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Commparison Between Different Methods of Control of Ball and Plate System with 6DOF Stewart Platform

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Abstract: In this paper, we tackle the control problem of ball and plate system (BPS) with 6 DOF Stewart platform. The BPS is a typical multi-variable nonlinear system which is a two dimensional expansion of the ball and beam system. Four strategies are proposed for static and dynamic position tracking: PID, LQR, Sliding Mode and Fuzzy controller. The results of simulation and also the validation on real system are provided. The comparison between the proposed strategies is presented based on the performance of the tracking.

Keywords: Nonlinear control, Ball and plate system; Stewart platform, Sliding mode control, Fuzzy control, LQR.

1. INTRODUCTION

The ball and beam system is a well-known problem for nonlinear control where the system is under-actuated and has two degrees of freedom, while the ball and plate system (BPS) can be considered as an extension of that system consisting of a ball that can roll freely on a plate. Therefore, the BPS has four degrees of freedom and it is more complicated due to the coupling between the variables. The complexity of the problem increases when the system is held on 6 DOF Stewart platform. In our case, the experimental system includes a plate fixed on the moving Stewart platform, a ball, six motors and their driving system. A touch pad is used to measure the position of the ball. This system can be considered as a big challenge to test various nonlinear strategies.

However, most of the previous works focused on the two dimensional Electro-mechanical ball and plate system (all the references). In Knuplez (2003), a controller design based on classical and modern control theory was proposed. A supervisory fuzzy controller of two layers was proposed in Ming (2006) to study the motion control in static and dynamic tracking. A state observer was used in Hongrui (2008) to estimate ball velocities while the position of the ball was regulated with double feedback loops in Wang (2008). The controllability on Poisson manifolds of the system was studied in Siyan (2009). The disturbance rejection topic was also tackled by some works. In Huida (2009), an active control is applied to the trajectory tracking and in Xiucheng (2009) PID neural network controller based on genetic algorithm was proposed. In Hai-Qi Lin (2014), a controller was designed to ensure the stability employing a loop shaping method based on Normalised Coprime Factors perturbation model. Besides, some works employed the sliding mode control (Dejun (2008), Hong Wei (2012)). In Ghiasi (2012), an optimal robust controller was designed for the trajectory tracking and only the results of simulation were presented. In this paper, we study the ball and plate system where the plate is mounted on Stewart Platform with 6 DOF. Four controllers designed and evaluated in static and dynamic tracking: Classical controller PID, Linear quadratic regulator (LQR), Sliding Mode Controller and Fuzzy Controller. The inverse kinematic of the Stewart platform was used to calculate the instantaneous inclination of the plate. The rest of the paper is organized as follows. Section 2 introduces the modeling of the ball and plate system. Section 3 discusses the design of the four methods of control and presents simulation. Finally, the validation on real system is presented in Section 4.

2. MATHEMATICAL MODELING

The following mathematical equations are based on (X.Fan (2004), K. Kyu Lee (2008)). The Euler-Lagrange equation of ball-plate system can be written as following:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_i} - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \tag{1}$$

,where q_i stands for i-direction coordinate, T is the kinetic energy of the system, V is potential energy of system and Q is composite force.

The system has four degrees of freedom; two derived from the motion of the ball and two for the inclination of the plate. Here we assume that the generalized coordinates of the system are x_b and y_b (the position of the ball on the plate) and α and β (the rotations of the plate). The kinetic energy of the ball consists of its both rotational with respect to its center of mass and translational energy:

$$T_b = \frac{1}{2}m_b\left(\dot{x}_b^2 + \dot{y}_b^2\right) + \frac{1}{2}I_b\left(\omega_x^2 + \omega_y^2\right) \tag{2}$$

Where m_b is the mass of the ball and I_b is the moment of inertia of the ball.

