhibit 9.10 Level 2 multi-level attribution effects

$$\frac{1+{}^2b_S}{1+{}^1b_S}-1=\frac{1.0588}{1.0598}-1=-0.09\%$$

Individual level 2 attribution effects:

$$\left({}^2w_i-{}^2W_i\right)\times\left(\frac{1+{}^2b_i}{1+{}^1b_i}-1\right)\times\frac{1+{}^1b_i}{1+{}^1b_S}$$

US equities	$(19\%-15\%)\times\left(\frac{1.075}{1.0895}-1\right)\times\frac{1.0895}{1.0598}=-0.05\%$
European equities	$(39.5\%-40\%)\times\left(\frac{1.095}{1.0895}-1\right)\times\frac{1.0895}{1.0598}=0.00\%$
Government bonds	$(12.5\%-10\%)\times\left(\frac{1.01}{1.0178}-1\right)\times\frac{1.0178}{1.0598}=-0.02\%$
Corporate bonds	$(29.0\%-35\%)\times\left(\frac{1.02}{1.0178}-1\right)\times\frac{1.0178}{1.0598}=-0.01\%$
Total	$-0.05\%+0.0\%-0.02\%-0.01\%=-0.09\%$

Exhibit 9.11 Level 3 multi-level attribution effects

$$\frac{1+{}^3b_S}{1+{}^2b_S}-1=\frac{1.0564}{1.0588}-1=-0.23\%$$

Individual level 3 attribution effects:

$$\left({}^3w_i-{}^3W_i\right)\times\left(\frac{1+{}^3b_i}{1+{}^2b_i}-1\right)\times\frac{1+{}^2b_i}{1+{}^2b_S}$$

Financials	$(10\%-5\%)\times\left(\frac{1.096}{1.075}-1\right)\times\frac{1.075}{1.0588}=0.10\%$
Software	$(9\%-10\%)\times\left(\frac{1.0645}{1.075}-1\right)\times\frac{1.075}{1.0588}=0.01\%$
Automobiles	$(22.5\%-25\%)\times\left(\frac{1.156}{1.095}-1\right)\times\frac{1.095}{1.0588}=-0.14\%$
Chemicals	$(17.0\%-15\%)\times\left(\frac{0.9933}{1.095}-1\right)\times\frac{1.095}{1.0588}=-0.19\%$
Total	$0.1\%+0.01\%-0.14\%-0.19\%=-0.23\%$

Exhibit 9.12 Security/Issue selection multi-level attribution effects

$$\frac{1+r}{1+{}^3b_S}-1=\frac{1.0594}{1.0564}-1=0.28\%$$

Security/issue selection effects:

$${}^3w_i\times\left(\frac{1+r_i}{1+{}^3b_i}-1\right)\times\frac{1+{}^3b_i}{1+{}^3b_S}$$

Financials	$10\%\times\left(\frac{1.0838}{1.096}-1\right)\times\frac{1.096}{1.0564}=-0.12\%$
Software	$9\%\times\left(\frac{1.02}{1.0645}-1\right)\times\frac{1.0645}{1.0564}=-0.38\%$
Automobiles	$22.5\%\times\left(\frac{1.1605}{1.156}-1\right)\times\frac{1.156}{1.0564}=0.09\%$
Chemicals	$17\%\times\left(\frac{1.02}{0.9933}-1\right)\times\frac{0.9933}{1.0564}=0.43\%$
Government bonds	$12.5\%\times\left(\frac{1.02}{1.01}-1\right)\times\frac{1.01}{1.0564}=0.12\%$
Corporate bonds	$29\%\times\left(\frac{1.025}{1.02}-1\right)\times\frac{1.02}{1.0564}=0.14\%$
Total	$-0.12\%-0.38\%+0.09\%+0.43\%+0.12\%+0.14\%=0.28\%$

Balanced attribution

In the previous example of multi-level balanced attribution the fixed income proportion of the portfolio was not adjusted for systematic risk or duration. The standard Brinson methodology can, however, be partially combined with weighted duration or van Breukelen-type attribution for the fixed income portion of the portfolio recognising the different investment processes in a combined equity and fixed income or balanced portfolio.

Lookthrough attribution (or fund of funds attribution)

Managers of fund of funds portfolios often require attribution analysis that "looks through" the individual fund holdings to the underlying positions. This type of analysis is only appropriate if the investment decision process also "looks through" the funds rather than treating each individual fund as a separate security. Lookthrough analysis requires that attribution is performed on each underlying fund and included in appropriate proportions in the fund of funds. Difficulties are likely to occur reconciling the underlying asset performance of each fund to its net unit price if transactions occur and if the underlying funds benchmarks are inconsistent with the overall benchmark.

ATTRIBUTION STANDARDS

Following the success of GIPS providing standards for the presentation of performance information there has been some debate about the desirability of attribution standards. I'm not yet sure we have reached the point at which attribution standards would be useful. Development of various attribution methodologies continues to gain pace and there is still some way to go. Standards will have the effect of slowing down future developments.

The very nature of attribution does not lend itself to the application of standards; asset managers are constantly seeking ways of differentiating their products, requiring continued development of attribution methodologies.

There are, however, a number of pitfalls that users of attribution analysis should avoid. I believe guidance to avoid these pitfalls providing information to the users of attribution is much more appropriate and beneficial from an educational viewpoint.

The European Investment Performance Council (EIPC, 2002) produced some basic guidance in 2002 reproduced in Appendix C and have updated this guidance (EIPC, 2004), reproduced in Appendix D.

All asset managers should be able to answer the 22 questions posed in the EIPC's original guidance for their own attribution reports.

The demand for attribution standards is a reflection of the rather narrow use asset managers make of attribution; it is a much broader and more powerful tool than generally believed. Of course, it's a key tool for performance measurers that allows them to participate in the investment decision process, to articulate their understanding of the sources of return and to verify the accuracy of the reported return. Attribution can also be used as an operational tool; daily attribution analysis can be used before valuations are signed off, to identify pricing and transaction errors which are normally painfully evident. In fact, attribution analysis can be used to improve the quality of the back office significantly. Attribution analysis is also useful for effective risk control, identifying rogue activities and acting as the interface between performance measurement and risk management teams.

EVOLUTION OF PERFORMANCE ATTRIBUTION METHODOLOGIES

The evolution of performance attribution methodologies is shown in Figure 9.2. The evolution down the page is not necessarily in chronological order but represents my preferences and interpretation of key contributions and insights.

The key stages are Brinson and Fachler in 1985, Karnosky and Singer in 1994 and the three geometric methodologies apparently developed in isolation – Burnie, Knowles and Teder, Bain and the geometric methodology shown in detail in Appendix A. The arithmetic smoothing methodologies are interesting but are ultimately unnecessary; Karnosky and Singer, although at first sight appearing to be arithmetic, is actually geometric because of the use of continuously compounded returns. The multi-currency geometric methodology is that detailed in Appendix B. The majority of fixed income methodologies are arithmetic in nature, no doubt

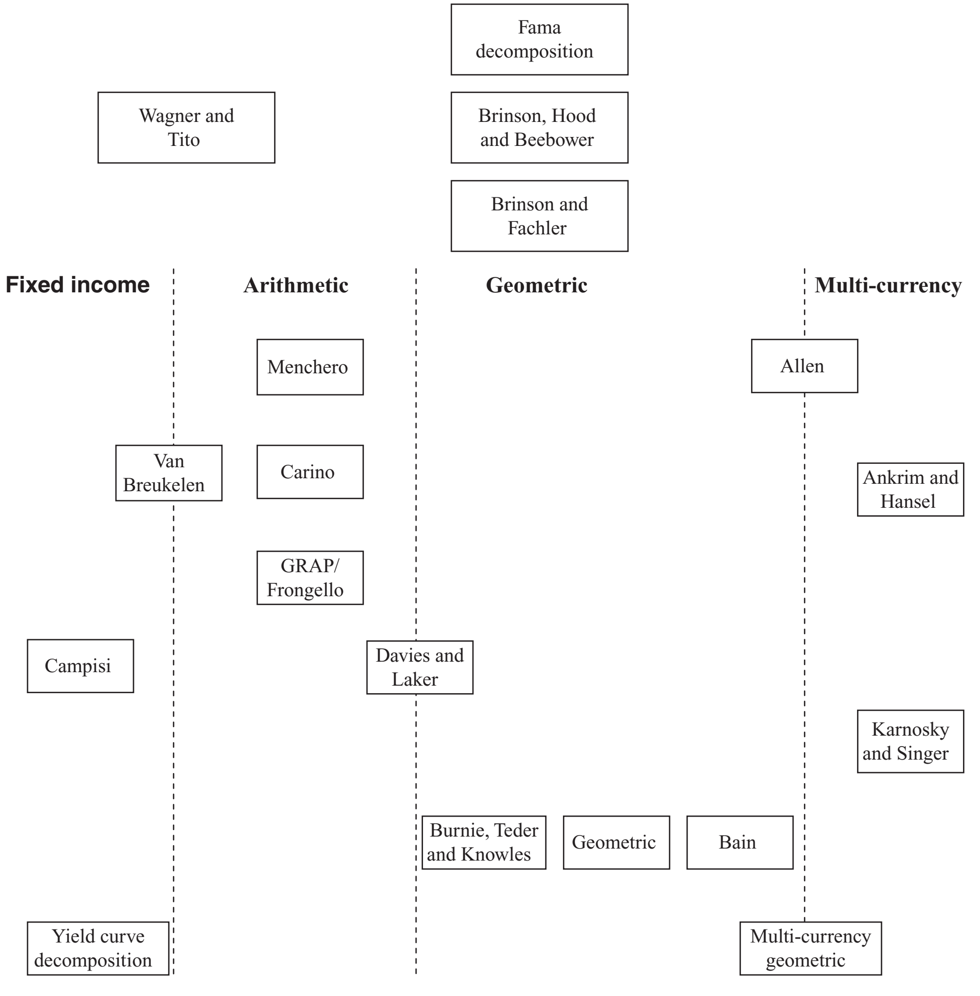

Figure 9.2 Evolution of performance attribution methodologies

in part due to smaller excess returns. Wagner and Tito is a variation of Fama decomposition using duration as a measure of systematic risk. Weighted duration (van Breukelen) attribution is a risk-adjusted form of the Brinson model. Campisi incorporates duration and credit spreads from the top down. Full yield curve decomposition analyses the impact of the change in shape of the yield curve for each instrument from the bottom up.

RISK-ADJUSTED ATTRIBUTION

Brinson *et al.* (1991) in a follow-up article to their original suggested that systematic risk measures such as beta or duration could be used in conjunction with their standard model. This is only appropriate if the portfolio manager is using systematic risk in the investment decision process. For example, an equity portfolio manager could choose to increase the beta of a category rather than or as well as going overweight to achieve an asset allocation effect. We can measure the impact of this decision in the standard Brinson model using the regression equation to calculate systematic risk-adjusted returns:

$$\text{Regression equation} \quad r_p - r_F = \alpha + \beta \times (b - r_F) + \varepsilon \tag{4.14}$$

Rearranging Equation (4.14) and ignoring the error term we can break down the return into selectivity or Jensen's alpha and the return derived from systematic risk:

$$r_p = \underbrace{\alpha}_{\text{Selectivity}} + \underbrace{r_F + \beta \times (b - r_F)}_{\text{Systematic risk}} \tag{9.7}$$

We can define an intermediate benchmark return adjusted for systematic risk b'_i:

$$b'_i = x_i + \beta_i \times (b_i - x_i) \tag{9.8}$$

where: x_i = risk-free rate in country i
b_i = benchmark return in country i
β_i = systematic risk in country i.

To identify the added value from asset allocation due to systematic risk we must calculate the return of an additional intermediate fund.

In this notional fund adjusted for systematic risk the asset allocation weights of the actual fund are applied to the risk-adjusted benchmark returns within each category.

$$\text{Systematic risk notional fund} \quad b'_S = \sum_{i=1}^{i=n} w_i \times b'_i \tag{9.9}$$

The total excess return is now the combination of three wealth ratios:

$$\frac{1+r}{1+b} - 1 = \underbrace{\frac{1+r}{1+b'_S}}_{\text{Selectivity}} \times \underbrace{\frac{1+b'_S}{1+b_S}}_{\substack{\text{Systematic}\\ \text{risk}}} \times \underbrace{\frac{1+b_S}{1+b}}_{\substack{\text{Asset}\\ \text{allocation}}} - 1 \tag{9.10}$$

Selectivity

To identify the total contribution to selectivity we can use the ratio of the portfolio return compared to the systematic risk-adjusted notional return:

$$\frac{(1+r)}{\left(1+b'_S\right)} - 1 \tag{9.11}$$

The contribution to selectivity in the ith category is now:

$$S'_i = w_i \times \left(\frac{1+r_i}{1+b'_i} - 1\right) \times \frac{\left(1+b'_i\right)}{\left(1+b'_S\right)} \tag{9.12}$$

To calculate the contribution from systematic risk we can use the ratio of the systematic risk notional fund with the semi-notional fund:

$$\frac{\left(1+b'_S\right)}{(1+b_S)} - 1 \tag{9.13}$$

The contribution from systematic risk in the ith category is now:

$$R_i = w_i \times \left(\frac{1+b'_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)} \tag{9.14}$$

The asset allocation term is unchanged.

Our standard data example is extended in Table 5.14 to include the risk-free rate for each market and the beta. We must make the assumption that the portfolio manager is using beta as part of the asset allocation process.

Using the regression equation we can calculate risk-adjusted benchmark returns and therefore calculate the attribution effects for selectivity and systematic risk as shown in Exhibit 9.13, 9.14 and 9.15, respectively.

Asset allocation effects are unchanged from before. The revised risk-adjusted attribution summary is shown in Table 9.5. The systematic risk asset allocation and asset allocation effects can be compounded to calculate the overall asset allocation effect. Note that there is now a contribution to UK equity allocation of 1.03% due to the high beta greater than 1 in a rising market. In effect, part of the 20.0% return is due to high systematic risk and is therefore not entirely stock selection.

Since portfolios are rarely managed in this manner risk-adjusted attribution for equities is seldom used.

Table 9.5 Standard data example with riskfree rate and beta

	Portfolio weight (%)	Benchmark weight (%)	Portfolio return (%)	Benchmark return (%)	Risk Free rate (%)	β
UK equities	40	40	20	10	1.0	1.3
Japanese equities	30	20	−5	−4	0.1	1.0
US equities	30	40	6	8	0.2	0.8
Total	*100*	*100*	*8.3*	*6.4*		1.06

Exhibit 9.13 Return adjusted for systematic risk

Revised benchmark returns adjusted for systematic risk:

UK equities $x_i + \beta_i \times (b_i - x_i) = 1.0\% + 1.3 \times (10.0\% - 1.0\%) = 12.7\%$

Japanese equities $0.1\% + 1.0 \times (-4.0\% - 0.1\%) = -4.0\%$

US equities $0.2\% + 0.8 \times (8.0\% - 0.2\%) = 6.44\%$

Systematic risk notional fund:

$$b'_S = 40\% \times 12.7\% + 30\% \times -4\% + 30\% \times 6.44\% = 5.81\%$$

Exhibit 9.14 Selectivity

$$\frac{(1+r)}{(1+b'_S)} - 1 = \frac{1.083}{1.0581} - 1 = 2.35\%$$

Individual country selectivity effects are:

UK equities $40\% \times \left(\frac{1.20}{1.127} - 1\right) \times \frac{1.127}{1.0581} = 2.76\%$

Japanese equities $30\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.0581} = -0.28\%$

US equities $30\% \times \left(\frac{1.06}{1.0644} - 1\right) \times \frac{1.0644}{1.0581} = -0.12\%$

Total $2.76\% - 0.28\% - 0.12\% = 2.35\%$

Exhibit 9.15 Systematic risk

$$\frac{(1+b'_S)}{(1+b_S)} - 1 = \frac{1.0581}{1.052} - 1 = 0.58\%$$

Individual country selectivity effects are:

UK equities $40\% \times \left(\frac{1.127}{1.10} - 1\right) \times \frac{1.10}{1.052} = 1.03\%$

Japanese equities $30\% \times \left(\frac{0.96}{0.96} - 1\right) \times \frac{0.96}{1.052} = 0.0\%$

US equities $30\% \times \left(\frac{1.0644}{1.08} - 1\right) \times \frac{1.08}{1.052} = -0.44\%$

Total $1.03\% + 0.0\% - 0.44\% = 0.58\%$

Table 9.6 Risk-adjusted attribution

	Portfolio weight w_i (%)	Benchmark weight w_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Systematic risk allocation $w_i \times \left(\frac{1+b'_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)	Selectivity $w_i \times \left(\frac{1+r_i}{1+b'_i} - 1\right) \times \frac{(1+b'_i)}{(1+b'_S)}$ (%)
UK equities	40	40	20	10	0.0	1.03	2.76
Japanese equities	30	20	−5	−4	−0.98	0.0	−0.28
US equities	30	40	6	8	−0.15	−0.44	−0.12
Total	*100*	*100*	*8.3*	*6.4*	*−1.13*	*0.58*	*2.35*

10

Performance Measurement for Derivatives

If all the rich people in the world divided up their money among themselves there wouldn't be enough to go around.

Christina Stead (1902–1983) *House of All Nations* (1938) "Credo"

Definition *A derivative instrument is an asset whose value is* derived *from the value of another underlying or reference asset.*

Typical derivative instruments include futures, forwards, swaps and options plus multiple variations and combinations. They are either traded on organised exchanges or agreed directly in the over-the-counter (OTC) market.

For some, derivatives have a scary reputation following the collapse of Barings Bank in 1995, the bail out of Long-Term Capital Management in 1998 and currency losses by Allied Irish Bank in 2002 to name but three.

Of course, derivatives can be used to generate extreme leverage and hence the possibility of large losses but they also provide the ability to effectively hedge and transfer risk.

FUTURES

A futures contract is an agreement transacted through an organised exchange to deliver an asset at a fixed price in the future.

Equity index future

An equity index futures contract is an agreement to exchange cash compensation payments based on the movements in the level of an equity index. Physical assets are not delivered.

Equity index futures are the most basic form of derivative. They have no market value in a portfolio; instead they have two legs of economic exposure each of equal value, one leg representing the equity index and the other leg economic or notional cash. Buying an equity index future will result in a long equity position and a short cash position; selling an equity index future will result in a short equity position and a long cash position.

The price of the equity index future implicitly includes the underlying dividend stream before expiry and the cost of carry from interest rates, therefore the futures price will naturally differ from the underlying index discounting the differential between dividend and interest income. The contract may also trade at a discount or premium to fair value depending on the supply and demand of the futures contract relative to the underlying reference or cash index, converging only at the time of expiry.

Equity index futures never generate any currency exposure. Gains and losses in the futures contract are realised daily in the margin account. Opening an equity futures position (long or short) requires the depositing of an initial margin (collateral) with a broker who handles payments made to and received from the clearing house. If the position is not closed by the end of the day it will be marked to market and profits and losses are added or deducted from the margin account. The margin account is an asset of the fund and should be included in all performance and attribution calculations generating currency exposure just like any cash asset. Since the initial margin is a great deal smaller than the economic exposure significant leverage can be achieved

Libor (London Interbank Offered Rate)

The British Banking Association (BBA) Libor benchmark, originally developed in the 1980s as a measure of the rate at which banks lend money to each other, is used to calculate the interest rates of a range of financial instruments and derivatives around the world in sterling, dollars, euros, yen and other currencies.

The BBA calculates the rates together with Reuters each day from a 16 bank panel at 11 am using an average of the middle 50%. Rates are created across 15 maturities (overnight to 12 months) for 10 currencies.

Attribution including equity index futures

Like any other asset the attribution of equity index futures must reflect the investment decision process.

The most effective way of representing this decision process is to use notional assets. Gains and losses in an equity index future are realised daily in the variation margin account which remains an asset of the portfolio. Both the economic exposures of the investment decision process and the return can be disaggregated as two notional assets of equal size; one long and one short, one representing the appropriate equity category and one cash. The effect of interest rates is included by applying Libor rates to the notional cash asset; this synthetic income, positive or negative, represents part of the realised return in the variation margin account.

The impacts of these notional assets are demonstrated in Exhibit 10.1 using data from Table 10.1

Exhibit 10.1 Return calculations

Using the data from **Table 10.1** the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 40\% \times 20\% + 55\% \times 6\% + 5\% \times 1.4\% + 5\% \times 9.5\% - 15\% \times 8.6\% - 5\% \times 1.5\% + 15\% \times 1.2 = 10.66\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 50\% \times 10\% + 40\% \times 8\% + 10\% \times 1.5\% = 8.35\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 40\% \times 10\% + 55\% \times 8\% + 5\% \times 1.5\% + 5\% \times 9.9\% - 15\% \times 8.5\% - 5\% \times 1.5\% + 15\% \times 1.2\% = 7.8\%$$

The data shown in Table 10.1 is in long form for illustration. The investment decision process is unlikely to treat the futures contracts in isolation, and we should find some way of consolidating the data consistent with the investment decision process. Analysing the attribution effects we notice that the return of UK futures has underperformed the UK futures benchmark; this

Table 10.1 Attribution with futures (long form)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	40	50	20.0	10.0	−0.15	3.71
US equities	55	40	6.0	8.0	−0.05	−1.02
Cash	5	10	1.4	1.5	0.32	0.00
UK equities	5	0	9.5	9.9	0.07	−0.02
US futures	−15	0	8.6	8.5	−0.02	−0.01
UK economic cash	−5	0	1.5	1.5	0.32	0.0
US economic cash	15	0	1.2	1.2	−0.99	0.0
Total	*100*	*100*	*10.66*	*8.35*	*−0.51*	*2.65*

is most likely due to timing but could also be due to the fact that the futures contract does not necessarily trade in line with the cash index. It's also worth noting that both the US and UK futures benchmarks are not the same as the underlying asset category benchmarks; this is fairly common as there may not be a futures index contract for the actual sector benchmark of the account. The impact from this mismatch is unavoidable but can be measured. Finally, its worth noting that the return on US economic cash is different from that on UK economic cash; this is because in this example US $ Libor is lower than £Libor.

Exhibit 10.2 Asset allocation with futures

Geometric asset allocation for data in Table 10.1 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.078}{1.0835} - 1 = -0.51\%$$

Individual country asset allocation effects are:

UK equities	$(40\% - 50\%) \times \left(\frac{1.10}{1.0835} - 1\right) = -0.15\%$
US equities	$(55\% - 40\%) \times \left(\frac{1.08}{1.0835} - 1\right) = -0.05\%$
Cash	$(5\% - 10\%) \times \left(\frac{1.015}{1.0835} - 1\right) = 0.32\%$
UK futures	$(5\% - 0\%) \times \left(\frac{1.095}{1.0835} - 1\right) = 0.07\%$
US futures	$(-15\% - 0\%) \times \left(\frac{1.085}{1.0835} - 1\right) = -0.02\%$
UK economic cash	$(-5\% - 0\%) \times \left(\frac{1.015}{1.0835} - 1\right) = 0.32\%$
US economic cash	$(15\% - 0\%) \times \left(\frac{1.012}{1.0835} - 1\right) = -0.99\%$
Total	$-0.15\% - 0.05\% + 0.32\% + 0.07\% - 0.02\% + 0.32\% - 0.99\% = -0.51\%$

Exhibit 10.3 Stock selection with futures

Geometric stock selection including interaction for data in Table 10.1 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.1066}{1.078} - 1 = 2.65\%$$

Individual country stock selection effects are:

UK equities	$40\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.078} = 3.71\%$
US equities	$55\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.078} = -1.02\%$
Cash	$5\% \times \left(\frac{1.014}{1.015} - 1\right) \times \frac{1.015}{1.078} = 0.00\%$
UK futures	$5\% \times \left(\frac{1.095}{1.099} - 1\right) \times \frac{1.099}{1.078} = -0.02\%$
US futures	$-15\% \times \left(\frac{1.086}{1.085} - 1\right) \times \frac{1.085}{1.078} = -0.01\%$
UK economic cash	$-5\% \times \left(\frac{1.015}{1.015} - 1\right) \times \frac{1.015}{1.078} = 0.00\%$
US economic cash	$15\% \times \left(\frac{1.012}{1.012} - 1\right) \times \frac{1.012}{1.078} = 0.00\%$
Total	$3.71\% - 1.02\% + 0.0\% - 0.02\% - 0.01\% + 0.0\% + 0.0\% = 2.65\%$

The presentation we use largely depends on the investment decision process; if the managers responsible for UK equities and US equities were responsible for the futures contract it would be reasonable to combine the effects as shown in Table 10.2 and Exhibits 10.4, 10.5 and 10.6.

However, if the asset allocator and not the individual sector manager is responsible for the futures contracts then we must reflect this in our attribution analysis including any futures index mismatch between the contract index used and the underlying benchmark, see Table 10.3. Asset allocation is identical to that in Exhibit 10.5, but reflecting the investment decision

Table 10.2 Attribution with futures (alternative presentation)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	45	50	18.83	10.0	−0.08	3.69
US equities	40	40	5.03	8.0	0.00	−1.10
Cash	15	10	1.17	1.5	−0.32	−0.05
Total	*100*	*100*	*10.66*	*8.35*	*−0.39*	*2.54*

Exhibit 10.4 Adjusted return calculations including futures

Combining sector returns using data from Table 10.1:

UK equities $\dfrac{40\% \times 20.0\% + 5\% \times 9.5}{40\% + 5\%} = 18.83\%$

US equities $\dfrac{55\% \times 6.0\% - 15\% \times 8.6}{55\% - 15\%} = 5.03\%$

Cash $\dfrac{5\% \times 1.4\% - 5\% \times 1.5 + 15\% \times 1.2}{5\% - 5\% + 15\%} = 1.17\%$

Using these combined returns the portfolio returns still reconcile as follows:

$$r = 45\% \times 18.83\% + 40\% \times 5.03\% + 15\% \times 1.17\% = 10.66\%$$

The benchmark return is clearly unchanged:

$$b = 50\% \times 10\% + 40\% \times 8\% + 10\% \times 1.5\% = 8.35\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$, however, must be revised to reflect this slightly different investment decision process:

$$b_S = 45\% \times 10\% + 40\% \times 8\% + 15\% \times 1.5\% = 7.93\%$$

Exhibit 10.5 Revised asset allocation with futures

Geometric asset allocation for data in Table 10.2 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.0793}{1.0835} - 1 = -0.39\%$$

Individual country asset allocation effects are:

UK equities $(45\% - 50\%) \times \left(\dfrac{1.10}{1.0835} - 1\right) = -0.08\%$

US equities $(40\% - 40\%) \times \left(\dfrac{1.08}{1.0835} - 1\right) = 0.00\%$

Cash $(15\% - 10\%) \times \left(\dfrac{1.015}{1.0835} - 1\right) = -0.32\%$

Total $-0.08\% + 0.0\% - 0.32\% = -0.39\%$

Exhibit 10.6 Revised stock selection with futures

Geometric stock selection including interaction for data in Table 10.2 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.1066}{1.0793} - 1 = 2.54\%$$

Individual country stock selection effects are:

UK equities $45\% \times \left(\dfrac{1.188}{1.10} - 1\right) \times \dfrac{1.10}{1.0793} = 3.69\%$

US equities $40\% \times \left(\dfrac{1.06}{1.08} - 1\right) \times \dfrac{1.08}{1.0793} = -1.10\%$

Cash $15\% \times \left(\dfrac{1.012}{1.015} - 1\right) \times \dfrac{1.015}{1.0793} = -0.05\%$

Total $3.69\% - 1.10\% - 0.05\% = 2.54\%$

Table 10.3 Attribution with futures (preferred alternative presentation)

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation (%)	Futures mismatch (%)	Stock selection (%)	Futures timing (%)
UK equities	45	50	20.0	10.0	−0.08	0.00	3.71	−0.02
US equities	40	40	6.0	8.0	0.00	−0.07	−1.02	−0.01
Cash	15	10	1.4	1.5	−0.32	−0.04	0.00	0.00
Total	*100*	*100*	*10.66*	*8.35*	*−0.39*	*−0.12*	*2.69*	*−0.03*

process the impact of futures is separated from stock selection. Stock selection for the physical assets is the same as that in Exhibit 10.3, but there are two new factors:

(1) Futures timing – this represents the timing effect, trading or difference between fair value of the futures contract and the selection effect calculated in Exhibit 10.3.
(2) Futures mismatch – this represents the difference between the futures index contract and the underlying benchmark and is calculated in Exhibit 10.7.

Exhibit 10.7 Futures mismatch

UK futures	$5\% \times \left(\frac{1.099}{1.10} - 1\right) \times \frac{1.10}{1.078} = 0.00\%$
US equities	$-15\% \times \left(\frac{1.085}{1.08} - 1\right) \times \frac{1.08}{1.078} = -0.07\%$
UK economic cash	$-5\% \times \left(\frac{1.015}{1.015} - 1\right) \times \frac{1.015}{1.078} = 0.00\%$
US economic cash	$15\% \times \left(\frac{1.012}{1.015} - 1\right) \times \frac{1.015}{1.078} = -0.04\%$
Total	$0.0\% - 0.07\% + 0.0\% - 0.04\% = -0.12\%$

The asset allocation effects, of course, include both physical and futures notional assets. The futures timing loss of 3 basis points is simply slight underperformance of the UK futures contract against its own index and a further loss of 1 basis point because the short US futures contract slightly outperformed. The 7 basis points loss due to index mismatch in the short US futures position was because the futures contract index outperformed the underlying benchmark index. The 4 basis points loss in cash was due to the fact that the net cash position was exposed to US $ Libor with a lower return than sterling Libor.

Leverage (or gearing)

Leverage is created if the total value of physical and economic assets (excluding notional cash) is greater than the total value of the portfolio. The amount of leverage is calculated as follows:

$$\text{Leverage} = \frac{\text{Total value of economic assets}}{\text{Total portfolio value}} - 1 \tag{10.1}$$

One is subtracted from Equation (10.1) because the natural exposure of the portfolio is 100%. In effect, leverage measures the degree to which an investor is utilising borrowed money.

The opposite of leverage is damping, representing the total amount of cash, notional cash and cash equivalent instruments relative to the total portfolio value:

$$\text{Damping} = \frac{\text{Total value of cash assets}}{\text{Total portfolio value}} \tag{10.2}$$

The representation of leverage in attribution analysis very much depends on the investment decision process. From the perspective of a UK investment trust it is often the trust board that takes decisions with regard to leverage (normally implemented using loans to buy further assets) while the management company takes the remaining investment decisions. This decision process can be replicated as a series of notional funds as shown in Exhibit 10.8:

Exhibit 10.8 Investment trust attribution (including gearing)

Benchmark		**+10.0%**
Stock selection	*+1.7*	
Asset allocation	*−0.5%*	
Currency	*−0.3%*	
Gross assets		**+11.0%**
Gearing	*+1.3%*	
Total assets (before fees)		**+12.4%**
Fees, expenses	*−0.4%*	
Total assets (net of fees)		**+12.0%**
Discount, other corporate	*+1.2%*	
Client performance		**+13.4**

The difference between each step in the decision process by definition defines the contribution for that factor. For example, the contribution from gearing is simply the geometric difference between the total asset performance including gearing and gross assets excluding gearing:

$$\frac{1.124}{1.11} - 1 = 1.3\%$$

Alternatively, leverage decisions may be part of the natural investment decision process at category or sector level. An alternative demonstration of leverage is shown in Exhibits 10.9, 10.10 amd 10.11 using revised data from Table 10.4.

Exhibit 10.9 Return calculations including leverage

Using the data from Table 10.4 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$$r = 150\% \times 20\% + 50\% \times -5\% + 100\% \times 6\% - 200\% \times 3\% = 27.50\%$$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$$b = 40\% \times 10\% + 20\% \times -4\% + 40\% \times 8\% = 6.4\%$$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$$b_S = 150\% \times 10\% + 50\% \times -4\% + 100\% \times 8\% - 200\% \times 3.0\% = 15.0\%$$

Exhibit 10.10 Asset allocation with leverage

Geometric asset allocation for data in Table 10.4 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.15}{1.064} - 1 = 8.08\%$$

Individual country asset allocation effects are:

UK equities	$(150\% - 40\%) \times \left(\frac{1.10}{1.064} - 1\right) = 3.72\%$
Japanese equities	$(50\% - 20\%) \times \left(\frac{0.96}{1.064} - 1\right) = -2.93\%$
US equities	$(100\% - 40\%) \times \left(\frac{1.08}{1.064} - 1\right) = 0.90\%$
Cash	$(-200\% - 0\%) \times \left(\frac{1.015}{1.064} - 1\right) = 6.39\%$
Total	$3.72\% - 2.93\% + 0.9\% + 6.39\% = 8.08\%$

Note that total leverage is $\frac{300\%}{100\%} - 1 = 200\%$

Exhibit 10.10 demonstrates that apart from using leverage to increase the weight in UK and US equities and generate positive asset allocation, additional value is added from the short position in cash or leverage of 200%. Stock selection in Exhibit 10.11 is also magnified by virtue of increased weights in outperforming categories (UK equities) and underperforming categories (US equities). Therefore value is added both from the leverage itself and by increased exposure to the assets bought with the cash made available.

Exhibit 10.11 Stock selection with leverage

Geometric stock selection including interaction for data in Table 10.4 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.275}{1.15} - 1 = 10.87\%$$

Individual country stock selection effects are:

UK equities	$150\% \times \left(\frac{1.20}{1.10} - 1\right) \times \frac{1.10}{1.15} = 13.04\%$
Japanese equities	$50\% \times \left(\frac{0.95}{0.96} - 1\right) \times \frac{0.96}{1.15} = -0.43\%$
US equities	$100\% \times \left(\frac{1.06}{1.08} - 1\right) \times \frac{1.08}{1.15} = -1.74\%$
Cash	$-200\% \times \left(\frac{1.03}{1.03} - 1\right) \times \frac{1.02}{1.15} = 0.00\%$
Total	$13.04\% - 0.43\% - 1.74\% + 0.0\% = 10.87\%$

Table 10.4 Attribution including leverage

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	150	40	20.0	10.0	3.72	13.04
Japanese equities	50	20	−5.0	−4.0	−2.93	−0.43
US equities	100	40	6.0	8.0	0.90	−1.74
Cash	−200	0	3.0	3.0	6.39	0.00
Total	*100*	*100*	*27.5*	*6.4*	*8.08*	*10.87*

FORWARD FOREIGN EXCHANGE (FFX) CONTRACT (OR CURRENCY FORWARD)

A forward foreign exchange contract is an agreement between two parties to exchange a specified amount of one currency at a specified foreign exchange rate on a future date. The theoretical forward rate for entering into a forward foreign exchange deal is established by the current spot exchange rate and the interest rates on the two currencies involved. For example, let's look at a euro denominated portfolio that wishes to hedge £ 1m of currency exposure for one month.

£ : € spot exchange rate =1.5
Sterling interest rate 0.5% per month
Euro interest rate 0.25% per month.

£ 1 000 000 invested for one month would be worth £ 1 005 000
€ 1 500 000 invested for one month would be worth € 1 503 750

Therefore the theoretical forward rate is $1\,503\,750/1\,005\,000 = 1.49627$ lower than the current spot price; the pound is at a discount relative to the euro because interest rates are higher in the UK. The investor has in effect sold or borrowed pounds, paying a higher rate of interest than received on the euros.

Currency forwards should be valued using the current forward rate at the point of valuation.

Unlike futures contracts there is no variation margin paid during the life of the contract; any gain or loss is unrealised.

The currency return between spot rates can be broken down into the forward currency return and the forward premium (or interest rate differential).

SWAPS

A swap is a contract between two parties agreeing to make payments to each other on specified future dates (typically quarterly) over an agreed time period, where the amount that each has to pay is calculated on a different basis.

Just like forward contracts there is no variation margin and therefore the swap itself might be positively or negatively valued. Each leg of the swap can be represented by an appropriate notional economic asset, the difference between the two representing the unrealised gain or loss on the contract. In effect, a swap is a cross between a futures contract and a forward contract; unrealised gains and losses are realised at each payment date, for most but not all types of swaps.

Interest rate swaps

The most common type of interest rate swap is the fixed/floating rate swap (or plain vanilla swap); the notional is normally fixed at a set amount. One party agrees to pay a fixed interest rate based on the notional amount and the other party agrees to pay a floating rate (Libor) at set times for an agreed time period.

Clearly, the fixed interest leg behaves exactly as a bond and should be valued as such with the notional asset in the correct category, and the other leg should be included as cash.

Total return swap

Total return swaps transfer the entire economic performance of a reference asset or index, including all cash flows. They are often used to transfer the whole credit risk of an instrument, in return receiving interest-related payments.

Credit default swap

A credit derivative is a contract whose payout depends on the creditworthiness of the reference asset. In the standard structure the buyer of protection pays a periodic premium to the seller of so many basis points per annum applied to the notional value of the reference asset. If during the term of the swap one of a set number of credit events occurs such as insolvency, failure to meet payments, credit ratings downgrade, etc. then the seller must take delivery of the assets at a set price or alternatively pay cash compensation.

The credit default premium is likely to be very close to the credit spread of the reference asset between government bonds.

The notional assets in this case are the appropriate corporate bond and government bond sectors, duration adjusted if that is appropriate to the investment decision process.

Equity index swaps

Equity index swaps are very similar to equity index futures contracts, the main difference being the lack of variation margin.

In a standard equity index swap one party agrees to make periodic payments based on the change in value of an equity index plus dividend yield applied to a notional value while receiving a fixed or floating rate of interest applied to the same notional value. The notional value can be set to remain constant or change in value in line with the equity index. Note that if the index falls in value the seller might receive additional payments for the negative change in value plus interest payments.

Equity index swaps can be represented by notional assets in a similar way to equity index futures contacts with the notable exception that the notional values may not be identical the difference representing unrealised losses and gains. These gains or losses are realised at each payment date.

Contracts for difference (CFD)

A contract for difference is an agreement between two parties, buyer and seller, in which the buyer will pay the seller the difference in value of a reference asset between the current value and its value at contract time (if the value is negative the seller will pay the buyer). CFDs avoid stamp duty, provide an opportunity for leverage and are less transparent allowing the building of large virtual positions in an individual security.

OPTIONS

The buyer of a European-style option contract has the right but not the obligation to buy (call option) or sell (put option) an agreed amount of a specified asset (the underlying), at a specified price, called the exercise or strike price on a future date, called the expiry or expiration date.

European options can only be exercised at expiry, whereas American-style options (the greater majority of options written) can be exercised on any business day up to and including expiry. A Bermudian option – halfway between America and Europe – is a halfway house allowing the option to be exercised on certain days up to expiry.

The purchaser of an option pays a premium to the seller or writer of the contract. Obviously, the writer of an option contract will receive a premium but retains a liability in the form of a negatively valued asset (for example, a short call or a short put).

The performance of option contracts are measured in exactly the same way as any other asset but they also generate economic exposures. Unlike futures contracts the economic exposure of option contracts is not linear, but convex, and will change depending on how close the price of the underlying asset is to the exercise or strike price.

The total value of an option is made up of intrinsic value and time value, see Figure 10.1. The intrinsic value of the option is the value that would be realised through immediately exercising the option. If no value would be realised the intrinsic value would be zero (never negative, the option holder can never be compelled to exercise) and the option is described as “out of the money”.

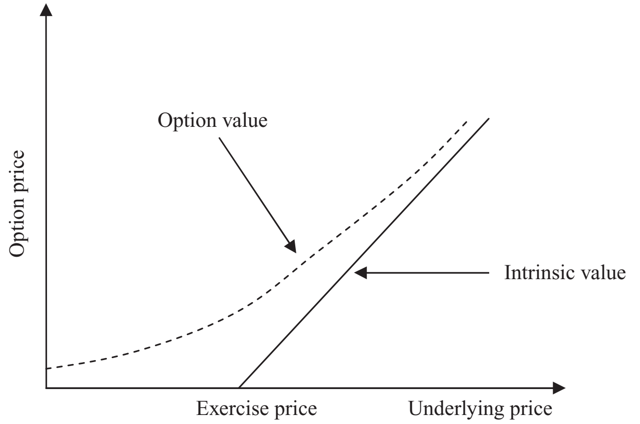

Figure 10.1 Option time value and intrinsic value

The time value reflects the possibility that the option may be "in the money" before expiry and is a function of time and volatility. Time value peaks around the exercise price; for a low underlying price well below the exercise price there is no intrinsic value and little time value. If the option is deeply in the money it is virtually certain to be exercised and time value will also be low.

Option price sensitivity (the Greeks)

Options can be valued using the Black–Scholes model which requires:

(1) The price of the underlying asset.
(2) The strike or exercise price of the option.
(3) Time to expiry.
(4) The variability (or volatility) of the underlying asset.
(5) The cost of carry.

