3.4. USING PARSER GENERATORS

```
1 P \rightarrow L

2 S \rightarrow \text{id} := \text{id}

3 S \rightarrow \text{while id do } S

4 S \rightarrow \text{begin } L \text{ end}

5 S \rightarrow \text{if id then } S

6 S \rightarrow \text{if id then } S \text{ else } S
```

GRAMMAR 3.30.

well understood. Most shift-reduce conflicts, and probably all reduce-reduce conflicts, should not be resolved by fiddling with the parsing table. They are symptoms of an ill-specified grammar, and they should be resolved by eliminating ambiguities.

3.4 USING PARSER GENERATORS

The task of constructing LR(1) or LALR(1) parsing tables is simple enough to be automated. And it is so tedious to do by hand that LR parsing for realistic grammars is rarely done except using parser-generator tools. Yacc ("Yet another compiler-compiler") is a classic and widely used parser generator; *Bison* and *occs* are more recent implementations.

A Yacc specification is divided into three sections, separated by %% marks:

```
parser declarations
%%
grammar rules
%%
programs
```

The *parser declarations* include a list of the terminal symbols, nonterminals, and so on. The *programs* are ordinary C code usable from the semantic actions embedded in the earlier sections.

The grammar rules are productions of the form

```
exp : exp PLUS exp { semantic action }
```

where \exp is a nonterminal producing a right-hand side of $\exp+\exp$, and PLUS is a terminal symbol (token). The *semantic action* is written in ordinary C and will be executed whenever the parser reduces using this rule.

CHAPTER THREE. PARSING

GRAMMAR 3.31. Yacc version of Grammar 3.30. Semantic actions are omitted and will be discussed in Chapter 4.

Consider Grammar 3.30. It can be encoded in Yacc as shown in Grammar 3.31. The Yacc manual gives a complete explanation of the directives in a grammar specification; in this grammar, the terminal symbols are ID, WHILE, etc.; the nonterminals are prog, stm, stmlist; and the grammar's start symbol is prog.

CONFLICTS

Yacc reports shift-reduce and reduce-reduce conflicts. A shift-reduce conflict is a choice between shifting and reducing; a reduce-reduce conflict is a choice of reducing by two different rules. By default, Yacc resolves shift-reduce conflicts by shifting, and reduce-reduce conflicts by using the rule that appears earlier in the grammar.

Yacc will report that this Grammar 3.30 has a shift-reduce conflict. Any conflict is cause for concern, because it may indicate that the parse will not be as the grammar-designer expected. The conflict can be examined by reading the verbose description file that Yacc produces. Figure 3.32 shows this file.

A brief examination of state 17 reveals that the conflict is caused by the familiar dangling else. Since Yacc's default resolution of shift-reduce conflicts is to shift, and shifting gives the desired result of binding an else to the nearest then, this conflict is not harmful.

