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Stabilizing of Ball and Plate System Using an Approximate Model

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Abstract: In this paper, the stabilization problem of the ball and plate system is considered. In order to derive the control rule, we propose a method to obtain an approximate solution the matching conditions which occurs as nonlinear partial differential equations (PDE's) using in the controlled Lagrangians method to stabilize under-actuated systems. The proposed approach is used an approximate model of Euler-Lagrange (EL) system and with this approach, it is only required a common solution of a set of linear PDE's, instead of to solve nonlinear PDE's. Therefore, the proposed method gives us an opportunity to find an approximate solution of the matching conditions and to derive the control rule to stabilize the system.

Keywords: Ball and plate system, matching conditions, EL system, approximate model.

1. INTRODUCTION

The methods were developed to control under-actuated systems for EL systems are proposed in Bloch (2000) and for the Hamiltonian systems are proposed in Ortega (2002). The idea of these approaches is based on to shape of the closed systems energy functions. Potential energy shaping is sufficient for Full-actuated systems. However kinetic energy shaping is also needed to be shaped for under-actuated case. To stabilize an under-actuated system via energy shaping, there is need to solve some nonlinear PDE's called matching conditions. This is compulsory part of energy shaping approaches and the exact solution is generally difficult to be found and closed-form solutions have appeared only for the case of under-actuation degree one. In this paper, a method is proposed to obtain an approximate solution the matching conditions. The method is developed using to benefit the simplicity of separable EL systems in solving the stabilization problem for the under actuated case. EL model is divided to linear parts, using the same idea proposed in the study Gören-Sümer and Şengör (2015) for under actuated Hamiltonian systems. Thus, matching conditions turns to a set of linear PDE's, instead of nonlinear ones. PDE's are solved for all parts and control rule is constructed for all parts.

Ball and plate system is one the most popular example of under-actuated system. There are many approaches to stabilize ball and plate system including neural networks (Dong, et al. 2009), fuzzy (Fan et al., 2004), sliding mode control (Liu et al., 2008), back-stepping (Hongrui et al., 2008) etc. To illustrate the effectiveness and the appropriateness of the proposed method, ball and plate system is considered.

2. PRELIMINARIES

EL system matching conditions are described Bloch, et al., (2000). Consider an EL system in n-dimensional configuration space Q:

$$\frac{d}{dt}\frac{\partial d}{\partial \dot{q}}L(q,\dot{q})-\frac{\partial d}{\partial q}L(q,\dot{q})=G(q)u \tag{1}$$

In (1) if $G(q)\in\mathbb{R}^{n\times m}$ and $n<m$, the system is called under-actuated.

Let define a closed loop EL system as:

$$\frac{d}{dt}\frac{\partial d}{\partial \dot{q}}L_c(q,\dot{q})-\frac{\partial d}{\partial q}L_c(q,\dot{q})=u_c \tag{2}$$

In (2) $u_c=0$ or this is defined as a gyroscopic force.

The Lagrangian function of simple mechanical system is defined as,

$$L(q,\dot{q})=\frac{1}{2}\dot{q}^TM(q)\dot{q}-V(q) \tag{3}$$

Based on (3), the motion equations of systems (1) and (3) have been:

$$M\ddot{q}+\frac{\partial M\dot{q}}{\partial q}\dot{q}-\frac{1}{2}\frac{\partial \dot{q}^TM\dot{q}}{\partial q}+\frac{\partial V}{\partial q}=Gu \tag{4a}$$

$$M_c\ddot{q}+\frac{\partial M_c\dot{q}}{\partial q}\dot{q}-\frac{1}{2}\frac{\partial \dot{q}^TM_c\dot{q}}{\partial q}+\frac{\partial V_c}{\partial q}=u_c \tag{4b}$$

