Digital Terrain Model Interpolation for Mobile Devices Using DTED Level 0 Elevation Data

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Abstract: *Digital elevation maps provide extensive information about regions on the Earth. The amount of data to be processed increases with $O(n^2)$ complexity when resolution detail of the map in use increases. As the storage and processing capacity of handheld mobile devices are very limited compared to regular computer systems, the amount of data that can be stored in the device memory is also much less than of more complex computer systems. DTED Level 0 data provides approximately 1km resolution of elevation data which requires smaller storage space. The points in between the sampled points can be interpolated making use of the interpolation algorithms, with the addition of two sets of extra points whose coordinates are determined according to the existing sample points.*

Key Words: Mobile 3D Navigation Terrain Interpolation

INTRODUCTION

DTED points come in a grid structure with sample points on the intersection of grid lines which form the cells of the region. Each cell has a special data set indicating maximum, minimum and average elevation values of points that exist in the DTED Level 1 elevation data of the same region.

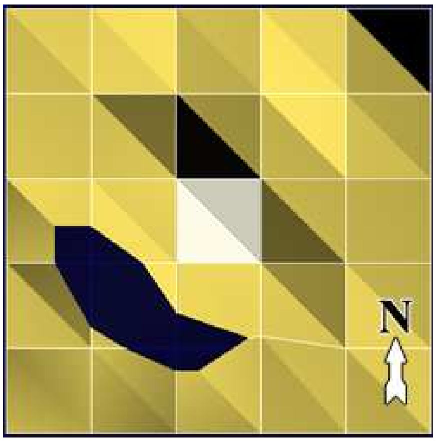

Fig. 1. Top View of a DTED Level 0 Grid of Terrain Cells

During DTED Level 0 data sampling from DTED Level 1 data, the maximum and minimum elevation values of the points in the cell are recorded with the average elevation value of all the points in the cell.

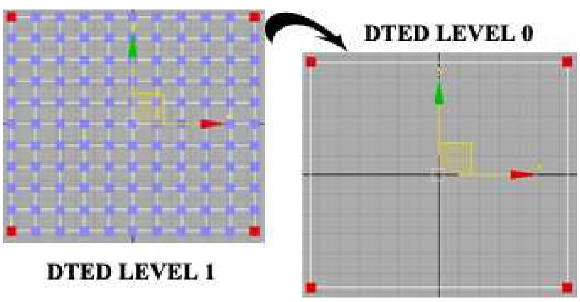

Fig. 2. Conversion from DTED Level 1 to DTED Level 0

The article is to propose an interpolation formulation that produces an elevation value for any point within the cell boundary, regardless of the resolution level preferred for the target application.

MOTIVATION

A 3D navigation application has been developed for mobile devices with pre-generated elevation data. Although the geographical are was 120x120 km^2 wide, the model had to be divided into 100 cells consisting of 50x50 elevation points each, in order to fit into the memory of the device. The elevation data for this small region takes more than 10MB of storage space and the distance between two points is nearly 240 meters.

Fig. 3. Mobile 3D Navigation Application

Limitations on storage capacity lead to alternative methods for terrain generation. Interpolation from fewer points provides ability to store larger geographical regions on small memories. Sampling elevation data points with a relevant resolution, storing only the sample points and interpolating the area between these points at any resolution whenever necessary is a solution approach for mobile 3D navigation storage problem.

DIGITAL TERRAIN MODELLING

Several methods exist for obtaining digital representation of Earth surface. Qualities of these methods are measured by their precision in elevation values and the percentage of data that are within the precision boundaries. Considering DTED Level 0 type of digital elevation data representation, data points are sampled with approximately 1km apart both in latitude and longitude. In such a case precision quality naturally becomes a rather secondary constraint for mobile device applications using that sort of model data, while storage and processing are more crucial resources.