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As the ball is assumed to rotate without slippage, therefore:

$$\dot{x}_b = r_b\omega_y \quad , \quad \dot{y}_b = r_b\omega_x \tag{3}$$

where r_b denotes the radius of the ball. By substituting equations (3) into equation (2) we will have:

$$T_b = \frac{1}{2}\left(m_b + \frac{I_b}{r_b^2}\right)\left(\dot{x}_b^2 + \dot{y}_b^2\right) \tag{4}$$

The kinetic energy of the plate (assuming that there is no spin around z axis)includes its rotational kinetic energy and the rotational kinetic energy of the ball and it can be written as:

$$T_p = \frac{1}{2}\left(I_p + I_b\right)\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + \frac{1}{2}m_b\left(x_b\dot{\alpha} + y_b\dot{\beta}\right)^2 \tag{5}$$

The relative potential energy of the ball to horizontal plane passing by the center of the inclined plate is:

$$V_x = m_b g x_b \sin\alpha \quad , \quad V_y = m_b g y_b \sin\beta$$

Where g is the gravity acceleration. By applying the Euler-Lagrange's equation, we obtain the mathematical model for the ball and plate system as follows:

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{x}_b - m_b\left(x_b\dot{\alpha}^2 + y_b\dot{\alpha}\dot{\beta}\right) + m_b g\sin\alpha = 0 \tag{6}$$

$$\left(m_b + \frac{I_b}{r_b^2}\right)\ddot{y}_b - m_b\left(y_b\dot{\beta}^2 + x_b\dot{\alpha}\dot{\beta}\right) + m_b g\sin\beta = 0 \tag{7}$$

$$\begin{aligned}\tau_x = \left(I_p + I_b + m_b x_b^2\right)\ddot{\alpha} + 2m_b x_b\dot{x}_b\dot{\alpha} + m_b x_b y_b\ddot{\beta} \\ + m_b\dot{x}_b y_b\dot{\beta} + m_b x_b\dot{y}_b\dot{\beta} + m_b g x_b\cos\alpha\end{aligned} \tag{8}$$

$$\begin{aligned}\tau_y = \left(I_p + I_b + m_b y_b^2\right)\ddot{\beta} + 2m_b y_b\dot{y}_b\dot{\beta} + m_b x_b y_b\ddot{\alpha} \\ + m_b\dot{x}_b y_b\dot{\alpha} + m_b x_b\dot{y}_b\dot{\alpha} + m_b g y_b\cos\beta\end{aligned} \tag{9}$$

Equations (6) and (7) describe the movement of the ball on the plate and they reflect the relationship between the acceleration of the ball and the rotational angle and the angular velocity of the plate and equations (8) and (9) show the effect of external torque on the whole system.
It is difficult to control such a highly nonlinear system. Besides that, in our system, the considered inputs are the angles α and β. Therefore, for simplification we can focus only on the equations (6) and (7).
In the steady state, the plate should be in the horizontal position, and the two angles α and β must equal to zero. We derive the linearised model of system in a neighbourhood of this working state assuming that the range of the rotational angles of the plate is $[-5°, +5°]$. Therefore, we can use the approximation of the sine function, i.e. $\sin\alpha \approx \alpha$,$\sin\beta \approx \beta$ and Equations (6) and (7) become:

$$x : (m_b + \frac{I_b}{r_b^2})\ddot{x} + m_b g\alpha = 0 \tag{10}$$

$$y : (m_b + \frac{I_b}{r_b^2})\ddot{y} + m_b g\beta = 0 \tag{11}$$

Where $I_b = \frac{2}{5}m_b r_b^2$.
If we consider $X = (x, \dot{x}, y, \dot{y})$ as a state vector of the system and the angles of the plate as the control input, we obtain the following state space representation:

$$\begin{aligned}\dot{X} &= AX + Bu \\ Y &= CX\end{aligned}$$

where:

$$A = \begin{pmatrix}0&1&0&0\\0&0&0&0\\0&0&0&1\\0&0&0&0\end{pmatrix}, B = -\frac{5}{7}g\begin{pmatrix}0&0\\1&0\\0&0\\0&1\end{pmatrix}, C = \begin{pmatrix}1&0&0&0\\0&0&1&0\end{pmatrix}$$

The above system is completely controllable and observable because the ranks of the controllability matrix and of the observability matrix are both 4 which equal to the dimensions of the considered system state. We consider this representation in the design of the controllers in the next section.