To obtain matching conditions equations (4a) and (4b) are calculated as follows (Blankenstein et al., 2002):

$$\ddot{q}=M^{-1}Gu-M^{-1}\frac{\partial M\dot{q}}{\partial q}\dot{q}+\frac{1}{2}M^{-1}\frac{\partial \dot{q}^TM\dot{q}}{\partial q}-M^{-1}\frac{\partial V}{\partial q} \tag{5}$$

$$\ddot{q}=M_c^{-1}u_c-M_c^{-1}\frac{\partial M_c\dot{q}}{\partial q}\dot{q}+\frac{1}{2}M_c^{-1}\frac{\partial \dot{q}^TM_c\dot{q}}{\partial q}-M_c^{-1}\frac{\partial V_c}{\partial q} \tag{6}$$

To match these two equations (5) and (6), the following conditions must be satisfied:

$$G^{\perp}\left[\left(\frac{\partial M\dot{q}}{\partial q}-MM_c^{-1}\frac{\partial M_c\dot{q}}{\partial q}\right)\dot{q} -\frac{1}{2}\left(\frac{\partial \dot{q}^TM\dot{q}}{\partial q}-MM_c^{-1}\frac{\partial \dot{q}^TM_c\dot{q}}{\partial q}\right)+MM_c^{-1}u_c\right]=0 \tag{7}$$

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$$G^{\perp}\left(\frac{\partial V}{\partial q}-MM_c^{-1}\frac{\partial V_c}{\partial q}\right)=0 \tag{8}$$

The equation (7) is called as kinetic energy matching condition and (8) is called potential energy matching condition. Where, $G^{\perp}:(\mathbb{R}^{n-m})^T \rightarrow (\mathbb{R}^n)^t$ is left annihilator of G.

The input signal which is defined in Blankenstein et al., (2010):

$$\begin{aligned} u &= (G^TG)^{-1}G^T\left[\left(\left(\frac{\partial M\dot{q}}{\partial q}-\frac{1}{2}\left(\frac{\partial M\dot{q}}{\partial q}\right)^T\right)\right.\right.\\ &\left.\left.-MM_c^{-1}\left(\frac{\partial M_c\dot{q}}{\partial q}-\frac{1}{2}\left(\frac{\partial M_c\dot{q}}{\partial q}\right)^T\right)\right)\dot{q}+\left(\frac{\partial V}{\partial q}-MM_c^{-1}\frac{\partial V_c}{\partial q}\right)\right] \end{aligned} \tag{9}$$

To obtain this control signal, $M_c(q)>0$ and $u_c(t)\dot{q}=0$ which provide matching conditions (7) and (8) is necessary but it is not sufficient to stabilize the system (Blankenstein et al., 2010).

There is not a general solution of matching conditions. In order to solve matching conditions, some methods are proposed in Hamberg (1999), Bloch (2000), Ortega (2002), Chang (2005), Auckly et al. (2002). Furthermore, Gören-Sümer and Şengör (2015) suggested an approach to find an approximate solution of the matching conditions occurred for Hamiltonian system.

In this study, Gören-Sümer and Şengör (2015)'s approach is applied to EL model of ball and plate system. Therefore, the ball and plate system is modelled and then matching conditions are obtained from approximate model of this application.

3. BALL AND PLATE SYSTEM MODEL

The inertia matrix of ball and plate system is described as in Nokhbeh and Khashabi (2011):

$$M(q)=\begin{bmatrix} m+\frac{J_b}{r^2} & 0 & 0 & 0\\ 0 & J+J_b+mq_1^2 & 0 & mq_1q_3\\ 0 & 0 & m+\frac{J_b}{r^2} & 0\\ 0 & mq_1q_3 & 0 & J+J_b+mq_3^2 \end{bmatrix} \tag{10}$$

and also potential energy function is described as:

$$V(q)=mg(q_1\sin(-q_2)+q_3\sin(-q_4)) \tag{11}$$

In which: m: mass of ball, J_b: inertia of ball, r: radius of ball, J: inertia of plate, q_1: angle of plate on x axis, q_2: displacement of ball on x axis, q_3: angle of plate on y axis q_4: displacement of ball on y axis.