The price sensitivities of options are measured by the "Greeks": delta, gamma, theta, vega and rho:

- *Delta* (Δ *or* δ). Delta is the change in value of an option for a small change in underlying asset price.
- *Gamma* (Γ *or* γ). Gamma measures the change in delta for a small change in underlying asset price.
- *Theta* (*theta*). Theta is the change in value of an option as time elapses, all other factors remaining constant.
- *Vega (sometimes called kappa* (κ)). Vega is the change in value of an option for a given change in volatility. Unlike the other Greeks vega is not a Greek letter.
- *Rho* (ρ). The change in value of an option for a given change in interest rates.

The impact of the change in price of an option for a positive change in each of the Greeks is summarised in Table 10.5.

Delta is the most important Greek from the point of view of performance measurement and attribution analysis. If a long call option is out of the money the delta will be less than 50% and converges to zero the more deeply out of the money it becomes. If the long call option is at the money the delta is approximately 50%; there is a 50%:50% change of the price moving either way. If the long call option is in the money the delta will be greater than 50% and converges to 100% if the option is deeply in the money, moving in line with the price of the underlying asset.

Table 10.5 Impact of positive change in Greek sensitivity

Option type	Delta	Gamma	Vega	Rho	Theta
Long call	+ve	+ve	+ve	+ve	+ve
Long put	−ve	+ve	+ve	−ve	+ve
Short call	−ve	−ve	−ve	−ve	−ve
Short put	+ve	−ve	−ve	+ve	−ve

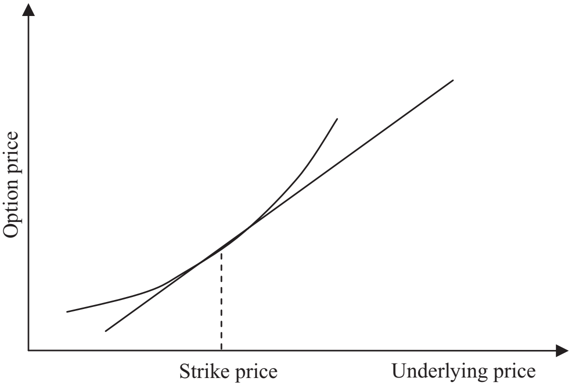

Figure 10.2 Option convexity

The delta is a first-order approximation of the change of price of the option; the actual relationship is convex as shown in Figure 10.2. The option delta is the slope of the line at a tangent to the curve at the strike price.

Delta works well for small change in value of underlying. Gamma measures the rate of change of this delta. For long calls the delta underestimates the gain in option price and overestimates the loss. For short calls and puts convexity is negative, overestimating gains and underestimating losses; clearly not a desirable situation.

WARRANTS

A warrant is a time-limited right to subscribe for shares, debentures, loan stock or government securities and is exercisable against the original issuer of the underlying securities. Some other instruments are also called warrants but are actually options; for example, a right to acquire securities, which is exercisable against someone other than the original issuer of the securities, is often called a covered warrant.

Convertible bonds

A convertible bond is a type of bond which can be converted into stock of the issuing company. In effect, a convertible is a bond with an embedded long call option and from the perspective of performance measurement and attribution analysis it is appropriate to separate the two types of exposure if consistent with the investment decision process.

Buyers of convertible bonds might be seeking both to arbitrage by selling the underlying security and to take advantage of pricing anomalies, or in a similarly way might be seeking to synthetically create the underlying security by buying the convertible but holding an appropriate amount of equitised cash to replicate the overall exposure of the desired asset at a cheaper price. This equitised cash, which may or may not be adjusted in line with the delta of the embedded option in the convertible, should be included as appropriate notional equity and notional (negative) cash assets for the purposes of attribution.

The participation rate of a convertible bond unlike delta describes the entire instrument's exposure to price changes in the underlying asset. Because much of the combined valuation is derived from the bond element it is likely that the upside participation will be greater than the downside participation – say, for example, 75% upside capture but only 65% participation on the downside.

Attribution analysis using options, warrants and convertible bonds

The key to attribution analysis with options is to include the appropriate economic exposure for which we can use the option's delta. The option itself will provide part of the valuation; the remainder of the option's economic value must be achieved again by using notional assets. The fundamentals of generating notional assets for options are laid out by Stannard (1996):

$$\begin{aligned}\text{Option economic exposure} &= \delta \times \text{number of options} \times \text{strike price} \\ &= \text{option valuation} + \text{notional exposure}\end{aligned} \tag{10.3}$$

The delta and hence the exposure is continuously changing. However, if the notional asset exposures are recalculated on a daily basis the impact of gamma can be significantly reduced.

Return calculations and attribution analysis for a portfolio with a single option plus cash are shown in Exhibits 10.12 and 10.13.

Exhibit 10.12 Performance measurement for options

Type of option	Long call
Underlying	Massive Firm Inc.
Current share price	\$ 100
Exercise price	\$ 100
Current option price	\$ 10
Option delta	50%
Share end price	\$ 102
Option end price	\$ 11

Libor for period of measurement = 0.1%

	SMV	EMV	Return (%)
Options	1000	1100	10.0
Cash	9000	9009	0.1
Total	10 000	10 109	1.09
Option economic value	0.5 × 100 × 100 = 5000		
Notional cash asset required	5000 − 1000 = 4000		

	SMV	EMV	Return (%)
Options	1000	1100	10.0
Notional exposure	4000	4004	0.1
Combined	5000	5104	2.08
Cash	9000	9009	0.1
Notional cash	(4000)	(4004)	0.1
Combined	5000	5005	0.1
Total	10 000	10 109	1.09

(Note that $\frac{5000}{10\,000} \times 2.08\% + \frac{5000}{10\,000} \times 0.1\% = 1.09\%$)

Exhibit 10.13 Attribution for options

Using data from Exhibit 10.12 and assuming a benchmark consisting of 40% exposure to the underlying asset of the option and 60% cash, attribution is calculated as follows:

Benchmark return $40\% \times 2.0\% + 60\% \times 0.1\% = 0.86\%$
Semi-notional return $50\% \times 2.0\% + 50\% \times 0.1\% = 1.05\%$

Geometric asset allocation:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.0105}{1.0086} - 1 = 0.19\%$$

Options $$(50\% - 40\%) \times \left(\frac{1.02}{1.0086} - 1\right) = 0.11\%$$

Cash $$(50\% - 60\%) \times \left(\frac{1.001}{1.0086} - 1\right) = 0.08\%$$

Geometric stock selection:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.0109}{1.0105} - 1 = 0.04\%$$

Options $$50\% \times \left(\frac{1.0208}{1.02} - 1\right) \times \frac{1.02}{1.0105} = 0.04\%$$

Cash $$50\% \times \left(\frac{1.001}{1.001} - 1\right) \times \frac{1.001}{1.0105} = 0.00\%$$

Correctly adjusting for the appropriate economic exposure the attribution analysis indicates the portfolio manager is overweight the reference asset and underweight cash, generating an added value of 19 basis points. Stock selection also made a small contribution of 4 basis points to added value; this might come from any of the Greeks other than delta – no attempt is made to identify which of the Greeks added the most value. For standard portfolios including options this type of analysis is sufficient to quantify the manager's decision process; for option-only portfolios more detailed analysis might be required.

MARKET NEUTRAL ATTRIBUTION

Market neutral funds as their name suggests are designed to be neutral against market movements. In theory, they consist of a 100% long allocation offset by a 100% short position in the same market. The resultant exposure should be 100% cash plus the excess return from stock selection on the long side and the excess return from the stock selection on the short side. Thus, good stock pickers should be able to generate double the excess return without any market exposure; a very attractive and easily understood absolute return strategy.

Attribution is relatively straightforward if the long and short parts of the portfolio are treated as separate asset categories. It is essential that the performance of underlying assets retains the appropriate sign. The profits generated by assets that are falling in value are the result of negative weights compounded with negative returns. Stock selection and asset allocation effects are calculated for a market neutral portfolio using data from Table 10.6 in Exhibits 10.15 and 10.16, respectively. Returns for the portfolio, benchmark and semi-notional fund are as before and shown in Exhibit 10.14.

Table 10.6 Market Neutral Attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
Long UK equities	30	40	20.0	10.0	−0.68	2.93
Long Japanese equities	70	60	−5.0	−4.0	−0.68	−0.68
Short UK equities	−30	−40	6.0	10.0	0.68	1.17
Short Japanese equities	−60	−60	−7.0	−4.0	0.0	1.76
Cash	90	100	3.0	3.0	0.0	0.0
Total	*100*	*100*	*7.6*	*3.0*	*−0.68*	*5.18*

Exhibit 10.14 Return calculations

Using the data from Table 10.6 the portfolio return $r = \sum_{i=1}^{i=n} w_i \times r_i$ is:

$r = 30\% \times 20\% + 70\% \times -5\% - 30\% \times 6\% - 60\% \times -7\% + 90\% \times 3\% = 7.6\%$

The benchmark return $b = \sum_{i=1}^{i=n} W_i \times b_i$ is:

$b = 40\% \times 10\% + 60\% \times -4\% - 40\% \times 10\% - 60\% \times -4\% + 100\% \times 3\% = 3.0\%$

The semi-notional return $b_S = \sum_{i=1}^{i=n} w_i \times b_i$ is:

$b_S = 30\% \times 10\% + 70\% \times -4\% - 30\% \times 10\% - 60\% \times -4\% + 90\% \times 3\% = 2.3\%$

Exhibit 10.15 Market neutral asset allocation

Geometric asset allocation for data in Table 10.6 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.023}{1.03} - 1 = -0.68\%$$

Individual country asset allocation effects are:

Long UK equities $(30\% - 40\%) \times \left(\frac{1.10}{1.03} - 1\right) = -0.68\%$

Long Japanese equities $(70\% - 60\%) \times \left(\frac{0.96}{1.03} - 1\right) = -0.68\%$

Short UK equities $(-30\% + 40\%) \times \left(\frac{1.10}{1.03} - 1\right) = 0.68\%$

Short Japanese equities $(-60\% + 60\%) \times \left(\frac{0.96}{1.03} - 1\right) = 0.0\%$

Cash $(90\% - 100\%) \times \left(\frac{1.03}{1.03} - 1\right) = 0.0\%$

Total $-0.68\% - 0.68\% + 0.68\% + 0.0\% + 0.0\% = -0.68\%$

Exhibit 10.16 Market neutral stock selection

Geometric stock selection including interaction for data in Table 10.6 is:

$$\frac{(1+r)}{(1+b_S)}-1=\frac{1.076}{1.023}-1=5.18\%$$

Individual country stock selection effects are:

Long UK equities	$30\% \times \left(\frac{1.20}{1.10}-1\right)\times\frac{1.10}{1.023}=2.93\%$
Long Japanese equities	$70\% \times \left(\frac{0.95}{0.96}-1\right)\times\frac{0.96}{1.023}=-0.68\%$
Short UK equities	$-30\% \times \left(\frac{1.06}{1.10}-1\right)\times\frac{1.10}{1.023}=1.17\%$
Short Japanese equities	$-60\% \times \left(\frac{0.93}{0.96}-1\right)\times\frac{0.96}{1.023}=1.76\%$
Cash	$90\% \times \left(\frac{1.03}{1.03}-1\right)\times\frac{1.03}{1.023}=0.0\%$
Total	$2.93\% - 0.68\% + 1.17\% + 1.76\% + 0.0\% = 5.18\%$

Interpreting the attribution results the long and short UK positions cancel out leaving a small negative allocation of −0.68% from a net 10% long position in Japanese equities. Asset allocation from cash will always be zero regardless of the weight since the overall benchmark return is also a cash benchmark. The excess return from this portfolio is derived from stock selection, positive on long UK equities and negative on long Japanese equities. In the short UK equity portion of the portfolio, although generating a positive return the portfolio manager has selected stocks that have underperformed the benchmark resulting in a positive contribution. In the short Japanese portfolio the manager again selected stocks that underperformed the benchmark which when combined with the negative weight generated a positive contribution. The stock selection from cash is unsurprisingly neutral; stock lending fees which are suffered in order to generate the short positions can be charged either to the cash sector or to the appropriate short equity sector as negative income.

Further variations of the market neutral strategies overlay the entire portfolio with index futures of entirely different markets thus generating the desired market exposure while trapping the excess return or alpha from different underlying marks where perhaps the portfolio manager retains more expertise.

Attribution for 130/30 funds (or extended short funds)

130/30 funds (including other weighting variations, e.g. 120/20, etc.) are a more recent variation of market neutral funds with a 130% short position offset by a 130% long position, thus retaining market exposure but allowing the manager to short a limited percentage of assets.

Attribution for a 130/30 fund is shown in Exhibits 10.17 and 10.18 based on data from Table 10.7.

Exhibit 10.17 130/30 fund asset allocation

Geometric asset allocation for data in Table 10.7 is:

$$\frac{(1+b_S)}{(1+b)} - 1 = \frac{1.093}{1.1} - 1 = -0.64\%$$

Individual country asset allocation effects are:

UK equities	$(120\% - 130\%) \times \left(\frac{1.10}{1.10} - 1\right) = 0.00\%$
Short UK equities	$(-30\% + 30\%) \times \left(\frac{1.10}{1.10} - 1\right) = 0.00\%$
Cash	$(10\% - 0\%) \times \left(\frac{1.03}{1.10} - 1\right) = -0.64\%$
Total	$0.0\% + 0.0\% - 0.64\% = -0.64\%$

Exhibit 10.18 130/30 fund stock selection

Geometric stock selection including interaction for data in Table 10.7 is:

$$\frac{(1+r)}{(1+b_S)} - 1 = \frac{1.144}{1.93} - 1 = 4.67\%$$

Individual country stock selection effects are:

UK equities	$120\% \times \left(\frac{1.14}{1.10} - 1\right) \times \frac{1.10}{1.093} = +4.39\%$
Short UK equities	$-30\% \times \left(\frac{1.09}{1.10} - 1\right) \times \frac{1.10}{1.093} = +0.27\%$
Cash	$10\% \times \left(\frac{1.03}{1.03} - 1\right) \times \frac{1.03}{1.093} = 0.0\%$
Total	$4.39\% + 0.27\% + 0.0\% = 4.67\%$

Table 10.7 130/30 fund attribution

	Portfolio weight w_i (%)	Benchmark weight W_i (%)	Portfolio return r_i (%)	Benchmark return b_i (%)	Asset allocation $(w_i - W_i) \times \left(\frac{1+b_i}{1+b} - 1\right)$ (%)	Stock selection $w_i \times \left(\frac{1+r_i}{1+b_i} - 1\right) \times \frac{(1+b_i)}{(1+b_S)}$ (%)
UK equities	120	130	14.0	10.0	0.00	4.39
Short UK equities	−30	−30	9.0	10.0	0.00	0.27
Cash	10	0	3.0	3.0	−0.64	0.00
Total	*100*	*100*	*14.4*	*10.0*	*−0.64*	*4.67*

It’s worth noting in this attribution that since the UK equity benchmark is equal to the overall benchmark there are no asset allocation effects in the long or short positions, only from any allocation to cash; in this case an overweight position leading to a negative allocation of −0.64%. However, outperformance in the long sector is leveraged by 20% with a 120% allocation and there is an additional contribution because the shorted stocks underperformed the benchmark. Notice that the benchmark retained 100% allocation to UK equities.

11

Performance Presentation Standards

There are three ways of losing money – horses, women and taking the advice of experts,

Horses – that is the quickest
women – that is the most pleasant; but
taking the advice of experts – that is the most certain.

Apocryphally attributed to M. Pompidou

WHY DO WE NEED PERFORMANCE PRESENTATION STANDARDS?

The CFA Institute (formally known as the Association for Investment Management and Research or AIMR) sponsored the creation of the Global Investment Performance Standards (GIPS, 2005) to provide an ethical framework for the calculation and presentation of the performance history (or track record) of asset management firms. GIPS are voluntary standards based on the fundamental principles of full disclosure and fair representation of performance returns.

The need for the standards first became apparent in the United States in the mid-1980s. Pension funds, in seeking firms to manage their assets, would obviously see a large number of presentations from asset managers, the overwhelming majority of which presented above average performance. This begged the question: "Where are the below average managers?" The answer, unfortunately, was that some of the below average managers were presenting above average returns.

Asset managers were very selective about the investment track records they presented to potential clients. Most marketing managers would be well aware over which time period they performed best and would consequently "cherry pick" the best period to show performance. Often single representative accounts would be used to calculate the firm's track record. Invariably, the representative account would be one of the better performing accounts for that investment strategy. If the representative account performed badly a rationale would be found to choose a new one. Managers might also be selective in the choice of calculation methodology as demonstrated in Chapter 2. Often the performance track records presented by assets managers were not a fair and honest representation of the performance they had delivered to their existing clients.

The result was the creation of the AIMR Performance Presentation Standards (1987) (AIMR-PPS), voluntary performance guidelines for the North American market. In the United Kingdom the National Association of Pension Funds (NAPF, 1992) produced its own guidelines for balanced pension funds encouraged by one of the principal recommendations of a Committee of Enquiry into Investment Performance Measurement also initiated by the NAPF (1990). My interest in the standards developed at this time. I wished to bring my firm, a London-based subsidiary of a large US bank, into compliance with both the AIMR-PPS and the NAPF guidelines, and it proved impossible to achieve both. The AIMR-PPS standards and the NAPF guidelines shared the same ethical objectives yet in certain regards they were contradictory.

With responsibility for a variety of European offices I could envisage the nightmare scenario of a separate set of contradictory standards in each European office. It was clear a set of global standards was required.

GLOBAL INVESTMENT PERFORMANCE STANDARDS (GIPS®)

In 1995 the CFA Institute sponsored and funded the Global Investment Performance Standards (GIPS®) Committee to develop a single standard for presenting investment performance. The objectives the committee set were to:

(1) Obtain worldwide acceptance of a standard for the calculation and presentation of investment performance in a fair, comparable format that provides full disclosure.
(2) Ensure accurate and consistent investment data for reporting, record keeping, marketing and presentation.
(3) Promote fair, global competition among investment firms for all markets without creating barriers to entry for new firms.
(4) Foster the notion of industry self-regulation on a global basis.

In a relatively short period this committee finalised the GIPS standards in February 1999 and, having met its objectives, disbanded.

The GIPS committee was replaced by the Investment Performance Council (IPC) whose principal goal was to have all countries adopt the GIPS standard.

To facilitate convergence the IPC promoted a transition from existing local standards to GIPS by approving Country Versions of GIPS (CVGs) while updating the GIPS standards to incorporate local best practice. The revised single standard was issued in February 2005 with an effective start date of 1 January 2006.

This latest version of GIPS includes a number of improvements plus new sections on real estate and private equity and effectively removes the need for CVGs. In 2006 the IPC was transformed into the GIPS Executive Committee which now serves as the effective decision-making authority for the standards. The GIPS Executive Committee has created four permanent standing subcommittees:

(1) GIPS Council.
(2) Interpretation subcommittee.
(3) Practitioners/Verifiers subcommittee.
(4) Investors/Consultants subcommittee.

The GIPS Council consists of country sponsors that have adopted the standards and provides four members of the GIPS executive committee.

As at 30 November 2007 28 countries including all the major capital markets have adopted GIPS, a full list is included in Table 11.1.

All subcommittee proposals are circulated for public comment (normally 60 days) prior to their adoption and incorporation into the standards.

ADVANTAGES FOR ASSET MANAGERS

The advantages of a global standard for clients are obvious; they can select asset managers based on good quality information with a certain level of confidence that the numbers presented are a fair and honest representation of that firm's track record. For their own protection pension fund

Table 11.1 GIPS Council country sponsors

Country	Sponsoring organization
Australia	Investment and Financial Services Association – Performance Analyst Group
Austria	(1) Österreichischen Verreinigung für Finanzanalyse und Asset Management and (2) Vereinigung österreichischer Investmentgesellschaften
Denmark	The Danish Society of Financial Analysts and CFA Denmark
Belgium	Belgium Asset Managers Association
France	French Experts Group on Performance Presentation Standards sponsored by (1) Société Française des Analystes Financiers and (2) Association Française de la Gestion Financière
Germany	German Asset Management Standards Committee: (1) BVI Bundesverband Investment und Asset Management e. V, (2) DVFA Deutsche Vereinigung für Finanzanalys und Asset Management and (3) German CFA Society
Hong Kong	The Hong Kong Society of Financial Analysts Limited
Hungary	CFA Society of Hungary
Ireland	Irish Association of Investment Managers
Italy	Italian Investment Performance Committee sponsored by: (1) L'Associazione Bancaria Italiana, (2) L'Associazione Italiana degli Analisti Finanziari, (3) Assogegtioni, (4) Sviluppo Mercato Fondi Pensione, (5) Assirevi, (6) Italian CFA Society
Japan	The Security Analysts Association of Japan
Kazakhstan	Association of Financial and Investment Analysts
Korea	CFA Korea Society
Liechtenstein	Liechtenstein Bankers Association
Micronesian Region	Asia Pacific Association for Fiduciary Studies
Netherlands	VBA – Beroepsvereniging van Beleggingsprofessionals
New Zealand	CFA Society of New Zealand
Norway	The Norwegian Society of Financial Analysts
Pakistan	CFA Association of Pakistan
Portugal	Associação Portuguesa de Analista Financeiros
Singapore	Investment Management Association of Singapore
South Africa	Investment Management Association of South Africa
Spain	Asociación Española de Presentación de Resultados de Gestión
Sweden	Swedish Society of Financial Analysts
Switzerland	Swiss Bankers Association
Ukraine	The Ukrainian Association of Investment Business
United Kingdom	UK Investment Performance Committee sponsored by: (1) Investment Management Association (IMA), (2) Association of British Insurers, and (3) The National Association of Pension Funds (NAPF)
United States and Canada	CFA Institute through its North American Investment Performance Committee

trustees should only hire asset managers that are compliant with the standards. Non-compliance with the standards may suggest a weaker commitment to ethical standards or weak internal controls insufficient to claim compliance. If performance measurement controls are not best practice, this may be an indication that other controls within the firm are weak.

For asset managers the advantages are less obvious and of course there is the cost of compliance to be offset; however, in my view the following advantages significantly outweigh the costs of compliance:

(1) *Marketing advantage*. Clearly, in the early stages of a standard an asset manager can gain a marketing advantage by claiming compliance with a good quality standard. In the US it has now become a marketing disadvantage not to be compliant, a situation that will arise in Europe at some stage.

    Many pension funds will not welcome firms into the selection process if they are not compliant and in some circumstances if their claim of compliance is not independently verified. Any trustee is taking the risk of future criticism or legal action by selecting an asset manager that does not comply to performance presentation standards if subsequently things go wrong and it is established that the original presentation was misleading.

(2) *Level playing field – international passport*. The standards are designed to encourage global competition and eliminate barriers to entry. In effect, GIPS allows asset managers to market their track record worldwide with the knowledge that they are presenting to the same standards as local competitors.

    The pressure to present misrepresentative performance is not so great if asset managers are confident that their competitors are operating to the same standard.

(3) *Increased professionalism*. To achieve compliance a firm must have good quality performance measurement processes and procedures in place and a commitment to the ethical presentation of performance track records. This naturally increases the profile and importance of performance measurers in the firm.

(4) *Risk control*. The standards require a basic level of risk control. Managers are required to investigate outliers in their track record to justify that accounts are being managed within the composite guidelines. This is good business practice, identifying poor performing accounts early, ensuring good performance is real and ensuring the entire firm is aware and understands the investment objectives and guidelines for each account.

(5) *Business efficiency and data quality*. Establishing effective procedures and improving performance measurement systems obviously requires investment; however, doing things right first time is obviously more efficient than calculating performance incorrectly and wasting time and resources investigating and correcting errors. Clearly, it's very inefficient if individual portfolio managers are wasting their time ensuring returns are calculated correctly.

THE STANDARDS

GIPS are ethical standards for investment performance presentation to ensure fair representation and full disclosure of a firm's performance track record. The core of the standard is reproduced in Appendix E.

Composites

GIPS require firms to include all discretionary portfolios in at least one "composite", defined according to similar style or investment strategy. A composite should be representative of the firm's performance with that investment strategy. All accounts managed to that strategy including lost accounts must be included thus eliminating the practice of "cherry picking" good performing accounts.

Initially, firms have complete flexibility to define their own composites. This flexibility allows firms to differentiate their product offering and encourages the development of new products.

The firm must decide between narrowly or widely defined composites. Widely defined composites will include minor variations in strategy (for example, the firm might conclude that the return series of a global equity account with a restriction disallowing investment in Australia is very similar to an unrestricted global equity account and therefore both accounts could coexist in a widely defined global equity composite). Widely defined composites are easier to administer and allow the asset manager to present composites with larger assets under management. Narrowly defined composites are smaller and because small changes of strategy must be closely monitored they are more difficult to administer; however, the dispersion of returns within the composite will be narrower indicating tighter investment controls.

Additional helpful guidance can be found in the Composite Construction Guidance Statement reproduced in Appendix F.

Presentation

GIPS require at least 5 years of performance history initially, presented annually, increasing to 10 years as the data becomes available. This avoids the "cherry picking" of time periods and provides some information about the consistency of performance. I would recommend the presentation of quarterly performance information although this is not required. The standards also require a number of disclosures designed to set the performance returns in their proper context.

By kind permission of GAM a typical GIPS performance presentation is shown in Appendix G.

Calculation

Some flexibility in the choice of calculation method is allowed but from 1 January 2001 portfolios must be valued at least monthly. From 1 January 2005 day-weighted cash flow (modified Dietz) methods must be used and from 1 January 2010 portfolios must be revalued and the performance chain linked in the advent of a large cash flow. The firm retains the flexibility to define what constitutes a large cash flow. Returns can only be regarded as true time-weighted if the portfolio is revalued for each cash flow no matter how small. My strong recommendation is to use true time-weighted returns to allow a fair comparison against other managers and benchmarks.

To create composites returns for use in presentations, individual portfolio returns are combined using one of three asset-weighted methodologies:

(1) Beginning period weights.
(2) Beginning period weight plus cash flows (in effect the denominator of the modified Dietz method).
(3) Total aggregation (basically combining the assets of all the qualifying portfolios and calculating the return as a single portfolio).

These methodologies will lead to different results but there is no structural bias and all are acceptable. For simplicity I would recommend beginning period weights. Clearly, larger portfolios have a more significant influence on composite returns. Equal-weighted composites were initially considered by the standard setters but declined because of the potential to “game” results in small portfolios.

Composites must be asset-weighted at least quarterly from 1 January 2005 and monthly from 1 January 2010. Monthly weighting is recommended.

More details are available in the Calculation Methodology Guidance Statement reproduced in Appendix H.

Claim of compliance

Once a firm has met all of the required elements of GIPS the firm may claim compliance. It is the firm that claims compliance; the claim is not specific to an individual presentation, composite or client type.

Firms cannot pick and choose to whom they present compliant presentations. GIPS are ethical standards, and in making a claim of compliance the firm is stating that all its performance presentations are a fair and honest representation of historic performance.

Structure of the standards

The standards are divided into eight sections:

(1) *Fundamentals of compliance*. The first section deals with the fundamentals of compliance, definition of the firm, documenting the firm’s policies and procedures, maintaining compliance with updates to the standards and making a correct claim of compliance.

Firms must be defined as an investment firm, subsidiary, or division held out to clients as a distinct business entity, thus larger firms may include a number of GIPS defined firms although these firms should not overlap.

The firm’s policies and procedures with reference to the standards and performance measurement are key documents required to achieve and maintain compliance.

(2) *Input data*. The Standards provide a blueprint for the consistency of input data crucial for effective compliance and full and fair comparisons of investment performance.

(3) *Calculation methodology*. The standards mandate the use of certain calculation methodologies utilising the time-weighted approach. True time weighting and linked modified Dietz are the two most common acceptable methodologies. Approximations using benchmarks such as analyst’s test, index substitution and the regression method are not acceptable. Internal rates of return are only acceptable for venture capital/private equity. Valuations will be required at the time of each cash flow from 1 January 2010; in effect true time-weighted returns.

Time-weighted returns are favoured in GIPS because of the need for comparability. For fair comparison the impact of cash flows must be removed. Requiring valuations at the point of cash flow increases the theoretical level of accuracy and removes the opportunity to game returns by self-selecting the approximate methodology most advantageously impacted by cash flow. At present the standards require firms to adopt a policy for the treatment of external cash flow. For example, for managers using linked monthly modified Dietz as standard, if the cash flow is above a certain level, say 10% of portfolio assets, then managers are required to change to a true time-weighted return by valuing the assets at the point of cash flow and chain linking the subperiod returns within the month.

(4) *Composite construction.* A composite is an aggregation of a number of portfolios into a single group that represents a particular investment strategy or objective.

Composite returns are asset-weighted using beginning period weights, beginning period weights plus day-weighted cash flows or simple aggregation. Equal weighting would allow smaller portfolios (more easily manipulated) to disproportionably impact the performance of the composite.

Appropriate documentation such as the investment management agreement or other communication with the client must support the inclusion of any portfolio in a composite. Every portfolio must belong to at least one composite (to avoid the performance record of a poor performing account being lost), therefore composite definitions may overlap.

(5) *Disclosures*. Disclosures allow firms to provide more information relevant to the performance presentation. The standards include both required and recommended disclosures. If in doubt the asset managers should add disclosure to assist the user of the performance presentation.

(6) *Presentation and reporting*. Finally, after gathering input data, calculating returns, constructing composites and determining appropriate disclosures firms must present data within the GIPS guidelines.

The standards are not so explicit at present but I would recommend all clients be provided with a compliant presentation initially even if the client is not concerned by the standards. A firm cannot pick and choose if it is compliant; the claim of compliance should mean that all performance presentations are a fair and honest representation of performance.

(7) *Real estate*. The 1999 version of the standards did not include specific requirements for real estate and private equity/venture capital asset categories. The 2005 version includes real estate and private equity.

(8) *Private equity*. In addition to extensive additional disclosures and requirements private equity is the only asset category allowed to use an internal rate of return methodology in recognition of the lack of appropriate valuations at the time of external cash flows.

VERIFICATION

Verification is the review of the firm's performance measurement processes and procedures by an independent third party or "verifier".

Verification tests:

(1) Whether the firm has complied with all the composite construction requirements firm wide.
(2) Whether the firm's processes and procedures are designed to calculate and present performance in compliance with the GIPS standards.

Verification is not yet mandatory but is strongly encouraged. Verification not only brings credibility to the claim of compliance but goes a long way to improve the performance measurement process and provides assurance to the board that their claim of compliance is accurate.

Compliance is non-trivial; claiming compliance without verification is high risk. An erroneous claim of compliance could cause both significant reputational damage and major problems with regulators. I would recommend that verification be undertaken at least annually.

The cost of verification will be determined not only by the number of composites and portfolios but also by the complexity of the business, the perceived quality of controls and the quality of performance measurement systems. Verification can cost from as little as a few thousand dollars to hundreds of thousands of dollars for large, complex businesses.

Verifiers need only be independent of the asset manager, and have a good understanding of the standards and relevant practical experience. The asset manager must consider the quality of the verifier not only in terms of attaching their name to performance presentations but also in the assurance given to the firm that the claim is accurate and the effectiveness of the verification process.

I see no conflict of interest in verification firms providing pre-compliance consultancy; I would certainly recommend that firms bring verifiers into the initial compliance process at a very early stage.

Verification/practitioners subcommittee

The verification/practitioners subcommittee is composed of third-party service providers, including verifiers, software developers and custodians. They are in an excellent position to identify areas of difficulty in the standards, misinterpretation (deliberate or otherwise) and issues that add little value yet are expensive and troublesome to comply with.

Global verification firms in particular are in a good position to ensure that there is no divergence of practice worldwide and that the claim of compliance in one country means much the same as the claim of compliance in another. Verifiers are able to identify areas in which many firms are struggling to comply with the standards; if these areas do not add much value then there is the opportunity to change the standards to encourage maximum uptake.

The writers of the standards face a constant dilemma between not making the standards too onerous, encouraging firms to participate and providing sufficient protection for the users of the standards.

INTERPRETATIONS SUBCOMMITTEE

On publication the standards were well received; however, a standard can only be successful in the long run if it is promoted and if it has the ability to respond to changes in market practice, correct errors in the standards and to provide interpretation were required.

The interpretation subcommittee has the responsibility of ensuring the integrity, consistency and applicability of the standards; in effect, it is the safety valve of standards. Errors, issues of interpretation or responses to new developments or market trends can be addressed by issuing "guidance statements".

Guidance statements

Guidance statements are formal additions to the standards. Asset management firms and verifiers are required to understand their content and keep up to date with the standards. The CFA Institute provides an e-mail alert facility providing notification of new guidance statements (standards@cfainstitute.org). Information on GIPS, the GIPS Executive Committee and GIPS Council, guidance statements and how to subscribe for the e-mail alert can be found on the CFA Institute's website at www.cfainstitute.org/standards. A selection of the more useful guidance statements are discussed in more detail below.

Definition of firm

The standards require firm-wide compliance to ensure poor performing accounts have not been excluded from the performance track record. Once the firm has been defined the exercise of allocating accounts to composites can begin – it determines the universe of portfolios to be allocated.

The firm definition must be meaningful, rational and fair. The definition cannot be used narrowly to exclude poor performing product areas. The standards recommend the broadest, most meaningful definition.

A firm may be defined as an investment firm, subsidiary, or division held out to clients or potential clients as a distinct business unit; or up to 1 January 2005 only, all assets managed to one or more base currencies.

The currency option is a throwback to the original AIMR-PPS standards. UK firms keen to participate in the AIMR standards successfully argued that they need only bring their US$ assets into compliance. This option is not available after 1 January 2005.

Although the broadest definition is recommended, it is acceptable to define a number of firms within the same organization provided they meet the above criteria, with a view to combining into one firm at a later date. Not all firms within a single organisation need be compliant simultaneously, allowing part of the organisation time to work on bringing its firm into compliance. This flexibility is often used geographically although it should never be used to exclude an underperforming part of the business.

The Definition of Firm Guidance Statement can be found in Appendix I.

Carve-outs

A carve-out is a subset of a portfolio's assets used to create a track record for a narrower mandate from a portfolio managed to a broader mandate. Carve-outs are permitted so that firms that manage assets to a particular strategy in a broader portfolio can demonstrate competency in that strategy even though they do not manage standalone portfolios in that strategy.

By their very nature carve-out returns offer greater potential to mislead than standalone portfolios. Carve-outs by definition are portions of a larger portfolio the criteria for which are determined by the firm. Because cash tends to act as a drag on performance (over the long term we would expect markets to outperform cash) if cash is not included in the calculation of the carve-out, the return may not be representative of what would have been achieved by a standalone portfolio. The standards require that cash is allocated consistently to carve-outs and at some future point will require that carve-outs be managed with their own cash balance.

Cash is one problem, there are others:

(1) *Concentration*. Because carve-outs are parts of larger portfolios they tend to contain a smaller number of securities than a standalone portfolio and consequently are potentially riskier.
(2) *Currency*. If the larger portfolio contains a currency overlay strategy it is very difficult, if not impossible, to isolate the currency allocation of the carve-out.
(3) *Asset allocation*. Within the larger portfolio, asset allocation "bets" are taken within the context of the overall benchmark not the implied benchmark of the carve-out. In other words, the carve-out may not be managed in the same way as a standalone portfolio.
(4) *Bet size*. Bet sizes are exaggerated with carve-outs – particularly if the carve-out is a small percentage of the overall strategy.

(5) *Composite administration*. Allocation of portfolios to composites becomes significantly more difficult and expensive. For standalone portfolios it is easy to identify the number of accounts and changes to investment guidelines. However, if carve-outs are used then all carve-outs managed to that strategy must be allocated to that composite – the firm must demonstrate that all carve-out strategies are included and that procedures are in place to identify changes to carve-out strategies within larger portfolios.

I would strongly recommend that firms avoid the use of carve-outs and if absolutely required only use if standalone portfolios are not available. I believe it is very difficult to demonstrate that the performance of a carve-out is representative of standalone performance. The Carve-outs Guidance Statement can be found in Appendix J.

Significant cash flows

Significant cash flows, not to be confused with large cash flows which are concerned with performance measurement calculations, are external cash flows of sufficient size to cause the portfolio not to be managed, for a temporary period, consistent with the investment objective of the composite.

Given certain market conditions portfolio managers may not wish to invest new money quickly particularly if they expect the market to fall. This may result in the portfolio not behaving in the same way as other existing portfolios in the composite, therefore it is logical to remove such portfolios from the composite. If the portfolio manager's timing decisions are correct there will be a positive bias to the returns. The Significant Cash Flow Guidance Statement, Appendix K, allows portfolios to be temporarily withdrawn. This is a sensible addition to the standards; prior to 1 July 2002 this activity was specifically not allowed in the US CVG because of the fear of the standard setters that such flexibility was too easy to game, with the result that it caused much confusion within the US.

Portability

The performance track record belongs to the firm not an individual. The standards take the view that performance is generated by many factors (e.g. the support and guidance of senior management, the research function, the dealing department, feedback from colleagues, the performance team, the asset allocation committee, etc.) and therefore all the drivers of performance are not portable. The portfolio manager may be the major contributor, but could the same performance have been delivered by that manager in a different environment?

In most cases the performance results of a prior firm cannot be used to represent the historical record of a new affiliation or a new firm. Performance information of a prior firm can be shown as supplemental information.

Prior performance can be linked with the performance of the new firm if all the following conditions apply:

(1) Substantially all the investment decision makers are employed by the new firm.
(2) The staff and decision-making process remain intact.
(3) The new firm discloses that the performance results from the old firm are linked with results from the new firm.
(4) The new firm has records that document and support the reported performance.

(5) And with regard to a specific composite, substantially all the assets from the original firm's composite transfer to the new firm.

The standard is written to ensure that portability is difficult to achieve and is most likely to occur in the event of a merger or acquisition. The guidance statement is reproduced in Appendix L.

Supplemental information

Supplemental information is defined as any performance-related information included as part of a compliant performance presentation that supplements or enhances the required/recommended disclosure and presentation provisions of GIPS.

Supplemental information is a powerful aid for firms that want to enhance the quality of their presentation by providing more information. Supplemental information must satisfy the spirit and principles of GIPS; it must not contradict a compliant presentation and must be clearly labelled as supplemental since it is not covered by verification.

Examples of supplemental information include attribution, *ex ante* risk analysis of a representative account and risk-adjusted performance.

Supplemental information must not be used to bypass the GIPS presentation standards, although the standards in no way restrict any information being presented that is specifically requested by the client. Given the opportunity to present potentially misleading supplemental information asset managers should take great care with its use – the guidance statement can be found in Appendix M.

Error correction

At the time of writing not all guidance statements had made their journey through the public consultation and revision process, including the guidance statement on error correction.

Errors will occur, including but not restricted to:

(1) Valuation errors.
(2) Missed trades.
(3) Incorrectly timed cash flows.
(4) Incorrect allocation of portfolios to composites.
(5) Missed or incorrect disclosures.
(6) Misprints.

Errors do not necessarily threaten the firm's claim of compliance but they must be handled correctly. Firms must decide whether previously reported returns should be recalculated and whether or not clients should be informed of the error.

The draft guidance statement encourages the development of:

(1) Written policy and procedures for handling errors.
(2) A definition of materiality.
(3) An error correction process.

Clearly, materiality is an issue; no value is gained reissuing presentations if the error amounts to one or two basis points but one or two whole percentage points might be a different matter. Firms are free to define their own levels of materiality but are required to make available their policies and procedures on error correction on request.

The error correction process should:

(1) Recalculate returns.
(2) Determine if the error is material.
(3) Decide what action to take.
(4) Document the original presentation, the corrected presentation and the action taken.

MEASURES OF DISPERSION

In reviewing the performance of a composite it's not only the asset-weighted returns that are of interest but also the spread of returns within the composite, otherwise known as dispersion or internal risk. A wide dispersion of returns might indicate weak investment controls within the firm.

The standards do not mandate a particular measure of dispersion and quite a few are available.

Equal-weighted standard deviation

$$S_D = \sqrt{\frac{\sum \left(R_i - \bar{R}\right)^2}{n}} \tag{11.1}$$

where: R_i = is the return on the ith portfolio
n = is the number of portfolios.

Asset-weighted dispersion

$$D = \sqrt{\sum W_i \times (R_i - R)^2} \tag{11.2}$$

where: W_i = weight of the ith portfolio in the composite
R = composite return.

High–low

High–low is simply the highest and lowest returns within the composite for the period under measurement. GIPS require a measure of dispersion for each year, therefore only the returns of portfolios for the entire annual period should be included.

Interquartile range

Interquartile range is the difference between the 1st and 3rd quartiles (25th and 75th percentiles).

Both high–low and equal-weighted standard deviation are common measures. High–low is too aggressive for me; either of the returns high or low might be outliers and hence not provide a true measure of dispersion; interquartile range is a good alternative to high–low.

Dispersion measures are only relevant if there are sufficient portfolios in the composite; for five or fewer portfolios a dispersion measure need not be calculated.

ACHIEVING COMPLIANCE

Achieving compliance is a non-trivial exercise. Performance measurers alone cannot achieve compliance; senior management must buy in to the exercise from the start.

To increase the chance of success I would recommend establishing a steering committee chaired by the project sponsor and tasked with monitoring progress, allocating resources, ensuring cooperation within the firm and addressing specific issues.