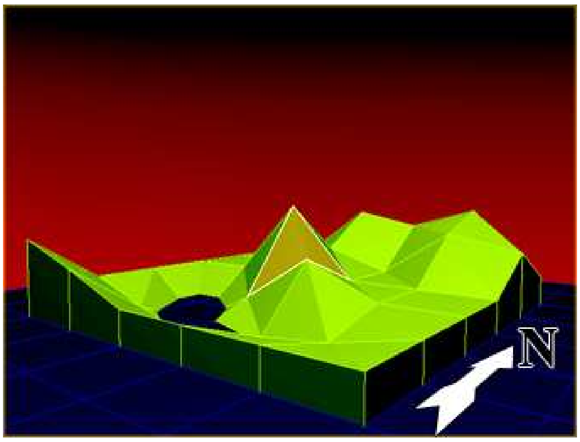

Fig. 4. Terrain Cells in 3D View

Interpolation algorithms can be used to estimate the elevation value of a certain point that has not been provided in DTED Level 0 data. Interpolation result is directly related to number of point parameters involved in the interpolation. Algorithms discussed in this paper make use of sample points; maximum, minimum and average elevations.

LOCATING MAXIMUM AND MINIMUM POINTS

Sample points are located on the corners of the cells. For each cell, maximum and minimum elevation values are defined but their location is unknown in detail Level 0. Considering a sample cell, the corner points can be labeled as “p1” corresponding to South-West corner, “p2” to South-East corner, “p3” to North-East corner and “p4” to North-West corner.

If at least one of the corner points has a different elevation value from the others, then a point “p5” is introduced as the maximum point, and another point “p6” is introduced as the minimum point of the cell. Locations of these two points are determined with respect to their elevation differences from all corner points. This method can be named as *“singular positioning of maximum and minimum points”*. Each cell with a maximum elevation value higher than all corner points contains at least one hill shaped geographic structure and each cell with a minimum elevation value lower than all corner points contains a hollow sunk shaped geographic structure.

The principle in locating the maximum elevation point “p5”, is *“the higher altitude has a corner point, the closer it is to the maximum point”* and similarly the principle in locating the minimum point “p6”, is *“the lower altitude has a corner point, the closer it is to the minimum point”*.

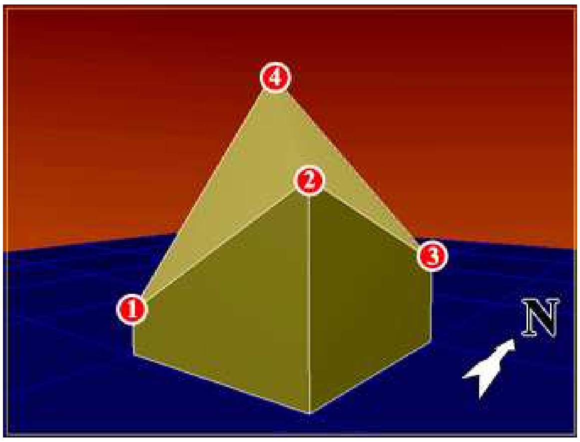

Fig. 5. Sample Cell with Corner Points Numbered

The reference system used throughout the paper for coordinates of the points is Cartesian coordinate system for simplicity of the explanation. Although the DTED implementation is a rather more complex algorithm involving longitudinal and latitudinal calculations, point coordinates are calculated relative to the existing points and this makes the algorithm applicable to any reference system.

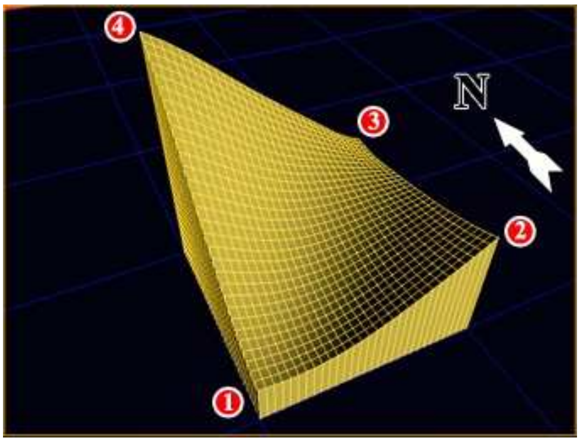

Fig. 6. Sample Smooth Interpolation with 4 Corner Points

With all units in meters, the sample cell used in the paper has the four corner points located as;

$p1_{x,y,z} = \{0, 0\ 10\}$,

$p2_{x,y,z} = \{1000, 0, 40\}$,

$p3_{x,y,z}$ = {1000, 1000, 20},
$p4_{x,y,z}$ = {0, 1000, 70}, and
maximum elevation = 80,
minimum elevation = 2,
average elevation = 50.