The normalized inertia matrix and potential energy function of system is calculated as below:

$$M(q)=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & L+L_2q_1^2 & 0 & L_2q_1q_3\\ 0 & 0 & 1 & 0\\ 0 & L_2q_1q_3 & 0 & L+L_2q_3^2 \end{bmatrix} \tag{12}$$

$$V(q)=L_2g(q_1\sin(-q_2)+q_3\sin(-q_4)) \tag{13}$$

where:

$$L \triangleq \frac{(J+J_b)r^2}{J_b+mr^2} \tag{14a}$$

$$L_2 \triangleq \frac{mr^2}{J_b+mr^2} \tag{14b}$$

The EL model of system is described as,

$$M(q)\ddot{q}+\frac{\partial M(q)\dot{q}}{\partial q}-\frac{1}{2}\left(\frac{\partial M(q)\dot{q}}{\partial q}\right)^T+\frac{\partial V(q)}{\partial q}=Gu \tag{15}$$

The inertia matrix can be written in the following form,

$$\begin{aligned} &M(q)=\begin{bmatrix} M_{11} & M_{12}\\ M_{21} & M_{22}\end{bmatrix} \quad \frac{\partial V(q)}{\partial q}=\begin{bmatrix}\nabla_{q1,2}V\\ \nabla_{q3,4}V\end{bmatrix}\\ &G^{\perp}=\begin{bmatrix}G_{1,2}^{\perp} & G_{3,4}^{\perp}\end{bmatrix} \quad q=\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix}=\begin{bmatrix}q_{1,2}\\ q_{3,4}\end{bmatrix} \end{aligned} \tag{16}$$

There can be seen that $M_{12}=M_{21}$ and the parts M_{11} and M_{22} are equal the model of ball and beam. If the parts M_{21} and M_{12} are neglected when constructing the control rule, this is same to control two separated ball and beam system. Ball and beam system is stabilized via IDA-PBC in Gordillo et al. 2002.

Control rule is constructed to separated parts of inertia matrix and they (2 parts of control rule) will be applied to the system separately.

Matching conditions of the separated system are below:

$$\begin{aligned} &G_{1,2}^{\perp}\left[\left(\frac{\partial M_{11}\dot{q}_{1,2}}{\partial \dot{q}_{1,2}}-M_{11}M_c^{-1}\frac{\partial M_c\dot{q}_{1,2}}{\partial \dot{q}_{1,2}}\right)\dot{q}_{1,2}\right.\\ &\left.-\frac{1}{2}\left(\frac{\partial \dot{q}_{1,2}{}^TM_{11}\dot{q}_{1,2}}{\partial \dot{q}_{1,2}}-M_{11}M_c^{-1}\frac{\partial \dot{q}_{1,2}{}^TM_c\dot{q}_{1,2}}{\partial \dot{q}_{1,2}}\right)\right]=0 \end{aligned} \tag{17a}$$

$$G_{1,2}^{\perp}\left(\nabla_{q1,2}V-M_{11}M_c^{-1}\frac{\partial V_c}{\partial q_{1,2}}\right)=0 \tag{17b}$$

On the next chapter matching conditions are reconstructed from approximate model:

4. MATCHING CONDITIONS

Let define r sub-region in EL configuration space:

$$\mathbb{S}_r \triangleq \{q|h_r(q)\geq h_l(q), l=1,2,\dots,n, r\neq l\} \tag{18}$$

$h_i(q)$'s are scalar functions which is $0<h_i(q)\leq 1$.The approximate of inertia matrix is described as,

$$\widehat{M}_{11}(q)=\sum\nolimits_i h_i(q)M_i \tag{19}$$

and also the approximate of controlled system inertia matrix is described as:

$$\widehat{M}_c(q)=\sum\nolimits_i h_{ci}(q)M_{ci} \tag{20}$$

The function $h_{ci}(q)$ is chosen as below:

$$h_{ci}(q)\triangleq\begin{cases}1 & q\in\mathbb{S}_i\\ 0 & q\notin\mathbb{S}_i\end{cases} \tag{21}$$

