Homework 2

1 Problem

Convert the following continuous-time system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$ with

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \qquad \text{and} \qquad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

into an exact discrete-time system

$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{G}\boldsymbol{u}_{k-1}$$

with sampling time Δt. You can check your answer using computer tools but should derive the discrete-time system by hand and show all of your work. Hint: The identity $e^{\boldsymbol{X}}e^{-\boldsymbol{X}} = \boldsymbol{I}$ (for an arbitrary matrix $\boldsymbol{X}$) is useful for computing $\boldsymbol{G}$.

2 Problem

Repeat the previous problem using the approximate discretization described in Lecture 4 with $\Delta t = 0.25$. Evaluate both system matrices $\boldsymbol{F}, \boldsymbol{G}$ and $\boldsymbol{F}_{\text{approx}}, \boldsymbol{G}_{\text{approx}}$ (where the former is the exact and the latter is the approximate system). Submit any MATLAB code you use.

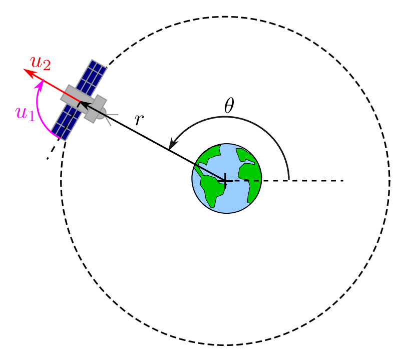
3 Problem

The motion of a satellite in orbit around the earth can be approximately described by the equations:

$$\ddot{r} = r\dot{\theta}^2 - \frac{\mu}{r^2} + 2u_2 \sin u_1 \tag{1}$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} + \frac{u_2}{2r}\cos u_1 \tag{2}$$

where r is the distance from the center of the earth, θ is the angle of the orbit (we're assuming the orbit is in a 2D plane), μ is the standard gravitational parameter, u_1 is the thrust angle and u_2 is a thrust magnitude.



A nominal trajectory for this system is a circular orbit where $r = r_0$ (a constant radius so that $\dot{r} = 0$) and $\dot{\theta} = \omega$ (a constant angular rate) where the radius and angular rate satisfy $\mu = r_0^3\omega^2$. In this nominal trajectory no thrust is needed $u_2 = 0$ and the nominal angle of the thrust can be assumed $u_1 = \pi/2$. Linearize the system around this nominal trajectory to give a system of the form:

$$\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}\Delta\boldsymbol{x} + \boldsymbol{B}\Delta\boldsymbol{u} \tag{3}$$

4 Problem

Use MATLAB's `lsim` tool to simulate the system from the previous problem assuming:

- An orbit 4,000 km above the Earth's surface. That is, $r_0 = h + r_{\text{Earth}}$ where $h = 4E6$ meters is the altitude of the orbit and $r_{\text{Earth}} = 6371000$ meters is the radius of the Earth.
- The gravitational parameter for the earth is $\mu = GM$ where $G = 6.674\text{E-}11\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$ is the gravitational constant and $M = 5.97219\text{E}24$ kg is the mass of the Earth.

Simulate the system starting along the reference trajectory while applying a thruster force of 0.25 Newtons for $T = 14400$ seconds (i.e., 4 hours). Plot the actual radius $r(t)$ and angle $\theta(t)$ (not deviations) during the simulation. Your plot should look something like the two left-most panels below (the right panel is optional).