Without determining coordinates of p5 and p6, a smooth interpolation algorithm would yield a surface that all points are elevated only according to the available corner points, missing the information provided as maximum elevation and minimum elevation. Modifications can be made for the parameters of the algorithm to obtain close average elevation values to the average elevation value provided in the cell, but still lacking close to real view of the cell.

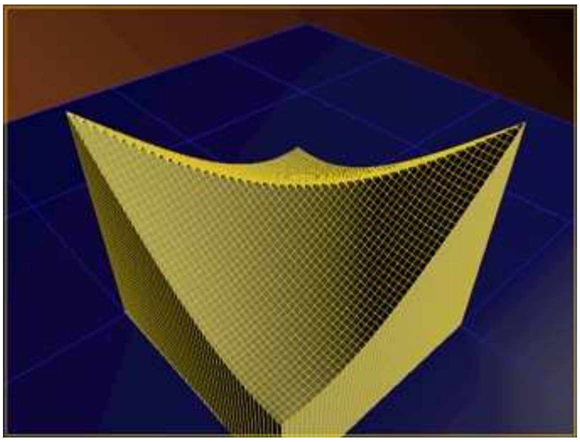

Fig. 7. Cell with Opposite Corners Having Same Elevation

For some conditions, maximum and minimum points are located in the same coordinate pair. These conditions are as follows:

a. All two non-adjacent sample point pairs have the same elevation values. (e.g. NW has same elevation with SE and NE has same elevation with SW)
b. All four sample points have the same elevation values.

For each of these conditions another locating method called "*circular positioning of maximum of minimum points*" can be proposed.

SINGULAR POSITIONING

Maximum and minimum points are positioned based on their elevation difference from sample points on the cells. First step in locating maximum or minimum point is to pair sample points with one of their neighboring points that has not been paired before to obtain two distinct pairs (e.g. North-West point paired with North-East point and South-West point paired with South-East point) and partition the distance between two points in each pair into two segments, where the length of each segment is directly proportional to its elevation difference from maximum or minimum elevation value.

When positioning the maximum elevated point, an elevation parameter E can be assigned the value of the maximum elevation. In case of positioning the minimum elevated point, E can be assigned the value of minimum elevation value.

$$\text{(I)} \quad E = \begin{cases} \text{maximum altitude,} \\ \text{if positioning maximum point,} \\ \text{minimum altitude,} \\ \text{if positioning minimum point.} \end{cases}$$

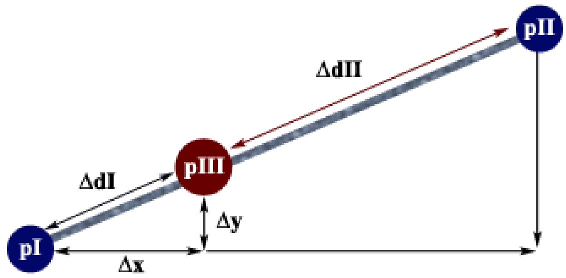

Fig. 8. Locating Segmentation Points. (Points Projected on x-y Plane)

If pI and pII are two points whose coordinates along with maximum and minimum elevation values of the cell will be used to calculate coordinates of an intermediate point pIII on the line connecting pI and pII, then the distance between pI and pII can be scaled to sum of difference between maximum or minimum elevation and elevation of pI, i.e. pIz, and difference between maximum or minimum elevation and pIIz, depending on case of positioning maximum point or minimum point.