Using this description, $\widehat{M}\widehat{M}_c^{-1}$ can be described as:

$$\widehat{M}\widehat{M}_c^{-1}=\sum\nolimits_i h_i(q)M_iM_{ci}^{-1} \tag{22}$$

The $h_i(q)$'s and M_i's must be chosen such that the following norms must be the minimum:

$$\begin{aligned} &min \left\| M(q) - \sum_i^r h_i(q) M_i \right\| \\ &min \left\| \frac{\partial M(q)}{\partial q_j} - \sum_i^r \frac{\partial h_i(q)}{\partial q_j} M_i \right\| \quad \forall j \end{aligned} \tag{23}$$

Particularly, for this example, $h_i(q)$ will be chosen as,

$$h_i(q) \triangleq \begin{cases} 1 & q \in \mathbb{S}_i \\ 0 & q \notin \mathbb{S}_i \end{cases} \tag{24}$$

With this choice, kinetic energy matching condition is always satisfied. And the potential energy condition will be:

$$G_{1,2}^{\perp} \left(\nabla_{q1,2} V - \sum_i M_i M_{ci}^{-1} \frac{\partial V_c}{\partial q_{1,2}} \right) = 0 \tag{25}$$

If there exist a matrix $M_{c1} > 0$ and a function $V_c(q)$, with the properties of $\nabla_{q1,2} V_c(q^*) = 0$ and hessian of $V_c(q^*)$ positive such that (25) satisfied, the lemma given in Gören-Sümer and Şengör (2015) is stated how to construct r number of matrices $M_{ci} > 0$, as

$$G_{1,2}^{\perp} \left(\nabla_{q1,2} V - M_1 M_{c1}^{-1} (\nabla_{q1,2} V_c) \right) = 0 \tag{26}$$

and

$$G_{1,2}^{\perp} \left(M_j M_{cj}^{-1} - M_i M_{ci}^{-1} \right) = 0, \forall i,j \tag{27}$$

5. SOLVING MATCHING CONDITIONS

The solution procedure will be given for ball and plate system as follows step by step. Since the control rules corresponded each part of system are calculated separately, the following procedure is given for first part, and for the second part the same way is used.

Step 1: Calculation of $V_c(q)$

As described before the potential energy matching condition for the approximate system is:

$$G_{1,2}^{\perp} \left(\nabla_{q1,2} V - \sum_i M_i M_{ci}^{-1} \left(\nabla_{q1,2} V_c \right) \right) = 0 \tag{28}$$

Let define a matrix which holds:

$$S_i = M_i M_{ci}^{-1} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \tag{29}$$

the variables are defined as in (28):

$$\nabla_{q1,2} V = \begin{bmatrix} g L_2 \sin(q_2) \\ g L_2 q_1 \cos(q_2) \end{bmatrix} \quad (\nabla_{q1,2} V_c) = \begin{bmatrix} \nabla_{q1} V_c \\ \nabla_{q2} V_c \end{bmatrix} \tag{30}$$

Matching condition is:

$$-s_{11} \nabla_{q1} V_c - s_{12} \nabla_{q2} V_c + g L_2 \sin(q_2) = 0 \tag{31}$$

When this PDE is solved:

$$\begin{aligned} V_c &= \frac{g L_2 \cos(q_2)}{s_{12}} + \Phi(z(q)) \\ z(q) &= q_2 - \frac{s_{12}}{s_{11}} (q_1 - q_1^*) \end{aligned} \tag{32}$$

where $\Phi(z(q))$ is an arbitrary differentiable function satisfying the condition $\nabla_q \Phi(z(q)) = 0$ for $q_2 = 0$ and arbitrary q_1^*. To ensure the closed loop potential energy V_c is being a quadratic function, $\Phi(z(q))$ is chosen as:

$$\Phi(z) = \frac{K_p}{2} z - L_2 \frac{g}{s_{12}} \tag{33}$$

Closed loop system potential energy function, its gradient and hessian for $q_1^* = 0$ will be:

$$V_c(q) = \frac{g L_2 (1 - \cos(q_2))}{s_{12}} + \frac{1}{2} K_p \left(q_2 - q_1 \frac{s_{12}}{s_{11}} \right)^2 \tag{34a}$$