3.4. USING PARSER GENERATORS

state 0:					
prog:.	stmlist	state 7:		state 14:	
ID	shift 6	stmlist :	stmlist SEMI . stm		EGIN stmlist END .
	Shift 5	ID	shift 6		
	shift 4	WHILE		•	reduce by rule 3
IF	shift 3	BEGIN		state 15:	
prog	goto 21	IF	shift 3	stm: W	HILE ID DO . stm
stm	goto 2	stm	goto 12	ID	shift 6
stmlist	goto 1		error	WHILE	shift 5
	error	state 8:		BEGIN	shift 4
-4-4- 1.			ID . THEN stm	IF	shift 3
state 1:	41:4		ID . THEN stm ELSE stm	stm	goto 18
prog : s				•	error
	stmlist . SEMI stm	THEN	shift 13	state 16:	
SEMI	shift 7	•	error		ASSIGN ID .
•	reduce by rule 0	state 9:			
state 2:		stm : BE	EGIN stmlist . END	•	reduce by rule 1
stmlist :	stm.	stmlist :	stmlist . SEMI stm	state 17: s	shift/reduce conflict
	reduce by rule 6	END	shift 14		(shift ELSE, reduce 4)
	reduce by fule o	SEMI	shift 7		ID THEN stm .
state 3:			error	stm: IF	ID THEN stm . ELSE stm
	. ID THEN stm	state 10:		ELSE	shift 19
stm : IF	. ID THEN stm ELSE stm		HILE ID . DO stm	÷	reduce by rule 4
ID	shift 8			state 18:	·
•	егтог	DO	shift 15		HILE ID DO stm .
state 4:		•	error	Suii . Wi	
	EGIN . stmlist END	state 11:		•	reduce by rule 2
		stm: ID	ASSIGN . ID	state 19:	
ID WHILE	shift 6	ID	shift 16	stm: IF	ID THEN stm ELSE . stm
BEGIN		•	error	ID	shift 6
IF	shift 3	state 12:		WHILE	
stm	goto 2		stmlist SEMI stm .	BEGIN	
stmlist	goto 9	sumst.	reduce by rule 7	IF	shift 3
othinst	error	•	reduce by rule /	stm	goto 20
		state 13:			error
state 5:	IIII E E DO		ID THEN . stm	state 20:	
stm:w	HILE . ID DO stm	stm: IF	ID THEN . stm ELSE stm		ID THEN stm ELSE stm.
ID	shift 10	ID	shift 6	Detail + II.	
•	еггог	WHILE	shift 5	•	reduce by rule 5
state 6:		BEGIN	shift 4	state 21:	
	. ASSIGN ID	IF	shift 3	EOF	accept
		stm	goto 17	201	-
ASSIGN	Vshift 11	•	error	٠	error
•	error				

FIGURE 3.32. LR states for Grammar 3.30.

	id	num	+	_	*	1	()	\$	E
1 [s2	s3					s4			g7
	84	3.5	r1	r1	r1	r1		r1	r1	
2 3			r2	r2	r2	r2		r2	r2	
	2	~?	12	12	1-		s4			g5
4	s2	s3						s6		
5			 7	r7	r7	r7		r 7	r 7	
6 7			r7	s10	s12	s14			a	
	_	•	s8	810	312	511	s4			g9
8	s2	s3	0 5	-10 -5	s12,r5	s14,r5	٥.	r5	r5	
9		_	s8,r5	s10,r5	812,13	317,13	s4	10		g11
10	s2	s3		10 (. 106	s14,r6	37	r6	r6	8
11			s8,r6	s10,r6	s12,r6	814,10	s4	10	10	g13
12	s2	s3			10.0	. 1.42	54	r3	r3	815
13			s8,r3	s10,r3	s12,r3	s14,r3	-1	13	13	g15
14	s2	s3					s4	4	-4	gij
15			s8,r4	s10,r4	s12,r4	s14,r4		<u>r4</u>	<u>r4</u>	L

TABLE 3.33. LR parsing table for Grammar 3.5.

Shift-reduce conflicts are acceptable in a grammar if they correspond to well understood cases, as in this example. But most shift-reduce conflicts, and all reduce-reduce conflicts, are serious problems and should be eliminated by rewriting the grammar.

PRECEDENCE DIRECTIVES

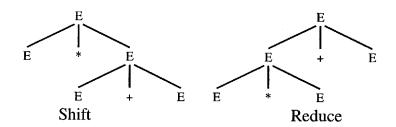
No ambiguous grammar is LR(k) for any k; the LR(k) parsing table of an ambiguous grammar will always have conflicts. However, ambiguous grammars can still be useful if we can find ways to resolve the conflicts.

For example, Grammar 3.5 is highly ambiguous. In using this grammar to describe a programming language, we intend it to be parsed so that \ast and / bind more tightly than + and -, and that each operator associates to the left. We can express this by rewriting the unambiguous Grammar 3.8.