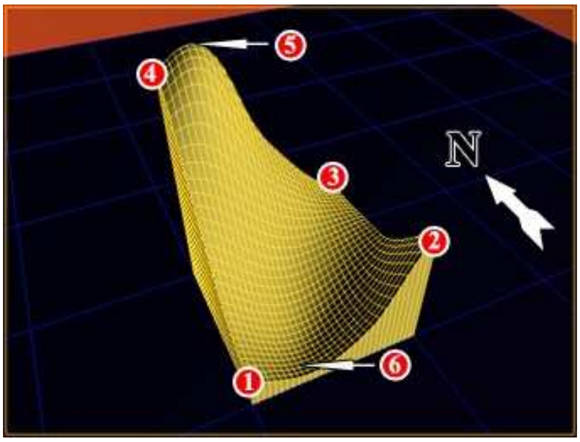

Fig. 9. Interpolation after Maximum and Minimum Points Located

If distance between pI and pIII is ΔdI and distance between pIII and pII is ΔdII then scaling according to elevation differences can be performed as;

$$\text{(II)} \qquad \frac{\Delta dI}{(\Delta dI + \Delta dII)} = \frac{(E - pIz)}{(E - pIz) + (E - pIIz)}$$

The coordinates of pIII, can be calculated as;

$$\text{(III)} \qquad pIIIx = pIx + \Delta x,$$
$$\text{(IV)} \qquad pIIIy = pIy + \Delta y,$$
$$\text{(V)} \qquad pIIIz = pIz + \Delta z$$

where;

$$\text{(VI)}\qquad \Delta x = \frac{\Delta dI}{(\Delta dI + \Delta dII)} \text{ X } (pIIx - pIx)\ ,$$

$$\text{(VII)}\qquad \Delta y = \frac{\Delta dI}{(\Delta dI + \Delta dII)} \text{ X } (pIIy - pIy)\ ,$$

and,

$$\text{(VIII)}\qquad \Delta z = \frac{\Delta dI}{(\Delta dI + \Delta dII)} \text{ X } (pIIz - pIz)\ .$$

Second step is to determine the elevation values of two points that are on the segmentation locations, linearly approximated from the two sample corner points constituting in the location of these points. Whenever these points are located and their elevation values are calculated, the distance between these points is also partitioned into two segments, where segment lengths are again proportional to elevation difference from maximum or minimum elevation value. The location that has just been found stands for longitude and latitude values of maximum or minimum point.

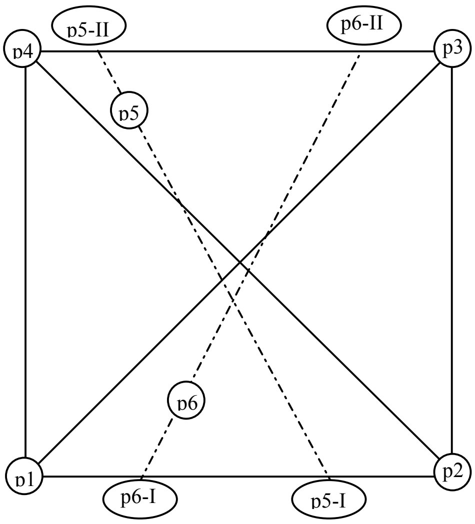

Fig. 10. Singular Positioning of Maximum and Minimum Points

In the sample cell, trying to locate p5, the maximum point; p1 and p2 are used to locate p5-I while p3 and p4 are used to locate p5-II. Afterwards p5-I and p5-II are used to locate p5. For p6, the minimum point; p1 and p2 are used to locate p6-I while p3 and p4 are used to locate p6-II. Later on p6-I and p6-II are used to locate p6.

CIRCULAR POSITIONING

If maximum elevation value is more close to average elevation value than minimum elevation value, this indicates that most of the elevation values of points in the cell are over average, and vice versa. If majority of the points are higher than average, the minimum point is located in the center of the cell and several maximum points are located on a concentric circle surrounding minimum point. If majority of the points are lower than average, the maximum point is located in the center, with several minimum points surrounding it.

$$\text{(IX)}\quad L = \begin{cases} \text{Longitudinal width,} \\ \text{if longitudinal width} < \text{latitudinal width} \\ \text{Latitudinal width, otherwise.} \end{cases}$$

Defining *MAX* as maximum elevation, *AVG* as average elevation, *MIN* as minimum elevation and *L* as the smaller of the latitudinal and longitudinal lengths of the cell, a circular region with radius *R,* that separates high altitude points from low altitude points, can be defined as;

$$\text{(X)}\qquad \frac{R^2}{(L/2)^2} = \frac{h1}{\text{MAX - MIN}}$$

where,

$$\text{(XI)}\qquad h1 = \min\{\text{MAX} - \text{AVG}, \text{AVG} - \text{MIN}\}\ .$$

