

PONTIFICIA UNIVERSIDAD CATÓLICA DEL PERÚ
Maestría en Control y Automatización



Sistemas Lineales ICA600

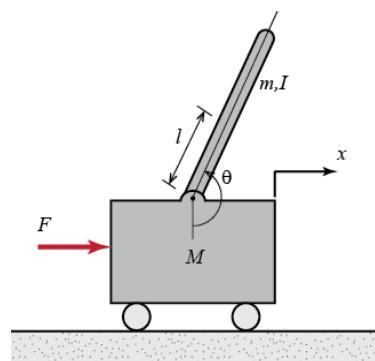
Título : Controladores de Estado con acción integral

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Código: 20156458

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2015

1. Planta motor + tornillo sínfín



Parámetros del modelo:

```

R = 1.1;
L = 0.0001;
Kt = 0.0573;
Kb = 0.05665;
I = 4.326e-5;
p = 0.004;
m = 1.00;
c = 40;
r = 0.015;
alfa = 45*pi/180;
voltmax = 60;
d = m + 2*pi*I*tan(alfa) / (p*r);
a22 = -c/d;
a23 = Kt*tan(alfa) / (r*d);
a32 = -2*pi*Kb / (p*L);
a33 = -R/L;
b31 = 1/L;
w21 = -1/d;
A = [ 0   1   0
      0   a22  a23
      0   a32  a33 ];
B = [ 0
      0
      b31 ];
Wf = [ 0
        w21
        0 ];
C=[1 0 0];
D=[0];

r = 0.5;           % Posición deseada
voltmax = 24;     % Voltaje máximo
Fseca = 0.5;       % Fricción 0, 0.5, 1.0, 1.5

```

Controladores de Estado con acción integral

```
q1 = 1e5; q2 = 1e3; q3 = 3;
Q = diag([ q1 q2 q3 ]);
RR = 1;
P = are(A,B*inv(RR)*B',Q);
K = inv(RR)*B'*P;
k1 = K(1,1); k2 = K(1,2); k3 = K(1,3);
ti = 0; tf = 10; dt = 0.001;
t = ti:dt:tf; t = t';
[ Ak Bk ] = c2d(A,B,dt);
[ Ak Wk ] = c2d(A,Wf,dt);
x = [ 0; 0; 0 ]; % Vector de estado inicial
k = 1;
for tt = ti:dt:tf
    pos(k,1) = x(1,1);
    vel(k,1) = x(2,1);
    cor(k,1) = x(3,1);
    u = -K*x + k1*r;
    if( u > voltmax)
        u = voltmax;
    elseif(u < -voltmax)
        u = -voltmax;
    end
    volt(k,1) = u;
    pot(k,1) = u*x(3,1);
    if(x(2,1) >= 0)
        Fs = Fseca;
    elseif(x(2,1) < 0)
        Fs = -Fseca;
    end
    x = Ak*x + Bk*u + Wk*Fs;
    k = k+1;
end

% Determinación de error estacionario
k = k - 1;
errest = ((pos(k,1) - r)/r)*100;
disp(['Error estacionario: ',num2str(errest),'%']);

% Determinación de sobreimpulso
posmax = max(pos);
if(posmax > r)
    sobreimpulso = (posmax-r)/r*100;
    disp(['Sobreimpulso: ',num2str(sobreimpulso),'%']);
else
    disp('Sobreimpulso 0.00%');
end

figure(1);
subplot(3,1,1); plot(t,pos); ylabel('Posicion');
title('Control sin Acción Integral');
subplot(3,1,2); plot(t,vel); ylabel('Velocidad');
subplot(3,1,3); plot(t,cor); ylabel('Corriente');
xlabel('Tiempo');
figure(2);
subplot(2,1,1), plot(t,volt); ylabel('Voltaje');
title('Control sin Acción Integral');
subplot(2,1,2); plot(t,pot); ylabel('Potencia');
xlabel('Tiempo');

% Diagrama de Bode
Acl = A - B*K;
Bcl = B*k1;
Ccl = C;
Dcl = D;
fre = 0:0.1:20; fre = fre';
wrs = 2*pi*fre;
[ mag fase ] = bode(Acl,Bcl,Ccl,Dcl,1,wrs);
figure(5);
subplot(2,1,1); loglog(fre,mag); grid; title('bode |G|');
axis tight
subplot(2,1,2); semilogx(fre,fase); grid; title('bode < G');
axis tight
```

1.1. Control sin acción integral

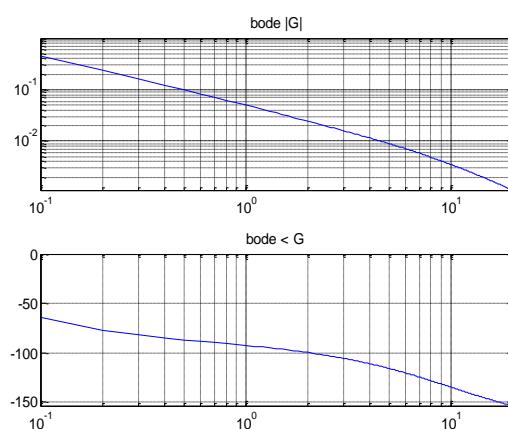
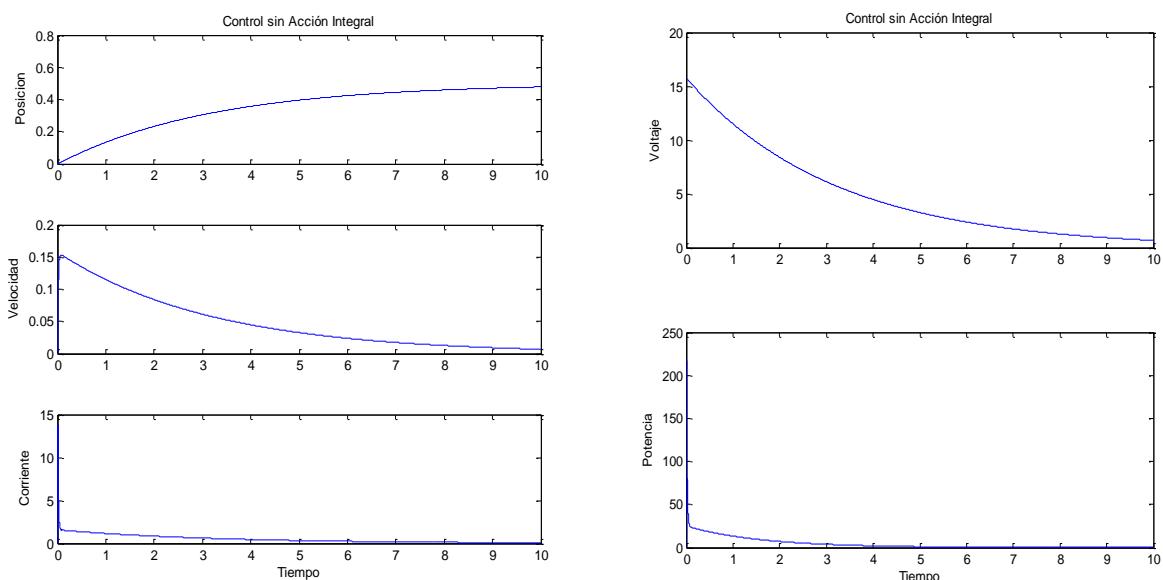
Fricción estática $F_s = 0$.

1.1.1. Caso: q_1 variable – q_2 cte. – q_3 cte.

$$q_1 = 1e3, q_2 = 0, q_3 = 0$$

Error estacionario: -4.3198%

Sobreimpulso 0.00%

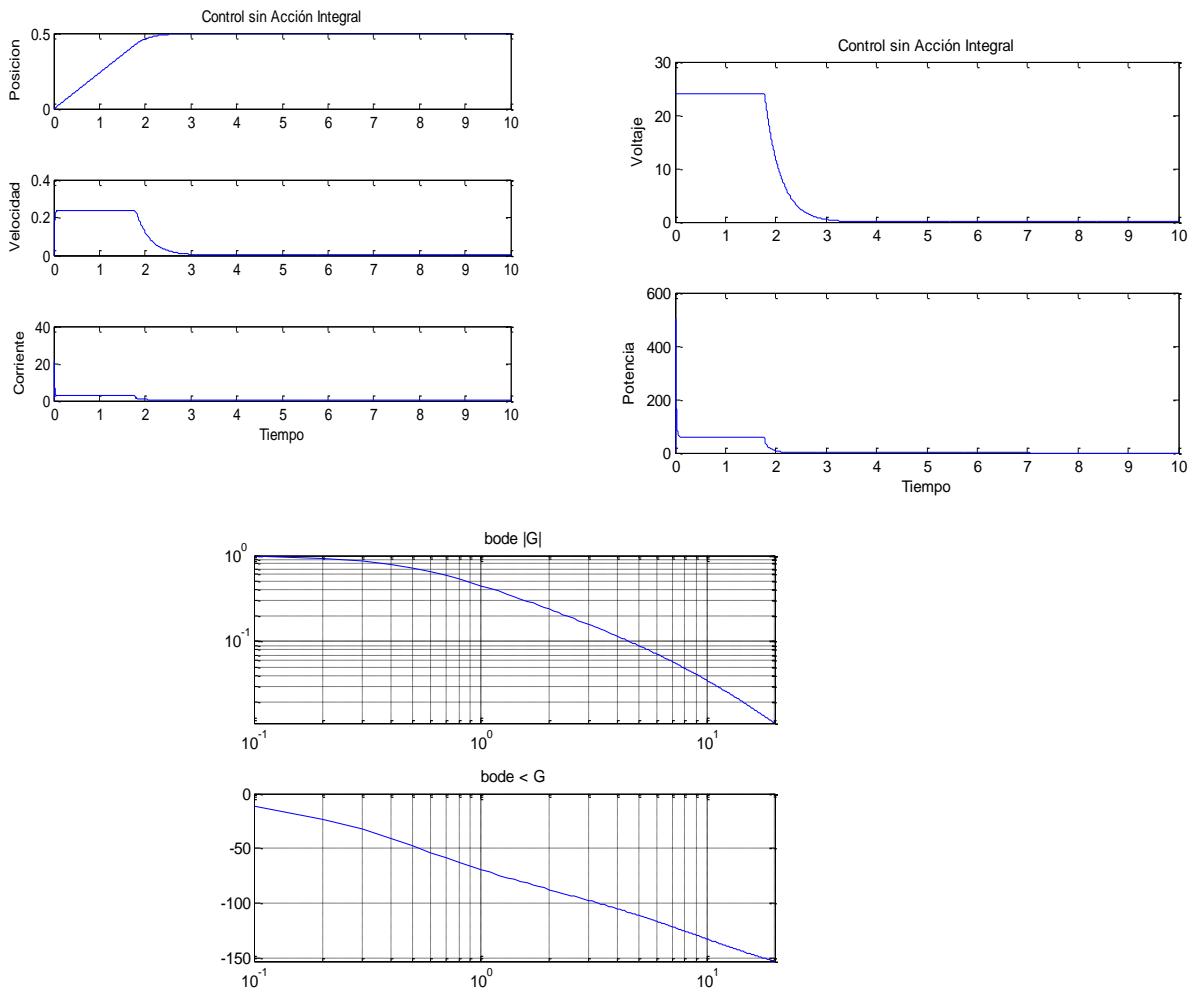


$$q_1 = 1e5, q_2 = 0, q_3 = 0$$

Error estacionario: -1.034e-009%

Sobreimpulso 0.00%

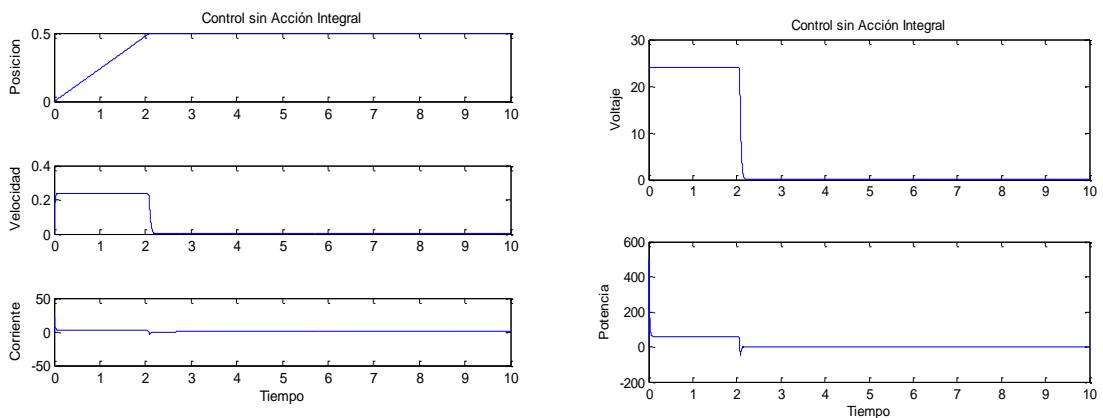
Controladores de Estado con acción integral



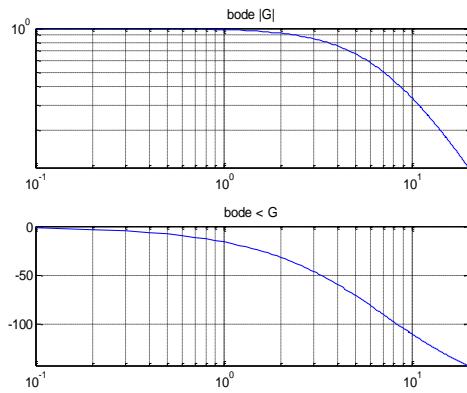
$$q1 = 1e7, q2 = 0, q3 = 0$$

Error estacionario: -1.2773e-010%

Sobreimpulso 0.00%

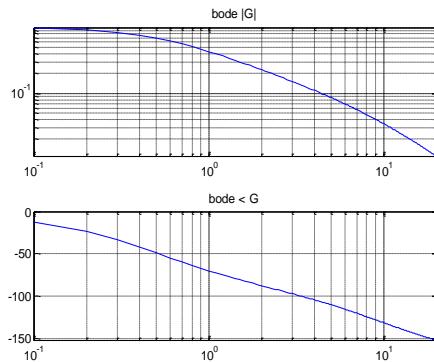
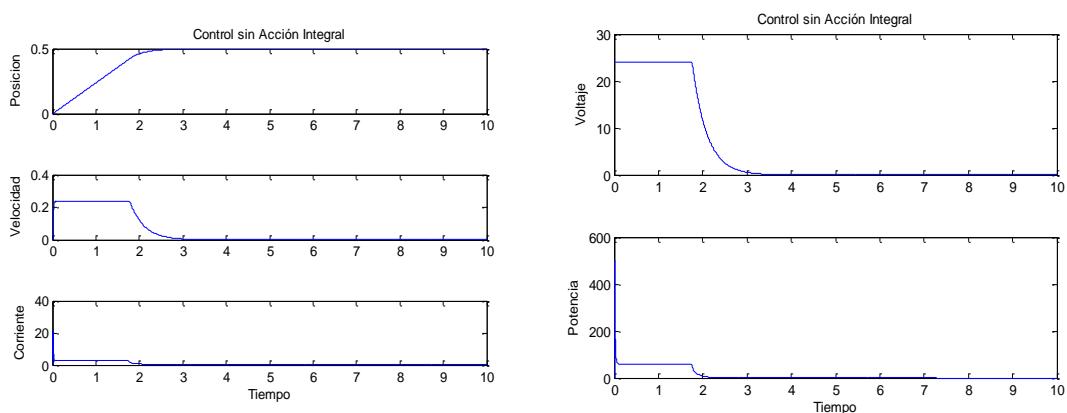


Controladores de Estado con acción integral



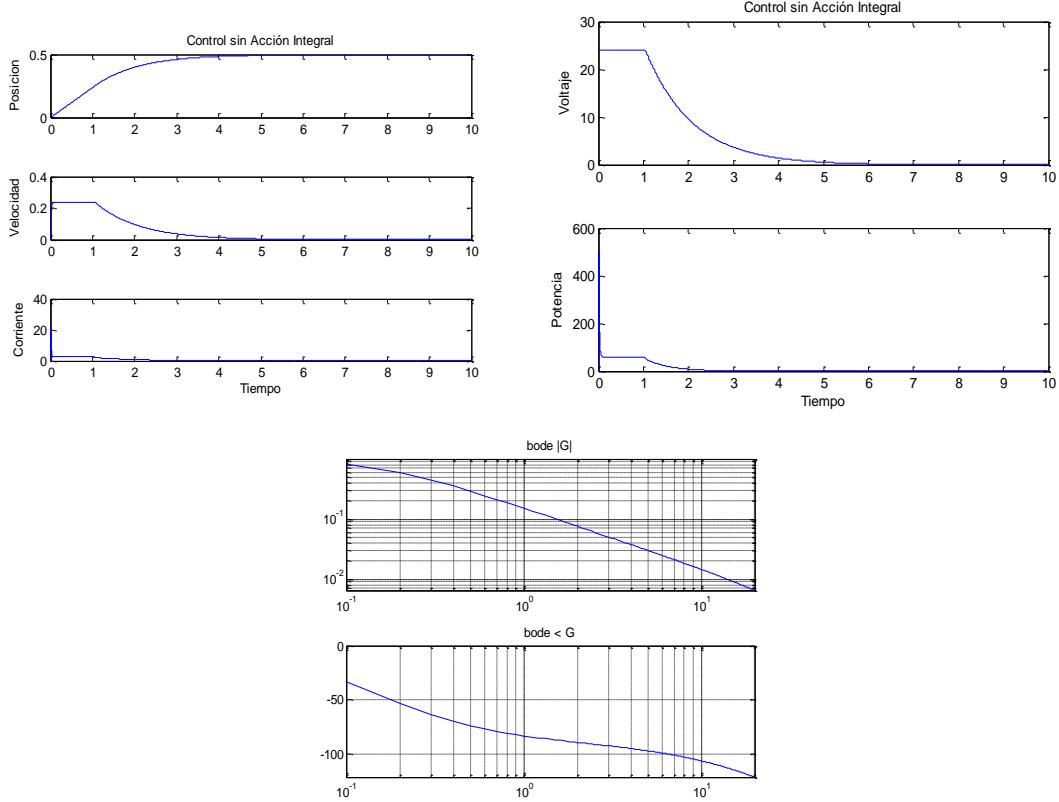
1.1.2. Caso: q1 cte. – q2 variable – q3 cte.