$$\nabla_q V_c(q) = \begin{bmatrix} \dfrac{K_p s_{12} (-q_2 s_{11} + q_1 s_{12})}{s_{11}^2} \\ K_p \left(q_2 - \dfrac{q_1 s_{12}}{s_{11}} \right) + \dfrac{g L_2 \sin(q_2)}{s_{12}} \end{bmatrix} \tag{34b}$$

$$\nabla_q^2 V_c(q) = \begin{bmatrix} \dfrac{K_p s_{12}^2}{s_{11}^2} & -\dfrac{K_p s_{12}}{s_{11}} \\ -\dfrac{K_p s_{12}}{s_{11}} & K_p - \dfrac{g L2 \cos(q_2)}{s_{12}} \end{bmatrix} \tag{34c}$$

$$|\nabla_q^2 V_c(q)| = -\left(\frac{g\, Kp\, L_2\, s_{12}}{s^2{}_{11}} \right) \tag{34d}$$

It is clearly seen that for $q_{1,2} = 0$, $V_c(q) = 0$ and $\nabla_q V_c(q) = 0$. for $s_{12} < 0$ and $s_{11} > 0$, $\nabla_q^2 V_c(q) > 0$ for $-\frac{\pi}{2} < q_2 < \frac{\pi}{2}$.

Step 2: Finding appropriate s_{12} and s_{11}:

$V_c(q)$ is found in last step (34a). Elements of S matrix can be chosen as follows,

$$s_{12} = -\frac{L_2}{g} \quad s_{11} = \frac{L_2 g}{10} \tag{35}$$

and also system parameters can be chosen:

$$L_2 = 1 \quad g = 9{,}81 \quad K_p = \frac{1}{2000} \tag{36}$$

Substitute the parameters to the $V_c(q)$ becomes,

$$V_c(q) = 1. + 0.000259778 \left(0.981\, q_2 - 9.81\, (-q_1) \right)^2 - \cos(q_2) \tag{37}$$

The closed loop potential energy function is shown in Fig 1 and the contour plot of it is given in Fig 2.

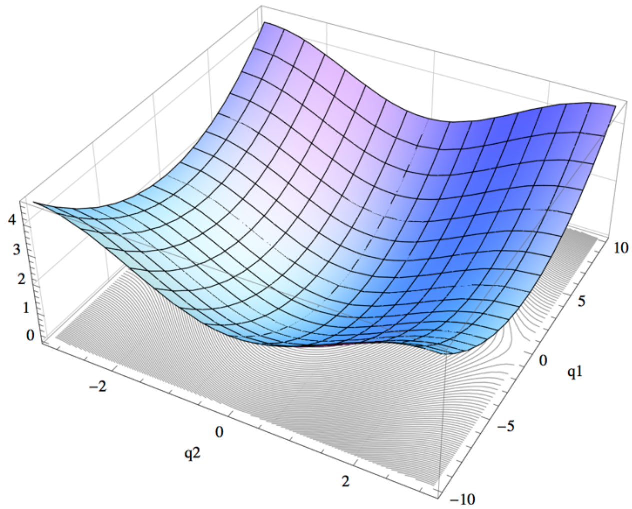

Fig. 1. Assigned potential energy function.

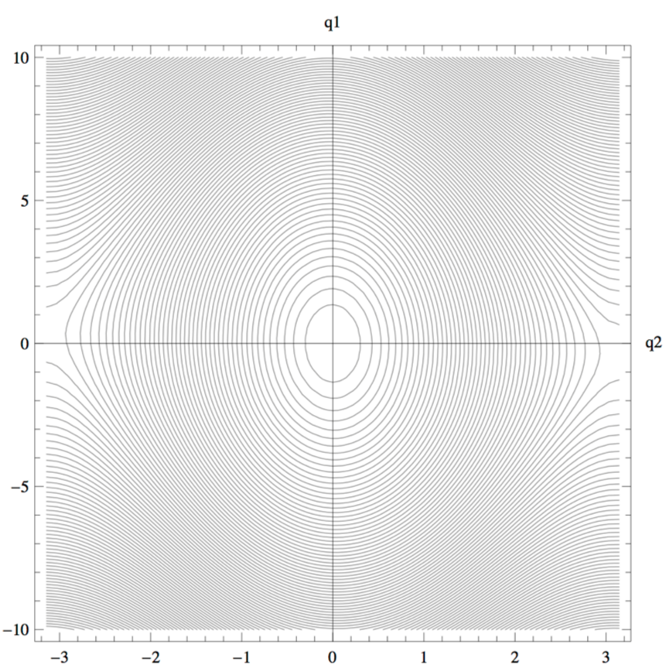

Fig. 2. Contour plot of potential energy function.

