Estadística

15 - 6 - 19

CAT	tamaño muestra	media muestral	cuasivarianza muestral
1	8	92,25	5,69
2	8	9,73	8,90

$\alpha = 5\%$

$$\left[\frac{1}{F_{n_x-1, n_y-1, 1-\frac{\alpha}{2}}} \frac{S_x^2}{S_y^2} , F_{n_x-1, n_y-1, 1-\frac{\alpha}{2}} \frac{S_x^2}{S_y^2} \right]$$

$\frac{\alpha}{2} = 0{,}025$

$1 - \frac{\alpha}{2} = 0{,}925$

$$\left[\frac{1}{4{,}995} \cdot \frac{(5{,}69)}{(8{,}9)} , 4{,}995 \cdot \frac{(5{,}69)}{(8{,}9)} \right]$$

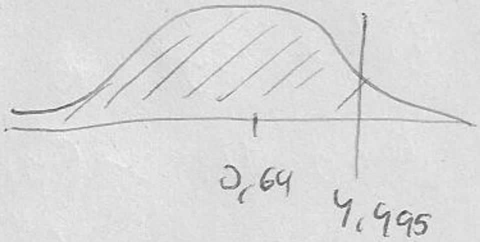
$$(0{,}128 , 3{,}19)$$

lo mismo pero con un contraste de hipótesis en lugar de intervalo

$H_0 = \sigma_x^2 = \sigma_y^2$

$H_1 = \sigma_x^2 \neq \sigma_y^2$

$$F_{exp} = \frac{S_x^2}{S_y^2} = \frac{5{,}69}{8{,}90} = 0{,}64$$



$$T = \frac{(\bar{x}-\bar{y}) - \mu}{S_p\left(\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}\right)} \; ; \; t = \frac{(92{,}25 - 92{,}73)}{2{,}7 \cdot \sqrt{\frac{1}{8} + \frac{1}{8}}} = -0{,}35$$

$H_0 = \mu_x - \mu_y = \mu_0 \overset{=0}{} \rightarrow \mu_x = \mu_y$

$H_1 = \mu_x - \mu_y \neq \mu_0$

$$S_p = \sqrt{\frac{(n_x - 1) S_x^2 + (n_y - 1) S_y^2}{n_x - 1 \; n_y - 2}} \; ; \; S_p = \sqrt{\frac{7 \cdot 5{,}64 + 7 \cdot 8{,}9}{14}} = 2{,}7$$

$T_{exp} = -0{,}35$

miramos la tabla t-student para $(7-1)+(7-1) = 14$

y $0{,}975 \rightarrow 2{,}145$

$0{,}35 \geq 2{,}145$? $\rightarrow$

$\rightarrow$ no rechazo

Relación 9

① b) $F(x,y) = x^3 + 3xy^2 - 15x - 12y$

$F'_x = 3x^2 + 3y^2 - 15 \rightarrow F''_{xx} = 6x$, $F''_{xy} = 6y$

$F'_y = 6xy - 12 \rightarrow F''_{yx} = 6y$, $F''_{yy} = 6x$

Siempre tienen que ser iguales

Puntos críticos

$$\begin{cases} f'_x = 0 & 3x^2 + 3y^2 - 15 = 0 \\ f'_y = 0 & 6xy - 12 = 0 \end{cases}$$

distintas formas de lo mismo

matriz Hessiana

$$H = \begin{pmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial x \partial y} \\ \frac{\partial^2}{\partial y \partial x} & \frac{\partial^2}{\partial y^2} \end{pmatrix} =$$

$$= \begin{pmatrix} 6x & 6y \\ 6y & 6x \end{pmatrix}$$

sustituimos los puntos

$$\text{Si } |H| > 0 \begin{cases} \text{si } \frac{\partial^2}{x^2} > 0 & \text{mínimo local} \\ \text{si } \frac{\partial^2}{\partial x^2} < 0 & \text{máximo local} \end{cases}$$

Si $|H| < 0$ punto de silla

Si $|H| = 0 \rightarrow$ no puede ser determinado

para 3 variables

$$H = \begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix}$$ difícil que caiga

Sacamos los puntos críticos de la función inicial

$$3x^2+3y^2-15=0 \;\; y \;\; 6xy-12=0$$

$$6yx=12$$

$$xy=2$$

$$x=\frac{2}{y}$$

$$3\left(\frac{2}{y}\right)^2+3y^2-15=0$$

$$\frac{12}{y^2}+3y^2-15=0$$

$$12+3y^4-15y^2=0$$

$$3y^4-15y^2+12=0$$

$$y^4-5y^2+4=0$$

$$t=y^2$$

$$t^2-5t+4$$

$$\frac{5\pm\sqrt{25-16}}{2}=\frac{5\pm 3}{2} \begin{cases} 4 \\ 1 \end{cases}$$

$$t=4 \to y \begin{cases} 2 \\ -2 \end{cases}$$

$$t=1 \to y \begin{cases} 1 \\ -1 \end{cases}$$

$A=(1,2) \to H=\begin{pmatrix} 6 & 12 \\ 12 & 6 \end{pmatrix} \to |H|<0 \to$ P silla