A sound project plan is absolutely essential. GIPS compliance projects can easily drift, and for a relatively complex business at least one year should be allowed. A new, relatively simple business may achieve compliance in a short period but 6 months would be a very aggressive target for most businesses. Allow plenty of contingency in the project plan which should also include time to educate the entire firm about what it means to be compliant. Many firms achieve compliance and verification but fail ultimately because client-facing individuals within the firm are unaware of their new responsibilities.

The most common issues firm struggle with are:

(1) *Definition of firm*. The firm definition determines the boundaries of the firm for establishing total firm assets – discretionary and non-discretionary.

The firm will have to justify its definition in terms of how it presents itself to the public. The firm will also have to demonstrate that all accounts that fall within the firm definition have been identified. Fee income is a useful indicator that an account existed and is particularly useful for demonstrating to verifiers that all accounts have been identified.

(2) *Definition of composite*. The firm will have almost complete flexibility to define composite guidelines initially, but once defined it is difficult to make changes.

My recommendation would be to start with narrow definitions – it is a relatively easy process to define a new composite that later encompasses a number of narrow composites. Firms are required to disclose the composite creation date to illustrate that the composite many have been created retrospectively.

(3) *Lack of data*. The performance claim must be supported by relevant data, which at a minimum will require periodic valuations and cash flows. Often one of the toughest barriers to compliance is the lack of data, particularly from accounts that are now closed. Recordkeeping guidance can be found in Appendix N.

(4) *Valuation of illiquid assets*. Clearly, valuation of assets is a key part of any performance return. The processes for valuing illiquid assets should be well documented, applied consistently and provide a fair estimate of value. Asset managers should be cognisant of their requirements to provide a fair and honest representation of performance.

MAINTAINING COMPLIANCE

Having achieved compliance, maintaining compliance is not that straightforward. Compliance is not only calculating the correct returns and maintaining the correct composites but also crucially presenting information in the correct format to prospective clients. In essence, the claim of compliance by an asset manager means that performance presented by that asset manager is a fair and honest representation of performance. This requires that the entire firm is educated about the meaning of the claim of compliance.

To help maintain compliance I would certainly appoint an individual responsible for ensuring the integrity of the firm's performance presentations and establish a quarterly review process in addition to an annual verification.

The environments in asset manager firms are so different and the flexibility built into the standards so great that it is very difficult to provide generic advice or rules that are suitable for all. However, if asset managers take as their touchstone the requirement to provide a fair and honest representation of the firm's track record they won't go far wrong. The standards help to lay down good practice and point the way to best practice. The foundations established by adopting the standards and establishing strong procedures and good quality controls will not only strengthen the performance measurement analysis within the firm but strengthen the firm itself.

Appendix A
Simple Attribution

A.1 ATTRIBUTION METHODOLOGY

The following methodology has been developed for use with single currency portfolios.

A.1.1 Scenario

Suppose we have a portfolio invested in n asset classes or industrial sectors. Then, suppose that the performance of this portfolio is measured against a benchmark.

A.1.2 Portfolio returns

Let the weight of the portfolio in the ith asset class be w_i, where $\sum w_i = 1$, and let the return of the portfolio assets in the ith asset class be r_i. Now the total portfolio return is:

$$r = \sum w_i r_i$$

A.1.3 Benchmark returns

Let the weight of the benchmark in the ith asset class be W_i, where $\sum W_i = 1$, and let the return of the benchmark for the ith asset class in the base currency of the portfolio be b_i. Now the total benchmark return is:
(as it is in the base currency of the portfolio):

$$b = \sum W_i b_i$$

A.1.4 Semi-notional returns

We define the semi-notional return of the ith asset class as $w_i b_i$. Now the total semi-notional return is:

$$b_S = \sum w_i b_i$$

A.1.5 Relative performance

We define the performance of the portfolio relative to the benchmark as:

$$\frac{1+r}{1+b} - 1$$

and it is this relative performance that we attribute in this methodology. We attribute it to two factors: stock selection and country allocation.

A.2 STOCK SELECTION

The term "stock selection" is used to describe the relative performance of the portfolio to the benchmark within a particular asset class. Intuitively, this seems to be the portfolio total return, r, relative to the semi-notional total return, b_S (remembering that b_S is the sum product of portfolio weights with the benchmark returns, so any difference between b_S and r is by definition due to stock selection).

We attribute relative performance to stock selection in the ith asset class as follows:

$$w_i \left(\frac{1+r_i}{1+b_i} - 1 \right) \left(\frac{1+b_i}{1+b_S} \right)$$

So, the total stock selection is:

$$\begin{aligned}
\sum_{i=1}^{n} w_i \left(\frac{1+r_i}{1+b_i} - 1 \right) \left(\frac{1+b_i}{1+b_S} \right) &= \sum \frac{w_i[(1+r_i)-(1+b_i)]}{1+b_S} \\
&= \sum \frac{w_i r_i - w_i b_i}{1+b_S} \\
&= \frac{\sum w_i r_i - b_S}{1+b_S} \\
&= \frac{1+\sum w_i r_i - (1+b_S)}{1+b_S} \\
&= \frac{1+r}{1+b_S} - 1
\end{aligned}$$

A.3 ASSET ALLOCATION

The term "asset allocation" is used to describe the effect of the relative weighting of the portfolio to the benchmark (or "bet") within a particular asset class.

We attribute relative performance to asset allocation in the ith asset class as follows:

$$(w_i - W_i) \left(\frac{1+b_i}{1+b} - 1 \right)$$

So, the total asset allocation is:

$$\begin{aligned}
\sum_{i=1}^{n} (w_i - W_i) \left(\frac{1+b_i}{1+b} - 1 \right) &= \sum (w_i - W_i) \left(\frac{1+b_i-1-b}{1+b} \right) \\
&= \sum \frac{w_i b_i - W_i b_i - w_i b + W_i b}{1+b} \\
&= \frac{\sum w_i b_i - \sum W_i b_i}{1+b} \\
&\quad \left(\text{because } \sum w_i = \sum W_i = 1 \Rightarrow \sum (W_i - w_i) b = 0\right) \\
&= \frac{\sum w_i b_i - \sum W_i b + \sum W_i b_i - b}{1+b}
\end{aligned}$$

$$= \frac{1 + \sum [(w_i - W_i)b_i + \sum W_i b_i] - (1 + b)}{1 + b}$$
$$= \frac{1 + b_S}{1 + b} - 1$$

A.4 SUMMARY

We now have attributed relative performance to the following factors:

$$\text{Stock selection} \quad \frac{1 + r}{1 + b_S} - 1$$

$$\text{Asset allocation} \quad \frac{1 + b_S}{1 + b} - 1$$

We can then see that these factors compound to give:

$$\left(\frac{1 + b_S}{1 + b}\right)\left(\frac{1 + r}{1 + b_S}\right) - 1 = \frac{1 + r}{1 + b} - 1$$

Thus, we have now accounted for all of our relative performance with no residuals (i.e. no "other" term). Once more, because these terms are geometric the relationship holds true over time.

Appendix B
Multi-currency Attribution Methodology

The following methodology has been developed for use with multi-currency portfolios.

B.1 SCENARIO

Suppose we have a portfolio invested in n asset classes with some asset classes having currencies other than the base currency of the portfolio. Then, suppose that the performance of this portfolio is measured against a benchmark.

B.1.1 Portfolio returns

Let the weight of the portfolio in the ith asset class be w_i, where $\sum w_i = 1$, and let the return of the portfolio assets in the ith asset class in local currency be r_{Li} and in the base currency of the portfolio be r_i. Now the total portfolio return in local currency is:

$$r_L = \sum w_i r_{Li} \quad \text{(or weighted average local return)}$$

and in the base currency of the portfolio:

$$r = \sum w_i r_i$$

B.1.2 Benchmark returns

Let the weight of the benchmark in the ith asset class be W_i, where $\sum W_i = 1$, and let the return of the benchmark for the ith asset class in local currency be b_{Li}, in the base currency of the portfolio be b_i and hedged into the base currency be b_{Hi}. Now the total benchmark return in local currency is:

$$b_L = \sum W_i b_{Li}$$

and in the base currency of the portfolio:

$$b = \sum W_i b_i$$

B.1.3 Semi-notional returns

We define the semi-notional return of the ith asset class in the local currency as $w_i b_{Li}$ and with any deviation from the index weightings ("bet") hedged into the base currency as $b_{SHi} = (w_i - W_i)b_{Hi} + W_i b_{Li}$. Now the total semi-notional return in the local currency is:

$$b_{SL} = \sum w_i b_{Li}$$

and with any deviation from the index weighting ("bet") hedged into the base currency:

$$b_{SH} = \sum \{(w_i - W_i)\, b_{Hi} + W_i b_{Li}\}$$

B.1.4 Relative performance

We define the performance of the portfolio relative to the benchmark as:

$$\frac{1+r}{1+b}-1$$

and it is this relative performance that we attribute in this methodology. We attribute it to three main factors: stock selection, country allocation and currency effects.

B.2 STOCK SELECTION

The term "stock selection" is used to describe the relative performance of the portfolio to the benchmark within a particular asset class. Intuitively, this seems to be the portfolio total local return, r_L, relative to the semi-notional total local return, b_S (remembering that b_S is the sum product of portfolio weights with the benchmark returns, so any difference between b_S and r_L is by definition due to stock selection).

We attribute relative performance to stock selection in the ith asset class as follows:

$$w_i\left(\frac{1+r_{Li}}{1+b_{Li}}-1\right)\left(\frac{1+b_{Li}}{1+b_{SL}}\right)$$

So, the total stock selection is:

$$\begin{aligned}
\sum_{i=1}^{n} w_i\left(\frac{1+r_{Li}}{1+b_{Li}}-1\right)\left(\frac{1+b_{Li}}{1+b_{SL}}\right) &= \sum\frac{w_i[(1+r_{Li})-(1+b_{Li})]}{1+b_{SL}}\\
&= \sum\frac{w_i r_{Li}-w_i b_{Li}}{1+b_{SL}}\\
&= \frac{\sum w_i r_{Li}-b_{SL}}{1+b_{SL}}\\
&= \frac{1+\sum w_i r_{Li}-(1+b_{SL})}{1+b_{SL}}\\
&= \frac{1+r_L}{1+b_{SL}}-1
\end{aligned}$$

B.3 ASSET ALLOCATION

The term "asset allocation" is used to describe the effect of the relative weighting of the portfolio to the benchmark (or "bet") within a particular asset class. Within this effect we include the cost of hedging the "bet" back to base currency, reasoning that this is a cost that should be borne by the decision-maker making the bet. In effect, we are saying that if the asset allocator causes a currency position in the portfolio, then that position must be notionally hedged back to the neutral benchmark exposure. The cost (or benefit) is reflected in the asset allocation calculation by using a fully hedged index to measure the impact.

We attribute relative performance to asset allocation in the ith asset class as follows:

$$(w_i-W_i)\left(\frac{1+b_{Hi}}{1+b_L}-1\right)$$

So, the total asset allocation is:

$$\begin{aligned}
\sum_{i=1}^{n}(w_i - W_i)\left(\frac{1+b_{Hi}}{1+b_L} - 1\right) &= \sum (w_i - W_i)\left(\frac{1+b_{Hi}-1-b_L}{1+b_L}\right) \\
&= \sum \frac{w_i b_i - W_i b_{Hi} - w_i b_L + W_i b_L}{1+b_L} \\
&= \frac{\sum w_i b_{Hi} - \sum W_i b_{Hi}}{1+b_L} \\
&\quad \text{(because } \sum w_i = \sum W_i = 1 \Rightarrow \sum (W_i - w_i) b_L = 0\text{)} \\
&= \frac{\sum w_i b_{Hi} - \sum W_i b_{Hi} + \sum W_i b_{Li} - b_L}{1+b_L} \\
&= \frac{1 + \sum [(w_i - W_i) b_{Hi} + \sum W_i b_{Li}] - (1+b_L)}{1+b_L} \\
&= \frac{1+b_{SH}}{1+b_L} - 1
\end{aligned}$$

B.4 CURRENCY EFFECTS

B.4.1 Naïve currency performance

The difference between the base currency return of the portfolio and the weighted average local return must by definition be the total currency effect. Therefore the currency return of the portfolio r'_C is:

$$r'_C = \frac{1+r}{1+r_L} - 1$$

Similarly, the currency return of the benchmark b'_C is:

$$b'_C = \frac{1+b}{1+b_L} - 1$$

Therefore the naïve currency attribution within the portfolio is the difference between the portfolio currency and the benchmark currency:

$$\frac{1+r'_C}{1+b'_C} - 1 = \left(\frac{1+r}{1+r_L} \Big/ \frac{1+b}{1+b_L}\right) - 1 \quad \text{or} \quad \frac{1+r}{1+r_L} \times \frac{1+b_L}{1+b} - 1$$

This is defined as naïve because it makes no allowance for the transfer of the cost of hedging discussed in asset allocation above or compounding effects between market or currency returns. It does not reflect the currency effect from the perspective of the “currency overlay manager”.

B.4.2 Measured currency returns

We can also derive the “measured” currency return from the bottom up using currency exposures and returns.

We define the currency return of the benchmark in the ith currency as:

$$c_i = \frac{1 + b_i}{1 + b_{Li}} - 1$$

Because most commercial international indexes use the WM Reuters 4 o'clock closing exchange rates this currency return can be derived from spot rates:

$$c_i = \frac{S_i^{t+1}}{S_{Si}^t} - 1$$

where: $S_i^t =$ the spot rate of currency i at time t.

Defining the benchmark forward rate of a forward currency contract as:

$$f_i = \frac{S_i^{t+1}}{F_i^{t+1}} - 1$$

where: $F_i^{t+1} =$ the forward exchange rate of currency i at time t for conversion through a forward contract at time $t + 1$.

Note the interest rate differential in currency i:

$$d_i = \frac{F_i^{t+1}}{S_i^t} - 1$$

The currency return is therefore:

$$\frac{S_i^{t+1}}{S_i^t} = \frac{S_i^{t+1}}{F_{Si}^{t+1}} \times \frac{F_i^{t+1}}{S_i^t} = (1 + f_i) \times (1 + d_i)$$

Note that the hedged index return is the combined effect of local return with interest rate differential:

$$b_{Hi} = (1 + b_{Li}) \times (1 + d_i) - 1 \quad \text{or} \quad b_{Hi} = \frac{1 + b_i}{1 + b_{Fi}} - 1$$

We define the total measured currency return of the benchmark as:

$$b_C = \sum W_i b_{Ci} + \sum \tilde{W}_i f_i$$

where: $\tilde{W}_i$ is the benchmark weight of forward currency contracts in currency.

The slight difference between b_C and b'_C is caused by compounding between market returns and currency. This difference is measured by:

$$\frac{1 + b_C}{1 + b'_C} - 1$$

We define the currency return of the portfolio in ith currency as:

$$c'_i = \frac{1 + r_i}{1 + r_{Li}} - 1$$

Currency returns in portfolios differ from benchmark currency returns because transactions naturally occur at spot rates different from the closing spot rates.

Forward currency returns in portfolios also differ from benchmark forward currency returns.

If we let the forward currency return of the portfolio in the ith currency be f_i', we can define the total measured currency return of the portfolio as:

$$b_{SC} = \sum w_i c_i' + \sum \tilde{w}_i f_i'$$

where: $\tilde{w}_i$ is the benchmark weight of forward currency contracts in currency i.

Similarly, there is a slight difference between b_{SC} and r_C' measured by:

$$\frac{1 + r_C'}{1 + b_{SC}} - 1$$

B.4.3 Compounding effects

Comparing the impact of compounding in the portfolio with that of the benchmark the combined impact is measured by:

$$\left(\frac{1 + r_C'}{1 + b_{SC}} \Big/ \frac{1 + b_C'}{1 + b_C} \right) - 1 \quad \text{or} \quad \frac{1 + r_C'}{1 + b_{SC}} \times \frac{1 + b_C}{1 + b_C'} - 1$$

This factor measures the impact of currency and market compounding invisible to the currency overlay manager, but nevertheless an effect within the total portfolio.

This factor may be shown separately or more commonly combined with the currency effect, particularly if the currency overlay manager is not independent of the investment decision process.

B.4.4 Currency attribution

We define the semi-notional currency return of the portfolio as:

$$r_{SC} = \sum w_i c_i + \sum \tilde{w}_i f_i$$

Note that the semi-notional currency return applies benchmark currency or spot returns to actual physical portfolio weights and benchmark currency forward returns to the actual portfolio forward currency weights.

We define the semi-notional currency return of the portfolio including the cost of hedging as:

$$b_{SC} = \sum W_i c_i + \sum [(\tilde{w}_i + w_i - W_i) f_i]$$

B.4.4.1 Currency timing

Currency timing is used to describe the difference between the real portfolio currency returns and benchmark currency returns caused by intra-day FX and FFX trades at spot and forward rates different from that assumed in the benchmark. Currency timing is analogous to stock selection:

$$w_i \left(\frac{1 + c_i'}{1 + c_i} - 1 \right) \left(\frac{1 + c_i}{1 + r_{SC}} \right)$$

and including forward contracts:

$$\tilde{w}_i \left(\frac{1 + f_i'}{1 + f_i} - 1 \right) \left(\frac{1 + f_i}{1 + r_{SC}} \right)$$

The total currency timing effect:

$$\frac{1 + r_C}{1 + r_{SC}}$$

B.4.4.2 Currency allocation

Currency managers generate currency exposure by using currency forward contracts or currency options whose price is derived from these forward currency contracts. They are priced by reference to spot rates and interest rate differentials between the two currencies. A forward currency contract will therefore generate two exposures: one long and one short.

It follows therefore that the currency manager can only generate a currency position by use of forward currency contracts and is therefore always exposed to interest rate differentials. To measure the impact of any currency bet we must use currency forward rates, not spot rates, to determine the impact of that currency bet. Currency allocation is analogous to asset allocation selection:

$$\left(w_i + \tilde{w}_i - W_i - \tilde{W}_i \right) \left(\frac{1 + f_i}{1 + b_C} - 1 \right)$$

So, the total currency allocation performance is:

$$\frac{1 + b_{SC}}{1 + b_C} - 1$$

The total currency effects from the currency overlay perspective are:

$$\frac{1 + r_C}{1 + r_{SC}} \times \frac{1 + b_{SC}}{1 + b_C} - 1$$

B.4.5 Cost of hedging

The "cost of hedging" represents the cost or benefit of hedging the asset allocator's decisions back to the "neutral" currency benchmark. The cost of hedging from the currency overlay perspective is:

$$\frac{1 + r_{SC}}{1 + b_{SC}} - 1$$

The cost of hedging from the asset allocator's perspective is:

$$\frac{1 + b_{SH}}{1 + b_{SL}} - 1$$

The asset allocator's perspective, including the compounding with market returns which causes a very slight mismatch, is measured by:

$$\left(\frac{1+r_{SC}}{1+b_{SC}}\bigg/\frac{1+b_{SH}}{1+b_{SL}}\right)-1 \quad \text{or} \quad \frac{1+r_{SC}}{1+b_{SC}}\times\frac{1+b_{SL}}{1+b_{SH}}-1$$

This impact is so small it can be ignored, unless you prefer to avoid all residuals.

B.4.6 Total currency effects

Combining all the currency effects in the portfolio we get:

$$\underbrace{\frac{1+r_C}{1+r_{SC}}\times\frac{1+b_{SC}}{1+b_C}}_{\text{Currency overlay}}\times\underbrace{\frac{1+b_{SL}}{1+b_{SH}}\times\frac{1+r_{SC}}{1+b_{SC}}}_{\text{Hedging mismatch}}\times\underbrace{\frac{1+r'_C}{1+r_C}\times\frac{1+b_C}{1+b'_C}}_{\text{Compounding}}-1$$

which simplifies to:

$$\frac{1+b_S}{1+b_{SH}}\times\frac{1+r'_C}{1+b'_C}-1$$

The naïve currency effect adjusted for the cost of hedging.

B.5 SUMMARY

We now have attributed relative performance to the following factors:

$$\text{Stock selection} \quad \frac{1+r_L}{1+b_{SL}}-1$$

$$\text{Asset allocation} \quad \frac{1+b_{SH}}{1+b_L}-1$$

$$\text{Total Currency Effects} \quad \frac{1+b_S}{1+b_{SH}}\times\frac{1+r'_C}{1+b'_C}-1 \quad \text{or} \quad \frac{1+b_{SL}}{1+b_{SH}}\times\frac{1+r}{1+r_L}\times\frac{1+b_L}{1+b}-1$$

We can then see that these factors compound to give:

$$\underbrace{\frac{1+r_L}{1+b_{SL}}}_{\text{Stock}}\times\underbrace{\frac{1+b_{SH}}{1+b_L}}_{\text{Asset}}\times\underbrace{\frac{1+b_{SL}}{1+b_{SH}}}_{\text{Hedging cost transferred}}\times\underbrace{\frac{1+r}{1+r_L}\times\frac{1+b_L}{1+b}}_{\text{Naïve currency attribution}}-1=\frac{1+r}{1+b}-1$$

Thus, we have now accounted for all of our relative performance with no residuals (i.e. no "other" term). Once more, because these terms are geometric the relationship holds true over time.

Appendix C
EIPC Guidance for Users of Attribution Analysis*

DEFINITION

Return attribution is a technique used to analyse the sources of excess returns of a portfolio against its benchmark into the active decisions of the investment management process.

PREAMBLE

Return attribution is becoming an increasing by valuable tool not only for assessing the abilities of asset managers and identifying where and how value is added but also for facilitating a meaningful dialogue between asset manager and client.

In this guidance we have chosen the term "return attribution" rather than the more common "performance attribution" to emphasise the distinction between return and risk, on the one hand, and to encourage the view of performance as a combination of risk and return or the other hand.

Risk and risk attribution are equally valuable tools for assessing the abilities of asset managers; however, in this note we have focused on the attribution of historic returns.

Over the years many different forms of attribution techniques have been developed with varying degrees of accuracy. Additionally, attribution results may be presented in a variety of different formats, which in some cases may lead to different conclusions being drawn.

The following list of questions has been provided to assist the user of attribution analysis to gain the maximum value from the presentation.

QUESTIONS:

1. Does the attribution model follow the investment decision process of the asset manager?

   Comment: Attributing factors that are not part of the asset manager's decision process add little value. It is essential the attribution process quantifies the actual decisions made by the asset manager.

2. Is the benchmark appropriate to the investment strategy?

   Comment: Does the benchmark adequately reflect the investment strategy and hence the investment decision process? Has it been used consistently over time? Is this the formal benchmark for the account?

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3. Has the benchmark or investment style changed during the period of analysis?

   Comment: Benchmark changes and changes in style and restrictions should be disclosed. It is not appropriate to attribute using a current benchmark if changes have occurred. The attribution should reflect the benchmark assigned at the time and attribution effects should be compounded consistently.

4. Has the attribution model changed during the period of analysis?

   Comment: Changes and the rationale for changes should be disclosed.

5. Does the model generate an unexplained performance residual?

   Comment: Many attribution models generate residuals or balancing items. Essentially, all factors of the investment decision process are attributable. Residuals may bring into question the quality of the analysis and bring into doubt any conclusions that may be drawn from it.

6. If a residual is generated is it:
   i. Shown separately as a residual, balancing, timing or transaction item?
   ii. Ignored?
   iii. Allocated between other factors?

   Comment: Because a large residual may be difficult to explain it may be renamed, ignored or even allocated to other factors. It is important to establish how the residual has been treated by the asset manager. It is not good practice to ignore residuals.

7. Is interaction specifically calculated?

   Comment: Interaction is a defined factor in early (classical) attribution models. It represents the combined impact (or cross product) of stock and asset selection. Often used when asset managers wish to derive the stock selection effect assuming the portfolio asset allocations are in line with the benchmark. Interaction is the remainder stock selection effect caused by asset allocations not in line with the benchmark.

8. If interaction is calculated is it:
   i. Shown separately?
   ii. Ignored?
   iii. Allocated to another factor?
   iv. Allocated to other factors consistently?

   Comment: Like residuals, large interaction effects are difficult to explain. They are often allocated to other factors or ignored. It is important to establish if interaction has been consistently applied to the same factor (i.e. stock selection) over time. It is not good practice to ignore residuals.

9. Is the attribution model arithmetic or geometric (multiplicative)?

   Comment: There are two common forms of expressing excess return: arithmetic $r - b$ and geometric $(1 + r)/(1 + b) - 1$. Different attribution models will be required to quantify arithmetic and geometric excess returns (r = portfolio return, b = benchmark return).

10. If the model used is arithmetic, has a smoothing algorithm been used to allocate residuals to other factors?

Comment: Arithmetic models are deficient for multi-period analysis, generating residuals over time. Smoothing algorithms have been developed in some cases which allocate in a systematic way this residual over time. The type of smoothing algorithm should be disclosed. Some geometric models are also deficient and hence should disclose any smoothing algorithm.

11. Is the attribution based on buy/hold snapshots or are transactions included?

    Comment: Stock level attribution in particular is data intensive. As an alternative buy/hold attributions may be performed based on holdings at the beginning of the period. Clearly, such attributions will not reconcile with real portfolio returns. Transactions and associated costs may be a significant factor in the portfolio return and are ignored in buy/hold-type analysis.

12. How are the weights of the elements of attribution defined?

    Comment: All methods rely on allocating weights to the sectors to be attributed. Only weight measures that ensure the weighted sum of returns is equal to the portfolio return will be accurate.

13. Is the model genuinely multi-currency? What FX rates are used for the portfolio and benchmark?

    Comment: Currency effects should only be allocated if the asset manager has a separate currency allocation process. Forward currency effects should be calculating reflecting the fact that local returns cannot be achieved–only base currency or hedged. If the timing of FX rates are different from the portfolio and benchmark this should be disclosed. Most international benchmarks use consistent FX rates.

14. How are asset allocation decisions outside of the benchmark treated?

    Comment: Any "bet" taken outside the benchmark will require an index to measure the impact of this decision. The choice of index will change the allocation between stock selection and asset allocation. The asset manager's approach should be determined and should be tested to ensure the approach taken is consistent with the investment process.

15. Are transaction costs included within stock selection or asset allocation, or are transaction costs treated as a separate attributable factor?

    Comment: Typically, all transaction costs are implicitly included in the calculation of stock level performance. However, asset allocation decisions may generate transaction costs which should be allocated to asset allocation. Some models allocate a notional transaction cost to asset allocation. Consideration should be given to attributing the impact of transactions in isolation and measuring the impact of dealing or the contribution of the dealing department.

16. Are the returns to be attributed net or gross of fees.

    Comment: If the returns are net of fees compared to a benchmark not adjusted for fees it is possible that the stock selection impact will include fees.

17. Is cash specifically included in the attribution? If so, has a cash benchmark been determined?

Comment: The user should establish if the attribution reflects all the assets within the portfolio. If cash is included, has an appropriate cash benchmark been selected? The exact use of cash (excluded, systematically allocated to sectors or managed) should be disclosed. Since cash is lowly correlated with most assets and often not included in the benchmark it is frequently one of the larger "bets" in the portfolio and hence a contributor to relative performance.

18. Does the attribution include gearing or leverage and if so is the attribution based on an all-cash analysis?

    Comment: If the asset manager is employing gearing this should be attributed according to investment decision process. Is the gearing at portfolio or asset level? Gearing should be disclosed.

19. Are derivatives included in the analysis? If yes, how?

    Comment: Just like any other asset class the impact of derivatives should be calculated in line with the investment decision process. It may not be appropriate to isolate the impact of derivatives alone. Attribution effects should be based on the economic exposure of derivatives if that accurately reflects the investment decision process.

20. Is the attribution derived directly from the asset manager's records? Is there a difference between the return used in the attribution and the formal portfolio return?

    Comment: It is important to determine the source of the attribution data, is it from the asset manager, custodian or other third party? Differences between the attribution calculated return and formal return should be identified. If top-level returns (portfolio and benchmark) can be reconciled to third parties then is it appropriate to use the asset manager's attribution model? (The third parties' attribution model may not follow the asset manager's decision process.)

21. Which methodology is used to calculate portfolio returns?

    Comment: The return calculation methodology (time-weighted or money-weighted) will determine the accuracy of the attribution results and the weights used to determine factor allocations. In a similar way that large cash flows effect return calculations, large cash flows both external and internal between sectors in a portfolio may impact attribution calculations. Typically, more frequent return calculations lead to more accurate attribution results.

22. If the attribution base is not a benchmark what is the rationale for this choice?

    Comment: Attributions can be performed against composites, representative accounts, model funds, carve-outs and peer groups. This should be disclosed together with the methodology used and the rationale for this type of presentation.

Appendix D
European Investment Performance Committee – Guidance on Performance Attribution Presentation*

SECTION 1 INTRODUCTION

Performance attribution has become an increasingly valuable tool not only for assessing asset managers' skills and for identifying the sources of value added but also for facilitating a meaningful dialogue between investment managers and their clients.

Like any other performance presentation, a presentation of performance attribution results provides meaningful information to the user only to the extent the user understands the assumptions and concepts underlying this presentation. That's why it is crucially important that the presentation of attribution results is provided in a way that does not mislead the users and contains all necessary disclosures to explain the underlying assumptions and concepts.

Given the aforementioned, the European Investment Performance Committee (EIPC) has decided to take the initiative and to address the demand of the investment management industry for specific guidance with respect to presentation of return and risk attribution analysis. The first step was the issue of the EIPC Working Paper "Guidance for Users of Attribution Analysis" in early 2002. The following Guidance on Performance Attribution Presentation represents the next milestone in this process and establishes a reporting framework, which provides for a fair presentation of return and risk attribution results with full disclosure. EIPC acknowledges that this Guidance is not the final step in this process and will have to be developed further to address any new matters arising in future.

Except for definition of some general terminology, the Guidance does not address methodological issues with respect to calculation of attribution results, nor attempts to present any prescriptive definitions. EIPC believes that setting any standard on performance attribution should primarily contribute to increasing the understanding of attribution through the necessary disclosures and transparency of the methodology and investment process. For details on various performance attribution methods and concepts, users should refer to the dedicated performance literature available. Being a "disclosure guidance", the Guidance can be generally applied to all types of investment portfolios (equity, fixed income or balanced).

The Guidance does not require investment managers to present return and risk attribution results. However, if investment managers do present attribution analysis, they are encouraged to provide full disclosure and to apply the provisions of the Guidance. As the importance of a particular piece of information may vary depending on the situation, EIPC believes that differentiation in the disclosures between required and recommended may be too subjective.

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EIPC regards it as the responsibility of users of performance attribution to duly inform themselves about performance attribution concepts and, when presented with performance attribution results, to ask relevant questions to understand the underlying assumptions and methods. Not doing this may lead to misinterpretations and misjudgment of the quality of investment managers presenting the attribution results.

The Guidance was approved by EIPC in January 2004. EIPC proposes that this Guidance be adopted by the Investment Performance Council (IPC) as a guidance for the investment management industry.

SECTION 2 DEFINITIONS

The purpose of the following definitions is to provide the user with an explanation on the terminology as it is used in this Guidance. The Guidance does not attempt to establish any absolute or dogmatic definitions and recognises that there may be various views and interpretations of these matters within the investment management industry.

Performance attribution

(1) Performance attribution techniques are generally understood as a process of decomposition of return and risk into the investment management decisions in order to measure the value added by active investment management and to communicate the risk components of the investment strategy.

(2) For the purposes of this Guidance term "Performance Attribution" refers both to attribution of historic returns and to risk attribution (ex-ante and ex-post). The Guidance emphasises the distinction between return and risk and encourages the view of performance as a combination of risk and return. As a rule, terms "*Return attribution*" and "*Risk attribution*" are explicitly used in this Guidance.

Excess/active return

The difference between a periodic portfolio return and its benchmark return. This value may be calculated either as an arithmetic or a geometric difference. Also called relative return.

Return attribution

(1) Return attribution techniques are generally understood as a process of decomposition of active (historic) returns into the investment management decisions in order to identify the sources of return.

(2) Return attribution can be applied to absolute returns (absolute attribution) or to relative/excess returns, being the difference between the portfolio and benchmark return (relative attribution).

Return contribution

Return contribution techniques are generally understood as a process of decomposition of returns in order to measure the contribution of each particular segment of the portfolio to the portfolio overall return.

Risk attribution

For the purpose of this Guidance, the following elements of risk attribution analysis are defined:

Risk measurement
The process of measurement of a portfolio's risk in absolute (e.g. volatility, value-at-risk) or relative (e.g. tracking error) terms, both ex-post (historic) and ex-ante (predicted).

Risk attribution
The first step of risk attribution is the risk decomposition, i.e. identifying the sources of a portfolio's risk, both ex-post (historic) and ex-ante (predicted), both in absolute terms and relative to the selected benchmark. This process may include decomposition into sources of systematic and specific risk or into various factors (e.g. industry, style, country, currency, credit quality, etc.) affecting a portfolio's risk; as well as determination of contribution of individual securities to the overall portfolio risk.

The further step of risk attribution is the process of measurement of contribution of investment management decisions to the active portfolio risk (e.g. to the portfolio tracking error).

Risk attribution for the purposes of this Guidance only refers to the analysis of investment risk and not to operational or other types of business risks.

SECTION 3 GUIDING PRINCIPLES

Investment managers are required to apply the following principles when calculating and presenting return and risk attribution results:

- Return and risk attribution analysis must follow the investment decision process of the investment manager and measure the impact of active management decisions. It is essential that the attribution analysis reflects the actual decisions made by the investment manager. Return and risk attribution analysis must mirror the investment style of the investment manager.
- For the attribution of relative return and risk, a benchmark appropriate to the investment strategy must be used. The employed benchmark should be specified in advance and meet such criteria as investability, transparency and measurability.
- If investment managers are not able to produce return and risk attribution results that comply with the above guiding principles, they still may use these results for internal purposes but should refrain from presenting attribution to external users or use it for the purposes of soliciting potential clients.

SECTION 4 DISCLOSURES

A Return attribution

The following disclosures are required to be provided, as long as they are applicable, when presenting return attribution results.

A.1 Investment process

A.1.1	Object of a return attribution analysis	Firms must disclose the object of a return attribution analysis, e.g. a particular portfolio, a representative portfolio, a model portfolio, a group of portfolios (composite), etc., and the reasons for selecting this particular object.
A.1.2	Investment management process and investment style	Firms must disclose the main elements of their investment management process, including the key investment decision factors employed.
A.1.3	Benchmark	Firms must disclose the composition of the benchmark used for the return attribution purposes. Benchmark rebalancing rules must also be disclosed. If there has been any change in benchmark, the old benchmark(s) and date(s) of change(s) are to be disclosed. In case of investments outside of the scope of the benchmark, firms must disclose the treatment of the impact of these investments, e.g. allocated to another attribution effect, presented separately, etc. If the attribution is not based on a benchmark, firms must disclose the rationale for this.

A.2. Return attribution model

A.2.1	Return attribution model and attribution effects	Firms must disclose a description of the return attribution model*. Attribution effects derived (e.g. depending on the portfolio type: timing, security selection, currency effects, or income, duration, spread effects, etc.) must be clearly identified. If the attribution model has changed during the period of analysis, these changes and the rationale for them must be disclosed. In addition, the implications for the attribution history, if any, as a result of this change must be disclosed.
A.2.2	Excess/active returns	Firms must disclose whether periodic excess returns are derived using an arithmetic or a geometric method.
A.2.3	Presentation period	Firms must disclose what time period the attribution analysis covers and why this period has been chosen.
A.2.4	Frequency of return attribution analysis	Firms must disclose the frequency of calculation of attribution effects (e.g. daily, monthly basis, etc.).
A.2.5	Linking methodology	If the attribution report provides effects which were calculated for subperiods (e.g. days) and linked to present results for longer periods (e.g. a month), then

*If the model is one which has been documented in an industry publication, its name and source reference must be disclosed If the model is a variation of a published model, the original name and source reference must be disclosed, as well as an explanation of the revisions which have been made. If the model is unpublished or proprietary, then a broad description of its details must be disclosed.

		the details of the linking methodology must be made available upon request. If a smoothing algorithm has been employed to allocate in a systematic way residual effects over time, the type of this algorithm is to be disclosed.
A.2.6	Buy-and-hold vs. transaction based approach	Firms must disclose whether the return attribution approach is buy-and-hold or transaction based.
A.2.7	Interaction effect and/or unexplained residuals	Some attribution models generate interaction effects or even unexplained residuals. Unexplained residuals may impair the quality of analysis and conclusions that may be drawn from it. If the model has an interaction term or an unexplained residual, details of its treatment must be disclosed, e.g. presented separately, ignored, allocated to other attribution effects, etc.
A.2.8	Derivatives	Firms must disclose to what extent derivatives are included and how they are treated in the return attribution analysis.
A.2.9	Effect of leverage	If leverage is employed, firms must disclose how leverage effects are attributed according to investment decision process.
A.2.10	Foreign currency effects	If investments in currencies other than the base currency of the portfolio are employed, treatment of foreign currency effects in terms of the currency management strategy must be disclosed.
A.2.11	Inclusion of cash	Firms must disclose whether cash is specifically included in the attribution analysis and whether a cash benchmark is determined. Firms also must disclose any difference in treatment of strategic cash allocation positions vs. temporary cash from realised income.
A.2.12	Transaction costs, fees	Firms must disclose the treatment of the impact of transaction costs, fees, etc. – e.g. allocated to a particular attribution effect, presented separately, etc.

A.3. Underlying input data

A.3.1	Portfolio returns	Firms must disclose: • methodology and frequency of calculation of portfolio and portfolio segment returns, • treatment of single performance components, such as management fees, custodian fees, taxes and transaction costs (gross vs. net treatment).

A.3.2	Benchmark returns	Firms must disclose: • methodology of calculation of benchmark returns, • any adjustments with respect to management fees, realised income positions, taxes etc., source of data. Firms are encouraged to disclose any other specific details that may be important.
A.3.3	Leveraged portfolios	If the underlying portfolio includes discretionary leverage, the firm must disclose whether calculation of portfolio returns is performed on an actual or "all-cash" basis.*
A.3.4	Underlying valuation data	Firms must disclose if there are any differences with respect to sources and timing of prices of underlying securities between the portfolio and the benchmark.
A.3.5	Foreign exchange rates	Firms must disclose if the sources or timing of foreign exchange rates are different between the portfolio and the benchmark.
A.3.6	Income positions	Firms must disclose if realised income from dividends and coupons is considered after or before deduction of applicable withholding taxes both for the portfolio and the benchmark.

Firms are encouraged to disclose any additional matters they find useful or relevant for the users of attribution analysis.

B. Risk attribution

The following disclosures are required to be provided when presenting risk attribution analysis results.

B.1 Investment process

B.1.1	Object of risk attribution	Firms must disclose the object of risk analysis, e.g. a particular portfolio, a representative portfolio, a model portfolio, a group of portfolios (composite), and the reasons for selecting this particular object.
B.1.2	Investment management process and investment style	Firms must disclose the main elements of their investment management process, including the key investment decision factors employed.
B.1.3	Benchmark	Firms must disclose the composition of the benchmarks used for the risk attribution purposes. Benchmark rebalancing rules must also be disclosed. If there has been any change in benchmark, the old benchmark(s) and date(s) of change(s) are to be disclosed. In case of investments outside of the scope or profile of the benchmark, firms must disclose the treatment of the impact of these investments. If the attribution is not based on a benchmark, firms must disclose the rationale for this.

*For details regarding "all-cash" basis calculations refer, for example, to *AIMR-PPS Handbook*, 1997, App. B, p. 117.

In case risk attribution is presented together with return attribution, the same benchmark as for return attribution should be used. If a different benchmark is used, the rationale for this must be disclosed.

B.2 Risk attribution model

B.2.1 Risk attribution model and attribution factors

Firms must disclose a general description of the risk attribution model, including description of the presented risk measures* and risk decomposition factors.

If the risk attribution model has changed during the period of analysis, these changes and the rationale for them are to be disclosed. In addition, the implications for the analysis history, if any, as a result of this change must be disclosed.

The risk attribution should, where possible, involve both ex-post and ex-ante analysis. This should also involve a reconciliation of the ex-post and ex-ante measures in order to assess the validity of the model.

B.2.2 Ex-ante risk measures

When presenting forward-looking risk measures, firms must provide a broad description with respect to the methods used to estimate portfolio holdings and/or likely magnitudes of relative returns for individual securities, sectors or markets and their correlation with each other.

Firms must also disclose the impact of the portfolio turnover and how this would influence their assumption regarding stability of the future portfolio asset structure.

B.2.3 Analysis period

When presenting risk measures, firms must disclose the reporting date of the analysis.

When presenting backward-looking risk measures, firms must disclose what time period the analysis covers and why this period has been chosen. In case ex-post risk attribution is presented together with return attribution, the analysis period should be the same as for the return attribution.

B.3 Underlying input data

B.3.1 Portfolio returns

Firms must disclose:

- methodology and frequency of calculation of portfolio and segment returns,
- treatment of single performance components, such as management fees, custodian fees, taxes, external cash flows and transaction costs (gross vs. net treatment).