Step 3: Constructing $M_{ci} > 0$'s:

M_{ci} matrices are solved $\forall i$ which are described respect to S_i matrix elements. First of $\widehat{M}_{11}(q)$ is:

$$M_{11} = \begin{bmatrix} 1 & 0 \\ 0 & L + L_2 q_1^2 \end{bmatrix} \Rightarrow M_i = \begin{bmatrix} 1 & 0 \\ 0 & m_i \end{bmatrix} \quad (38)$$

And M_{ci} is obtained as,

$$M_{ci} = \begin{bmatrix} \dfrac{s_{22}}{gL_2\, s_{21} + \dfrac{gL_2 s_{22}}{10}} & \dfrac{gL_2 m_i}{gL_2 s_{21} + \dfrac{gL_2 s_{22}}{10}} \\ \dfrac{-s_{21}}{gL_2\, s_{21} + \dfrac{gL_2 s_{22}}{10}} & \dfrac{gL_2 m_i}{10\left(gL_2\, s_{21} + \dfrac{gL_2 s_{22}}{10}\right)} \end{bmatrix} \quad (39)$$

It is clearly seen that $s_{21} = -gL_2 m_i$ and $s_{22} > \frac{981\, m_i}{10}$ for $M_{ci} > 0$. We can choose the rest of matrix and S matrix will be:

$$S = \begin{bmatrix} \dfrac{gL_2}{10} & -gL_2 \\ -gL_2 m_i & 1000 \end{bmatrix} \quad (40)$$

Step 4: Calculation of $u_{es}(t)$ and $u_d(t)$

The control input, obtained from matching conditions is defined in (9).

After the calculations of the last steps are substituted, the control signal is described as:

$$u_{es} = (G^T G)^{-1} G^T \left[\left(\frac{\partial V}{\partial q} - M^{-1}(q) \widehat{M}_c^{-1} \frac{\partial V_c}{\partial q} \right) \right] \quad (41)$$

To provide system asymptotic stability, damping added:

$$\begin{gathered} u = u_{es} + u_d \\ u_d = -K_v G^T M^{-1}(q)\, \widehat{M}_c \dot{q} \end{gathered} \quad (42)$$

5. SIMULATION RESULTS

The simulation parameters are set as (36) and $K_v = 1000$. For the approximation of $M(q)$ as

$$\begin{gathered} \widehat{M}(q) = \sum_{i=1}^{r} h_i(q) M_i\,, \quad r = 120 \\ M_i = M\big(q_1(i)\big), \quad q_1(i) = 2 - i\frac{2}{r}, \qquad i = 1, \dots, r \end{gathered} \quad (43)$$

The sub regions are taken as,

$$\mathbb{S} = \left\{ q_1 \middle| q_1(i) - i\frac{2}{r} < q_1 < q_1(i) + i\frac{2}{r} \right\}, \quad i = 1, \dots, r$$

with the $h_i(q)$'s are as follows:

$$h_i(q) \triangleq \begin{cases} 1 & q_1 \in \mathbb{S}_i \\ 0 & q_1 \notin \mathbb{S}_i \end{cases}$$

The initial conditions $\big(q_1(0), q_2(0), q_3(0), q_4(0)\big) = (1,0,1,0)$ and the desired position is $(q_1^*, q_2^*, q_3^*, q_4^*) = (0,0,0,0)$. The results are presented in Figure 3, 4 and 5 which illustrate time domain responses:

