Homework 1

1 Problem

(a) Write down a state equation for a dynamical system in state-space form that you obtain from a research paper of your choosing (e.g., search Google Scholar or IEEE Xplore for a paper that is related to your research or interests). Your system may or may not include a control input. If needed, reduce the equation to a system of first order ODEs and show your work. Explain what are the components of the state vector and what process/system the equation represents. Define any symbols introduced.

(b) State the measurement equation for the system. If a measurement equation is not included in the paper hypothesize what sensor could be used to observe the behavior of the system and what the corresponding measurement equation would look like.

(c) Provide the citation for the paper you referred to.

2 Problem

Use the Peano-Baker series to prove that the state transition matrix $\boldsymbol{\Phi}(t,0)$ for the system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ with

$$A = \begin{bmatrix} t & t \\ 0 & t \end{bmatrix} \qquad \text{is} \qquad \boldsymbol{\Phi}(t,0) = \begin{bmatrix} e^{\frac{1}{2}t^2} & \frac{1}{2}t^2 e^{\frac{1}{2}t^2} \\ 0 & e^{\frac{1}{2}t^2} \end{bmatrix}$$

Hint: Use the series definition of the exponential function.

3 Problem

Consider the system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ with initial condition $\boldsymbol{x}_0 = [1,0]^{\mathrm{T}}$. Find the solution $\boldsymbol{x}(t)$ using the Laplace inverse approach with partial fractions.

$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

4 Problem

For the same system as given in the problem above, find the solution $\boldsymbol{x}(t)$ using the eigenvalue approach (with Cayley-Hamilton Theorem).

5 Problem

Consider the following system of coupled second-order equations

$$\ddot{y} - a(\dot{z} - \dot{y}) - b(z - y) = cu_1 + du_2$$
$$\ddot{z} - e\dot{z} - f(y + z) = gu_1$$

where (a, b, c, d, e, f, g) are all constants and $z(t)$ and $y(t)$ are states and $u_1(t)$ and $u_2(t)$ are control inputs. Define an appropriate state vector $\boldsymbol{x} \in \mathbb{R}^n$ and control vector $\boldsymbol{u} \in \mathbb{R}^m$ and then re-write this system as n first-order differential equations in the form $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$.

6 Problem

An equilibrium point for a nonlinear system $\dot{\boldsymbol{x}} = f(\boldsymbol{x})$ is a state $\boldsymbol{x}^*$ for which the state-rate is zero (i.e., $\dot{\boldsymbol{x}} = f(\boldsymbol{x}^*) = 0$). For the following nonlinear system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 - 2x_1x_2 \\ -x_1 + x_1^2 + x_2^2 \end{bmatrix}$$

determine the four equilibrium points $\boldsymbol{x}_a^*$, $\boldsymbol{x}_b^*$, $\boldsymbol{x}_c^*$ and $\boldsymbol{x}_d^*$. (Hint: one of them is $[1/2, -1/2]^{\mathrm{T}}$). Then state the linearized dynamics $\Delta\dot{\boldsymbol{x}} = \boldsymbol{A}\Delta\boldsymbol{x}$ around each nominal equilibrium. Define exactly what you mean by $\Delta\boldsymbol{x}$ for each linearized system.