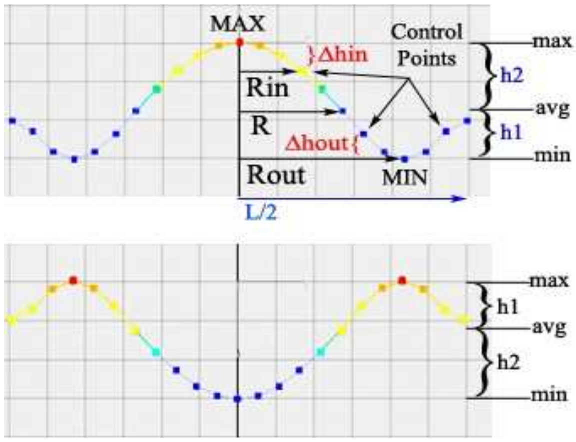

Fig. 11. Central Cross-Sections for MAX or MIN in the Center

Comparing area inside this circle to the area outside, the inner portion is proportional to the smaller of the difference between *MAX* and *AVG* elevations, and difference between *AVG* and *MIN* elevations.

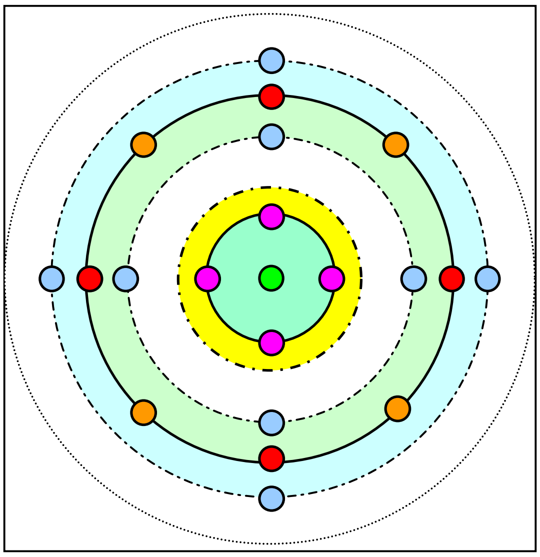

Fig. 12. Circular Positioning

If $MAX–AVG$ is smaller than $AVG–MIN$, this means that the number of points with elevation values higher than average elevation, are more than the number of points with elevation values lower than average elevation. This requires the smaller portion of the cell area to be occupied with low altitude points and the rest to be occupied with higher points, meaning that the minimum point will be placed in the center of the cell with several maximum points surrounding it.

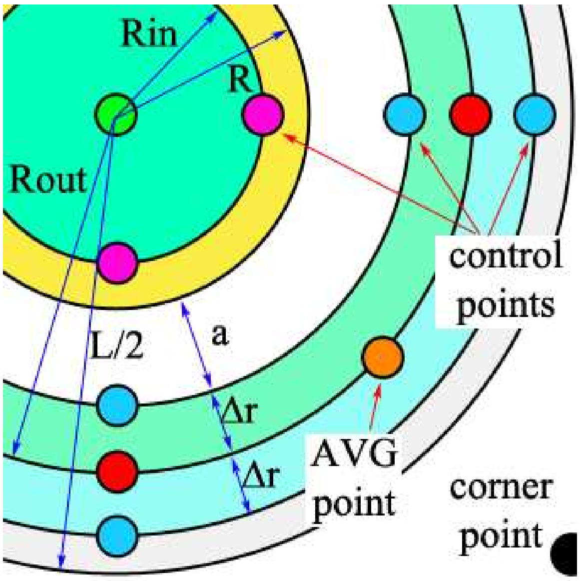

Fig. 13. Circular Positioning

The algorithm includes control points around minimum and maximum points with intermediate elevation values, that are used to keep the interpolation algorithm smooth. Having calculated *R*, two other radius values, namely *Rin* and *Rout*, are obtained for inner and outer control points respectively, where;