3. THE DESIGN OF THE CONTROLLERS

We proposed four methods to control the static and dynamic position of the ball on the plate. We fixed some requirements in order to compare between the performances of these methods. We consider the response time t_s, the maximum overshoot D_{100} and the steady state error e_{ss}. The limits are given by:

$$\begin{cases} t_s & \leq 3sec \\ D_{100} & < 5\% \\ e_{ss} & \leq 2mm \end{cases}$$

The following simulation results are tested in MATLAB 2009 program.

3.1 PID Controller

In order to stabilize the ball in a desired position with respect to the requirements, we designed PID controller for x-axis and another one for y-axis as they are expressed in two separate differential equations. Fig.1 shows the simulation result for step response with the obtained controller for the position of the ball on the x-axis. We got: $t_s = 2.5sec$, $D_{100} < 5\%$ and $e_{ss} = 0$, with : $K_p = -0.3$, $K_d = -0.337$.

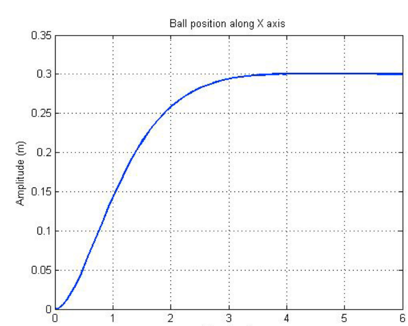

Fig. 1. Position of the ball on the x-axis (PID).

In Fig.2, the simulation result for the tracking of a desired circle with radius of 5 cm is presented where we obtained an error less than 3 mm using these parameters ($K_p = -0.52$, $K_d = -0.35$)

3.2 LQR Controller

The second control method is a Linear Quadratic controller. Here, the feedback gain is based upon the minimization of a quadratic cost function:

$$J = \frac{1}{2}\int_0^\infty \left(x^T Qx + u^T Ru\right) dt$$

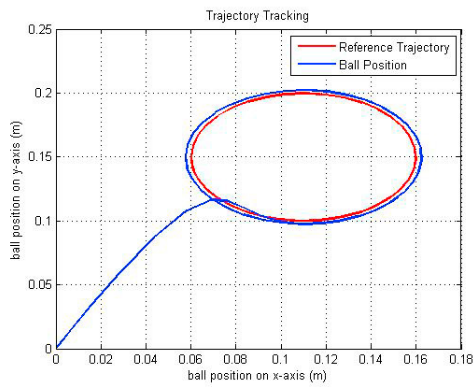

Fig. 2. Circular Trajectory Tracking with PID controller.

We chose the Q, R matrices as follows:

$$Q = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0.05 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0.05 \end{pmatrix}, R = \begin{pmatrix} 20 & 0 \\ 0 & 20 \end{pmatrix}$$

Sloving Riccati equation we get the state-feedback gain K:

$$K = \begin{bmatrix} -0.3162 & -0.3047 & 0 & 0 \\ 0 & 0 & -0.3162 & -0.3047 \end{bmatrix}$$

Fig.3 shows simulation result for stabilizing the ball in a desired position satisfying the desired performance requirements. In Fig.4 we present the tracking of a desired circular trajectory where we obtained an error less than 5mm.

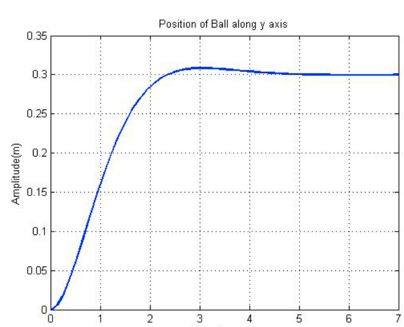

Fig. 3. The ball position on the x-axis (LQR).