$B=(-1,-2) \to H=\begin{pmatrix} -6 & -12 \\ -12 & -6 \end{pmatrix} \to |H|<0 \to$ P silla

$C=(2,1) \to H=\begin{pmatrix} 12 & 6 \\ 6 & 12 \end{pmatrix} \to |H|>0$ y $\frac{\partial^2}{\partial x^2}>0$ mínimo local

$D=(-2,-1) \to H=\begin{pmatrix} -12 & -6 \\ -6 & -12 \end{pmatrix} = |H|>0$ y $\frac{\partial^2}{\partial x^2}<0$

máximo local

Ejercicio de examen de 2017

$$F(x,y) = x^2 + xy + y^3 - 3x - 2y + 1$$

$\partial/\partial x = 2x + y - 3$ $\quad \partial^2/\partial x^2 = 2$ $\quad \partial^2/\partial x \partial y = 1$

$\partial/\partial y = x + 3y^2 - 2$ $\quad \partial^2/\partial y \partial x = 1$ $\quad \partial^2/\partial y^2 = 6y$

$H = \begin{pmatrix} 2 & 1 \\ 1 & 6y \end{pmatrix}$ $\quad \left.\begin{array}{l} 2x + y - 3 = 0 \\ x + 3y^2 - 2 = 0 \quad (-2) \end{array}\right\}$

$$\begin{array}{r} 2x + y - 3 = 0 \\ -2x - 6y^2 + 4 = 0 \\ \hline 6y^2 + y + 1 = 0 \end{array}$$

$y = \frac{-1 \pm \sqrt{1+24}}{-12} = \frac{1 \pm 5}{-12}$ $\; \frac{1}{2}$; $\frac{-4}{12} = \frac{-1}{3}$

$x = \frac{3-y}{2}$

$x = \frac{3 - \frac{1}{2}}{2} = \frac{\frac{6-1}{2}}{2} = \frac{5}{4}$

$x = \frac{3 - (-\frac{1}{3})}{2} ; \frac{\frac{9+1}{3}}{2} ; \frac{10}{6} = \frac{5}{3}$

puntos críticos

$\left(\frac{5}{4}, \frac{1}{2}\right)$

$\left(\frac{5}{3}, -\frac{1}{3}\right)$

$|H| = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} \rightarrow |H| > 0 \quad \partial^2/\partial x^2 > 0$ mínimo local $\left(\frac{5}{4}, \frac{1}{2}\right)$

$|H| = \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = |H| < 0 \rightarrow$ P silla en $\left(\frac{5}{3}, -\frac{1}{3}\right)$

Maximizar $x_1 + 2x_2$

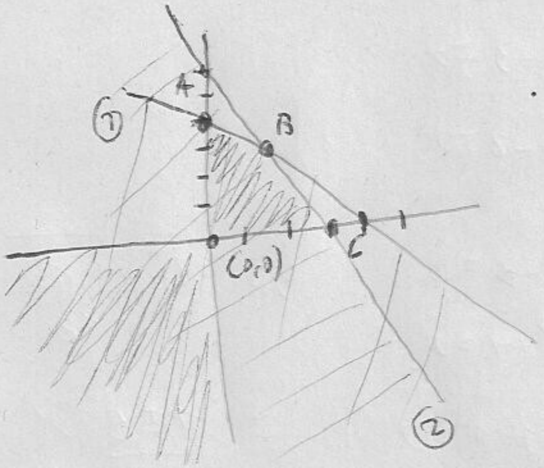
$$\begin{cases} x_1 + x_2 \leq 4 & ① \\ 2x_1 + x_2 \leq 6 & ② \\ x_1, x_2 \geq 0 & ③ \end{cases}$$

$x_1 + x_2 \leq 4$

x_1	x_2
0	4
4	0

$2x_1 + x_2 \leq 6$

x_1	x_2
0	6
3	0



la restricción ③ nos limita la zona al primer y tercer cuadrante

A = (0,4) ← miramos la gráfica

B = (2,2)

C = (3,0)

B $\begin{cases} x_1 + x_2 = 4 \\ 2x_1 + x_2 = 6 \end{cases}$

$$\begin{array}{r} -x_1 - \cancel{x_2} = -4 \\ 2x_1 + \cancel{x_2} = 6 \\ \hline x_1 = 2 \end{array}$$

$x_2 = 2$

$f(0,4) = 8$

$f(2,2) = 6$

$f(3,0) = 3$

una almazara produce aceite virgen y aceite de orujo

el triple de . . . .