(XII) $$\frac{(Rin)^2}{(R)^2} = \frac{h2}{MAX - MIN}$$

with,

(XIII) $$h2 = MAX – MIN – h1 ,$$

and,

(XIV) $$Rout = (R + L/2) / 2$$

Inner control points are 4 points located with 90 degrees apart on the *Rin* circle with their elevations offset as much as Δhin from elevation of the central point in the direction of AVG elevation, where,

(XV) $$\frac{\Delta hin}{h2} = \frac{h1}{MAX - MIN}$$

Outer control points are 8 points, 4 of which are located 90 degrees apart on the circle with radius Rout – Δr, and 4 of which are located 90 degrees apart on the circle with radius Rout + Δr, where;

(XVI) $$\Delta r = Rout – (R + a)$$

with *a* obtained from,

(XVII) $$\frac{(R)^2}{(R+a)^2} = \frac{h2}{MAX - MIN}$$

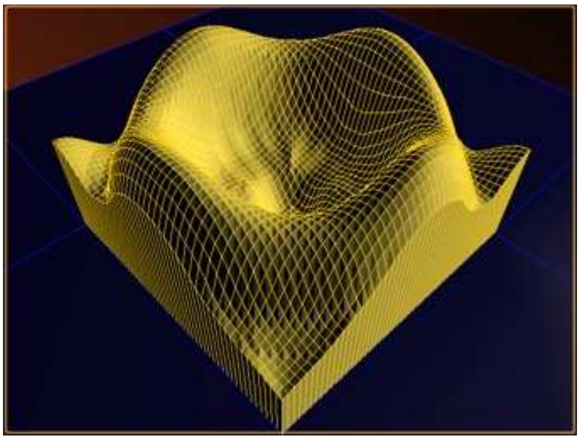

Fig. 14. Elevations: Max – Avg > Avg - Min

Elevations of outer control points are offset as much as $\Delta hout$ from elevation of the points on the *Rout* circle in the direction of AVG elevation, where,

(XVIII) $$\frac{\Delta hout}{h1} = \frac{h1}{MAX - MIN}$$

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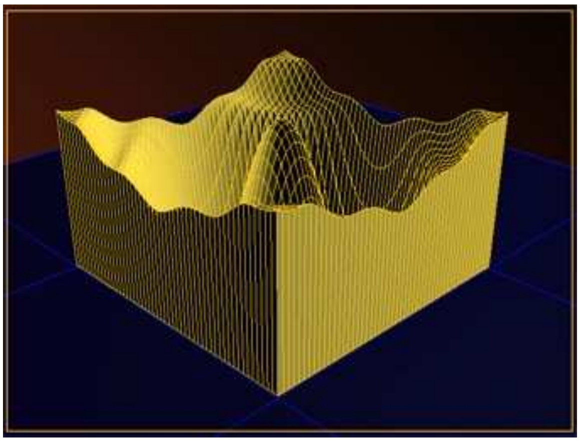

Fig. 15. Elevations: Max – Avg < Avg – Min

Having calculated positions and elevations of all maximum, minimum and control points, 4 more points are to be located on the *Rout* circle with 45 degrees offset from the points on *Rout* circle and with 90 degrees apart from each other, with elevation values equal to that of the average elevation value of the cell. These points avoid abrupt elevation differences between the corner points and the rest of the cell.

CELL GENERATION WITH INTERPOLATION

As soon as all maximum and minimum points are located, inverse distance weighting algorithm (IDW) is utilized to find elevation value of a specific coordinate within the cell. Reconstruction of higher level DTED data with this methodology yields a close value to the average value specified in the DTED Level 0 data of the cell.

IDW algorithm is based on weights that are calculated relative to the distance values of the point of which the elevation value is to be interpolated from the reference points with known elevation values. A weighted elevation value can be calculated as sum of weight values multiplied by the corresponding elevation value of the reference point that the weight has been calculated for.