But we can avoid introducing the T and F symbols and their associated "trivial" reductions $E \to T$ and $T \to F$. Instead, let us start by building the LR(1) parsing table for Grammar 3.5, as shown in Table 3.33. We find many conflicts. For example, in state 13 with lookahead + we find a conflict between *shift into state* 8 and *reduce by rule* 3. Two of the items in state 13 are:

$$E \rightarrow E * E . + E \rightarrow E . + E \rightarrow E . + E$$
 (any)

In this state the top of the stack is $\cdots E * E$. Shifting will lead to a stack $\cdots E * E +$ and eventually $\cdots E * E + E$ with a reduction of E + E to E. Reducing now will lead to the stack $\cdots E$ and then the + will be shifted. The parse trees obtained by shifting and reducing are:



If we wish * to bind tighter than +, we should reduce instead of shift. So we fill the (13, +) entry in the table with r3 and discard the s8 action.

Conversely, in state 9 on lookahead *, we should shift instead of reduce, so we resolve the conflict by filling the (9, *) entry with s12.

The case for state 9, lookahead + is

$$E \to E + E . + E \to E . + E$$
 (any)

Shifting will make the operator right-associative; reducing will make it left-associative. Since we want left associativity, we fill (9, +) with r5.

Consider the expression a - b - c. In most programming languages, this associates to the left, as if written (a - b) - c. But suppose we believe that this expression is inherently confusing, and we want to force the programmer to put in explicit parentheses, either (a - b) - c or a - (b - c). Then we say that the minus operator is *nonassociative*, and we would fill the (11, -) entry with an error entry.

The result of all these decisions is a parsing table with all conflicts resolved (Table 3.34).

Yacc has *precedence directives* to indicate the resolution of this class of shift-reduce conflicts. A series of declarations such as

%nonassoc EQ NEQ
%left PLUS MINUS
%left TIMES DIV
%right EXP

	+	-	*	1	
			:		
9 11	r5	r5	s12 s12	s14 s14	
13	r3	r3	r3	r3	
15	r4	r4			
			<u>:</u>		

TABLE 3.34. Conflicts of Table 3.33 resolved.

indicates that + and - are left-associative and bind equally tightly; that * and / are left-associative and bind more tightly than +; that $^$ is right-associative and binds most tightly; and that = and \neq are nonassociative, and bind more weakly than +.

In examining a shift-reduce conflict such as

$$E \rightarrow E * E . + E \rightarrow E . + E \rightarrow E . + E$$
 (any)

there is the choice of shifting a *token* and reducing by a *rule*. Should the rule or the token be given higher priority? The precedence declarations (%left, etc.) give priorities to the tokens; the priority of a rule is given by the last token occurring on the right-hand side of that rule. Thus the choice here is between a rule with priority * and a token with priority +; the rule has higher priority, so the conflict is resolved in favor of reducing.

When the rule and token have equal priority, then a <code>%left</code> precedence favors reducing, <code>%right</code> favors shifting, and <code>%nonassoc</code> yields an error action.

Instead of using the default "rule has precedence of its last token," we can assign a specific precedence to a rule using the *prec directive. This is commonly used to solve the "unary minus" problem. In most programming languages a unary minus binds tighter than any binary operator, so -6 * 8 is parsed as (-6) * 8, not -(6 * 8). Grammar 3.35 shows an example.

The token UMINUS is never returned by the lexer; it is merely a place-holder in the chain of precedence (%left) declarations. The directive %prec UMINUS gives the rule exp: MINUS exp the highest precedence, so reducing by this rule takes precedence over shifting any operator, even a minus sign.

3.4. USING PARSER GENERATORS

GRAMMAR 3.35.

Precedence rules are helpful in resolving conflicts, but they should not be abused. If you have trouble explaining the effect of a clever use of precedence rules, perhaps instead you should rewrite the grammar to be unambiguous.

SYNTAX VERSUS SEMANTICS

Consider a programming language with arithmetic expressions such as x + y and boolean expressons such as x + y = z or a & (b = c). Arithmetic operators bind tighter than the boolean operators; there are arithmetic variables and boolean variables; and a boolean expression cannot be added to an arithmetic expression. Grammar 3.36 gives a syntax for this language.