$q1 = 1e5$, $q2 = 1e3$, $q3 = 0$
 Error estacionario: -1.2759e-009%
 Sobreimpulso 0.00%



$q1 = 1e5$, $q2 = 1e5$, $q3 = 0$
 Error estacionario: -0.0099853%
 Sobreimpulso 0.00%

Controladores de Estado con acción integral

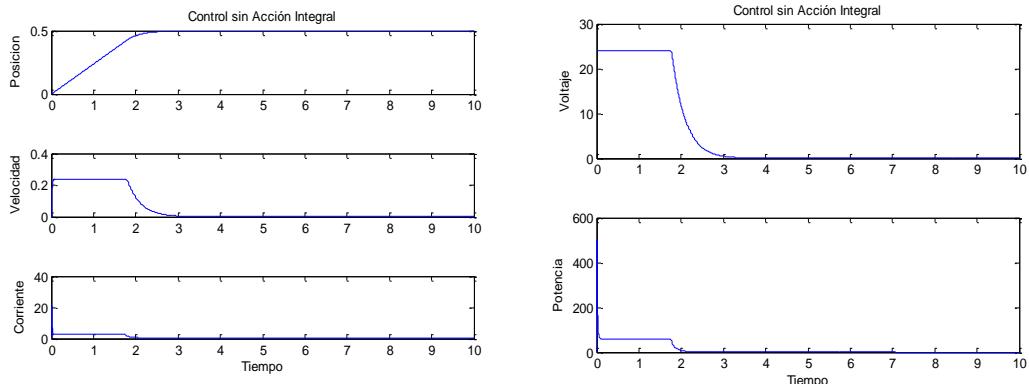


1.1.3. Caso: q1 cte. – q2 cte. – q3 variable

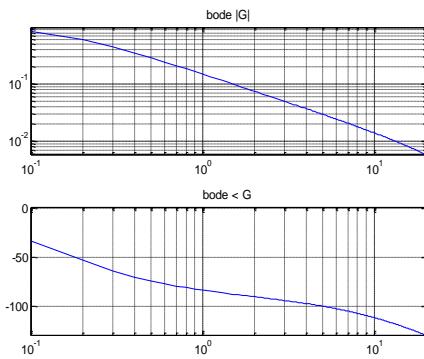
$q1 = 1e5$, $q2 = 1e5$, $q3 = 1$

Error estacionario: $-1.0651e-009\%$

Sobreimpulso 0.00%



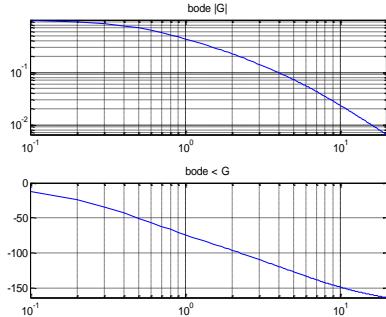
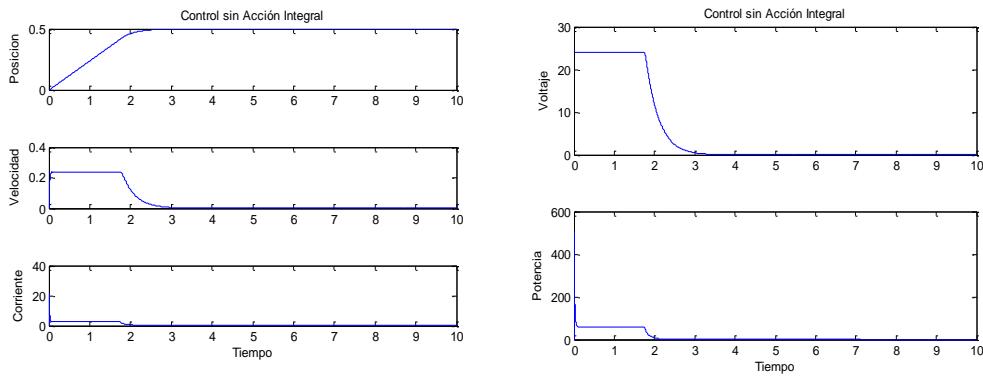
Controladores de Estado con acción integral



$$q_1 = 1e5, q_2 = 0, q_3 = 3$$

Error estacionario: -1.0651e-009%

Sobreimpulso 0.00%



Fricción estática $F_s = 0.5$

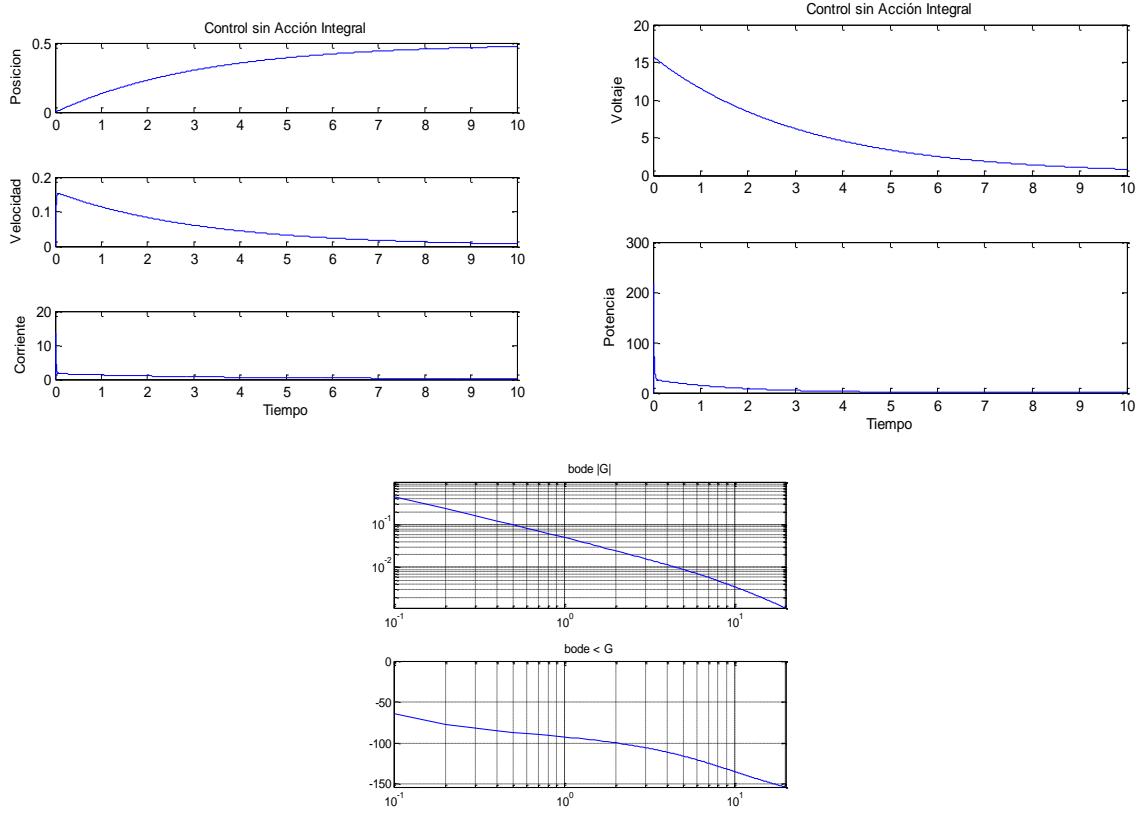
1.1.4. Caso: q_1 variable – q_2 cte. – q_3 cte.

$$q_1 = 1e3, q_2 = 0, q_3 = 0$$

Error estacionario: -5.1911%

Sobreimpulso 0.00%

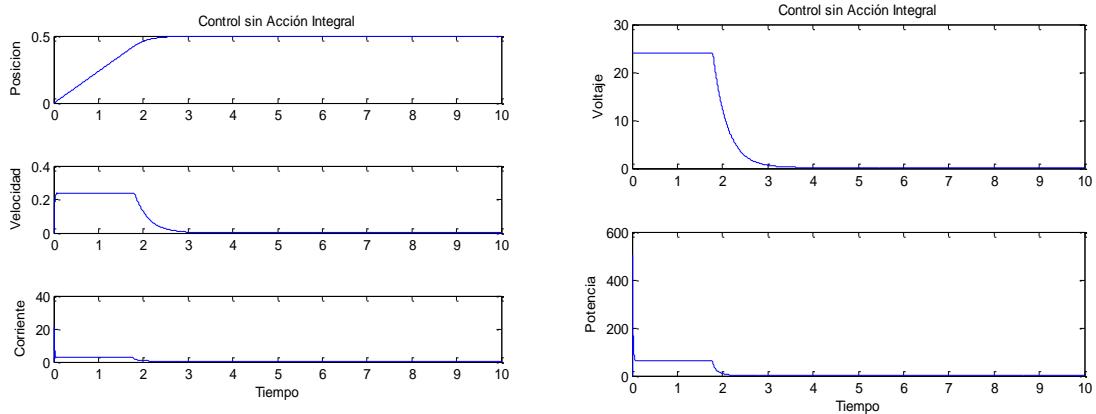
Controladores de Estado con acción integral



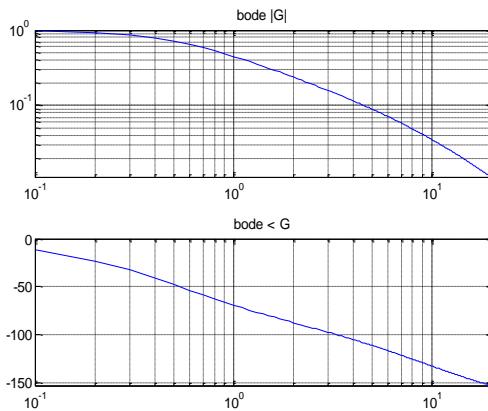
$$q1 = 1e5, q2 = 0, q3 = 0$$

Error estacionario: -1.034e-009%

Sobreimpulso 0.00%



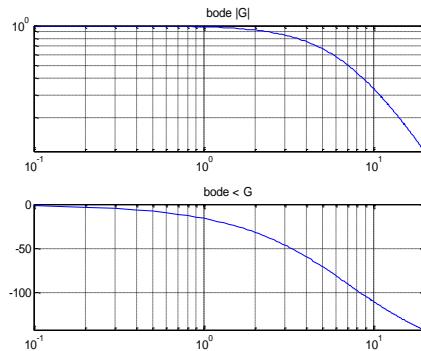
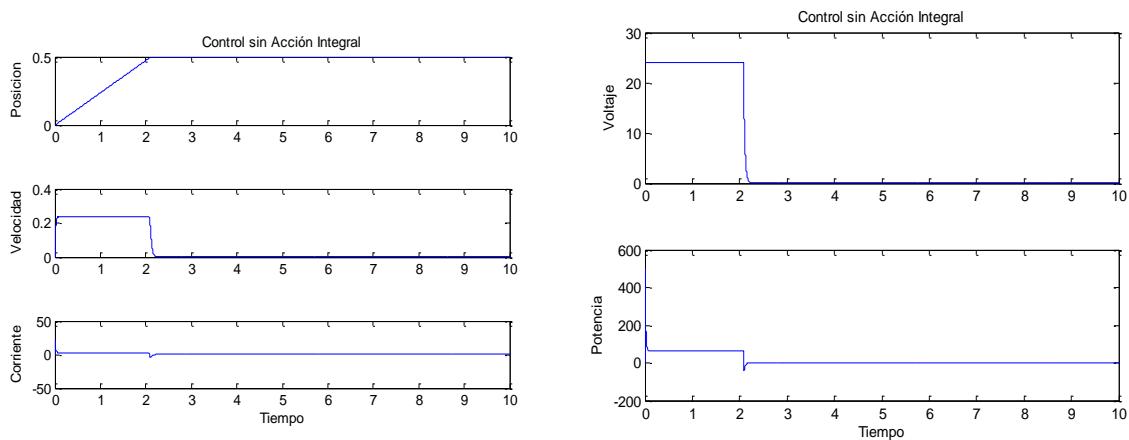
Controladores de Estado con acción integral



$$q1 = 1e7, q2 = 0, q3 = 0$$

Error estacionario: -0.0091276%

Sobreimpulso 0.00%



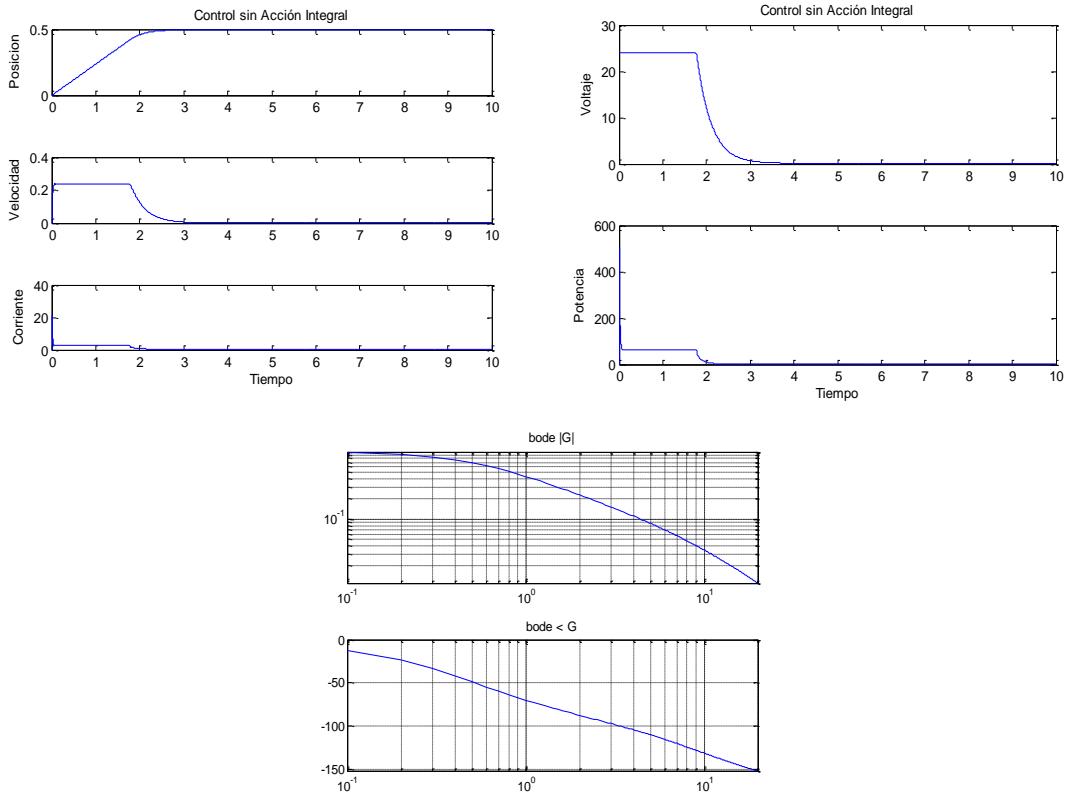
1.1.5. Caso: $q1$ cte. – $q2$ variable – $q3$ cte.

$$q1 = 1e5, q2 = 1e3, q3 = 0$$

Error estacionario: -0.09111%

Sobreimpulso 0.00%

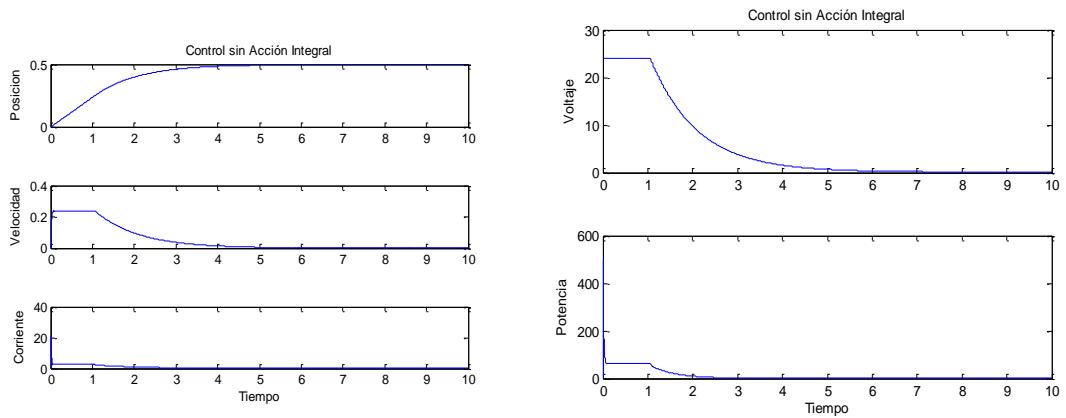
Controladores de Estado con acción integral