For "*n*" reference points, if *wi* indicates the weight value of a reference point and *pzi* indicates the elevation value of that point, weighted elevation *we* is obtained from:

$$\text{(XIX)} \qquad w_e = \sum_{i=1}^{n} w_i \text{ X } pz_i$$

A vector *W* of size "n" for weight values of reference points can be obtained by multiplying an n-by-n inverted distance matrix D^{-1} of the reference points with a distance vector *V* of the point that is to be interpolated. *V* contains distance values calculated as distance of the point to be interpolated, from reference points, on the x-y plane, while *D* contains the distance values of each reference point from all other reference points.

$$\text{(XX)} \qquad D = \begin{bmatrix} d11, d12, \dots, d1n \\ d21, d22, \dots, d2n \\ . \quad . \quad . \\ . \quad . \quad . \\ dn1, dn2, \dots, dnn \end{bmatrix}$$

In order to construct *D,* x-y distances of each reference point with other reference points are represented as a row in the matrix where *dij* on the *ith* row and *jth* column of the D matrix indicates the x-y distance of the *ith* reference point to the *jth* reference point.

Similarly the *V* column vector is constructed with values *di* which is the x-y distance of the interpolation point to the *ith* reference point.

$$\text{(XXI)} \qquad V = \begin{bmatrix} d1 \\ d2 \\ . \\ . \\ dn \end{bmatrix}$$

The n-by-1 weights vector W with *wi* in each row indicating weight value of the *ith* reference value is then available in the form:

$$\text{(XXII)} \qquad \mathbf{W} = \mathbf{D}^{-1} \text{ x } \mathbf{V}.$$

After all reference points are determined, either using linear interpolation or circular interpolation, elevation value of a certain point can be estimated from the available reference points using the formulation above.

RESULTS

In order to simulate a DTED Level 0 data set, a grey scale image of size 6000x6000 pixels has been generated where darker pixels indicating lower altitudes and the area is considered to be 120x120 km^2 wide. Since the cells of DTED level 0 are generated from 120x120 points in the simulation yielding 20m resolution for higher level DTED data, the sample region covers 50x50 cells. Maximum, minimum and average values of each cell are calculated from the available points in the sample region.

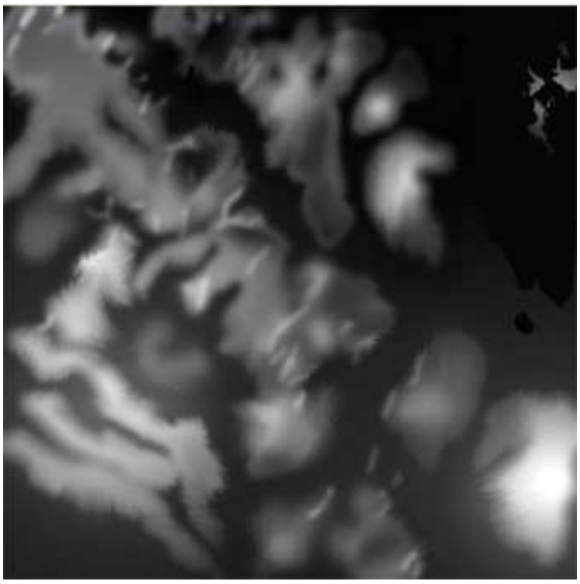

Fig. 16. Sample region with 2500 cells.

As soon as all cells are generated in DTED Level0 format, interpolation algorithm is run to obtain interpolated cells with various resolutions. For each detail level IDW algorithm is utilized to generate interpolation values from both the proposed estimation algorithm and IDW that uses only the four corner points of the cells.

As each cell has a fixed average value calculated from real points, the two methodologies are compared in terms of closeness to the real average value. For each cell, all points within the cell are interpolated with both algorithms and their average are calculated. For different number of points in the cells, the interpolation results for closeness to the average value can be observed on the following table as percentage of each algorithm yielding closer values over 2500 cell average values.

Table 1 Closeness to real average.

Cell Size (# of points)	IDW with four reference points	IDW with extra reference points
6x6	25.16%	74.84%
11x11	23.64%	76.36%
16x16	22.92%	77.08%
21x21	22.84%	77.16%
26x26	22.56%	77.44%
31x31	22.48%	77.52%
36x36	22.52%	77.48%
41x41	22.60%	77.40%
46x46	22.56%	77.44%
51x51	22.56%	77.44%
61x61	22.52%	77.48%
71x71	22.52%	77.48%
81x81	22.44%	77.56%
96x96	22.32%	77.68%
111x111	22.36%	77.64%
121x121	22.40%	77.60%

CONCLUSION AND FUTURE WORK

Adding more points to the four corner points of each DTED Level 0 cell data, it is possible to obtain better interpolation values for elevations of unknown points. Using inverse distance weighting algorithm in interpolation with additional reference points yields a closer average value for an entire cell, to the real cell average, in more than 75% of 2500 sample cells, compared to IDW algorithm applied only to four corner points of DTED Level0 data.