The grammar has a reduce-reduce conflict, as shown in Figure 3.37. How should we rewrite the grammar to eliminate this conflict?

Here the problem is that when the parser sees an identifier such as a, it has no way of knowing whether this is an arithmetic variable or a boolean variable – syntactically they look identical. The solution is to defer this analysis until the "semantic" phase of the compiler; it's not a problem that can be handled naturally with context-free grammars. A more appropriate grammar is:

$$S \rightarrow id := E$$

 $E \rightarrow id$
 $E \rightarrow E \& E$
 $E \rightarrow E = E$
 $E \rightarrow E + E$

Now the expression a + 5&b is syntactically legal, and a later phase of the compiler will have to reject it and print a semantic error message.

CHAPTER THREE. PARSING

```
%{ declarations of yylex and yyerror %}
%token ID | ASSIGN | PLUS | MINUS | AND | EQUAL
%start stm
%left OR
%left AND
%left PLUS
옹옹
stm : ID ASSIGN ae
    | ID ASSIGN be
be
   : be OR be
    be AND be
      ae EQUAL ae
      ID
    : ae PLUS ae
ae
    | ID
```

GRAMMAR 3.36.

3.5 ERROR RECOVERY

LR(k) parsing tables contain shift, reduce, accept, and error actions. On page 59 I claimed that when an LR parser encounters an error action it stops parsing and reports failure. This behavior would be unkind to the programmer, who would like to have *all* the errors in her program reported, not just the first error.

RECOVERY USING THE ERROR SYMBOL

Local error recovery mechanisms work by adjusting the parse stack and the input at the point where the error was detected in a way that will allow parsing to resume. One local recovery mechanism – found in many versions of the Yacc parser generator – uses a special error symbol to control the recovery process. Wherever the special error symbol appears in a grammar rule, a sequence of erroneous input tokens can be matched.

For example, in a Yacc grammar for Tiger, we might have productions such as

3.5. ERROR RECOVERY

```
state 0:
                                           state 5: reduce/reduce conflict
                                                                                           state 9:
 stm:. ID ASSIGN ae
                                                     between rule 6 and
                                                                                            be: ae EQUAL ae.
 stm:. ID ASSIGN be
                                                     rule 4 on EOF
                                                                                            ae: ae. PLUS ae
                                            be: ID.
 ID
          shift 1
                                                                                            PLUS
                                                                                                     shift 7
                                            ae: ID.
          goto 14
 stm
                                                                                                     reduce by rule 3
          error
                                            PLUS
                                                     reduce by rule 6
                                                                                           state 10:
                                            AND
                                                     reduce by rule 4
state 1:
                                                                                            ae: ID.
                                            EQUAL reduce by rule 6
 stm: ID . ASSIGN ae
                                                                                                     reduce by rule 6
                                            EOF
                                                     reduce by rule 4
 stm: ID . ASSIGN be
                                                     error
                                                                                           state 11:
 ASSIGN shift 2
                                                                                            ae: ae. PLUS ae
                                           state 6:
         error
                                                                                            ae: ae PLUS ae.
                                            be: ae EQUAL. ae
state 2:
                                                                                                     reduce by rule 5
                                            ID
                                                     shift 10
 stm: ID ASSIGN. ae
                                                     goto 9
                                            ae
                                                                                           state 12:
 stm: ID ASSIGN. be
                                                     error
                                                                                            be: ae. EQUAL ae
ID
         shift 5
                                                                                            ae: ae. PLUS ae
                                           state 7:
be
         goto 4
                                            ae: ae PLUS. ae
                                                                                            PLUS
                                                                                                     shift 7
         goto 3
ae
                                                                                            EQUAL shift 6
         error
                                            ID
                                                     shift 10
                                                                                                     error
                                                     goto 11
                                            ae
state 3:
                                                     error
stm: ID ASSIGN ae.
                                                                                           state 13:
be: ae. EQUAL ae
                                                                                            be: be. AND be
                                           state 8:
ae: ae. PLUS ae
                                                                                            be: be AND be.
                                            be: be AND. be
PLUS
         shift 7
                                                                                                     reduce by rule 2
                                            ID
                                                     shift 5
EQUAL shift 6
                                                     goto 13
                                            be
                                                                                           state 14:
         reduce by rule 0
                                            ae
                                                     goto 12
                                                                                            EOF
                                                                                                     accept
                                                     error
state 4:
                                                                                                     error
stm: ID ASSIGN be.
be: be. AND be
AND
         shift 8
         reduce by rule 1
```