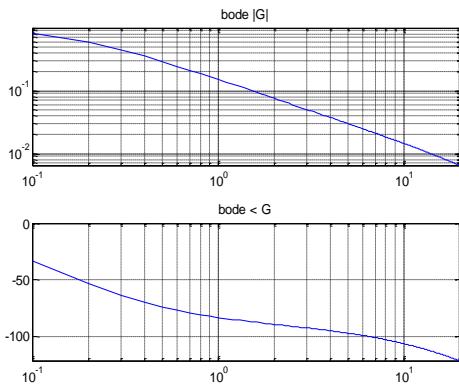


$$q1 = 1e5, q2 = 1e5, q3 = 0$$

Error estacionario: -0.10229%

Sobreimpulso 0.00%



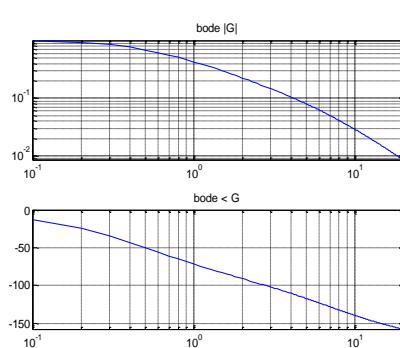
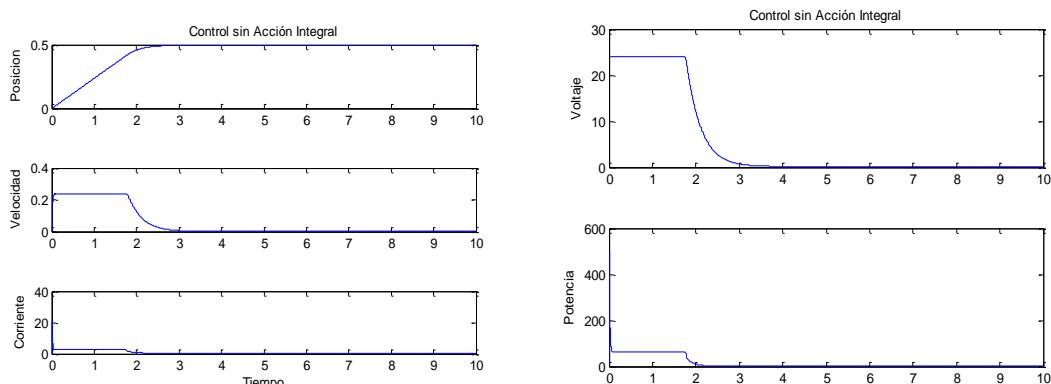


1.1.6. Caso: q1 cte. – q2 cte. – q3 variable

$$q1 = 1e5, q2 = 1e3, q3 = 1$$

Error estacionario: -0.12309%

Sobreimpulso 0.00%

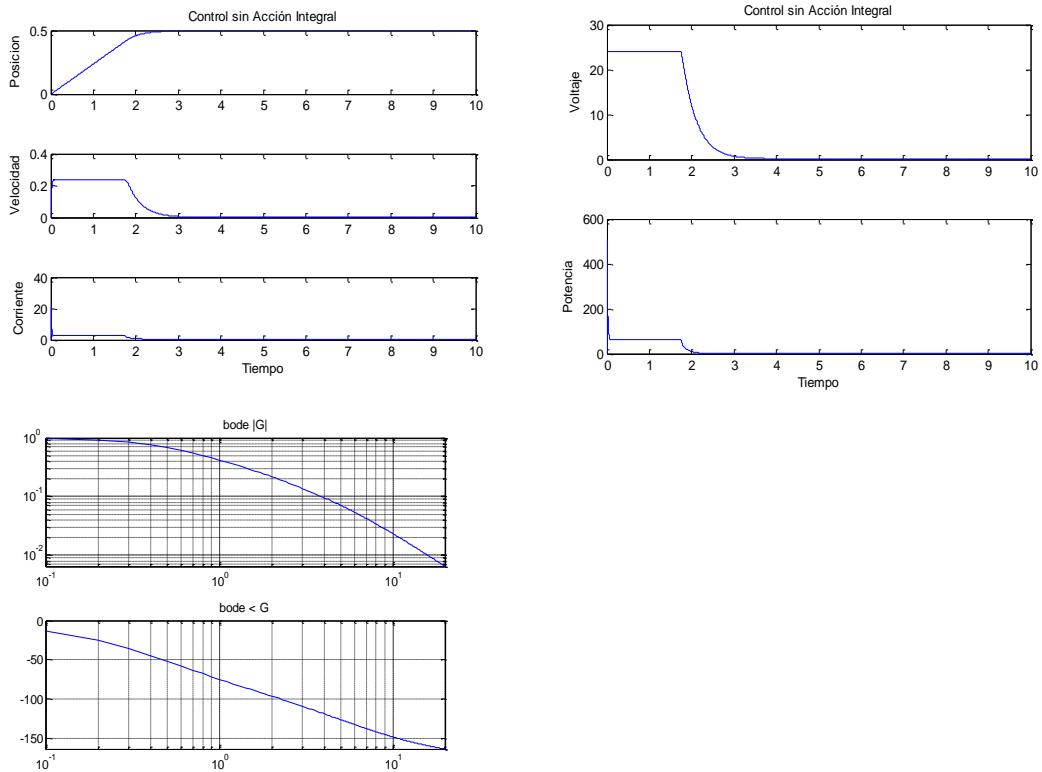


$$q1 = 1e5, q2 = 1e3, q3 = 3$$

Error estacionario: -0.16987%

Sobreimpulso 0.00%

Controladores de Estado con acción integral



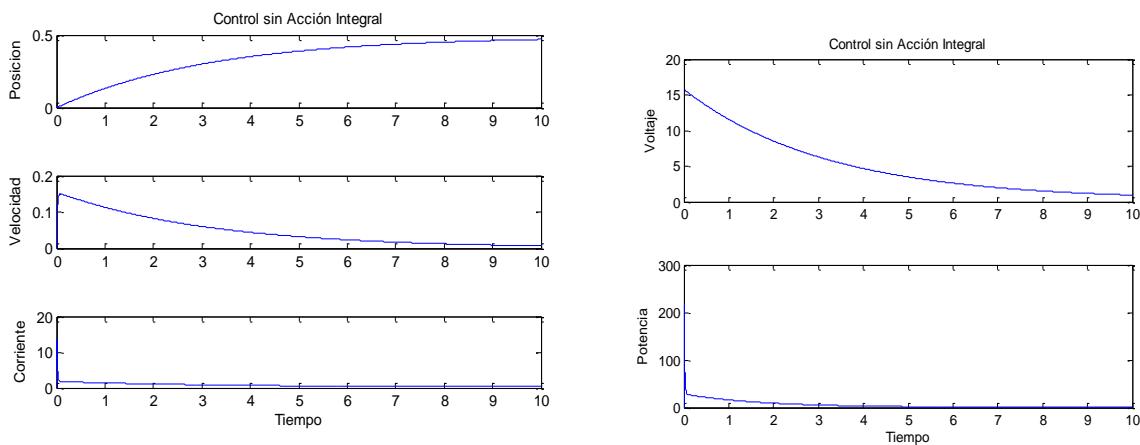
Fricción estática $F_s = 1$

1.1.7. Caso: q_1 variable – q_2 cte. – q_3 cte.

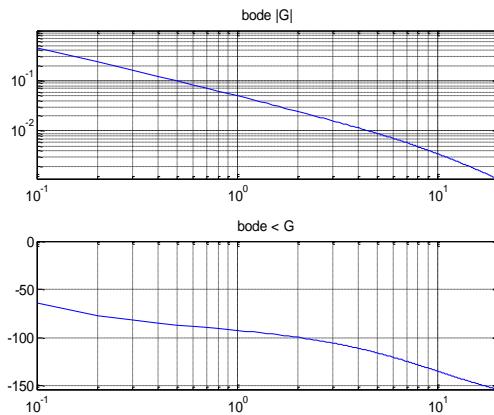
$$q_1 = 1e3, q_2 = 0, q_3 = 0$$

Error estacionario: -6.0624%

Sobreimpulso 0.00%



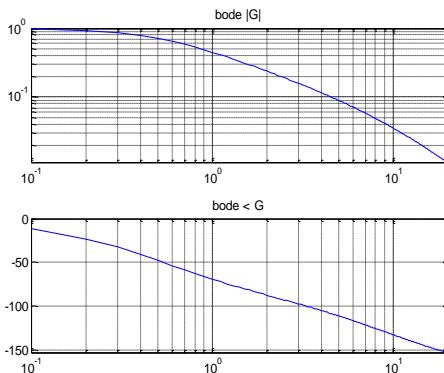
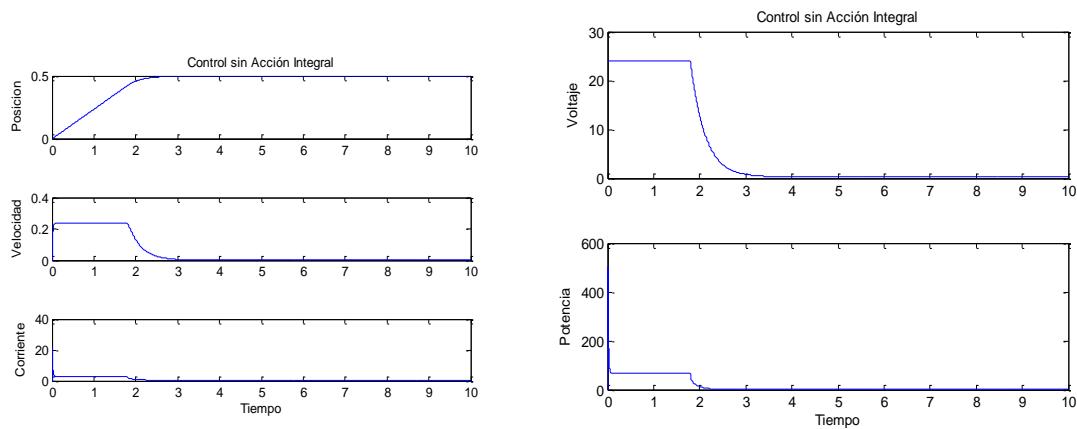
Controladores de Estado con acción integral



$$q1 = 1e5, q2 = 0, q3 = 0$$

Error estacionario: -0.18217%

Sobreimpulso 0.00%

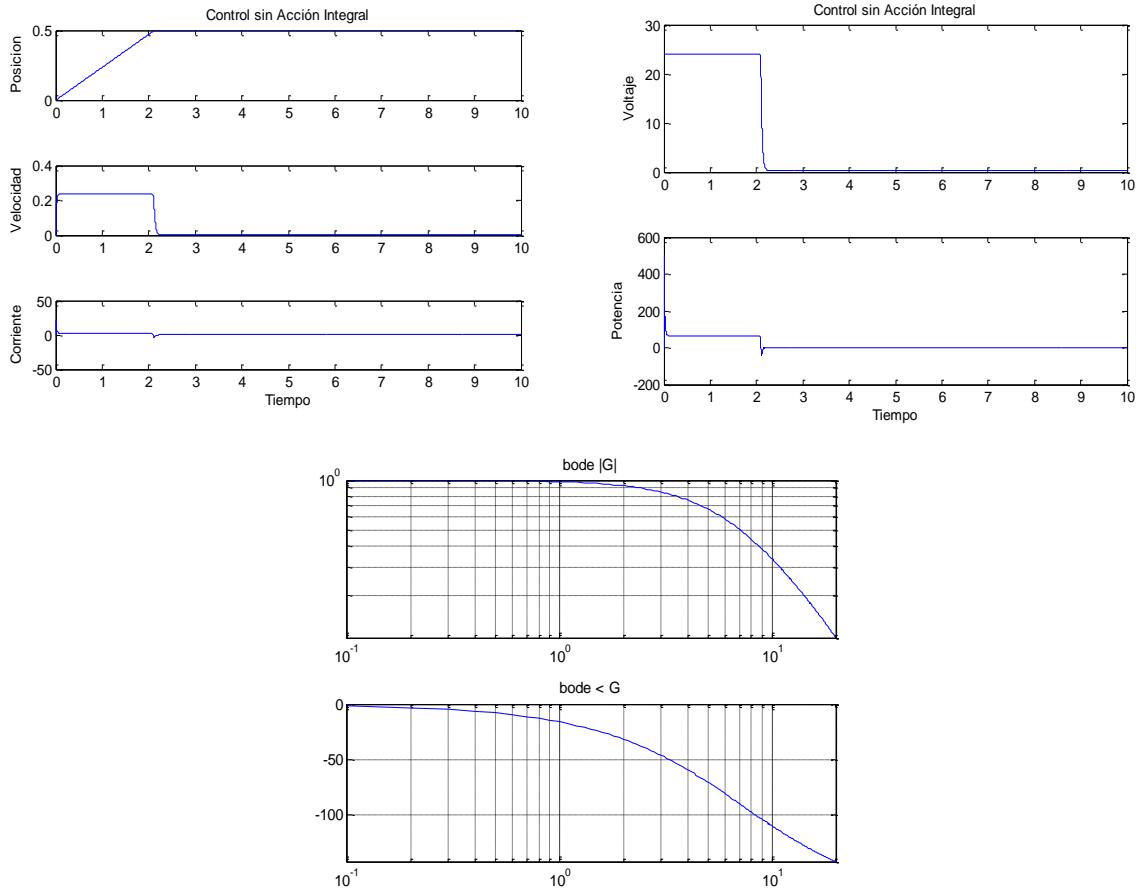


$$q1 = 1e7, q2 = 0, q3 = 0$$

Error estacionario: -0.018255%

Sobreimpulso 0.00%

Controladores de Estado con acción integral



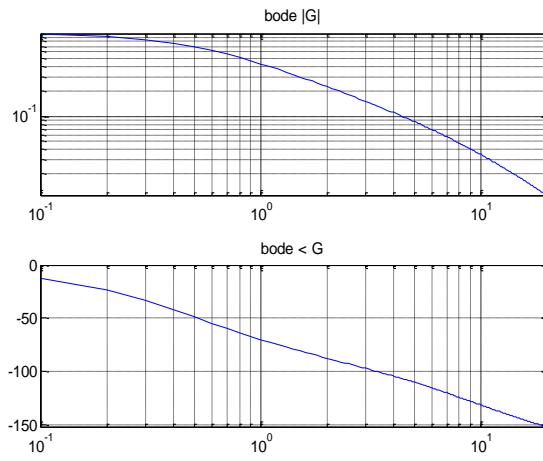
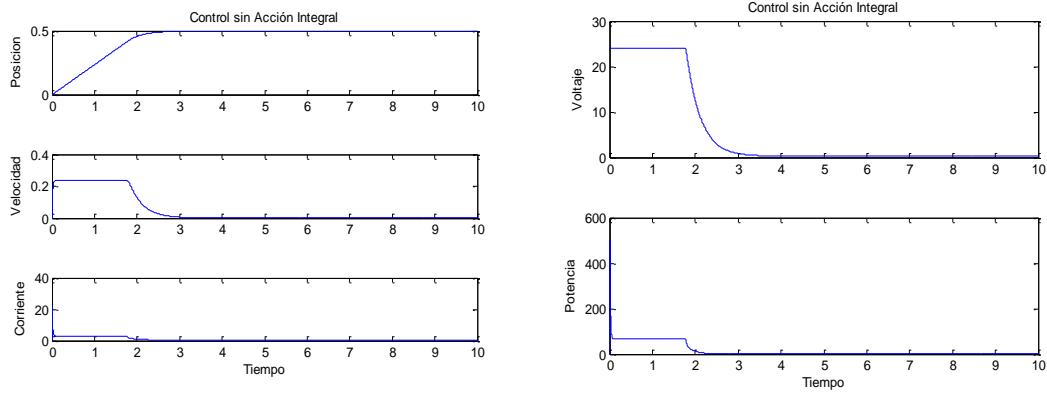
1.1.8. Caso: q1 cte. – q2 variable – q3 cte.

$$q1 = 1e5, q2 = 1e3, q3 = 0$$

Error estacionario: -0.18222%

Sobreimpulso 0.00%

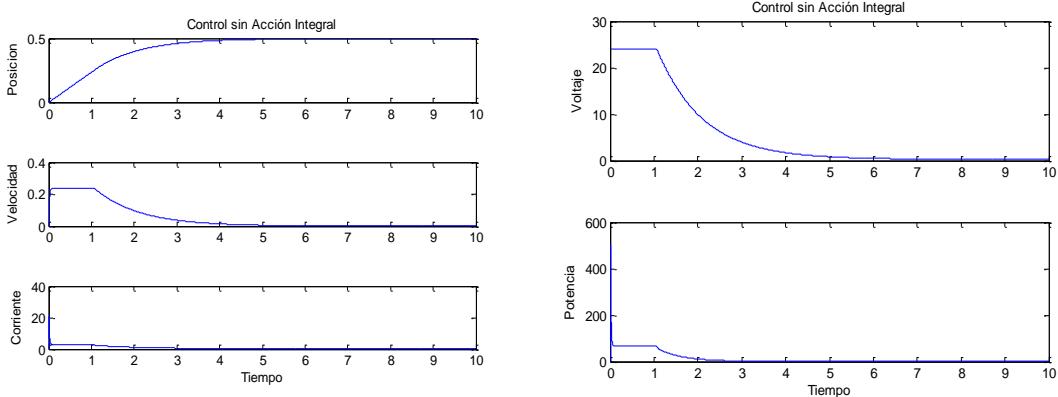
Controladores de Estado con acción integral