*If the model is one which has been documented in an industry publication, its name and source reference must be disclosed. If the model is a variation of a published model, the original name and source reference must be disclosed, as well as an explanation of the revisions which have been made. If the model is unpublished or proprietary, then a broad description of its details must be disclosed.

*For details regarding "all-cash" basis calculations refer, for example, to *AIMR-PPS Handbook*, 1997, App. B, p. 117.

B.3.2	Benchmark returns	Firms must disclose: • methodology of calculation of benchmark returns, • any adjustments with respect to management fees, realised income positions, taxes, etc., • source of data. Firms are encouraged to disclose any other specific details that may be important.
B.3.3	Leveraged portfolios	If the underlying portfolio includes discretionary leverage, the firm must disclose whether calculation of portfolio returns is performed on an actual or "all-cash" basis.*
B.3.4	Underlying valuation data	Firms must disclose if there are any differences with respect to sources and timing of prices of underlying securities and foreign exchange rates between the portfolio and the benchmark.
B.3.5	Foreign exchange rates	Firms must disclose if the sources or timing of foreign exchange rates are different between the portfolio and the benchmark.
B.3.6	Income positions	Firms must disclose if realised income from dividends and coupons is considered after or before deduction of applicable withholding taxes.

Firms are encouraged to disclose any additional matters they find useful or relevant for the users of attribution analysis.

SECTION 5 RELATION TO THE GLOBAL INVESTMENT PERFORMANCE STANDARDS (GIPS™)

EIPC does not currently view this Guidance as a part of the Global Investment Performance Standards (GIPS™) compliance framework. However, the Guidance can obviously be considered as a part of a broader ethical code of conduct for investment managers. Firms claiming GIPS compliance and presenting performance attribution analysis are encouraged to follow this Guidance. However, users should be aware that some GIPS requirements may not always be applicable for attribution analysis purposes, e.g. return calculation methods for individual client reporting.

Attribution analysis results may also be presented as a supplemental information to a GIPS compliant performance presentation. If attribution analysis is presented as a part of a GIPS compliant performance presentation, users should also refer to the GIPS Guidance Statement on the Use of Supplemental Information for guidance.

APPENDIX 1 EXAMPLE OF RETURN AND RISK ATTRIBUTION REPORT IN COMPLIANCE WITH THIS GUIDANCE

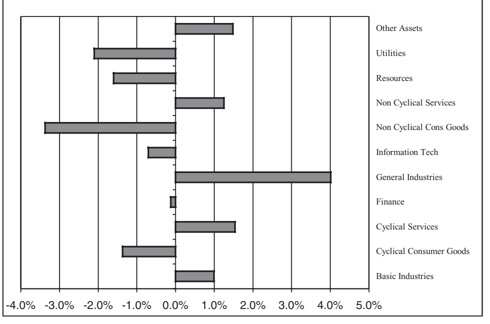
The sample attribution analysis report shown overleaf refers to an equity portfolio and is *an example* of how a performance attribution presentation in compliance with this Guidance could look like. This sample report is absolutely not intended to serve as a "best practice" benchmark to present performance attribution in terms of methodology or layout.

Investment Manager ABC

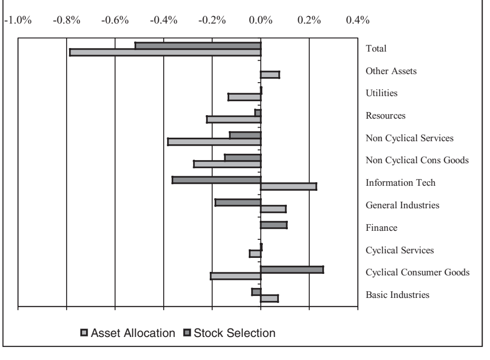
Return Attribution and Risk Attribution Report for Equity Portfolio XYZ as of 31.03.2001

Return and Risk Attribution Report for:	**PORTFOLIO XYZ**
Period: .	**1.1.2000 - 30.03.2001**
Reference Currency:	**EUR**
Benchmark:	**Customised (refer to Disclosures)**

Sector Overweights

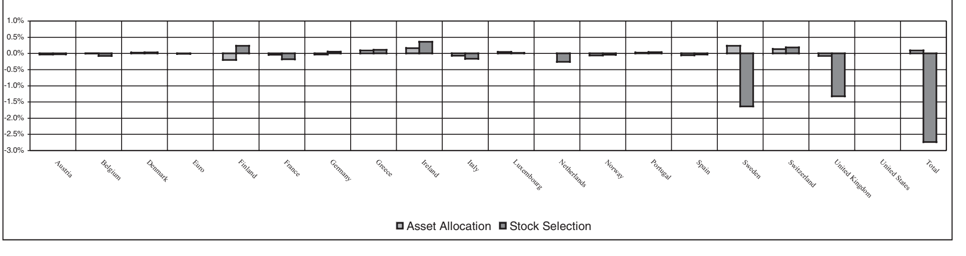


Attribution Analysis by Industry Sector



Return		
Portfolio		-4.45%
Benchmark		-2.89%
Active (Relative) Return		-1.56%
Return Attribution Analysis by Industry Sector		
Asset Allocation		-0.79%
Stock Selection		-0.52%
Other Effect		-0.25%
Total		**-1.56%**
Return Attribution Analysis by Region		
Asset Allocation		0.09%
Stock Selection		-2.74%
Other Effect		1.09%
Total		**-1.56%**
Risk Analysis (end of period)	**Portfolio**	**Benchmark**
Number of Securities	99	576
Number of Currencies	2	2
Portfolio Value	227'447'728	
Total Risk (ex-ante)	15.76%	15.31%
- Factor Specific Risk	15.53%	15.20%
- Style	4.91%	4.29%
- Industry	11.95%	11.80%
- Stock Selection Risk	2.72%	1.83%
Tracking Error (ex-post)	2.29%	
Tracking Error (ex-ante)	2.35%	
Value at Risk (at 97.7%)	10'878'425	
R-squared	0.98	
Beta-adjusted Risk	15.59%	15.31%
Predicted Beta	1.02	
Predicted Dividend Yield	2.22	2.37

Attribution Analysis by Region



Disclosures

Investment Process

Object of the attribution analysis	The return and risk attribution analysis is performed for Portfolio XYZ as an integral part of the periodic client reporting to company XYZ.
Investment management process and investment style	Portfolio XYZ is a discretionary equity mandate with reference currency EUR managed in an active way against the customised benchmark specified by company XYZ as described below. In addition, the following specific client guidelines apply: outperform the defined benchmark (basis EUR) by 2% p.a. over a rolling 2-year period with a tracking error of max. 3% p.a. Investment Manager ABC applies a top-down investment approach by actively modifying the portfolio asset allocation and taking active decisions with respect to stock selection. Foreign currency positions are not actively hedged. The inception date of portfolio XYZ is 1.1.2000.
Benchmark	The benchmark for portfolio XYZ is given as follows: • EUR Cash Index Z: 5% • EUR Stock Index X 60% • World Stock Index Y 35% A monthly rebalancing is applied. Results from investments in single stocks outside of the scope of the benchmark are allocated to the stock selection effect. There were no changes in the benchmark since inception of the mandate.

Attribution model

Return attribution model	Return attribution is performed under the *Brinson-Fachler* method. Details and explanations to this model are available upon request. Returns are attributed to asset allocation (timing) and stock selection effects and presented according to the industry sector and region. Please refer also to disclosure "Interaction effect and/or unexplained residuals". There has been no change in the model since inception of the portfolio.
Excess/active returns	Periodic excess returns are derived using an arithmetic method.
Presentation period	The return attribution and risk attribution analyses cover the period from 1.1.2000 to 31.03.2001 and is performed within the regular quarterly since-inception reporting.
Frequency of return attribution analysis	The attribution effects are calculated on a monthly basis

Linking methodology	The monthly attribution effects are multiplicatively linked to show the attribution results for the whole presentation period. No smoothing algorithms are employed to systematically allocate the residual effects over time. Details on the methodology are available upon request.
Treatment of transactions	The return attribution model is based on a "buy-and-hold" approach. However, as transactions in the portfolio usually occur at the beginning of the month and the attribution effects are calculated on a monthly basis, porttolio manager ABC believes that potential distortions should be minimal.
Interaction effect and/or unexplained residuals	The model generates a residual effect due to multiplicative linking of arithmetically derived attribution effects over time. This effect is presented separately as "Other effect". The model does not generate any other unexplained residuals.
Derivatives	Derivatives are not employed in this portfolio.
Use of leverage	Leverage is not employed in this portfolio.
Inclusion of cash	According to the defined portfolio benchmark, cash represents a strategic position and is specifically included in the attribution analysis against a specified cash benchmark index. There is no difference in treatment of the strategic cash allocation position comparing to temporary cash from realised income as the realised income cash is deemed to be immaterial.
Foreign currency positions	Foreign currency positions are not hedged into the portfolio reference currency. Foreign exchange effects of these positions are included in the return attribution analysis within the stock selection effect.
Transaction costs and fees	Returns are calculated net of transaction costs and gross of fees. The impact of transaction costs vis-à-vis the benchmark return is not calculated specifically as the model is not transaction based. The model implicitly includes transaction costs on a cash level.
Risk attribution analysis	The presented risk attribution analysis includes both ex-post and ex-ante risk measurement and risk decomposition. Ex-post analysis includes calculation of the historical annualised tracking error. Ex-ante analysis includes calculation of the predicted total risk of the portfolio (annualised volatility) and its decomposition into factor-specific (style and industry) and stock selection components. In addition ex-ante annualised tracking error and value-at-risk (VaR) measures are presented. The predicted VaR measure is calculated on the basis of the parametric (variance/covariance) method.

The methodology and assumptions used for calculation of ex-ante (predicted) risk measures are developed and implemented in the proprietary model of company WWW, broad details of which are available upon request. For the purposes of the ex-ante risk analysis, an assumption is taken that the portfolio strategic asset structure remains stable (with monthly rebalancing) over time.

While reasonable care is exercised when predicting risk parameters, users of this report should be aware of inherent limitations of such forecast methods as well as of the assumptions underlying the calculation of risk measures (such as normality of return distributions, etc.).

A periodic reconciliation of the ex-post and ex-ante measures is performed on a quarterly basis to assess the model risk. The historic reconciliation results (since portfolio inception) show that an average model error lies within the bandwidth of 200–300 b.p.

Underlying input data

Underlying portfolio returns

The underlying portfolio returns are calculated in EUR on a montly basis according to the true time-weighted rate of return method and under application of the total-return concept. Returns are calculated net of transaction costs and withholding taxes on interest and dividend income and gross of management and custodian fees.

The underlying portfolio data are derived from the accounting records of Investment Manager ABC. The source of securities prices and foreign exchange rates is data provider ZZZ.

Benchmark returns

The underlying benchmark returns are calculated on a monthly basis under application of the total-return concept and monthly rebalancing. The benchmark returns are calculated on the basis of EUR as reference currency. The source of the benchmark data is data provider ZZZ.

Appendix E
The Global Investment Performance Standards*

I. PREFACE: BACKGROUND OF THE GIPS

A. Preamble – why is a global standard needed?

1. The financial markets and the investment management industry are becoming increasingly global in nature. Given the variety of financial entities and countries involved, this globalization of the investment process and the exponential growth of assets under management demonstrate the need to standardize the calculation and presentation of investment performance.
2. Prospective clients and investment management firms will benefit from an established standard for investment performance measurement and presentation that is recognized worldwide. Investment practices, regulation, performance measurement, and reporting of performance results vary considerably from country to country. Some countries have guidelines that are widely accepted within their borders, and others have few recognized standards for presenting investment performance.
3. Requiring investment management firms to adhere to performance presentation standards will help assure investors that the performance information is both complete and fairly presented. Investment management firms in countries with minimal presentation standards will be able to compete for business on an equal footing with investment management firms from countries with more developed standards. Investment management firms from countries with established practices will have more confidence that they are being fairly compared with "local" investment management firms when competing for business in countries that have not previously adopted performance standards.
4. Both prospective and existing clients of investment management firms will benefit from a global investment performance standard by having a greater degree of confidence in the performance numbers presented by the investment management firms. Performance standards that are accepted in all countries enable all investment management firms to measure and present their investment performance so that clients can readily compare investment performance among investment management firms.

B. Vision statement

5. A global investment performance standard leads to readily accepted presentations of investment performance that (1) present performance results that are readily comparable among investment management firms without regard to geographical location and (2) facilitate a dialogue between investment managers and their prospective clients about the critical

issues of how the investment management firm achieved performance results and determines future investment strategies.

C. Objectives

6. To obtain worldwide acceptance of a standard for the calculation and presentation of investment performance in a fair, comparable format that provides full disclosure.
7. To ensure accurate and consistent investment performance data for reporting, record keeping, marketing, and presentations.
8. To promote fair, global competition among investment management firms for all markets without creating barriers to entry for new investment management firms.
9. To foster the notion of industry "self-regulation" on a global basis.

D. Overview

10. The Global Investment Performance Standards ("GIPS standards" or "Standards") have several key characteristics:
    a. For the purpose of claiming compliance with the GIPS standards, investment management FIRMS MUST define an entity that claims compliance ("FIRM"). The FIRM MUST be defined as an investment FIRM, subsidiary, or division held out to clients or potential clients as a DISTINCT BUSINESS ENTITY.
    b. The GIPS standards are ethical standards for investment performance presentation to ensure fair representation and full disclosure of a FIRM'S performance.
    c. The GIPS standards REQUIRE FIRMS to include all actual fee-paying, discretionary PORTFOLIOS in COMPOSITES defined according to similar strategy and/or investment objective and REQUIRE FIRMS to initially show GIPS-compliant history for a minimum of five (5) years or since inception of the FIRM or COMPOSITE if in existence less than 5 years. After presenting at least 5 years of compliant history, the FIRM MUST add annual performance each year going forward up to ten (10) years, at a minimum.
    d. The GIPS standards REQUIRE FIRMS to use certain calculation and presentation methods and to make certain disclosures along with the performance record.
    e. The GIPS standards rely on the integrity of input data. The accuracy of input data is critical to the accuracy of the performance presentation. For example, BENCHMARKS and COMPOSITES SHOULD be created/selected on an EX ANTE basis, not after the fact.
    f. The GIPS standards consist of provisions that FIRMS are REQUIRED to follow in order to claim compliance. FIRMS are encouraged to adopt the RECOMMENDED provisions to achieve best practice in performance presentation.
    g. The GIPS standards MUST be applied with the goal of full disclosure and fair representation of investment performance. Meeting the objectives of full disclosure and fair representation will likely require more than compliance with the minimum REQUIREMENTS of the GIPS standards. If an investment FIRM applies the GIPS standards in a performance situation that is not addressed specifically by the Standards or is open to interpretation, disclosures other than those REQUIRED by the GIPS standards may be necessary. To fully explain the performance included in a presentation, FIRMS are encouraged to present all relevant ADDITIONAL INFORMATION and SUPPLEMENTAL INFORMATION.
    h. All requirements, clarifications, updated information, and guidance MUST be adhered to when determining a FIRM'S claim of compliance and will be made available via the *GIPS Handbook* and the CFA Institute website (www.cfainstitute.org).

i. In cases where applicable local or country-specific law or regulation conflicts with the GIPS standards, the Standards REQUIRE FIRMS to comply with the local law or regulation and make full disclosure of the conflict.

j. The GIPS standards do not address every aspect of performance measurement, valuation, attribution, or coverage of all asset classes. The GIPS standards will evolve over time to address additional aspects of investment performance. Certain RECOMMENDED elements in the GIPS standards may become REQUIREMENTS in the future.

k. Within the GIPS standards are supplemental REAL ESTATE and PRIVATE EQUITY provisions that MUST be applied to these asset classes. (See sections II.6 and II.7.)

E. Scope

11. *Application of the GIPS standards*: FIRMS from any country may come into compliance with the GIPS standards. Compliance with the GIPS standards will facilitate a FIRM'S participation in the investment management industry on a global level.

12. *Historical performance record*:

    a. FIRMS are REQUIRED to present, at a minimum, 5 years of annual investment performance that is compliant with the GIPS standards. If the FIRM or COMPOSITE has been in existence less than 5 years, the FIRM MUST present performance since the inception of the FIRM or COMPOSITE; and

    b. After a FIRM presents 5 years of compliant history, the FIRM MUST present additional annual performance up to 10 years, at a minimum. For example, after a FIRM presents 5 years of compliant history, the FIRM MUST add an additional year of performance each year so that after 5 years of claiming compliance, the FIRM presents a 10-year performance record.

    c. FIRMS may link a non-GIPS-compliant performance record to their compliant history so long as no noncompliant performance is presented after 1 January 2000 and the FIRM discloses the periods of noncompliance and explains how the presentation is not in compliance with the GIPS standards.

    d. FIRMS previously claiming compliance with an Investment Performance Council-endorsed Country Version of GIPS (CVG) are granted reciprocity to claim compliance with the GIPS standards for historical periods prior to 1 January 2006. (See "Background of GIPS standards" for more details on CVGs.) If the FIRM previously claimed compliance with a CVG, at a minimum, the FIRM MUST continue to show the historical CVG-compliant track record up to 10 years (or since inception).

    Nothing in this section shall prevent FIRMS from initially presenting more than 5 years of performance results.

F. Compliance

13. *Effective date*: The GIPS standards were amended by the IPC on 7 December 2004 and adopted by the CFA Institute Board of Governors on 4 February 2005. The effective date of the revised Standards is 1 January 2006. All presentations that include performance results for periods after 31 December 2005 MUST meet all the REQUIREMENTS of the revised GIPS standards. Performance presentations that include results through 31 December 2005 may be prepared in compliance with the 1999 version of the GIPS standards. Early adoption of these revised GIPS standards is encouraged.

14. *REQUIREMENTS*: FIRMS MUST meet all the REQUIREMENTS set forth in the GIPS standards to claim compliance with the GIPS standards. Although the REQUIREMENTS MUST be met immediately by a FIRM claiming compliance, the following REQUIREMENTS do not go into effect until a future date:
    a. For periods beginning 1 January 2008, REAL ESTATE investments MUST be valued at least quarterly.
    b. For periods beginning 1 January 2010, FIRMS MUST value PORTFOLIOS on the date of all LARGE EXTERNAL CASH FLOWS.
    c. For periods beginning 1 January 2010, FIRMS MUST value PORTFOLIOS as of the calendar month-end or the last business day of the month.
    d. For periods beginning 1 January 2010, COMPOSITE returns MUST be calculated by asset weighting the individual PORTFOLIO returns at least monthly.
    e. For periods beginning 1 January 2010, CARVE-OUT returns are not permitted to be included in single asset class COMPOSITE returns unless the CARVE-OUTS are actually managed separately with their own cash balances.

    Until these future REQUIREMENTS become effective, these provisions SHOULD be considered RECOMMENDATIONS. FIRMS are encouraged to implement these future REQUIREMENTS prior to their effective dates. To ease compliance with the GIPS standards when the future REQUIREMENTS take effect, the industry should immediately begin to design performance software to incorporate these future REQUIREMENTS.
15. *Compliance check*: FIRMS MUST take all steps necessary to ensure that they have satisfied all the REQUIREMENTS of the GIPS standards before claiming compliance with the GIPS standards. FIRMS are strongly encouraged to perform periodic internal compliance checks and implement adequate business controls on all stages of the investment performance process – from data input to presentation material – to ensure the validity of compliance claims.
16. *Third-party performance MEASUREMENT and COMPOSITE construction*: The GIPS standards recognize the role of independent third-party performance measurers and the value they can add to the FIRM'S performance measurement activities. Where third-party performance measurement is an established practice or is available, FIRMS are encouraged to use this service as it applies to the FIRM. Similarly, where the practice is to allow third parties to construct COMPOSITES for FIRMS, FIRMS can use such COMPOSITES in a GIPS-compliant presentation only if the COMPOSITES meet the REQUIREMENTS of the GIPS standards.
17. *Sample presentations*: Sample presentations, shown in Appendix A, provide examples of what a compliant presentation might look like.

G. Implementing a global standard

18. In 1999, the Investment Performance Council (IPC) was created and given the responsibility to meet the ongoing needs for maintaining and developing a high quality global investment performance standard. The IPC provides a practical and effective implementation structure for the GIPS standards and encourages wider public participation in an industry-wide standard.
19. One of the principal objectives of the IPC is for all countries to adopt the GIPS standards as the common method for calculating and presenting investment performance. As of December 2004, more than 25 countries around the world had adopted or were in the process of adopting the GIPS standards. The IPC believes the establishment and acceptance of the GIPS standards are vital steps in facilitating the availability of comparable investment

performance history on a global basis. GIPS compliance provides FIRMS with a "passport" and creates a level playing field where all FIRMS can compete on equal footing.

20. The presence of a local sponsoring organization for investment performance standards is essential for their effective implementation and on-going operation within a country. Such country sponsors also provide an important link between the IPC, the governing body for the GIPS standards, and the local markets where investment managers operate.

    The country sponsor, by actively supporting the GIPS standards and the work of the IPC, will ensure that the country's interests can and will be taken into account as the GIPS standards are developed going forward. Compliance with the GIPS standards is voluntary, but support from the local country sponsor will help drive the success of the GIPS standards.
21. The IPC strongly encourages countries without an investment performance standard in place to accept the GIPS standards as the local standard and translate them into the local language when necessary, thus promoting a "translation of GIPS" (TG).
22. Compliance with the GIPS standards will provide FIRMS with a "right of access" to be considered alongside all investment managers, thereby allowing all FIRMS to be evaluated on equal terms.
23. Although the GIPS standards may be translated into many languages, if a discrepancy arises between the different versions of the Standards (e.g. TGs), the English version of GIPS standards is controlling.
24. The IPC will continue to develop the GIPS standards so that they maintain their relevance within the changing investment management industry and has committed to evaluating the Standards every 5 years.
25. The self-regulatory nature of the GIPS standards necessitates a strong commitment to ethical integrity. Self-regulation also assists regulators in exercising their responsibility for ensuring the fair disclosure of information to and within the financial markets in general. Regulators are encouraged to:
    - recognize the benefit of voluntary compliance with standards that represent global best practices,
    - give consideration to adopting a function favored by some regulators, namely to enforce sanctions upon false claims of compliance with the GIPS standards as fraudulent advertising, and
    - recognize and encourage independent verification services.
26. Where existing laws or regulations already impose performance presentation standards, FIRMS are strongly encouraged to comply with the GIPS standards in addition to those local requirements. Compliance with applicable law or regulation does not necessarily lead to compliance with the GIPS standards. When complying with the GIPS standards and local law or regulation, FIRMS MUST disclose any local laws and regulations that conflict with the GIPS standards.

II. PROVISIONS OF THE GLOBAL INVESTMENT PERFORMANCE STANDARDS

The GIPS standards are divided into eight sections that reflect the basic elements involved in presenting performance information: fundamentals of compliance, input data, calculation methodology, COMPOSITE construction, disclosures, presentation and reporting, REAL ESTATE, and PRIVATE EQUITY.

The provisions for each section are divided between REQUIREMENTS, listed first in each section, and RECOMMENDATIONS. FIRMS MUST meet all the REQUIREMENTS to claim compliance with the GIPS standards. FIRMS are strongly encouraged to adopt and implement the RECOMMENDATIONS to ensure that the FIRM fully adheres to the spirit and intent of the GIPS standards. Examples of GIPS-compliant presentations are included as Appendix A. A Glossary is included as Appendix E to serve as a reference and provide brief descriptions of key words and terms in the GIPS standards. Words appearing in CAPITAL letters are defined in the GIPS Glossary.

0. *Fundamentals of compliance*: Critical issues that a FIRM MUST consider when claiming compliance with the GIPS standards are defining the FIRM, documenting FIRM policies and procedures, maintaining compliance with updates to the GIPS standards, and properly using the claim of compliance and references to verification. The definition of the FIRM is the foundation for FIRM-wide compliance and creates defined boundaries whereby TOTAL FIRM ASSETS can be determined. Once a FIRM meets all of the REQUIREMENTS of the GIPS standards, it MUST appropriately use the claim of compliance to state compliance with the GIPS standards.
1. *Input data*: Consistency of input data is critical to effective compliance with the GIPS standards and establishes the foundation for full, fair, and comparable investment performance presentations.
2. *Calculation methodology*: Achieving comparability among FIRMS' performance presentations requires uniformity in methods used to calculate returns. The Standards mandate the use of certain calculation methodologies for both PORTFOLIOS and COMPOSITES. [corrected January 2006]
3. *Composite construction*: A COMPOSITE is an aggregation of one or more PORTFOLIOS into a single group that represents a particular investment objective or strategy. The COMPOSITE return is the asset-weighted average of the performance results of all the PORTFOLIOS in the COMPOSITE. Creating meaningful, asset-weighted COMPOSITES is critical to the fair presentation, consistency, and comparability of results over time and among FIRMS.
4. *Disclosures*: Disclosures allow FIRMS to elaborate on the raw numbers provided in the presentation and give the end user of the presentation the proper context in which to understand the performance results. To comply with the GIPS standards, FIRMS MUST disclose certain information about their performance presentation and policies adopted by the FIRM. Disclosures are to be considered static information that does not normally change from period to period. Although some disclosures are REQUIRED of all FIRMS, others are specific to certain circumstances and thus may not be REQUIRED. No "negative assurance" language is needed for nonapplicable disclosures.
5. *Presentation and reporting*: After gathering the input data, calculating returns, constructing the COMPOSITES, and determining the necessary disclosures, the FIRM MUST incorporate this information in presentations based on the REQUIREMENTS set out in the GIPS standards for presenting the investment performance returns. No finite set of provisions can cover all potential situations or anticipate future developments in investment industry structure, technology, products, or practices. When appropriate, FIRMS have the responsibility to include other information not necessarily covered by the Standards in a GIPS-compliant presentation.
6. *Real estate*: These provisions apply to all investments where returns are primarily from the holding, trading, development, or management of REAL ESTATE assets. REAL ESTATE

includes land, buildings under development, completed buildings, and other structures or improvements held for investment purposes. The provisions apply regardless of the level of control the FIRM has over management of the investment. The provisions apply irrespective of whether a REAL ESTATE asset or investment is producing revenue. They also apply to REAL ESTATE investments with leverage or gearing.

7. *Private equity*: These provisions apply to all PRIVATE EQUITY investments other than OPEN-END or EVERGREEN FUNDS (which MUST follow the main GIPS provisions). PRIVATE EQUITY investments MUST be valued according to the GIPS PRIVATE EQUITY Valuation Principles found in Appendix D. PRIVATE EQUITY refers to investments in nonpublic companies that are in various stages of development and encompasses venture investing, buyout investing, and mezzanine investing. Fund-of-funds investing as well as secondary investing are also included in PRIVATE EQUITY. Investors typically invest in PRIVATE EQUITY assets either directly or through a fund of funds or LIMITED PARTNERSHIP.

0. Fundamentals of compliance

0.A Definition of the firm – requirements

0.A.1 The GIPS standards MUST be applied on a FIRM-wide basis.
0.A.2 FIRMS MUST be defined as an investment firm, subsidiary, or division held out to clients or potential clients as a DISTINCT BUSINESS ENTITY.
0.A.3 TOTAL FIRM ASSETS MUST be the aggregate of the MARKET VALUE of all discretionary and nondiscretionary assets under management within the defined FIRM. This includes both fee-paying and non-fee-paying assets.
0.A.4 FIRMS MUST include the performance of assets assigned to a subadvisor in a COMPOSITE provided the FIRM has discretion over the selection of the subadvisor.
0.A.5 Changes in a FIRM'S organization are not permitted to lead to alteration of historical COMPOSITE results.

0.B Definition of the firm – recommendations

0.B.1 FIRMS are encouraged to adopt the broadest, most meaningful definition of the FIRM. The scope of this definition SHOULD include all geographical (country, regional, etc.) offices operating under the same brand name regardless of the actual name of the individual investment management company.

0.A Document policies and procedures – requirements

0.A.6 FIRMS MUST document, in writing, their policies and procedures used in establishing and maintaining compliance with all the applicable REQUIREMENTS of the GIPS standards.

0.A Claim of compliance – requirements

0.A.7 Once a FIRM has met all the REQUIRED elements of the GIPS standards, the FIRM MUST use the following compliance statement to indicate that the FIRM is in compliance with the GIPS standards:

> "[Insert name of FIRM] has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®)."

0.A.8 If the FIRM does not meet all the REQUIREMENTS of the GIPS standards, the FIRM cannot represent that it is "in compliance with the Global Investment Performance Standards except for. . .".

0.A.9 Statements referring to the calculation methodology used in a COMPOSITE presentation as being "in accordance [or compliance] with the Global Investment Performance Standards" are prohibited.

0.A.10 Statements referring to the performance of a single, existing client as being "calculated in accordance with the Global Investment Performance Standards" are prohibited except when a GIPS-compliant FIRM reports the performance of an individual account to the existing client.

0.A Firm fundamental responsibilities – requirements

0.A.11 FIRMS MUST make every reasonable effort to provide a compliant presentation to all prospective clients. That is, FIRMS cannot choose to whom they want to present compliant performance. (As long as a prospective client has received a compliant presentation within the previous 12 months, the FIRM has met this REQUIREMENT.)

0.A.12 FIRMS MUST provide a COMPOSITE list and COMPOSITE DESCRIPTION to any prospective client that makes such a request (a sample list and COMPOSITE DESCRIPTION are included in Appendix B). FIRMS MUST list "discontinued" COMPOSITES on the FIRM'S list of COMPOSITES for at least 5 years after discontinuation.

0.A.13 FIRMS MUST provide a compliant presentation for any COMPOSITE listed on the FIRM'S list and a COMPOSITE DESCRIPTION to any prospective client that makes such a request.

0.A.14 When the FIRM jointly markets with other FIRMS, the FIRM claiming compliance with the GIPS standards MUST be sure that it is clearly defined and separate relative to any other FIRMS being marketed and that it is clear which FIRM is claiming compliance.

0.A.15 FIRMS are encouraged to comply with the RECOMMENDATIONS and MUST comply with all applicable REQUIREMENTS of the GIPS standards, including any updates, reports, guidance statements, interpretations, or clarifications published by CFA Institute and the Investment Performance Council, which will be made available via the CFA Institute website (www.cfainstitute.org) as well as the *GIPS Handbook*.

0.B Verification – recommendations

0.B.2 FIRMS are encouraged to undertake the verification process, defined as the review of a FIRM'S performance measurement processes and procedures by an independent third-party verifier. A single verification report is issued in respect to the whole FIRM; verification cannot be carried out for a single COMPOSITE. The primary purpose of verification is to establish that a FIRM claiming compliance with the GIPS standards has adhered to the Standards.

0.B.3 FIRMS that have been verified are encouraged to add a disclosure to COMPOSITE presentations or advertisements stating that the FIRM has been verified. FIRMS MUST disclose the periods of verification if the COMPOSITE presentation includes results for periods that have not been subject to FIRM-wide verification. The verification disclosure language SHOULD read:

> "[Insert name of FIRM] has been verified for the periods [insert dates] by [name of verifier]. A copy of the verification report is available upon request."

1. Input data

1.A Input data – requirements

1.A.1 All data and information necessary to support a FIRM'S performance presentation and to perform the REQUIRED calculations MUST be captured and maintained.
1.A.2 PORTFOLIO valuations MUST be based on MARKET VALUES (not cost basis or book values).
1.A.3 For periods prior to 1 January 2001, PORTFOLIOS MUST be valued at least quarterly. For periods between 1 January 2001 and 1 January 2010, PORTFOLIOS MUST be valued at least monthly. For periods beginning 1 January 2010, FIRMS MUST value PORTFOLIOS on the date of all LARGE EXTERNAL CASH FLOWS.
1.A.4 For periods beginning 1 January 2010, FIRMS MUST value PORTFOLIOS as of the calendar month-end or the last business day of the month.
1.A.5 For periods beginning 1 January 2005, FIRMS MUST use TRADE DATE ACCOUNTING.
1.A.6 ACCRUAL ACCOUNTING MUST be used for fixed-income securities and all other assets that accrue interest income. MARKET VALUES of fixed-income securities MUST include accrued income.
1.A.7 For periods beginning 1 January 2006, COMPOSITES MUST have consistent beginning and ending annual valuation dates. Unless the COMPOSITE is reported on a noncalendar fiscal year, the beginning and ending valuation dates MUST be at calendar year-end (or on the last business day of the year).

1.B Input data – recommendations

1.B.1 ACCRUAL ACCOUNTING SHOULD be used for dividends (as of the ex-dividend date).
1.B.2 When presenting NET-OF-FEES RETURNS, FIRMS SHOULD accrue INVESTMENT MANAGEMENT FEES.
1.B.3 Calendar month-end valuations or valuations on the last business day of the month are RECOMMENDED.

2. Calculation methodology

2.A Calculation methodology – requirements

2.A.1 Total return, including realized and unrealized gains and losses plus income, MUST be used. [corrected September 2005]
2.A.2 TIME-WEIGHTED RATES OF RETURN that adjust for EXTERNAL CASH FLOWS MUST be used. Periodic returns MUST be geometrically linked. EXTERNAL CASH

FLOWS MUST be treated in a consistent manner with the FIRM'S documented, COMPOSITE-specific policy. At a minimum:

a. For periods beginning 1 January 2005, FIRMS MUST use approximated rates of return that adjust for daily-weighted EXTERNAL CASH FLOWS.
b. For periods beginning 1 January 2010, FIRMS MUST value PORTFOLIOS on the date of all LARGE EXTERNAL CASH FLOWS.

2.A.3 COMPOSITE returns MUST be calculated by asset weighting the individual PORTFOLIO returns using beginning-of-period values or a method that reflects both beginning-of-period values and EXTERNAL CASH FLOWS.

2.A.4 Returns from cash and cash equivalents held in PORTFOLIOS MUST be included in TOTAL RETURN calculations.

2.A.5 All returns MUST be calculated after the deduction of the actual TRADING EXPENSES incurred during the period. Estimated TRADING EXPENSES are not permitted.

2.A.6 For periods beginning 1 January 2006, FIRMS MUST calculate COMPOSITE returns by asset weighting the individual PORTFOLIO returns at least quarterly. For periods beginning 1 January 2010, COMPOSITE returns MUST be calculated by asset weighting the individual PORTFOLIO returns at least monthly.

2.A.7 If the actual direct TRADING EXPENSES cannot be identified and segregated from a BUNDLED FEE:

a. when calculating GROSS-OF-FEES RETURNS, returns MUST be reduced by the entire BUNDLED FEE or the portion of the BUNDLED FEE that includes the direct TRADING EXPENSES. The use of estimated TRADING EXPENSES is not permitted.
b. when calculating NET-OF-FEES RETURNS, returns MUST be reduced by the entire BUNDLED FEE or the portion of the BUNDLED FEE that includes the direct TRADING EXPENSES and the INVESTMENT MANAGEMENT FEE. The use of estimated TRADING EXPENSES is not permitted.

2.B Calculation methodology – recommendations

2.B.1 Returns SHOULD be calculated net of nonreclaimable withholding taxes on dividends, interest, and capital gains. Reclaimable withholding taxes SHOULD be accrued.

2.B.2 FIRMS SHOULD calculate COMPOSITE returns by asset weighting the member PORTFOLIOS at least monthly.

2.B.3 FIRMS SHOULD value PORTFOLIOS on the date of all LARGE EXTERNAL CASH FLOWS.

3. Composite construction

3.A Composite construction – requirements

3.A.1 All actual, fee-paying, discretionary PORTFOLIOS MUST be included in at least one COMPOSITE. Although non-fee-paying discretionary PORTFOLIOS may be included in a COMPOSITE (with appropriate disclosures), nondiscretionary PORTFOLIOS are not permitted to be included in a FIRM'S COMPOSITES.

3.A.2 COMPOSITES MUST be defined according to similar investment objectives and/or strategies. The full COMPOSITE DEFINITION MUST be made available on request.

3.A.3 COMPOSITES MUST include new PORTFOLIOS on a timely and consistent basis after the PORTFOLIO comes under management unless specifically mandated by the client.
3.A.4 Terminated PORTFOLIOS MUST be included in the historical returns of the appropriate COMPOSITES up to the last full measurement period that the PORTFOLIO was under management.
3.A.5 PORTFOLIOS are not permitted to be switched from one COMPOSITE to another unless documented changes in client guidelines or the redefinition of the COMPOSITE make it appropriate. The historical record of the PORTFOLIO MUST remain with the appropriate COMPOSITE.
3.A.6 Convertible and other hybrid securities MUST be treated consistently across time and within COMPOSITES.
3.A.7 CARVE-OUT segments excluding cash are not permitted to be used to represent a discretionary PORTFOLIO and, as such, are not permitted to be included in COMPOSITE returns. When a single asset class is carved out of a multiple asset class PORTFOLIO and the returns are presented as part of a single asset COMPOSITE, cash MUST be allocated to the CARVE-OUT returns in a timely and consistent manner. Beginning 1 January 2010, CARVE-OUT returns are not permitted to be included in single asset class COMPOSITE returns unless the CARVE-OUT is actually managed separately with its own cash balance.
3.A.8 COMPOSITES MUST include only assets under management within the defined FIRM. FIRMS are not permitted to link simulated or model PORTFOLIOS with actual performance.
3.A.9 If a FIRM sets a minimum asset level for PORTFOLIOS to be included in a COMPOSITE, no PORTFOLIOS below that asset level can be included in that COMPOSITE. Any changes to a COMPOSITE-specific minimum asset level are not permitted to be applied retroactively.

3.B Composite construction – recommendations

3.B.1 CARVE-OUT returns SHOULD not be included in single asset class COMPOSITE returns unless the CARVE-OUTS are actually managed separately with their own cash balance.
3.B.2 To remove the effect of a significant EXTERNAL CASH FLOW, the use of a TEMPORARY NEW ACCOUNT is RECOMMENDED (as opposed to adjusting the COMPOSITE composition to remove PORTFOLIOS with significant EXTERNAL CASH FLOWS).
3.B.3 FIRMS SHOULD not market a COMPOSITE to a prospective client who has assets less than the COMPOSITE'S minimum asset level.

4. Disclosures

4.A Disclosures – requirements

4.A.1 FIRMS MUST disclose the definition of "FIRM" used to determine the TOTAL FIRM ASSETS and FIRM-wide compliance.
4.A.2 FIRMS MUST disclose the availability of a complete list and description of all of the FIRM'S COMPOSITES.

4.A.3 FIRMS MUST disclose the minimum asset level, if any, below which PORTFOLIOS are not included in a COMPOSITE. FIRMS MUST also disclose any changes to the minimum asset level.
4.A.4 FIRMS MUST disclose the currency used to express performance.
4.A.5 FIRMS MUST disclose the presence, use, and extent of leverage or derivatives (if material), including a sufficient description of the use, frequency, and characteristics of the instruments to identify risks.
4.A.6 FIRMS MUST clearly label returns as GROSS-OF-FEES or NET-OF-FEES.
4.A.7 FIRMS MUST disclose relevant details of the treatment of withholding tax on dividends, interest income, and capital gains. If using indexes that are net-of-taxes, the FIRM MUST disclose the tax basis of the BENCHMARK (e.g. Luxembourg based or U.S. based) versus that of the COMPOSITE.
4.A.8 FIRMS MUST disclose and describe any known inconsistencies in the exchange rates used among the PORTFOLIOS within a COMPOSITE and between the COMPOSITE and the BENCHMARK.
4.A.9 If the presentation conforms with local laws and regulations that differ from the GIPS REQUIREMENTS, FIRMS MUST disclose this fact and disclose the manner in which the local laws and regulations conflict with the GIPS standards.
4.A.10 For any performance presented for periods prior to 1 January 2000 that does not comply with the GIPS standards, FIRMS MUST disclose the period of noncompliance and how the presentation is not in compliance with the GIPS standards.
4.A.11 For periods prior to 1 January 2010, when a single asset class is carved out of a multiple asset PORTFOLIO and the returns are presented as part of a single asset COMPOSITE, FIRMS MUST disclose the policy used to allocate cash to the CARVE-OUT returns.
4.A.12 FIRMS MUST disclose the FEE SCHEDULE appropriate to the presentation.
4.A.13 If a COMPOSITE contains PORTFOLIOS with BUNDLED FEES, FIRMS MUST disclose for each annual period shown the percentage of COMPOSITE assets that is BUNDLED FEE PORTFOLIOS.
4.A.14 If a COMPOSITE contains PORTFOLIOS with BUNDLED FEES, FIRMS MUST disclose the various types of fees that are included in the BUNDLED FEE.
4.A.15 When presenting GROSS-OF-FEES RETURNS, FIRMS MUST disclose if any other fees are deducted in addition to the direct TRADING EXPENSES.
4.A.16 When presenting NET-OF-FEES RETURNS, FIRMS MUST disclose if any other fees are deducted in addition to the INVESTMENT MANAGEMENT FEE and direct TRADING EXPENSES.
4.A.17 FIRMS MUST disclose that ADDITIONAL INFORMATION regarding policies for calculating and reporting returns is available upon request.
4.A.18 Beginning 1 January 2006, FIRMS MUST disclose the use of a subadvisor(s) and the periods a subadvisor(s) was used.
4.A.19 FIRMS MUST disclose all significant events that would help a prospective client interpret the performance record.
4.A.20 FIRMS MUST disclose the COMPOSITE DESCRIPTION.
4.A.21 If a FIRM is redefined, the FIRM MUST disclose the date and reason for the redefinition.
4.A.22 If a FIRM has redefined a COMPOSITE, the FIRM MUST disclose the date and nature of the change. Changes to COMPOSITES are not permitted to be applied retroactively.
4.A.23 FIRMS MUST disclose any changes to the name of a COMPOSITE.