FIGURE 3.37. LR states for Grammar 3.36.

$$exp \rightarrow ID$$

 $exp \rightarrow exp + exp$
 $exp \rightarrow (exps)$
 $exps \rightarrow exp$
 $exps \rightarrow exps$; exp

Informally, we can specify that if a syntax error is encountered in the middle of an expression, the parser should skip to the next semicolon or right parenthesis (these are called *syncronizing tokens*) and resume parsing. We do this by adding error-recovery productions such as

```
exp \rightarrow (error)

exps \rightarrow error ; exp
```

What does the parser-generator do with the *error* symbol? In parser generation, *error* is considered a terminal symbol, and shift actions are entered in the parsing table for it as if it were an ordinary token.

When the LR parser reaches an error state, it takes the following actions:

- **1.** Pop the stack (if necessary) until a state is reached in which the action for the *error* token is *shift*.
- 2. Shift the error token.
- 3. Discard input symbols (if necessary) until a state is reached that has a non-error action on the current lookahead token.
- 4. Resume normal parsing.

In the two *error* productions illustrated above, we have taken care to follow the *error* symbol with an appropriate synchronizing token – in this case, right parenthesis or semicolon. Thus, the "non-error action" taken in step 3 will always *shift*. If instead we used the production $exp \rightarrow error$, the "non-error action" would be *reduce*, and (in an SLR or LALR parser) it is possible that the original (erroneous) lookahead symbol would cause another error after the reduce action, without having advanced the input. Therefore, grammar rules that contain *error* not followed by a token should be used only when there is no good alternative.

Caution. One can attach *semantic actions* to Yacc grammar rules; whenever a rule is reduced, its semantic action is executed. Chapter 4 explains the use of semantic actions. Popping states from the stack can lead to seemingly "impossible" semantic actions, especially if the actions contain side effects. Consider this grammar fragment:

"Obviously" it is true that whenever a semicolon is reached, the value of nest is zero, because it is incremented and decremented in a balanced way according to the grammar of expressions. But if a syntax error is found after some left parentheses have been parsed, then states will be popped from the stack without "completing" them, leading to a nonzero value of nest. The best solution to this problem is to have side-effect-free semantic actions that build abstract syntax trees, as described in Chapter 4.

GLOBAL ERROR REPAIR

What if the best way to recover from the error is to insert or delete tokens from the input stream at a point *before* where the error was detected? Consider the following Tiger program:

```
let type a := intArray [ 10 ] of 0 in ...
```

A local technique will discover a syntax error with := as lookahead symbol. Error recovery based on *error* productions would likely delete the phrase from type to 0, resynchronizing on the in token. Some local repair techniques can insert tokens as well as delete them; but even a local repair that replaces the := by = is not very good, and will encounter another syntax error at the [token. Really, the programmer's mistake here is in using type instead of var, but the error is detected two tokens too late.

Global error repair finds the smallest set of insertions and deletions that would turn the source string into a syntactically correct string, even if the insertions and deletions are not at a point where an LL or LR parser would first report an error. In this case, global error repair would do a single-token substitution, replacing type by var.