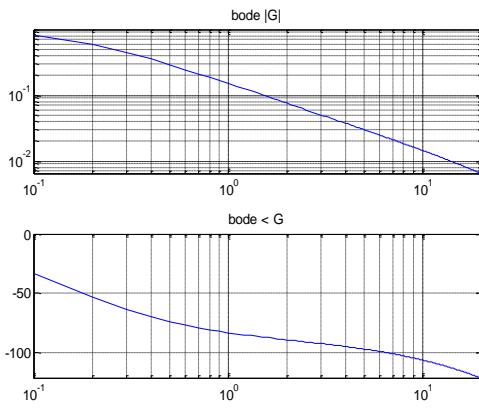
$$q1 = 1e5, q2 = 1e5, q3 = 0$$

Error estacionario: -0.19459%

Sobreimpulso 0.00%



Controladores de Estado con acción integral

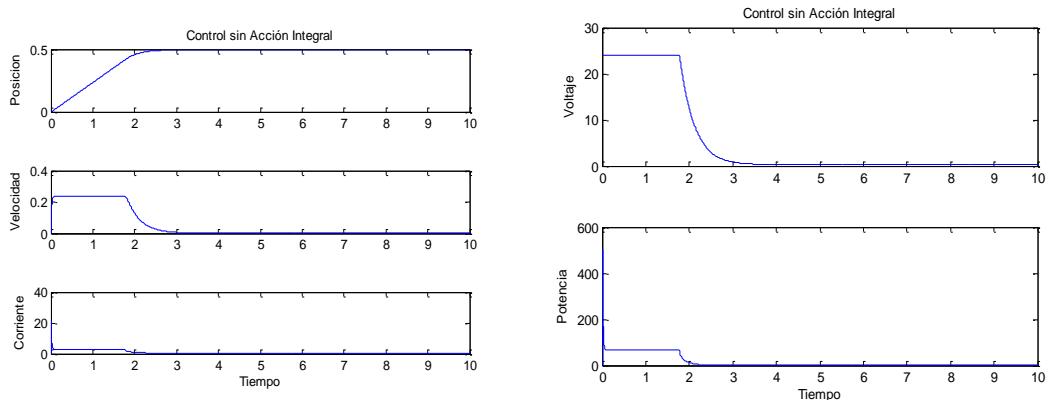


1.1.9. Caso: q1 cte. – q2 cte. – q3 variable

$q1 = 1e5, q2 = 1e3, q3 = 1$

Error estacionario: -0.24619%

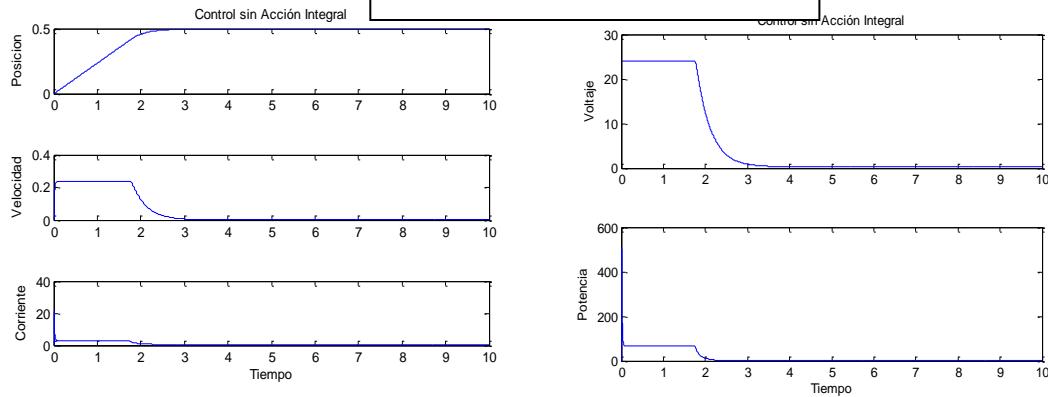
Sobreimpulso 0.00%



$q1 = 1e5, q2 = 1e3, q3 = 3$

Error estacionario: -0.33974%

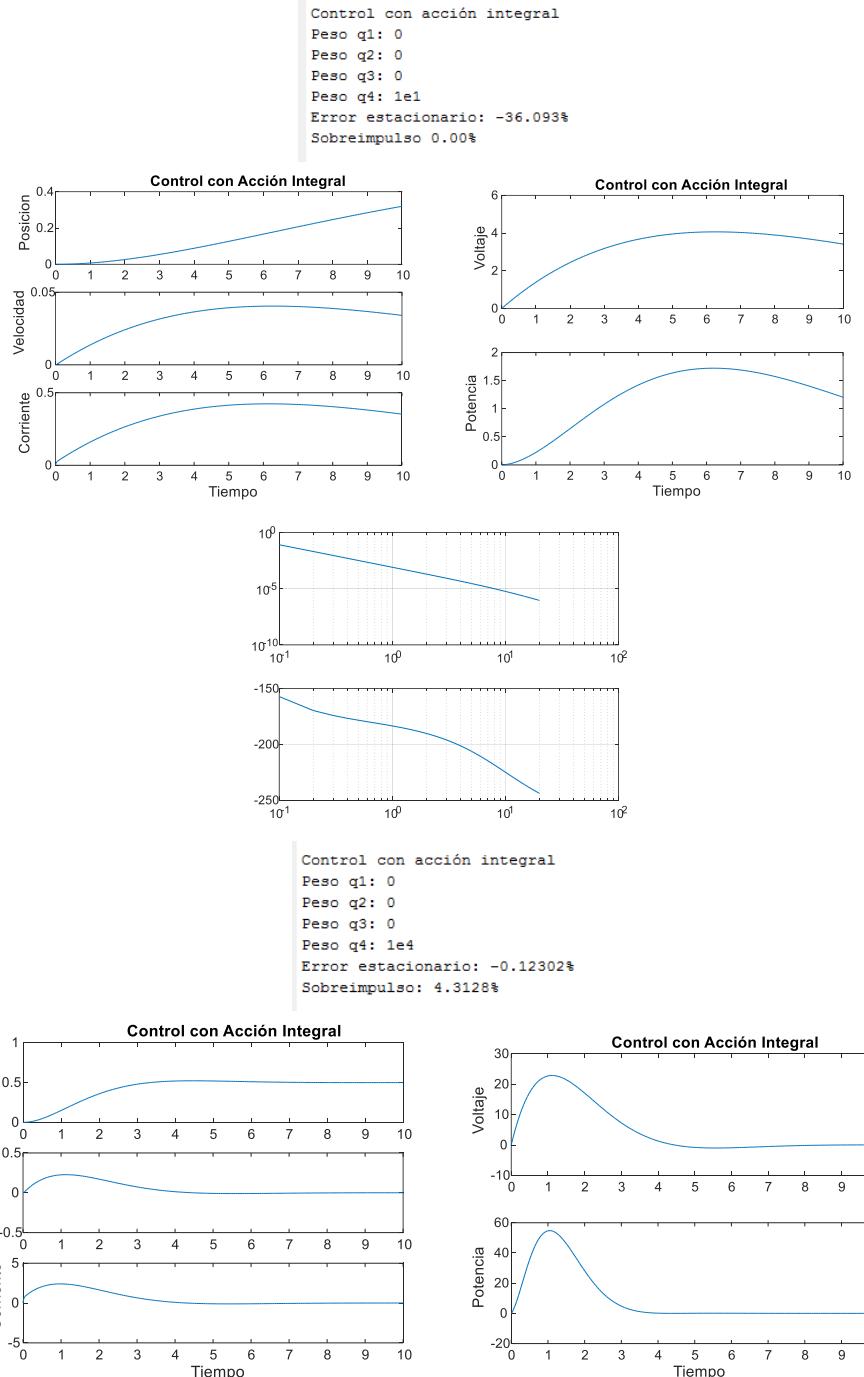
Sobreimpulso 0.00%



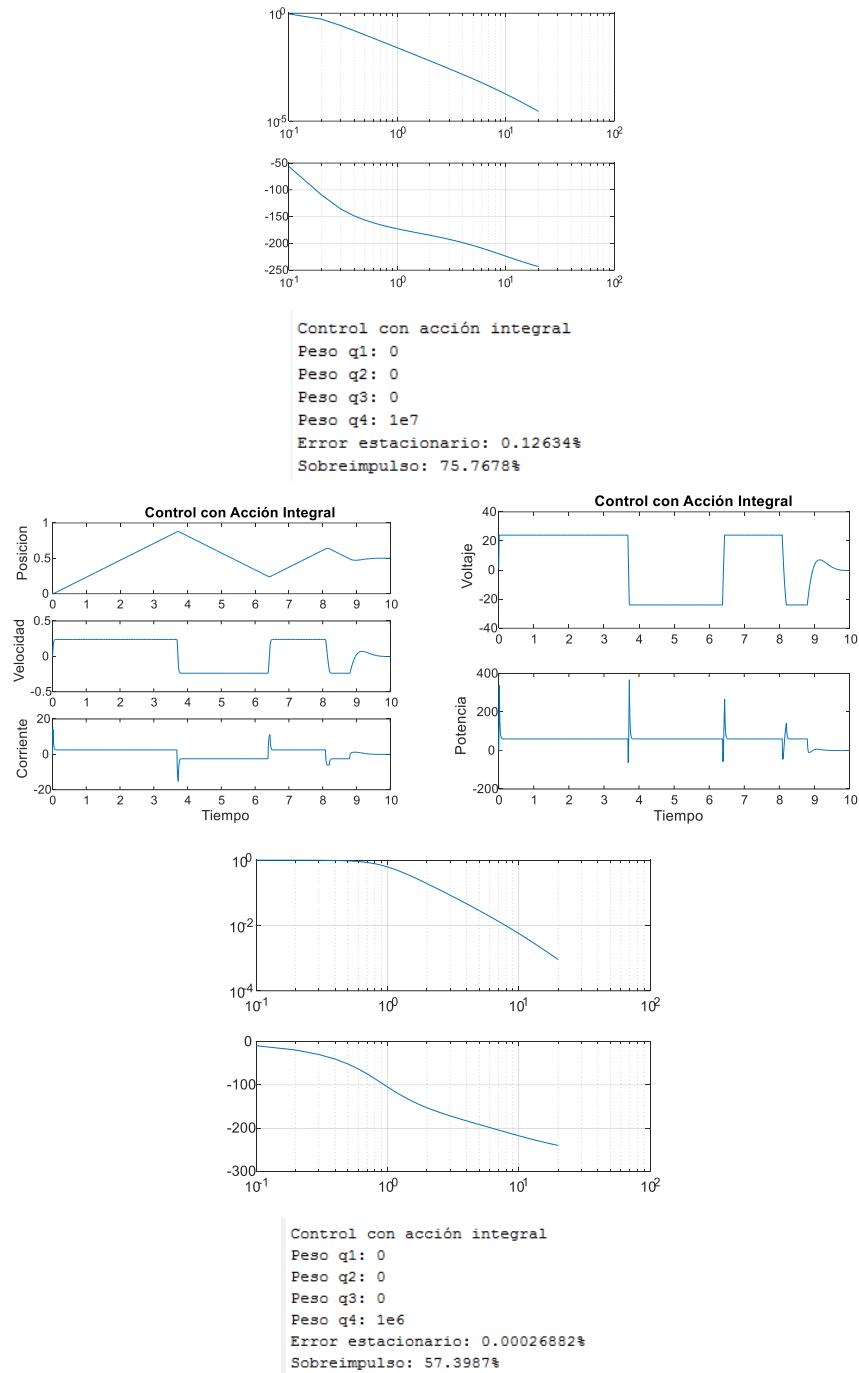
1.2. Control con acción integral

Fricción estática $F_s = 0$.

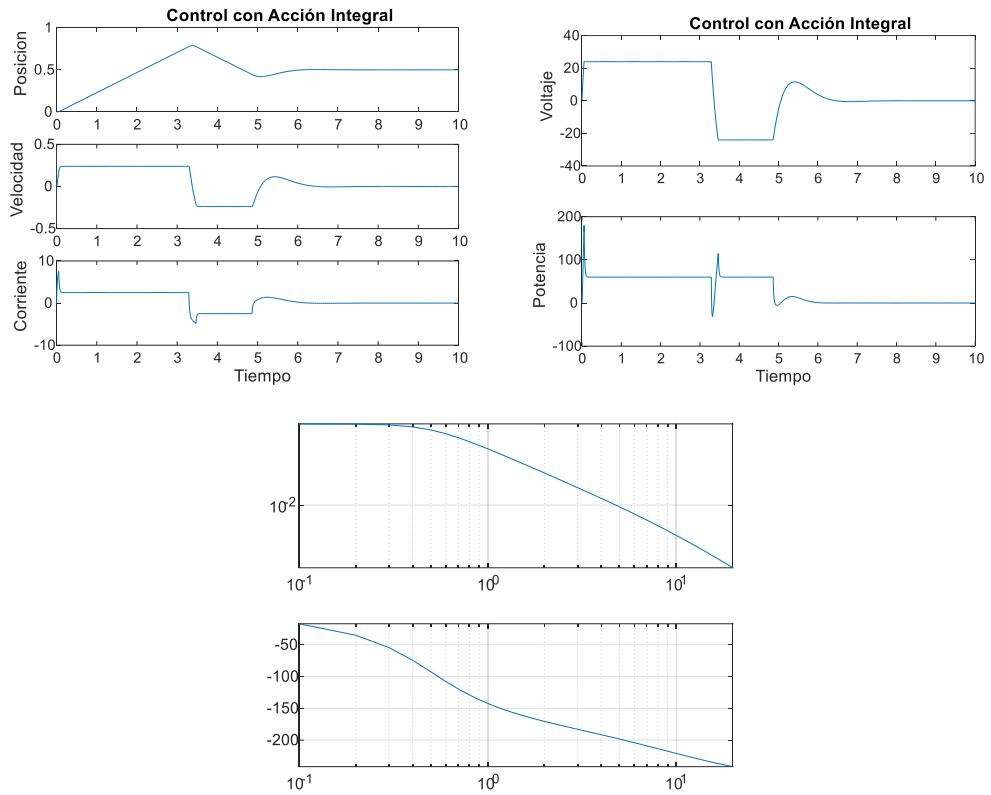
1.2.1. $q_1 = 0, q_2 = 0, q_3 = 0, q_4$ variable



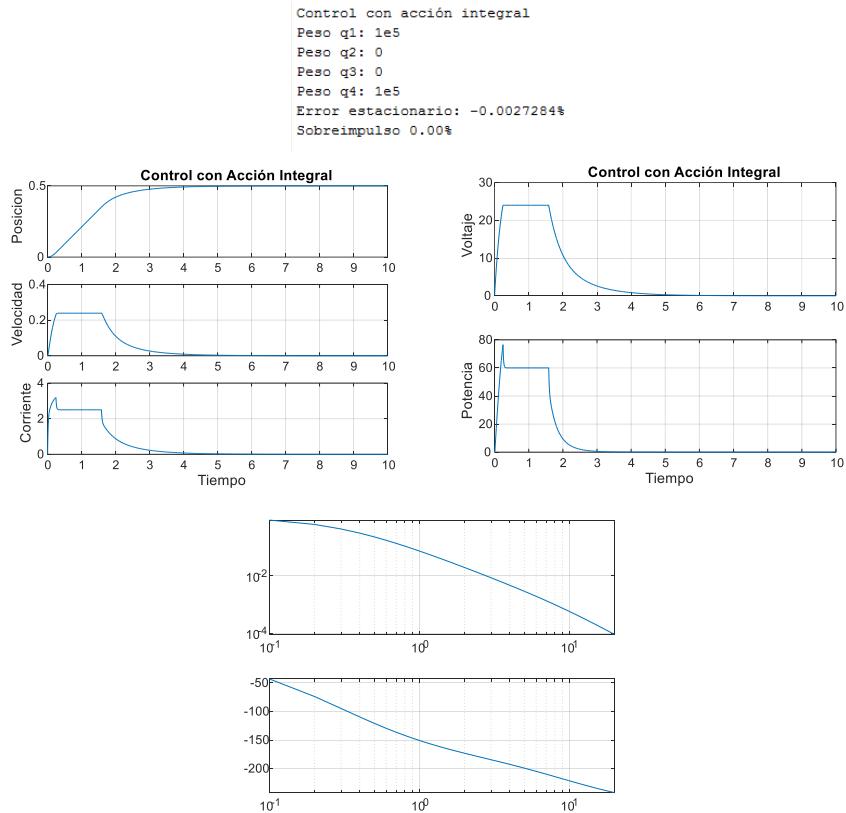
Controladores de Estado con acción integral



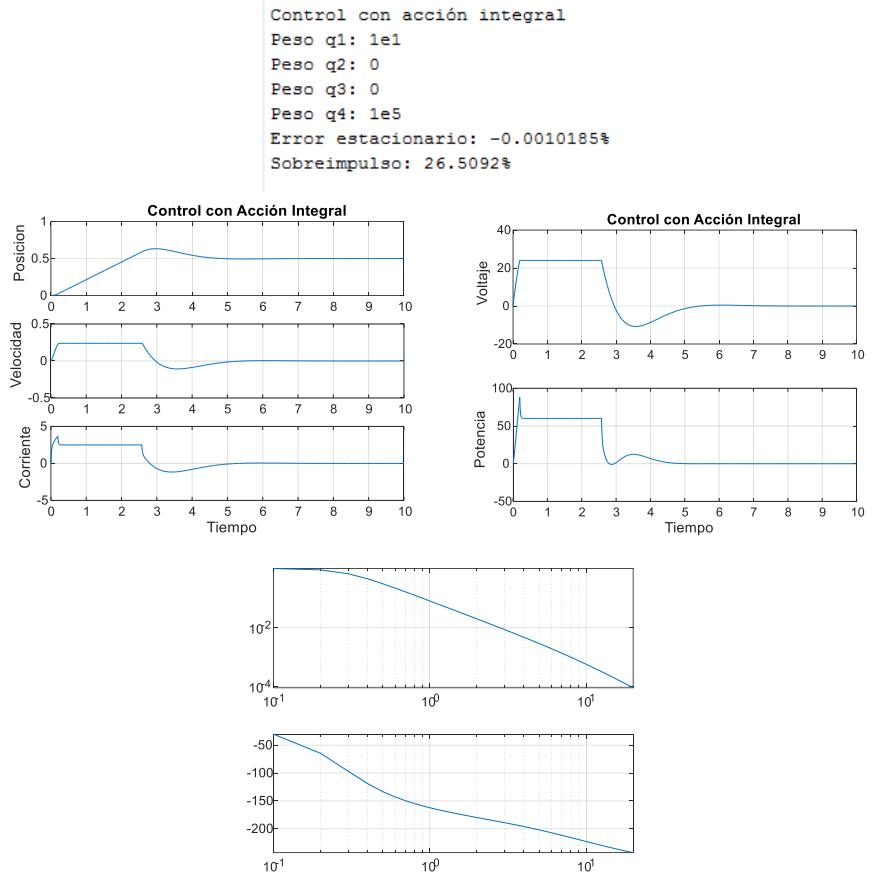
Controladores de Estado con acción integral



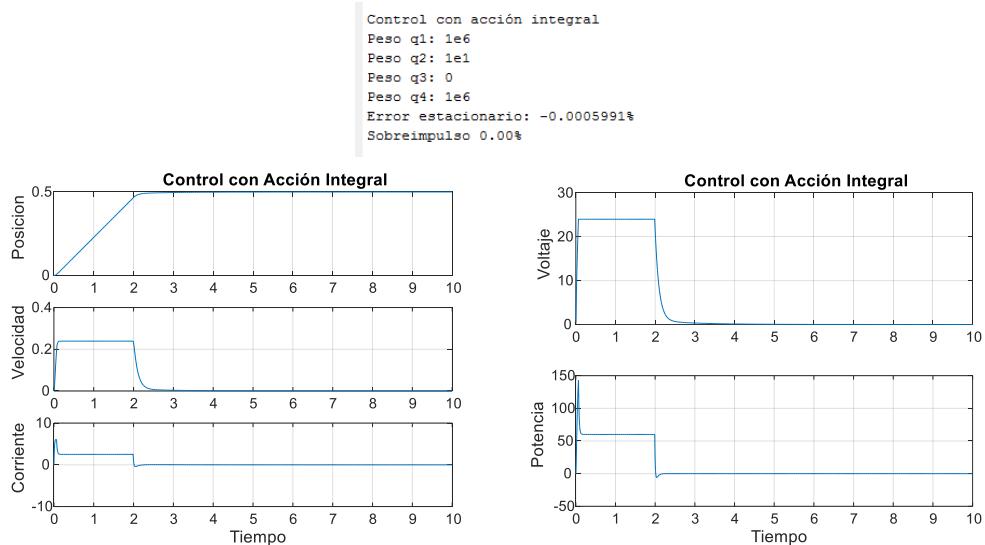
1.2.2. q_1 variable, $q_2 = 0$, $q_3 = 0$, q_4 cte.