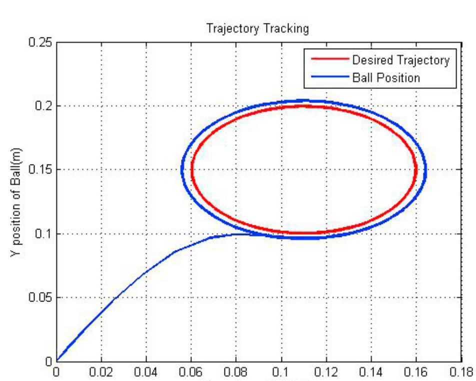

Fig. 4. Circular Trajectory Tracking with LQR controller.

3.3 Sliding Mode Controller

Sliding mode controller design provides a systematic approach to the problem of maintaining stability facing the modeling imprecision, parameter variations and disturbances. Here, we define a sliding surface and satisfying sliding constraints to force the system to slide on the sliding surface. As for the ball and plate system, we defined two sliding surfaces; one for each axis.
Let's us define a time-varying surface $s(x,t) = 0$ where:

$$s(x,t) = \dot{\tilde{x}} + \lambda \tilde{x} \quad ; \quad \tilde{x} = x - x_{desired}, \quad \lambda > 0$$

The continuous part of the control law that keeps the state of system on the sliding surface is given by:

$$\dot{s} = 0 \Rightarrow \hat{\alpha} = \frac{7}{5g}\left(\lambda \dot{\tilde{x}} - \ddot{x}_d\right)$$

To force system trajectories to tend toward the defined sliding surface, a condition called "*Sliding Condition*" must be satisfied:

$$\frac{1}{2}\frac{d}{dt}s^2 < -\eta |s| \quad ; \quad \eta > 0 \tag{12}$$

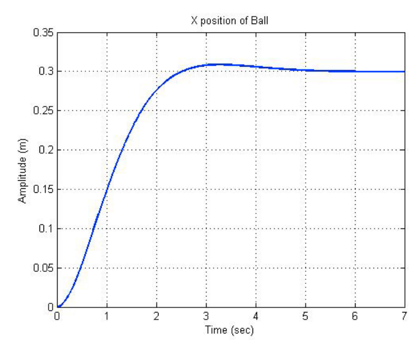

Fig. 5. The ball position on the x-axis (Sliding Mode).

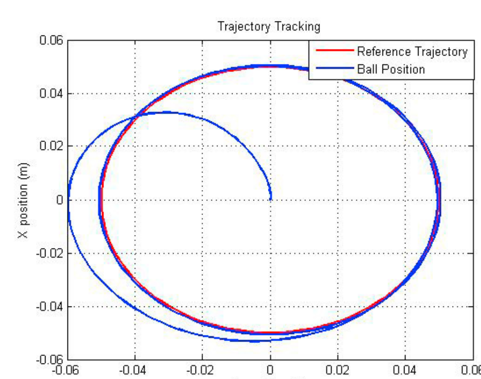

Fig. 6. Circular Trajectory Tracking with Sliding Mode Controller.

To satisfy the sliding condition (12), a discontinuous term is added to $\hat{\alpha}$ across the surface s=0:

$$\alpha = \hat{\alpha} - ksgn(s)$$

We can see that the control laws are not continuous across the surface $s(t)$, and that leads in practice to undesirable phenomenon called *Chattering*, which can be eliminated by smoothing out the control law with a saturation function.

Fig.5 shows the step response for the position of the ball on the x-axis using this methodology of control and Fig.6 shows circular trajectory tracking where the error is less than 2 mm knowing that the radius of the desired circle is 5 cm. We choose parameters as follows: K = 0.33; $\lambda = 1$; and saturation function with upper limit = 1 and lower limit = -1.