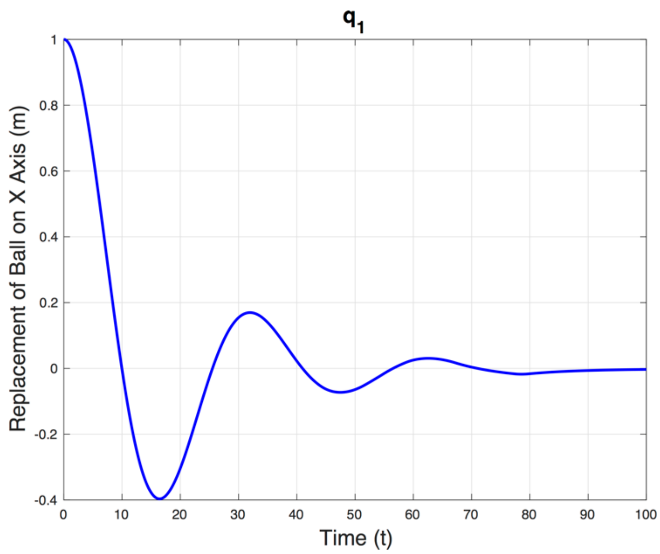

Fig. 3.q_1: Replacement of Ball on X-Axis .

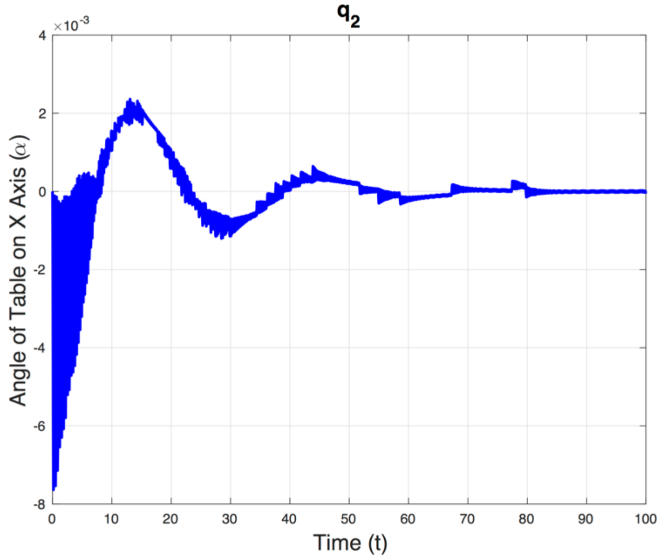

Fig. 4. q_2: Angle of table on X-Axis .

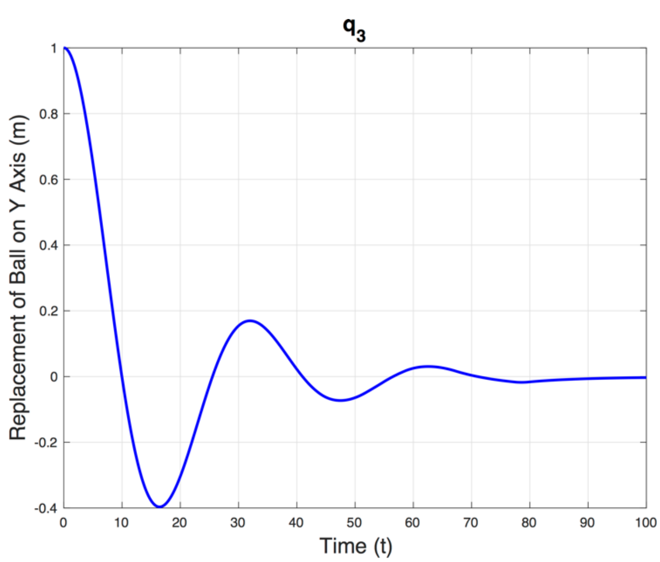

Fig. 5. q_3: Replacement of Ball on Y-Axis.