4.A.24 FIRMS MUST disclose the COMPOSITE CREATION DATE.
4.A.25 FIRMS MUST disclose if, prior to 1 January 2010, calendar month-end PORTFOLIO valuations or valuations on the last business day of the month are not used.
4.A.26 FIRMS MUST disclose which DISPERSION measure is presented.

4.B Disclosures – recommendations

4.B.1 If a parent company contains multiple defined FIRMS, each FIRM within the parent company is encouraged to disclose a list of the other FIRMS contained within the parent company.
4.B.2 FIRMS SHOULD disclose when a change in a calculation methodology or valuation source results in a material impact on the performance of a COMPOSITE return.
4.B.3 FIRMS that have been verified SHOULD add a disclosure to their COMPOSITE presentation stating that the FIRM has been verified and clearly indicating the periods the verification covers if the COMPOSITE presentation includes results for periods that have not been subject to FIRM-wide verification.

5. Presentation and reporting

5.A Presentation and reporting – requirements

5.A.1 The following items MUST be reported for each COMPOSITE presented:
  a. At least 5 years of performance (or a record for the period since FIRM or COMPOSITE inception if the FIRM or COMPOSITE has been in existence less than 5 years) that meets the REQUIREMENTS of the GIPS standards; after presenting 5 years of performance, the FIRM MUST present additional annual performance up to 10 years. (For example, after a FIRM presents 5 years of compliant history, the FIRM MUST add an additional year of performance each year so that after 5 years of claiming compliance, the FIRM presents a 10-year performance record.)
  b. Annual returns for all years.
  c. The number of PORTFOLIOS and amount of assets in the COMPOSITE, and either the percentage of the TOTAL FIRM ASSETS represented by the COMPOSITE or the amount of TOTAL FIRM ASSETS at the end of each annual period. If the COMPOSITE contains 5 PORTFOLIOS or less, the number of PORTFOLIOS is not REQUIRED. [corrected September 2005]
  d. A measure of DISPERSION of individual PORTFOLIO returns for each annual period. If the COMPOSITE contains 5 PORTFOLIOS or less for the full year, a measure of DISPERSION is not REQUIRED. [corrected September 2005]
5.A.2 FIRMS may link non-GIPS-compliant returns to their compliant history so long as the FIRMS meet the disclosure REQUIREMENTS for noncompliant performance and only compliant returns are presented for periods after 1 January 2000. (For example, a FIRM that has been in existence since 1995 and that wants to present its entire performance history and claim compliance beginning 1 January 2005 MUST present returns that meet the REQUIREMENTS of the GIPS standards at least from 1 January 2000 and MUST meet the disclosure REQUIREMENTS for any noncompliant history prior to 1 January 2000.)
5.A.3 Returns of PORTFOLIOS and COMPOSITES for periods of less than 1 year are not permitted to be annualized.

5.A.4 a. Performance track records of a past FIRM or affiliation MUST be linked to or used to represent the historical record of a new FIRM or new affiliation if:
   i. Substantially all the investment decision makers are employed by the new FIRM (e.g. research department, PORTFOLIO managers, and other relevant staff),
   ii. The staff and decision-making process remain intact and independent within the new FIRM, and
   iii. The new FIRM has records that document and support the reported performance.
b. The new FIRM MUST disclose that the performance results from the past FIRM are linked to the performance record of the new FIRM,
c. In addition to 5.A.4.a and 5.A.4.b, when one FIRM joins an existing FIRM, performance of COMPOSITES from both FIRMS MUST be linked to the ongoing returns if substantially all the assets from the past FIRM'S COMPOSITE transfer to the new FIRM.
d. If a compliant FIRM acquires or is acquired by a noncompliant FIRM, the FIRMS have 1 year to bring the noncompliant assets into compliance.

5.A.5 Beginning 1 January 2006, if a COMPOSITE includes or is formed using single asset class CARVE-OUTS from multiple asset class PORTFOLIOS, the presentation MUST include the percentage of the COMPOSITE that is composed of CARVE-OUTS prospectively for each period.

5.A.6 The total return for the BENCHMARK (or BENCHMARKS) that reflects the investment strategy or mandate represented by the COMPOSITE MUST be presented for each annual period. If no BENCHMARK is presented, the presentation MUST explain why no BENCHMARK is disclosed. If the FIRM changes the BENCHMARK that is used for a given COMPOSITE in the performance presentation, the FIRM MUST disclose both the date and the reasons for the change. If a custom BENCHMARK or combination of multiple BENCHMARKS is used, the FIRM MUST describe the BENCHMARK creation and re-balancing process. [corrected January 2006]

5.A.7 If a COMPOSITE contains any non-fee-paying PORTFOLIOS, the FIRM MUST present, as of the end of each annual period, the percentage of the COMPOSITE assets represented by the non-fee-paying PORTFOLIOS.

5.B Presentation and reporting – recommendations

5.B.1 It is RECOMMENDED that FIRMS present the following items:
a. COMPOSITE returns gross of INVESTMENT MANAGEMENT FEES and ADMINISTRATIVE FEES and before taxes (except for nonreclaimable withholding taxes),
b. Cumulative returns for COMPOSITE and BENCHMARKS for all periods,
c. Equal-weighted mean and median returns for each COMPOSITE,
d. Graphs and charts presenting specific information REQUIRED or RECOMMENDED under the GIPS standards,
e. Returns for quarterly and/or shorter time periods,
f. Annualized COMPOSITE and BENCHMARK returns for periods greater than 12 months,
g. COMPOSITE-level country and sector weightings.

5.B.2 It is RECOMMENDED that FIRMS present relevant COMPOSITE-level risk measures, such as beta, tracking error, modified duration, information ratio, Sharpe ratio, Treynor ratio, credit ratings, value at risk (VaR), and volatility, over time of the COMPOSITE and BENCHMARK. [corrected September 2005]

5.B.3 After presenting the REQUIRED 5 years of compliant historical performance, the FIRM is encouraged to bring any remaining portion of its *historical* track record into compliance with the GIPS standards. (This does not preclude the REQUIREMENT that the FIRM MUST add annual performance to its track record on an *ongoing* basis to build a 10-year track record.)

6. Real estate

Following are provisions that apply to the calculation and presentation of REAL ESTATE assets. The REAL ESTATE provisions supplement all the REQUIRED and RECOMMENDED elements of the GIPS standards (outlined in Section II.0 through Section II.5), except the REAL ESTATE provisions that override the existing GIPS provisions for valuation: II.6.A.1, II.6.A.2, II.6.B.1, and II.6.B.2. Investment types not considered as REAL ESTATE and, therefore, addressed elsewhere in the general provisions of the GIPS standards include: [corrected January 2006]

- Publicly traded REAL ESTATE securities, including any listed securities issued by public companies,
- Commercial mortgage-backed securities (CMBS),
- Private debt investments, including commercial and residential loans where the expected return is solely related to contractual interest rates without any participation in the economic performance of the underlying REAL ESTATE.

If a PORTFOLIO includes a mix of REAL ESTATE and other investments that are not REAL ESTATE, then these REQUIREMENTS and RECOMMENDATIONS only apply to the REAL ESTATE portion of the PORTFOLIO, and when the FIRM CARVES OUT the REAL ESTATE portion of the PORTFOLIO, the GIPS CARVE OUT provisions (see II.3.A.7) MUST also be applied.

6.A Real estate input data – requirements

6.A.1 REAL ESTATE investments MUST be valued at MARKET VALUE at least once every 12 months. For periods beginning 1 January 2008, REAL ESTATE investments MUST be valued at least quarterly.

6.A.2 REAL ESTATE investments MUST be valued by an external PROFESSIONALLY DESIGNATED, CERTIFIED, OR LICENSED COMMERCIAL PROPERTY VALUER/ APPRAISER at least once every 36 months. In markets where neither professionally designated nor appropriately sanctioned valuers or appraisers are available and valuers or appraisers from other countries bearing such credentials do not commonly operate, then the party responsible for engaging such services locally shall take necessary steps to ensure that only well-qualified property valuers are used.

6.B Real estate input data – recommendations

6.B.1 REAL ESTATE investments SHOULD be valued at least quarterly.

6.B.2 REAL ESTATE investments SHOULD be valued by an external valuer or appraiser at least once every 12 months.

6.B.3 If calculating the INTERNAL RATE OF RETURN, FIRMS SHOULD use quarterly cash flows at a minimum.

6.A Real estate disclosures – requirements

6.A.3 In addition to the other disclosure REQUIREMENTS of the GIPS standards, performance presentations for REAL ESTATE investments MUST disclose:

a. The calculation methodology for component returns – that is, component returns are (1) calculated separately using chain-linked TIMEWEIGHTED RATES OF RETURN, or (2) adjusted such that the sum of the INCOME RETURN and the CAPITAL RETURN is equal to the TOTAL RETURN, [corrected September 2005]
b. The FIRM'S description of discretion,
c. The valuation methods and procedures (e.g. discounted cash flow valuation model, capitalized income approach, sales comparison approach, the valuation of debt payable in determining the value of leveraged REAL ESTATE),
d. The range of performance returns for the individual accounts in the COMPOSITE,
e. The source of the valuation (whether valued by an external valuer or INTERNAL VALUATION or whether values are obtained from a third–party manager) for each period,
f. The percent of total MARKET VALUE of COMPOSITE assets (asset weighted not equally weighted) to total REAL ESTATE assets valued by an EXTERNAL VALUATION for each period, and [corrected September 2005]
g. The frequency REAL ESTATE investments are valued by external valuers.

6.B Real estate disclosures – recommendations

6.B.4 If since-inception INTERNAL RATE OF RETURN performance results are shown, the FIRM SHOULD disclose the time period that is covered as well as the frequency of the cash flows used in the calculation.

6.A Real estate presentation and reporting – requirements

6.A.4 The income and capital appreciation component returns MUST be presented in addition to TOTAL RETURN.

6.B Real estate presentation and reporting – recommendations

6.B.5 When available, the capital and income segments of the appropriate REAL ESTATE BENCHMARK SHOULD be presented.

6.B.6 It is RECOMMENDED that FIRMS present the since-inception INTERNAL RATE OF RETURN for the COMPOSITE.

6.B.7 It is RECOMMENDED that the following items be presented, especially in those circumstances when the investment manager has the ability to control the timing of investor capital call tranches during the fund's or PORTFOLIO'S initial acquisition period:

a. GROSS- and NET-OF-FEES (including incentive allocations) annualized since inception TIME-WEIGHTED RATE OF RETURN and INTERNAL RATE OF RETURN (terminal value based on ENDING MARKET VALUE net assets of the COMPOSITE) to the last year reported for the COMPOSITE.

b. GROSS- and NET-OF-FEES (including incentive allocations) annualized since inception TIME-WEIGHTED RATE OF RETURN and INTERNAL RATE OF RETURN (based on realized cash flows only, excluding unrealized gains) to the last year reported for the COMPOSITE.
c. In addition, other performance measures may provide additional useful information for both prospective and existing investors. The GIPS PRIVATE EQUITY Provisions (see GIPS standards II.7) provide guidance with regard to such additional measures as investment and REALIZATION MULTIPLES and ratios relating to PAID-IN-CAPITAL.

7. Private equity

Following are provisions that apply to the calculation and presentation of PRIVATE EQUITY investments other than OPEN-END or EVERGREEN FUNDS (which MUST follow the main GIPS provisions). The PRIVATE EQUITY provisions supplement all the REQUIRED and RECOMMENDED elements of the GIPS standards (outlined in Section II.0 through Section II.5), except these PRIVATE EQUITY provisions that override the existing GIPS provisions for valuation (II.7.A.1 and II.7.B.1), calculation methodology (II.7.A.2 and II.7.A.3), fees (II.7.A.4 and II.7.A.5), and presentation and reporting of returns (II.7.A.20). [corrected January 2006]

7.A Private equity input data – requirements

7.A.1 PRIVATE EQUITY investments MUST be valued (preferably quarterly but at least annually) according to the GIPS PRIVATE EQUITY Valuation Principles provided in Appendix D.

7.B Private equity input data – recommendations

7.B.1 PRIVATE EQUITY investments SHOULD be valued quarterly.

7.A Private equity calculation methodology – requirements

7.A.2 FIRMS MUST calculate the annualized since-inception INTERNAL RATE OF RETURN (SI-IRR).
7.A.3 The annualized SI-IRR MUST be calculated using either daily or monthly cash flows and the period-end valuation of the unliquidated remaining holdings. Stock DISTRIBUTIONS MUST be valued at the time of DISTRIBUTION.
7.A.4 NET-OF-FEES RETURNS MUST be net of INVESTMENT MANAGEMENT FEES, CARRIED INTEREST, and TRANSACTION EXPENSES.
7.A.5 For INVESTMENT ADVISORS, all returns MUST be net of all underlying partnership and/or fund fees and CARRIED INTEREST. NET-OF-FEES RETURNS MUST, in addition, be net of all the INVESTMENT ADVISOR'S fees, expenses, and CARRIED INTEREST.

7.A. Private equity composite construction – requirements

7.A.6 All CLOSED-END PRIVATE EQUITY investments, including, but not limited to, fund of funds, partnerships, or DIRECT INVESTMENTS, MUST be included in a COMPOSITE defined by strategy and VINTAGE YEAR.

7.A.7 Partnership/fund investments, DIRECT INVESTMENTS, and OPEN-END PRIVATE EQUITY investments (e.g. EVERGREEN FUNDS) MUST be in separate COMPOSITES.

7.A. Private equity disclosures – requirements

7.A.8 FIRMS MUST disclose the VINTAGE YEAR of the COMPOSITE.

7.A.9 For all closed (discontinued) COMPOSITES, FIRMS MUST disclose the final realization (liquidation) date of the COMPOSITE.

7.A.10 FIRMS MUST disclose the unrealized appreciation/depreciation of the COMPOSITE for the most recent period.

7.A.11 FIRMS MUST disclose the total COMMITTED CAPITAL of the COMPOSITE for the most recent period.

7.A.12 For the most recent period, FIRMS MUST disclose the valuation methodologies used to value their PRIVATE EQUITY investments. If any change occurs in either valuation basis or methodology from the prior period, the change MUST be disclosed.

7.A.13 If the presentation complies with any local or regional valuation guidelines in addition to the GIPS PRIVATE EQUITY Valuation Principles, FIRMS MUST disclose which local or regional guidelines have been used.

7.A.14 FIRMS MUST document the FIRM'S valuation review procedures and disclose that the procedures are available upon request.

7.A.15 FIRMS MUST disclose the definition of the COMPOSITE investment strategy (e.g. early stage, development, buy-outs, generalist, turnaround, mezzanine, geography, middle market, and large transaction).

7.A.16 If a BENCHMARK is used, FIRMS MUST disclose the calculation methodology used for the BENCHMARK.

7.A.17 If a valuation basis other than FAIR VALUE is used to value investments within the COMPOSITE, FIRMS MUST disclose for the most recent period presented their justification for why FAIR VALUE is not applicable. Additionally, FIRMS MUST disclose the following:

a. The carrying value of non-FAIR-VALUE-basis investments relative to total fund.
b. The number of holdings valued on a non-FAIR-VALUE basis.
c. The absolute value of the non-FAIR-VALUE-basis investments.

7.A.18 FIRMS MUST disclose whether they are using daily or monthly cash flows in the SI-IRR calculation.

7.A.19 If a FIRM does not use a calendar year period-end, a disclosure MUST be made indicating the period-end used.

7.A Private equity presentation and reporting – requirements

7.A.20 FIRMS MUST present both the NET-OF-FEES and GROSS-OF-FEES annualized SI-IRR of the COMPOSITE for each year since inception.

7.A.21 For each period presented, FIRMS MUST report:
   a. PAID-IN CAPITAL to date (cumulative DRAWDOWN),
   b. Total current INVESTED CAPITAL, and
   c. Cumulative DISTRIBUTIONS to date.

7.A.22 For each period presented, FIRMS MUST report the following multiples:
   a. TOTAL VALUE to PAID-IN CAPITAL (INVESTMENT MULTIPLE or TVPI),
   b. Cumulative DISTRIBUTIONS to PAID-IN CAPITAL (REALIZATION MULTIPLE or DPI),
   c. PAID-IN CAPITAL to COMMITTED CAPITAL (PIC MULTIPLE), and
   d. RESIDUAL VALUE TO PAID-IN CAPITAL (RVPI).

7.A.23 If a BENCHMARK is shown, the cumulative annualized SI-IRR for the BENCHMARK that reflects the same strategy and VINTAGE YEAR of the COMPOSITE MUST be presented for the same periods for which the COMPOSITE is presented. If no BENCHMARK is shown, the presentation MUST explain why no BENCHMARK is disclosed.

7.B Private equity presentation and reporting – recommendations

7.B.2 FIRMS SHOULD present the average holding period of the investments (PORTFOLIO companies) over the life of the COMPOSITE.

III. VERIFICATION

The primary purpose of verification is to establish that a FIRM claiming compliance with the GIPS standards has adhered to the Standards. Verification will also increase the understanding and professionalism of performance measurement teams and consistency of presentation of performance results.

The verification procedures attempt to strike a balance between ensuring the quality, accuracy, and relevance of performance presentations and minimizing the cost to FIRMS of independent review of performance results. FIRMS SHOULD assess the benefits of improved internal processes and procedures, which are as significant as the marketing advantages of verification.

The goal of the IPC in drafting the verification procedures is to encourage broad acceptance of verification.

Verification is strongly encouraged and is expected to become mandatory at a future date. The IPC will re-evaluate all aspects of mandatory verification by 2010 and provide the industry sufficient time to implement any changes.

A. Scope and purpose of verification

1. Verification is the review of an investment management FIRM'S performance measurement processes and procedures by an independent third-party "verifier." Verification tests:
   a. Whether the FIRM has complied with all the COMPOSITE construction REQUIREMENTS of the GIPS standards on a FIRM-wide basis, and
   b. Whether the FIRM'S processes and procedures are designed to calculate andSOFTPWSHANKARBREAK;present performance results in compliance with the GIPS standards.

A single verification report is issued in respect of the whole FIRM; verification cannot be carried out for a single COMPOSITE.

2. Third-party verification brings credibility to the claim of compliance and supports the overall guiding principles of full disclosure and fair representation of investment performance.
3. The initial minimum period for which verification can be performed is 1 year of a FIRM'S presented performance. The RECOMMENDED period over which verification is performed is that part of the FIRM'S track record for which GIPS compliance is claimed.
4. A verification report must confirm that:
   a. The FIRM has complied with all the COMPOSITE construction REQUIREMENTS of the GIPS standards on a FIRM-wide basis, and
   b. The FIRM'S processes and procedures are designed to calculate and present performance results in compliance with the GIPS standards.

   Without such a report from the verifier, the FIRM cannot state that its claim of compliance with the GIPS standards has been verified.
5. After performing the verification, the verifier may conclude that the FIRM is not in compliance with the GIPS standards or that the records of the FIRM cannot support a complete verification. In such situations, the verifier must issue a statement to the FIRM clarifying why a verification report was not possible.
6. A principal verifier may accept the work of a local or previous verifier as part of the basis for the principal verifier's opinion.
7. The minimum GIPS verification procedures are described in section III.B Required Verification Procedures.

B. Required verification procedures

The following are the minimum procedures that verifiers must follow when verifying an investment FIRM'S compliance with the GIPS standards. Verifiers must follow these procedures prior to issuing a verification report to the FIRM:

1. Pre-verification procedures
   a. *Knowledge of the FIRM*: Verifiers must obtain selected samples of the FIRM'S investment performance reports and other available information regarding the FIRM to ensure appropriate knowledge of the FIRM.
   b. *Knowledge of GIPS standards*: Verifiers must understand all the REQUIREMENTS and RECOMMENDATIONS of the GIPS standards, including any updates, reports, guidance statements, interpretations, and clarifications published by CFA Institute and the Investment Performance Council, which will be made available via the CFA Institute website (www.cfainstitute.org) as well as the *GIPS Handbook*. All clarification and update information must be considered when determining a FIRM'S claim of compliance.
   c. *Knowledge of the performance standards*: Verifiers must be knowledgeable of country-specific laws and regulations applicable to the FIRM and must determine any differences between the GIPS standards and the country-specific laws and regulations.
   d. *Knowledge of FIRM policies*: Verifiers must determine the FIRM'S assumptions and policies for establishing and maintaining compliance with all applicable REQUIREMENTS of the GIPS standards. At a minimum, verifiers must determine the following policies and procedures of the FIRM:
      i. Policy with regard to investment discretion. The verifier must receive from the FIRM, in writing, the FIRM'S definition of investment discretion and the FIRM'S guidelines for determining whether accounts are fully discretionary;

ii. Policy with regard to the definition of COMPOSITES according to investment strategy. The verifier must obtain the FIRM'S list of COMPOSITE DEFINITIONS with written criteria for including accounts in each COMPOSITE;
iii. Policy with regard to the timing of inclusion of new accounts in the COMPOSITES;
iv. Policy with regard to timing of exclusion of closed accounts in the COMPOSITES;
v. Policy with regard to the accrual of interest and dividend income;
vi. Policy with regard to the market valuation of investment securities;
vii. Method for computing the TIME-WEIGHTED-RATE OF RETURN for the portfolio;
viii. Assumptions on the timing of capital inflows/outflows;
ix. Method for computing COMPOSITE returns;
x. Policy with regard to the presentation of COMPOSITE returns;
xi. Policies regarding timing of implied taxes due on income and realized capital gains for reporting performance on an after-tax basis;
xii. Policies regarding use of securities/countries not included in a COMPOSITE'S BENCHMARK;
xiii. Use of leverage and other derivatives; and
xiv. Any other policies and procedures relevant to performance presentation.

e. *Knowledge of valuation basis for performance calculations*: Verifiers must ensure that they understand the methods and policies used to record valuation information for performance calculation purposes. In particular, verifiers must determine that:
   i. The FIRM'S policy on classifying fund flows (e.g. injections, disbursements, dividends, interest, fees, and taxes) is consistent with the desired results and will give rise to accurate returns;
   ii. The FIRM'S accounting treatment of income, interest, and dividend receipts is consistent with cash account and cash accruals definitions;
   iii. The FIRM'S treatment of taxes, tax reclaims, and tax accruals is correct and the manner used is consistent with the desired method (i.e. gross- or net-of-tax return);
   iv. The FIRM'S policies on recognizing purchases, sales, and the opening and closing of other positions are internally consistent and will produce accurate results; and
   v. The FIRM'S accounting for investments and derivatives is consistent with the GIPS standards.

2. Verification procedures
   a. *Definition of the FIRM*: Verifiers must determine that the FIRM is, and has been, appropriately defined.
   b. *COMPOSITE construction*. Verifiers must be satisfied that:
      i. The FIRM has defined and maintained COMPOSITES according to reasonable guidelines in compliance with the GIPS standards;
      ii. All the FIRM'S actual discretionary fee-paying PORTFOLIOS are included in a COMPOSITE;
      iii. The FIRM'S definition of discretion has been consistently applied over time;
      iv. At all times, all accounts are included in their respective COMPOSITES and no accounts that belong in a particular COMPOSITE have been excluded;
      v. COMPOSITE BENCHMARKS are consistent with COMPOSITE DEFINITIONS and have been consistently applied over time;
      vi. The FIRM'S guidelines for creating and maintaining COMPOSITES have been consistently applied; and
      vii. The FIRM'S list of COMPOSITES is complete.

c. *Nondiscretionary accounts*. Verifiers must obtain a listing of all FIRM PORTFOLIOS and determine on a sampling basis whether the manager's classification of the account as discretionary or nondiscretionary is appropriate by referring to the account's agreement and the FIRM'S written guidelines for determining investment discretion.
d. *Sample account selection*: Verifiers must obtain a listing of open and closed accounts for all COMPOSITES for the years under examination. Verifiers may check compliance with the GIPS standards using a selected sample of a FIRM'S accounts. Verifiers SHOULD consider the following criteria when selecting the sample accounts for examination:
   i. Number of COMPOSITES at the FIRM;
   ii. Number of PORTFOLIOS in each COMPOSITE;
   iii. Nature of the COMPOSITE;
   iv. Total assets under management;
   v. Internal control structure at the FIRM (system of checks and balances in place);
   vi. Number of years under examination; and
   vii. Computer applications, software used in the construction and maintenance of COMPOSITES, the use of external performance measurers, and the calculation of performance results.

   This list is not all-inclusive and contains only the minimum criteria that SHOULD be used in the selection and evaluation of a sample for testing. For example, one potentially useful approach would be to choose a PORTFOLIO for the study sample that has the largest impact on COMPOSITE performance because of its size or because of extremely good or bad performance. The lack of explicit record keeping or the presence of errors may warrant selecting a larger sample or applying additional verification procedures.
e. *Account review*: For selected accounts, verifiers must determine:
   i. Whether the timing of the initial inclusion in the COMPOSITE is in accordance with policies of the FIRM;
   ii. Whether the timing of exclusion from the COMPOSITE is in accordance with policies of the FIRM for closed accounts;
   iii. Whether the objectives set forth in the account agreement are consistent with the manager's COMPOSITE DEFINITION as indicated by the account agreement, PORTFOLIO summary, and COMPOSITE DEFINITION;
   iv. The existence of the accounts by tracing selected accounts from account agreements to the COMPOSITES;
   v. That all PORTFOLIOS sharing the same guidelines are included in the same COMPOSITE; and
   vi. That shifts from one COMPOSITE to another are consistent with the guidelines set forth by the specific account agreement or with documented guidelines of the FIRM'S clients.
f. *Performance measurement calculation*: Verifiers must determine whether the FIRM has computed performance in accordance with the policies and assumptions adopted by the FIRM and disclosed in its presentations. In doing so, verifiers SHOULD:
   i. Recalculate rates of return for a sample of accounts in the FIRM using an acceptable return formula as prescribed by the GIPS standards (e.g. TIME-WEIGHTED RATE OF RETURN); and

ii. Take a reasonable sample of COMPOSITE calculations to assure themselves of the accuracy of the asset weighting of returns, the geometric linking of returns to produce annual rates of returns, and the calculation of the DISPERSION of individual returns around the aggregate COMPOSITE return.

g. *Disclosures*: Verifiers must review a sample of COMPOSITE presentations to ensure that the presentations include the information and disclosures REQUIRED by the GIPS standards.

h. *Maintenance of records*: The verifier must maintain sufficient information to support the verification report. The verifier must obtain a representation letter from the client FIRM confirming major policies and any other specific representations made to the verifier during the examination.

C. Detailed examinations of investment performance presentations

Separate from a GIPS verification, a FIRM may choose to have a further, more extensive, specifically focused examination (or performance audit) of a specific COMPOSITE presentation.

FIRMS cannot make any claim that a particular COMPOSITE has been independently examined with respect to the GIPS standards unless the verifier has also followed the GIPS verification procedures set forth in section III.B. FIRMS cannot state that a particular COMPOSITE presentation has been "GIPS verified" or make any claim to that affect. GIPS verification relates only to FIRM-wide verification. FIRMS can make a claim of verification only after a verifier has issued a GIPS verification report.

To assert a verification report has been received, a detailed examination of a COMPOSITE presentation is not REQUIRED. Examinations of this type are unlikely to become a REQUIREMENT of the GIPS standards or become mandatory.

APPENDIX A: SAMPLE GIPS-COMPLIANT PRESENTATIONS

Example 1

Sample 1 Investment Firm
Balanced Composite
1 January 1995 through 31 December 2004

	Gross-of-Fees Return	Net-of-Fees Return	Benchmark Return	Number of Portfolios	Internal Dispersion	Total Composite	Total Firm Assets
1995	16.0	15.0	14.1	26	4.5	165	236
1996	2.2	1.3	1.8	32	2.0	235	346
1997	22.4	21.5	24.1	38	5.7	344	529
1998	7.1	6.2	6.0	45	2.8	445	695
1999	8.5	7.5	8.0	48	3.1	520	839
2000	−8.0	−8.9	−8.4	49	2.8	505	1014
2001	−5.9	−6.8	−6.2	52	2.9	499	995
2002	2.4	1.6	2.2	58	3.1	525	1125
2003	6.7	5.9	6.8	55	3.5	549	1225
2004	9.4	8.6	9.1	59	2.5	575	1290

Sample 1 Investment Firm has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®).

Notes:

1. Sample 1 Investment Firm is a balanced portfolio investment manager that invests solely in Canadian securities. Sample 1 Investment Firm is defined as an independent investment management firm that is not affiliated with any parent organization. For the periods from 2000 through 2004, Sample 1 Investment Firm has been verified by Verification Services Inc. A copy of the verification report is available upon request. Additional information regarding the firm's policies and procedures for calculating and reporting performance results is available upon request.
2. The composite includes all nontaxable balanced portfolios with an asset allocation of 30% S&P TSX and 70% Scotia Canadian Bond Index Fund, which allow up to a 10% deviation in asset allocation.
3. The benchmark: 30% S&P TSX; 70% Scotia Canadian Bond Index Fund rebalanced monthly.
4. Valuations are computed and performance reported in Canadian dollars.
5. Gross-of-fees performance returns are presented before management and custodial fees but after all trading expenses. Returns are presented net of nonreclaimable withholding taxes. Net-of-fees performance returns are calculated by deducting the highest fee of 0.25% from the quarterly gross composite return. The management fee schedule is as follows: 1.00% on first CAD25M; 0.60% thereafter.
6. This composite was created in February 1995. A complete list and description of firm composites is available upon request.
7. For the periods 1995 and 1996, Sample 1 Investment Firm was not in compliance with the GIPS standards because portfolios were valued annually.
8. Internal dispersion is calculated using the equal-weighted standard deviation of all portfolios that were included in the composite for the entire year.

Compliance statement

Sample 2 Asset Management Company has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®).

Example 2

Sample 2 Asset Management Company
Equities World BM MSCI Active Mandates Direct
Reporting Currency CHF — Creation Date 01 July 1999

Period	MSCI World Total Return (%)	(ri) in CHF Benchmark Return (%)	Number of Portfolios	Composite Dispersion (Range)	Total Composite Assets (millions)	Percentage of Firm Assets (%)
2004	18.0	19.6	6	0.2	84.3	<0.1
2003	−35.3	−33.0	8	0.7	126.6	0.1
2002	−16.0	−14.5	8	1.5	233.0	0.2
2001	−13.5	−11.8	7	1.3	202.1	0.2
2000	60.2	46.1	<5	N/A	143.7	0.2
1999	21.3	17.5	<5	N/A	62.8	<0.1
1998	22.5	26.3	<5	N/A	16.1	<0.1

Definition of the firm

Sample 2 Asset Management Company is an independent investment management firm established in 1997. Sample 2 Asset Management Company manages a variety of equity, fixed income, and balanced assets for primarily Swiss and European clients. Additional information regarding the firm's policies and procedures for calculating and reporting performance returns is available upon request.

Benchmark

Sources of foreign exchange rates may be different between the composite and the benchmark.

Fees

Performance figures are presented gross of management fees, custodial fees, and withholding taxes but net of all trading expenses.

List of composites

A complete listing and description of all composites is available on request.

Verification

Sample 2 Asset Management Company has been verified by an independent verifier on an annual basis from 1998 through 2003.

Fee schedule

The standard fixed management fee for accounts with assets under management of up to CHF50 million is 0.35% per annum.

Minimum account size

The minimum portfolio size for inclusion in Equities World BM MSCI composite is CHF1 million.

Example 3: Sample 3 Realty management firm – DISCLOSURES

Compliance statement

Sample 3 Realty Management Firm has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®).

The firm

Sample 3 Realty Management Firm (the "Firm"), a subsidiary of ABC Capital, Inc., is a registered investment adviser under the Investment Advisors Act of 1940. The Firm exercises complete discretion over the selection, capitalization, asset management, and disposition of investments in wholly-owned properties and joint ventures. A complete list and description of the Firm's composites is available upon request.

Schedule of Performance Results
Sample 3 Realty Management Firm
Core Real Estate Composite
January 1, 1995 through December 31, 2004

Year	Income Return	Capital Return	TOTAL	Range of Returns	Composite Dispersion	NCREIF Property Index Benchmark −5.6%	Number of Portfolios	Net Assets (USD Millions)	Percent Leverage	External Valuation	Firm Total Net Assets (USD) Million	Percent of Firm Assets
1995	5.1%	−4.0%	0.8%	0.7–1.0	NA		5	$79	43%	100%	$950	8%
1996	5.5%	−0.9%	4.5%	4.0–5.0	NA		5	$143	49%	100%	$989	14%
1997	6.9%	−1.5%	5.3%	5.0–5.4	NA		5	$217	56%	100%	$1,219	18%
1998	8.1%	0.9%	9.1%	8.9–9.7	NA		5	$296	54%	100%	$1,375	22%
1999	8.9%	1.7%	10.8%	9.9–11.0	NA		5	$319	50%	100%	$1,425	22%
2000	9.0%	0.5%	9.6%	9.1–10.9	0.7		5	$367	45%	100%	$1,532	24%
2001	9.1%	1.2%	10.5%	10.0–10.7	0.3		5	$349	39%	100%	$1,712	20%
2002	7.9%	1.8%	9.9%	9.8–10.5	0.3		5	$398	31%	100%	$1,796	22%
2003	8.5%	2.9%	11.5%	10.9–12.0	0.5		5	$425	28%	100%	$1,924	22%
2004	8.2%	2.5%	10.8%	9.9–11.8	0.8		5	$432	22%	100%	$1,954	22%
Annualized Since Inception Time-Weighted Returns:												
	7.7%	0.5%	8.0%									
Annualized Since Inception Internal Rate of Return:												
			7.8%									

The composite

The Core Real Estate Composite (the "Composite") comprises all actual fee-paying discretionary portfolios managed by the Firm with a core investment and risk strategy with an income focus having a minimum initial portfolio size of $10 million. Portfolios that initially qualify are excluded later from the composite if their asset size decreases below the minimum requirement due to capital distributions. The Composite was created in 1998. Composite dispersion is measured using an asset-weighted standard deviation of returns of the portfolios.

Valuation

Assets are valued quarterly by the Firm and appraised annually by an independent Member of the Appraisal Institute. Both the internal and external property valuations rely primarily on the application of market discount rates to future projections of free cash flows (unleveraged cash flows) and capitalized terminal values over the expected holding period for each property. Property mortgages, notes, and loans are marked to market using prevailing interest rates for comparable property loans if the terms of existing loans preclude the immediate repayment of such loans. Loan repayment fees, if any, are considered in the projected year of sale.

Calculation of performance returns

Returns presented are denominated in United States dollars. Returns are presented net of leverage. Composite returns are calculated on an asset-weighted average basis using beginning-of-period values. Returns include cash and cash equivalents and related interest income. Income return is based on accrual recognition of earned income. Capital expenditures, tenant improvements, and lease commissions are capitalized and included in the cost of the property, are not amortized, and are reconciled through the valuation process and reflected in the capital return component. Income and capital returns may not equal total returns due to chainlinking of quarterly returns. Annual returns are time-weighted rates of return calculated by linking quarterly returns. For the annualized since-inception time-weighted return, terminal value is based on ending market value of net assets of the Composite. For the since-inception internal rate of return, contributions from and distributions to investors since January 1, 1995, and a terminal value equal to the composite's ending market value of net assets as of December 31, 2004, are used. The IRR is calculated using monthly cash flows. Additional information regarding policies for calculating and reporting returns in compliance with the GIPS standards is available upon request.

Investment management fees

Some of the portfolios pay incentive fees ranging between 10% and 20% of IRR in excess of established benchmarks. Current annual investment advisory fees are as follows:

- Up to $30 million: 1.6%
- $ 30–$50 million: 1.3%
- over $50 million: 1.0%

NCREIF Property Index benchmark

The National Council of Real Estate Investment Fiduciaries (NCREIF) Property Index benchmark has been taken from published sources. The NCREIF Property Index is unleveraged, includes various real estate property types, excludes cash and other nonproperty related assets and liabilities, income, and expenses. The calculation methodology for the index is not consistent with calculation methodology employed for the Composite because the benchmark computes the total return by adding the income and capital appreciation return on a quarterly basis.

Example 4

Sample 4 Private Equity Partners
Buy-out Composite
1 January 1995 through 31 December 2002

Year	Annualized SI-IRR Gross-of-Fees (%)	Annualized SI-IRR Net-of-Fees (%)	Benchmark Return (%)	Composite Assets (USD$ mil)	Total Firm Assets (USD$ mil)
1995	(7.5)	(11.07)	(9.42)	4.31	357.36
1996	6.2	4.53	2.83	10.04	402.78
1997	13.8	10.10	14.94	14.25	530.51
1998	13.1	9.28	14.22	25.21	613.73
1999	53.2	44.53	37.43	54.00	871.75
2000	40.6	26.47	32.97	24.25	1,153.62
2001	29.9	21.86	27.42	8.25	1,175.69
2002	25.3	17.55	25.24	10.25	1,150.78

Year	Paid-in Capital (USD$ mil)	Invested Capital (USD$ mil)	Cumulative Distributions (USD$ mil)	Investment Multiple (TVPI)	Realization Multiple (DPI)	PIC	RVPI
1995	4.68	4.68	0.00	0.92	0.00	0.19	0.92
1996	9.56	9.56	0.00	1.05	0.00	0.38	1.05
1997	14.54	12.91	2.55	1.16	0.18	0.58	0.98
1998	23.79	22.15	2.55	1.17	0.11	0.95	1.06
1999	25.00	19.08	15.78	2.79	0.63	1.00	2.16
2000	25.00	17.46	27.44	2.07	1.10	1.00	0.97
2001	25.00	14.89	39.10	1.89	1.56	1.00	0.33
2002	25.00	13.73	41.25	2.06	1.65	1.00	0.41

TVPI = Total Value to Paid-in Capital
DPI = Distributed Capital to Paid-in Capital
PIC = Paid-in Capital to Committed Capital
RVPI = Residual Value to Paid-in Capital

Sample 4 Private Equity Partners has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®).

Example 4: Sample 4 Private Equity Partners – disclosures

Sample 4 Private Equity Partners is an independent private equity investment firm, having offices in London, New York, and San Francisco. The Sample 4 Buy-out Composite invests in private equity buyouts and was created in January 1995.

The Sample 4 Buy-out Composite complies with the XYZ Venture Capital Association's valuation guidelines. Valuations are prepared by Sample 4's valuations committee and reviewed by an independent advisory board. Sample 4 follows the fair value basis of valuation as recommended in the GIPS Private Equity Valuation Principles. All investments within the Sample 4 Buy-out Composite are valued either using a most recent transaction or an earnings multiple. Sample 4's valuation review procedures are available upon request.

The GP-BO index is used as the benchmark and is constructed as the QRS index return plus 500 basis points. The benchmark return is calculated using monthly cash flows. There is only one fund in the composite for all time periods, and the dispersion of portfolio returns within the composite, therefore, is zero for all years.

The vintage year of the Sample 4 Buy-out Fund is 1995, and total committed capital is USD$25 million. The total composite assets (unrealized gains) are USD$10.25 million as of 31 December 2002.

The fund's SI-IRR calculation incorporates monthly cash flows.

A complete list of firm composites and composite performance results is available upon request. Additional information regarding the firm's policies and procedures for calculating and reporting performance results is available upon request.

The standard fee schedule currently in effect is as follows: 1.00% of assets under management. In addition, there is a 20% incentive fee for all assets. The incentive fee is applied to the value added in excess of fees, expenses, and the return of the GP-BO Index. [corrected October 2006]

APPENDIX B: SAMPLE LIST AND DESCRIPTION OF COMPOSITES

Sample Asset Management Firm

List and description of composites

The *Small Cap Growth Composite* includes all institutional portfolios invested in U.S. equities with strong earnings and growth characteristics and small capitalizations. The benchmark is the Russell 2000® Growth Index.

The *Large Cap Growth Composite* includes all institutional portfolios invested in U.S. equities with strong earnings and growth characteristics and large capitalizations. The benchmark is the Russell 1000® Growth Index.

The *Core Fixed Income Composite* includes all institutional portfolios invested in fixed securities. Portfolios within the composite will have a duration that is plus or minus 20 percent of the benchmark. The benchmark is the Lehman Brothers Aggregate Bond Index.