3.4 Fuzzy Controller

In the design of a fuzzy controller we have to:

- Choose state and control variables.
- Derive IF-THEN rules that relates the state variables with the control variables.

We chose five membership functions for each state variable and seven for the output variable. The rules used to map the input and output fuzzy sets are shown in table 1.

Table 1. Fuzzy rule base

Output		**Error**				
		nb	**ns**	**z**	**ps**	**pb**
derror	**nb**	nb	nb	nm	ns	z
	ns	nb	nm	ns	z	ps
	z	nm	ns	z	ps	pm
	ps	ns	z	ps	pm	pb
	pb	z	ps	pm	pb	pb

Fig.7 shows the step response for the position of the ball on the x-axis with the fuzzy controller. In tracking a circle with radius of 5 cm, the error was less than 3 mm, as illustrated in Fig.8.

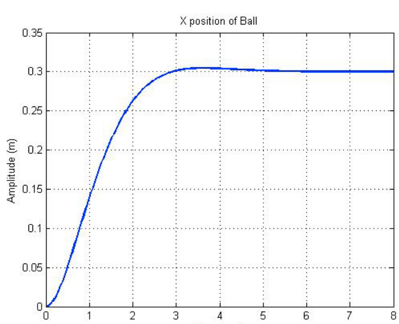

Fig. 7. The position of the ball on the x-axis (Fuzzy Controller).

4. EXPERIMENTAL RESULTS

The validation of our results was applied on a platform that was developed at HIAST(Fig.9). The platform consists of a 6 DOF 'Stewart Platform' with 6 servo motors (HS-765HB) with resolution of 1 degree. An electronic board based on the microcontroller Atmeg1280 was designed to drive the motors of the platform and to communicate with the host computer via USB port. A resistive touch pad is mounted on the moving plate of the Stewart platform to measure the position of the ball on the plate. The Touch pad is connected to the host PC via the serial port and the resolution of touch pad is ($0.5{\times}0.6\ mm^2$). All high-level control algorithms including

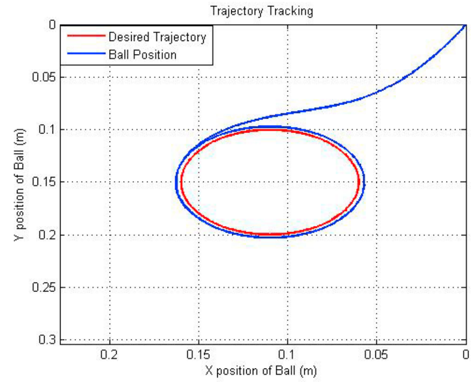

Fig. 8. Circular Trajectory Tracking with Fuzzy controller.

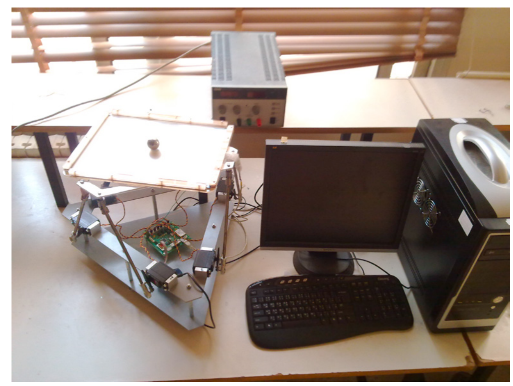

Fig. 9. Experimental Work Space

the four proposed controllers are performed on a host computer ($2.4GHz, 2GRAM$) running $WindowsXP$. The algorithms were programmed in $Visual\ \ Studio$ 2008.