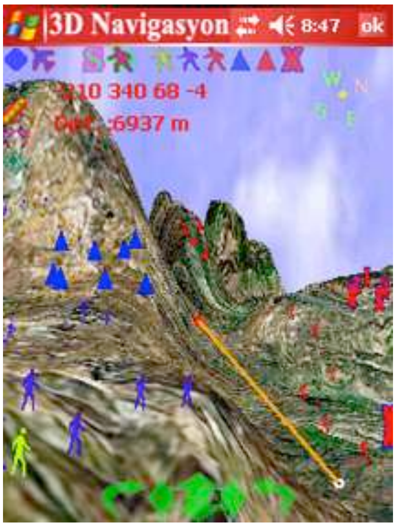

Fig. 17. 3D Navigation with static data.

This algorithm will be utilized in the mobile implementation of a 3D Navigation system. For the time being, the application uses static points at a resolution of 240m and this algorithm will bring in the flexibility of resolution variability.

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REFERENCES

[1] Lu,George Y., Wong, David W. 2008. An adaptive inverse-distance weighting spatial interpolation technique. *Computers & Geosciences* 34 (2008) 1044– 1055

[2] OSTMAN, A. (1987): Accuracy Estimation of Digital Elevation Data Banks. *PhotogrammetricEngineering and Remote Sensing*, Vol. 53(4), pp. 425 - 430.

[3] WEIBEL, R., AND M. HELLER (1990): A Framework for Digital Terrain Modelling. *Proceedings 4th International Symposium on Spatial Data Handling*, 1, Zürich, pp. 219 - 229.

[4] WEIBEL, R. (1997): Digital Terrain Modelling for Environmental Applications: A Review of Techniques and Trends. *Joint European Conference on Geographical Information (JECGI)*, Vienna, Austria, pp. 464 - 474.

[5] Tomczak, M., 1998. Spatial interpolation and its uncertainty using automated anisotropic inverse distance weighting (IDW)-crossvalidation/jackknife approach. *Journal of Geographic Information and Decision Analysis* 2 (2), 18–30.

[6] Ozyurt, M., Tugcu, T., Alagoz, F,. Effective Large Area Tactical Land Marking on 3D Mobile WLAN and GPS Enabled Devices. *Proceedings International Workshop On Small Satellites, New Missions And New Technologies*, Istanbul 2008

ADDITIONAL READINGS

[1] Geoengine.nga. National Geospatial Agency, 2007
http://geoengine.nga.mil/geospatial/SW_TOOLS/NIMAMUSE/webinter/rast_roam.html

[2] Nga. National Geospatial Agency, 2007
http://www.nga.mil/ast/fm/acq/89020B.pdf :

[3] Openmap, BBN Technologies, 2008
http://www.openmap.org/doc/api/com/bbn/openmap/

[4] Przegl d Geologiczny. Janusz Badura, Bogusaw Przybylski, 2005
http://www.pgi.gov.pl/pdf/pg_2005_10_2_22.pdf

[5] GeoComputation Conference Proceedings. David Kidner1, Mark Dorey1 & Derek
Smith2, 1999 http://www.geocomputation.org/1999/082/gc_082.htm

[6] AIAA JOURNAL. Jay D. Martin, Timothy W. Simpson, 2005
http://edog.mne.psu.edu/pdfs/AIAA05.w-jay.pdf

[7] Sun Microsystems. Dennis J Bouvier, 2003
http://java.sun.com/developer/onlineTraining/java3d/j3d_tutorial_ch1.pdf

[8] J3d.org. J3d.org, 2006
http://code.j3d.org/javadoc/org/j3d/renderer/java3d/terrain/package-summary.html

[9] Mat.uniroma. Daniel Selman, 2002
http://www.mat.uniroma2.it/~picard/SMC/didattica/materiali_did/Java/Java_3D/Java_3D_Pro
gramming.pdf