The *Intermediate Fixed Income Composite* includes all institutional portfolios invested in fixed securities. Portfolios within the composite will have a duration that is plus or minus 20 percent of the benchmark. The benchmark is the Lehman Brothers Intermediate Aggregate Bond Index.

The *High Yield Fixed Income Composite* includes all institutional portfolios invested in high yield debt securities. The benchmark is the Lehman Brothers U.S. Corporate High Yield Bond Index.

The *Balanced Growth Composite* includes all institutional balanced portfolios that have a 50–70% allocation to growth equities, with a typical allocation between 55–65%. The benchmark is 60% S&P 500® and 40% Lehman Brothers Aggregate Bond Index. Only portfolios greater than $5 million are included in the composite.

Terminated composites

The *GARP Equity Composite* includes all institutional portfolios invested in growth stocks that are reasonably priced and valued "cheap" compared with their peers. The benchmark is the S&P 500® Index. The composite terminated in November 2003.

The *Small-Mid Cap Growth Composite* includes all institutional portfolios invested in U.S. equities with strong earnings and growth characteristics. The benchmark is the Russell 2500® Growth Index. The composite terminated in February 2004.

APPENDIX C: GIPS ADVERTISING GUIDELINES

A. Purpose of the GIPS advertising Guidelines

The Global Investment Performance Standards provide the investment community with a set of ethical standards for FIRMS to follow when presenting their performance results to potential clients. The Standards serve to provide greater uniformity and comparability among investment managers without regard to geographical location and to facilitate a dialogue between FIRMS and their prospective clients about the critical issues of how the FIRM achieved historical performance results and determines future investment strategies.

The GIPS Advertising Guidelines attempt to serve as industry global best practice for the advertisement of performance results. The GIPS Advertising Guidelines do not replace the GIPS standards nor do they absolve FIRMS from presenting performance presentations that adhere to the REQUIREMENTS of the full GIPS standards. The guidelines only apply to FIRMS that already satisfy all the REQUIREMENTS of the Standards on a FIRM-wide basis and claim compliance with the Standards. FIRMS that claim compliance can choose to advertise that claim using the GIPS Advertising Guidelines.

The guidelines are mandatory for FIRMS that include a claim of compliance with the GIPS Advertising Guidelines in their advertisements. The guidelines are voluntary for FIRMS that do not include a claim of compliance in their advertisements. All FIRMS are encouraged to abide by these ethical guidelines.

Definition of advertisement

For the purposes of these guidelines, an advertisement includes any materials that are distributed to or designed for use in newspapers, magazines, FIRM brochures, letters, media, or any other written or electronic material addressed to more than one prospective client. Any written material (other than one-on-one presentations and individual client reporting) distributed to maintain existing clients or solicit new clients for an advisor is considered an advertisement.

Relationship of GIPS Advertising Guidelines to regulatory requirements

The GIPS Advertising Guidelines are guidelines that promote an ethical framework for advertisements. They do not change the scope of the activities of local regulatory bodies regarding the regulation of advertisements. FIRMS advertising performance results MUST also adhere to all applicable regulatory rules and requirements governing advertisements. FIRMS are encouraged to seek legal or regulatory counsel because it is likely that additional disclosures are REQUIRED. In cases where applicable law or regulation conflicts with the GIPS Advertising Guidelines, the guidelines REQUIRE FIRMS to comply with the law or regulation. FIRMS MUST disclose any conflicts between laws/regulations and the GIPS Advertising Guidelines.

The calculation and advertisement of pooled unitized products, such as mutual funds and open-ended investment companies, are regulated in most markets. These advertising guidelines are not intended to replace the regulations when a FIRM is advertising performance solely for a pooled unitized product. However, should a GIPS-compliant FIRM choose to advertise performance results, the FIRM MUST apply all applicable laws and regulations as well as the GIPS Advertising Guidelines in order to include a claim of compliance with the GIPS standards.

B. Requirements of the GIPS Advertising Guidelines

All advertisements that include a claim of compliance with the GIPS Advertising Guidelines MUST include the following:

1. A description of the FIRM.
2. How an interested party can obtain a presentation that complies with the REQUIREMENTS of GIPS standards and/or a list and description of all FIRM COMPOSITES.
3. The GIPS Advertising Guidelines compliance statement:

   [Insert name of firm] claims compliance with the Global Investment Performance Standards (GIPS®).

All advertisements that include a claim of compliance with the GIPS Advertising Guidelines and that present performance results MUST also include the following information (the relevant information MUST be taken/derived from a presentation that adheres to the REQUIREMENTS of the GIPS standards):

4. A description of the strategy of the COMPOSITE being advertised.
5. Period-to-date COMPOSITE performance results in addition to either:
   a. 1-, 3-, and 5-year cumulative annualized COMPOSITE returns with the end-of-period date clearly identified (or annualized period since COMPOSITE inception if inception is greater than 1 and less than 5 years). Periods of less than 1 year are not permitted to be annualized. The annualized returns MUST be calculated through the same period of time as presented in the corresponding compliant presentation; or
   b. years of annual COMPOSITE returns with the end-of-period date clearly identified (or since COMPOSITE inception if inception is less than 5 years). The annual returns MUST be calculated through the same period of time as presented in the corresponding compliant presentation.

6. Whether performance is shown gross and/or net of INVESTMENT MANAGEMENT FEES.
7. The BENCHMARK TOTAL RETURN for the same periods for which the COMPOSITE return is presented and a description of that BENCHMARK. (The appropriate COMPOSITE BENCHMARK return is the same BENCHMARK TOTAL RETURN as presented in the corresponding GIPS-compliant presentation.) If no BENCHMARK is presented, the advertisement MUST disclose why no BENCHMARK is presented.
8. The currency used to express returns.
9. The description of the use and extent of leverage and derivatives if leverage or derivatives are used as an active part of the investment strategy (i.e. not merely for efficient PORTFOLIO management) of the COMPOSITE. Where leverage/derivatives do not have a material effect on returns, no disclosure is REQUIRED.
10. When presenting noncompliant performance information for periods prior to 1 January 2000 in an advertisement, FIRMS MUST disclose the period(s) and which specific information is not compliant as well as provide the reason(s) the information is not in compliance with the GIPS standards.

Additional and supplemental information

FIRMS are encouraged to present SUPPLEMENTAL INFORMATION or ADDITIONAL INFORMATION (in addition to the information REQUIRED under the GIPS Advertising Guidelines) provided the SUPPLEMENTAL INFORMATION is clearly labeled as such and shown with equal or lesser prominence than the information REQUIRED under the guidelines. Where such SUPPLEMENTAL INFORMATION is included for noncompliant periods, these periods MUST be disclosed together with an explanation of what information is not compliant and why it is not in compliance with the GIPS standards.

SUPPLEMENTAL and ADDITIONAL INFORMATION is the subject of the "Guidance Statement on the Use of Supplemental Information" and users should refer to that guidance for further clarification on how to disclose such data.

Sample advertisements

Sample advertisement without performance returns

Sample 4 Investments

Sample 4 Investments is the institutional asset management division of Sample 4 Plc and is a registered investment advisory firm specializing in qualitative, growth-oriented investment management.

Sample 4 Investments claims compliance with the Global Investment Performance Standards (GIPS®).

To receive a complete list and description of Sample 4 Investments' composites and/or a presentation that adheres to the GIPS standards, contact John Doe at (800) 555-1234, or write to Sample 4 Investments, 123 Main Street, Resultland 12345, or e-mail jdoe@sample4investments.com

Sample advertisement including performance returns (1-, 3-, and 5-year annualized)

Sample 4 Investments: Global Equity Growth Composite Performance				
	Ending 31 Mar 04	Ending 31 Dec 03		
Results shown in US$ before fees	Period to date (3 mths)	1 year	3 years per annum	5 years per annum
Global Equity Growth	–3.84%	–19.05%	–14.98%	0.42%
MSCI World Index	–4.94%	–19.54%	–16.37%	–1.76%

Sample 4 Investments is the institutional asset management subsidiary of Sample 4 plc and is a registered investment advisor specializing in qualitative, growth-oriented investment management. The Global Equity Growth Composite strategy focuses on earnings, growth of earnings, and key valuation metrics.

- Sample 4 Investments claims compliance with the Global Investment Performance Standards (GIPS®).
- To receive a complete list and description of Sample 4 Investments' composites and/or a presentation that adheres to the GIPS standards, contact Jean Paul at + 12 (034) 5678910, or write to Sample 4 Investments, One Plain Street, Resultland 12KJ4, or e-mail jpaul@sample4inv.com.re.

OR the firm may present:

Sample advertisement including performance returns (5 years of annual returns)

Sample 4 Investments: Global Equity Growth Composite Performance

Results are shown in US$ before fees	Period to date (3 mths to 31 Mar 0 4)	31 Dec 2003	31 Dec 2002	31 Dec 2001	31 Dec 2000	31 Dec 1999
Global Equity Growth Composite	−3.84%	−19.05%	−17.05%	−8.47%	31.97%	25.87%
MSCI World Index	−4.94%	−19.54%	−16.52%	−12.92%	25.34%	24.80%

Sample 4 Investments is the institutional asset management subsidiary of Sample 4 plc and is a registered investment advisor specializing in qualitative, growth-oriented investment management. The Global Equity Growth Composite strategy focuses on earnings, growth of earnings, and key valuation metrics.

Sample 4 Investments claims compliance with the Global Investment Performance Standards (GIPS®).

To receive a complete list and description of Sample 4 Investments' composites and/or a presentation that adheres to the GIPS standards, contact Jean Paul at +12 (034) 5678910, or write to Sample 4 Investments, One Plain Street, Resultland 12KJ4, or e-mail:jpaul@sample4inv.com.re

APPENDIX D: PRIVATE EQUITY VALUATION PRINCIPLES

Introduction

Opinions among INVESTMENT ADVISORS, practitioners, and investors differ regarding the valuation of PRIVATE EQUITY assets. The margin of error for a particular valuation methodology may often be greater than the difference between alternative methodologies. The volatility of asset values is also often high, increasing the perception that a historical valuation was "wrong." Although cash-to-cash returns are the principal metric, PRIVATE EQUITY funds raise capital in part based on unrealized interim returns. The valuation of unrealized assets underpinning these interim returns is critical to this analysis.

Although many points are contested, some common ground exists:

- The PRIVATE EQUITY industry must strive to promote integrity and professionalism in order to improve investor confidence and self-regulation.
- Consistency and comparability are important in reporting to investors, and many aspects of valuation SHOULD be transparent. More information, however, does not always equal greater transparency, and there are legal and practical constraints on the dissemination of information.
- Each PRIVATE EQUITY investment is based on a set of assumptions. It is reasonable for investors to expect interim valuations to reflect factors that, at a minimum, adversely impact these assumptions.
- When a PRIVATE EQUITY asset becomes publicly traded, arguments against interim valuations fall away, although practical considerations may remain where there are restrictions on trading or trading volumes are low.

Beyond these issues are the debates on valuation basis and methodology. The move toward a FAIR VALUE basis has been gathering momentum in most areas of financial reporting. Particularly for early stage venture investments that may not achieve profitability for a number of years, practical problems remain and the utility of the FAIR VALUE basis must garner greater support before a consensus on detailed guidelines is likely to be possible.

Guidelines for valuation

The following MUST be applied to all forms of investment vehicles making PRIVATE EQUITY investments. These principles do not apply to OPEN-END or EVERGREEN FUNDS.

1. Valuations MUST be prepared with integrity and professionalism by individuals with appropriate experience and ability under the direction of senior management.
2. FIRMS MUST document their valuation review procedures.
3. FIRMS MUST create as much transparency as deemed possible in relation to the valuation basis used to value fund investments. For the latest period presented, the valuation methodologies used to value PRIVATE EQUITY investments MUST be clearly disclosed, including all key assumptions.
4. The basis of valuation MUST be logically cohesive and applied rigorously. Although a FAIR VALUE basis is RECOMMENDED, all valuations MUST, at a minimum, recognize when assets have suffered a diminution in value. (Please see the "Additional Considerations" section for further guidance on diminution circumstances.)
5. Valuations MUST be prepared on a consistent and comparable basis from one reporting period to the next. If any change is deemed appropriate in either valuation basis or method, the change MUST be explained. When such a change gives rise to a material alteration in the valuation of the investments, the effect of the change SHOULD also be disclosed.
6. Valuations MUST be prepared at least annually. (Quarterly valuations are RECOMMENDED.)

Fair value recommendation

It is RECOMMENDED that the FAIR VALUE basis, which is consistent with international financial reporting principles, be used to value PRIVATE EQUITY investments. This valuation SHOULD represent the amount at which an asset could be acquired or sold in a current transaction between willing parties in which the parties each acted knowledgeably, prudently, and without compulsion.

The accuracy with which the value of an individual PRIVATE EQUITY asset can be determined will generally have substantial uncertainty. Consequently, it is RECOMMENDED that a valuation method that involves the least number of estimates is preferred over another method that introduces additional subjective assumptions. However, if the latter method results in more accurate and meaningful valuation, then it SHOULD be used instead of the former method.

Valuation hierarchy

The following hierarchy of FAIR VALUE methodologies SHOULD be followed when valuing PRIVATE EQUITY investments:

1. *Market transaction*. Where a recent independent third-party transaction has occurred involving a material investment as part of a new round of financing or sale of equity, it would provide the most appropriate indication of FAIR VALUE.
2. *Market-based multiples*. In the absence of any such third-party transactions continuing to have relevance, the FAIR VALUE of an investment may be calculated using earnings or other market-based multiples. The particular multiple used SHOULD be appropriate for the business being valued. Market-based multiples include, but are not limited to, the following: price to earnings, enterprise value to EBIT, enterprise value to EBITDA, and so on.
3. *Discounted expected future cash flows*. This method SHOULD represent the present value of risk-adjusted expected cash flows, discounted at the risk-free rate.

Additional considerations

1. Where a third-party transaction has taken place other than at arm's length, or where the new investor's objectives in making the investment are largely strategic in nature (i.e. the new investor was not acting solely as a financial investor), the manager SHOULD consider ignoring the valuation or applying an appropriate discount to it.
2. A material diminution in the value of an investment may result from, among other things, a breach of covenant, failure to service debt, a filing for creditor protection or bankruptcy, major lawsuit (particularly concerning intellectual property rights), or a loss or change of management. Other events may include fraud within the company, a material devaluation in an investment currency that is different from the fund currency, substantial changes in quoted market conditions, or any event resulting in profitability falling significantly below the levels at the time of investment or the company performing substantially and consistently behind plan. Estimating the extent of the diminution in most cases will generally involve both quantitative and qualitative analysis and SHOULD be performed with as much diligence as possible.
3. The FIRM SHOULD have policies in place for informing clients/prospects when a material diminution has taken place within the PORTFOLIO. Waiting until a quarterly update may often not provide the prospective investor with this critical information soon enough to make an informed decision.
4. Within the valuation hierarchy there will be certain industries where very specific valuation methodologies become applicable. Within the correct industry, either of these methods could be considered the primary valuation methodology in the absence of an applicable third-party transaction. Whenever one of these methods is used, the FIRM MUST justify the measure as representing the most appropriate and accurate method for calculating a FAIR VALUE.
   a. *Net assets*: For FIRMS that derive a majority of their value from their underlying assets rather than the company's earnings, this method may be preferred.
   b. *Industry BENCHMARKS*: In particular industries, there are metrics, such as "price per subscriber", that can be used to derive the value of a FIRM. These measures are very specialized to the industries they represent and must not be carried over to more diversified FIRMS.
5. It is RECOMMENDED that valuations be reviewed by a qualified person or entity that is independent from the valuer. Such parties would include third-party experts, an independent advisory board, or a committee independent of the executives responsible for the valuations.
6. As stated in the "Valuation Hierarchy" section of this document, FAIR VALUE allows for the use of a recent transaction as the primary methodology for valuation. Accordingly, when an investment is first made, this "cost" represents the most recent transaction and, therefore, the FAIR VALUE. In this case, the cost is permitted to be used not because it represents the cost of the investment but, rather, because it represents the value of the most recent transaction.

Cost as a basis of valuation is only permitted when an estimate of FAIR VALUE cannot be reliably determined. Although a FAIR VALUE basis SHOULD always be attempted, the PRIVATE EQUITY provisions do recognize that there may be situations when a non-FAIR-VALUE basis is necessary. Ultimately, FIRMS must keep in mind that investors make decisions based on FAIR VALUES, not out-of-date historical cost-based measures.

In any case, when a non-FAIR-VALUE basis is used, the FIRM MUST disclose its justification for why a FAIR VALUE basis cannot be applied. In addition, for each COMPOSITE, the FIRM MUST disclose the number of holdings to which a non-FAIR-VALUE basis is applied, the TOTAL VALUE of those holdings, and the value of those holdings as a percentage of the total COMPOSITE/fund assets.

7. Where companies have activities that span more than one sector, making it impractical to find comparable companies or sectors, each earnings stream may be valued independently. Sector average multiples, based on companies of comparable size, can be used where it is not practical or possible to identify a sufficient number of directly comparable companies.
8. The entry multiple(s) for an investment SHOULD only be used as a last resort when comparable quoted companies are not available.
9. All quasi-equity investments SHOULD be valued as equity unless their realizable value can be demonstrated to be other than the equity value.
10. When a PRIVATE EQUITY FIRM has invested in loan stock and preference shares alongside an equity investment, these instruments SHOULD not generally be valued on the basis of their yield. They SHOULD be valued at cost plus any premium or rolled up interest only to the extent it has fully accrued, less any provision/discount where appropriate.

Appendix F
Guidance Statement on Composite Definition*

Revised Effective Date: 1 January 2006
Adoption Date: 13 March 2002
Effective Date: 1 April 2002
Retroactive Application: Required
Public Comment Period: Sep–Dec 2001

GUIDANCE STATEMENT ON COMPOSITE DEFINITION (REVISED)

Introduction

Three of the most fundamental issues that a firm must consider when becoming compliant with the GIPS® standards are the definition of the firm, the firm's definition of discretion, and the firm's composite definition principles and guidelines. The definition of the firm is the foundation for firm-wide compliance and creates defined boundaries whereby total firm assets can be determined. The firm's definition of discretion establishes criteria to judge which portfolios should be in a composite to accurately reflect the application of the firm's investment strategy. Once the firm and discretion have been defined, composites can be constructed based on the strategies implemented by the firm. Firms are reminded that, under the GIPS standards, they must comply with all applicable laws and regulations.

A composite is an aggregation of individual portfolios representing a similar investment mandate, objective or strategy and is the primary vehicle for presenting performance to prospective clients. The firm must include all actual, fee-paying, discretionary portfolios in at least one composite. In this way, firms cannot "cherry-pick" their best performing portfolios to present to prospective clients. Non-fee-paying portfolios may be included in the firm's composites; however, firms are required to disclose the percentage of composite assets represented by non-fee-paying portfolios as of the end of each annual period. If the firm includes non-fee-paying portfolios in its composites, they are subject to the same rules as fee-paying portfolios (e.g. the firm must not move the non-fee-paying portfolio into and out of a composite without documented changes in client guidelines or the redefinition of the composite make it appropriate). Firms are permitted to include a portfolio in more than one composite, provided it satisfies the definition of each composite.

Before defining composites, the firm must establish reasonable criteria that support the fundamental principle of fair representation. A variety of criteria must be analyzed to identify whether portfolios are similar and should be grouped together into a composite.

Guiding principles

The GIPS standards encourage firms to develop objective criteria for defining composites. The following are Guiding Principles that firms must consider when defining composites:

- Composites must be defined according to similar investment objectives and/or strategies. Composites should enable clients to compare the performance of one firm to another. The firm should also consider the definition and construction of similar products found within the competitive universe. Composites must be representative of the firm's products and be consistent with the firm's marketing strategy.
- Firms must apply the criteria for defining composites consistently (e.g. the firm may not select only certain, specific portfolios (i.e. "cherry-picking") that meet the composite definition, but must include all portfolios that satisfy the criteria for inclusion).
- Firms are not permitted to include portfolios with different investment strategies or objectives in the same composite. The performance of such a composite is meaningless. In the case where there are many portfolios with unique, defining investment characteristics, it may be necessary for the firm to create numerous single-portfolio composites.
- Portfolios must not be moved into and out of composites except in the case of documented, client-driven changes in investment objectives or guidelines or in the case of the redefinition of the composite. The historical record of the portfolio must remain with the appropriate composite.

Discretion

Discretion is the ability of the firm to implement its intended strategy. If documented client-imposed restrictions significantly hinder the firm from fully implementing its intended strategy the firm may determine that the portfolio is non-discretionary. Non-discretionary portfolios must not be included in a firm's discretionary composites. There are degrees of discretion and not all client-imposed restrictions will necessarily cause a portfolio to be non-discretionary. The firm must determine if the restrictions will, or could, interfere with the implementation of the intended strategy to the extent that the portfolio is no longer representative of the strategy. For example, if a client requests that the firm not purchase any tobacco stocks in their portfolio, the firm should first consider if this restriction will hinder the implementation of the intended strategy. If so, the firm could either classify this portfolio as non-discretionary (and all other portfolios with this restriction) or could choose to classify it as discretionary and create a composite for portfolios with tobacco restrictions. Firms should, where possible, consider classifying these portfolios as discretionary and grouping them with portfolios with similar restrictions in a separate composite.

Firms must document, in writing, their policies and procedures used in establishing and maintaining compliance with all the applicable requirements of the GIPS standards. As such, each firm must document its definition of discretion and must apply the definition consistently. Ideally, discretion is defined at the firm level, but may be defined at the composite level or by asset class. Firms should also document the reasons for classifying a portfolio as non-discretionary. It is the firm's responsibility to ensure that all of its actual, fee-paying discretionary portfolios are included in at least one composite. Accordingly, firms should review each of their portfolios (both discretionary and non-discretionary) on a regular basis to determine whether any portfolios should be re-classified. According to the GIPS verification procedures, available in the GIPS standards at Section III. Verification, a verifier must determine if the firm's definition of discretion is appropriate and has been applied consistently over time.

Examples of client-imposed restrictions that may cause a portfolio to be classified as non-discretionary include, but are not limited to:

- Restricting trading activities due to conditional client approval,
- Restricting asset allocation (i.e. firm cannot alter asset allocation established by client),
- Tax considerations (e.g. low-cost basis stocks, etc.),
- Limiting the sale of certain securities (e.g. sentimental holdings),
- Restricting the purchase of certain securities or types of securities (e.g., firm cannot buy tobacco stocks, firm cannot buy futures, firm cannot buy securities below a specific quality, etc.),
- Cash flow requirements (e.g. the client requires large cash distributions on a regular basis), or
- Legal restrictions.

Few of these restrictions are reason to *automatically* classify a portfolio as non-discretionary, as the firm must determine if the restriction will significantly hinder the implementation of the intended strategy. In addition, the outsourcing of performance measurement or record keeping by a third-party does not negate the firm's responsibility related to compliance and is not a sufficient reason to classify assets as non-discretionary.

In the case of client-restricted securities (e.g. low-cost basis stocks, held to maturity securities, etc.), the firm may choose to classify the restricted portion of the portfolio as non-discretionary (also commonly referred to as "un-managed" or "un-supervised") and keep the remaining discretionary portion of the portfolio in the composite, provided the remaining portion is representative of the composite's strategy. When considering if a portion of a portfolio should be classified as non-discretionary, firms should consider if the asset(s) affect the management of the portfolio's investment strategy. All calculation and composite construction requirements would apply to the remaining discretionary portion of the portfolio.

Non-discretionary portfolios are not permitted to be included in the firm's composites (i.e. composites consisting of discretionary portfolios). Some firms, however, may group some or all of the firm's non-discretionary portfolios together to simplify composite administration. According to the Standards, this is *not* a composite and must not be included on the firm's list of composites.

Minimum asset level

The GIPS standards contain two provisions that refer to a minimum asset level. GIPS provision 3.A.9 provides that if the firm sets a minimum asset level for portfolios to be included in a composite, no portfolios below that asset level can be included in that composite. GIPS provision 4.A.3 provides that a firm must disclose the minimum asset level, if any, below which portfolios are not included in a composite. Firms may establish a minimum asset level for a composite to identify portfolios that are too small to be representative of the intended strategy. Firms should not market a composite to a prospective client who has assets to invest which are less than the composite's minimum asset level. Firms must disclose the minimum asset level of the composite, if one exists, in each respective composite presentation and must consistently apply the minimum. Firms must document and disclose changes to the minimum asset level and must not retroactively apply the new limit. Portfolios below the minimum are not necessarily non-discretionary; however, asset level can affect discretion.

Portfolios may fall below the minimum due to client withdrawals or depreciation in market value. Firms must determine, as part of their policies regarding minimum asset levels, which market value will be used to evaluate composite portfolios against the minimum asset level (e.g. beginning market value, ending market value, beginning market value plus cash flows, etc.). If a firm establishes a minimum, it must document its policies regarding how portfolios will be treated if they fall below the minimum and must apply these policies consistently. Firms should consider establishing a threshold for the application of the minimum asset level and a minimum time period in order to minimize portfolio movement into or out of a composite. For example, the firm establishes a range of $+-5\%$ of the minimum asset level when determining when to remove a portfolio from the composite and/or the firm establishes that a portfolio must remain above/below the minimum for at least two periods prior to removal/addition. If a portfolio is removed from a composite, the prior history of the portfolio must remain in the composite. Like all policies, once the firm establishes a policy regarding the minimum asset level it must be applied consistently. Once a portfolio is removed, the firm must determine if the portfolio meets any other composite definition and must include it in the appropriate composite(s) in a timely and consistent manner.

Firms should bear in mind that if all the portfolios in a composite fall below the minimum level and, according to the firm's policies, are removed from the composite, the performance record of the composite would come to an end. If after a period of time, portfolios move above the minimum or new portfolios are added to the composite, the prior performance history of the composite should be shown but not linked to the on-going composite performance results.

Composite creation date

Firms must disclose the creation date of the composite, which is the date when the firm first groups the portfolios to create a composite. This is not necessarily the earliest date for which performance is reported for the composite (i.e. composite inception date).

Composite definition

Creating meaningful composites is critical to fair presentation, consistency, and comparability of results over time and among firms. A composite's definition must include detailed criteria that determine the allocation of portfolios to composites and must be made available upon request. Firms must document principles and polices related to composite definition.

While investment strategies can change over time, in most cases firms should not change the definition of a composite. Generally, changes in strategy result in the creation of a new composite. In some very rare cases, however, it may be appropriate to redefine a composite. If a firm determines that it is appropriate to redefine a composite, it must disclose the date and nature of the change. Changes to composites must not be applied retroactively. It is required that firms disclose any changes to the name of a composite. Discontinued composites must continue to be listed on the firm's list of composites for five years after discontinuation. When requested, firms must provide a compliant presentation for any composite on the firm's list of composites.

Firms are only permitted to move portfolios into and out of composites due to documented changes in client guidelines or in the case where the re-definition of a composite make it appropriate. For purposes of the GIPS standards, documentation can include, but is not limited to, letters, faxes, emails, and/or internal memorandums documenting conversations with clients. The historical record of the portfolio must remain with the appropriate composite.

Composite definition criteria

In addition to the Guiding Principles above, firms may choose to define their composites according to relevant criteria and must document the definition of each composite, including any criteria or constraints. It is constructive to consider a hierarchical structure of criteria for composite definition that promotes primary and secondary strategy characteristics. It is also important to understand the defining characteristics commonly found in the marketplace for investment products. Comparability of similar strategies or products is a fundamental objective of the Standards and benefits current and prospective clients when firms define strategies similarly, using clear and unambiguous terminology.

Suggested hierarchy for composite definition

The following suggested hierarchy may be helpful as firms consider how to define composites. Firms are not required to define their composites according to each level of the hierarchy.

Investment mandate

Composites based on the summary of strategy or product description.

Example: "Large-cap global equities"

Asset classes

Composites based on a broad asset class are the most basic and should be representative of the firm's products. Firms may further define the asset class by country or region.

Example: Equity, fixed income, balanced, real estate, venture capital, U.S. fixed income, European equities, etc.

Style or strategy

Firms may further define a composite based on the style or strategy in order to provide investors with additional insight and allow for increased comparability.

Example: Growth, value, active, indexed, asset class sector (e.g. telecommunications), etc.

Benchmarks

Firms may define composites on the basis of the portfolios' benchmark or index provided the benchmark reflects the investment objective or strategy and there are no other composites with the same characteristics. This is often the case if the benchmark also defines the investment universe.

Example: Swiss Market Index, S&P 500, Lehman Aggregate, etc.

Risk/return characteristics

Portfolios with different risk characteristics (e.g. targeted tracking error, beta, volatility, information ratio, etc.) and return objectives may be grouped together into different composites.

Example: Japanese Equity Composite with a targeted excess return of 1% and targeted tracking error of 2% would be in a separate composite from a Japanese Equity Composite with a targeted excess return of 3% and targeted maximum volatility of 6%.

Constraints/guidelines

In addition to the fundamental criteria above, firms may choose to further define their composites based on relevant client constraints or guidelines. The following are examples of constraints or guidelines that could result in materially different strategies and, therefore, justify separate composites.

Extent of the use of derivatives, hedging and/or leverage

In general, portfolios that use derivatives, leverage and/or hedging have a unique investment strategy from those portfolios that do not utilize these techniques or instruments. Accordingly, firms must consider whether portfolios that use leverage, derivatives, and/or hedging should be included in separate composites from portfolios that are restricted from using such instruments or strategies.

Treatment of taxes

The firm should define separate composites for portfolios with specific tax treatments if the treatment of taxes hinders the firm's ability to implement a specific investment strategy as compared to similar portfolios without specific tax treatments. For example, the different tax situations of corporate or insurance clients and private clients may require different investment strategies in terms of emphasizing growth versus yield or dividend versus interest income. If so, firms are required to define separate composites appropriate to the different strategies.

Type of client (e.g. pension fund, private client, endowment, etc.)

Client type alone must not be used as the primary criteria for defining a composite. In some cases, the client type determines the investment strategy because of characteristics that are unique to the client type. If portfolios of different client types have materially different investment strategies and/or styles that are specific to the type of client, the firm must create separate composites representing each of the different strategies.

Instruments used (e.g. invest only in pooled vehicles versus individual securities)

If portfolios use specific instruments the firm may define separate composites.

Size of portfolios

Differences in portfolio size may result in meaningful, material differences in investment strategy and justify the creation of separate composites. For example, an index strategy may be implemented via sampling (i.e. holding a sample of the index securities) for smaller portfolios, while the strategy may be implemented via a full replication of the index for larger portfolios. In this case, the strategy is actually different based on the size of portfolio.

Client characteristics (e.g. cash flow needs, risk tolerances)

Firms may create composites based on multiple client characteristics. For example, a firm may choose to create a composite composed of growth equity, taxable clients that allow leverage and have a targeted tracking error of 4%.

Portfolio types (e.g. segregated (separate) portfolios, pooled portfolios (mutual funds))

Pooled funds, including mutual funds and unit trusts, may be treated as separate composites or combined with other portfolios into one or more composites of the same strategy, style, or objective.

Base currency

Base currency must not be a criteria used for composite definition unless it is specific to the investment strategy.

Additional considerations

- ***Multiple asset class portfolios***. Multi-asset or balanced portfolios are portfolios that consist of more than one asset class. Composites should be constructed according to strategic ranges of asset mixes provided in the client investment guidelines, not according to the tactical percentage of assets invested in the different asset classes. Portfolios with varying, but similar strategic asset allocations can be grouped together if they collectively have the same strategy or style. Firms often have discretion to tactically alter the asset allocation in an effort to add value. Portfolios must not be moved into or out of composites due to changes in the tactical asset allocation. Only in the case of client-documented strategic asset allocation changes can portfolios be moved into different composites.
- ***Inception date***. In general, firms are not permitted to create composites based solely on inception date. However, in very specific situations, it may be appropriate to group portfolios into composites according to inception date (e.g. venture capital composites, after-tax composites, municipal bond composites).
- ***Firms with multiple offices, branches, or investment divisions.*** Firms are only permitted to define different composites for offices, branches, or investment divisions of a firm *if* the portfolios are managed according to investment objectives, styles or strategies that are unique to each particular office, branch, or division. Thus, it is the style or strategy that determines the composite, not the location or group. Composite definition cannot span multiple firms. For additional guidance regarding how the firm can be defined, please refer to the Guidance Statement on Definition of the Firm.
- ***Dispersion of the portfolio returns within a composite.*** While dispersion is one measure to determine how consistently the firm has implemented its strategy across the portfolios in the composite, it can only be measured on an ex-post basis and, therefore, must not be used as a criterion to define a composite. A dispersion figure may serve as a good indicator of whether the criteria for composite definition are suitable and whether or not to redefine

the composite. There is no general rule for a maximum amount of composite dispersion. The firm should contemplate the definition of a broad, "inclusive" composite with a wide dispersion of portfolio returns versus a narrow, "exclusive" composite with a more narrow dispersion measure.

- ***Treatment of fees***. Different types of management fees should not be used as criteria for composite definition.

Effective date

This Guidance Statement was originally effective 1 April 2002 and was revised to reflect the changes to the GIPS standards effective as of 1 January 2006. Firms are required to apply this revised guidance to all periods.

Key GIPS provisions specifically applicable to composite definition

3.A.1 All actual, fee-paying, discretionary portfolios must be included in at least one composite. Although non-fee-paying discretionary portfolios may be included in a composite (with appropriate disclosures), nondiscretionary portfolios are not permitted to be included in a firm's composites.

3.A.2 Composites must be defined according to similar investment objectives and/or strategies. The full composite definition must be made available on request.

3.A.3 Composites must include new portfolios on a timely and consistent basis after the portfolio comes under management unless specifically mandated by the client.

3.A.4 Terminated portfolios must be included in the historical returns of the appropriate composites up to the last full measurement period that the portfolio was under management.

3.A.5 Portfolios are not permitted to be switched from one composite to another unless documented changes in client guidelines or the redefinition of the composite make it appropriate. The historical record of the portfolio must remain with the appropriate composite.

3.A.9 If a firm sets a minimum asset level for portfolios to be included in a composite, no portfolios below that asset level can be included in that composite. Any changes to a composite-specific minimum asset level are not permitted to be applied retroactively.

3.B.3 Firms should not market a composite to a prospective client who has assets less than the composite's minimum asset level.

4.A.2 Firms must disclose the availability of a complete list and description of all of the firm's composites.

4.A.3 Firms must disclose the minimum asset level, if any, below which portfolios are not included in a composite. Firms must also disclose any changes to the minimum asset level.

4.A.22 If a firm has redefined a composite, the firm must disclose the date and nature of the change. Changes to composites are not permitted to be applied retroactively.

4.A.23 Firms must disclose any changes to the name of a composite.

4.A.24 Firms must disclose the composite creation date.

Applications:

1. *Firm A has a client that has multiple accounts (e.g. personal trust and a personal investment account) and manages these accounts as one "master" portfolio. For purposes of the Standards, can Firm A treat these accounts as one portfolio and include them in an appropriate composite?*

If multiple portfolios are managed as one "master" portfolio, the firm can treat this "master" portfolio as any other portfolio and include it in an appropriate composite. Firms must treat this as one portfolio for purposes of calculating the dispersion measure and the number of portfolios within the composite. Firms must consider if account restrictions, such as tax considerations, of any of the individual portfolios affect the overall asset allocation process or the implementation of the firm's strategy for the "master" portfolio. Firm must be careful not to double count assets (e.g. counting both the "master" portfolio and underlying portfolio assets) when calculating composite and total firm assets.

2. *If we currently have a composite for a particular strategy, and the strategy changes, can the performance track record continue to be associated with the new strategy? The change in question is the addition of resources to the investment process. To be precise, we have added a fundamental portfolio manager to a strategy that was previously run using a quantitative (models) process. The new portfolio manager is an additional layer added on top to further refine the stock picks. The quant models will still be used as before.*

As most firms evolve, they modify their investment process through the use of new technologies and resources. It would seem clients would expect their firm to refine and improve the investment process.

A composite is an aggregation of a number of portfolios into a single group that represents a particular investment objective or strategy. The Standards require that composites must be defined according to similar investment objectives and/or strategies. In the situation you present, if the investment objective of the portfolios in the composite remains constant as the firm modifies its investment process, the firm should not create a new composite. If, however, the investment objective/strategy of the portfolios in the composite has changed, the firm should create a new composite, and the performance track record starts for that new composite when portfolios meeting the definition of the new composite are added to it. The firm should clearly document its decision and decision-making process in the event the creation of a new composite is questioned by a verifier/regulator.

3. *We are starting an investment strategy but currently do not have any clients. As such, we would like to start the strategy with a proprietary portfolio. However, once we receive a client account managed in this strategy (which may happen up to 2–3 years later) we will close the proprietary portfolio.*

   Can we create a composite and continue to use the historical performance of the proprietary portfolio by simply adding any subsequent client accounts we receive to the composite? The composite would contain the historical performance of the proprietary portfolio but would continue with only the new client portfolios, just as a normal composite would.

Yes, the situation you describe is appropriate. Performance of a proprietary portfolio (also known as a seed or incubator fund) may be included in a composite if the fund is composed of actual assets under management. As with any other terminated account, the performance history of the proprietary portfolio will remain in the composite up to the last full measurement period that the proprietary portfolio was under management. As other portfolios managed to the same strategy are added to the composite, the historical performance of the composite will continue to include the proprietary portfolio's performance.

4. *If a fund invests in publicly traded equities for both limited partnerships and for separately managed accounts, should the manager set up different composites for each legal structure?*

A composite should include all portfolios that are managed according to the same strategy. Differences in legal structure alone would not warrant separate composite definitions. However, it is up to the firm to decide how results can be presented in the most meaningful way, and if differences in legal structure cause the results of portfolios to differ, then the manager would split limited partnerships and separately managed accounts into separate composites.

5. *Can a firm include a single portfolio in more than one of the firm's composites?*

Yes. The Standards state that firms must include all discretionary fee-paying portfolios in at least one of the firm's composites. Portfolios must be included in each composite for which it meets the prescribed criteria for inclusion. For example, a firm may have an all-cap equity composite and a large-cap equity composite. If the firm manages a portfolio that meets the criteria for inclusion in the all-cap equity composite as well as the large-cap equity composite, the firm must include the portfolio in both composites.

6. *Why should each discretionary fee-paying portfolio be included in at least one composite? If a portfolio represents a style we never plan to market in the future, why should we have to include it in a composite?*

The Standards are ethical guidelines for firms to follow when presenting their performance results. The Standards are based on the principles of fair representation and full disclosure. They are not marketing guidelines.

The requirement for firms to include all fee-paying discretionary portfolios in at least one composite ensures that firms record an accurate picture of the firm's complete performance record. Without this requirement, there is a potential for firms to exclude poor performing portfolios from the appropriate composites. Portfolios that might otherwise belong in the composite could be grouped with "unmarketed" portfolios. Because the intent of the Standards is to accurately and fairly represent firm performance, all fee-paying discretionary portfolios must be included in at least one of the firm's composites.

Firms are also required to disclose that a complete list of the firm's composites is available on each compliant presentation. Potential clients can review descriptions of all composites to determine if any similarities exist. Prospective clients can also request to see additional information on the firm's historical performance record through other composites on the list. These requirements exist to provide prospective clients with a complete picture of the firm's investment performance achieved on all accounts under the firm's discretion.

Appendix G

Sample Global Investment Performance Standards Presentation

GAM European Equity Composite (G025)

GAM has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®). A complete list and description of composites is available on request.

Composite Performance	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	YTD
Composite Returns %	48.76	13.62	43.94	15.44	−15.13	−26.23	13.72	16.76	28.56	20.56	9.00
Benchmark Returns* %	44.90	19.41	36.16	−1.93	−15.26	−30.50	15.76	12.65	26.68	20.18	6.52
Number of Portfolios in Composite	<6	8	<6	<6	<6	<6	<6	<6	<6	<6	<6
Composite Asset value (DEXm)	41	148	60	148	444	333	581	647	874	1255	1414
Total Firm Assets (DEXm)	18 673	20 897	14 579	12 595	15 427	16 064	21 464	27 844	46 881	51 702	53 621
High Return %	N/A	18.28	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
Low Return %	N/A	15.20	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A

1. GAM delivers active investment management to private clients, institutions and intermediaries. Established in 1983, GAM was owned by UBS AG from 1999 until Dec 2005 and is now a company of the Julius Baer Group. GAM continues to have a distinctive style and culture. All GAM's assets are included in the GIPS definition of the firm.
2. (Name deleted) have carried out a verification of the firm's compliance with GIPS for the period from 30 Jun 1996 to 31 Dec 2006. A verification includes obtaining an understanding of the policies and procedures applied by management in compiling performance reports.
3. The composite consists of actively managed portfolios of European equities (including the United Kingdom). Mid to large capitalisation stocks are selected from attractive sectors based on existence of or potential for above average Cash Flow Return on Investment.
4. Derivatives may be used from time to time in the portfolios within the composite to gain market exposure as well as for hedging purposes.
5. Composite results are presented gross of investment management fees and net of trading expenses and of net of withholding taxes on dividends, capital gains and interest. Benchmarks are gross of withholding taxes on dividends.
6. The maximum investment management fee for accounts is 1.5% per annum. Management fees may vary by product and jurisdiction.
7. The composite was created in Nov 2002 and applied retrospectively.
8. High and low returns (for those constituents present in the composite throughout each period) are presented above to demonstrate dispersion within the composite. Dispersion information is only required by GIPS where there are 6 or more portfolios in the composite.
9. The benchmark shown is MSCI Europe Index.
10. *The benchmark shown is for comparative purposes only. The composite is not managed to a specific benchmark.
11. DEX - The reporting currency for this composite is EUR. Prior to Dec 1998, this composite was denominated in DEM.
12. Additional information regarding policies for calculating and reporting returns is available on request.
13. Prior to Nov 2002 GAM's compliance with GIPS was restricted to the institutional and private client assets which were defined as two separate firms for GIPS purposes.
14. As of 2 Dec 2005, GAM changed its methodology for calculating its Total Firm Assets to be in accordance with Julius Baer Group policies, which follow the principles and guidelines of the Federal Banking Commission of Switzerland (Table Q). This change in methodology resulted in Total Firm Assets as of Nov 2005 being increased by 38%. Please note that the year to date total firm asset data is as at 30 Jun 2007.
15. FX rates used for valuation of funds and portfolios within the composite are GMT23.00, FX rates for benchmark and composition calculation are GMT16.00.