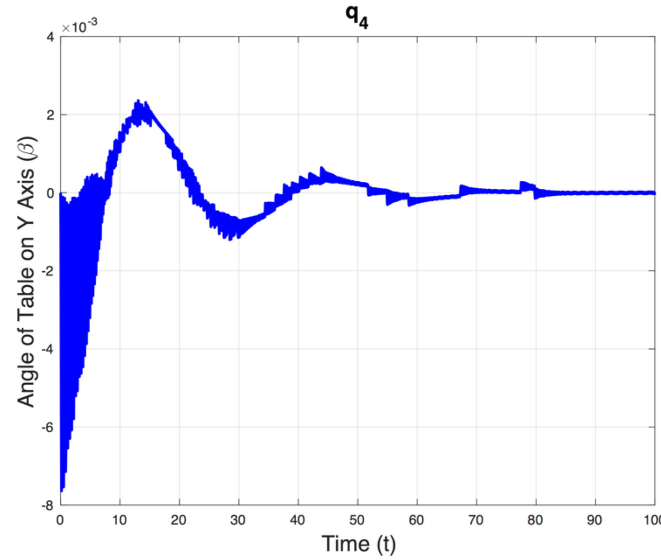

Fig. 6. q_4: Angle of table on Y-Axis .

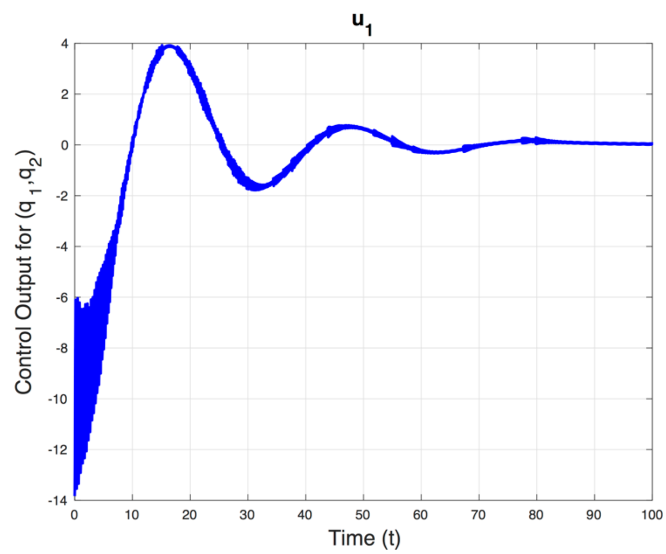

Fig. 7. Control input for states (q_1, q_2).

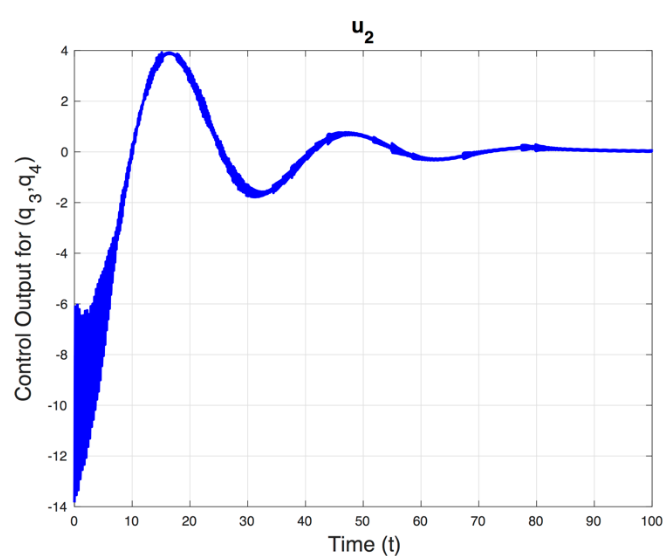

Fig. 8. Control input for states (q_3, q_4).

6. CONCLUSIONS

Controlling of under-actuated systems is a difficult problem. Recently, this problem is solved via Hamiltonian approach with approximate model in Gören-Sümer and Şengör (2015). This approach provided simplicity for the solution of the problem. In this study, controlling of under-actuated Euler Lagrange system is analysed. To overcome the difficulties of solving the problem of controlling under-actuated system, model approximation approach is used. By advantages of this approach, matching conditions is constructed using approximate model as a set of linear PDE's instead of nonlinear ones, then these are solved and the control rule is obtained. The approximate model of ball and palate system is constructed first two separated ball and beam system and then each ball and beam system is approximating by blending some linear EL systems.

Future work, a better approximation of the model can be used to solve tracking problem.

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