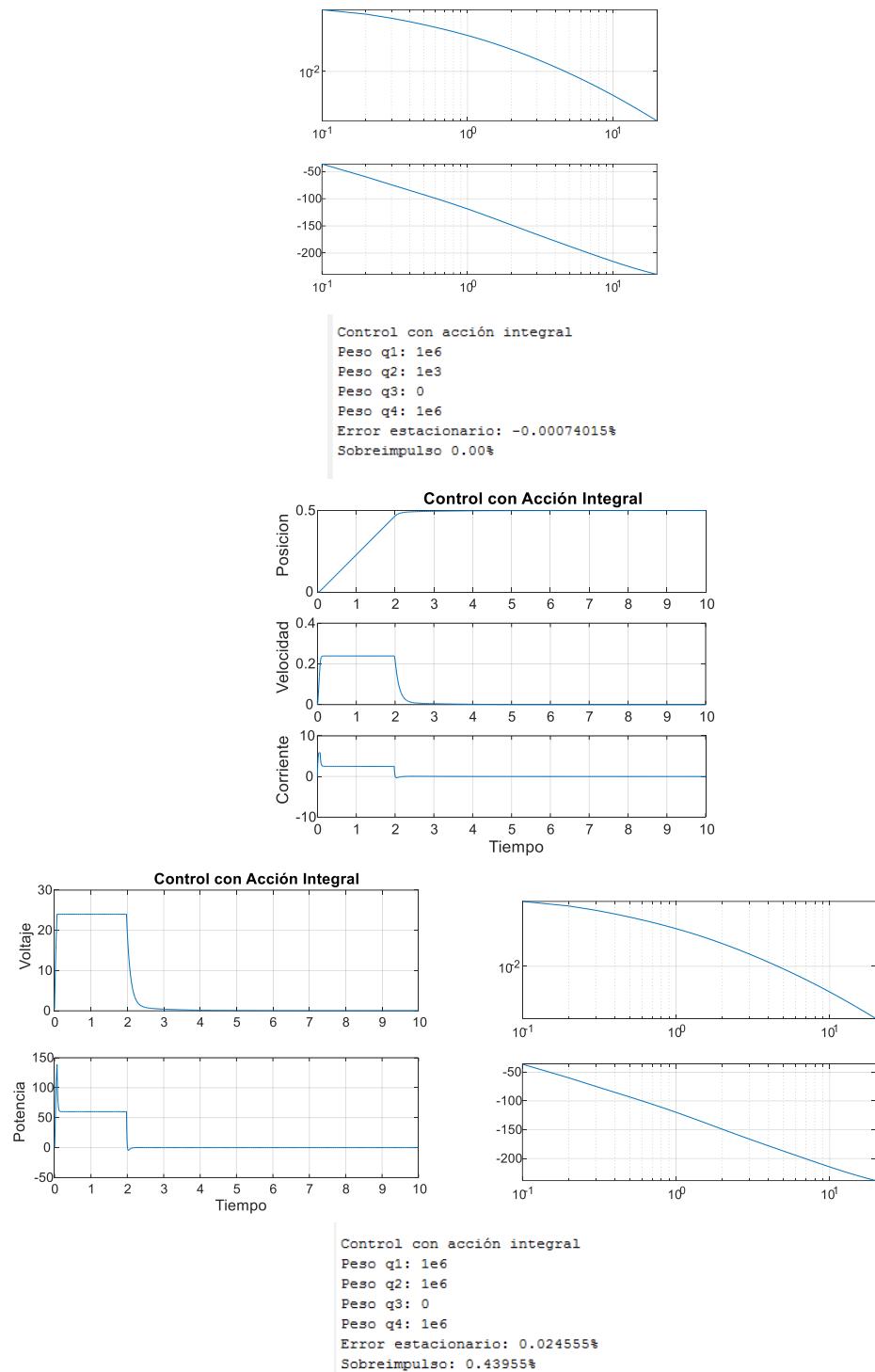
Controladores de Estado con acción integral



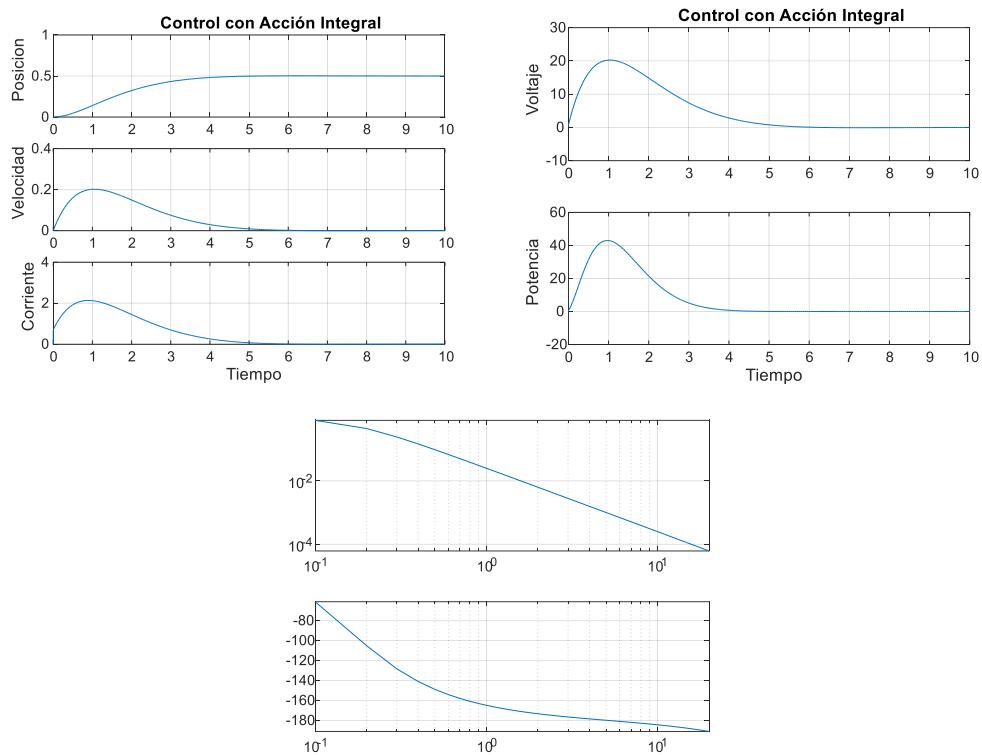
1.2.3. q_1 cte., q_2 variable, $q_3 = 0$, q_4 cte.



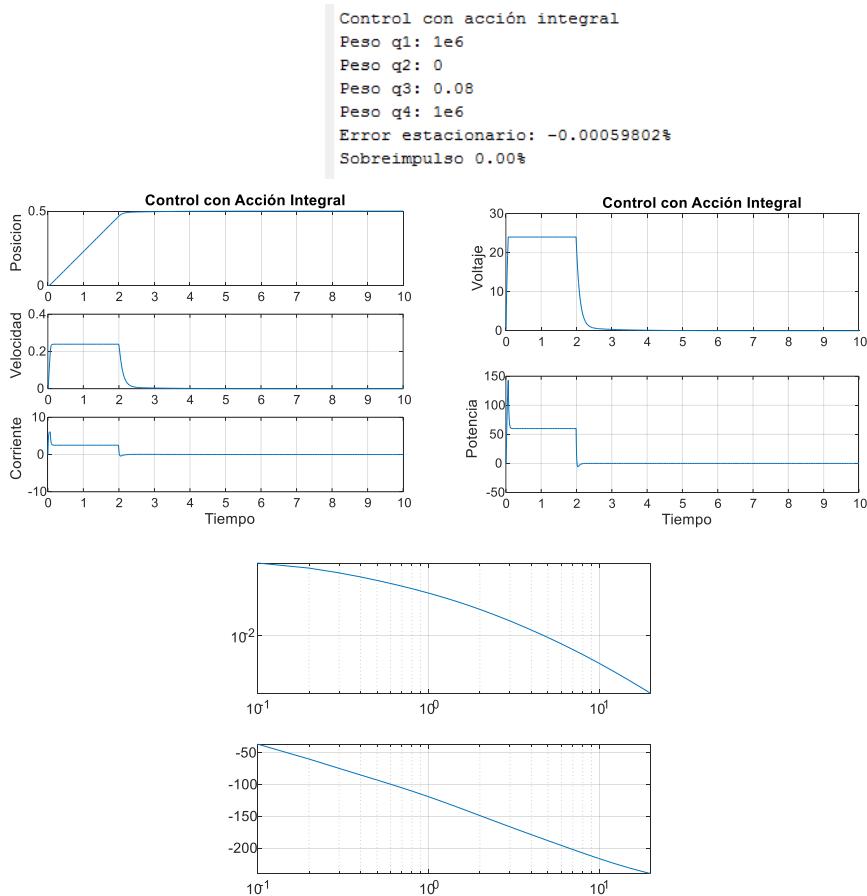
Controladores de Estado con acción integral



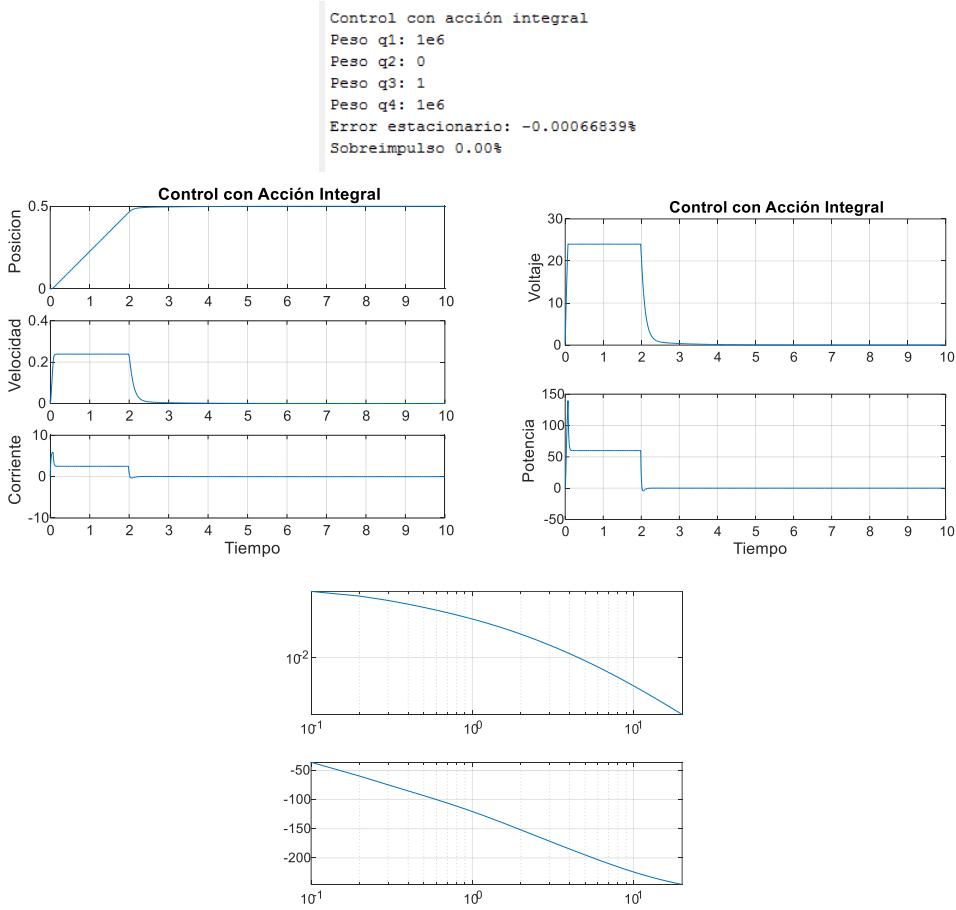
Controladores de Estado con acción integral



1.2.4. q_1 cte., $q_2 = 0$, q_3 = variable, q_4 cte.



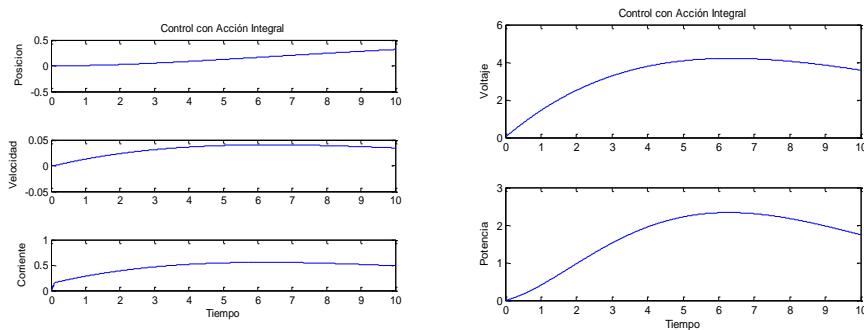
Controladores de Estado con acción integral



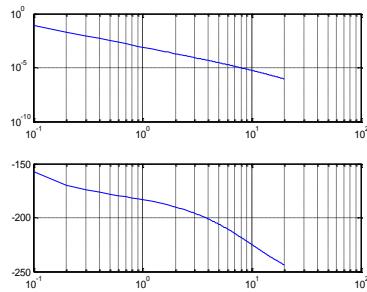
Fricción estática $F_s = 0.5$

1.1.1. $q_1 = 0, q_2 = 0, q_3 = 0, q_4$ variable

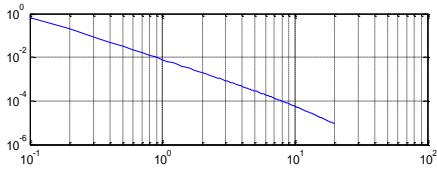
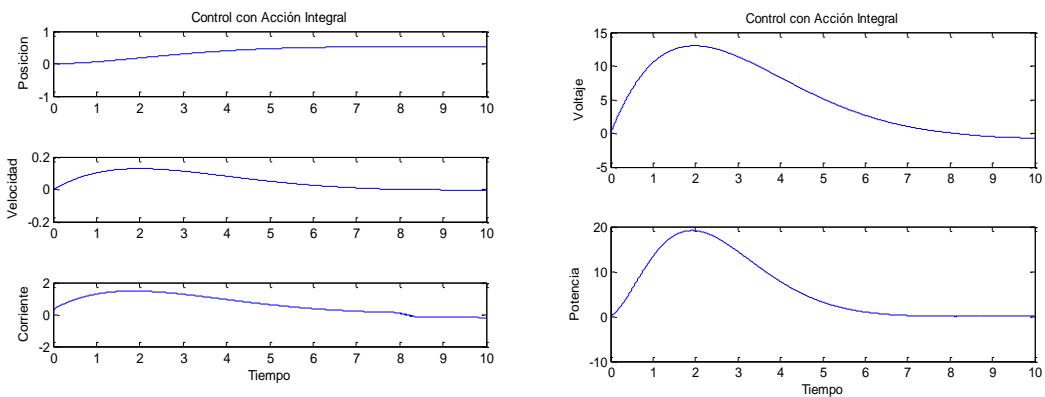
$q_1 = 0, q_2 = 0, q_3 = 0, q_4 = 10$
Error estacionario: -36.7156%
Sobreimpulso 0.00%



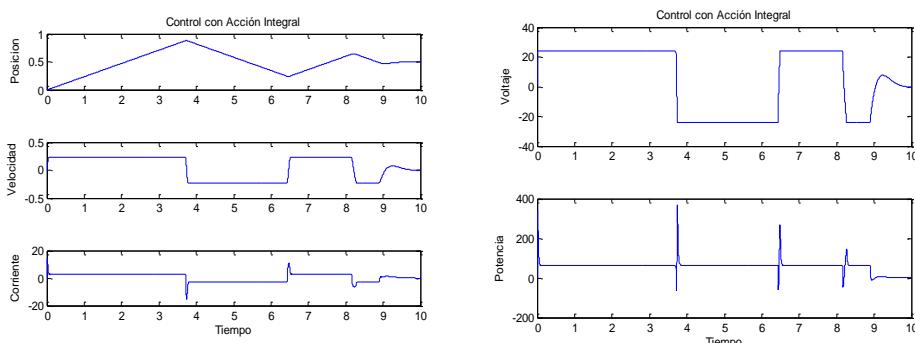
Controladores de Estado con acción integral

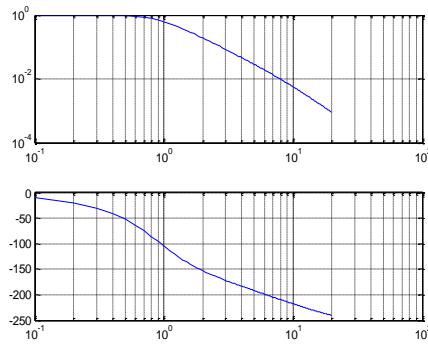


$q1 = 0, q2 = 0, q3 = 0, q4 = 1e3$
Error estacionario: 3.1633%
Sobreimpulso: 4.318%



$q1 = 0, q2 = 0, q3 = 0, q4 = 1e7$
Error estacionario: 0.23426%
Sobreimpulso: 75.9112%



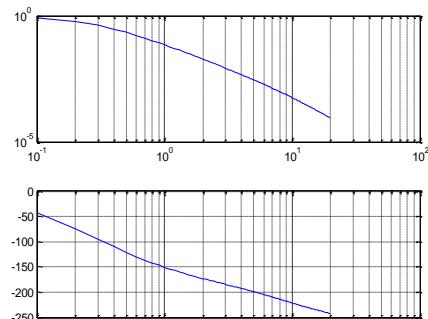
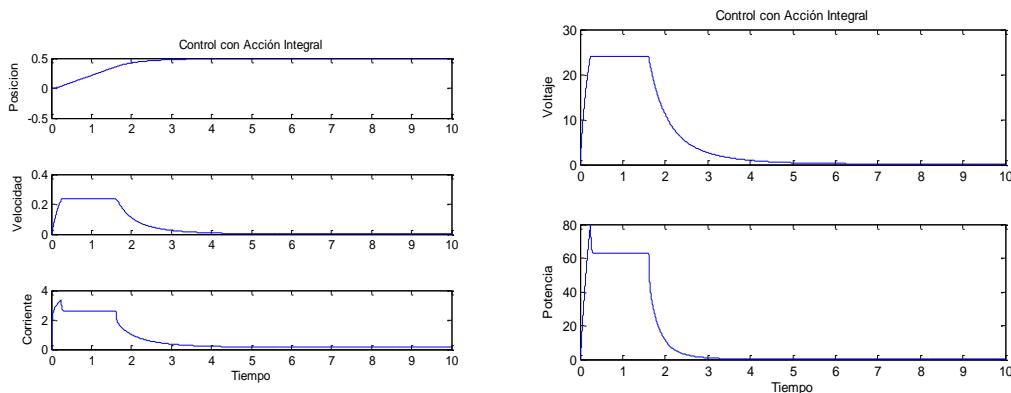


1.1.2. q_1 variable, $q_2 = 0$, $q_3 = 0$, q_4 cte.