4.1 PID

Fig.10 shows the response on the x-axis for stabilizing the ball at a desired position using the PID controller with ($K_p = -1.1, K_d = -0.45$). The initial position is 1.57 cm and the desired position is 11.01 cm. However, a tuning was carried out on the controller parameters to achieve a response with the least error and the minimum settling time. The best behaviour we obtained has a response time of 0.82 second and the error was less than 1 mm and the results are better than those obtained in simulation due to the parameters tuning. A circular trajectory with radius

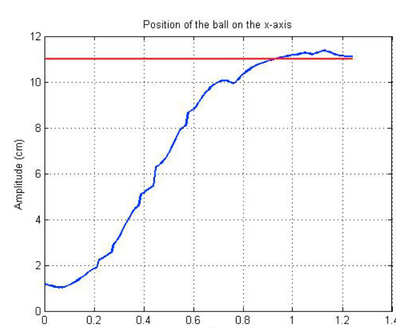

Fig. 10. The position of Ball on x-axis(PID) on real system.

of 5 cm is chosen to test the tracking. Fig.11 shows that the tracking is achieved with an error less than 6 mm. However, the error is bigger than the error obtained in the simulation stage due to the parameters tuning.

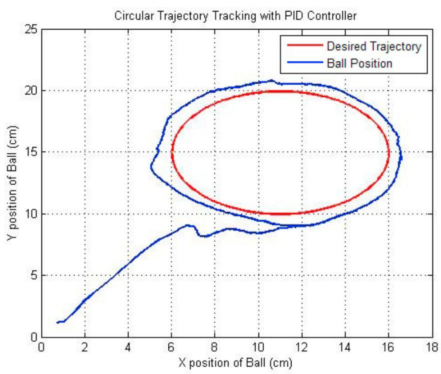

Fig. 11. Circular Trajectory Tracking with PID Controller on real system.

4.2 LQR Controller

We test the performance of the static and dynamic position tracking using the LQR controller. Fig.12 shows the response of the position of the ball on the x-axis and we obtained $e_{ss} = 1mm$ and $t_s = 0.85second$. The state-

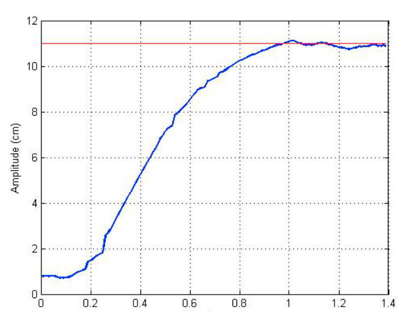

Fig. 12. The position of Ball on x-axis (LQR) on real system.

feeback gain is:

$$K = \begin{bmatrix} -0.3162 & -0.3047 & 0 & 0 \\ 0 & 0 & -0.3162 & -0.3047 \end{bmatrix}$$

As for the circular trajectory, Fig.13 shows the tracking of the desired circle and the error was less than 6 mm.

4.3 Sliding Mode Controller

In order to stabilize the ball in a desired position, we apply the designed sliding mode controller and we obtained the results shown in Fig.14 with $e_{ss} = 0.5mm$ and $t_s = 0.52second$.

The performance with the circular trajectory is presented in Fig.15 where we can see that the tracking was achieved with error less than 2 mm.

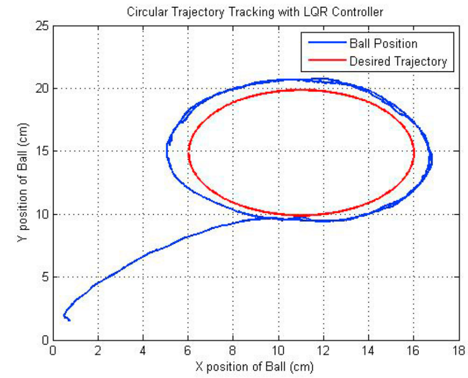

Fig. 13. Circular Trajectory Tracking with LQR Controller on real system.