Burke-Fisher error repair. I will describe a limited but useful form of global error repair, which tries every possible single-token insertion, deletion, or replacement at every point that occurs no earlier than K tokens before the point where the parser reported the error. Thus, with K=15, if the parsing engine gets stuck at the 100th token of the input, then it will try every possible repair between the 85th and 100th token.

The correction that allows the parser to parse furthest past the original reported error is taken as the best error repair. Thus, if a single-token substitution of var for type at the 98th token allows the parsing engine to proceed past the 104th token without getting stuck, this repair is a successful one.

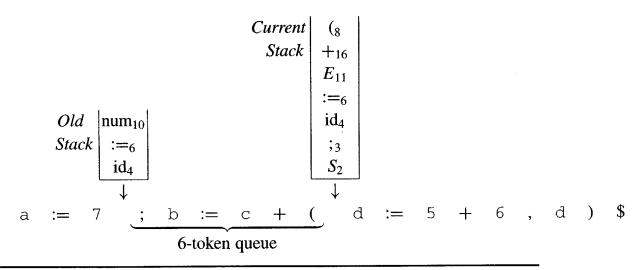


FIGURE 3.38. Burke-Fisher parsing, with an error-repair queue. Figure 3.18 shows the complete parse of this string according to Table 3.19.

Generally, if a repair carries the parser R=4 tokens beyond where it originally got stuck, this is "good enough."

The advantage of this technique is that the LL(k) or LR(k) (or LALR, etc.) grammar is not modified at all (no *error* productions), nor are the parsing tables modified. Only the parsing engine, which interprets the parsing tables, is modified.

The parsing engine must be able to back up K tokens and reparse. To do this, it needs to remember what the parse stack looked like K tokens ago. Therefore, the algorithm maintains two parse stacks: the current stack and the old stack. A queue of K tokens is kept; as each new token is shifted, it is pushed on the current stack and also put onto the tail of the queue; simultaneously, the head of the queue is removed and shifted onto the old stack. With each shift onto the old or current stack, the appropriate reduce actions are also performed. Figure 3.38 illustrates the two stacks and queue.

Now suppose a syntax error is detected at the *current* token. For each possible insertion, deletion, or substitution of a token at any position of the queue, the Burke-Fisher error repairer makes that change to within (a copy of) the queue, then attempts to reparse from the *old* stack. The success of a modification is in how many tokens past the *current* token can be parsed; generally, if three or four new tokens can be parsed, this is considered a completely successful repair.

In a language with N kinds of tokens, there are $K + K \cdot N + K \cdot N$ possible deletions, insertions, and substitutions within the K-token window. Trying

this many repairs is not very costly, especially considering that it happens only when a syntax error is discovered, not during ordinary parsing.

Semantic actions. Shift and reduce actions are tried repeatly and discarded during the search for the best error repair. Parser generators usually perform programmer-specified semantic actions along with each reduce action, but the programmer does not expect that these actions will be performed repeatedly and discarded – they may have side effects. Therefore, a Burke-Fisher parser does not execute any of the semantic actions as reductions are performed on the *current* stack, but waits until the same reductions are performed (permanently) on the *old* stack.

This means that the lexical analyzer may be up to K + R tokens ahead of the point to which semantic actions have been performed. If semantic actions affect lexical analysis – as they do in C, compiling the typedef feature – this can be a problem with the Burke-Fisher approach. For languages with a pure context-free grammar approach to syntax, the delay of semantic actions poses no problem.

Semantic values for insertions. In repairing an error by insertion, the parser needs to provide a semantic value for each token it inserts, so that semantic actions can be performed as if the token had come from the lexical analyzer. For punctuation tokens no value is necessary, but when tokens such as numbers or identifiers must be inserted, where can the value come from? The ML-Yacc parser generator, which uses Burke-Fischer error correction, has a %value directive, allowing the programmer to specify what value should be used when inserting each kind of token:

```
%value ID ("bogus")
%value INT (1)
%value STRING ("")
```

Programmer-specified substitutions. Some common kinds of errors cannot be repaired by the insertion or deletion of a single token, and sometimes a particular single-token insertion or substitution is very commonly required and should be tried first. Therefore, in an ML-Yacc grammar specification the programmer can use the %change directive to suggest error corrections to be tried first, before the default "delete or insert each possible token" repairs.

```
%change EQ -> ASSIGN | ASSIGN -> EQ | SEMICOLON ELSE -> ELSE | -> IN INT END
```

Here the programmer is suggesting that users often write "; else" where they mean "else" and so on.