Source: GAM as at 30 Sep 2007

Appendix H
Calculation Methodology Guidance Statement*

Revised Effective Date: 1 January 2006
Adoption Date: 4 March 2004
Effective Date: 1 June 2004
Retroactive Application: Not Required
Public Comment Period: Aug–Nov 2002

GUIDANCE STATEMENT ON CALCULATION METHODOLOGY (REVISED)

Introduction

Achieving comparability among investment management firms' performance presentations requires as much uniformity as possible in the methodology used to calculate portfolio and composite returns. The uniformity of the return calculation methodology is dependent on accurate and consistent input data, a critical component to effective compliance with the GIPS® standards. Although the GIPS standards allow flexibility in return calculation, the return must be calculated using a methodology that incorporates the time-weighted rate of return concept for all assets (except private equity assets). For information on calculating performance for these assets, see the separate Private Equity Provisions and Guidance.

The Standards require a time-weighted rate of return because it removes the effects of cash flows, which are generally client-driven. Therefore, a time-weighted rate of return best reflects the firm's ability to manage the assets according to a specified strategy or objective, and is the basis for the comparability of composite returns among firms on a global basis.

In this Guidance Statement, the term "return" is used rather than the more common term "performance" to emphasize the distinction between return and risk and to encourage the view of performance as a combination of risk and return. Risk measures are valuable tools for assessing the abilities of asset managers; however, this Guidance Statement focuses only on the return calculation.

Money- or dollar-weighted returns may add further value in understanding the impact to the client of the timing of external cash flows, but are less useful for return comparison and are therefore not covered by this Guidance Statement.

Guiding principles

Valuation principles – The following are guiding principles that firms must use when determining portfolio values as the basis for the return calculation:

- Portfolio valuations must be based on market values (not cost basis or book values).
- For periods prior to 1 January 2001, portfolios must be valued at least quarterly. For periods between 1 January 2001 and 1 January 2010, portfolios must be valued at least monthly. For periods beginning 1 January 2010, firms must value portfolios on the date of all large external cash flows.
- For periods beginning 1 January 2010, firms must value portfolios as of calendar month-end or the last business day of the month.
- Firms must use trade-date accounting for periods beginning 1 January 2005. (Note: for purposes of the Standards, trade-date accounting recognizes the transaction on the date of the purchase or sale. Recognizing the asset or liability within at least 3 days of the date the transaction is entered into satisfies this requirement.)
- Accrual accounting must be used for fixed income securities and all other assets that accrue interest income. Market values of fixed-income securities must include accrued income.
- Accrual accounting should be used for dividends (as of the ex-dividend date).

Calculation principles – The following are guiding principles that firms must use when calculating *portfolio* returns:

- Firms must calculate all returns after the deduction of the actual trading expenses incurred during the period. Estimated trading expenses are not permitted.
- Firms must calculate time-weighted total returns, including income as well as realized and unrealized gains and losses.
- The calculation method chosen must represent returns fairly, must not be misleading, and must be applied consistently.
- Firms must use time-weighted rates of return that adjust for external cash flows. External cash flows are defined as cash, securities, or assets that enter or exit a portfolio (capital additions or withdrawals) and are generally client-driven. Income earned on a portfolio's assets is not considered an external cash flow.
- The chosen calculation methodology must adjust for daily-weighted external cash flows for periods beginning 1 January 2005, at the latest. An example of this methodology is the modified Dietz method.
- For periods beginning 1 January 2010, at the latest, firms must calculate performance for interim periods between all large external cash flows and geometrically link performance to calculate period returns. (Note: as such, at 1 January 2010, or before if appropriate, each firm must define, prospectively, on a composite-specific basis, what constitutes a large external cash flow.) For information on calculating a "true" time-weighted return (see below).
- External cash flows must be treated in a consistent manner with the firm's documented, composite-specific policy.
- Firms must calculate portfolio returns at least on a monthly basis. For periods prior to 2001, firms may calculate portfolio returns on a quarterly basis.
- Periodic returns must be geometrically linked.

Calculation principles – The following are guiding principles that firms must use when calculating *composite* returns:

- Composite returns must be calculated by asset weighting the individual portfolio returns using beginning-of-period values or a method that reflects both beginning-of-period values and external cash flows.
- The aggregate return method, which combines all the composite assets and cash flows to calculate composite performance as if the composite were one portfolio, is acceptable as an asset-weighted approach.
- For periods prior to 1 January 2010, firms must calculate composite returns by asset weighting the individual portfolio returns at least quarterly. For periods beginning 1 January 2010, composite returns must be calculated by asset weighting the individual portfolio returns at least monthly.
- Periodic returns must be geometrically linked.

Cash flow principles – The following are guiding principles that firms must consider when defining their cash flow policies:

- An *external cash flow* is a flow of cash, securities, or assets that enter or exit a portfolio, which are generally client driven. When calculating approximated rates of return, where the calculation methodology requires an adjustment for the daily-weighting of cash flows, the formula reflects a weight for each external cash flow. The cash flow weight is determined by the amount of time the cash flow is held in the portfolio.
- When calculating a more accurate time-weighted return, a *large external cash flow* must be defined by each firm for each composite to determine when the portfolios in that composite are to be revalued for performance calculations. It is the level at which a client-initiated external flow of cash and or securities into or out of a portfolio may distort performance if the portfolio is not revalued. Firms must define the amount in terms of the value of the cash/asset flow, or in terms of a percentage of portfolio or composite assets.
- The large external cash flow (described above) determines when a portfolio is to be revalued for performance calculations. This is differentiated from a *significant cash flow*, which occurs in situations where cash flows disrupt the implementation of the investment strategy. Please see the Guidance Statement on the Treatment of Significant Cash Flows, which details the procedures and criteria that firms must adhere to and offers additional options for dealing with the impact of significant cash flows on portfolios.

Time-weighted rate of return

Valuing the portfolio and calculating interim returns each time there is an external cash flow ought to result in the most accurate method to calculate the time-weighted rates of return, referred to as the "true" time-weighted rate of return method.

A formula for calculating a true time-weighted portfolio return whenever cash flows occur is:

$$R_i = \frac{(EMV_i - BMV_i)}{BMV_i},$$

where EMV_i is the market value of the portfolio at the end of sub-period i, excluding any cash flows in the period, but including accrued income for the period. BMV_i is the market value at the end of the previous sub-period (i.e. the beginning of the current sub-period), plus any cash flows at the end of the previous sub-period, where an inflow is positive and an outflow is negative, and including accrued income up to the end of the previous period. The cash inflow is included in the BMV (previous period EMV + positive cash inflow) of the sub-period when the cash inflow is available for investment at the start of the sub-period; a cash outflow is reflected in the BMV (previous period EMV + negative cash outflow) of the sub-period when the cash outflow is no longer available for investment at the start of the sub-period.

The sub-period returns are then geometrically linked to calculate the period's return according to the following formula:

$$R_{TR} = ((1 + R_1) \times (1 + R_2) \dots (1 + R_n)) - 1,$$

where R_{TR} is the period's total return and R_1, $R_2 \dots R_n$ are the sub-period returns for sub-period 1 through n respectively.

Approximation of time-weighted rate of return

As mentioned in the Introduction, the GIPS standards require firms to calculate returns using a methodology that incorporates the time-weighted rate of return concept (except for private equity assets). The Standards allow flexibility in choosing the calculation methodology, which means that firms may use alternative formulas, provided the calculation method chosen represents returns fairly, is not misleading, and is applied consistently.

Calculating a true time-weighted rate of return is not an easy task and may be cost intensive. For these reasons, firms may use an approximation method to calculate the total return of the individual portfolios for the periods and sub-periods. The most common approximation methods combine specific rate of return methodologies (such as the original Dietz method, the modified Dietz method, the original internal rate of return (IRR) method, and the modified IRR method) for sub-periods and incorporate the time-weighted rate of return concept by geometrically linking the sub-period returns.

Just as the GIPS standards transition to more frequent valuations, the Standards also transition to more precise calculation methodologies. Therefore, the GIPS standards require firms to calculate approximated time-weighted rates of return that adjust for daily-weighted cash flows by 1 January 2005 (e.g. modified Dietz method) and will require the calculation of a more accurate time-weighted rate of return with valuations occurring at each large external cash flow as well as calendar month-end or the last business day of the month for periods beginning 1 January 2010.

This Guidance Statement does not contain details on the different formulas for calculating approximate time-weighted rates of return.

Composite return calculation

Provision 2.A.3 requires that composite returns must be calculated by asset weighting the individual portfolio returns using beginning-of-period values or a method that reflects both beginning-of-period values and external cash flows.

The intention is to show a composite return that reflects the overall return of the set of the portfolios included in the composite.

To calculate composite returns, firms may use alternative formulas so long as the calculation method chosen represents returns fairly, is not misleading, and is applied consistently.

According to the *beginning market value-weighted method* the composite return, R_{BMV}, can be calculated using the formula:

$$R_{BMV} = \frac{\sum_{i=1}^{n} (BMV_i \times R_i)}{BMV_{TOTAL}},$$

where BMV_i is the beginning market value (at the start of the period) for a portfolio, R_i is the rate of return for portfolio i, and BMV_{TOTAL} is the total market value at the beginning of the period for all the portfolios in the composite.

The *beginning market value plus cash flow-weighted method* represents a refinement to the asset-weighted approach. Consider the case in which one of two portfolios in a composite doubles in market value as the result of a contribution on the third day of a performance period. Under the asset-weighted approach, this portfolio will be weighted in the composite based solely on its beginning market value (i.e. not including the contribution). The beginning market value and cash flow-weighted method resolves this problem by including the effect of cash flows in the weighting calculation as well as in the market values. Assuming that cash flows occur at the end of the day, the weighting factor for each cash flow is calculated as:

$$W_{i,j} = \frac{\left(CD - D_{i,j}\right)}{CD},$$

where CD is the total number of calendar days in the period and $D_{i,j}$ is the number of calendar days since the beginning of the period in which cash flow j occurred in portfolio i.

The beginning market value plus cash flow-weighted composite return, R_{BMV+CF}, can be calculated as follows:

$$R_{BMV+CF} = \frac{\sum_{i=1}^{n} \left\{\left(BMV_i + \left(\sum_{j=1}^{m} CF_{i,j} \times W_{i,j}\right)\right) \times R_i\right\}}{\sum_{i=1}^{n} \left(BMV_i + \left(\sum_{j=1}^{m} CF_{i,j} \times W_{i,j}\right)\right)},$$

where $CF_{i,j}$ is the cash flow j within the period for portfolio i (contributions to the portfolio are positive flows, and withdrawals or distributions are negative flows) and R_i is the return for portfolio i.

The *aggregate return method* combines all the composite assets and cash flows before any calculations occur to calculate returns as if the composite were one portfolio. The method is also acceptable as an asset-weighted approach.

Geometric linking of the periodic composite returns

To calculate the composite return over more than one (sub-)period, the composite return over the total period is calculated by geometrically linking the individual composite sub-period returns using the following formula:

$$R_{CT} = ((1 + R_{C1}) \times (1 + R_{C2}) \ldots (1 + R_{Cn})) - 1,$$

where R_{CT} is the composite return over the total period and R_{C1}, R_{C2}, and R_{Cn} are the individual composite returns for the sub-periods 1, 2, and n, respectively.

Additional considerations

Changes to the methodology – Where appropriate, in the interest of fair representation and full disclosure, firms should disclose when a change in a calculation methodology or valuation source results in a material impact on the composite return.

Third-party performance measurement – Firms may use portfolio returns calculated by a third-party performance measurer as long as the methodology adheres to the requirements of the GIPS standards.

Different valuation and/or calculation method – Firms are permitted to include portfolios with different valuation and/or calculation methodologies within the same composite (as long as the methodologies adhere to the requirements of the GIPS standards). Firms must be consistent in the methodology used for a portfolio (e.g. firms cannot change the methodology for a portfolio from month to month).

Month end valuations – Firms must be consistent in defining the (monthly) valuation period. The valuation period must end on the same day as the reporting period. In other words, firms must value the portfolio/composite on the last day of the reporting period (or the nearest business day). Aggregating portfolios with different ending valuation dates in the same composite is not permitted after 1 January 2006.

Trading expenses – Returns must be calculated after the deduction of all trading expenses. Trading expenses are the costs of buying or selling a security, and include brokerage commissions and any other regulatory fee, duty, etc. associated with an individual transaction.

Trade date accounting – Firms must use trade-date accounting for periods beginning 1 January 2005. Trade-date accounting recognizes an asset or liability on the date the transaction is entered into. Recognizing the asset or liability within at least 3 days of the date the transaction is entered into satisfies the trade-date accounting requirement. As a result, the account will recognize any change between the price of the transaction and the current market value.

Taxes – Firms must disclose relevant details of the treatment of withholding tax on dividends, interest income, and capital gains. Returns should be calculated net of non-reclaimable withholding taxes on dividends, interest, and capital gains. Reclaimable withholding taxes should be accrued.

Grossing-up or netting-down of investment management fees – Firms are allowed to include portfolios with different grossing-up methodologies within the same composite. Firms must be consistent in the methodology used for a portfolio (e.g. firms cannot change the methodology for a portfolio from month-to-month). Please see the guidance on Fees for the GIPS standards.

Large cash flows – The firm must have an established policy on defining and adjusting for large cash flows and apply this policy consistently. Actual valuation at the time of any large external cash flow is required for periods beginning 1 January 2010.

Disclosures – Firms must disclose that additional information regarding policies for calculating and reporting returns is available upon request. Generally, the firm's policies and procedures on calculating and reporting returns could serve as the basis for this information.

Effective date

This Guidance Statement was originally effective 1 June 2004 and was revised to reflect the changes to the GIPS standards effective as of 1 January 2006.

Firms are encouraged, but not required, to apply this guidance prior to the original Effective Date of 1 June 2004; however, the original guidance must be applied to all presentations that include performance for periods on and after that date.

The revisions made to this guidance (effective 1 January 2006) must be applied to all presentations that include performance for periods after 31 December 2005.

Key GIPS provisions specifically applicable to calculation methodology

1.A.2 Portfolio valuations must be based on market values (not cost basis or book values).

1.A.3 For periods prior to 1 January 2001, portfolios must be valued at least quarterly. For periods between 1 January 2001 and 1 January 2010, portfolios must be valued at least monthly. For periods beginning 1 January 2010, firms must value portfolios on the date of all large external cash flows.

1.A.4 For periods beginning 1 January 2010, firms must value portfolios as of the calendar month-end or the last business day of the month.

1.A.5 For periods beginning 1 January 2005, firms must use trade date accounting.

1.A.6 Accrual accounting must be used for fixed-income securities and all other assets that accrue interest income. Market values of fixed-income securities must include accrued income.

1.A.7 For periods beginning 1 January 2006, composites must have consistent beginning and ending annual valuation dates. Unless the composite is reported on a noncalendar fiscal year, the beginning and ending valuation dates must be at calendar year-end (or on the last business day of the year).

2.A Calculation methodology – requirements

2.A.1 Total return, including realized and unrealized gains and losses plus income, must be used.

2.A.2 Time-weighted rates of return that adjust for external cash flows must be used. Periodic returns must be geometrically linked. External cash flows must be treated in a consistent manner with the firm's documented, composite-specific policy. At a minimum:

   a. For periods beginning 1 January 2005, firms must use approximated rates of return that adjust for daily-weighted external cash flows.
   b. For periods beginning 1 January 2010, firms must value portfolios on the date of all large external cash flows.

2.A.3 Composite returns must be calculated by asset weighting the individual portfolio returns using beginning-of-period values or a method that reflects both beginning-of-period values and external cash flows.

2.A.4 Returns from cash and cash equivalents held in portfolios must be included in total return calculations.

2.A.5 All returns must be calculated after the deduction of the actual trading expenses incurred during the period. Estimated trading expenses are not permitted.

2.A.6 For periods beginning 1 January 2006, firms must calculate composite returns by asset weighting the individual portfolio returns at least quarterly. For periods beginning

1 January 2010, composite returns must be calculated by asset weighting the individual portfolio returns at least monthly.

2.A.7 If the actual direct trading expenses cannot be identified and segregated from a bundled fee:

a. when calculating gross-of-fees returns, returns must be reduced by the entire bundled fee or the portion of the bundled fee that includes the direct trading expenses. The use of estimated trading expenses is not permitted.

b. when calculating net-of-fees returns, returns must be reduced by the entire bundled fee or the portion of the bundled fee that includes the direct trading expenses and the investment management fee. The use of estimated trading expenses is not permitted.

2.B Calculation methodology – recommendations

2.B.1 Returns should be calculated net of nonreclaimable withholding taxes on dividends, interest, and capital gains. Reclaimable withholding taxes should be accrued.

2.B.2 Firms should calculate composite returns by asset weighting the member portfolios at least monthly.

2.B.3 Firms should value portfolios on the date of all large external cash flows.

Applications:

1. *Does the firm violate the GIPS standards by reporting money-weighted rates of return to an existing client for their portfolio (which contains no private equity assets)?*

No, the Standards would not be violated if the firm reported money-weighted rates of return to an existing client for their portfolio. The Standards are primarily based on the concept of presenting the firm's composite performance to a prospective client rather than presenting individual portfolio returns to an existing client. The IRR (or money-weighted return) represents the performance of the specific client's fund holdings (i.e. influenced by the client's timing and amount of cash flows) and measures the performance of the fund rather than the performance of the fund manager. Money-weighted returns may add further value in understanding the impact to the client of the timing of external cash flows, but are less useful for comparison purposes.

IRRs are only required in the GIPS standards when calculating performance for private equity assets where the investment firm controls the cash flows.

2. *The GIPS standards currently state that firms are required to use trade-date accounting as of 1 January 2005. How should trade date be defined?*

For the purposes of the GIPS standards, trade-date accounting is defined as "recognizing the asset or liability within at least 3 days of the date the transaction is entered into." Settlement-date accounting is defined as "recognizing the asset or liability on the date in which the exchange of cash, securities, and paperwork involved in a transaction is completed."

When using settlement-date accounting, any movement in value between the trade date or booking date and the settlement date will not have an impact on performance return until settlement date; whereas for trade-date accounting, the change in market value will be reflected for each valuation between trade date and settlement date. If the trade and settlement dates straddle a performance measurement period-end date, then performance return comparisons between portfolios that use settlement-date accounting and those that use trade-date accounting may not be valid. The same problem occurs when comparing settlement-date portfolios and benchmarks.

The principle behind requiring trade-date accounting is to ensure there is not a significant lag between trade execution and reflecting the trade in the performance of a portfolio. For the purposes of compliance with the GIPS standards, portfolios are considered to satisfy the trade-date accounting requirement provided that transactions are recorded and recognized consistently and within normal market practice – typically, a period between trade date and up to three days after trade date ($T + 3$). After 1 January 2005, all firms must recognize transactions on trade date as defined herein.

3. *Given the following information, calculate the rate of return for this portfolio for January, February, March, and the first quarter of 2000, using a true time-weighted rate of return:*

Date	Market Value (€)	Cash Flow (€)	Market Value Post Cash Flow (€)
12/31/99	500 000		
1/31/00	509 000		
2/19/00	513 000	+50 000	563 000
2/28/00	575 000		
3/12/00	585 000	–20 000	565 000
3/31/00	570 000		

Solution:

January

$$R = \frac{(509\ 000 - 500\ 000)}{500\ 000} = 1.80\%$$

February

$$1/31/00\text{–}2/19/00 \quad R = \frac{(513\ 000 - 509\ 000)}{509\ 000} = 0.79\%$$

$$2/19/00\text{–}2/28/00 \quad R = \frac{(575\ 000 - 563\ 000)}{563\ 000} = 2.13\%$$

$$1/31/00\text{–}2/28/00 \quad R_{FEB} = ((1 + 0.008) \times (1 + 0.021)) - 1 = 2.92\%$$

March

$$2/28/00\text{–}3/12/00 \quad R = \frac{(585\ 000 - 575\ 000)}{575\ 000} = 1.74\%$$

$$3/12/00\text{–}3/31/00 \quad R = \frac{(570\ 000 - 565\ 000)}{565\ 000} = 0.88\%$$

$$2/28/00\text{–}3/31/00 \quad R_{Mar} = ((1 + 0.017) \times (1 + 0.009)) - 1 = 2.62\%$$

Quarter 1

$$R_{QT1} = ((1 + 0.018) \times (1 + 0.029) \times (1 + 0.026)) - 1 = 7.48\%$$

Appendix I
Definition of Firm Guidance Statement*

Revised Effective Date: 1 January 2006
Adoption Date: 13 March 2002
Effective Date: 1 April 2002
Retroactive Application: see "Effective Date" section
Public Comment Period: Sep–Dec 2001

GUIDANCE STATEMENT ON DEFINITION OF THE FIRM (REVISED)

Introduction

Three of the most fundamental issues that a firm must consider when becoming compliant with the GIPS® standards are the definition of the firm, the firm's definition of discretion, and the firm's composite definition principles and guidelines. The definition of the firm is the foundation for firm-wide compliance and creates defined boundaries whereby total firm assets can be determined. The firm's definition of discretion establishes criteria to judge which portfolios should be in a composite to accurately reflect the application of the firm's investment strategy. Once the firm and discretion have been defined, composites can be constructed based on the strategies implemented by the firm.

The GIPS standards must be applied on a firm-wide basis. As the first step in complying with the Standards, the firm must be defined fairly and appropriately. Compliance with the GIPS standards relies on a clear and consistent definition of the firm. The Standards require that the definition of the firm be disclosed on the composite presentations, and the verification principles require that verifiers determine if the firm is, and has been, appropriately defined.

In addition, the definition of the firm delineates the universe of "all" portfolios that must be included in total firm assets under management. Fundamental to the Standards is the premise that all actual, fee-paying discretionary portfolios must be included in at least one composite.

As merger and acquisition activity can affect the definition of the firm, the Guidance Statement on Performance Record Portability should also be considered.

Guiding principles

When defining the firm, it is important to consider the following:

- How the firm holds itself out to the public.
- The firm definition must be appropriate, rational and fair.

- Firms are encouraged to adopt the broadest, most meaningful definition of the firm.
- Firms must not use the definition of the firm as a substitute for defining composites (e.g. defining the firm too narrowly, as to only encompass one product).

Defining the firm

The Standards require that firms must be defined as an investment firm, subsidiary, or division held out to clients or potential clients as a distinct business entity.

A distinct business entity is a unit, division, department, or office that is organizationally and functionally segregated from other units, divisions, departments, or offices and retains discretion over the assets it manages and should have autonomy over the investment decision-making process.

Possible criteria that can be used to determine this include:

- being a legal entity,
- having a distinct market or client type (e.g. institutional, retail, private client, etc.), or
- using a separate and distinct investment process.

As previously stated, firms are encouraged to adopt the broadest, most meaningful definition of the firm and consider how it is held out to the public. The scope of this definition should include all geographic (country, regional, etc.) offices operating under the same brand name regardless of the actual name of the individual investment management company. These include, but are not limited to:

- All offices operating under the same brand name (e.g. XYZ Asset Management),
- Other names resulting from mergers, acquisitions, etc., trading under a different name for branding purposes,
- Financial service holding companies defined as one global firm with multiple brands, several legal entities, multiple offices, investment teams, and investment strategies,
- An investment management firm with one brand, but multiple strategies and investment teams,
- All offices trading under a globally recognizable trading name with regional/country specific additions (e.g. XYZ Asset Management Asia),
- Investment management firms in most countries must register with one or more governmental agencies or regulators. The GIPS standards recognize a regulatory registration as a possible definition of a firm for purposes of compliance, but also require firms consider the manner in which they are holding themselves out to the public when determining the firm definition.

Additional considerations

In addition to the Guiding Principles listed above, firms should consider the following when defining the firm:

- The Standards require that when the firm jointly markets with other firms, the firm claiming compliance with the GIPS standards must be sure that it is clearly defined and separate relative to any other firms being marketed and that it is clear which firm is claiming compliance.
- The Standards recommend that if a parent company contains multiple defined firms, each firm within the parent company is encouraged to disclose a list of the other firms contained within the parent company.

- The use of a third party (e.g. custodian, broker/dealer, etc.) to perform record keeping or performance measurement is not a valid reason for excluding assets from the definition of the firm.
- Systems incompatibilities cannot be used as a reason for excluding assets from the definition of the firm (i.e. a firm cannot make the claim of compliance for only those assets that are measured and monitored on compatible systems).

Inception of the firm/redefinition of the firm

In some cases, due to corporate restructuring and merger and acquisition activities, the changes within the firm may be so significant that it is held out to the public as a new firm. The new firm must determine if there is a continuation from the prior firm or if the restructuring is so substantial that it is essentially a new firm.

Changes in investment style or personnel are not valid reasons for redefining the firm, unless the changes are such that the firm is held out to the public in a significantly different way. A simple name change is not sufficient reason to redefine the firm and restart the performance record. In some cases, a firm definition may change without the firm losing its performance history. Please refer to the Guidance Statement on Performance Record Portability for related guidance. In all cases, the underlying principles of the Standards must be considered: fair representation and full disclosure. If a firm is redefined, provision 4.A.21 requires that the firm disclose the date and reason for the redefinition. The Standards require that changes in a firm's organization are not permitted to lead to alteration of historical composite results.

Total firm assets

The definition of the firm also determines the boundaries for determining total firm assets. This includes all assets for which a firm has investment management responsibility and includes assets managed outside the firm (e.g. by subadvisors) for which the firm has discretionary authority. The Standards state that total firm assets must be the aggregate of the market value of all discretionary and non-discretionary assets under management within the defined firm. This includes both fee-paying and non-fee-paying assets.

Assets to which the Standards cannot be applied are not to be considered by firms when claiming compliance and are not to be included in total firm assets. Such assets include investment vehicles that are based on cost or book values rather than market values.

Subadvisors

Some firms utilize a subadvisor to manage part or all of a particular strategy. For example, if a firm specializes in managing equities, it might hire a subadvisor to manage the fixed income portion of its balanced portfolios. The Standards require that firms must include the performance of assets assigned to a subadvisor in a composite provided the firm has the authority to allocate the assets to a subadvisor.

If a firm has discretion over the selection of the subadvisor (i.e. can hire and/or fire), the firm must claim the subadvisor's performance as part of its performance history and include the assets in the firm's total assets. Because the subadvisor has discretion over the actual investment of the assets and the firm has discretion over the selection of the subadvisor, both the firm and the subadvisor are able to claim the performance of the assets as their own. The firm is able to claim this performance because the subadvised portion of the portfolio is essentially viewed as an asset (similar to purchasing a mutual fund within the portfolio) and the firm must be held responsible for its decision to utilize a subadvisor. The firm can only include the subadvisor's performance record relevant to those assets assigned by the firm. If a firm does not have discretion over subadvisor selection, it must not include the subadvisor's performance in its performance history.

The Standards require that beginning 1 January 2006, firms must disclose the use of a subadvisor(s) and the periods a subadvisor(s) was used.

Effective Date

This Guidance Statement was originally effective 1 April 2002 and was revised to reflect the changes to the GIPS standards effective as of 1 January 2006.

Firms claiming compliance prior to 1 January 2006 were required to retroactively apply the original guidance for all periods prior to 31 December 2005; however, the revisions to this guidance (effective 1 January 2006) are not required to be retroactively applied. Firms have until 1 January 2006 to redefine themselves as necessary to comply with this Guidance Statement.

All firms coming into compliance on or after 1 January 2006 must apply this revised guidance to all periods.

Key GIPS provisions specifically applicable to definition of the firm:

0.A Definition of the firm – requirements

0.A.1 The GIPS standards must be applied on a firm-wide basis.

0.A.2 Firms must be defined as an investment firm, subsidiary, or division held out to clients or potential clients as a distinct business entity.

0.A.3 Total firm assets must be the aggregate of the market value of all discretionary and nondiscretionary assets under management within the defined firm. This includes both fee-paying and non-fee-paying assets.

0.A.4 Firms must include the performance of assets assigned to a subadvisor in a composite provided the firm has discretion over the selection of the subadvisor.

0.A.5 Changes in a firm's organization are not permitted to lead to alteration of historical composite results.

0.B Definition of the firm – recommendations

0.B.1 Firms are encouraged to adopt the broadest, most meaningful definition of the firm. The scope of this definition should include all geographical (country, regional, etc.) offices operating under the same brand name regardless of the actual name of the individual investment management company.

0.A Document policies and procedures — requirements

0.A.6 Firms must document, in writing, their policies and procedures used in establishing and maintaining compliance with all the applicable requirements of the GIPS standards.

Applications:

1. *An investment management firm uses a subadvisor to manage a portion of the assets for a strategy that the firm manages. Should the assets managed by the subadvisor be excluded from the composite performance?*

If the firm has discretion over the selection of the subadvisor, then the assets managed by a subadvisor on behalf of the firm must be included in the firm's composite performance and total firm assets. Once the assets are given to a subadvisor to manage, the firm will not have control over exactly how those assets are invested by the subadvisor. Nevertheless, the firm chose to invest the assets by placing them with a subadvisor and has the discretion to hire or fire the subadvisor.

The firm can only include the subadvisor's performance record relevant to those assets assigned by the firm.

2. *According to this Guidance Statement, "Assets to which the Standards cannot be applied are not to be considered by firms when claiming compliance and are not to be included in total firm assets. Such assets include investment vehicles that are based on cost or book values rather than market values." Does this mean that cash substitutes, such as CDs and money market funds held in portfolios, are not to be considered by firms when claiming compliance and are not to be included in total firm assets?*

Money market funds, CDs and other cash substitutes ARE to be considered by firms when claiming compliance and are to be included in total firm assets. A market value for these assets can be determined.

3. *Previously, our firm was a department within a larger organization. Recently, the department was able to complete a buy-out, and we are now an independent investment advisor. What must the firm do in order to continue the claim of compliance?*

A firm's definition reflects how it holds itself out to the public. As the department was previously within the larger organization, if the department claimed compliance as a separate firm without the parent organization being included in that firm definition, there may not need to be any changes. The department may have been and may continue to be a distinct business entity held out to the public as such. It may be necessary for the department to disclose any significant events that would help prospective clients interpret the performance record.

However, if the department and parent organization were historically combined in the same firm definition for purposes of claiming compliance, the definition of the firm has changed with the buy-out and the claim of compliance with the Standards must be reevaluated.

4. *An investment management firm has a Euro-zone fixed income composite that contains the following three portfolios:*
   a. *a fund that is invested in European bonds with net assets of 20,*
   b. *a fund that is invested in bonds of one country of the Euro-zone with net assets of 30, and*
   c. *a private portfolio invested entirely in the two above mentioned funds. Net assets of this portfolio are 10.*

How should GIPS 5.A.1.c be interpreted?

- *Is it correct to say that the number of portfolios of the composite described above is 3?*
- *Is it correct to say that the amount of assets in the composite is 20+30+10=60?*
- *Is it correct to say that the total firm's assets are 50 or that the total firm's assets are 60 (in the case this firm only owns that one composite)?*

The question is that of eliminating double counting assets. Is it correct to say that showing the above composite's asset level of 60 is not misleading to the reader? (The firm will also disclose what % the composite represents of the firm's assets, therefore showing 110% of firm's assets means that some assets are counted more than once. For example, because the management of the private portfolio above is not geared towards choosing the securities in the portfolio, but rather geared towards realizing the proper asset allocation between both funds.)

The GIPS standards are based on the principles of fair representation and full disclosure. Double counting assets would not fairly represent the firm's assets. The composite would have three portfolios and have net assets of 50. If there are no other assets within the firm, then total firm assets would be 50.

Appendix J
Treatment of Carve-outs Guidance Statement*

Revised Effective Date: 1 January 2006
Adoption Date: 4 March 2004
Effective Date: 1 June 2004
Retroactive Application: Not required
Public Comment Period: Aug–Nov 2002

GUIDANCE STATEMENT ON THE TREATMENT OF CARVE-OUTS (REVISED)

Introduction

A carve-out is defined as a single or multiple asset class segment of a multiple asset class portfolio. It is used to create a track record for a narrower mandate from a portfolio managed to a broader mandate. For example, the Asian securities from a Euro-Pacific portfolio or the equity portion of a balanced portfolio could be considered a carve-out. Carve-outs are generally based on asset class, geographic region, or industry sector.

Inherent problems

Carve-outs have several problems associated with them. Because they represent only a portion of a broader, more diversified strategy, carve-out returns are only a valid track record if they are representative of what would have been achieved in a portfolio dedicated to the carved-out strategy. The use of carve-outs gives the impression that the firm has experience managing portfolios dedicated to a particular strategy, when this may not be the case. For example, a carve-out of the U.K. equities in a global equity portfolio that holds only two U.K. equities is not representative of a diversified U.K.-only portfolio.

A second problem occurs if cash is not accounted for separately and, therefore, must be allocated to the carved-out segment. If the carve-out is not accounted for separately, then the calculation of the return is potentially less accurate. The Standards require that returns from cash and cash equivalents held in the portfolio must be included in the total return. Unless the carved-out portion is accounted for as a separate portfolio there will be no cash associated with the returns. For periods prior to 1 January 2010, cash must be allocated to the returns in a timely manner using a consistent, objective methodology.

Beginning 1 January 2010, if the firm intends to carveout an asset class, sector, industry, size range (e.g. large cap) or style type (e.g. value), each carved-out segment must have either its own cash balance or be accounted for separately, with its own associated cash position.

The rationale behind the inclusion of cash at all times in total returns is based on the principle of fair representation – a composite that includes portfolios without any cash would not be representative to the typical prospective client who hires the firm on a fully discretionary basis where cash allocation and management would be implicit. It would be misleading to present returns without cash, since this does not fairly represent how a separate portfolio would be managed.

Guiding principles

Firms must remember the fundamental principles of the Standards, fair representation and full disclosure, and must avoid presenting misleading information. Any carve-out used as a track record must be representative of an actual segregated portfolio managed to that strategy. The carved-out segment must be discretionary and structured materially the same as a portfolio dedicated to that strategy and have a risk profile that is substantially similar. For example, the equity segment of a balanced portfolio may be structured differently than a separately managed equity portfolio because additional risks taken in the equity segment may be offset by lower risk taken in the fixed income segment. The firm must determine if the carved-out segment is representative of a separately managed portfolio dedicated to the same strategy.

Firms must establish a policy for the creation, use, and calculation of carve-outs and apply the policy consistently. The calculation methodology used to calculate and allocate the return achieved on cash should be determined by the firm, documented, and applied consistently.

The GIPS standards state, "Beginning 1 January 2010, carve-out returns are not permitted to be included in single asset class composite returns unless the carve-out is actually managed separately with its own cash balance." Accordingly, it is necessary to clarify what is permissible prior to 1 January 2010 and after the provision becomes effective 1 January 2010.

The following guiding principles must be met when a firm considers creating a carve-out:

Prior to 1 January 2010

- The carve-out should be managed separately (i.e. the segment should be managed as if it were a separate portfolio, rather than a segment of a larger portfolio).
- The carve-out must be representative of a stand-alone portfolio managed to the same strategy.
- If a firm creates a carve-out of a particular strategy, then all similar portfolio segments managed to that strategy must also be carved out and included in the composite (e.g. if the equity segment of a balanced portfolio is carved out and included in an equity composite, then all similar equity segments of the firm's portfolios must be carved out and included in the equity composite, provided the conditions outlined in this Guidance Statement are met).
- If a firm chooses to carve out a portion of a portfolio, they are not compelled to carve out other parts of the portfolio.
- When presenting net-of-fees performance of composites containing carve-outs, fees must be deducted from the carved out returns. The fees must be representative of the fees charged for a separately managed portfolio for the asset class carved out considering the fee schedule for the composite containing the carve-outs.

- The carve-out should have its cash accounted for separately. If the segment does not have its own cash, cash must be allocated to the segment on a consistent basis. Acceptable allocation methods include:
  1. *Beginning of period allocation.* Identify the cash allocation percentage for each portfolio segment at the beginning of the period. For example, at the beginning of January, identify the percentage of residual cash that will be allocated to the carve-outs at month end.
  2. *Strategic asset allocation.* Base the allocation directly upon the target strategic asset allocation. For example, if the portfolio is targeted to have 40% in equities and 60% in bonds, then the allocation will relate to the *actual* amounts invested.

If the portfolio had a target allocation of 40%, but at the beginning of the period only held 35% in equities, then the cash return would constitute the difference (5%). Firms must determine which method to use, document it, and apply consistently:

As of 1 January 2010

- The carve-out must be managed separately (i.e. the segment must be managed as if it were a separate portfolio, rather than a segment of a larger portfolio).
- The carve-out must be representative of a stand-alone portfolio managed to the same strategy.
- If a firm creates a carve-out of a particular strategy, then all similar portfolio segments managed to that strategy must also be carved out and included in the composite (e.g. if the equity segment of a balanced portfolio is carved out and included in an equity composite, then all similar equity segments of the firm's portfolios must be carved out and included in the equity composite, provided the conditions above are met).
- If a firm chooses to carve out a portion of a portfolio, they are not compelled to carve out other parts of the portfolio.
- When presenting net-of-fees performance of composites containing carve-outs, fees must be deducted from the carved-out returns. The fees must be representative of the fees charged for a separately managed portfolio for the asset class carved out considering the fee schedule for the composite containing the carve-outs.
- The carve-out must have its own cash. Possible methods for properly accounting for the cash positions include:
  1. Separate portfolios: cash and securities are actually segregated into a separate portfolio at the custodian.
  2. Multiple cash accounts: each segment's cash is accounted for separately (e.g. equity cash account, fixed-income cash account, etc.).
  3. Sub-portfolios: each segment of a portfolio is accounted for as if it were a separate portfolio.

Performance record for discontinued carve-outs

When a firm, which has created carve-outs using cash allocation methods for periods prior to 1 January 2010, does not choose to apply any method for accounting for the cash position to the carve-outs and thus discontinues the carve-outs for periods after 1 January 2010, then the firm must meet all of the following conditions:

- The past performance record of the carve-outs using cash allocation methods must be left unchanged within the same composites in which the carve-outs were included,

- In the composite presentation, the firm must disclose the historical inclusion of carve-outs and the period of inclusion, and
- If the firm has a composite consisting of only carve-outs using cash allocation methods and does not apply any method for accounting for the cash position to any of the carve-outs in the composite for periods after 1 January 2010, the composite is discontinued but must continue to be listed on the firm's list of composites for five years after discontinuation.

Acceptable uses

Effective 1 January 2010, carve-outs must be managed separately with their own cash (i.e. allocation of cash will no longer be allowed as of 1 January 2010). This change will not be retroactive, so the history of existing carve-outs must not change. Carve-out track records that are representative of the composite strategy may be used like any other portfolio provided that the carve-out is accounted for separately with its own cash. Firms are not permitted to combine different carve-outs or composites to create a new, simulated strategy composite for purposes of compliance with the GIPS standards. For example, a firm may not combine an equity carve-out and a fixed income carve-out to create a simulated balanced composite. Although comprised of actual returns, this type of composite is hypothetical because it does not reflect real asset allocation decisions and therefore is viewed as model or simulated results under the GIPS standards. This information can be presented as supplemental information only but must not be linked to actual returns.

Disclosures

According to Provision 4.A.11, when a single asset class is carved out of a multiple-asset portfolio and the returns are presented as part of a single asset composite, firms must disclose the cash allocation method that was used for periods prior to 1 January 2010. It is recommended that firms disclose any change in the cash allocation methods.

In addition, Provision 5.A.5 requires that beginning 1 January 2006, if a composite includes or is formed using single asset class carve-outs from multiple asset class portfolios, the presentation must include the percentage of the composite that is composed of carve-outs prospectively for each period.