Aceite $\to x$

Orujo $\to y$

sa $\begin{cases} 3y \leq x + 10 & (1) \\ 2y + 3x \leq 18 & (2) \\ x \geq 0 \\ y \geq 0 \end{cases}$

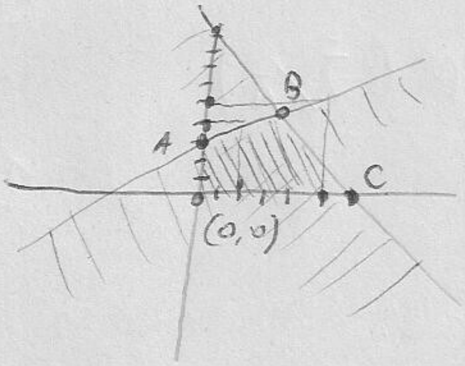
$F(x,y) = 5000x + 1000y$

(1)

x	y
2	4
5	5

(2)

x	y
0	9
6	0



$A = (0, 10/3)$

$B =$

$C = (6,0)$

$x = 3y - 10$

$2y + 3(3y-10) = 18 \to$

$2y + 9y - 30 = 18$

$11y = 48 \to y = \frac{48}{11}$

$x = 3\left(\frac{48}{11}\right) - 10 \to \frac{144}{11} - 10 = \frac{144-110}{11} = \frac{34}{11}$

Repaso

1 pob — media T; — var x^2; — prop Z

2 pob — diferencia medias; — cociente varianza; — diferencia de proporciones

(4)

$P(M) = 0{,}4 \qquad P(\bar{M}) = 0{,}6$

$P(J) = 0{,}25$

$P(M \cap J) = 0{,}12$

$P(M \cap \bar{J}) = P(M) - P(M \cap J) = 0{,}4 - 0{,}12 = 0{,}28$

$$P(\bar{J}/\bar{M}) = \frac{P(\bar{J} \cap \bar{M})}{P(\bar{M})} = \frac{P(\overline{J \cup M})}{P(\bar{M})} = \frac{1 - P(J \cup M)}{P(\bar{M})} =$$

$$= \frac{1 - 0{,}53}{0{,}6} = \frac{0{,}47}{0{,}6} = 0{,}78$$

$n = 200 \qquad B(n, p)$

$P[X \geq 75] \qquad B(200, 0'4)$

$\Big\}$ pasamos a una distribución normal

$$B(n,p) \sim N(\mu, \sigma) \sim N(n \cdot p, \sqrt{n \cdot p \cdot q}) \sim$$

$$\sim N(200 \cdot 0{,}4, \sqrt{200 \cdot 0{,}4 \cdot 0{,}6}) = (80, 6'93)$$

Tipificamos $P\left[Z \geq \frac{75-80}{6'93}\right] = P\left[Z \geq \frac{-5}{6{,}93}\right] = P[Z \geq -0{,}721] =$

$= 1 - P[Z \leq -0{,}721] \simeq 0{,}7642$

	población	media	S
A	11	56	12,1
B	13	49	11,5

$1-\alpha = 0{,}9$

$\alpha = 0{,}1$

$\alpha/2 = 0{,}05$

$1-\alpha/2 = 0{,}95$

$$\left[\frac{1}{F}\cdot\frac{S_A^2}{S_B^2},\ F\cdot\frac{S_A^2}{S_B^2}\right]$$

$$\left[\frac{1}{2{,}753}\cdot\frac{12{,}1^2}{11{,}5^2},\ 2{,}913\cdot\frac{12{,}1^2}{11{,}5^2}\right]$$

$$[0{,}402,\ 3{,}225]$$

como el 1 está en el intervalo asumimos que la variabilidad es la misma

b)

$H_0 \to \mu_A = \mu_B$

$H_1 \to \mu_A \neq \mu_B$

$$S_P = \sqrt{\frac{(n_a-1)\cdot S_A^2+(n_b-1)\cdot S_b^2}{n_a+n_b-2}} = \sqrt{\frac{10\cdot(12{,}1)^2+12(11{,}3)^2}{22}}$$

$$T_{exp} = \frac{(A-B)}{S_P\sqrt{\frac{1}{n_A}+\frac{1}{n_B}}} = \frac{56-49}{11{,}78\sqrt{\frac{1}{11}+\frac{1}{13}}} =$$

$$T_{exp} = \frac{7}{4{,}83} = 1{,}45$$

$T_{tabla} = 1{,}72$

tabla → 0,95 y $22 = 11-1+13-1$

aceptamos la hipótesis nula