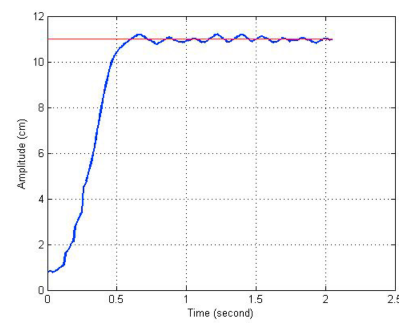

Fig. 14. The position of ball on the x-axis with Sliding Mode Controller on real system.

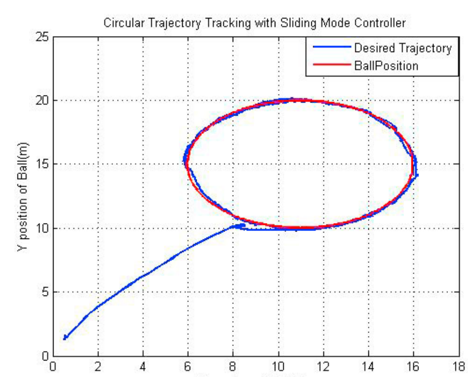

Fig. 15. Circular Trajectory Tracking with Sliding Mode Controller on real system.

4.4 Fuzzy Controller

Using the fuzzy controller, the ball was able to reach the steady state values of the desired position with an error $e_{ss} = 1mm$ and $t_s = 0.88seconds$ as illustrated in Fig.16.

On Fig.17, we see that the ball was able also to track the desired circle with an error less than 6 mm.

Table 2 presents a comparison between the four proposed methods of control for stabilizing the ball in a desired position. We can say that the sliding mode scheme had the best performance with a response time of 0.52 second and an error less than $1mm$. Moreover, the best performance in trajectory tracking was achieved using the designed

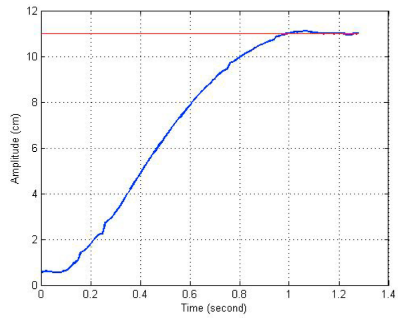

Fig. 16. The position of Ball on x-axis (Fuzzy Controller) on real system.

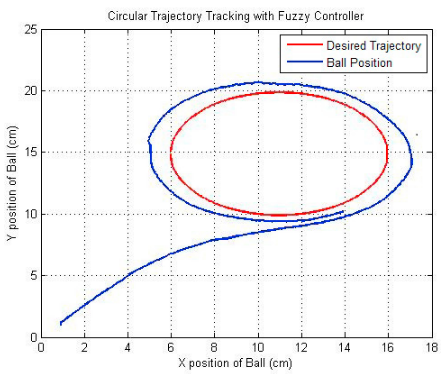

Fig. 17. Circular Trajectory Tracking with Fuzzy Controller on real system.

sliding mode controller due to the least steady state error (table 3). These results prove the capability of the sliding mode scheme to solve the problem of controlling nonlinear systems with high complexity as it overcomes modeling imprecision and parameter variations.

Table 2. Static position tracking comparison

	e_{ss} (mm)	t_s (second)
PID	1	0.82
Sliding Mode	0.5	0.52
LQR	1	0.85
Fuzzy	1	0.88

Table 3. Dynamic position tracking comparison

	Maximum Steady State Error (mm)
PID	6
Sliding Mode	2
LQR	6
Fuzzy	6

5. CONCLUSION

The ball and plate system is a typical multi-variable nonlinear system which is a two dimensional expansion of the ball and beam system. The complexity of this system increases when the plate is mounted on a Stewart platform with 6DOF which makes the system a big challenge in the control field. This paper proposes four strategies for static and dynamic position tracking: PID, LQR, Sliding Mode and Fuzzy controller. The results of simulation and also the validation on real system are presented. The comparison between the proposed strategies illuminates the capability of the Sliding Mode control to solve the problem of static and dynamic tracking for nonlinear systems with high complexity as it overcomes modeling imprecision and parameter variations.

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