Effective Date

This Guidance Statement was originally effective 1 June 2004 and was revised to reflect the changes to the GIPS standards effective as of 1 January 2006.

Firms are encouraged, but not required, to apply this guidance prior to the original Effective Date of 1 June 2004; however, the original guidance must be applied to all presentations that include performance for periods on and after that date.

The revisions made to this guidance (effective 1 January 2006) must be applied to all presentations that include performance for periods after 31 December 2005.

Key GIPS provisions specifically applicable to carve-outs:

3.A.7 Carve-out segments excluding cash are not permitted to be used to represent a discretionary portfolio and, as such, are not permitted to be included in composite returns.

When a single asset class is carved out of a multiple asset class portfolio and the returns are presented as part of a single asset composite, cash must be allocated to the carve-out returns in a timely and consistent manner. Beginning 1 January 2010 carve-out returns are not permitted to be included in single asset class composite returns unless the carve-out is actually managed separately with its own cash balance.

3.B.1 Carve-out returns should not be included in single asset class composite returns unless the carve-outs are actually managed separately with their own cash balance.

4.A.11 For periods prior to 1 January 2010, when a single asset class is carved out of a multiple asset portfolio and the returns are presented as part of a single asset composite, firms must disclose the policy used to allocate cash to the carve-out returns.

5.A.5 Beginning 1 January 2006, if a composite includes or is formed using single asset class carve-outs from multiple asset class portfolios, the presentation must include the percentage of the composite that is composed of carve-outs prospectively for each period.

Application:

1. *Firm B manages balanced portfolios and would like to carve out the equities to create an equity composite. Firm B charges 0.75% for its fixed income strategy, 1.50% for its equity strategy, and 1.00% for its balanced strategy. How should the investment management fee be allocated to the equity carve-out for presenting a net-of-fees return?*

Firms must allocate fees to each segment that are appropriate to the asset class. In this case, the firm must use the 1.50% that it charges for equity management.

Appendix K
Significant Cash Flow Guidance Statement*

Revised Effective Date: 1 January 2006
Adoption Date: 13 March 2002
Effective Date: 30 June 2002
Retroactive Application: see “Effective Date” section
Public Comment Period: Jul–Oct 2001

GUIDANCE ON THE TREATMENT OF SIGNIFICANT CASH FLOWS (REVISED)

Introduction

Dealing with large external cash flows in a portfolio is a common struggle for most investment managers. These large flows, of cash and/or securities, can make a significant impact on investment strategy implementation and, thus, on a portfolio’s and composite’s performance. Accordingly, this Guidance Statement clarifies the issues related to the treatment of significant cash flows under the Global Investment Performance Standards (GIPS®).

Background

GIPS provision II.3.A.3 requires that composites must include new portfolios on a timely and consistent basis after the portfolio comes under management unless specifically mandated by the client (e.g. a client mandates a schedule of initial cash flows over several time periods and can prolong the length of time needed to implement the strategy). GIPS provision 3.A.4 states that terminated portfolios must be included in the historical returns of the appropriate composites up to the last full measurement period that the portfolio was under management.

The GIPS standards were developed with the understanding that new portfolios may require a period of time (a “grace period”) for a firm to fully implement the intended investment management strategy. During the grace period, the portfolio is not required to be included in a composite. The necessary length of this grace period may vary from composite to composite, depending on a number of factors that impact the implementation of an investment strategy. It is also reasonable that when cash flows to a portfolio are significantly large, the same process applies that governs the introduction of a new portfolio into a composite.

External cash flow definition

For the purposes of the GIPS standards, an External Cash Flow is defined as "cash, securities or assets that enter or exit a portfolio" [i.e. capital additions or withdrawals that are generally client initiated]. Transfers of assets between asset classes within a portfolio or manager initiated flows must not be used to move portfolios out of composites on a temporary basis. The "cash flow" may be defined by the firm as a single flow or an aggregate of a number of flows within a stated period of time.

In cases of multiple cash flows over an extended period of time, firms should refer to the Discretion section of the Guidance Statement on Composite Definition and consider whether the portfolio should be classified as non-discretionary.

For a discussion of the distinction between external/large external cash flows and Significant Cash Flows, please see the Guidance Statement on Calculation Methodology.

Significant Cash Flows

Firms that wish to remove portfolios from composites in cases of Significant Cash Flows must define what is meant by "significant" on a composite-by-composite basis. The definition may be influenced by the characteristics of the asset class(es) within the strategy, such as market liquidity, market volatility, and/or by the trading capabilities of the investment manager. (For instance, a Significant Cash Flow may be considered 10% of a portfolio's market value for an Emerging Market Fixed-Income composite but may be in excess of 50% of a portfolio's market value for a more liquid composite, such as European Equities.) In theory, the determination of significance should primarily be based on the liquidity of the asset class and the investment strategy employed. Because of the dynamic nature of global markets and the inherent subjectivity involved, it is impractical to establish absolute levels of significance for each asset class. Theoretically, cash flows that are relatively small on a composite level, but relatively large on a portfolio level, can potentially distort the portfolio's performance and skew the measure of composite dispersion. The measure of significance must be determined as either a specific monetary amount (e.g. €50 000 000) or a percentage of portfolio assets (based on the most recent valuation).

Grace periods for the treatment of Significant Cash Flows

Each composite should have a portfolio inclusion policy for new portfolios and an exclusion policy for terminated portfolios. It is the responsibility of the firm to set reasonable guidelines for each composite regarding the inclusion/exclusion of portfolios in the composite. Firms are encouraged to establish a policy that includes new portfolios in composites as soon as possible, preferably at the start of the next full performance measurement period. Firms are also encouraged to establish a policy that includes terminated portfolios through the last full measurement period the portfolio is under management. Similarly, these policies should be replicated for the purpose of addressing significant cash flows in composites.

Grace period policies, as well as definitions and policies concerning Significant Cash Flows, must be established and documented for each composite by the firm *before* they are implemented (preferably at the time each composite is created) and firms must not retroactively apply these policies to restate performance. Once implemented, the firm must consistently apply these policies (i.e. if a cash flow in a portfolio occurs that meets the definition of significant for that composite, the portfolio must be removed according to the guidelines). Firms must not reconsider whether a portfolio should be removed from a composite on an ex-post basis (after the fact) when it can be determined whether the cash flow has helped or hurt performance. It should be noted that the removal of a portfolio due to a Significant Cash Flow will not affect the specific portfolio's performance history. The definitions and policies for Significant Cash Flows and Grace Periods for new or cash flow-impacted portfolios can be amended, but the changes must not be applied retroactively. It is expected that the removal of portfolios due to Significant Cash Flows will be an infrequent occurrence, particularly in composites that are invested in the larger, most liquid asset classes.

Firms are recommended to periodically review their policies regarding Significant Cash Flows, especially if firms find that they are frequently removing portfolios due to Significant Cash Flows. Firms are encouraged to establish Significant Cash Flow policies and definitions for all of their composites, but are not required to do so.

Temporary removal of entire portfolios with Significant Cash Flows

If a firm wishes to make use of the option to remove portfolios when Significant Cash Flows occur, then it *must* disclose the following items *in each composite presentation*:

1. how the firm defines a "Significant Cash Flow" for that composite,
2. the grace period for the composite,
3. if the definitions, policies, or grace periods for handling Significant Cash Flows have been redefined, firms must disclose the date and nature of the change, and
4. that additional information regarding the treatment of Significant Cash Flows is available upon request, which must include:
   a. the number of portfolios removed during a given period,
   b. the number of times portfolios were removed during a given period, and
   c. the amount of composite assets represented by the portfolios affected by the application of these policies.

It is important to note that if all of a composite's portfolios were removed during one or more periods due to Significant Cash Flows, there would be a break in the composite performance record. Firms that have composites with only a few portfolios should strongly consider either defining the measure of significance at a very high level or possibly determining that a Significant Cash Flows policy is not appropriate for that composite. If a composite loses all of its member portfolios (whether that is due to Significant Cash Flows, portfolio termination, or some other reason), the performance record stops. If portfolios are later added to that composite, the two periods cannot be linked.

It is also important to note that removing a portfolio due to a Significant Cash Flow removes the portfolio when transaction costs are expected to be high. The intent of this Guidance Statement is not to allow firms to "hide" transaction costs, but rather to remove the potentially more disruptive effects that occur as a result of a Significant Cash Flow.

Documentation

Firms must document each time a portfolio moves into or out of a composite due to a Significant Cash Flow. Documentation must be part of the firm's record keeping process and, at a minimum, must include:

1. the date of the cash flow; the date the firm removes the portfolio from the composite, and the date the firm returns the portfolio to the composite,

2. depending upon the firm's definition of Significant Cash Flow: the amount of the cash flow or the amount of the cash flow as a percentage of the most recent portfolio market value, and
3. if the cash flow is moving into or out of the portfolio.

Documentation will allow third parties to easily determine whether firms have followed their grace period policy and definition of Significant Cash Flow.

Firms must document their definitions and policies regarding Significant Cash Flows, including the definition of the Grace Period and measure of Significance. Firms must also document any changes that are made to the definitions or policies.

Temporary New Accounts

The use of Temporary New Accounts remains the most direct method for dealing with Significant Cash Flows. Under this methodology, when Significant Cash Flows occur in a portfolio, the firm may treat these cash flows as temporary "new" accounts. For example, if a Significant Cash Flow is withdrawn from a portfolio at the end of the month, the firm would move the necessary cash and/or securities into a Temporary New Account for liquidation and/or distribution to the client. Temporary New Accounts are not required to be included in any composite. The portfolio would reflect the withdrawal of funds and/or securities as a cash outflow of the portfolio, and the performance would be calculated to include this cash outflow at the date of transfer to the temporary account. The Temporary New Account would receive the funds and/or securities as a cash inflow. The assets would remain in this account until the funds are distributed. The same principles would hold true with a cash inflow. In this example, the funds would remain in the temporary account until all funds in the Temporary New Account are invested in line with the manager's standard policy for the mandate and then be transferred to the main portfolio.

Firms that are currently able to use a Temporary New Account methodology are encouraged to continue to do so. Although technology at the present time does not readily allow for the use of this method, the GIPS standards recommend the use of Temporary New Accounts to remove the effects of significant external cash flows. The removal of portfolios due to Significant Cash Flows may no longer be permitted at some point in the future. The firm's policy for the use of Temporary New Accounts must be defined and consistently applied in the same manner as the policy for the temporary exclusion of an account from a composite.

Effective Date

This Guidance Statement was originally effective 30 June 2002 and was revised to reflect the changes to the GIPS standards effective as of 1 January 2006.

Firms must not apply these guidelines prior to the implementation date of the firm's Significant Cash Flow policy as described above and must not be used to retroactively restate performance. Firms currently coming into compliance must not apply this guidance to composite performance for periods prior to 30 June 2002.

The revisions made to this guidance (effective 1 January 2006) must be applied to all presentations that include performance for periods after 31 December 2005.

Key GIPS provisions specifically related to significant cash flows:

3.A.3 Composite must include new portfolios on a timely and consistent basis after the portfolio comes under management unless specifically mandated by the client.
3.A.4 Terminated portfolios must be included in the historical returns of the appropriate composites up to the last full measurement period that the portfolio was under management.
3.B.2 To remove the effect of a significant external cash flow, the use of a Temporary New Account is recommended (as opposed to adjusting the composite composition to remove portfolios with significant external cash flows).

Appendix L
Guidance Statement on Performance Record Portability*

Revised Effective Date: 1 January 2006
Adoption Date: 18 May 2001
Effective Date: 18 May 2001
Retroactive Application: Not Required
Public Comment Period: Sep–Dec 2000

GUIDANCE STATEMENT ON PERFORMANCE RECORD PORTABILITY (REVISED)

Introduction

In the current global market for merger, acquisition, and consolidation of investment management firms, past performance records are increasingly valuable assets for their owners. But historical records are the result of many factors (e.g. people, process, discipline, and strategy) that may not be easily transferred to a new entity and still warrant having the same label as the old entity. The applicability and integrity of the performance record is only as good as the ongoing integrity of the strategy and all the contributing factors. Portability of performance records is a very important area that should be clarified within the Global Investment Performance Standards (GIPS®). In addition, because the legal issues and requirements surrounding portability can be particularly complex, firms are reminded that under the GIPS standards they must comply with all applicable laws and regulations regarding portability before applying this Guidance Statement.

Performance is the record of the firm, not of the individual

Changes in a firm's organization are not permitted to lead to alteration of historical composite results. Therefore, composites must include all accounts managed by a member of a firm, even if the individual responsible for the past results is no longer with the firm. Composites must not include portfolios managed by members of the firm before they joined the firm, unless provision 5.A.4 (provided below) is met. If the provision is met, performance track records must be used by the manager, or group of managers, to represent the historical record of a new affiliation or a newly formed entity. Using the performance data from a prior firm or affiliation as supplemental information is permitted as long as the past record is identified clearly as such and is not linked to the results of the new affiliation. If the provisions for portability are met, then it is possible for multiple firms to claim the same performance history as their own.

Provision 0.A.2 of the GIPS standards provides how a firm is to be defined within the context of the Standards. "Firm" mergers can happen within an affiliated group and this Guidance Statement will apply to such situations. As provision II.5.A.4.(d) of the GIPS standards reads, "If a compliant firm acquires or is acquired by a noncompliant firm, the firms have 1 year to bring the noncompliant assets into compliance." However, the important determinant of allowable performance record portability is not a firm's former compliance with the GIPS standards but whether the acquiring firm continues the original strategy that defined the composite with all of its continuing factors.

Performance data from a prior firm may be used, with the proper disclosures, as supplemental information. If the conditions of provision 5.A.4 (listed below) are not met, this supplemental information must not be linked to the ongoing performance of the new firm. The key issue is the linking of prior performance results to the ongoing performance record at the new affiliation.

When a manager, group of managers, or an entire firm joins a new firm, provision 5.A.4 requires:

a. Performance track records of a past firm or affiliation must be linked to or used to represent the historical record of a new firm or new affiliation if:
    i. Substantially all the investment decision-makers are employed by the new firm (i.e. research department, portfolio managers, and other relevant staff),
    ii. The staff and decision-making process remain intact and independent within the new firm, and
    iii. The new firm has records that document and support the reported performance.
b. The new firm must disclose that the performance results from the past firm are linked to the performance record of the new firm.
c. In addition to 5.A.4.(a) and 5.A.4.(b), when one firm joins an existing firm, performance of composites from both firms must be linked to the ongoing returns if substantially all the assets from the past firm's composite transfer to the new firm.
d. If a compliant firm acquires or is acquired by a noncompliant firm, the firms have one year to bring the noncompliant assets into compliance.

If all of the above requirements are not met, the past performance record of the former firm or manager or group of managers from the former firm cannot be linked to the ongoing performance record at the new firm. However, the past performance record may be presented as supplemental information when relevant.

In the case where two firms join and two composites are to be merged, the new firm must first determine if there is a "surviving" composite. A "surviving" composite is the composite that represents the continuity of investment strategy, process, and personnel. In order to be a "surviving" composite, the staff and decision-making process of the composite must remain intact and independent at the new firm.

If the firm identifies a "surviving" composite, its performance history can be presented and linked to the ongoing performance of the merged composite. It is recommended that the performance of the "non-surviving" composite be made available as supplemental information upon request. For example, as a result of a merger, two composites ("C" and "D") are combined in a merged composite "CD." If the firm is able to satisfy all the rules of portability and determines that composite "C" is the "surviving" composite, then the performance history from composite "C" may be linked to the ongoing record of composite "CD." Although the assets from composite "D" are included in composite "CD," the performance history of composite

"D" is not linked to the ongoing record of composite "CD" but should be made available upon request.

If the firm determines that neither composite maintains all the elements of continuity, then there has not been a merger of composites and neither historic performance record can be linked to the ongoing composite performance record, but it is recommended that both of the "non-surviving" composites be presented as supplemental information. For example, if the staff of two firms are combined into one and the investment decision-making process is shared (and thus changed), the historical performance records of both of the non-surviving composites should be presented as supplemental information and must not be linked to the ongoing results of the new composite.

If the presenting firm is a "manager of managers" and is hired by its clients because of the presenting firm's manager selection skills and the firm maintains discretion of the underlying assets (has the control to hire or fire the subadvisor), the firm must include those assets in the total firm assets and present the performance of the underlying assets in the presenting firm's composites. Similarly, if the presenting firm replaces one subadvisor with another, the presenting firm must include within the same composite the performance of the assets assigned to the new firm going forward and leave the results from the former firm unchanged. Provision 4.A.18 requires that beginning 1 January 2006, firms must disclose the use of a subadvisor(s) and the periods a subadvisor(s) was used.

If the presenting firm does not have discretion of the underlying assets managed by the subadvisor, then the performance record of the underlying assets must not be included in the presenting firm's performance composites.

Firms must keep in mind that this Guidance Statement falls under the over-riding principles of the GIPS standards: fair representation and full disclosure. Provision 4.A.19 requires that firms must disclose all significant events that would help a prospective client interpret the performance record. As such, events that impact the firm's operations and/or investment process (for example, change in ownership, merger or acquisition, departure of key investment professional, etc.) must be disclosed.

Effective Date

This Guidance Statement was originally effective 18 May 2001 and was revised to reflect the GIPS standards effective as of 1 January 2006.

Firms are encouraged, but not required, to apply this guidance prior to the original Effective Date of 18 May 2001; however, the original guidance must be applied to all presentations that include performance for periods on and after that date.

The revisions made to this guidance (effective 1 January 2006) must be applied to all presentations that include performance for periods after 31 December 2005.

Applications:

1. *If Firm A acquires Firm B and all of the portability requirements are met, is Firm A required to present Firm B's historical performance, or can Firm A choose to not present Firm B's historical performance?*

The GIPS standards are based on the fundamental principles of fair representation and full disclosure. If all of the portability requirements are satisfied and Firm B is included in the definition of Firm A, Firm A must use Firm B's historical performance. If Firm A were permitted to exclude Firm B's historical performance, it would be cherry-picking which is against the spirit of fair representation of the GIPS standards.

2. *The Guidance Statement on Performance Record Portability states "Performance track records of a past firm or affiliation must be linked to or used to represent the historical record of a new firm or new affiliation if," certain conditions are met. What does performance track record refer to in this instance? Can a firm create the composite history based on the portfolios that transfer to the new firm if the history is representative of the old composite?*

The concept of portability revolves around the ability to bring a track record from one firm to another. It would not be representative to recreate a record with only selected portfolios. The performance track record refers to the entire composite from the old firm. In addition to meeting all the elements of the Guidance Statement, in order for a firm to be able to link the composite from the old firm to the ongoing performance of the new firm, the entire composite performance history, including all portfolios, must be used. The firm must have all the records needed to substantiate that performance history.

3. *Manager A previously worked for Firm X that was not compliant with the GIPS standards. Manager A left Firm X and was hired by Firm Z. Firm Z and Manager A are satisfied that the situation meets all the rules of portability. Can Manager A's performance history at Firm X become compliant or can it not be used because it is non-compliant?*

To clarify, the performance history can never become "compliant." Compliance with the Standards can only be achieved on a firm-wide basis.

If the manager (or management team) satisfies all the requirements of the Guidance Statement on Performance Record Portability (including bringing all the supporting documentation and records to Firm Z), then Firm Z can use the records to construct a composite history that can be used. As provided in the Guidance Statement on Performance Record Portability, the important determinant of allowable performance record portability is not a firm's former compliance with the GIPS standards but whether the acquiring firm continues the original strategy that defined the composite with all of its continuing factors.

Most cases will not meet all of the portability requirements, in which case the past performance record of the manager cannot be linked to the ongoing performance record of the firm. The past performance record of the manager can be presented as supplemental information when relevant.

Please note one of the rules of portability requires that the staff and decision-making process remain intact and independent at the new firm. If the investment process is somehow changed, or if the investment staff changes, the historical records of the new manager can only be presented as supplemental information and cannot be linked to the ongoing record of the firm.

Appendix M
Guidance Statement on the Use of Supplemental Information*

Revised Effective Date: 1 January 2006
Adoption Date: 1 October 2003
Effective Date: 1 January 2004
Retroactive Application: Not Required
Public Comment Period: Aug–Nov 2002

GUIDANCE STATEMENT ON THE USE OF SUPPLEMENTAL INFORMATION (REVISED)

Introduction

In preparing performance reports, firms must keep in mind the spirit and objectives of the Standards: fair representation and full disclosure. Meeting the intent of the Standards may necessitate including information in the performance reports beyond the required and recommended provisions of the Standards to adequately cover the firm's specific situations. Firms that claim compliance with the Standards are encouraged to present all relevant information, beyond that required and recommended in the Standards, to fully explain their performance.

What is Supplemental Information?

Supplemental Information is defined as any performance-related information included as part of a compliant performance presentation that supplements or enhances the required and/or recommended disclosure and presentation provisions of the GIPS standards. Supplemental Information should provide users of the composite presentation with the proper context in which to better understand the performance results. Because Supplemental Information could have the potential to be misleading in relation to the firm's claim of compliance, this Guidance Statement defines and addresses the proper use of Supplemental Information.

What is *not* Supplemental Information?

- *Additional Information* that is required or recommended under the GIPS standards is not considered "Supplemental Information" for the purposes of compliance. Additional Information is not required to be labeled/identified as supplemental or separate from the required compliant information.

- *Non-performance Related Information* is also omitted from this Guidance Statement and is not required to be labeled/identified as supplemental or separate from the required compliant information. Non-performance information includes, but is not limited to, general information regarding the firm, a description of the investment strategy, or details about the investment process.
- *Misleading Information*. Firms that claim compliance with the GIPS standards must not present information that may mislead or deceive. For example, the following two items are misleading and unrepresentative; therefore, compliant firms are prohibited from presenting this information (unless specifically requested from the firm by a prospective or current client in a one-on-one presentation):
  1. Model, hypothetical, backtested, or simulated results *linked* to actual performance results.
  2. Non-portable performance from a prior firm *linked* to current ongoing performance results.

This is not an exhaustive list and is only provided to show examples of potentially misleading information.

Guiding principles

If a firm chooses to show Supplemental Information, it is important to consider the following guiding principles:

- Supplemental Information must satisfy the spirit and principles of the GIPS standards: i.e. fair representation and full disclosure.
- Supplemental Information must *not* contradict or conflict with the information provided in the compliant composite presentation.
- Supplemental Information must be clearly labeled and identified as supplemental to a particular composite presentation.

This Guidance Statement does *not* prohibit firms from preparing and presenting information according to specific requests from prospective clients. However, firms are required to provide a fully compliant presentation prior to or accompanying any Supplemental Information.

Examples of Supplemental Information

Supplemental Information must relate directly to the compliant presentation. Examples of Supplemental Information include, but are not limited to:

- Carve-out returns that exclude cash
- Non-portable returns
- Model, hypothetical, backtested, or simulated returns
- Representative account information, such as:
  1. Portfolio-level country weightings
  2. Portfolio-level sector weightings
  3. Portfolio-level risk measures
- Attribution
- Composite or portfolio-level specific holdings
- Peer group comparisons
- Risk-adjusted performance

Location of Supplemental Information

Supplemental Information must be clearly labeled and identified as supplemental to a particular composite presentation. The presentation and location of Supplemental Information in relation to the GIPS required or recommended data depends on the type of Supplemental Information and its potential to mislead prospective clients.

There are certain situations that allow for the presentation of both compliant and Supplemental Information on the same page; however, firms should consider that there are also many situations that call for the separation of compliant and Supplemental Information. When in doubt, firms are encouraged to place the compliant and Supplemental Information on separate pages.

Firms must provide a fully compliant presentation prior to or accompanying any Supplemental Information. Firms must identify all Supplemental Information to a specific compliant presentation, for example:

- place Supplemental Information on the same or back of the page as the compliant data–if appropriate, or
- include a statement indicating that the Supplemental Information supplements the XYZ Composite presentation (as provided on pg. 11 *or* provided on 15 March 200X).

This Guidance Statement does *not* restrict firms from providing any specific information requested by prospective clients or their agents.

Supplemental Information – Verification

Supplemental Information is not subject to verification under the GIPS standards. It is the ultimate responsibility of the firm claiming compliance to ensure that it abides by the ethical principles and spirit of the GIPS standards each time it presents performance results.

Effective Date

This Guidance Statement was originally effective 1 January 2004 and was revised to reflect the changes to the GIPS standards effective as of 1 January 2006.

Firms are encouraged, but not required, to apply this guidance prior to the original Effective Date of 1 January 2004; however, the original guidance must be applied to all presentations that include performance for periods on and after that date.

The revisions made to this guidance (effective 1 January 2006) must be applied to all presentations that include performance for periods after 31 December 2005.

Applications:

1. *Can Supplemental Information be presented on the same page as the compliant presentation?*

Yes. Supplemental Information can be presented on the same page as long as it satisfies the guiding principles of the Supplemental Information Guidance Statement (i.e. is not misleading, does not contradict or conflict with the required compliant information, is clearly labeled, and references the appropriate composite presentation that it supplements).

2. *Firm A has a marketing brochure that describes the firm, staff, and investment process. One of the pages contains the compliant presentation for a specific composite. Do all the other pages of the brochure need to be labeled as Supplemental Information?*

No. Supplemental Information is defined as any performance-related information included as part of a compliant performance presentation that supplements or enhances the required and/or recommended disclosure and presentation provisions of the GIPS standards. Supplemental Information does not include general information regarding the firm or the investment strategy or process.

3. *XYZ firm created a presentation booklet that primarily highlights performance information that is considered supplemental. The booklet shows an appropriate compliant presentation in the back of the book as an appendix. Is it acceptable for the supplemental information to precede the compliant presentation?*

Yes, provided the Supplemental Information:

a. is not misleading,
b. does not contradict or conflict with the compliant information, and
c. is clearly labeled as supplemental.

The firm must include a statement indicating that the Supplemental Information supplements the compliant composite presentation.

4. *Are there any limits to what can be shown as Supplemental Information?*

The definition and guiding principles of this Guidance Statement on the Use of Supplemental Information are very specific about the types of information that should and should not be shown in conjunction to a fully-compliant GIPS composite presentation. When in doubt, firms should always turn their focus to the first guiding principle of the Guidance Statement, which is also the fundamental objective of the Standards: to ensure fair representation and full disclosure of performance results. By continually using this principle to guide the calculation and presentation of performance, firms are sure to satisfy the spirit and provisions of the GIPS standards.

Specifically, firms that claim compliance with the GIPS standards must not present information that may mislead or deceive. For example, the following two items are misleading and unrepresentative; therefore, compliant firms are prohibited from presenting this information (unless specifically requested from the firm by a prospective or current client in a one-on-one presentation):

1. Model, hypothetical, backtested, or simulated results *linked* to actual performance results
2. Non-portable performance from a prior firm *linked* to current ongoing performance results

This is not an exhaustive list and is only provided to show examples of potentially misleading information.

Appendix N

Guidance Statement on Recordkeeping Requirements of the GIPS Standards*

Adoption Date:	July 2007
Effective Date:	31 October 2007
Retroactive Application:	Not Required
Public Comment Period:	Oct–Dec 2004

GUIDANCE STATEMENT ON RECORDKEEPING REQUIREMENTS OF THE GIPS STANDARDS

Introduction

GIPS® standards Provision 1.A.1 states: "All data and information necessary to support a firm's performance presentation and to perform the required calculations must be captured and maintained."

The following guidance relates only to records necessary to satisfy the recordkeeping requirements of the GIPS standards. In all instances, either paper (hard-copy) records or electronically stored records will suffice. If records are stored electronically, the records must be easily accessible and printable if needed. Although most firms are looking for a very precise list of the minimum supporting documents that must be maintained to support all parts of the GIPS-compliant performance presentation, including the ability to recreate the firm's performance history, there is not a single list of records that will suffice in all situations.

Guiding principles

1. Above all else, a firm must meet any and all applicable regulatory requirements addressing records that must be maintained.
2. For each performance period presented in the GIPS-compliant presentation, a firm must maintain sufficient records that allow for the recalculation of portfolio-level returns. Depending on the system and methods used for calculating portfolio-level returns, one firm may need different records than another firm needs. Records to support portfolio-level returns might include a combination of the following (this list is not meant to be an exhaustive list):
   - associated bank/custodial statements and reconciliations;
   - portfolio statements of assets and valuations, including pricing calculations for securities such as:
     i. not-readily priced securities; or
     ii. thinly-traded securities;

- portfolio transactions reports;
- outstanding trades reports;
- corporate action reports;
- income received/earned reports;
- accrued income reports;
- foreign or other withholding tax reclaim reports;
- cash flow/weighted cash flow reports;
- information on calculation methodology used;
- information provided by a third party (for example, the Sponsor in a wrap fee/SMA relationship) where it may be necessary for a firm to take additional steps to ensure the information provided by the third party can be relied on to meet the requirements of the Standards;
- investment management fee information.

3. For each performance period presented in the GIPS-compliant presentation, a firm must maintain sufficient records that allow for the recalculation of composite-level returns and data. Depending on the system and methods used for calculating composite-level returns and data, one firm may need different records than another firm needs. Records to support composite-level returns and other composite-level data might include a combination of the following (this list is not meant to be an exhaustive list):
   - portfolios included in/excluded from the composite;
   - when each portfolio entered (and exited, if applicable) the composite;
   - each portfolio's return;
   - market value used to weight each portfolio (Beginning Market Value (BMV) or BMV plus weighted cash flows);
   - number of portfolios in the composite and the composite's assets at the end of each period presented;
   - dispersion calculation data;
   - investment management fee information.
4. A firm must maintain records to support why a portfolio was assigned to a specific composite, or was excluded from all composites. Supporting records might include a combination of the following (this list is not meant to be an exhaustive list):
   - composite definition, particularly related to the composite inclusion criteria, including the definition of discretion;
   - portfolios excluded from composites, and the reasons for exclusion;
   - investment management agreements and amendments thereto;
   - reports provided to clients, including attribution information, if utilized to help determine composite assignment;
   - e-mail/other correspondence with clients regarding investment management strategy amendments.
5. A firm must maintain records to support their claim of compliance on a firm-wide basis. Information should be maintained to establish (this list is not meant to be an exhaustive list):
   - definition of the firm, historically and current;
   - total firm assets for all periods reported;
   - composite definitions and creation dates;
   - complete list and description of the firm's composites;
   - compliant presentations, and supporting information for all composites.
6. Provision 0.A.6 requires that "firms must document, in writing, their policies and procedures used in establishing and maintaining compliance with all the applicable requirements

of the GIPS standards." Therefore, firms must maintain all policies and procedures (both current and previous versions) that support the claim of compliance.

7. When firms utilize third-party service providers, firms are encouraged to ensure that they have adequate service-level agreements to provide the historical records necessary, both currently and at a date in the future. If a firm utilizes a third party to provide any service (such as subadvisor, custodian, performance measurement, etc.), the firm is responsible for its claim of compliance, and must ensure the records and information provided meet the requirements of the Standards.
8. A firm should maintain any additional records necessary to support a claim of compliance, which might include a combination of the following (this list is not meant to be an exhaustive list):
   - marketing output/Request For Proposal (RFP) responses;
   - externally reviewed system and control reports (such as accounting reports or other internal controls/compliance reports for the client and/or custodians);
   - third-party (subadvisory, custodial, performance data provider, etc.) agreements;
   - minutes of relevant decision-making committees, such as Board, Investment Committee or Composite/GIPS Compliance Committee;
   - client fee schedules/agreements;
   - fee data, including custody and administrative fees;
   - systems manuals especially for the systems that generate the portfolio and composite reports (including returns and additional disclosures/statistics);
   - documentation of efforts made to provide all prospective clients with compliant presentations;
   - documentation of providing the following to any prospective client who requested:
     i. a list and description of the firm's composites; or
     ii. a composite presentation;
   - underlying benchmark data (if not publicly available).
9. All records deemed necessary by the firm must be maintained for each year that is presented in a GIPS-compliant presentation. At some point in time, a firm may feel it may be able to reduce the amount of records stored, as long as the firms can maintain the ability to recalculate the required returns. For example, an annual portfolio transaction report may be maintained instead of individual monthly detail reports. The summary report could be used to recreate period-specific information if needed. Electronically-stored records/reports are acceptable.

Effective Date

Firms are required to apply this guidance beginning 31 October 2007. Firms claiming compliance with the GIPS standards have historically been required to maintain records to support their claim. However, this Guidance Statement does not identify any additional specific records or documents that are required to be maintained by firms. Firms are encouraged to evaluate their historical documentation while considering the guidance provided herein. Firms are encouraged, but not required, to apply this guidance prior to the Effective Date.

Applications:

1. *We have custodial records, trade confirmations, portfolio holdings and valuations, transactions reports, corporate action reports, income received/earned reports, accrued income reports, and cash flow/weighted cash flow reports. Must we maintain* all *these records for all portfolios (both discretionary and non-discretionary) in order to satisfy GIPS 1.A.1?*

No. For all composites the firm must maintain sufficient records to support the firm's performance presentations, including the composite's performance record. This might include a combination of the types of records you suggested. A firm is not required to maintain records to support performance calculations for portfolios excluded from composites, including non-discretionary portfolios. However, a firm is required to maintain support to prove that all portfolios that were excluded from composites were properly excluded. The firm must determine which records would suffice and maintain these records for all portfolios/composites for each period of performance presented.

2. *How long must I keep the records? For example, we show a 25-year track record for our Large Cap Value Equity strategy in our GIPS standards compliant presentation. Must we still maintain the records?*

Yes. The firm must maintain records to support any performance period presented in the GIPS-compliant presentation. If your firm presents a 25-year track record of the Large Cap Value Equity strategy in the GIPS-compliant presentation, you would need to maintain sufficient records for the full 25-year period. If the composite has a 25-year track record and only 15 years are presented in the compliant presentation, you must maintain records to support the 15 years presented.

3. *I have a composite that I don't use for advertising or marketing purposes. Must I keep the records for the portfolios in this composite as well as records for the composite itself, even if I don't market the composite's performance?*

Yes. The firm must maintain sufficient records that allow for the recalculation of composite-level returns (as well as the underlying portfolio-level returns) for all the firm's composites, regardless of whether the composites are used for marketing or not. All composites of actual, fee-paying, discretionary portfolios, whether marketed or not, must be listed on the firm's list of composites. The firm must be prepared to provide a composite presentation in compliance with the Standards for all composites on the list when requested to do so by a prospective client. The presentation should be available within a reasonable time frame if not immediately available.

4. *Certain types of records (e.g. thermal printed faxes) have a limited life and in many cases such documents are more than 10 years old. What happens if these records begin to disintegrate and are no longer readable?*

Original documents are not required to be maintained. With the advent of technology, firms can rely on electronic scans of paper documents in order to satisfy the recordkeeping requirements.

5. *Do we have to keep trade tickets?*

The determination of which records are necessary to support the performance record is left up to the firm. In certain cases, the trade tickets may prove a useful resource to capture and maintain some of the data needed to support the record. In other cases, this data may be captured elsewhere. Since records capturing duplicate information are not necessary, the firm must make the determination of which records to maintain.

6. *I have records to support my performance; however, the records are stored in a system that is not operable and I do not have access to the records. Is this acceptable?*

Records stored in a system that is not operable will not satisfy the requirements of the GIPS standards to maintain data and information necessary to support the firm's performance presentation.

7. *I have a question that has not been answered directly by this Guidance Statement on the recordkeeping requirements. What should I do?*

Temporarily, the firm should maintain as much data as possible and seek advice on the most appropriate data to keep, which might include referring to the GIPS question and answer database.

8. *How does provision 0.A.6, which requires that firms must document, in writing, their policies and procedures used in establishing and maintaining compliance with all the applicable requirements of the GIPS standards, relate to provision 1.A.1?*

The data and information required to be collected and maintained by provision 1.A.1 forms part of the policies and procedures required to be documented by provision 0.A.6. The firm must determine what data and information is necessary to support a firm's performance presentation and to perform the required calculations. Once this is determined, the firm should document in its policies and procedures how the data and information required to construct the performance presentation will be gathered, how the data and information will be maintained, and the length of time it will be maintained.

9. *We present risk-adjusted returns as Supplemental Information within our presentation in compliance with the GIPS standards for our Growth and Income composite. Must we keep the data and information to support this Supplemental Information?*

Firms are required to maintain records to support any data and information necessary to support the firm's performance presentation (required and recommended provisions) and to perform the required calculations. Supplemental Information can take many forms. It is considered best practice for firms to maintain the records to support any data and information presented as Supplemental Information. Firms should also consider any regulatory requirements addressing recordkeeping requirements for any data and information presented to prospective clients or presented in advertisements.

10. *Why isn't this Guidance Statement applicable retroactively?*

Firms have been required, since the inception of the Standards, to maintain the necessary records to support their performance presentations. The guidance provided in this document is not intended to require firms to go back and change the records they have maintained to satisfy provision 1.A.1. This guidance is provided so that firms are aware there may be different documents available to the firm to use for meeting the recordkeeping requirements of the Standards.

11. *Firm A claims compliance with the GIPS standards. It maintains hard copies of the records supporting compliance for three years and discards all records older than three years in an effort to save office space and does not maintain any electronic copies of the discarded records. The performance reported on all its composite presentations shows five years of history. Is the firm in compliance with the GIPS standards?*

No. A firm must maintain data necessary to support a firm's claim of compliance. Because Firm A presents five years of performance history, it must maintain the records to support the firm's five-year history and all other relevant data on the firm's composite presentations. Because the Standards require firms to build a 10-year performance history, the firm must continue to maintain the records to support the firm's eventual 10-year performance history.

12. *Are recordkeeping requirements different for compliant and non-compliant performance within the presentation in compliance with the Standards?*

In certain circumstances a firm may present non-compliant performance in a compliant presentation. If the non-compliant performance is included appropriately with the compliant performance because the non-compliant performance is from periods prior to the required effective date of 1 January 2000, the firm must have the supporting documentation for the non-compliant performance. If a report contains non-compliant performance, the firm must also disclose how the presentation is not in compliance with the GIPS standards. If a firm does not have records to support the non-compliant performance it wishes to include in the presentation, the firm can either: (1) remove the non-compliant information from the report, or (2) report this non-compliant information as supplemental information.

13. *Firm B does not have the information necessary to substantiate the required calculations that would support the firm's performance record prior to the date of a fire in its storage facility that destroyed its records and electronic back-up systems. Is the firm precluded from claiming compliance with the GIPS standards?*

The GIPS standards require firms to present, at a minimum, five years of annual investment performance that is compliant with the GIPS standards. If any year prior to the date of the fire is required to be included in the presentation to meet the initial five-year requirement, as long as a firm can recapture the information that would support its performance record for those years, the firm could claim compliance and include performance for years prior to the date of the fire in its performance presentations. For instance, Firm B could reconstruct the information necessary by obtaining the information from clients, custodians, consultants, verifiers or any other party outside the firm that might have duplicate copies of those records. If the firm were able to collect data to support a five-year record, then it would be able to claim compliance with the GIPS standards, provided all the other requirements of the Standards are also met. If the records cannot be obtained through alternate sources, the firm may have options for continuing to claim compliance. See questions 14 and 15 for more information on such scenarios.

14. *Can a firm previously claiming compliance with the GIPS standards continue claiming compliance if it does not have records to substantiate a portion of its past performance due to a catastrophic event?*

The firm should try to reconstruct the information necessary by obtaining the information from clients, custodians, consultants, verifiers or any other party outside the firm that might have duplicate copies of those records. However, if the underlying data to support the performance presentation or to perform the required calculations were destroyed because of extreme circumstances beyond the control of the manager and unavailable from other sources, the firm may continue to claim compliance and show performance if the lack of records for the unavailable period(s) is disclosed. The disclosure must include the reason why the records are unavailable and that the firm is unable to duplicate the records. For example, if the records for Firm A from its inception through 31 December 2002 were destroyed under extreme circumstances beyond the control of the manager, the firm can claim compliance with the GIPS standards by showing its performance going forward from 1 January 2003 in compliance with the Standards, and disclosing that the firm's records for the period from 1 January 1999 through 31 December 2002 were destroyed under extreme circumstances beyond the control of the manager and the data is unavailable from other sources. The firm must also consider any applicable regulatory requirements and must remember that the GIPS standards are ethical standards based on the principles of fair representation and full disclosure. Any performance information that is presented must adhere to these principles.

15. *What events would qualify as a catastrophic event, and would exempt a firm from record-keeping requirements?*

A catastrophic event is an extreme circumstance beyond the control of the manager, and would include natural disasters such as floods and earthquakes. Other catastrophic events that are not natural disasters would include fire or terrorist attack. Events within the control of the firm, such as a change in IT systems, do not qualify as catastrophic events.

Appendix O
Useful Websites

www.investment-performance.com
www.statpro.com

- Carl.bacon@statpro.com

www.i-performance-analysis.com
www.tsg.com
www.andreassteiner.net
www.cfainstitute.com

– www.cfainstitute.org/standards	GIPS Standards
– standards@cfainstitute.org	Email alerts
– gips@cfainstitute.org	Helpdesk
– www.cipm.com	

www.compoundinghappens.com
www.indexuniverse.com

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