$q_1 = 1e5, q_2 = 0, q_3 = 0, q_4 = 1e5$

Error estacionario: -0.0026552%

Sobreimpulso 0.00%

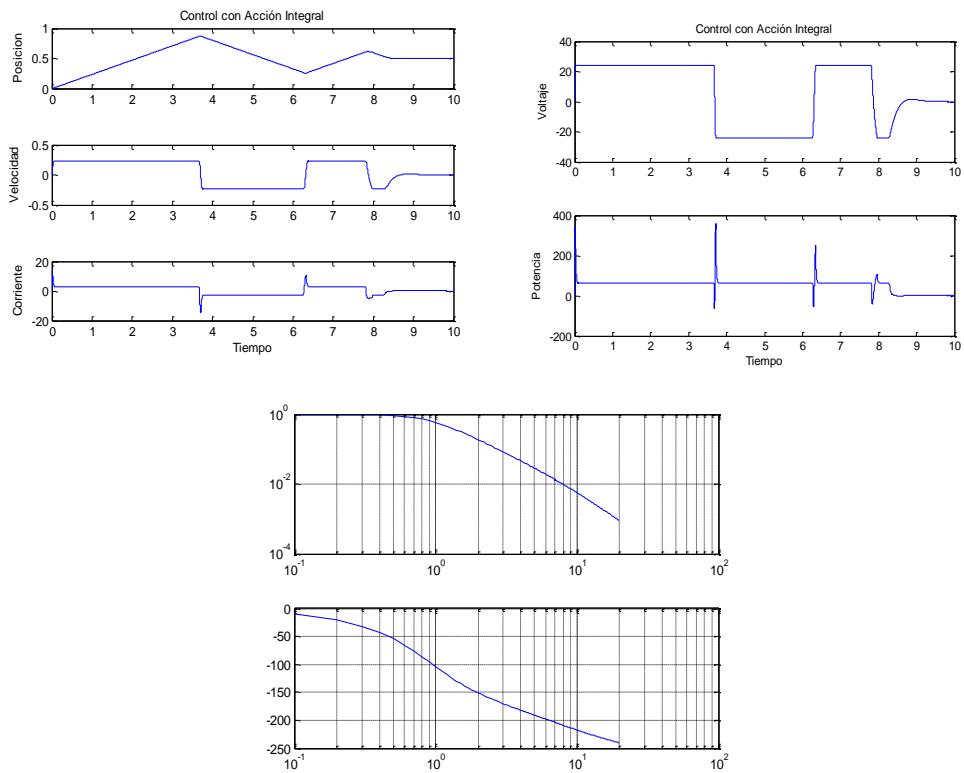


$q_1 = 1e5, q_2 = 0, q_3 = 0, q_4 = 1e7$

Error estacionario: 0.029466%

Sobreimpulso: 74.1133%

Controladores de Estado con acción integral

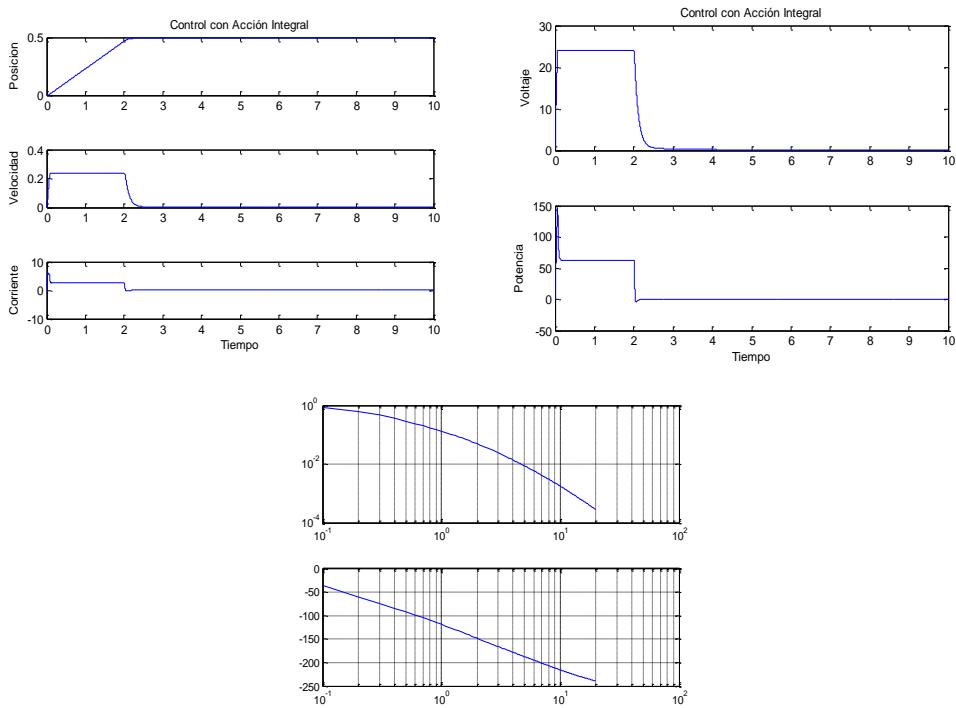


1.1.3. q_1 cte., q_2 variable, $q_3 = 0$, q_4 cte.

$$q_1 = 1e6, q_2 = 10, q_3 = 0, q_4 = 1e6$$

Error estacionario: -0.00039684%

Sobreimpulso 0.00%

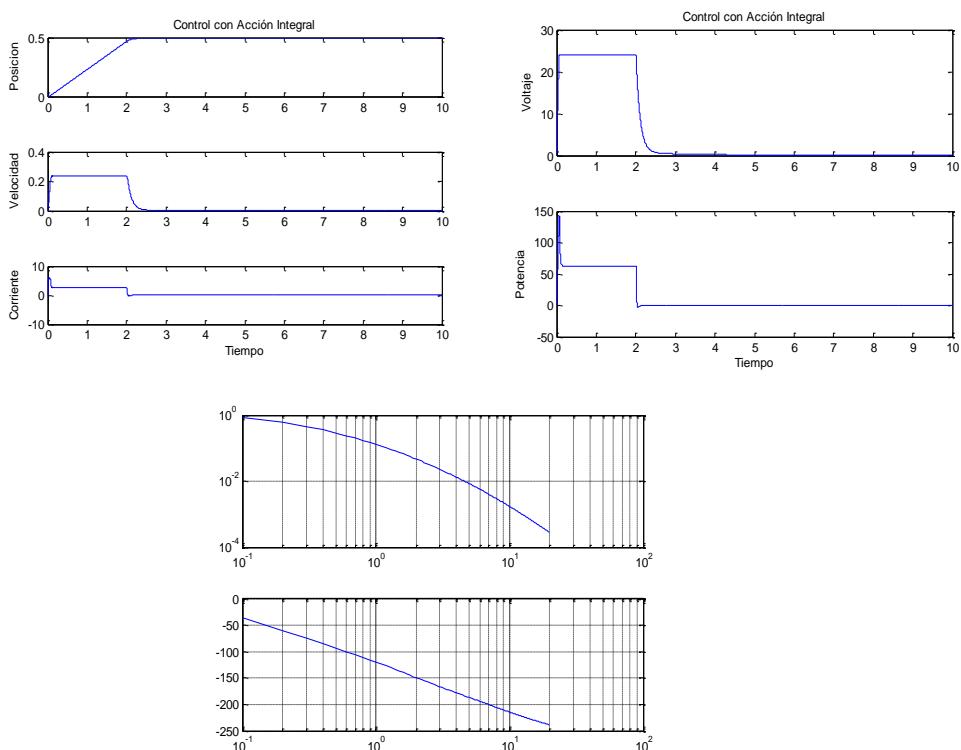


Controladores de Estado con acción integral

$q1 = 1e6, q2 = 1e3, q3 = 0, q4 = 1e6$

Error estacionario: -0.00047589%

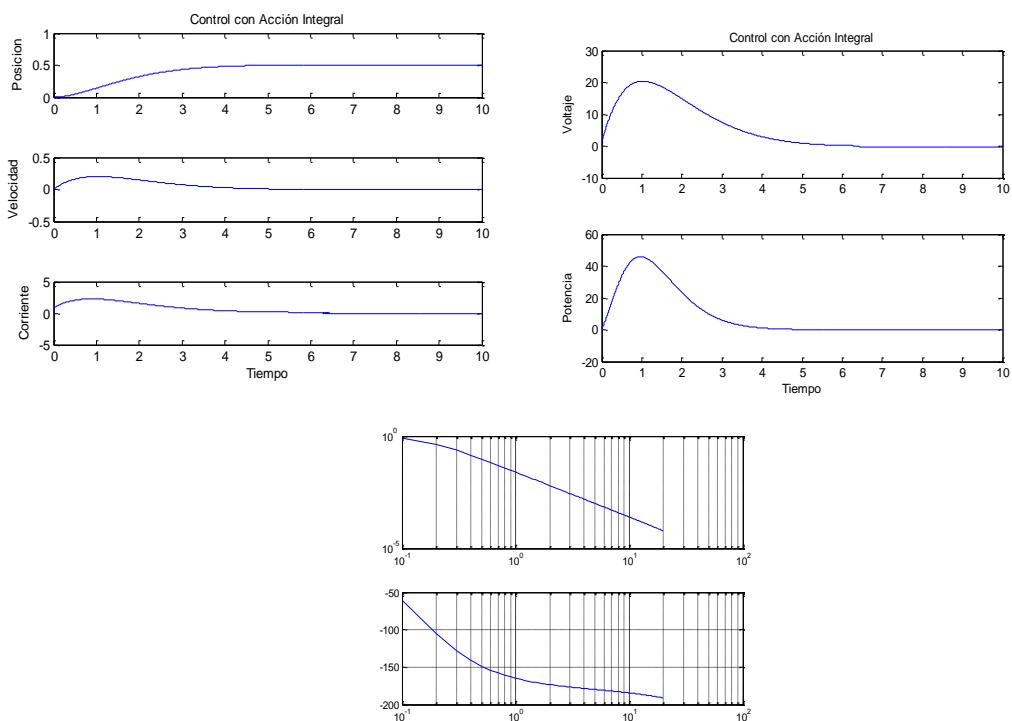
Sobreimpulso 0.00%



$q1 = 1e6, q2 = 1e6, q3 = 0, q4 = 1e6$

Error estacionario: 0.03013%

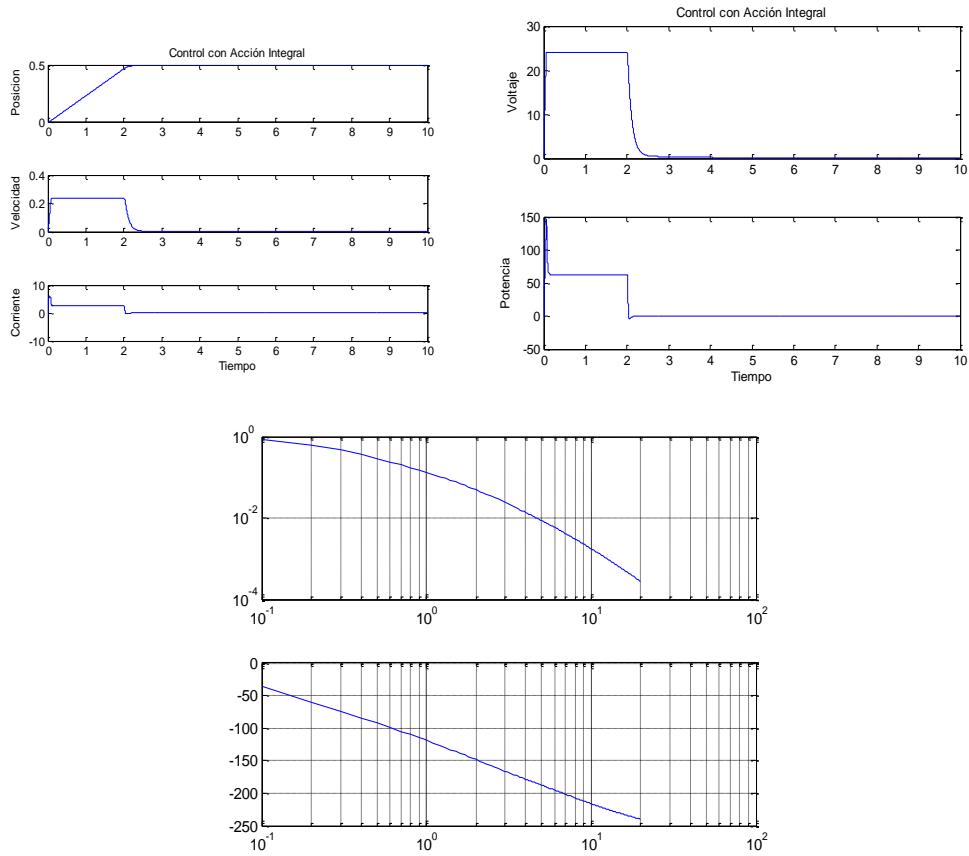
Sobreimpulso: 0.43928%



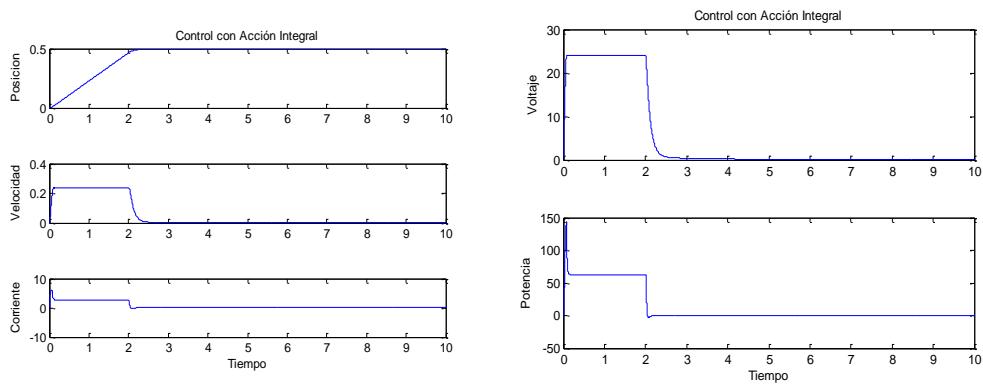
1.1.4. $q1$ cte., $q2 = 0$, $q3$ = variable, $q4$ cte.