The insertion of in 0 end is a particularly important kind of correction, known as a *scope closer*. Programs commonly have extra left parentheses or right parentheses, or extra left or right brackets, and so on. In Tiger, another kind of nesting construct is let ··· in ··· end. If the programmer forgets to close a scope that was opened by left parenthesis, then the automatic single-token insertion heuristic can close this scope where necessary. But to close a let scope requires the insertion of three tokens, which will not be done automatically unless the compiler-writer has suggested "change *nothing* to in 0 end" as illustrated in the %change command above.

PROGRAM

PARSING

Use Yacc to implement a parser for the Tiger language. Appendix A describes, among other things, the syntax of Tiger.

You should turn in the file tiger.grm and a README.

Supporting files available in \$TIGER/chap3 include:

makefile The "makefile."

errormsg.[ch] The Error Message structure, useful for producing error messages with file names and line numbers.

lex.yy.c The lexical analyzer. I haven't provided the source file tiger.lex, but I've provided the output of Lex that you can use if your lexer isn't working. parsetest.c A driver to run your parser on an input file. tiger.grm The skeleton of a file you must fill in.

You won't need tokens.h anymore; instead, the header file for tokens is y.tab.h, which is produced automatically by Yacc from the token specification of your grammar.

Your grammar should have as few shift-reduce conflicts as possible, and no reduce-reduce conflicts. Furthermore, your accompanying documentation should list each shift-reduce conflict (if any) and explain why it is not harmful.

My grammar has a shift-reduce conflict that's related to the confusion between

variable [expression] type-id [expression] of expression

In fact, I had to add a seemingly redundant grammar rule to handle this confusion. Is there a way to do this without a shift-reduce conflict?

FURTHER READING

Use the precedence directives (%left, %nonassoc, %right) when it is straightforward to do so.

Do not attach any semantic actions to your grammar rules for this exercise.

Optional: Add *error* productions to your grammar and demonstrate that your parser can sometimes recover from syntax errors.

FURTHER READING

Conway [1963] describes a predictive (recursive-descent) parser, with a notion of FIRST sets and left-factoring. LL(k) parsing theory was formalized by Lewis and Stearns [1968].

LR(k) parsing was developed by Knuth [1965]; the SLR and LALR techniques by DeRemer [1971]; LALR(1) parsing was popularized by the development and distribution of Yacc [Johnson 1975] (which was not the first parser-generator, or "compiler-compiler," as can be seen from the title of the cited paper).

Figure 3.29 summarizes many theorems on subset relations between grammar classes. Heilbrunner [1981] shows proofs of several of these theorems, including $LL(k) \subset LR(k)$ and $LL(1) \not\subset LALR(1)$ (see Exercise 3.14). Backhouse [1979] is a good introduction to theoretical aspects of LL and LR parsing.

Aho et al. [1975] showed how deterministic LL or LR parsing engines can handle ambiguous grammars, with ambiguities resolved by precedence directives (as described in Section 3.4).

Burke and Fisher [1987] invented the error-repair tactic that keeps a *K*-token queue and two parse stacks.

EXERCISES

- **3.1** Translate each of these regular expressions into a context-free grammar.
 - a. $((xy^*x)|(yx^*y))$?
 - b. $((0|1)^{+}$ "." $(0|1)^{*}$) $|((0|1)^{*}$ "." $(0|1)^{+}$)
- ***3.2** Write a grammar for English sentences using the words

time, arrow, banana, flies, like, a, an, the, fruit