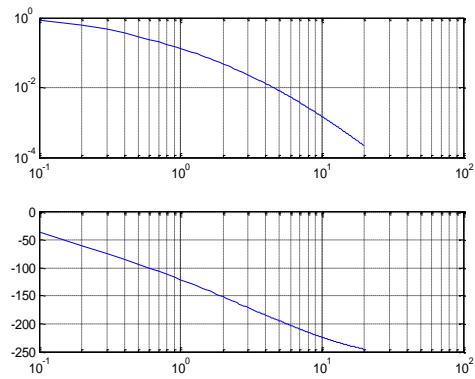
Controladores de Estado con acción integral

$q1 = 1e6, q2 = 0, q3 = 0.05, q4 = 1e6$
Error estacionario: -0.000379%
Sobreimpulso 0.00%



$q1 = 1e6, q2 = 0, q3 = 1, q4 = 1e6$
Error estacionario: -0.00043343%
Sobreimpulso 0.00%





2. Planta 2

$$\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} F_s$$

r=0.5
 -20<u<20
 F_s de 0.1 a 1

2.1. Control sin acción integral

```

clear;
close all;
clc;
A = [0 1
      -3 -5];
B = [ 0
      1 ];
Wf = [ 0
      0.1 ];
C = [1 0];
D = [0];
r = 0.5;           % Posición deseada
voltmax = 20;      % Voltaje máximo
Fseca = 0.*12;     % Fricción 0, 0.5, 1.0, 1.5
%%%%%%%%%%%%%
disp('Control sin acción integral');
q1 = input('Peso q1: '); %1e7
q2 = input('Peso q2: '); %1e4 ---- 1e5
Q = diag([q1 q2]);
RR = 1;
P = are(A,B*inv(RR)*B',Q);
K = inv(RR)*B'*P;
k1 = K(1,1); k2 = K(1,2);
ti = 0; tf = 10; dt = 0.001;
t = ti:dt:tf; t = t';
[ Ak Bk ] = c2d(A,B,dt);
[ Ak Wk ] = c2d(A,Wf,dt);
x = [ 0 ; 0 ]; % Vector de estado inicial
k = 1;
for tt = ti:dt:tf
    x1(k,1) = x(1,1);
    x2(k,1) = x(2,1);
    u = -K*x + k1*r;
    if( u > voltmax)
        u = voltmax;
    elseif(u < -voltmax)
        u = -voltmax;
    end
    volt(k,1) = u;
    if(x(2,1) >= 0)
        Fs = Fseca;
    elseif(x(2,1) < 0)
        Fs = -Fseca;
    end
    x = Ak*x + Bk*u + Wk*Fs;
    k = k+1;
end
% Determinación de error estacionario
k = k - 1;
errest = ((x1(k,1) - r)/r)*100;
disp(['Error estacionario: ',num2str(errest),'%']);
% Determinación de sobreimpulso
x1max = max(x1);
if(x1max > r)
    sobreimpulso = (x1max-r)/r*100;
    disp(['Sobreimpulso: ',num2str(sobreimpulso),'%']);
else
    disp('Sobreimpulso 0.00%');
end

```

Controladores de Estado con acción integral

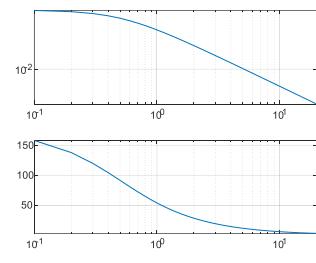
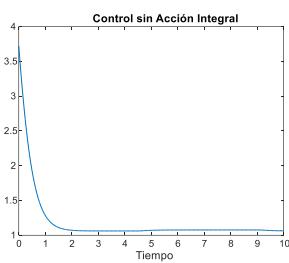
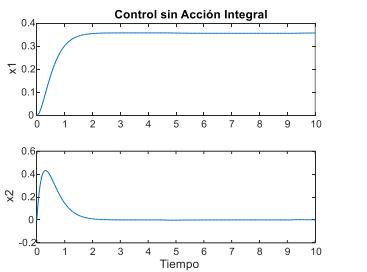
```

figure(1);
subplot(2,1,1); plot(t,x1); ylabel('x1');
title('Control sin Acción Integral');
subplot(2,1,2); plot(t,x2); ylabel('x2');
%subplot(3,1,3); plot(t,cor); ylabel('Corriente');
xlabel('Tiempo');
figure(2);
subplot(1,1,1), plot(t,volt); ylabel('u');
title('Control sin Acción Integral');

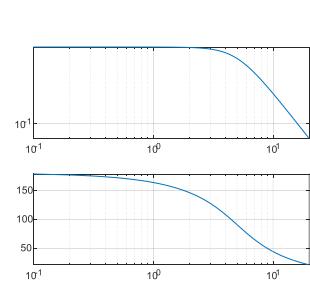
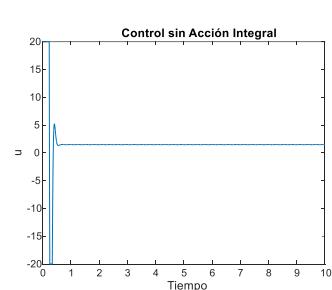
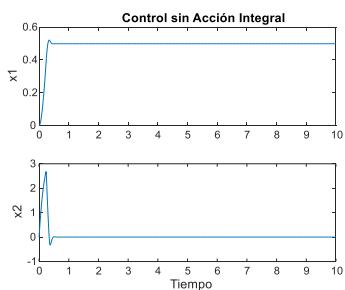
%subplot(2,1,2); plot(t,pot); ylabel('Potencia');
xlabel('Tiempo');
% Diagrama de Bode
Acl = A - B*K;
Bcl = Acl(:,1);
Ccl = C;
Dcl = D;
fre = 0:0.1:20; fre = fre';
wrs = 2*pi*fre

```

Control sin acción integral
 Peso q1: 1e2
 Peso q2: 0
 Error estacionario: -28.6233%
 Sobreimpulso 0.00%



Control sin acción integral
 Peso q1: 1e6
 Peso q2: 0
 Error estacionario: -0.30217%
 Sobreimpulso: 4.1115%



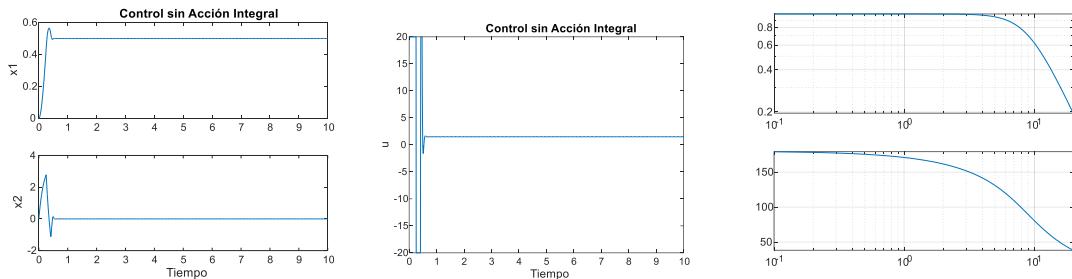
Controladores de Estado con acción integral

```

Control sin acción integral
Peso q1: 1e7
Peso q2: 0
Error estacionario: -0.095334%
Sobreimpulso: 12.8174%

```

F_s=0.1

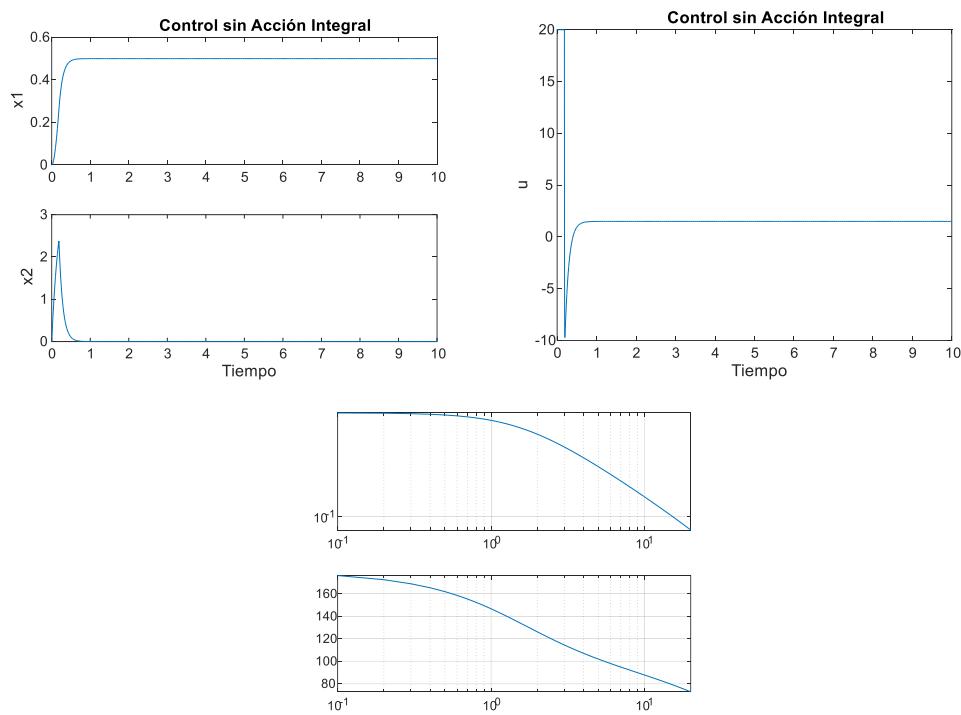


Se ve que para $q_1=1e7$ el error estacionario se reduce significativamente con lo que tomaremos ese valor.

```

Control sin acción integral
Peso q1: 1e7
Peso q2: 1e5
Error estacionario: -0.094868%
Sobreimpulso 0.00%

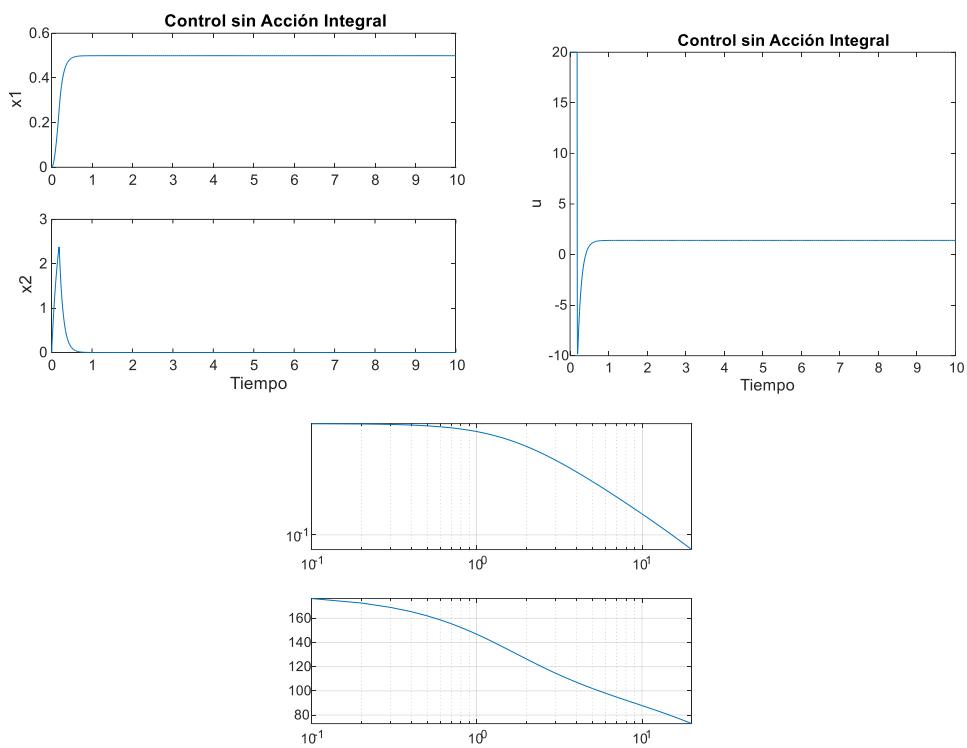
```



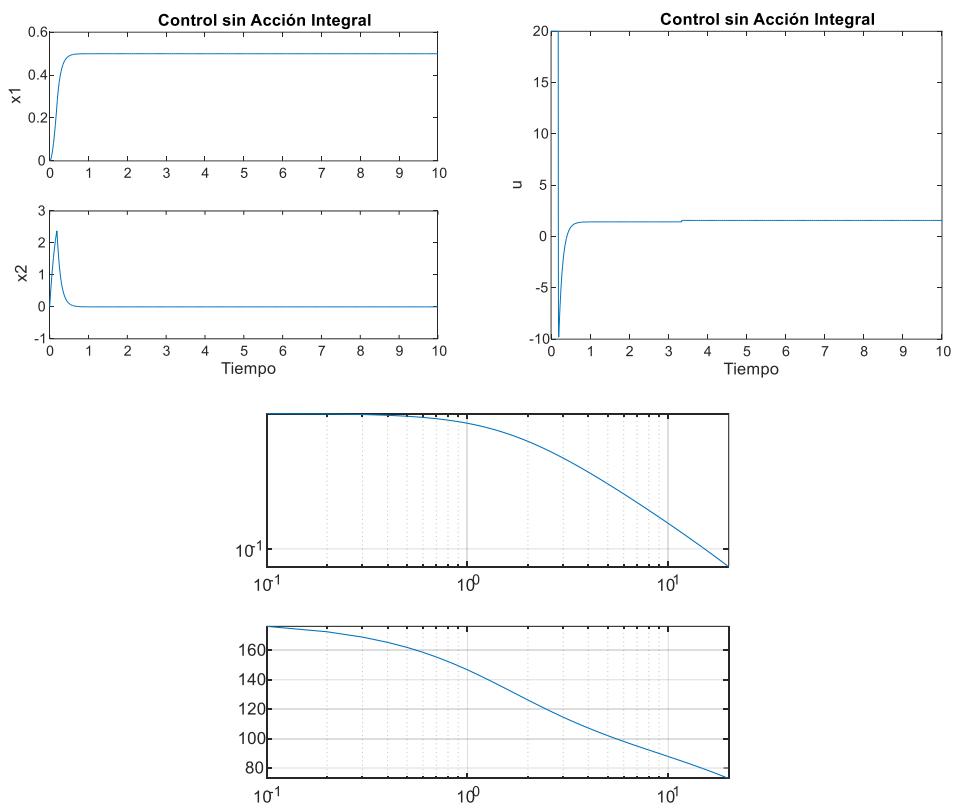
Simulamos las salidas para diferentes valores de Fricción Seca.

Controladores de Estado con acción integral

Para $f_s=1$

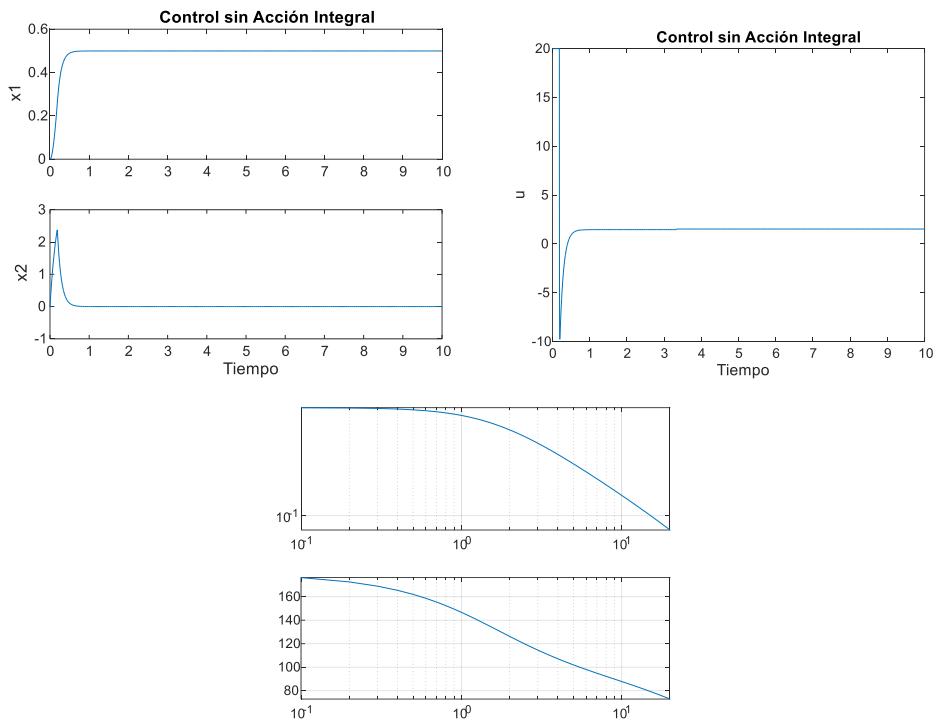


Para $f_s=0.7$

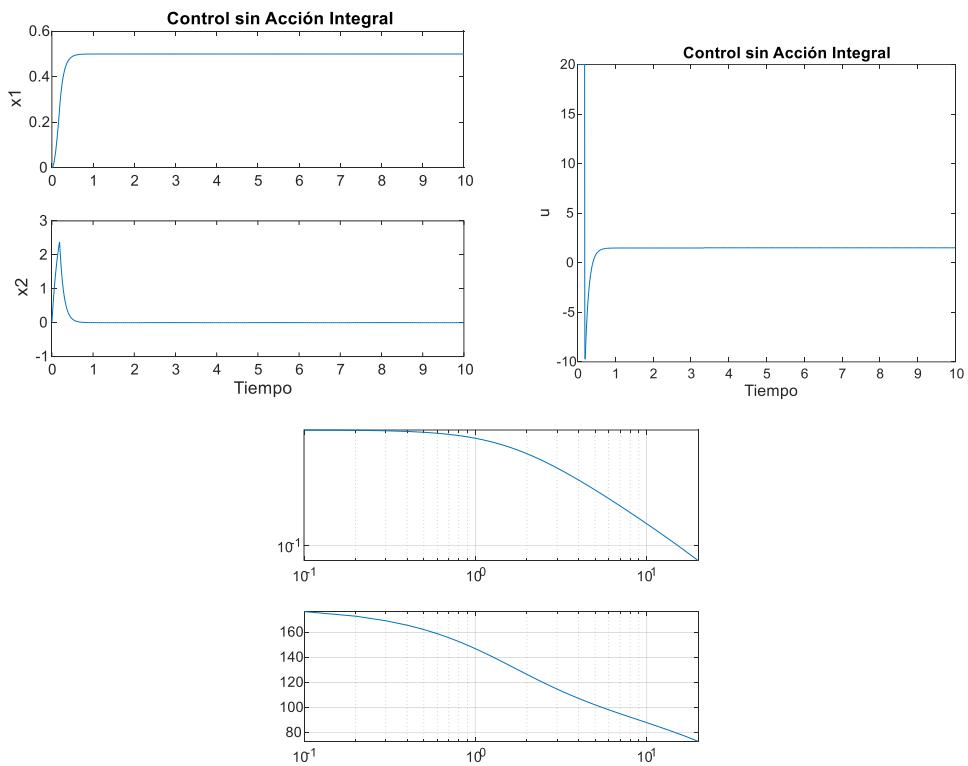


Controladores de Estado con acción integral

Para $f_s=0.3$



Para $f_s=0.1$



2.2. Control con acción integral

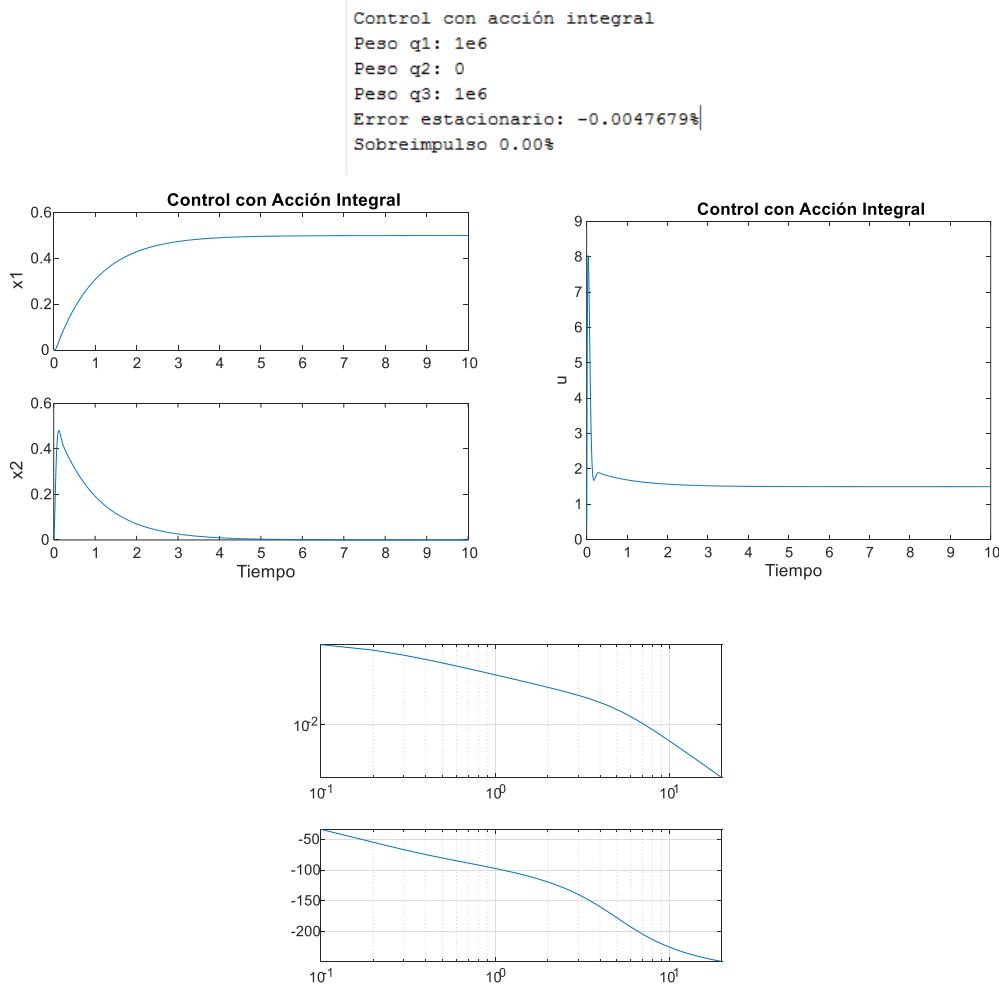
```
% Control con acción integral
clear;
close all;
clc;
A = [0 1
      -3 -5];
B = [ 0
      1 ];
C = [1 0];
D = [0];
r = 0.5;           % Posición deseada
voltmax = 20;     % Voltaje máximo
Fseca = 0*12;     % Fricción 0, 0.5, 1.0, 1.5
% Control con acción integral
Ai = [0 1 0
      -3 -5 0
      1 0 0];
Wf = [ 0
      0.1];
Bi = [ 0
      1
      0 ];
Wri = [ 0
      0
      -1 ];
Ci = [ 1 0 0 ];
Di = [ 0 ];
disp(' ');
disp('Control con acción integral');
q1 = input('Peso q1: ');    % 1.e6
q2 = input('Peso q2: ');    % 0
q3 = input('Peso q3: ');    % 1e6
Qi = diag([ q1 q2 q3 ]);
RRi = 1;
Pi = are(Ai,Bi*inv(RRi)*Bi',Qi);
Ki = inv(RRi)*Bi'*Pi;
ti = 0;    tf = 10;    dt = 0.001;
t = ti:dt:tf;    t = t';
[ Ak Bk ] = c2d(A,B,dt);
[ Ak Wk ] = c2d(A,Wf,dt);
x = [ 0 ; 0 ];    % Vector de estado inicial
int_err = 0;    % Valor inicial de integral del error
k = 1;
for tt = ti:dt:tf
    x1(k,1) = x(1,1);
    x2(k,1) = x(2,1);
    int_err = int_err + (x(1,1)-r)*dt;
    u = -Ki(1,1:2)*x - Ki(1,3)*int_err;
    if( u > voltmax)
        u = voltmax;
    elseif(u < -voltmax)
        u = -voltmax;
    end
    volt(k,1) = u;
    if(x(2,1) >= 0)
        Fs = Fseca;
    elseif(x(2,1) < 0)
        Fs = -Fseca;
    end
    x = Ak*x + Bk*u + Wk*Fs;
    k = k+1;
end
% Determinación de error estacionario
k = k - 1;
errest = ((x1(k,1) - r)/r)*100;
disp(['Error estacionario: ',num2str(errest), '%']);
% Determinación de sobreimpulso
x1max = max(x1);
```

Controladores de Estado con acción integral

```

disp(' ');
figure(3);
subplot(2,1,1); plot(t,x1); ylabel('x1');
title('Control con Acción Integral');
subplot(2,1,2); plot(t,x2); ylabel('x2');
%subplot(3,1,3); plot(t,cor); ylabel('Corriente');
xlabel('Tiempo');
figure(4);
subplot(1,1,1), plot(t,volt); ylabel('u');
title('Control con Acción Integral');
%subplot(2,1,2); plot(t,pot); ylabel('Potencia');
xlabel('Tiempo');
% Diagrama de Bode
Acl = Ai - Bi*Ki;
Bcl = Wri;
Ccl = Ci;
Dcl = Di;
fre = 0:0.1:20; fre = fre';
wrs = 2*pi*fre;
[mag fase] = bode(Acl,Bcl,Ccl,Dcl,1,wrs);
figure(5);
subplot(2,1,1); loglog(fre,mag); grid;
axis tight
subplot(2,1,2); semilogx(fre,fase); grid;
axis tight

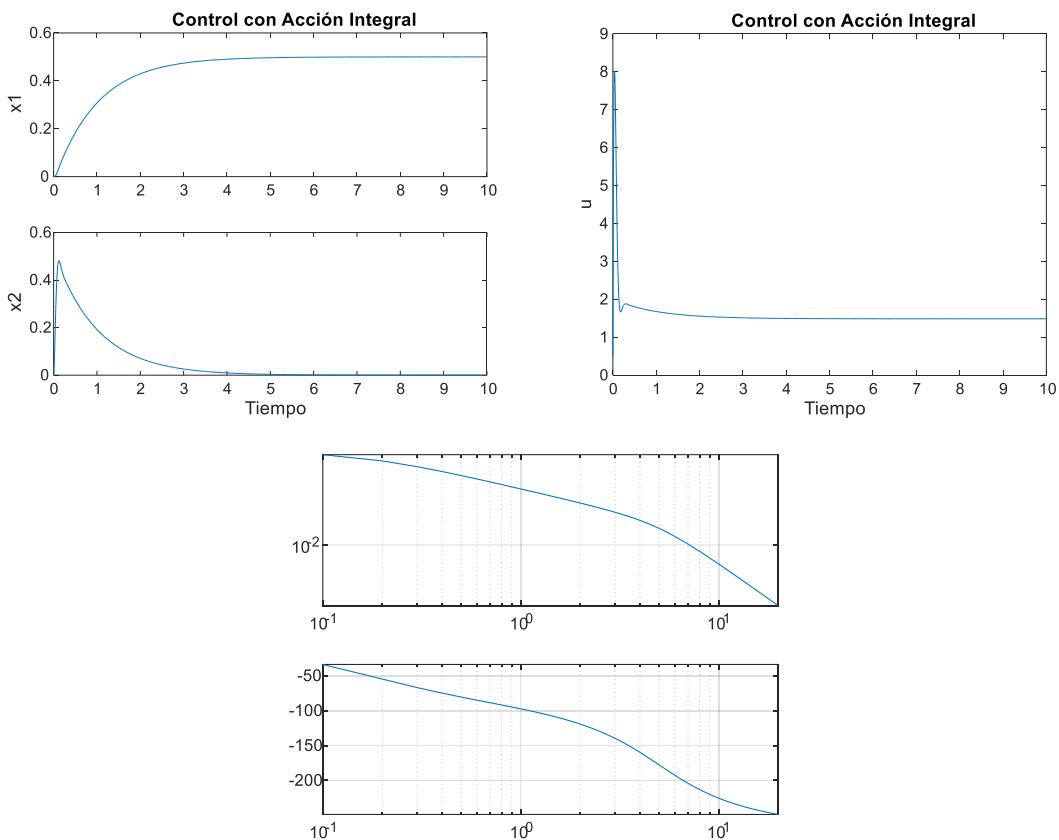
```



Controladores de Estado con acción integral

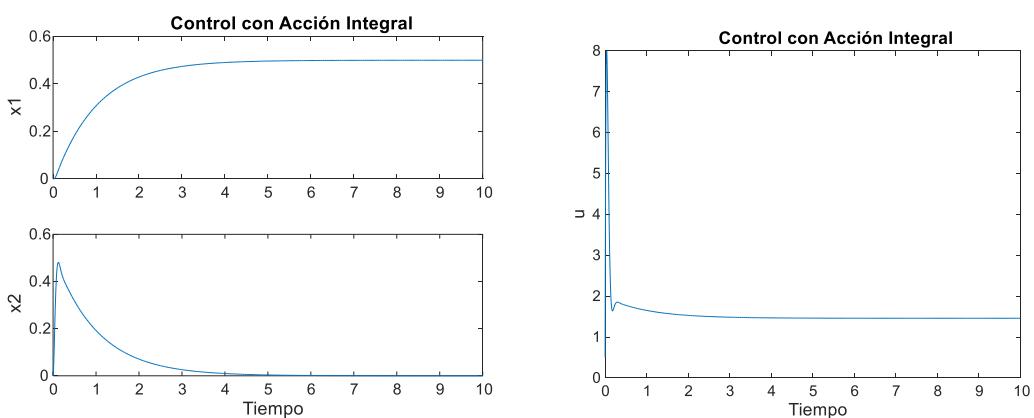
Para $f_s=0.1$

```
Control con acción integral
Peso q1: 1e6
Peso q2: 0
Peso q3: 1e6
Error estacionario: -0.0047678%
Sobreimpulso 0.00%
```

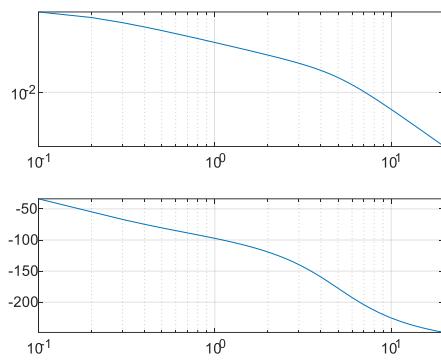


Para $f_s=0.4$

```
Control con acción integral
Peso q1: 1e6
Peso q2: 0
Peso q3: 1e6
Error estacionario: -0.0047675%
Sobreimpulso 0.00%
```

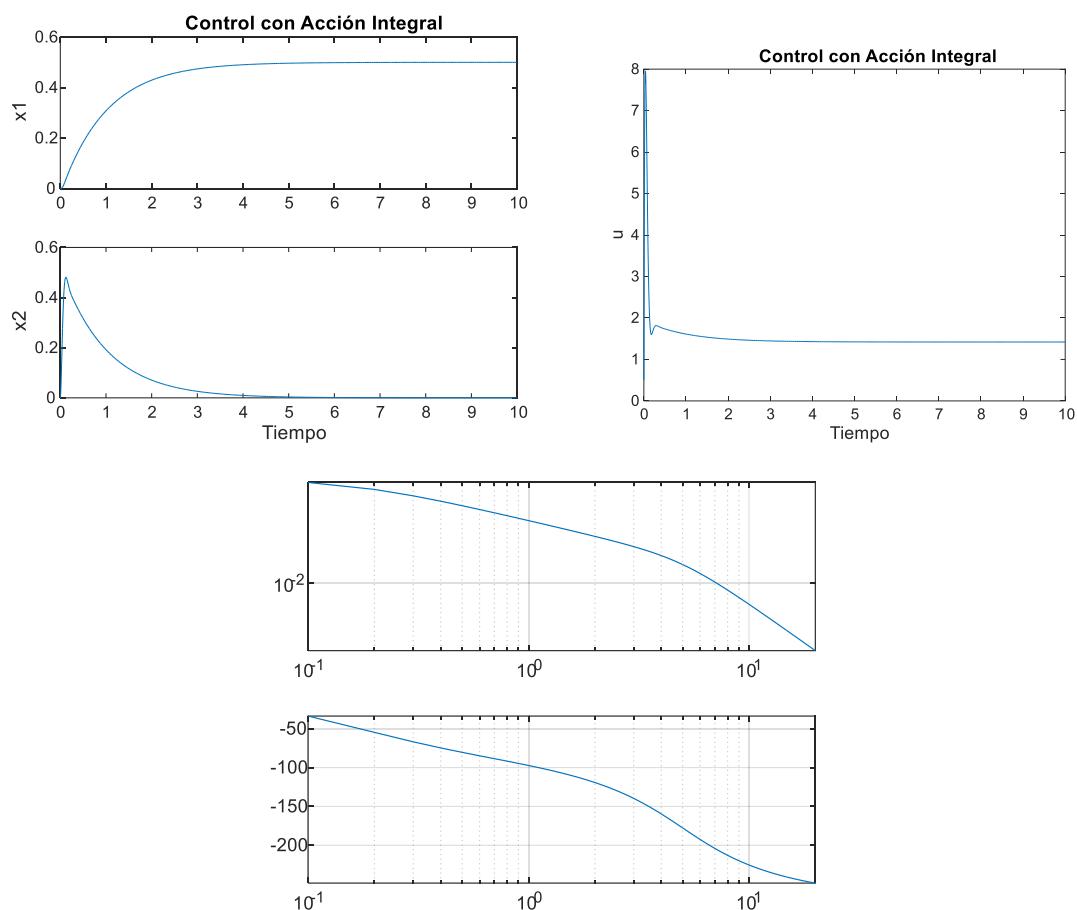


Controladores de Estado con acción integral



Para fs=0.8

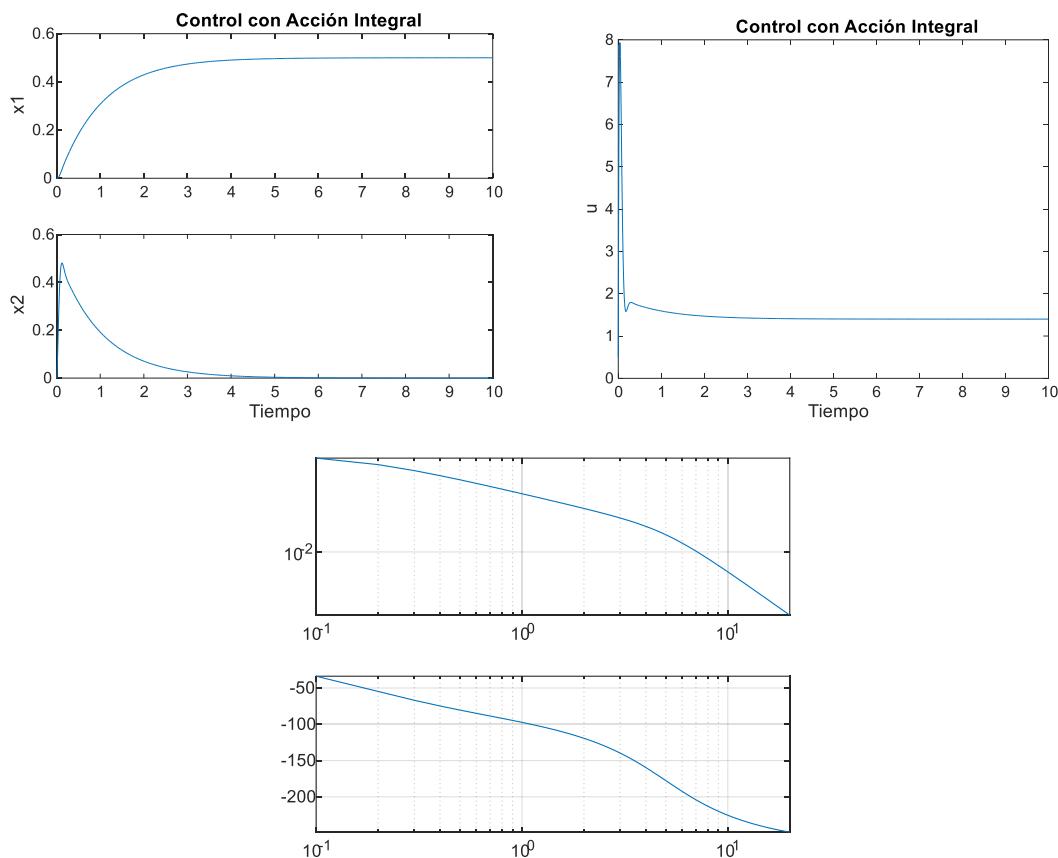
```
Control con acción integral
Peso q1: 1e6
Peso q2: 0
Peso q3: 1e6
Error estacionario: -0.0047671%
Sobreimpulso 0.00%
```



Controladores de Estado con acción integral

Para fs=1

```
Control con acción integral
Peso q1: 1e6
Peso q2: 0
Peso q3: 1e6
Error estacionario: -0.0047669%
Sobreimpulso 0.00%
```



```
Control con acción integral
Peso q1: 50
Peso q2: 3
Peso q3: 50
Error estacionario: -0.0010296%
Sobreimpulso 0.00%
```

